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OF THE

AMERICAN SOCIETY

OF

CIVIL ENGINEERS

(INSTITUTED 1852)

VOLUME 89

1926

Edited by the Secretary, under the direction of the Committee on Technical Activities and Publications.

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NEW YORK
PUBLISHED BY THE SOCIETY

1926

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Paper No. 1574

SECONDARY STRESSES IN BRIDGES*

By Cecil Vivian von Abo,† Jun. Am. Soc. C. E.

With Discussion by Messrs. O. H. Ammann, William Cain, Almon H. Fuller, K. P. Bahjejian, C. A. P. Turner, P. L. Pratley, Henry S. Jacoby, Charles Evan Fowler, H. M. MacKay, George E. Beggs, D. B. Steinman, Thomas C. Shedd, Hardy Cross, W. M. Wilson, Thomson E. Mao, Edward Godfrey, Cyrus C. Fishburn, F. P. Shearwood, Laurence J. Waller, R. McC. Beanfield, and Cecil Vivian von Abo.

Synopsis

The paper first describes all the methods which have been proposed for the complete analysis of secondary stresses due to the rigidity of the joints in bridge trusses. Approximate methods, however, such as those of Engesser and Landsberg, which neglect the effect of the web system, are not included.

A typical through railway bridge of six panels is analyzed by each method in turn, using unsymmetrical loading, in order to test more thoroughly the advantages and disadvantages of each method. A critical comparison is then made of the methods thus applied.

Influence lines are developed for the secondary stresses in all the members of the main trusses, in the top chords as effected by the top lateral system, and in a typical cross-frame. These influence lines are then combined for the purpose of computing the maximum secondary stresses in certain typical members.

PART I

1.—Introduction

Prior to 1879, no form of mathematical analysis was known, and no empirical results were available, which would enable an engineer to calculate

* Presented at the meeting of November 5, 1924. This paper was presented by Mr. von Abo in September, 1922, as a thesis to the Committee on Graduate Studies of McGill University for the degree of Doctor of Philosophy.

† Care, Chf. Civ. Engr., South African Rivers and Harbors, Johannesburg, Transvaal, South Africa.

with reasonable accuracy the stresses in a structure as built. Even after that date, when Manderla gave his excellent solution of secondary stresses due to the restraint of members meeting at a joint, little attention was paid to such stresses for many years. The labor involved in the calculations was usually the reason given for this neglect; but it is possible that lack of familiarity with the subject played an equally important part.

In recent years the construction of such phenomenal bridges as the Quebec cantilever, the Sciotoville continuous truss, and the Hell Gate arch, has awakened new interest in the subject. The Engineering Profession owes much to those in charge of these structures for the accounts of their investigations. Of particular interest is the paper by D. B. Steinman, M. Am. Soc. C. E., on "Stress Measurements on the Hell Gate Arch Bridge" (33),* with the full discussion by leading bridge engineers.

In recent specifications the tendency is to recognize the importance of secondary stresses. Higher unit stresses are properly permitted for combinations of secondary and primary stresses. This undoubtedly leads to a better distribution of metal, and to a more uniform strength throughout the structure. Although many think that the calculation of secondary stresses in ordinary spans will seldom be necessary, the writer believes that if the Engineering Profession at large becomes thoroughly versed in the matter, the strength and economy of all bridges will be superior to any that may be built with what has been so appropriately called "the factor of ignorance".

Definition of Secondary Stresses.—The most general definition of "secondary stress" would include shearing and torsional stresses. For the purposes of this paper, however, these are not considered, but rather the direct stresses, tensile or compressive, of the fibers parallel to the center line of a member. This member, under loading, transmits a certain total stress, which, if divided by the area of cross-section, gives an average unit stress; this is the uniform unit stress in the member of the ideal bridge. If, however, in the actual case, some particular fiber be considered and it were possible to determine its elongation, and, therefore, its unit stress, in general this unit stress would differ from the average. Therefore, at any particular cross-section of a member in a bridge, the amount by which the unit stress in any longitudinal fiber differs from the average unit stress, found by dividing the total direct stress by the area, is called the secondary stress of that fiber.

This paper is concerned with the maximum variation, at the four edges of the member, as far away from the center line as possible, for it is there that stresses are most severe.

Classification.—In 1914 a committee of the American Railway Engineering Association classified the secondary stresses in bridges.† The writer, however, suggests the following classification:

(1) Secondary stresses, arising from rigid or partly rigid connections.
(2) Secondary stresses, arising from the beam action of members.
(3) Secondary stresses developed in axially loaded members.
(4) Secondary stresses due to faulty design.

* Figures in parentheses refer to numbered articles in the Bibliography.
† *Proceedings*, Am. Ry. Eng. Assoc., Vol. 15 (1914).

Class (1) would take care of any secondary stresses that may arise in the various plane frames due to the rigidity of the joints in those planes. The bridge trusses, the top lateral system, the floor system, and the various cross-frames would contribute to this class. Imagine a bridge to be in equilibrium under a stationary train load. Certain secondary stresses will be found in each of the trusses. The top chords of both trusses change in length, while the other top lateral members remain constant in length; hence, secondary stresses are found in the top lateral system. In the same way, the changes in length of the bottom chords, with no changes in length of the floor-beams and stringers, introduce secondary stresses in the floor system. Finally, in the cross-frames the floor-beams bend under loads in the adjoining panels; if these floor-beams were pin-connected to the two trusses, the bending stresses in the floor-beams would be simply those of Class (2), but, as the floor-beams are rigid parts of the cross-frames, secondary stresses exist in the frames as a whole, which should be regarded, therefore, as in Class (1).

Imagine that a gale is blowing. It has the effect of changing this arrangement, which is symmetrical about the vertical plane dividing the bridge in half along its length, making it unsymmetrical, while certain direct stresses induced by the wind in the top lateral and the floor systems superimpose secondary stresses on the ones already found. In the case of a wind, too, the end portals come into play and are placed in Class (1).

Class (2) needs little explanation, for a member has bending stresses developed by its own weight and by any wind pressure against it.

Class (3) is seldom, if ever, reckoned as part of a study of secondary stresses. In leading up to the definition of secondary stresses, the assumption was made that the compression members in the ideal bridge were to behave just opposite to those in tension. Euler showed, however, that a column does not transmit a uniform compression under an axial load. Since then many investigators have striven after solutions of this problem, which is all the more complex when the column is a built-up one with diaphragms, latticing, etc. It must be remembered that this member suffers bending under transverse loads, such as dead weight or wind; any axial load, which may now be regarded as an eccentric load, induces further bending stresses.

Mathematical analysis, supported by practical investigation, shows that, in the case of eye-bars transmitting an axial load, the fibers across the section through the center of the pin do not suffer uniform deformation. In fact, the unit stress of the fibers on the circumference of the hole is three times the average stress across the section. This property of an eye-bar is, therefore, a cause of secondary stresses.

Under Class (4) appear eccentric connections. A careful set of measurements would indicate the values of these eccentricities fairly accurately, in which case they could be judiciously incorporated in analyses of Class (1). There is in this class, however, a condition which defies the engineer, such as occurs, for example, when a compression member is made up of two channels which are supposed to have the same cross-section and to share the load equally. Generally, such an equal distribution is not found. This, of course, would

cause a variation in the secondary stress from that found by assuming the load to be shared equally.

The Superposition Theory, Applied to Secondary Stress Analysis.—In analyzing for the primary stresses in a structure, that is, regarding it as ideal and the stresses as pure tensions or pure compressions, it is possible to solve the problem in three dimensions, but it is far easier to regard the structure as being made up of a number of planes, solving for the stresses in each plane produced by forces that act in that plane, and simply adding algebraically the stresses for all the planes of which the member forms a part. This is the superposition theorem—the method of analysis commonly used by engineers.

In analyzing for secondary stresses, the only known method is to solve in a plane structure; even if it were possible to solve for secondary stresses in a space frame, the case of the primary stresses is conclusive evidence that the corresponding secondary stress problem will be most complicated and laborious.

Suppose for the moment that, by some method or other, the secondary stresses have been determined in the members belonging to one of the plane frameworks, for instance, one of the trusses, due to a certain loading in that plane. The so-called neutral axes of the members are really neutral planes. These planes pass through the centers of gravity of the members and are perpendicular to the plane of the truss. The fibers, the secondary stresses of which presumably have been found, are those lying along the planes on both sides of, parallel to, and farthest remote from, the neutral planes.

This same loading also induces secondary stresses in the top lateral system, but in this case the plane is horizontal, and the neutral planes are vertical. (This assumes that the top chords are straight, but the argument applies to the case where the top chords are curved or made up of a number of straight parts.) The top chords of the trusses are also parts of this new plane. In the former case, the neutral planes were planes perpendicular to the vertical truss plane; in the latter, the neutral planes are perpendicular to the horizontal top lateral plane. Whereas the secondary stresses of the top chord, as part of the truss, were for the fibers that lie along its top and bottom sides, the secondary stresses of the top chord, as part of the top lateral system, are for the vertical sides. The four corner fibers are affected in both analyses and, in order to obtain the greatest variations from the average stress, it is necessary to compound the secondary stresses in the two cases. This overlapping effect should be kept in mind throughout the analysis of a bridge, in order to arrive at a true estimate of the secondary stresses.

Another Definition of Secondary Stress.—A good practical definition of these stresses is as follows:

"The secondary stress in any member at any section is equal to one-half the difference between the two extreme fiber stresses measured at the same section; the primary stress is then equal to one-half the sum of the same extreme fiber stresses."

This is the case where the member is part of a single plane framework—no other planes influence it. The two extreme fiber stresses are those belonging to the two sets of fibers on opposite sides of the neutral plane. This definition is justified only if all the stresses are within the elastic limit and if Bernoulli's

assumption holds. Let $ABCD$ (Fig. 1), be any cross-section of the member; the definition is true if the fibers on one side of the neutral plane, XX, and at equal distances from it are stressed equally. That is, ab represents the stress in every longitudinal fiber passing through AB, and cd represents the stress in every longitudinal fiber passing through CD.

Therefore, the cross-section, $ABCD$, may be replaced by any line parallel to AD and BC, and the problem regarded as an investigation of the stresses along this line. Hence the secondary stress is $\frac{1}{2}(ab - cd)$ and the primary stress $\frac{1}{2}(ab + cd)$.

It is somewhat different in practice, however. Following the assumption of Bernoulli, the diagram will be shown in Fig. 2. The trapezoid, $abb'a'$, repre-

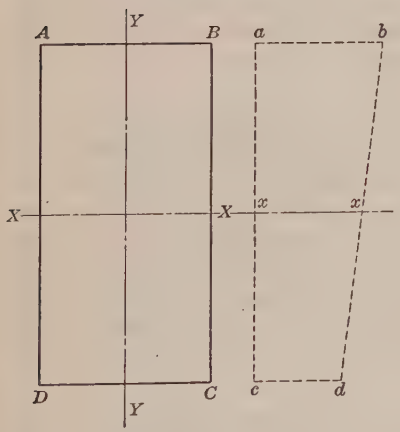


Fig. 1.

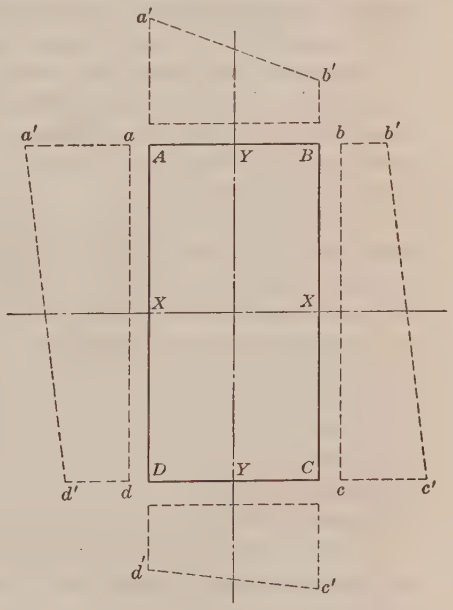


Fig. 2.

sents the stresses along the fibers that cross AB, $bb'c'c$ those along the fibers that cross BC, etc. In the case considered, $abb'a'$ and $cdd'c'$ were rectangles. Assuming that the variations along AB, BC, CD, and DA are uniform, it appears that, as the bb' of $abb'a'$ is the bb' of $bb'c'c$, etc., the primary stress is $\frac{1}{4}(aa' + bb' + cc' + dd')$, and the secondary stress at A is $\frac{1}{4}(aa' + bb' + cc' + dd') - aa'$, with similar expressions at B, C, and D.

This affords a more comprehensive definition:

"The primary stress is the average of the actual stresses found at the four corners of a member; the secondary stress is then the difference between this average and the actual stress of the fiber considered."

2.—Historical Notes

Derivation of Names.—The term "secondary stresses" is due to German writers, who classed all stresses in a bridge under (1) *Grundspannungen* (primary stresses), and (2) *Sekundarspannungen* (secondary stresses).

The *Grundspannungen* are themselves divided into (*a*) *Hauptspannungen* (principal stresses), and (*b*) *Zusatzspannungen* (additional stresses). The *Hauptspannungen* arise from the dead load and the live load, the latter being considered as a static load, while the *Zusatzspannungen* are due to wind pressure, traction, centrifugal force, and impact. The *Grundspannungen* are the stresses of the ideal structure, all pure tensions and pure compressions.

The fixedness of the joints, the weights of the members (distributed and not concentrated at the ends), the wind pressure acting in the same way, the property of axially loaded columns—all contribute to the distortion of the uniform stress distribution of the ideal case, the amount of distortion for a fiber being its *Sekundarspannung* (secondary stress).

Account of the Theoretical Investigations for Bridge Trusses.—In 1877, the Technical University of Munich offered a prize for the solution of a problem, worded by Professor Asimont as follows: What stresses arise in the members of a bridge truss owing to the fact that the angles of the truss triangles suffer no change?

In 1879, H. Manderla, an Assistant in the Technical University of Munich, submitted an excellent solution (3). His mathematical analysis is most exact, for the effects of the direct or primary stresses on the bending moments at the ends of members are taken into account. Hyperbolic functions are necessary for the solution of the differential equation, and infinite series are found in the expressions for the bending moments. As the solution of a problem by the exact method is laborious, and the values of the bending moments generally differ little from those found by a method in which the direct stress is not considered, it is this latter more approximate method that is generally known and always used.

Professor Engesser, of Karlsruhe, Germany, has published* a method in which he omitted the effect of the web system, the bending stresses of which are generally small compared with those in the heavy chord members.

The first detailed computations of secondary stresses are to be found in Part I of Winkler's "Theorie der Brücken", 1881 (4). In this painstaking work, the author gives methods for determining the magnitude of secondary stresses, as well as means of reducing them. He also points out the existence of secondary stresses in pin-connected trusses; Manderla confirmed this by extensometer measurements on the Waltenhofen Bridge.

In 1885, Professor Landsberg, of Hannover, Germany, gave an approximate graphical solution, based on the assumption that only the chords are riveted (5). This, like Engesser's method, neglects the web system. During the same year, Professor Müller-Breslau, of Berlin, Germany, contributed an analytical solution (6). In 1890, Professor W. Ritter, of Zurich, Switzerland, gave his graphical solution (7), and, in 1892, Professor O. Mohr presented his semi-

* *Zeitschrift für Baukunde*, 1879.

graphical one. In 1893, Professor Engesser published a thorough treatise on secondary stresses (8).

A remarkable work on secondary stresses was produced in 1902 by Professor E. Patton, of Moscow, Russia (10). His aim was to prove the great advantages of the double intersection trusses, then in common use in Russia. Professor Patton made a summary of the results obtained by computing the maximum primary and secondary stresses of five different bridge trusses, together with those obtained by other prominent scientists. These data enabled him to draw certain curves and study the relations between secondary stresses and the stiffness of the members. The Patton curves, however, give only a rough approximation, as the effect of the truss system and of the distribution of the loads on secondary stresses had not been taken into consideration.

It was not until 1905 that a written discussion in English appeared. In that year, Professor Isami Hiroi, of the Tokyo Imperial University, Tokyo, Japan, published his "Statically-Indeterminate Stresses in Frames Commonly Used for Bridges" (12). He uses the method of least work to find the secondary stresses in bridge trusses and also in the cross-frames. The method is long, and has no checks; therefore, it is not often used. The German periodical, *Der Eisenbau,* contains some remarkable treatises on the subject of secondary stresses, including a method of solving secondary stresses in various types of frames by Professor Mohr, of Dresden, Germany. He makes use of what is known as elastic weights, a method purely analytical (19).

A second method found in *Der Eisenbau* is due to Dr. Pirlet, of Aachen (20). He treats pairs of bending moments at the ends of adjacent members entering a joint as the unknown quantities; there are as many such pairs as there are angles in the truss, or 3 (n-2), in which n is the number of joints and (n-2) the number of triangles. He obtains equations that are of Gauss' normal type. His method is interesting and shows how closely the solutions of secondary stresses and of indeterminate stresses are related. The practical use, however, is small, as Manderla's method requires the solution of only n simultaneous equations of Gauss' normal type, and, therefore, is far superior to that of Pirlet.

Turneaure has developed invaluable methods of tabulating the various quantities, of solving the simultaneous equations, and especially of choosing the panel-point loads for influence lines (25).

In 1913, Mr. José Páez submitted a remarkably fine thesis to Cornell University, entitled "A Comparative and Critical Study of the Different Methods of Computing Secondary Stresses in Bridge Trusses" (27). In 1917, Dr. T. E. Mao also presented a thesis to Cornell University on the study of a large spandrel-braced arch (28); he made extensive use of Gauss' method to solve the equations leading up to the secondary stresses. Two years later, he presented a thesis on "Secondary Stresses" (34) to the Carnegie Institute of Technology, Pittsburgh, Pa., when he obtained the degree of Doctor of Engineering.

Practical Investigations for Bridge Trusses.—The foregoing is a brief account of the theoretical investigations of secondary stresses in bridge trusses

due to the rigidity of the joints. Turning to practice for verification, it is most gratifying to know that many investigations in the field have been made. Manderla was the first investigator; he experimented on the Waltenhofen Bridge. In 1883, Frankel made the first direct measurement of secondary stresses. In 1899, Mesnager published an account of the measurement of stresses in a Pratt truss of 180-ft. span on the Orleans Railway in France.* In 1901, M. Rabut described a series of stress measurements which had been made on a bridge of the Orleans Railway. During 1905 and 1906, W. Gehler conducted a series of tests on a railway bridge, of 128-ft. span, near Elsterwerda, Saxony. In addition to extensometers of the Frankel type, he used sensitive levers. A detailed report of the computations, experiments, and results is to be found in Professor Gehler's book (15). During 1907, 1908, and 1909, extensive secondary stress measurements were made by Professors Crandall and Turneaure, under the auspices of the American Railway Engineering Association. This investigation was primarily to determine the "additional" stress that had to be allowed for impact. A number of plate girders and truss bridges, ranging in span from 30 to 440 ft., were tested for both impact and secondary stresses, but in the latter case the investigations were performed only when the test load was traveling over the bridge at speeds that produced no impact.

In 1915, Mr. Steinman carried out extensive measurements on the Hell Gate Arch (33). Some results were found to conflict with theory, but on the whole the investigation is a remarkable verification of the theory. Recently, quite a number of investigations have been carried out in Switzerland and in England (36). The results of many investigations have not been published, and, therefore, are not available to the profession.

Cross-Frames.—Dr. Hiroi explains the method of solution of a cross-frame, using the principle of least work (12). Dr. Cyril Batho, of McGill University, has a neat way of applying the slope-deflection method in his own solution. Professor Mohr makes use of elastic weights, not only to solve problems of cross-frames, but also the Vierendeel truss, a type found in Europe, principally in Belgium; this is a truss that has no diagonals, but depends on the stiffness of the chords to transmit the shear.†

Column Action.—A history of secondary stress investigations must include an account of the investigations on columns. Such an account would practically mean a résumé of the theory of columns, but only two investigations will be mentioned. The treatise by H. M. Westergaard, Assoc. M. Am. Soc. C. E., on "Buckling of Elastic Structures"(35), not only throws much light on the subject but also has a useful bibliography. Perhaps the finest practical contribution was the series of investigations on columns and eyebars, carried out under the direction of H. M. MacKay, M. Am. Soc. C. E., of McGill University, in connection with the Quebec Cantilever Bridge.‡

* "Les fatigues réelles et fatigues calculées dans un pont à grandes mailles," *Annales des Ponts et Chaussées*, 1899.

† Professor Mohr's method may be found in *Der Eisenbau*, III, 1912, p. 85.

‡ Report of Board of Engineers, Quebec Bridge; also, R. Mayer, "Die Knickfestigkeit," Berlin, 1921.

PART II.—ANALYSIS OF SECONDARY STRESSES IN A BRIDGE TRUSS DUE TO THE RIGIDITY OF ITS JOINTS

1.—Manderla's Method

Introduction.—Consider the analysis of a truss made up of triangular elements. If quadrilaterals or other polygons are involved, as in the case of trusses with subdivided panels, the introduction of imaginary members, with zero cross-section and zero load, transforms such polygons into triangles. The assumption is that, if there are two trusses, identical except that one has riveted joints while the other has perfectly frictionless hinges, and if these trusses could be interchanged, the positions assumed by the joints of the riveted truss under a certain load and those by the corresponding joints of the hinged truss under the same load are identical.

When a truss with frictionless hinges is loaded every member suffers some definite elongation or contraction, and the positions of the joints change to agree with these deformations. The important facts to remember are that (1) all the members remain perfectly straight; (2) no bending stresses are introduced; (3) the angles of each triangular element are changed; and (4) the angles between adjacent members at a joint are likewise changed.

If the joints were fixed, and some means were used for forcing the members to positions where the angles at the joints again assume the values they had before loading, the members of the hinged truss would take up the exact positions and shape of the corresponding members in the riveted truss under the same load, for the effect of a gusset-plate at a joint is to keep the angle between any pair of members at the joint constant at all times. The problem, therefore, is to calculate the changes of angle between adjacent members of the hinged truss, and also the value of the bending moment at each end of every member necessary to make these changes zero.

Changes of Angle in a Triangular Element.—Knowing the unit axial stress in each member of a given triangular element, as well as the modulus of elasticity of the material, the angular changes may be computed by any one of several methods. The following is a neat graphical method due to Ritter. Let ABC (Fig. 3) be any triangular truss element and s_1, s_2, and s_3, the unit stress in the sides, BC, CA, and AB, respectively.

Draw a line through A perpendicular to the opposite side, BC, and cut off a length, AA', equal to s_1, the stress, BC. Through A' draw a line parallel to BC and terminated by the sides, BC, AB. Repeat this operation for the other two vertices.

For a change of length in the member, BC, the contributions toward the changes of angle will be found to be:

$$E \cdot \Delta \alpha_1 = +a$$
$$E \cdot \Delta \beta_1 = -a''$$
$$E \cdot \Delta \gamma_1 = -a',$$

for a change in the member, CA:

$$E \cdot \Delta \alpha_2 = -b'$$
$$E \cdot \Delta \beta_2 = +b$$
$$E \cdot \Delta \gamma_2 = -b'',$$

and, finally, for a change in the member, AB:

$$E \cdot \Delta \alpha_3 = -c''$$
$$E \cdot \Delta \beta_3 = -c'$$
$$E \cdot \Delta \gamma_3 = +c.$$

The total change, $\Delta \alpha$, of α due to a simultaneous change in length in all the members will be found by compounding the three contributions to α in the previous equations, namely:

$$\left.\begin{aligned} E \cdot \Delta \alpha &= E \cdot \Delta \alpha_1 + E \cdot \Delta \alpha_2 + E \cdot \Delta \alpha_3 \\ &= +a - b' - c'' \end{aligned}\right\}$$

Similarly,

$$\left.\begin{aligned} E \cdot \Delta \beta &= +b - c' - a'' \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (1)$$

and,

$$\left.\begin{aligned} E \cdot \Delta \gamma &= +c - a' - b'' \end{aligned}\right\}$$

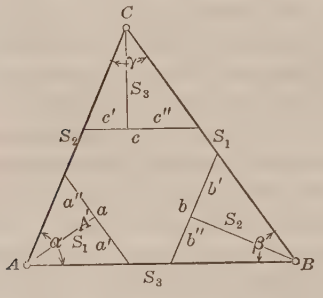


FIG. 3.

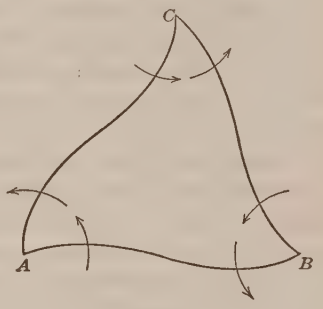


FIG. 4.

To make these results suitable for analytical computation it is to be noted that:

$$a' = s_1 \cot \beta \qquad b' = s_2 \cot \gamma \qquad c' = s_3 \cot \alpha$$
$$a'' = s_1 \cot \gamma \qquad b'' = s_2 \cot \alpha \qquad c'' = s_3 \cot \beta$$
$$a = s_1 (\cot \beta + \cot \gamma) \qquad b = s_2 (\cot \gamma + \cot \alpha) \qquad c = s_3 (\cot \alpha + \cot \beta)$$

Therefore, Equations (1) become:

$$\left.\begin{aligned} E \cdot \Delta \alpha &= (s_1 - s_2) \cot \gamma + (s_1 - s_3) \cot \beta \\ E \cdot \Delta \beta &= (s_2 - s_3) \cot \alpha + (s_2 - s_1) \cot \gamma \\ E \cdot \Delta \gamma &= (s_3 - s_1) \cot \beta + (s_3 - s_2) \cot \alpha \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (2)$$

These are the fundamental equations for the changes of angle when the intensities of stress in the three sides are known. There are many ways of arriving at these equations, Manderla himself using the differential calculus. Because of the application to graphics in Ritter's method, and in that of Mao, and because the method mentioned is the easiest, it will be the only one reproduced.

Equations (2) are easily remembered. Fix attention on one angle of a triangle. The difference between the stresses in the opposite side and one of the adjacent sides is multiplied by the co-tangent of the included angle. Two such terms are found, their sum being the product of the required change in the angle considered and E, the modulus of elasticity.

Bending Moments Introduced at the Joints.—Consider the triangle, ABC (Fig. 4), representing the shape that the frame of Fig. 3 would assume if the members were forced round so that the angles at A, B, and C have the same value as before loading. Fig. 4 also represents the shape that the triangular framework of the riveted truss actually assumes under load. The member, AB, is represented by itself in Fig. 5.

Let the bending moments at A and B be M_1 and M_2, both counterclockwise. There must be shearing forces at A and B; let these be V_1 upward at A and V_2 downward at B. They are, of course, equal in magnitude, for AB is in equilibrium and V_1 and V_2 are the only two vertical forces.

Finally, there is some force, P, along AB—this is the total stress transmitted by the member and assumed to be tension in this case.

The moment at any point, N, at a distance, x, from A is:

$$M_x = M_1 - P \cdot y - V_1 \cdot x \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which y is the offset of the center of gravity of Section N from AB.

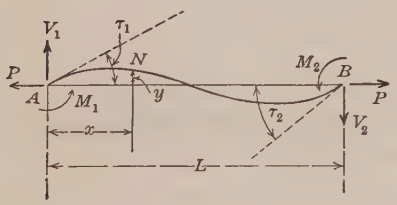


FIG. 5.

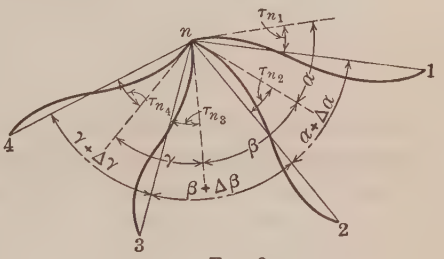


FIG. 6.

The differential equation of the elastic line is:

$$\frac{d^2 y}{d x^2} = -\frac{M_x}{E I} = -\frac{M_1}{E I} + \frac{P \cdot y}{E I} + \frac{V_1 \cdot x}{E I} \quad \ldots\ldots\ldots\ldots (4)$$

in which, I is the moment of inertia of the member. If the primary stress, P, is compression, the only change in Equation (4) is $-\frac{P \cdot y}{E I}$ instead of $+\frac{P \cdot y}{E I}$.

Let $\frac{P}{E I} = Q^2$, which is, therefore, a constant as long as the condition of loading remains.

Equation (4) now becomes:

$$\frac{d^2 y}{d x^2} - Q^2 \cdot y = -\frac{M_1}{E I} + \frac{V_1 \cdot x}{E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

This is a standard differential equation, solved by finding (a) its complementary function by assuming $y = e^{mx}$; and (b) its particular integral by inspection.

The solution is:

$$y = C_1 \cdot e^{+Qx} + C_2 \cdot e^{-Qx} + \frac{M_1}{P} - \frac{V_1 x}{P} \quad \ldots\ldots\ldots\ldots (6)$$

in which, C_1 and C_2 are constants to be determined from the end conditions. As $y = 0$ both when $x = 0$ and when $x = L$,

$$C_1 = -\frac{M_1}{P} \cdot \frac{(1 - e^{-QL})}{(e^{+QL} - e^{-QL})} + \frac{V_1 L}{P} \cdot \frac{1}{(e^{+QL} - e^{-QL})} \quad \ldots\ldots\ldots (7)$$

and,

$$C_2 = +\frac{M_1}{P} \cdot \frac{(1 - e^{-QL})}{(e^{+QL} - e^{-QL})} - \frac{V_1 L}{P} \cdot \frac{1}{(e^{+QL} - e^{-QL})} \quad \ldots\ldots\ldots (8)$$

The slope of the elastic line at any point, N, is:

$$\frac{d\,y}{d\,x} = +\,Q\,C_1\,e^{+Qx} - Q\,C_2\,e^{-Qx} - \frac{V_1}{P} \quad \ldots\ldots\ldots (9)$$

but,

$$r_1 = \frac{d\,y}{d\,x} \text{ for } x = 0, \text{ and } r_2 = \frac{d\,y}{d\,x} \text{ for } x = L$$

hence,

$$r_1 = +\,Q\,C_1 - Q\,C_2 - \frac{V_1}{P} \quad \ldots\ldots\ldots (10)$$

and,

$$r_2 = +\,Q\,C_1\,e^{+QL} + Q\,C_2\,e^{-QL} - \frac{V_1}{P} \quad \ldots\ldots\ldots (11)$$

after substitution in Equation (9).

On substituting the values of C_1 and C_2 found from Equation (7) and Equation (8) in Equation (10) and Equation (11):

$$r_1 = \frac{M_1}{P} \cdot Q \cdot \frac{(e^{+QL} - 1)}{(e^{+QL} + 1)} + \frac{V_1}{P}\left(\frac{2}{e^{+QL} - e^{-QL}} \cdot QL - 1\right) \quad \ldots\ldots (12)$$

and

$$r_2 = \frac{M_1}{P} \cdot Q \cdot \frac{(e^{+QL} - 1)}{(e^{+QL} + 1)} + \frac{V_1}{P}\left(\frac{e^{+QL} + e^{-QL}}{e^{+QL} - e^{-QL}} \cdot Q\,L - 1\right) \quad \ldots\ldots (13)$$

Making use of hyperbolic functions, these equations become:

$$r_1 = \frac{M_1}{P} \cdot Q \tanh \cdot \frac{Q\,L}{2} + \frac{V_1}{P}\left(\frac{1}{\sinh \cdot Q\,L} \cdot Q\,L - 1\right) \quad \ldots\ldots (14)$$

$$r_2 = \frac{M_1}{P} \cdot Q \tanh \cdot \frac{Q\,L}{2} + \frac{V_1}{P}\left(\frac{\cosh \cdot Q\,L}{\sinh \cdot Q\,L} \cdot Q\,L - 1\right) \quad \ldots\ldots (15)$$

Eliminating V_1 from Equation (14) and Equation (15), a relation is found between M_1 and the deflection angles, r_1 and r_2. This is:

$$M_1 = \frac{2\,E\,I}{L}(2\,a\,r_1 + b\,r_2) \quad \ldots\ldots\ldots (16)$$

in which,

$$a = \frac{1}{8}\left(\frac{Q^2 L^2}{Q\,L \coth \cdot \frac{Q\,L}{2} - 2} + Q\,L \cdot \coth \cdot \frac{Q\,L}{2}\right)$$

and,

$$b = \frac{1}{4}\left(\frac{Q^2 L^2}{Q\,L \coth \cdot \frac{Q\,L}{2} - 2} - Q\,L \cdot \coth \cdot \frac{Q\,L}{2}\right)$$

Replacing M_1 by M_2 and interchanging τ_1 and τ_2,

$$M_2 = \frac{2\,E\,I}{L}\,(2\,a\,\tau_2 + b\,\tau_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

These values of a and b may be expressed in the form of infinite series by the use of the known series:

$$\coth . x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \frac{2\,x^5}{945}, \text{ etc.,} \quad \ldots\ldots\ldots\ldots (18)$$

Thus,

$$a = 1 + \frac{(Q\,L)^2}{30} - \frac{11\,(Q\,L)^4}{25\,000} +, \text{ etc.,} \quad \ldots\ldots\ldots\ldots (19)$$

$$b = 1 - \frac{(Q\,L)^2}{60} + \frac{13\,(Q\,L)^4}{25\,000} -, \text{ etc.,} \quad \ldots\ldots\ldots\ldots (20)$$

Equation (19) and Equation (20) are for the case of P as tension. If P is compression, Q^2 is negative, and the signs of the terms in the even places in Equation (19) and Equation (20) must be changed, as follows:

$$a = 1 - \frac{(Q\,L)^2}{30} - \frac{11\,(Q\,L)^4}{25\,000}, \text{ etc.,} \quad \ldots\ldots\ldots\ldots (21)$$

$$b = 1 + \frac{(Q\,L)^2}{60} + \frac{13\,(Q\,L)^4}{25\,000}, \text{ etc.,} \quad \ldots\ldots\ldots\ldots (22)$$

Summarizing, the fundamental deflection formulas are:

$$\left.\begin{aligned} M_{12} &= \frac{2\,E\,I}{L}\,(2\,a\,\tau_{12} + b\,\tau_{21}) \\ M_{21} &= \frac{2\,E\,I}{L}\,(2\,a\,\tau_{21} + b\,\tau_{12}) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (23)$$

in which,

$$\left.\begin{aligned} a &= 1 \pm \frac{(Q\,L)^2}{30} - \frac{11\,(Q\,L)^4}{25\,000} \pm \cdots \\ b &= 1 \mp \frac{(Q\,L)^2}{60} + \frac{13\,(Q\,L)^4}{25\,000} \mp \cdots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (24)$$

the upper signs in Equation (24) being used when P is tension, and the lower when P is compression.

A slight change in the notation has been introduced. The member, AB, is called 12, and the bending moment and deflection angle at End 1 are denoted by M_{12} and τ_{12}, respectively, while those of End 2 are M_{21} and τ_{21}, respectively.

Expressing τ_1 and τ_2 in terms of M_{12} and M_{21}, Equation (23) will give:

$$\left.\begin{aligned} \tau_{12} &= \frac{L}{6\,E\,I}\,(2\,c\,M_{12} - d\,M_{21}) \\ \text{and,} \qquad \tau_{21} &= \frac{L}{6\,E\,I}\,(2\,c\,M_{21} - d\,M_{12}) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (25)$$

in which,

$$c = 1 \mp \frac{(Q\,L)^2}{15} + 2\,(Q\,L)^4 \mp \frac{(Q\,L)^6}{1\ 575} + \cdots$$

and,

$$d = 1 \mp \frac{7\,(Q\,L)^2}{60} + \frac{3\,(Q\,L)^4}{2\ 520} \mp \frac{127\,(Q\,L)^6}{100\ 800} + \cdots \quad \Big\} \ldots\ldots\ldots (26)$$

the upper signs in Equation (26) being used when P is tension, and the lower when P is compression.

Relation Between the Deflection Angles of Members Meeting in a Joint.— Let any number of members, $n\,1$, $n\,2 \ldots nm$, meet at a joint, n. It may readily be shown* that the τ of any of these members can be expressed in terms of the τ of any other, chosen for purpose of reference, and the angular changes taking place between the different members at n.

In general,

$$\tau_{nm} = \tau_{n1} + \sum_{1}^{m-1} \Delta\,\alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

It is usually convenient to take as the reference, τ, that of the member first reached in passing around the joint from the outside of the frame in a clockwise direction.

The Sum of the Bending Moments About Any Joint is Zero.—In order to find these unknown τ's, use is made of the fact that each joint is in equilibrium under all the bending moments about the joint. If an eccentric external force were present, it can be accounted for, too, but for the time being it is assumed that there is none.

Hence, from Fig. 6:

$$M_{n_1} + M_{n_2} + \ldots + M_{nm} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which, M_{n_1} is the moment at n in Member $n1$, etc.

Referring now to Equation (23):

$$M_{n1} = \frac{2\,E\,I_{n1}}{L_{n1}}\,(2\,a_{n1}\,\tau_{n1} + b_{n1}\,\tau_{1n})$$

in which,

I_{n_1} = the moment of inertia of Member $n1$;
L_{n_1} = the length of Member $n1$;
τ_{n_1} = the deflection angle of Member $n1$ at End n;
τ_{1n} = the deflection angle of Member $n1$ at End 1.

$$a_{n1} = 1 \pm \frac{(Q_{n1}\,L_{n1})^2}{30} - \frac{11\,(Q_{n1}\,L_{n1})^4}{25\ 000} \pm \ldots$$

$$b_{n1} = 1 \mp \frac{(Q_{n1}\,L_{n1})^2}{60} + \frac{13\,(Q_{n1}\,L_{n1})^4}{25\ 000} \mp \ldots$$

the upper signs being used when P is tension, and the lower when P is compression, a and b being constants.

$$Q_{n1}{}^2 = \frac{P_{n1}}{E\,I_{n1}}$$

in which, P_{n_1} is the total primary stress transmitted by Member $n1$.

* "Modern Framed Structures," by Messrs. Johnson, Bryan and Turneaure, Part II.

Similarly,

$$M_{n2} = \frac{2\,E\,I_{n2}}{L_{n2}}\,(2\,a_{n2}\,\tau_{n2} + b_{n2}\,\tau_{2n})$$

in which the various quantities have exactly the same significance as before, and so on for the other bending moments.

There is, therefore, for every joint an equation that will contain the deflection angles of all the members that enter the joint. Furthermore, this equation is linear. For example, Equation (28) will take the form:

$$\left.\begin{aligned} &c_{n1}\,\tau_{n1} + c_{n2}\,\tau_{n2} + c_{n3}\,\tau_{n3} + c_{n4}\,\tau_{n4} + \cdots \\ &+ c_{1n}\,\tau_{1n} + c_{2n}\,\tau_{2n} + c_{3n}\,\tau_{3n} + c_{4n}\,\tau_{4n} + \cdots = 0 \end{aligned}\right\} \cdots\cdots(29)$$

in which, c_{n1}, etc., are constants.

Referring now to Equation (27), the τ's of Equation (29) must be changed into what are termed the "reference deflection angles" of the joints, of which Equation (29) makes use. For the purpose of consistency, take each member separately in a clockwise direction around every joint beginning outside the truss; the τ of the member first met will be the reference deflection angle of the joint. Taking τ_{n1} of Equation (29) as the reference deflection angle of the joint, n (Fig. 6), call it τ_n. Making use of Equation (27),

$$\tau_{n2} = \tau_n + \Delta\,\alpha_{1n2}$$

in which, α_{1n_2} is the angle between $1n$ and $2n$,

$$\tau_{n3} = \tau_n + \Delta\,\alpha_{1n2} + \Delta\alpha_{2n3}, \text{ etc.},$$

while,

$$\tau_{1n} = \tau_1 + \Sigma\,\Delta\,\alpha_1,$$

in which, τ_1 is the reference deflection angle for Joint (1), and $\Sigma\,\Delta\,\alpha_1$ is the sum of the changes of angles that lie between the reference member of Joint (1) and the member, $n1$, etc.

Summarizing, it is possible to change Equation (29) into an equation, as follows:

$$k_n\,\tau_n + k_1\,\tau_1 + k_2\,\tau_2 + k_3\,\tau_3 + k_4\,\tau_4 + \cdots\, S = 0 \cdots\cdots(30)$$

in which, all the k's and S's are constants.

A repetition of this for every joint will finally give a set of simultaneous linear equations, equal in number to the joints, and the unknowns are the reference deflection angles. On solving these, substitute the values in Equation (27) to determine all the deflection angles in the truss. Making use of the latter in Equation (23), the two bending moments at the ends of every member are found. A check must be applied here; the sum of the bending moments at every joint must be zero. If this is satisfactorily done, the secondary stresses are obtained from the formula,

$$f = \frac{M\,y}{I},$$

in which,

f = the secondary stress;
M = the bending moment at one end of a particular member;
y = the distance of the extreme fiber from the center of gravity; and
I = the moment of inertia of the member in question.

It must be remembered that the extreme fiber represents the set of extreme fibers that lie in a plane parallel to the neutral plane of the member.

Mathematically, the problem is solved. In practice, however, the labor is excessive; certain approximations, therefore, are necessary.

2.—Modification of Manderla's Method

An improvement in Manderla's method is due to Winkler. The analysis is much the same as that just explained with no change as far as the derivation of Equation (3).

The Bending Moments Introduced at the Joints.—According to Equation (3) and Fig. 5:

$$M_x = M_1 - Py - V_1 x_1$$

Now, y, the offset of N from the line, A-B, is generally so small that $-Py$ may be neglected. Therefore,

$$M_x = M_1 - V_1 x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

giving rise to the following equation of the elastic line:

$$E\,I \cdot \frac{d^2 y}{d x^2} = V_1 x - M_1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

Equation (32) can be easily solved by direct integration. Integrating once with respect to x,

$$E\,I \frac{d y}{d x} = + \frac{V_1 x^2}{2} - M_1 x + C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33)$$

which represents the slope of the elastic line at any point, N, distant x from A. The constant, C, will be determined later. Integrating again,

$$E\,I \cdot y = + \frac{V_1 x^3}{6} - \frac{M_1 x^2}{2} + C x + D \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (34)$$

which gives the distance of the point, N, from AB. The constants, C and D, are now determined from the fact that $y = 0$ when $x = 0$ and $x = L$. Therefore,

$$D = 0, \text{ and } C = + \frac{M_1 L}{2} - \frac{V_1 L^2}{6}$$

Substituting now in Equation (33), the equation of the slope of the elastic line becomes:

$$E\,I \cdot \frac{d y}{d x} = + \frac{V_1 x^2}{2} - M_1 x + \frac{M_1 L}{2} - \frac{V_1 L^2}{6} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

The slope at $x = 0$ is the deflection angle, τ_1. Hence,

$$E\,I \cdot \tau_1 = + \frac{M_1 L}{2} - \frac{V_1 L^2}{6} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (36)$$

The moments about B are now taken, giving $V_1 L = M_1 + M_2$. Hence, Equation (36) becomes:

$$\tau_1 = \frac{L}{6\,E\,I} (2\,M_1 - M_2) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (37)$$

Starting at the other end of the member, there would have resulted a similar expression,

$$\tau_2 = \frac{L}{6\,E\,I}(2\,M_2 - M_1) \qquad (38)$$

(Compare Equations (37) and (38) with Equations (25); the difference is that the c and d of Equations (25) are each replaced by unity in Equations (37) and (38). In other words, the Q in Equations (26) is here considered as zero, that is, the force, P, is regarded as zero. That is exactly what would be expected, for at the start of this analysis the effect of the direct stress on the bending moments was disregarded.)

Making use of Equations (37) and (38) to express M_1 and M_2 in terms of τ_1 and τ_2:

$$\left.\begin{aligned} M_1 &= \frac{2\,E\,I}{L}(2\,\tau_1 + \tau_2) \\ M_2 &= \frac{2\,E\,I}{L}(2\,\tau_2 + \tau_1) \end{aligned}\right\} \qquad (39)$$

(Again, comparing with Equations (23), it is evident that the a and b are each replaced by unity, for the same reasons that c and d were.)

Relation Between the Deflection Angles of Members Meeting in a Joint.—This is the same as was given for Manderla's original method.

The Sum of the Bending Moments Around Any Joint is Zero.—Again, as previously (see Fig. 6):

$$M_{n1} + M_{n2} + M_{n3} + M_{n4} = 0$$

Making use of Equations (39), this becomes:

$$\left.\begin{aligned} &\frac{2\,E\,I_{n1}}{L_{n1}}(2\,\tau_{n1} + \tau_{1n}) + \frac{2\,E\,I_{n2}}{L_{n2}}(2\,\tau_{n2} + \tau_{2n}) \\ &+ \frac{2\,E\,I_{n3}}{L_{n3}}(2\,\tau_{n3} + \tau_{3n}) + \frac{2\,E\,I_{n4}}{L_{n4}}(2\,\tau_{n4} + \tau_{4n}) = 0 \end{aligned}\right\} \qquad (40)$$

Let the quantity, $\frac{I}{L}$, be called K. Hence,

$$\begin{aligned} K_{n1}(2\,\tau_{n1} + \tau_{1n}) + K_{n2}(2\,\tau_{n2} + \tau_{2n}) + K_{n3}(2\,\tau_{n3} + \tau_{3n}) \\ + K_{n4}(2\,\tau_{n4} + \tau_{4n}) = 0 \end{aligned} \qquad (41)$$

Use may then be made of the reference deflection angles. Thus, in Fig. 6, the τ of the joint, n, is τ_{n1}, which without fear of confusion may be called τ_n, while τ_{n2} becomes $\tau_n + \Delta\,\alpha$, τ_{n3} becomes $\tau_n + \Delta\,\alpha + \Delta\,\beta$, etc. (See Equation (27)).

As many equations of the type of Equation (41) are found as there are joints. There will be a large number of τ's of the nature, τ_{n1}, τ_{n2} .... τ_{1n}, τ_{2n} ...., etc., but by making use of Equation (27), there results finally a set of as many simultaneous linear equations in the reference τ's as there are joints. After solving these, Equation (39) will express the bending moments at the ends of all the members. Finally, the use of the equation, $f = \frac{M\,y}{I}$, gives the required secondary stresses.

Professor Turneaure's method will be followed in arriving at the set of simultaneous linear equations for the τ's, and the steps that lead to the equations with comparative ease will be explained more fully in the practical application which Manderla's original method lacked.

It is evident that the value of E does not affect the secondary stresses, for the simple reason that in this analysis E appears in the denominator of an expression in one place, only to be cancelled out in the numerator later. As all the methods that will be applied show this property, E will be considered as unity throughout. It is another matter, of course, in testing the theory in the field; there, in determining what stress a certain deformation represents, E is essential.

Manderla solves his simultaneous equations by a method of approximation. It will be found that the coefficient of one of the τ's in every equation (the one belonging to the reference, τ, of the joint for which that particular equation is formed) is much larger than any of the others. By first imagining all the small coefficients to be zero, approximate values are obtained for the τ's that are retained. Taking each equation as given, and substituting in all the terms except that with the large coefficient the approximate values of the τ's just found, a fresh set of values for the τ's is determined. This process is repeated until a stage is reached where the τ's of the final approximation differ little from those of the previous approximation—these are then the values required. This method lends itself to slide-rule computation and is, therefore, much in favor with engineers; the great objection is that many approximations are necessary when there are many joints—a tedious task. This method, however, will be illustrated by an example.

Professor Turneaure has a definite method of elimination (25). Winkler has also given such a method (4). Several investigators have noted the fact that all these sets of simultaneous equations are, as they appear, of the type known as Gauss' normal equations. Most books on least squares describe Gauss' method of solving a set of simultaneous linear equations, in which he re-arranges them in what he calls the "normal" form, to which he then applies a most effective and absolutely self-checking method of solution. A solution of secondary stresses, using Gauss' method of normal equations, will also be given.

3.—Müller-Breslau's Method

Introduction.—A simple truss made up of triangular elements is such that, if it has n joints, the number of members is $(2\,n\text{-}3)$. This is the criterion of what is known as a statically determinate plane framework. In the present problem every member has a certain bending moment at each end; these bending moments are really the unknown quantities that are to be determined. Their number is, therefore, $2\ (2\,n\text{-}3) = (4\,n\text{-}6)$.

The underlying principles of this method are exactly those of the modified Manderla method; the manner of attacking the problem is different. As many linear equations as there are joints are obtained from the property that the sum of the moments about a joint is zero. As will be shown later, there are three independent linear relations between the three changes of angle in every

triangular element and the six moments, two for each of the three sides. As there are $(n\text{-}2)$ triangular elements, the total number of these relations is $3\ (n\text{-}2) = (3\,n\text{-}6)$. There are, therefore, $n + (3\,n\text{-}6) = (4\,n\text{-}6)$ linear equations to solve the $(4\,n\text{-}6)$ unknown bending moments.

Manderla has this number of equations too, but, by making use of the fact that all the deflection angles at a joint can be expressed in terms of one at that joint, he is able to tie up, as it were, the $(3n\text{-}6)$ relations in the n simultaneous equations; after these are solved he reverts to the expression for all the τ's at a joint in terms of the one, and then uses the remaining $(3\,n\text{-}6)$ relations.

Müller-Breslau has a special way of setting down the total $(4\,n\text{-}6)$ relations, one after the other. By assuming two particular bending moments at the left end of a truss to be known, he proceeds to express every successive one in terms of the preceding ones until, finally, he arrives at two equations in which every bending moment is already in terms of the two assumed ones. In other words, he obtains two linear equations with which to find the bending moments. Repeating the work and substituting these in the expressions, all the bending moments become known. It is a neat method of elimination.

All moments in a clockwise direction will be reckoned positive in this method.

Procedure.—A triangle, ABC (Fig. 7) under such a system of moments will be bent in a form as shown. Hence,

$$\tau_2 + \alpha + \Delta\alpha = \tau_3 + \alpha.$$

Therefore,

$$\Delta\alpha = \tau_3 - \tau_2.$$

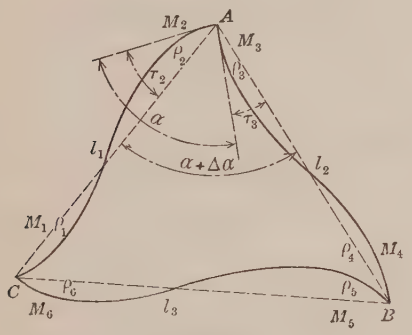


FIG. 7.

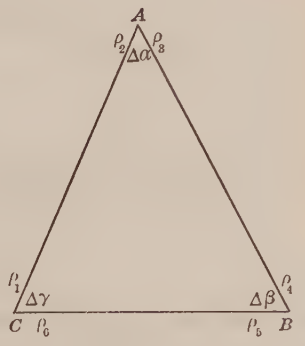


FIG. 8.

From Equation (37) however:

$$\tau_2 = 2\,M_2 - \frac{M_1}{6\,E\,I_1} \cdot l_1$$

and,

$$\tau_3 = 2\,M_3 - \frac{M_4}{6\,E\,I_2} \cdot l_2$$

(The analysis for the deflection angles in terms of the end bending moments is of course the same as that used in the modified Manderla method. Note that Müller-Breslau also neglects the effect of the primary stress on the bending moment.)

Therefore,

$$\Delta\alpha = 2M_3 - \frac{M_4}{6EI_2} \cdot l_2 - 2M_2 - \frac{M_1}{6EI_1} \cdot l_1 \quad \ldots\ldots (42)$$

For convenience let,

$$\frac{M_1 l_1}{I_1} = \rho_1;\ \frac{M_2 l_1}{I_1} = \rho_2;\ \frac{M_3 l_2}{I_2} = \rho_3;$$
$$\frac{M_4 l_2}{I_2} = \rho_4;\ \frac{M_5 l_3}{I_3} = \rho_5;\ \frac{M_6 l_3}{I_3} = \rho_6$$

Insert these expressions in Equation (42) and multiply by $6E$, obtaining:

$$6E.\Delta\alpha = (2\rho_3 - \rho_4) - (2\rho_2 - \rho_1),$$

or,

$$6E.\Delta\alpha = \rho_1 - 2(\rho_2 - \rho_3) - \rho_4 \quad \ldots\ldots (43)$$

Similarly,

$$6E.\Delta\beta = \rho_3 - 2(\rho_4 - \rho_5) - \rho_6 \quad \ldots\ldots (44)$$

and,

$$6E.\Delta\gamma = \rho_5 - 2(\rho_6 - \rho_1) - \rho_2 \quad \ldots\ldots (45)$$

Between the changes of the angle, $\Delta\alpha$, $\Delta\beta$, and $\Delta\gamma$, of the triangle there exists the relation:

$$\Delta\alpha + \Delta\beta + \Delta\gamma = 0$$

or,

$$\rho_1 - \rho_2 + \rho_3 - \rho_4 + \rho_5 - \rho_6 = 0 \quad \ldots\ldots (46)$$

Express ρ_4, ρ_5, and ρ_6, in terms of ρ_1, ρ_2, and ρ_3, and the changes of angle. Hence Equation (43) gives:

$$\rho_4 = -6E.\Delta\alpha + \rho_1 - 2(\rho_2 - \rho_3)$$

or,

$$(2\rho_3 - \rho_4) = 6E\Delta\alpha + (2\rho_2 - \rho_1) \quad \ldots\ldots (47)$$

Equations (44) and (46) give:

$$\rho_5 = 6E\Delta\beta + (\rho_1 - \rho_2) + \rho_4$$

or,

$$(\rho_5 - \rho_4) = 6E\Delta\beta + (\rho_1 - \rho_2) \quad \ldots\ldots (48)$$

and Equations (45) and (46) give:

$$\rho_6 = 6E\Delta\gamma - (\rho_3 - \rho_4) + \rho_1$$

or,

$$(\rho_1 - \rho_6) = 6E\Delta\gamma + (\rho_3 - \rho_4) \quad \ldots\ldots (49)$$

In using Equations (47), (48), and (49), it is convenient to bear in mind the following rules:

1.—When any three adjacent values, ρ_1, ρ_2, and ρ_3, are known, the value, ρ_4, is readily obtained by Equation (47), in which $(2\rho_3 - \rho_4)$ is the deflection angle of AB at A, that is, of the member first met in passing around the

joint, A, in a clockwise direction, and $(2\rho_2 - \rho_1)$ is the deflection angle of AC at A.

2.—When any four adjacent values, ρ_1, ρ_2, ρ_3, and ρ_4, are given, a fifth value, ρ_5 or ρ_6, may be found from Equation (48) or Equation (49), in which $(\rho_5 - \rho_4)$ is the difference of the ρ-values at B of the sides, BC and BA, which include the angle, B, and $(\rho_1 - \rho_2)$ is the difference of the values of the side, AC, opposite to B.

In both cases the difference is found by passing around the joint in a clockwise direction and then subtracting the second one of the values in question from the first.

Equations (47), (48), and (49) and the three fundamental equations, one for each joint, namely, that the sum of the bending moments around each joint is zero, comprise six independent relations between the six unknowns, $\rho_1 \ldots \rho_6$, which therefore can be determined.

(This is for the case of a structure composed of a single triangle. Note that the changes of angle are previously determined in exactly the same manner as was shown for Manderla's method.)

If instead of a single triangle the structure is as shown in Fig. 9, the process of calculating the values, $\rho_1 \ldots \rho_{14}$, is as follows: Consider any Joint (1) in which only two members intersect, and assume for the present that the ρ_1 and ρ_2 of the member, 1-2, are known.

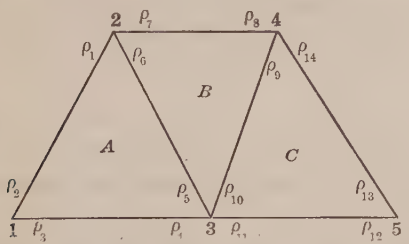


FIG. 9.

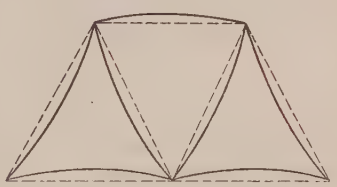


FIG. 10.

The fundamental equation, $M = 0$, for Joint (1) gives:

$$M = M_2 + M_3 = 0$$

Turning back to the notation adopted, this equation becomes

$$\rho_3 \cdot \frac{I_2}{l_2} = -\rho_2 \cdot \frac{I_1}{l_1}$$

After ρ_3 is known, its value is substituted in Equation (47), thus giving ρ_4 as a function of ρ_1 and ρ_2. By substituting the values of ρ_3 and ρ_4 in Equations (48) and (49), ρ_5 and ρ_6 also are expressed in terms of ρ_1 and ρ_2.

In passing to Joint (2), ρ_7 is the only quantity not known; but,

$$M = M_7 + M_6 + M_1 = 0$$

or,

$$\rho_7 \cdot \frac{I_4}{l_4} + \rho_6 \cdot \frac{I_3}{l_3} + \rho_1 \cdot \frac{I_1}{l_1} = 0$$

whence, ρ_7 is expressed in terms of ρ_1 and ρ_2.

By applying Equations (47), (48), and (49) to the triangle, B, of which ρ_5, ρ_6, and ρ_7 are now expressed in terms of ρ_1 and ρ_2, the remaining quantities, ρ_8, ρ_9, and ρ_{10}, may first be expressed in terms of ρ_5, ρ_6, and ρ_7, and then, in turn, in terms of ρ_1 and ρ_2.

Knowing ρ_4, ρ_5, and ρ_{10}, ρ_{11} at Joint (3) is found from,

$$M_4 + M_5 + M_{10} + M_{11} = 0$$

and, finally, ρ_{12}, ρ_{13}, and ρ_{14}, are found by means of Equations (47), (48), and (49), which are to be applied to the triangle, C.

The preceding analysis shows that each ρ may be expressed as a linear function of ρ_1 and ρ_2, which were assumed to be known at the start. There are two equations left that have not been used, namely,

$$\Sigma M_{(4)} = \Sigma M_{(5)} = 0$$

These are:

$$\left.\begin{aligned} \rho_{14} \cdot \frac{I_7}{l_7} + \rho_9 \cdot \frac{I_5}{l_5} + \rho_8 \cdot \frac{I_4}{l_4} &= 0 \\ \text{and,} \qquad \rho_{12} \cdot \frac{I_6}{l_6} + \rho_{13} \cdot \frac{I_7}{l_7} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots (50)$$

Making use of the expressions for ρ_{14}, etc., in terms of ρ_1 and ρ_2, Equations (50) become two simultaneous linear equations in terms of ρ_1 and ρ_2 which are then solved. All the other ρ's, therefore, are also determined. The secondary stress, f, is then found from,

$$f = \frac{M\,y}{I} = \rho \cdot \frac{I}{l} \cdot \frac{y}{I} = \rho \cdot \frac{y}{l} \quad \ldots\ldots\ldots\ldots\ldots (51)$$

in which, y is the distance of the extreme fiber from the center of gravity and l is the length of the member.

Remarks.—Müller-Breslau's method is not exactly like this one, which is due to Páez. Müller-Breslau assumes that the triangles alternate in shape as in Fig. 10.

This requires a changing of signs in the moments passing from one triangle to the next, a method that is apt to create confusion and is the source of many errors. The method of Páez is to reckon all the moments to be positive and to remain positive in the analysis. It is then known what shapes the triangles assume theoretically. If, in the calculations, a moment at one end of a member is found to be negative the shape of the member should be contrary to that assumed. Thus, Páez' modification is far superior in that the signs are taken care of in the work.

4.—Ritter's Method.

Introduction.—The graphical method of computing secondary stresses developed by Ritter is also based on the fundamental equation of Manderla. Its aim is to solve graphically the $(4n\text{-}6)$ equations for the $(4n\text{-}6)$ bending moments, one at each end of every member.

Procedure.—Consider any joint of a structure as Joint (5) in Fig. 11, at which the four members, 5-3, 5-4, 5-6, and 5-7, meet. Imagine all the members to be bent under positive (clockwise) bending moments. The two bending moments of the member, 3-5, are designated M_3 at Joint (5) and M_3' at Joint (3). For the member, 4-5, the moments are M_4 at Joint (5) and M_4' at Joint (4), etc. When Joint (4) is considered, the bending moments of the member, 4-5, are M_5 at Joint (5) and M_5' at Joint (5).

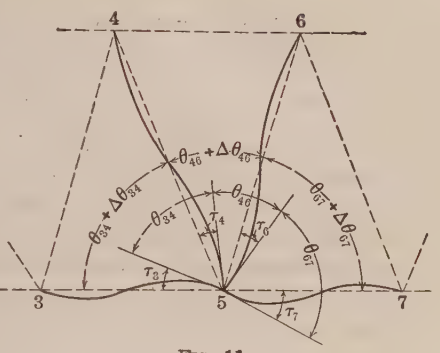


FIG. 11.

It follows, therefore, that:

$$M_4 \text{ for Joint (5)} = M_5' \text{ for Joint (4)}$$
$$M_4' \text{ " " (5)} = M_5 \text{ " " (4)}$$

which means that each moment in a truss is designated in two different ways.

From Fig. 11,

$$\theta_{3.4} + \Delta\,\theta_{3.4} + \tau_4 = \theta_{3.4} + \tau_3$$

or,

$$\Delta\,\theta_{3\cdot4} = \tau_3 - \tau_4$$

Substituting the values of τ_3 and τ_4 as found from Equation (37),

$$E \,.\, \Delta\,\theta_{3.4} = \frac{2\,M_3 - M_3'}{6\,I_3} \,.\, l_3 - \frac{2\,M_4 - M_4'}{6\,I_4} \,.\, l_4$$

in which, M_3 and M_4 are the moments at the near ends, and M_3' and M_4' those at the far ends of the members, 5-3 and 5-4, which include the angle, $\theta_{3\cdot4}$. Similar expressions can be written for $E \,.\, \Delta\,\theta_{4\cdot6}$ and $E \,.\, \Delta\,\theta_{6\cdot7}$.

The sum of the moments around Point (5) is zero and, therefore,

$$M_3 + M_4 + M_6 + M_7 = 0.$$

For convenience, let $\dfrac{M\,l}{6\,I} = \mu$. The four equations connected with Joint (5) can now be written as follows:

$$\left.\begin{aligned} E \,.\, \Delta\,\theta_{3.4} &= (2\,\mu_3 - \mu_3') - (2\,\mu_4 - \mu_4') \\ E \,.\, \Delta\,\theta_{4.6} &= (2\,\mu_4 - \mu_4') - (2\,\mu_6 - \mu_6') \\ E \,.\, \Delta\,\theta_{6.7} &= (2\,\mu_6 - \mu_6') - (2\,\mu_7 - \mu_7') \\ \frac{I_3}{l_3} \,.\, \mu_3 + \frac{I_4}{l_4} \,.\, \mu_4 + \frac{I_6}{l_6} \,.\, \mu_6 + \frac{I_7}{l_7} \,.\, \mu_7 &= 0 \end{aligned}\right\} \ldots\ldots\ldots (52)$$

For every joint there are as many equations as there are members intersecting at the joint. The total number of these equations is equal to the number of ends of all the members and, therefore, equal to that of the unknown μ-values.

The changes of angle are determined either graphically or analytically by Ritter's method as used previously,* and are, therefore, regarded as known.

* See p. 9.

In order to explain how Ritter solves his problem, assume all the μ'-values to be known for the present. The μ-values are then determined easily by force and equilibrium polygons, as follows:

Consider the values, $\frac{I}{l}$, as vertical forces; plot them in order on a vertical line and draw a force polygon with any arbitrary pole, Fig. 12. The distances, $E \,.\, \Delta\, \theta$, between the forces are then laid out horizontally as shown in Fig. 13. Each of the forces is then displaced to the left by the amount, μ', that belongs to the member to which the so-called force corresponds. The equilibrium polygon is now constructed for these displaced forces. The distances of the displaced forces from their resultant will then be the double values of the μ's.

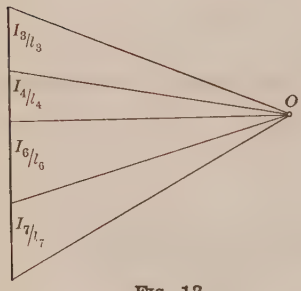


FIG. 12.

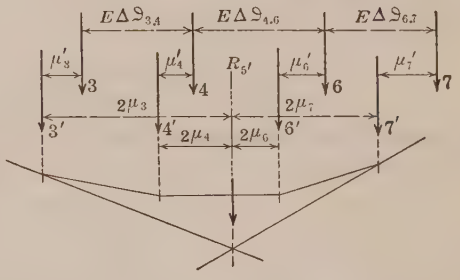


FIG. 13.

A study of Equations (52) and Fig. 13 will prove the truth of this statement. The distance of a displaced force from the resultant, is 2 μ for a displacement, μ'; consequently, the distance of an undisplaced force from the resultant of the displaced forces is equal to $(2\,\mu - \mu')$, and the difference between the distances of two adjacent undisplaced forces must equal $E \,.\, \Delta\, \theta$. The first three of the Equations (52) are, therefore, satisfied. The fourth is also satisfied, as the algebraic sum of the moments $(= \Sigma \frac{I}{l} \,.\, 2\,\mu)$ of the parallel forces, $\frac{I}{l}$, about any point on the line of the resultant is equal to zero.

The difficulty, however, is that neither the μ-values nor the μ'-values are known. Let Figs. 14 and 15 represent the equilibrium polygons for Joints (4) and (5), R_4' and R_5', the resultants of the displaced forces, 2′, 3′ ..... If, now, the distance between an undisplaced force and the resultant of the displaced forces is denoted by γ, we have:

$$2\,\mu_4 = \mu_4' + \gamma_4$$
$$2\,\mu_5 = \mu_5' + \gamma_5$$

but,

$$\mu_4 = \mu_5' \text{ and } \mu_5 = \mu_4'$$

therefore,

$$2\,\mu_5' = \mu_4' + \gamma_4$$
$$2\,\mu_4' = \mu_5' + \gamma_5$$

By solving these equations:

and,

$$\left.\begin{aligned} \mu_4' &= \frac{(\gamma_4 + 2\,\gamma_5)}{3} \\ \mu_5' &= \frac{(\gamma_5 + 2\,\gamma_4)}{3} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (53)$$

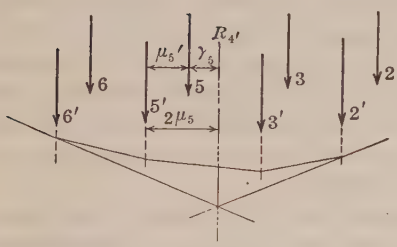


FIG. 14.

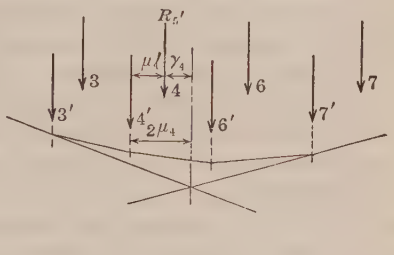


FIG. 15.

If the positions of the resultants, R_4' and R_5', of the displaced forces were known, the values of 2 μ could be found at once. These positions can only be found by trial. As the position of the resultant, R', undergoes little displacement (that is, the changes of the values, γ, are generally small), a roughly approximate position of the resultant, R', may be assumed as that of the resultant, R, of the undisplaced forces.

Measuring now the distances of the undisplaced forces from their resultant, and calling these distances the first approximation to the true γ's, these values are substituted in Equations (53) to give the first approximation to the μ's, the amounts by which the forces are displaced. Next determine the resultant of this first set of displaced forces.

The distances of the undisplaced forces to the resultant of the first set of displaced forces are the second approximation to the γ-values. Carrying through the process, a second resultant of displaced forces is found. This process is repeated several times, until it is found that, for every joint, the resultant of the displaced forces from the last trial differs very little from that of the former trial. The values of 2 μ are then the distances of the last set of displaced forces from their resultant.

The secondary stress is, therefore,

$$f = \frac{M\,y}{I} = \mu \cdot \frac{6\,I}{l} \cdot \frac{y}{I} = \frac{6\,y}{l} \cdot \mu \quad \ldots\ldots\ldots\ldots\ldots\ldots (54)$$

Remarks.—In order to avoid errors strict attention must be paid to the signs of $\Delta\,\theta$, μ, and μ'. The succession of the members around a joint should be taken in a clockwise direction; a $\Delta\,\theta$ which has a positive sign should be laid out at the right, and a negative $\Delta\,\theta$ to the left. Further, any quantity, μ or μ', to be transferred into a corresponding equilibrium polygon should be

laid out to the left, if it is situated to the left of the resultant of the forces, $\frac{I}{l}$, and *vice versa*. Finally, the values, $2\ \mu$, situated to the left of the resultant are positive and those to the right are negative.

5.—Mohr's Semi-Graphical Method

Introduction.—The preceding methods of Manderla, Müller-Breslau, and Ritter are based on the equations giving the changes of angle in the triangular elements, and on the equation arising from the fact that the sum of the bending moments around a joint is zero; these were the two sets of fundamental equations. In deriving his method, Mohr introduced two sets of angles, ψ and ϕ, on which the bending moments are dependent, thus avoiding in his computations the calculation of the changes of the angle, $\Delta\theta$, and of the deflection angles, τ, on which the previous methods depend. It is nevertheless true that this method can be easily changed into any of the others by establishing the relations between ψ and ϕ on the one hand and $\Delta\theta$ and τ on the other, and then substituting for ψ and ϕ in terms of $\Delta\theta$ and τ.

Let 1-2 (Fig. 16) represent any member before the exterior forces begin to act. Under the action of any loading the joints, 1 and 2, will be displaced to 1′ and 2′. The lines, 1′-2″ and 1″-2′, are parallel to each other and to the member, 1-2; 1′ T_1 and 2′ T_2 indicate the end tangents drawn to the elastic line of the curved member after deformation has taken place.

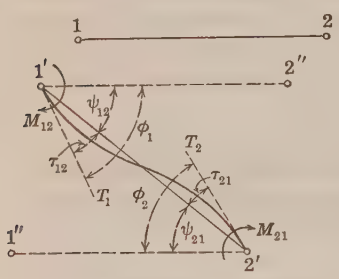


Fig. 16.

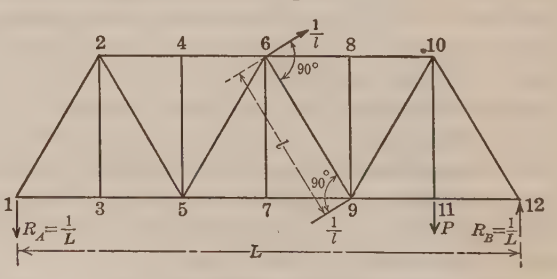


Fig. 17.

The angle, $2'' - 1' - 2' = \psi_{12} = \psi_{21}$, included between the original direction of the member and the chord, 1′-2′, of the curved member after deformation, is the angle through which a member turns under load, assuming that the truss is provided with frictionless joints. More clearly expressed, suppose the truss is one with frictionless joints; the load causes it to deflect, taking a member like 1-2 into a different position, namely, 1′-2′, the member remaining straight. Now, suppose that the gusset-plates at the ends, 1′ and 2′, start to act, introducing the bending moments, M_{12} and M_{21}, and causing the member to become curved. The assumption is that this bending takes place in such a manner, that the ends, 1 and 2, of the member are not moved. Mohr calls the angle, ψ_{12} and ψ_{21}, "the change in slope".

The angle, ϕ_1, included between the original direction, 1-2 or 1′-2″, of the member and its end tangent, 1′ T_1, is called by Mohr "the angle of rotation of the joint". This angle is the same for the ends of all members at

any joint, owing to the assumption that the ends are riveted rigidly together. In other words, the angle between any two members meeting at a joint is assumed to be constant during deformation of the truss.

To express the bending moments as functions of ϕ and ψ, start with the fundamental Equation (39),

$$M_{12} = \frac{2EI}{l} \cdot (2\tau_{12} + \tau_{21}).$$

From Fig. 16,

$$\tau_{12} = \phi_1 - \psi_{12},$$

and,

$$\tau_{21} = \phi_2 - \psi_{21} = \phi_2 - \psi_{12}.$$

Substituting these values in the fundamental equation,

$$\left.\begin{aligned} M_{12} &= \frac{2EI}{l} \cdot (2\phi_1 - 2\psi_{12} + \phi_2 - \psi_{12}) \\ \text{that is,} \quad M_{12} &= \frac{2EI}{l} \cdot (2\phi_1 + \phi_2 - 3\psi_{12}) \\ \text{and, similarly,} \quad M_{21} &= \frac{2EI}{l} \cdot (\phi_1 + 2\phi_2 - 3\psi_{12}) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (55$$

The quantities, ψ, are really the properties of the truss considered as having frictionless joints, and are easily obtained as the truss is statically determinate. It has been proved that there is only one ϕ for each joint; also, the sum of the moments around a joint is zero. Hence, if this fact is expressed for each joint, making use of Equations (55), there will result as many simultaneous linear equations involving ϕ's as there are joints, that is, as there are ϕ's, the ψ's having been determined previously. These equations are solved for the ϕ's. The secondary stresses then follow easily.

Calculation of the Changes in the Slope, ψ.—This calculation may be done analytically or graphically.

(1).—Analytical Method.—The values of ψ are obtained analytically from the equation, $M\psi = \Sigma s \cdot \Delta l$, or, by assuming $M = 1$,

$$1 \cdot \psi = \Sigma s \cdot \Delta l \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (56)$$

in which, $M = 1$ designates any moment acting on a truss, s the stress in a member due to the moment, and ψ the angle of rotation of the member under the loads that produce the change in length, Δl, of the member in question.

Equation (56) follows from the theorem that, if a system in equilibrium is acted on by a set of external forces, thereby causing the system to move into another position of equilibrium, the work done by the external forces is equal to that done by the internal stresses, these stresses having been called into existence by the externally applied forces.

To illustrate the analytical method of determining s, consider the truss shown in Fig. 17. Suppose the change in the slope of the member, 6-9, under the action of the loading, P, at Joint (11), is required. For this purpose, first assume that P does not exist, but that two forces, $\frac{1}{l}$, act at the ends

of the member, 6-9, as shown (Fig. 17), introducing a couple of moment $= +1$. This moment requires a downward left-hand reaction, $\frac{1}{L}$, and an upward right-hand reaction of the same magnitude.

Compute the stresses, s, set up in the members under these two pairs of forces, namely, $\frac{1}{l}$ and $\frac{1}{L}$; also compute all the Δl-values of the members caused by the given loading, P, at Joint (11). The products, $s \cdot \Delta l$, are then computed for each member and added, giving the value, $\Sigma\, s \cdot \Delta l$, which is then the angle of revolution or change in the slope, ψ, of the member, 6-9, due to the load, P, at Joint (11).

It must be evident that the calculation of all the slopes of all the members—a process of repeating the method given as many times as there are members—is extremely laborious. It is safe to say that engineers will always avoid the analytical method and make use of the graphical, when the results are obtained with greater ease and reasonable accuracy.

(2).—Graphical Method.—The graphical method of determining the ψ-values consists in the construction of a Williot displacement diagram for the truss under the given loading, that is, the truss with the imaginary frictionless joints.

For the sake of simplicity consider any triangle, ABC, Fig. 18, and imagine that some loading produces the stresses, $-s_1$, $+s_2$, and $+s_3$, in BC, CA, and AB, thus changing their lengths, l_1, l_2, and l_3, by the amounts, $-\Delta l_1$, $+\Delta l_2$, and $+\Delta l_3$.

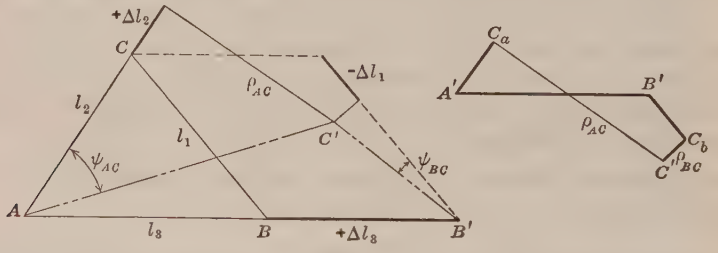


Fig. 18.

By regarding the joint, A, and the direction, A to B, as fixed, the displaced positions, B' and C', of B and C are found as shown. From Fig. 18, and also because the elastic deformations, Δl, of the members are small, so that the tangent of the arc may be substituted for the arc itself,

$$\rho_{AC} = l_2\, \psi_{AC}$$

or,

$$\psi_{AC} = \frac{\rho_{AC}}{l_2} \qquad (57)$$

The change in the slope, ψ_{AC}, of any member, AC, is, therefore, equal to the length, ρ_{AC}, of the perpendicular, $C_a\, C'$, to AC, which locates C', divided by the length, l_2, of AC. ψ is positive when the member turns in a clockwise direction.

If, instead of a single triangle, there is a truss composed of a series of triangles, the process of determining ψ for each member for any given loading will be the same, that is, by first constructing a displacement diagram (which gives the ρ-values), and by dividing the values of ρ of each member by its corresponding length, l.

The graphical method is more expeditious than the analytical method; besides, the construction of a displacement diagram is a process well known to engineers. For this reason the graphical method of computing ψ, which has been commonly used by previous computers, will be used here. In constructing a displacement diagram of a truss for any given loading, assume an arbitrary point, a, and an arbitrary direction, $a\,b$, as fixed. This generally gives different vertical displacements, ϑ_A and ϑ_B, for A and B, the points of supports. For the calculations of the vertical displacements of the panel points, it is necessary to introduce a correction (as suggested by Mohr), in the Williot diagram. The supports, A and B, Fig. 19, do not move vertically. In order to satisfy this condition in the displacement diagram, the truss must be rotated as a whole about A' or B' by an amount,

$$\psi'' = \frac{(\vartheta_B - \vartheta_A)}{L} \qquad (58)$$

thereby bringing $A'B'$ parallel to AB.

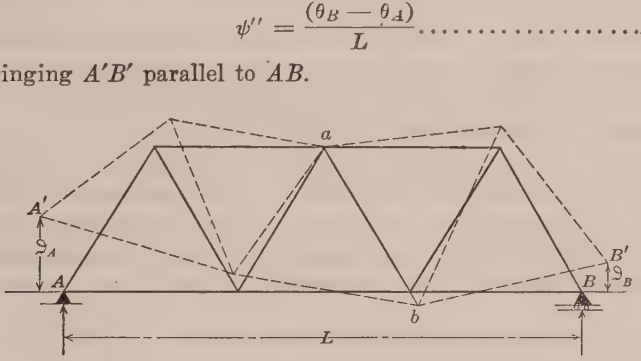


FIG. 19.

In the case of secondary stresses this is unnecessary. For the time being, however, suppose it is desirable to know the true angles, ψ and ϕ that is, the angles which the chord and the tangent line to a member at its one end, after loading, make with the direction of the member before loading. It will then be necessary to have a displacement diagram in which no relative vertical displacement between A and B has occurred. Therefore, the diagram as a whole must be rotated about either A or B by an amount, ψ'', as given in Equation (58). All values, ψ', obtained by means of the displacement diagram must be corrected by this amount, constant for a given loading, giving $\psi = \psi' + \psi''$, the true changes in the slopes with reference to the original direction of the members. The next step depends on whether it is desired to obtain only the secondary stresses, or also the correct values of ψ and ϕ for purposes of experimental verification.

It is the writer's opinion that it is better to omit this correction, to solve for the bending moments, and, if these satisfy the fundamental equations—that the sum of the bending moments about each joint is zero—then to add

the constant correction to all the ψ's and ϕ's in order to obtain the true quantities, this procedure only in the cases where experimental verification is to follow.

The reason why this correction is unnecessary is simple. The bending moment at one end of a member is (see Equation (55)):

$$M_{nm} = \frac{2\,E\,I}{l} \cdot (2\,\phi_n + \phi_m - 3\,\psi_{nm})$$

Deducting the constant quantity, ψ'', from both the ϕ's and the ψ, does not affect M_{nm}, because the correction is $2\psi'' + \psi'' - 3\,\psi''$. In other words, it is also true that:

$$M_{nm} = \frac{2\,E\,I}{l} \cdot (2\,\phi_{n'} + \phi_{m'} - 3\,\psi'_{nm}).$$

Hence, the use of the uncorrected values of ψ, namely, ψ', would give a set of simultaneous linear equations for the ϕ's, and these would have the property that they, too, differ from the true ϕ's by the constant amount, ψ''. As the secondary stresses depend on the M's, it is, therefore, justifiable to use the ψ''s and ϕ''s.

In this investigation it is intended to omit the prime marks, and designate the uncorrected changes of slope as ψ, and the uncorrected angles of rotation as ϕ, remembering that these, corrected by the constant quantity, ψ'', would give the values for investigation in the field.

Calculation of the ϕ's.—Assuming that the values, ψ, in Equation (55) have been found either analytically or graphically as indicated, the only real unknowns of the problem are, therefore, ϕ. The application of the fundamental equation, $\Sigma\,M = 0$, to each joint gives as many equations as there are joints, and unknowns, ϕ. Thus, for Joint (4), Fig. 17, by substituting the values of M as expressed in Equation (55) and putting $\frac{I}{l} = K$,

$$\Sigma\,M_{4\xi} = 2\,E\,(2\,\phi_4\,\Sigma\,K_{4\xi} + \Sigma\,\phi_\xi\,K_{4\xi} - 3\,\Sigma\,\psi_{4\xi}\,K_{4\xi}) = 0$$

or,

$$2\,\phi_4\,\Sigma\,K_{4\xi} + \Sigma\,\phi_\xi\,K_{4\xi} - 3\,\Sigma\,\psi_{4\xi} \cdot K_{4\xi} = 0 \quad \ldots\ldots\ldots\ldots (59)$$

in which the numbers of the adjacent joints—in this case 2, 3, 5, 7, 6—are to be substituted for the subscripts, ξ.

The solution of the n simultaneous linear equations of the form of Equation (59) gives the values of the n unknowns, ϕ, where n denotes the number of joints. It is important to note that these are again of the type known as Gauss' normal equations and may be solved according to his method.

Solution of the Stresses.—The next step consists in substituting the values, ϕ and ψ, in Equation (59), thus giving the values of the moments, M, acting at each end of every member. Before finding the secondary stresses, verify the values of the moments by applying the check, $\Sigma\,M = 0$, at each joint.

The secondary stress is then readily obtained from,

$$f = \frac{M\,y}{I} = 2\,E\,I\,(2\,\phi_a + \phi_n - 3\,\psi_{an})\,\frac{y}{l\,I}$$

or by putting $E = 1$,

$$f = \frac{2\,y}{l} \cdot (2\,\phi_a + \phi_n - 3\,\psi_{an}) \quad \ldots\ldots\ldots\ldots\ldots\ldots (60)$$

in which,

ϕ_a = the value of ϕ for the near end of the member;
ϕ_n = the value of ϕ for the far end of the member; and
ψ_{an} = the change in the slope of the member.

(Note that the ψ's from the deflection diagram are regarded as E multiplied by the true ψ's. The resulting ϕ's are, therefore, also the true ϕ's multiplied by E. Hence, although E is always regarded as equal to unity it will still give the true secondary stresses.)

6.—Mohr's Elastic Weight Method

Small Movements of a Chain of Elastic Weights.—In Fig. 20, 1, 2, 3, 4 ...., represent the joints of a plane chain of members. These members are caused to move in the plane while Joint (1) remains fixed.

Let the members, 1–2, 2–3, 3–4, ..., have the lengths, l_1, l_2, l_3, l_4, ..., before any stresses are present, and let these lengths become $l_1 + d\,l_1$, $l_2 + d\,l_2$, $l_3 + d\,l_3$, $l_4 + d\,l_4$, ..., under loading. Then, $\frac{d\,l_1}{l_1}$, $\frac{d\,l_2}{l_2}$, $\frac{d\,l_3}{l_3}$, ..., are the strains, positive if the members are increased in length. Let these strains be denoted by λ_1, λ_2, λ_3, ..., that is,

$$\lambda_1 = \frac{d\,l_1}{l_1};\ \lambda_2 = \frac{d\,l_2}{l_2};\ \lambda_3 = \frac{d\,l_3}{l_3};\ \text{etc.} \quad \ldots\ldots\ldots\ldots (61)$$

The initial positions of the joints are determined by their distances, (u_1, v_1), (u_2, v_2), (u_3, v_3), ..., from two axes at right angles to each other. These axes may be chosen anywhere, but must fulfill one condition, namely, if the u-axis is once chosen in direction, the v-axis must be situated in such a way, that a clockwise rotation through 90° of the u-axis will make it coincide with the v-axis (Fig. 20). The co-ordinate of a point with regard to an axis is considered positive if, looking along that axis in its positive

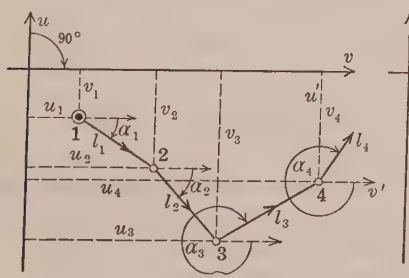


Fig. 20.

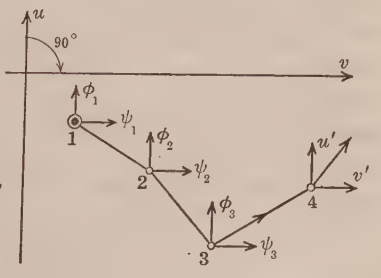


Fig. 21.

direction, the point lies on the right-hand side. Thus, all the co-ordinates in Fig. 20 are positive. The angles, α_1, α_2, α_3, ..., between the v-axis and the members, 1–2, 2–3, 3–4, ..., are the amounts of turning, in a clockwise direc-

tion, that would bring the v-axis into coincidence with the members, 1–2, 2–3, 3–4, ...

When $l_1, l_2, l_3, \ldots$, are changed by the small amounts, $d\,l_1, d\,l_2, d\,l_3, \ldots$, the angles, $\alpha_1, \alpha_2, \alpha_3, \ldots$, are changed by the corresponding small amounts, $d\,\alpha_1, d\,\alpha_2, d\,\alpha_3, \ldots$ Let,

$$\phi_1 = d\,\alpha_1,\ \phi_2 = d\,\alpha_2 - d\,\alpha_1,\ \phi_3 = d\,\alpha_3 - d\alpha_2 \ldots\ldots\ldots\ldots (62)$$

and, therefore,

$$d\,\alpha_1 = \phi_1,\ d\,\alpha_2 = \phi_1 + \phi_2,\ d\,\alpha_3 = \phi_1 + \phi_2 + \phi_3 \ldots\ldots\ldots (63)$$

The co-ordinates (u, v), of a joint, for example (u_4, v_4) of Joint (4), are expressed by,

$$\left.\begin{aligned} u_4 &= u_1 + l_1 \cos \alpha_1 + l_2 \cos \alpha_2 + l_3 \cos \alpha_3 = u_1 + \sum_1^3 l \cos \alpha \\ v_4 &= v_1 + l_1 \sin \alpha_1 + l_2 \sin \alpha_2 + l_3 \sin \alpha_3 = v_1 + \sum_1^3 l \sin \alpha \end{aligned}\right\} \ldots (64)$$

The small changes in the co-ordinates of Joint (4), due to the combined changes in the lengths of the members and the angles which the members make with the v-axis, are found by differentiating Equations (64). Keeping in mind that u_1 and v_1 are constant quantities,

$$\left.\begin{aligned} d\,u_4 &= \sum_1^3 d\,l\,.\cos \alpha - \sum_1^3 l \sin \alpha\,.\,d\,\alpha \\ d\,v_4 &= \sum_1^3 d\,l\,.\sin \alpha + \sum_1^3 l \cos \alpha\,.\,d\,\alpha \end{aligned}\right\} \ldots\ldots\ldots (65)$$

As $d\,l_1 = l_1\,\lambda_1$, from Equations (61),

$$\cos \alpha_1 = (u_2 - u_1)\,.\,l_1,$$

from Fig. 20, and,

$$\sin \alpha_2 = (v_2 - v_1)\,.\,l_1$$

the summations in Equations (65) become,

$$\left.\begin{aligned} \sum_1^3 d\,l\,.\cos \alpha &= (u_2 - u_1)\,\lambda_1 + (u_3 - u_2)\,\lambda_2 + (u_4 - u_3)\,\lambda_3 \\ -\sum_1^3 l \sin \alpha\,.\,d\,\alpha &= -(v_2 - v_1)\,d\,\alpha_1 - (v_3 - v_2)\,d\,\alpha_2 - (v_4 - v_3)\,d\,\alpha_3 \\ \sum_1^3 d\,l\,.\sin \alpha &= (v_2 - v_1)\,\lambda_1 + (v_3 - v_2)\,\lambda_2 + (v_4 - v_3)\,\lambda_3 \\ \sum_1^3 l\,.\cos \alpha\,.\,d\,\alpha &= (u_2 - u_1)\,d\,\alpha_1 + (u_3 - u_2)\,d\,\alpha_2 + (u_4 - u_3)\,d\,\alpha \end{aligned}\right\} (66)$$

Letting,

$$\psi_1 = \lambda_1, \psi_2 = \lambda_2 - \lambda_1,\ \psi_3 = \lambda_3 - \lambda_2$$

or,

$$\lambda_1 = \psi_1,\ \lambda_2 = \psi_1 + \psi_2,\ \lambda_3 = \psi_1 + \psi_2 + \psi_3 \ldots\ldots\ldots\ldots (67)$$

and using the relations expressed by Equations (62), and (63), Equations (66) take the following neat forms:

$$\left.\begin{aligned}
&\sum_1^3 d\,l\,.\,\cos\alpha = (u_4 - u_1)\,\psi_1 + (u_4 - u_2)\,\psi_2 + (u_4 - u_3)\,\psi_3 = +u_{4\psi}, \text{ say}\\
-&\sum_1^3 l \sin\alpha\,.\,d\,\alpha = -(v_4 - v_1)\,\phi_1 - (v_4 - v_2)\,\phi_2 - (v_4 - v_3)\,\phi_3 = -v_{4\phi}, \text{ say}\\
&\sum_1^3 d\,l\,.\,\sin\alpha = (v_4 - v_1)\,\psi_1 + (v_4 - v_2)\,\psi_2 + (v_4 - v_3)\,\psi_3 = +v_{4\psi}, \text{ say}\\
&\sum_1^3 l\cos\alpha\,.\,d\,\alpha = (u_4 - u_1)\,\phi_1 + (u_4 - u_2)\,\phi_2 + (u_4 - u_3)\,\phi_3 = +u_{4\phi}, \text{ say}
\end{aligned}\right\} \quad (68)$$

Therefore, the changes in the co-ordinates of Joint (4), Equations (65), have the values:

$$\left.\begin{aligned} d\,u_4 &= u_{4\psi} - v_{4\phi}\\ d\,v_4 &= v_{4\psi} + u_{4\phi}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (69)$$

Now, imagine Joint (1), Fig. (21), to be loaded with a positive weight, ϕ_1, parallel to the u-axis, and a positive weight, ψ_1, parallel to the v-axis; and Joint (2) to be loaded with a positive weight, ϕ_2, parallel to the u-axis, and a positive weight, ψ_2, parallel to the v-axis, etc. It will be seen that $u_{4\phi}$ of Equations (69) is the negative value of the sum of the moments of ϕ_1, ϕ_2, ϕ_3, about the axis, u', through Joint (4) parallel to the u-axis, while $v_{4\psi}$ is the negative of the sum of the moments of ψ_1, ψ_2, ψ_3, about the axis, v', through Joint (4) parallel to the v-axis. Further, if all the axes are turned in a clockwise direction through 90°, the u-axis and the u'-axis take the places and sense of the v-axis and the v'-axis, respectively, while the v-axis and the v'-axis take the places of the u-axis and the u'-axis, but are opposite to them in sense. Therefore, $u_{4\psi}$ of Equations (69) may be regarded as the negative of the sum of the moments of the ψ's about the new u'-axis, while $v_{4\phi}$ is the sum of the moments of the ϕ's, about the new v'-axis. The negative sign of $v_{4\phi}$ in the equation means that $-v_{4\phi}$ is the negative of the sum of the moments of the ϕ's, about the new v'-axis. Remembering that a positive weight on the right-hand side of an axis, and a negative weight on the left-hand side of an axis give positive moments, the argument becomes clear.

Therefore, considering two sets of positive weights, (ϕ_1 ϕ_2 ϕ_3) and (ψ_1 ψ_2 ψ_3), perpendicular to each other and acting at the Joints (1), (2), and (3), as shown in Fig. 21, the negative of $d\,v_4$ in Equations (69) may be obtained by taking the sum of the moments of the ϕ's about an axis, u', parallel to the u-axis, and those of the ψ's about an axis, v', parallel to the v-axis. If both axes are turned 90° in a clockwise direction, the negative of $d\,u_4$ in Equations (69) is the sum of the moments of the ϕ's, about the new v'-axis, and those of the ψ's, about the new u'-axis.

It is important to note that the argument is unaltered if, instead of Joint (4), the origin is any other point rigidly connected with the member, 3-4.

Small Movements of a Closed Chain.—For a closed chain, 1, 2, 3,..... n, 1 (Fig. 22), where Joint (1) is fixed in position and where the co-ordinates are as before, proceeding round the chain in the order 1, 2, 3, 4,.....n, 1, Equations (69) become:

$$\left.\begin{aligned} d\,u_n &= u_{n\psi} - v_{n\phi}\\ d\,v_n &= v_{n\psi} + u_{n\phi}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (70)$$

for Joint (n), and the axes, u', v', now pass through Joint (n). It is possible, however, to go further and reach Joint (1). Considering Joint (1), not as the start but as the end of the route, Equations (69) become:

$$\left.\begin{aligned} d\,u_1 &= u_{1\psi} - v_{1\phi} \\ d\,v_1 &= v_{1\psi} + u_{1\phi} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (71)$$

and the axes, u', v', now pass through Joint (1). As Joint (1), however, is fixed, $d\,u_1 = d\,v_1 = 0$ and Equations (71) are really.

$$\left.\begin{aligned} u_{1\psi} - v_{1\phi} &= 0 \\ v_{1\psi} + u_{1\phi} &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (72)$$

If Joint (n) had been the terminus, it would have been necessary to consider all the ψ's, from ψ_1 to ψ_{n-1}, and all the ϕ's, from ϕ_1 to ϕ_{n-1}, in Equations (70). Proceeding to Joint (1) and obtaining Equations (72) adds ψ_n to the list of ψ's, and ϕ_n to the list of ϕ's. In other words, Equations (72) are true

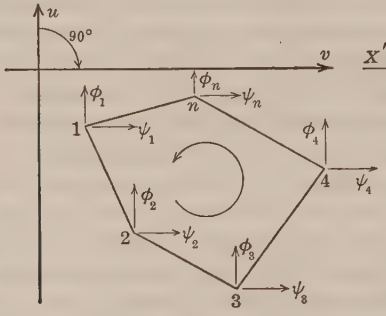


Fig. 22.

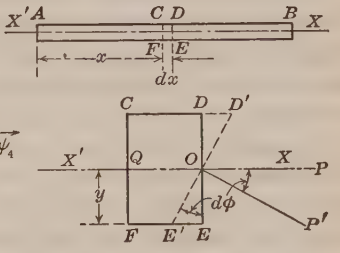


Fig. 23.

for all the weights, ψ and ϕ, of all the joints in the chain. Turning to Equations (62) and (67), it is evident that until now the ϕ's and ψ's have been taken as follows:

$$\left.\begin{aligned} \phi_1 &= d\,\alpha_1 && \text{and} && \psi_1 = \lambda_1 && \text{for Joint (1)} \\ \phi_2 &= d\,\alpha_2 - d\,\alpha_1 && \text{and} && \psi_2 = \lambda_2 - \lambda_1 && \text{for Joint (2)} \\ \phi_3 &= d\,\alpha_3 - d\,\alpha_2 && \text{and} && \psi_3 = \lambda_3 - \lambda_2 && \text{for Joint (3)} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \phi_n &= d\,\alpha_n - d\,\alpha_{n-1} && \text{and} && \psi_n = \lambda_n - \lambda_{n-1} && \text{for Joint } (n) \end{aligned}\right\} \quad \ldots\ldots (73)$$

The moments of all the weights about the axes, u', v', through Joint (1) are under consideration; the moments of ϕ_1 and ψ_1, which act at Joint (1), are, therefore, zero. In other words, Equations (72) are not affected if either the axes, ϕ_1, ψ_1, are neglected or some other values substituted for ϕ_1 and ψ_1. A study of Equations (73) would suggest a completion of the cyclic order, by making,

$$\phi_1 = d\,\alpha_1 - d\,\alpha_n \text{ and } \psi_1 = \lambda_1 - \lambda_n$$

This gives two new relations, for on adding all the ϕ's,

$$\Sigma\,\phi = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (74)$$

and adding all the ψ's,

$$\Sigma\,\psi = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (75)$$

Now, the set of ψ's is a horizontal one, while the set of ϕ's is a vertical one, therefore, each set resolves itself into a couple and as the moment of a couple is constant, no matter where the axis is taken, the origin may be chosen anywhere. The direction of the u-axis is arbitrary, but, once chosen, it controls the position of the v-axis as that into which the u-axis falls when it is turned 90° clockwise. In that case, Equations (72) hold, as well as Equations (74) and (75). These are four linear relations that exist between the $2n$-quantities, ϕ and ψ. Hence, if $(2n\text{-}4)$ of the $2n$-quantities, ϕ and ψ, of a closed chain are known, the remaining four may be found.

A Beam Subjected to Bending.—Consider a beam, AB, Fig. 23, and let it be subjected to bending. Consider a small length, dx, of the beam, distant x from A, magnified as shown. Before bending, its shape is $C\,D\,E\,F$; after bending, it becomes $C\,D'\,E'\,F$, $C\,D$ suffering an elongation, $D\,D'$, and $E\,F$ a contraction, $E\,E'$, while the neutral axis remains unchanged. Imagine a perpendicular, $O\,P$, rigidly attached to the face, $D\,E$; that perpendicular will be carried to the position, $O\,P'$, when the beam is bent. The change in angle between $O\,P$ and $O\,P'$, $d\,\phi$, is what has been regarded as a weight in the foregoing discussion. If $d\,x$ becomes infinitesimally small, $Q\,O$ may be regarded as one of an infinite number of links, while the $d\,\phi$'s at their right-hand ends, O, become what are known as "elastic weights", and, as is well known,

$$d\,\phi = \frac{M_x \,.\, d\,x}{E\,I} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (76)$$

in which, M is the bending moment at the section, x.

A Beam Subjected to a Bending Moment at Each End.—Consider a beam, 1-2, Fig. 24. Let a counterclockwise (positive) moment, M_{12}, act at End (1); similarly, let a positive moment, M_{21}, act at End (2). The shape into which

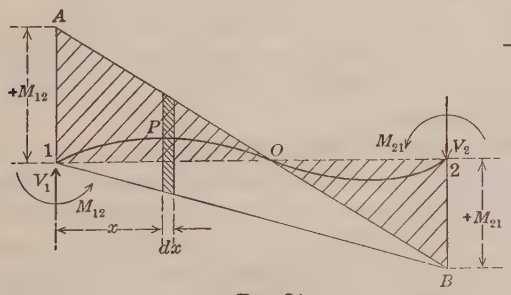


Fig. 24.

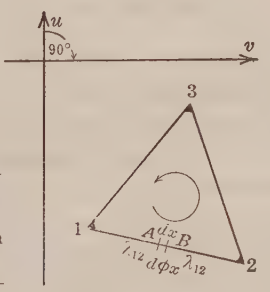


Fig. 25.

the beam is bent is somewhat as shown in the diagram. These moments induce a shear, V_1, upward at End (1), and a shear, V_2, downward at End (2). Taking moments about any point, P, distant x from End (1),

$$M_x = M_{12} - V_1 x$$

This proves that the variation of the moment is a linear one. Somewhere, however, between End (1) and End (2), there is a point of inflection, marked O, where the bending moment is zero. Hence, the bending moment diagram is as shown (shaded). Clearly, the bending moment area, $A\ O\ 1$, is positive,

while $B\ O\ 2$ is negative. If the portion, $1\ O\ B$, were added to $A\ O\ 1$, to make a positive area, $A\ B\ 1$, and also the same portion, $1\ O\ B$, to $B\ O\ 2$, to make a negative area, $1\ 2\ B$, these two newly found areas would have the same effect as the two original areas.

From Equation (76), however,

$$d\ \phi = \frac{M_x\ dx}{E\ I}$$

which represents the small bending moment area at x of width, $d\ x$. E is constant and the member is assumed to have a constant moment of inertia, I. Now, regard the positive bending moment area, $AB1$, as being replaced by all its small elastic weights, $d\ \phi$; these weights can themselves be replaced by their sum, acting at their centers of gravity, before treating them in any moment problem; but,

$$\phi_{12} = \int d\ \phi \text{ from End (1) to End (2)},$$

$$= \int_o^{l_{12}} \frac{M_x\ dx}{E\ I}$$

$$= + \frac{M_{12}\ l_{12}}{2\ E\ I}.$$

That is, it is permissible to replace all the elastic weights, $d\ \phi$, belonging to the positive area, $AB1$, by one elastic weight the value of which is ϕ_{12}, acting at the first "third" point from End (1) to End (2).

Similarly, all the elastic weights belonging to the negative area, $12B$, may be replaced by one elastic weight, $\phi_{21} = -\frac{M_{21}\ l_{12}}{2\ E\ I}$, acting at the second "third" point from End (1) to End (2).

Application of Elastic Weights to a Triangular Frame with Riveted Joints.—In the triangle, 1 2 3 (Fig. 25), the ends are riveted together so that the angle between the elastic lines of adjacent members at a joint, is constant under all conditions of loading.

Let the triangle be subjected to some loading, causing unit stresses, λ_{12}, in the member, 1-2, λ_{23} in the member, 2-3, and λ_{31} in the member, 3-1. Because of the riveted joints, some bending stresses will also be introduced throughout the frame. Consider End (1) fixed and u, v to be a pair of rectangular axes through any point in the plane of the frame; these axes are subject, however, to the condition that u must turn through 90° in a clockwise direction to coincide with v. It is assumed that anywhere in a member the direction of the displaced direct stress, due to the bending effect, deviates so little from that of the line joining the ends of the member, that the direction of the direct stress coincides with that line.

The triangle now becomes a closed chain, composed of an infinite number of links, dx, the joints, (1), (2), and (3), of the triangle serving as joints between adjacent links of two adjacent members. The riveted connections make this continuity possible. Following round the chain in a counterclockwise direction, take any one of the links of the member, 1-2, except the first,

such as AB, in Fig. 25. Joint (A) has a unit stress, λ_{12}, likewise Joint (B). Thus, in Equations (73), as applied to a closed chain, $\psi_B = 0$, while ϕ_B is the small elastic weight, $\frac{M_B\, dx}{E\, I}$, in which, M_B is the bending moment in the member at B, etc. If all the links but the first are taken, all the ψ's are zero while all the ϕ's are present.

In connection with the first link of the member, 2-3, the λ first met is the λ_{12}, while the second λ is λ_{23}. Therefore, at a point on the member, 2-3, adjacent to Joint (2), the ψ is $\lambda_{23} - \lambda_{12}$, and as the link is infinitesimally small, $\lambda_{23} - \lambda_{12}$ may be regarded as acting at Joint (2). Here, however, the ϕ is zero, as the end tangents to the elastic lines at Joint (2) are assumed to retain their relative positions. In considering the remaining links of the member, 2-3, all the ψ's are again zero, while the ϕ's are the elastic weights due to the bending of the member, 2-3.

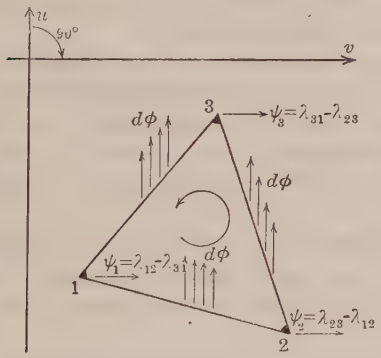


FIG. 26.

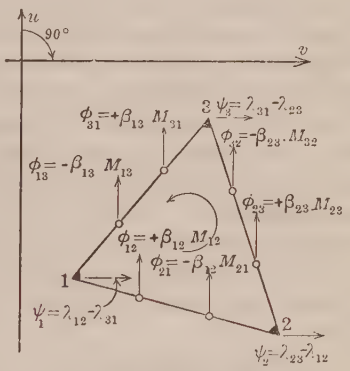


FIG. 27.

Arguments similar to the preceding, hold for Joints (3) and (1) and for the member, 1-3. Therefore (Fig. 26), it is required to account for only three ψ's, namely,

$$\left.\begin{aligned} \psi_1 &= \lambda_{12} - \lambda_{31} \text{ at Joint (1)} \\ \psi_2 &= \lambda_{23} - \lambda_{12} \text{ " " (2)} \\ \psi_3 &= \lambda_{31} - \lambda_{23} \text{ " " (3)} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (77)$$

while there is an infinite number of $d\,\phi$'s along the members, 1-2, 2-3, and 3-1. It was proved in the preceding discussion that the infinite number of elastic weights, $d\,\phi$, of the member, 1-2, can be replaced by one elastic weight $= + \frac{M_{12}\, l_{12}}{2\, E\, I}$, acting at the first "third" point from the member, 1-2, and one elastic weight $= - \frac{M_{21}\, l_{12}}{2\, E\, I_{12}}$, acting at the second "third" point from the member, 1-2, in which $+ M_{12}$ is the bending moment at End (1), and $+ M_{21}$ is the bending moment at End (2), of the member, 1-2.

A similar argument holds for the members, 2-3 and 3-1. It is important to remember, that, as in the analysis of a closed chain, it is necessary to go

round the circuit in one direction. In Fig. 26, this direction is counterclockwise, hence, the first elastic weight met along the member, 1-2, is $+\frac{M_{12}\, l_{12}}{2\, E\, I_{12}}$, while the second is $-\frac{M_{21}\, l_{12}}{2\, E\, I_{12}}$, etc.

For convenience, let,

$$\beta = \frac{l}{2\, E\, I} \qquad (78)$$

The problem finally becomes a question of establishing enough relations between the ψ's and ϕ's, shown in Fig. 27. It is assumed that the direct stresses in the riveted frame are the same as if it were pin-connected; the λ's, and, therefore, the ψ's, are assumed to be known. There are six unknown bending moments, M, contained in the ϕ's. Assuming three of these bending moments to be known, moments of all the weights about three sets of axes, their origins not being collinear, will give three equations which will determine the remaining three unknown bending moments. The axes, of course, are chosen in such a way as to obtain the simplest relations.

Application of Elastic Weights to the Solution of the Moments in a Riveted Truss.—In the case of a riveted truss, made up of triangular elements and statically determinate, the joints being frictionless hinges, there are n joints, $(n\text{-}2)$ triangles, and $(2n\text{-}3)$ members. The number of bending moments to be determined is, therefore, $2\ (2n\text{-}3) = (4n\text{-}6)$, one of each end of each member.

The sum of the bending moments found at a joint is zero. As there are n joints, this gives n relations between the $(4n\text{-}6)$ bending moments. Therefore, there are needed $(4n\text{-}6) - n = 3\ (n\text{-}2)$ additional relations to solve the problem.

One assumes, as in the previous methods, that the direct or primary stresses in the members of the riveted truss are also those of a truss with frictionless hinges. Under any load, these stresses, therefore, are easily found, and, together with the dimensions of the truss, give the λ's and, therefore, the ψ's of Equations (77). The dimensions also determine the β's of Equation (78).

Each triangle is now considered by itself. The ϕ's belonging to the members of that triangle are placed at the third points, as in Fig. 27. The ψ's that arise from the stresses in the three members of the triangle, are placed at the vertices. It should be noted that the ϕ's of a side remain the same, in magnitude and sign, for the two triangles to which that side is common. Hence, after following round the first triangle in a clockwise direction, it is necessary, in order to preserve the signs of the ϕ's, to go round the next triangle in a counterclockwise direction, etc. The ψ's and ϕ's are regarded as weights. As each triangle is a closed chain, the sum of the bending moments of all the ϕ's about one axis, and the bending moments of all the ψ's about a perpendicular axis, is zero, the condition for the two axes being that the ϕ-axis must turn through 90° in a clockwise direction in order to coincide with the ψ-axis. Three such sets of axes are chosen for each triangle, to give the three relations required between the ϕ's and ψ's of each triangle. The three origins must not be collinear.

The method of procedure is first to assume the end moments of one of the two members making the first joint to be known. However, the sum of the bending moments at a joint is zero, therefore, that bending moment at the joint of the remaining member is also known. The remaining three bending moments of the first triangle are then solved by choosing three sets of axes. This being done in a definite way, as will be shown in the example, each additional equation will contain only one new unknown bending moment, all the others having already been expressed in the earlier equations. It must be borne in mind that as soon as the stage is reached where all but one of the bending moments around a joint are expressed in the equations already found, the relation, $\Sigma M = 0$, for the joint is introduced. It will be found that the $\Sigma M = 0$'s for the last two joints introduce no fresh bending moments; their use is found later. Call this whole set of equations, $(2n\text{-}6)$ in number, Set No. 1.

The next step is to express every M of Set No. 1 in terms of the M's found previously. Call this new set, Set No. 2.

It is then a matter of substitution to change Set No. 2 into Set No. 3, where every M is expressed in terms of the two M's assumed to be known at the start. Here, the last two equations of Set No. 1 are of value, for they are found to be two simultaneous linear equations with the two assumed M's which, therefore, are found. Substituting in Set No. 3 gives the values of all the M's from which to determine the secondary stresses. The method is easy to follow, as will be shown in an example.

7.—Mao's Methods

Fundamental Principles

The Fundamental Conception.—Quoting from Dr. Mao's thesis (28): Secondary stresses are fiber stresses, produced in the members, due to bending moments developed around the joints. There are many sources from which the moments are derived, but the principal one is the rigidity of connections. If $M =$ the bending moment, $I =$ the moment of inertia of the member, $y =$ the distance from neutral axis to the fiber, the stress of which is required, and $f =$ the secondary stress, then,

$$f = \frac{M\,y}{I} = \frac{M}{S}$$

in which, S is the section modulus. Therefore,

$$M = S \,.\, f \qquad (79)$$

In other words, if S is assumed as a force, and M is the moment it produces about a point, then the offset of the force from the point gives the value of the secondary stress.

Let AB be a member, connecting the Joints (A) and (B), and S its section modulus (Fig. 28). If S is assumed to be a force acting in this member, like the primary stress, it will produce moments at Joints (A) and (B) if its line of action is displaced from the axis of AB, and the offsets, f_a and f_b, will then be the stresses produced at Joints (A) and (B) (Equation (79)).

Particularly, if the moments about Joints (A) and (B) are those due to rigidity of joints, the offsets, f_a and f_b, will be the secondary stresses. The line of action of S will be called hereafter the "secondary stress line" or briefly "stress line", no confusion being caused thereby, as the primary stress line is not shown or is understood to be the axis of the member.

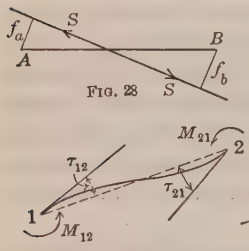


FIG. 28

FIG. 30.

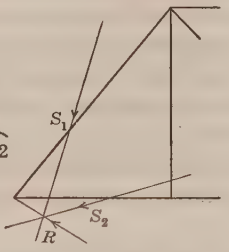


FIG. 29.

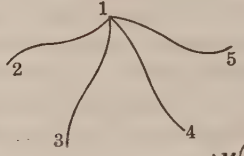


FIG. 31.

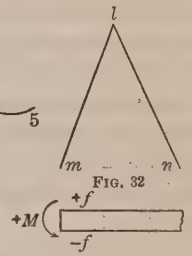


FIG. 32

FIG. 33.

It is now evident that the values of f depend only on the location of the stress line, S, while the moment, M, will also depend on its magnitude. As the magnitude of S is constant, being the section modulus, the position and direction of S, and, consequently, f, will depend solely on M.

Let this conception be extended to every member of the truss. There will then be as many stress lines as there are members. Each stress line will be displaced to an extent proportionate to the bending moments produced. To utilize this fact, by reversing the process, it is at once evident that, if the stress lines are located so as to satisfy various imposed conditions, the offsets of the stress lines will give directly the secondary stresses. This conception is fundamental.

The first condition, to which the various stress lines are subjected, is that the total resulting moments around every joint must be zero. (Eccentric moments are excluded here, but could be taken care of very readily.) Graphically, this means that, if a force diagram is drawn of all the stress lines in the truss, and an equilibrium polygon is likewise constructed on the truss diagram, these two polygons must, respectively, close. This, however, is impossible for the stress lines adopted previously, because, the values of S being constant, they could not be made to balance each other. Thus, in Fig. 29, the stress lines, S_1 and S_2, could not balance each other, unless they are on the same straight line, which is impossible. To overcome this difficulty, an external ideal force will be applied to the joint, with such magnitude and direction that the equilibrium is maintained around the joint. Thus, in Fig. 29, a force, R, may be introduced which will balance the forces, S_1 and S_2. In general, the magnitude and direction of R is equal to the resultant of the stress lines acting on the joint, but with opposite sense. It is important to notice that the position of R must be such that its line of action passes through the joint under consideration, so that there will be no external moment. In case of eccentric connections, the position of R may be adjusted so that its value and the offset from the joint will give the eccentric moment.

As a convenience in terminology, the term, "stress line", will be understood to be the internal secondary stress line, S, while the external force will be understood as the balancing force, R.

With the conceptions of S and R thus established, it is now possible to draw a force diagram around every joint from which to construct an equilibrium polygon. If these two polygons are made to close in any given case, the first condition, that the sum of internal moments must be zero around any joint, is everywhere satisfied. Further, from the condition that the total external moment is zero for all the joints of the truss, it may be inferred that both the force and equilibrium polygons of the external ideal forces, R, must, respectively, close.

The Fundamental Equation.—Dr. Mao's method for establishing this relation is unnecessarily complicated; an easier way, therefore, will be adopted as follows:

It has been shown (Fig. 5 and Equations (37) and (38)) that, if a member, 1-2, Fig. 30, is subjected to two positive bending moments (counterclockwise being the positive sense), M_{12} and M_{21}, at its ends, and if τ_{12} and τ_{21} are the angles between the end tangents and the chord at Member 1 and Member 2, respectively, then,

$$\tau_{12} = \frac{l_{12}}{6\,E\,I_{12}}\,(2\,M_{12} - M_{21})$$

and,

$$\tau_{21} = \frac{l_{12}}{2\,E\,I_{12}}\,(2\,M_{21} - M_{12})$$

Now, imagine a number of members, 1-2, 1-3, 1-4, 1-5, entering Joint (1), (Fig. 31),

$$\tau_{12} = \frac{L_{12}}{6\,E\,I_{12}}\,(2\,M_{12} - M_{21})$$

$$\tau_{13} = \frac{L_{13}}{6\,E\,I_{13}}\,(2\,M_{13} - M_{31})$$

Therefore,

$$\tau_{12} - \tau_{13} = \frac{L_{12}}{6\,E\,I_{12}}\,(2\,M_{12} - M_{21}) - \frac{L_{13}}{6\,E\,I_{13}}\,(2\,M_{13} - M_{31})$$

However, $\tau_{12} - \tau_{13} = \Delta\,\alpha_{213}$, this being the change in the angle, 213, under the loading, considering the joints as frictionless hinges. (See Fig. (6) and Equation (27)). Therefore,

$$L_{12}\left(\frac{2\,M_{12}}{I_{12}} - \frac{M_{21}}{I_{21}}\right) - L_{13}\left(\frac{2\,M_{13}}{I_{13}} - \frac{M_{31}}{I_{31}}\right) = 6\,E\,\Delta\,\alpha_{213}$$

or, by using Equation (79),

$$\frac{L_{12}}{y_{12}}\,(2\,f_{12} - f_{21}) - \frac{L_{13}}{y_{13}}\,(2\,f_{13} - f_{31}) = 6\,E\,.\,\Delta\,\alpha_{213}$$

In general, if two members, l-m and l-n, meet in a joint, Fig. 32, and if the order of consideration is in a counterclockwise direction,

$$\frac{L_{lm}}{y_{lm}}\,(2\,f_{lm} - f_{ml}) - \frac{L_{ln}}{y_{ln}}\,(2\,f_{ln} - f_{nl}) = 6\,E\,.\,\Delta\,\alpha_{mln} \quad \ldots\ldots\ldots (80)$$

in which,

L = length of the member;
y = extreme fiber distance;
f_{lm} = secondary stress of the member, l-m, at the end, l; and
$\Delta \alpha_{nlm}$ = change of angle between the members, l-n and l-m.

This is Dr. Mao's "Fundamental Equation" which, he says, forms the basis for his "Theory of Deformation Contour" (see page 48).

Analytical Equations

In taking the bending moment as positive when it acts in a counterclockwise direction, it is seen that f, the stress of the fiber first met, is negative while that of the one passed over last in this counterclockwise route, is positive (Fig. 33).

On substituting U for $\frac{L}{y}$ and K for $6\,E\,.\,\Delta\,\alpha$, Equation (80) is changed to,

$$U_{lm}\,(2\,f_{lm} - f_{ml}) - U_{ln}\,(2\,f_{ln} - f_{nl}) = K_{mln}$$

or,

$$2\,f_{lm} - f_{ml} = \frac{U_{ln}}{U_{lm}}\,(2\,f_{ln} - f_{nl}) + \frac{K_{mln}}{U_{lm}} \quad \ldots\ldots\ldots\ldots (81)$$

Now, let,

$$r_{lm} = 2\,f_{lm} - f_{ml}$$

and,

$$r_{ml} = 2\,f_{ml} - f_{lm} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (82)$$

Equation (81) can then be written:

$$r_{lm} = \frac{U_{ln}}{U_{lm}} \cdot r_{ln} + \frac{K_{mln}}{U_{lm}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (83)$$

From Equation (82),

$$r_{lm} = 3\,f_{ml} - 2\,r_{ml} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (84)$$

and, also,

$$f_{ml} = \frac{1}{3}(2\,r_{ml} + r_{lm}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (85)$$

This shows that the value of r_{lm} may be obtained from the values of f and r at the other end of the member, without knowing the value of f_{lm}.

Graphical Constructions

Preliminary Considerations.—Quoting again from Dr. Mao's thesis:

Scales.—For graphical constructions, the scale of f is evidently dependent on that of $E\,\Delta\,\alpha$. From the fundamental Equation (81), it is seen that the scale of f is of the same dimensional degree as that of $\frac{K_{mln}}{U_{lm}}$, or of $\left(\frac{6\,E\,\Delta\,\alpha}{\frac{L}{y}}\right)_{lm}$.

As L is always expressed in feet and y in inches, L and y may be represented by the same unit, if $6\,E\,\Delta\,\alpha$ is changed into $\frac{E\,\Delta\,\alpha}{2}$. Hence, hereafter, K is to

be understood as $\frac{E \Delta \alpha}{2}$, with the understanding that y is expressed in inches, but represented as if it were in feet. As the values of $E \Delta \alpha$ are found from the primary stresses, by graphic or analytic methods, it is convenient to consider only one-half the primary stresses. In graphic methods, the scale of primary stresses may be made only one-half as large as that used for measuring the values of $E \Delta \alpha$. This will give the value of K directly.

The scale of S, the section modulus, used in constructing the force diagrams, may be any value independent of f, as it is only the direction that is required. The scale used for r must be the same as that for K or f.

Signs.—In the present work there are two instances where a conception of signs is necessary. One is rotational and the other is linear. The first applies to moments, angles, circular sections, and directions of contours, while the second applies to the stress lines, their offsets, representations of f and r, and the co-ordinates of variables. The rotational direction will always be considered positive if it is counterclockwise. For linear directions there are many considerations, but the principle is, with the exception of the stress lines, that if ab is the base of a straight line through a and b, with a as origin, the distance from a to any point, x, on ab, is positive, if x is laid off from a toward b, and negative, if away from b. That is, it is positive if it is between a and b, and negative if it is on the side of a away from b.

For stress lines the following convention is necessary: It has been shown that a positive moment is that, which will rotate a member in a counterclockwise direction; as the moment is represented by the product of f and S, the direction of S should be such that a counterclockwise moment would give a positive stress, f. For the sake of uniformity, all the stress lines, S, will be considered as compressive stresses in the members. Under this convention, if the stress lines in the members, 1-2 and 1-3, are as shown, Fig. 34, f_{12} and f_{21} will be negative (moment is clockwise about Joint (1) and also about Joint (2)), and f_{13} and f_{31} will be positive (moments about Joints (1) and (3) are counterclockwise). Hence, when the position of the stress line

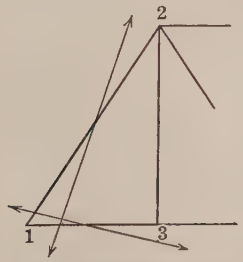


FIG. 34.

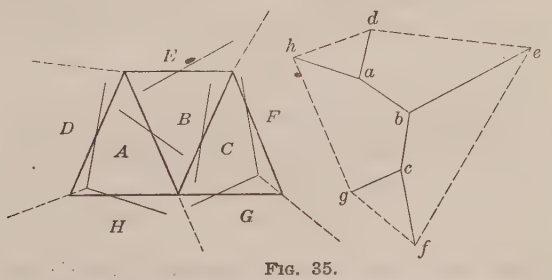


FIG. 35.

is known, both the sign and the magnitude of f are at once obtained. Further, by this convention, the force diagram of stress lines, S, assumes a definite form which tends to facilitate its construction. Consider the simple truss, as shown in Fig. 35. The letters, a, b, and c, in the force diagram correspond to the spaces, A, B, and C, of the truss diagram. It will be seen that the forces da, ab, bc, and cf, may be considered, for illustration, as the trunk of

a tree, while the forces, *ah*, *bc*, and *cg*, are its branches. This makes it easy to remember that all the stress lines in web members and end posts form the trunk of a tree, while those of the chord members form the spreading branches.

Truss Diagrams.—Two or three truss diagrams are necessary in the graphic methods presented in this paper. One of the diagrams will be used to record given information and to construct the values of K. The use of this diagram will be described, while the construction on the remaining diagrams will be considered subsequently.

The quantities, S, and primary stresses, in pounds per square inch, are first marked on all the members of the truss. The "y-circles" are then constructed. These are the circles, with radii equal to the y's of the members, drawn at one end of each member. Let L be the length of the member, in feet, and y, in inches (Fig. 36). At one end of the member draw a circle with radius equal to y, expressed in inches. The scale used is the same as that for L, which is in feet, the value of y being thus magnified twelve times. From the other end of the member draw a line tangent to this circle. This line will be known as the "U-line." To find the value of $\frac{K}{U} = \frac{\frac{K}{L}}{y}$, lay off a segment equal to K from the end of the member, from which the U-line is drawn. At the end of the segment, K, draw a circle, tangent to the U-line. The radius of this circle then gives the value required.

Changes of Angle.—The method of Ritter (page 9) is followed. The purely graphical construction of Mao is dangerous, because of possible mistakes in signs. All the triangles of the truss to be considered have a right angle; let P_{13}, P_{23}, and P_{12} (Fig. 37) be the stresses in the sides, 1-3, 2-3, and 1-2, respectively. These are drawn, to a certain scale, from the opposite vertices perpendicularly toward the sides, 1-3, 2-3, and 1-2, respectively.

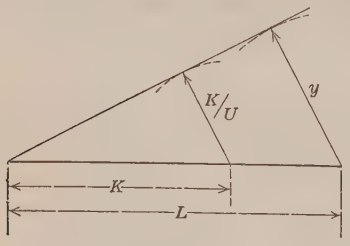


Fig. 36.

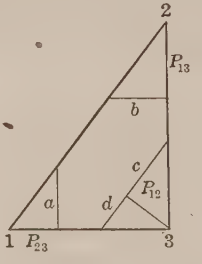


Fig. 37.

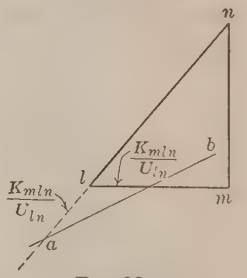


Fig. 38.

The lines, drawn through the ends of the P's and parallel to the sides opposite, give the quantities, a, b, c, and d. Now, the change of angle at Joint (2) is given by $(b\text{-}d)$. If P_{13} and P_{12} are both positive, it would be possible to superimpose d on b graphically, and thus mark off a length of line (positive if b is greater than d, and *vice versa*) that represents the change of angle, $\Delta\ \alpha_{123}$. It is readily seen that, with the P's differing in sign, such a process is extremely troublesome. It is wiser to introduce Mao's analytical method of

obtaining the changes of angle into his graphical solutions. This tabulation will be shown later.

Deformation and Property Lines.—Consider Equation (83):

$$r_{lm} = \frac{U_{ln}}{U_{lm}} \cdot r_{ln} + \frac{K_{mln}}{U_{lm}}$$

the quantities, $\left(\frac{U_{ln}}{U_{lm}}\right)$ and $\left(\frac{K_{mln}}{U_{lm}}\right)$, are constants under the particular loading of the truss, whereas r_{lm} and r_{ln} may be regarded as two unknowns. There is, therefore, a linear relation between them.

If lm and ln (Fig. 38) be the two axes (oblique in general), mark off $+\frac{K_{mln}}{U_{lm}}$ along lm and $-\frac{K_{mln}}{U_{lm}}$ along ln; this gives the straight line, a-b, which is called the "property line" of the triangle for the angle, mln. It is, in fact, the graph of Equation (83) on the two members considered as axes. There are three such deformation lines for every triangle. With regard to the convention of drawing them, it must be remembered that the U's are positive quantities, and the K's can be either positive or negative. If the K of an angle is positive, lay off a positive quantity along the member first met in a counterclockwise direction, and a negative quantity along the second member; if the K is negative, lay off a negative quantity along the first member and a positive quantity along the second.

Dr. Mao introduces what he terms the "property lines". From the fact that the sum of the three angles of a triangle remains constant, $\Sigma K = 0$ for every triangle. Using this, it can be shown analytically, that the three straight lines, through the vertices of a triangle and parallel to the corresponding deformation lines, are concurrent. Such a line would be:

$$r_{lm} = \frac{U_{ln}}{U_{lm}} \cdot r_{ln} \qquad (86)$$

the property line for the vertex, l, of the triangle, mln (Fig. 38). These lines may be regarded as the deformation lines for zero loading, when all the K's are zero. Use is made of this property to check the drawing of the deformation lines.

For the construction of the property and deformation lines, proceed as follows: Consider the truss diagram, Fig. 39, with all the U-lines drawn. Describe an arc of a circle with as large a radius as it is convenient to take and with Joint (1) as the center. Let this arc cut the member, 1-2, in a and the member, 1-3, in b. Draw perpendiculars to U_{12} and U_{13} from a and b, respectively. Let these be u_{12} and u_{13}.

In the second diagram, Fig. 40, lay off u_{13} from Joint (1) along the member, 1-3. Call the point, c. From c, lay off the length, $cd = u_{12}$, along the line through c, parallel to the member, 1-2. Join 1 to d. This is the property line for the angle $A1$, as $\frac{u_{12}}{u_{13}} = \frac{U_{12}}{U_{13}}$, fulfilling the requirements of Equation (86). This is repeated for every angle in the truss. In order to obtain the best results

the quantities, u_{12}, u_{13}, and, therefore, the radius of the circle, $a\,b$, must be as large as possible.

For the deformation lines, instead of any radius, select K_{312} to the scale of the f's, and describe an arc of a circle about 1 as the center. As the whole solution depends on the accuracy of locating these deformation lines, special care must be taken in this part of the work. Let the new points of intersection in Fig. 39 be a' and b', and the corresponding perpendiculars, u_{12}' and u_{13}'. Now, lay off u_{13}' along the member, 1-3, and u_{12}' along the member, 2-1, produced, if K_{312} is positive, whereas, lay off u_{13}' along the member, 3-1, produced, and u_{12}' along the member, 1-2, if K_{312} is negative. This is shown in Fig. 41 for convenience; in practice, the construction of the two sets of lines is on one diagram. A study of Equation (83) will prove that the line, *ef*, is the deformation line of the angle, *A*1. These deformation lines, now, must be parallel to the corresponding property lines, of which those belonging to a triangle must be concurrent.

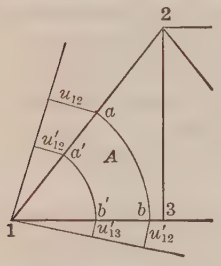


Fig. 39.

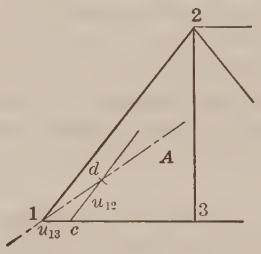


Fig. 40.

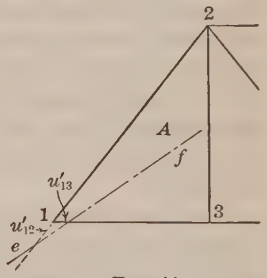


Fig. 41.

The Equilibrium Contour.—Quoting again from Dr. Mao's thesis: Under "Fundamental Conceptions" (page 40) (Fig. 29), it has been shown, that the stress lines, *S*, could be made to balance each other around any joint by the introduction of an ideal external force, *R*. In the process of construction, however, the conception of stress lines as there adopted is not convenient, as it would involve a different force diagram for each change of the stress lines, which is unavoidable in the graphical construction. A speedy method demands that one force diagram shall serve the whole process, that is, the direction of the stress lines is to be kept constant. This fixed direction of the stress lines is best chosen perpendicular to the axis of the member, for then the offset, $(= f)$, would be found along the axis of the member. For example, let S_{12}, Fig. 42, cut the member, 1-2, in *c*. The stress, f_{12}, is then equal to 1-*c*, positive if *c* is between 1 and 2, and negative if on the line, 2-1, produced. For the end, 2, the same force, S_{12}, should be shifted to a point the distance of which from the end, 2, gives the stress, f_{21}. By this modification, the direction of stress lines, *S*, is made constant for the members, and one force diagram is sufficient for different values of f at the same end of the same member. Here, these *S* lines are no longer the "Stress Lines", in the correct use of the term, but they will be known as the "Equilibrium Lines".

The direction of the equilibrium lines should be such, that they give a counterclockwise moment around the joint when the values of f are positive.

that is, when the lines cut the members somewhere between the joints. Thus, the direction of S_{12} should be upward, whether it is on the right-hand or the left-hand side of Joint (1). If on the left side of Joint (1), the moment is clockwise, and f is then negative.

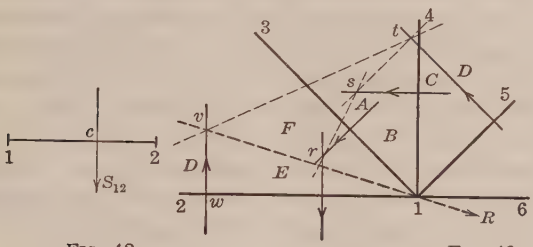


FIG. 42

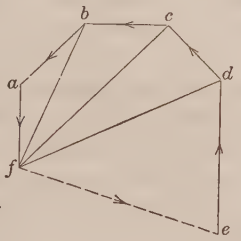


FIG. 43.

Consider the joint of a truss, as shown in Fig. 43. The force diagram is assumed to have been constructed. If the stresses in the members, 1-2, 1-3, 1-4, and 1-5, are known, the positions of the equilibrium lines are known. Then the moment of the member, 1-2, equals the force, *af*, multiplied by the perpendicular distance, f_{12}. If f_{12} is negative, it should be laid off on the side of 1 remote from 2, that is, along the member, 1-6. The closing line, *fe*, furnishes the external force, *R*, at the joint. The direction of the force is constant, and *R* may be drawn on the diagram as a part of the truss.

Suppose that the stress in the member, 1-6 ($= f_{16}$), is required, that is, the location of the force, *de*, the direction of which is upward, found by passing around the joint in a counterclockwise direction. In the force diagram, take *f* as the pole, and draw the rays, *fb*, *fc*, and *fd*. At the intersection, *r*, of the forces, *FA* and *AB*, draw a line parallel to *fb*, meeting the force, *BC*, at *s*. From *s* draw a line parallel to the ray, *fc*, meeting the force, *cd*, in *t*. From *t*, draw a line parallel to the ray, *fd*, meeting the force, *FE*, that is, *R*, at *v*. Then, the force, *DE*, must pass through *v*. Therefore, draw a perpendicular from *v* on the side, 1-6, and the distance, 1*w*, will give the value of f_{16}. It is negative here as it is on the side of 1, that is, remote from 6. Thus, it will be seen that only three lines are necessary for a joint of five members in order to determine the stress in the member, 1-6. For joints with a fewer number of members, only one or two lines are necessary. If there are eccentric connections at the joint, the eccentric moments, *M*, can be taken care of by displacing the force, *R*, a distance equal to $\frac{M}{R}$.

By a similar process the stress in any member may be found, if the stresses in all the other members meeting at the joint are known. The contour, formed by the force, *R*, and the equilibrium lines of the different members, will be known as the "Equilibrium Contour".

To cultivate speed and accuracy in drawing the force diagram for the equilibrium lines, the following method will be found useful. For a truss as shown, Fig. 44, first draw a line perpendicular to the member, 1-2, and measure $ha = S_{12}$. Then, from *a*, draw a line perpendicular to the member, 2-3, and

measure off $ab = S_{23}$. From b, draw a line perpendicular to the member, 2-5, and measure off $bc = S_{25}$. Continue for all the web members of the truss, until the points, e and f, are obtained. Only one-half the truss will be compared, as the other half will be symmetrical. From a, draw a line perpendicular to the member, 1-3, and measure off $ag = S_{13}$. The point, g, should be below a, so that the force, ag, will be downward. Similarly, from all the points, b, c, d, and e, draw lines perpendicular to the respective chord members, and measure off the segments, bk, cj, etc., equal to S_{35}, S_{24}, etc. All the values of the equilibrium lines are then constructed. Next, join the ends, l, j, h, g, etc., with the dotted lines shown. These will give the directions of the external forces, R, which are to be transferred to the truss diagram, as shown. Lastly, the rays, hb, hc, etc., are completed. They are the lines which join the left end of the external force, R, to the inner points of the diagram.

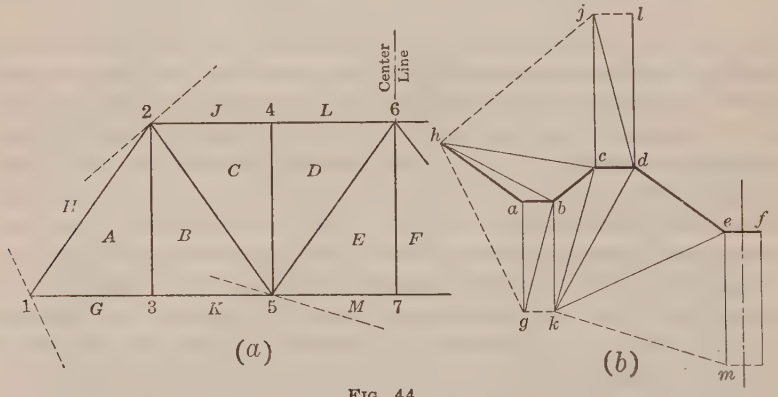


FIG. 44.

The Theory of Deformation Contour

Equation (83) applied to the deformation line of B-5, Fig. 44(a), shows that this deformation line is a graphical representation of the relation between r_{53} and r_{52}. Hence, if r_{53} is known, r_{52} is at once found; but knowing r_{52}, the deformation line, C-5, gives r_{54}, etc. In other words, if r_{53} is known, the deformation lines around Joint (5) permit all the other r's at Joint (5) to be found. This can be expressed analytically, by using Equation (83) in the following manner:

$$r_{52} = \frac{U_{53}}{U_{52}} \cdot r_{53} + \frac{K_{B5}}{U_{52}} \qquad (87)$$

$$r_{54} = \frac{U_{52}}{U_{54}} \cdot r_{52} + \frac{K_{C5}}{U_{54}}$$

and substituting from Equation (87):

$$r_{54} = \frac{U_{53}}{U_{54}} \cdot r_{53} + K_{B5} + \frac{K_{C5}}{U_{54}} \qquad (88)$$

$$r_{56} = \frac{U_{54}}{U_{56}} \cdot r_{54} + \frac{K_{D5}}{U_{56}}$$

and substituting from Equation (88):

$$r_{56} = \frac{U_{53}}{U_{56}} \cdot r_{53} + K_{B5} + K_{C5} + \frac{K_{D5}}{U_{56}} \quad \ldots\ldots\ldots\ldots (89)$$

$$r_{57} = \frac{U_{55}}{U_{57}} \cdot r_{56} + \frac{K_{E5}}{U_{57}}$$

and substituting from Equation (89):

$$r_{57} = \frac{U_{53}}{U_{57}} \cdot r_{53} + K_{B5} + K_{C5} + K_{D5} + \frac{K_{E5}}{U_{57}} \quad \ldots\ldots\ldots\ldots (90)$$

or,

$$r_{57} = a \cdot r_{53} + b \quad \ldots\ldots\ldots\ldots (91)$$

where a and b are constants; similarly, for r_{52}, r_{54}, and r_{56}.

This is utilized in a simple yet elegant way. In Fig. 45, let 5-a represent r_{53} in magnitude and sign, as determined by earlier construction (assumed) similar to the one to follow.

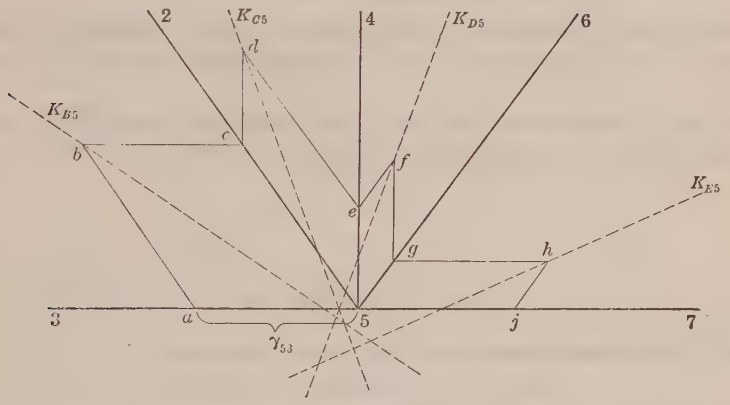


FIG. 45.

Draw ab parallel to the member, 5-2, to cut K_{B5} in b, K_{B5} being the deformation line of the angle, 5, in the triangle, B, and, therefore, supposed to be drawn by methods explained earlier. Draw bc through b parallel to the member, 5-3, to cut the member, 5-2, in c; 5-c is then r_{52}. Draw cd parallel to the member, 5-4, to cut K_{C5} in d, and then draw de parallel to the member, 5-2, to cut the member, 5-4, in e; 5-e is then r_{54}. Repeat this construction to find 5 j = r_{57}. This star-shaped diagram is the "deformation contour".

(Note that Dr. Mao introduces two 45° lines toward each other and downward from a and j. The whole diagram enclosed by these and the rays already drawn he calls the "deformation contour". These 45° lines are omitted from this investigation, as the writer fails to see their use; besides, the whole construction in an actual example becomes such a maze of lines that it is advisable to minimize the number.)

In order to use these deformation contours, Mao selects a certain member as the "reference" member for the deformation contour under consideration. It is always the member on the left of the joint. Thus, 5-3, Fig. 44(a), is the

reference member for Joint (5), 2-1 is the reference member for Joint (2), etc.

Suppose, the correct deformation contours for Joints (1) and (2) are known, that is, those contours that give the r's which, when substituted in Equation (85), will yield the desired secondary stresses, f. Substituting r_{12} and r_{21} in Equation (85),

$$f_{12} = \frac{1}{3}\,(2\,r_{12} + r_{21}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (92)$$

Now, because Joint (1) is in equilibrium under $S_{12} \cdot f_{12}$ and $S_{13} \cdot f_{13}$, the two moments at Joint (1),

$$f_{13} = -\frac{S_{12}}{S_{13}} \cdot f_{12} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (93)$$

Therefore, f_{13} and r_{13} are known, the former from Equation (93), and the latter from the assumed deformation contour of Joint (1); but r_{31} is related to f_{13} and r_{13} by Equation (84), therefore,

$$r_{31} = 3\,f_{13} - 2\,r_{13}.$$

By marking off r_{31} along 31, the deformation contour for Joint (3) may be completed in the same manner as in constructing the contour for Joint (5), Fig. 45.

To arrive at the correct value for r_{53}, the reference quantity for Joint (5), note that f_{31} and f_{32} are easily found, Equation (85), from,

$$f_{31} = \frac{1}{3}\,(2\,r_{31} + r_{13})$$

and,

$$f_{32} = \frac{1}{3}\,(2\,r_{32} + r_{23})$$

all the r's being known quantities. As $(\Sigma\, M)_3 = 0$, or,

$$S_{31} \cdot f_{31} + S_{32} \cdot f_{32} + S_{35} \cdot f_{35} = 0$$

or,

$$f_{35} = -\frac{1}{S_{35}}\,(S_{31} \cdot f_{31} + S_{32} \cdot f_{32})$$

The equilibrium polygon of the S's for Joint (3) gives the f_{35} in the quickest way. As r_{35} has already been determined,

$$r_{53} = 3\,f_{35} - 2\,r_{35}$$

from Equation (84), enabling the drawing of the deformation contour for Joint (5).

It is important to note that all this is possible because the relation between any r's and f's, or combinations of r's and f's, is a linear one.

Method of Successive Deduction

With regard to Mao's "Method of Successive Deduction", of which the writer has not carried out an actual example—this method depends on how closely a draftsman can guess the correct values of f_{12} and f_{21}. Suppose two values have been chosen. The foregoing analysis indicates the method of proceeding from joint to joint and fitting in all the deformation contours. When

the last two joints are reached, it will be found that the two relations, $\Sigma M = 0$, for these have not been used at all. After computing the values of ΣM (that is, $\Sigma S \cdot f$), for these joints, making use of the r's found, in general, the resulting ΣM's will differ from zero. Now, assume two new values of f_{12} and f_{21}, repeat the preceding work and obtain a fresh set of values for the ΣM's of the last two joints; if these are nearer zero than the two from the first process, a judicious choice of the f_{12} and f_{21} will give a third approximation of ΣM that will be closer to zero than either of the former two solutions. This process may be continued until the ΣM's for the last two joints are as close to zero as desired. The final set of deformation contours will then give the required secondary stresses. It may be stated here that the element of chance, as in Ritter's method, is not attractive when there are straightforward methods available.

"Base" and "Vertex" Lines

In order to grasp the idea of Mao's "base" and "vertex" lines, it should be remembered that, by drawing the deformation contours for the r's and the equilibrium polygons for the f's, as already explained, linear relations are established between any desired set of r's, or f's, or combinations of them.

Consider the triangle, B, Fig. 44(a). Mao defines the member, 3-5, as its base (that member of the triangle, that lies in the boundary of the truss). Joint (2) is then its vertex; but Joint (2) is also a joint the reference contour of which is r_{21}. The base will carry the two contours, r_{35} and r_{53}.

Now,

$$r_{53} = a \cdot r_{21} + b \cdot r_{35} + c \ldots\ldots\ldots\ldots\ldots\ldots (94)$$

in which, a, b, and c are constants.

A proof of this is simple. Suppose, r_{21} and r_{35} to be known; r_{21} establishes a deformation contour round Joint (2), and r_{35} does the same for Joint (3), for, of course, the deformation contour round a joint can be done in either a clockwise or a counterclockwise manner. Now r_{23} and r_{32}, from these contours, give f_{32}, linear in r_{23} and r_{32} and, therefore, linear in r_{21} and r_{35}. Here one must take for granted something that will be proved later, namely, that r_{13} is found from r_{21} and r_{31}, also through a linear relation. Assuming this, as r_{31} and r_{35} are linearly connected, r_{13} is, therefore, a linear expression involving r_{35} and r_{21}. This then gives f_{31}, which is, in turn, only a linear function of r_{21} and r_{35}. Now, f_{35} is a linear function of f_{31} and f_{32} ($f \cdot S = 0$), and therefore, a linear function of r_{21} and r_{35}. Finally, $r_{53} = (3 f_{35} - 2 r_{35})$, and is, therefore, linear with respect to r_{21} and r_{35}.

Suppose, now, that a certain constant value, say, k, be assigned to r_{21}. The relationship, then, between r_{53} and r_{35} will be graphically represented by a straight line. Equation (94) becomes:

$$r_{53} = b \cdot r_{35} + (c + a\,k) \ldots\ldots\ldots\ldots\ldots\ldots (95)$$

in which, $(c + a\,k)$ is a constant. If, now, some value is given to r_{35}, the corresponding value of r_{53} is at once determined.

Lay out these corresponding values along the member, 3-5, as shown in Fig. (46). Through the end points, p and q, draw two lines at 45° to meet in P.

$$x = y + r_{35}$$

and,

$$y = \frac{1}{2}(d - r_{35} - r_{53}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (96)$$

These relations are evident from a study of Fig. 46. Eliminating r_{35} and r_{53} from Equations (95) and (96),

$$y = \frac{(b+1)}{(b-1)} \cdot x + \frac{(a\,k + c - d)}{(b-1)} \ldots\ldots\ldots\ldots\ldots\ldots (97)$$

This proves that, for a constant value of r_{21}, P lies on some definite straight line. If r_{21} be given the constant value of zero, the locus of P is called the "base line".

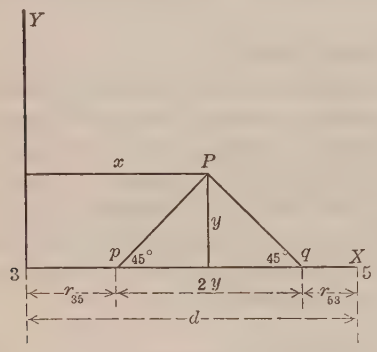


Fig. 46.

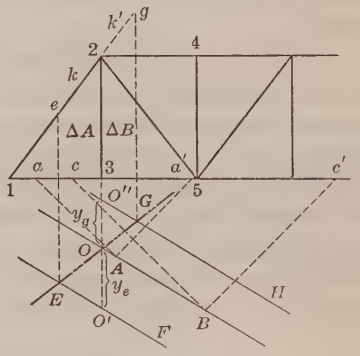


Fig. 47.

It is at once evident from Equation (97) that every value of r_{21} defines a certain straight line, but as this value of r_{21} is incorporated in the constant term of Equation (97) all these lines are parallel to the "base line".

Let $O\,A\,B$ (Fig. 47) be the base line of $\Delta\,B$. In order to obtain this, r_{21} is assumed to be zero, while two sets of corresponding values of r_{35} and r_{53} are determined. The graphical method of doing this will be shown later.

Let two such sets be ($3a$, $5a'$) and ($3c$, $5c'$). Lines at 45° through a and a' give A, and through c and c' determine B. Let the point of intersection of AB and the member, 2-3, be O. Let $E\,F$ be the line that gives the relationship between r_{35} and r_{53} for a value of, say, $r_{21} = 2e = k$, and let $G\,H$ be that when, say, $r_{21} = 2g = k'$. Draw the perpendiculars, $e\,E$, $g\,G$ to the member, 3-5, and let these cut $E\,F$ and $G\,H$ in E and G, respectively. Join $E\,O$ and $O\,G$. The horizontal distances from E and G to the member, 2-3, are proportional to k and k'. Now, 3-O, 3-O', and 3-O'' are the y-intercepts for the lines, $A\,B$, $E\,F$, and $G.\ H$, respectively, and are, therefore, from Equation (97), respectively, equal to:

$$\frac{c-d}{b-1}, \quad \frac{a\,k + c - d}{b-1}, \text{ and } \frac{a\,k' + c - d}{b-1}.$$

Hence,

$$O\,O' = \frac{a\,k + c - d}{b-1} - \frac{c-d}{b-1} = \frac{a\,k}{b-1},$$

and,

$$O\,O'' = \frac{a\,k'}{b - 1}.$$

The algebraic signs are correct. Therefore, the ratio of $O\,O'$ to $O\,O''$ is $\frac{k}{k'}$. Further, as the angles, $O\ O'\ E$ and $O\ O''\ G$, are equal, because $G\ H$ and $E\ F$ are parallel, the two triangles, $O\ O'\ E$ and $O\ O''G$, are similar. The angles, $E\ O\ O'$ and $G\ O\ O''$, are, therefore, equal, and, hence, E, O, and G are collinear.

If any other value for r_{21} be chosen, the perpendicular from the end point to the "base", 3-5, will cut the corresponding line of the parallel family in a point on the straight line, $E\ G$. This is Dr. Mao's "vertex line".

(Note that it is now possible to verify the assumption used in the proof for Equation (94). The base and vertex lines of $\Delta\ A$ may be obtained in exactly the same way as for $\Delta\ B$. This means that the relationship between r_{21}, r_{13}, and r_{31} is linear, just as that for r_{21}, r_{35}, and r_{53}, the quantity, r_{21}, being the reference contour in each case. This justifies the statement that f_{31} is a linear function in r_{21} and r_{35}, etc.)

Construction of Base and Vertex Lines

As the base line is a straight line, two points will determine it completely. A splendid check, however, can be introduced here, for if three points are found independently of each other, and are collinear, there is little chance of the base line being wrong. Hence, throughout the work of locating base lines, three points are to be determined for every line.

In determining a vertex line, its accompanying base line having already been found, note that one point on the vertex line is the point of intersection of the base line and the perpendicular from the vertex (see Fig. 47). It remains, therefore, to find two more points, which must be collinear with that on the base line.

Let the base and vertex lines for a truss, shown in Fig. 44(a), be required. This is a Warren truss with verticals, but the method is applicable to all kinds of trusses, the only difference being found in the order of procedure, which is governed by the number of members meeting at the joints. The base of a triangle is always chosen as the member that forms the outline of the truss.

Base and Vertex Lines for $\Delta\ A$.—Either the member, 1-2, or the member, 1-3, may be chosen as the base, as each is part of the outline of the truss. Although only one set of base and vertex lines is necessary for the solution of the problem, it offers a neat check to introduce both sets; this will be explained in the example.

Member 1-3 as Base.—As the vertex of $\Delta\ A$ is Joint (2), the base line is obtained by assuming r_{21} equal to zero, as the member, 2-1, is the reference member for Joint (2).

Some arbitrary value is now assigned to r_{13}, and the deformation contour for Joint (1) completed. This gives the value of r_{12}. As $2\,r_{12} + r_{21} = 3\,f_{12}$ (see Equation (85)), and $r_{21} = 0$ here, let the force, S_{12}, act perpendicularly to

the member, 1-2, at a distance, $2\,r_{12}$, from it. Through the point of intersection of S_{12} and R_1 (the line of action of which has already been determined from the force polygon), draw a line perpendicular to the member, 1-3; the distance from Joint (1) along the member, 1-3, thus found gives the value $3\,f_{13}$; but $r_{31} = 3\,f_{13} - 2\,r_{13}$ (see Equation (84)). Then r_{31} is laid off from Joint (3) along the member, 3-1, care being taken that, if it is negative, it must be measured along the member, 1-3, produced. A line, at 45° to the member, 1-3, and downward to the right, is now drawn through the end of r_{13}, and a line, at 45° to the member, 1-3, downward but toward the left, is drawn through the end of r_{31}; the point of intersection is one of the points on the required base line.

The process is twice repeated, that is, r_{21} remains zero while two more arbitrary values are given to r_{13}. This gives two more points for the base line. If care has been taken with regard to the signs, and the draftsmanship has been accurate, these three points will be found to be collinear, determining the base line for ΔA with the member, 1-3, as base.

By producing the member, 2-3, to cut the base line just found, the first of the three points for the required vertex line is obtained. To get a second point, assume a fairly large value for r_{21} (no longer zero, as for the case of the base line), and some arbitrary value for r_{13}. Using the latter, construct the deformation contour for Joint (1), to find r_{12}. As $3\,f_{12} = 2\,r_{12} + r_{21}$, it is possible to complete the equilibrium polygon round Joint (1), giving $3\,f_{13}$. Now $r_{31} = 3\,f_{13} - 2\,r_{13}$. The two corresponding values of r_{13} and r_{31}, therefore, are known, and by drawing the two 45° lines downward toward each other through the ends of r_{13} and r_{31}, their point of intersection is a point on the straight line parallel to the base line already found. Draw this parallel line; where it cuts the vertical through the end of r_{21}, is one of the points on the required vertex line. Repeat this process with a different r_{21}, fairly large and preferably with a sign different from that already used, this greatly improves the accuracy. The three points should be collinear, giving the required vertex line for ΔA with the member, 1-3, as base.

(Note that Dr. Mao draws his equilibrium polygons with the offsets equal to the values of the f's. It will be noticed that in changing from r's to f's and *vice versa* (see Equation (85)), it is most convenient to work with three times the f-values. An equilibrium polygon around a joint is magnified, therefore, just three times, and not only is time saved, but great accuracy is obtained. Throughout this investigation, the equilibrium polygons are to be understood as being drawn with the $3\,f$-values.)

Member 1-2 as Base.—The vertex is now Joint (3) which has the member, 3-1, for reference member. To obtain the base line, assume r_{31} to be zero, and r_{12} as some arbitrary value. With this r_{12} the deformation contour for Joint (1) is constructed. Now $3\,f_{13} = 2\,r_{13} + r_{31}$, and as $r_{31} = 0$, the equilibrium polygon for Joint (1) is completed by starting with S_{13}, acting at the end of $3\,f_{13} = 2\,r_{13}$. This construction then gives $3\,f_{12}$, but as r_{12} was assumed at the start, the corresponding r_{21} results from $r_{21} = 3\,f_{12} - 2\,r_{12}$. Drawing the 45° lines (to the member, 1-2, in this case) through the ends of r_{12} and r_{21}, deter-

mines the first point on the base line of ΔA, Vertex (3). This process, repeated twice for two new and arbitrary values of r_{12}, gives the two additional points. If the three points are found to be collinear, it is safe to say that the base line has been determined.

To determine the vertex line, assume a fairly large value of r_{31} and use it in conjunction with an arbitrarily chosen r_{12} to obtain the corresponding r_{21} eventually. The difference between this and the analysis for the base line is that the r_{31} in $3 f_{13} = 2 r_{13} + r_{31}$ is no longer zero. The two 45° lines through the ends of r_{12} and r_{21} now determine the point through which a line is drawn parallel to the base line already found. The perpendicular from the end of r_{31} to the base, 1-2, intersects this parallel in a point on the required vertex line. The point where the perpendicular from the Vertex (3) cuts the base line is, of course, another point on the vertex line. A conclusive check is to take another fairly large value of r_{31} and proceed to find a third point, and if this is collinear with the two already found, the vertex line is located.

Base and Vertex Lines for ΔB.—The base for ΔB is the member, 3-5, and the vertex, Joint (2), so that the base line is obtained by making $r_{21} = 0$, the deformation contour for which is drawn. As before, assume a value for r_{35} and find the corresponding value for r_{53}; these two values will determine a point on the base line. On account of the greater number of members entering Joint (3), the process is a little more complicated than for ΔA, but the principle is the same.

For the assumed value of r_{35}, construct a deformation contour for Joint (3), giving r_{32} and r_{31}. Now, as $r_{21} = 0$, the value of r_{13} is found by drawing a 45° line down to the left from the end of r_{31} to the base line of ΔA, Vertex (2), and then a 45° line from this point of intersection to cut the member, 1-3, giving r_{13}. Of all the r's connected with the members entering Joint (3), r_{53} is now the only one unknown. To determine it, the equilibrium polygon for Joint (3) is drawn by making S_{31} act perpendicularly to the member, 3-1, at a distance $3 f_{31} = 2 r_{31} + r_{13}$ from Joint (3) along the member, 3-1, and by making S_{32} act perpendicularly to the member, 3-2, at a distance $3 f_{32} = 2 r_{32} + r_{23}$ from Joint (3) along the member, 3-2; these, together with the resultant R_3, determine the line of action of S_{35}, giving $3 f_{35} = 2 r_{35} + r_{53}$. Hence, $r_{53} = 3 f_{35} - 2 r_{35}$, and the point on the base line of ΔB is found. Two more points are determined, and if the three are collinear the construction for the base line is correct.

The vertex line is determined by assuming fairly large values for r_{21}. These values give certain deformation contours for Joint (2), while they also belong to definite lines parallel to the base line of ΔA, Vertex (2). These parallel lines are found by drawing perpendiculars from the ends of r_{21} to cut the vertex line of ΔA, the parallel lines passing through the points just found. It is these parallel lines that must now be utilized to obtain the values of r_{13} from those of r_{31}, which have themselves come from arbitrary values of r_{35}. The equilibrium polygon for Joint (3) follows, and, hence, r_{53} is determined. The points, determined from corresponding values of r_{35} and r_{53}, are on lines parallel to the base line of ΔB. The points where these parallels cut the verticals from the respective r_{21}'s are on the vertex line required. If two such

points and the point where the perpendicular from Vertex (2) to the base, 3-5, cuts the base line of ΔB, are collinear, the vertex line for ΔB is determined.

Base and Vertex Lines for ΔC.—The base for ΔC is the member, 2-4, and the vertex, Joint (5). The base line, therefore, is obtained by making $r_{53} = 0$ (5-3 being the reference member) and assuming three different values for r_{24}, from which three corresponding values of r_{42} are obtained. In each of these cases r_{53} is zero.

To arrive at a value of r_{42} from a given r_{24} and $r_{53} = 0$, follow the same principle as before. A glance at Fig. 44(*a*) will show that, in order to get to r_{42}, $3f_{25}$, $3f_{23}$, and $3f_{21}$ must first be obtained, with which to draw an equilibrium polygon for Joint (2) that will give $3f_{24}$; then only is $r_{24} = 3f_{24} - 2r_{24}$ obtained. The deformation contour around Joint (2) for the r_{24} under consideration, and the deformation contour around Joint (3) for $r_{53} = 0$, give r_{25} and r_{52}, respectively. Then, $3f_{25} = 2r_{25} + r_{52}$. The value of r_{21}, from the former contour, also determines the parallels to the base lines of ΔA and ΔB, 2-1 being the reference member for both. A 45° line, from Joint (5) to the parallel to the base line of ΔB, gives the point from which to draw upward at 45° to obtain r_{35}; this is because $r_{53} = 0$. Use this value of r_{35} for a deformation contour for Joint (3), and with the r_{31} thus found and the parallel to the base line of ΔA, obtain r_{13}. This gives the deformation contour around Joint (1). The quantities, $3f_{25} = 2r_{25} + r_{52}$, $3f_{23} = 2r_{23} + r_{32}$, and $3f_{21} = 2r_{21} + r_{12}$, are now known, and the problem is solved.

The vertex line is obtained by assuming for r_{53} two values of different magnitude, preferably of different sign. These values give two deformation contours around Joint (5). By giving an arbitrary value to r_{24}, it is possible to proceed exactly as before to get the corresponding r_{42}, the only point to notice being that r_{52} is not zero now, so that the 45° lines must start, not from Joint (5), but from the end of r_{53}. The third point on the vertex line is where the perpendicular from Vertex (5) to the base, 2-4, cuts the base line of ΔC.

(Note that all the 45° lines are drawn away from the truss, so that the base and vertex lines for ΔC, ΔD, ΔG, and ΔH will be situated somewhere above the truss diagram, while those of ΔA, with Joint (3) as its vertex, lie to the left, those of ΔJ, with Joint (11) as vertex, to the right, and the remainder below the truss diagram. This is done to keep the drawing as clear as possible.)

Solution of Problem by Deformation Contour

The solution depends on the determination of all the base and vertex lines of all the triangles. The foregoing explanation appears to be difficult because it is long; this seems to be unavoidable. The writer found that, by merely reading Dr. Mao's thesis, his method seemed very difficult, but after going through the computations of an actual example the whole principle became exceedingly clear and simple. Any one who is eager to know Mao's work thoroughly is strongly advised, therefore, to work out a simple example; if time will not permit, a neat freehand sketch, accompanying the reading, will serve the purpose.

The base and vertex lines for the triangles, Fig. 44(*a*), are found in the following order, the vertex line of a triangle being determined immediately after the base line of the triangle in question: ΔA with either Joint (2) or Joint (3) as vertex, the order of procedure being immaterial; ΔB with Joint (2) as vertex; ΔC with Joint (5) as vertex; ΔD with Joint (5) as vertex, etc.

When this stage is reached, it becomes evident that ΔJ has the member, 10-12, in the outline of the truss, but instead of obtaining only one set of base and vertex lines with the vertex at Joint (11), two are obtained, one by considering the equilibrium of Joint (12), and the second that of Joint (10). It is on this property that the solution of the whole problem depends. It is the same idea that underlies the method of Müller-Breslau and that of Mohr (by elastic weights), where the two last joints give two equations wherewith to solve the two values assumed in the beginning.

Assume that the two sets of base and vertex lines of ΔJ, Vertex (11), are drawn and, of course, all the preceding ones. The reference member for ΔJ, Vertex (11), is 11-9. Assume any arbitrary value for $r_{11\cdot9}$. A perpendicular, from the end point of $r_{11\cdot9}$ to the member, 12-10, will cut the two vertex lines of the respective sets for ΔJ, Vertex (11), in two points, through which parallels to the respective base lines are then drawn. These parallels determine a point, most important inasmuch as it is common to the two sets. Proceed to draw 45° lines to the member, 10-12, from this point. Two values are determined, namely, $r_{10\cdot12}$ and $r_{12\cdot10}$. Complete the deformation contours round Joints (10) and (12). The value of $r_{10\cdot8}$ found will enable the determination of a parallel to the base line of ΔJ, Vertex 10 (10-8 is the reference member in this case); this parallel must be used in conjunction with $r_{12\cdot11}$, from Contour (12), to obtain $r_{11\cdot12}$. Complete the deformation contour for Joint (11). The value of $r_{11\cdot9}$ thus found will, in general, be quite different from that assumed at the start. The object is to obtain an exact coincidence. A second value of $r_{11\cdot9}$ is judiciously assumed; it is advisable that this value approximate the one found in the process just described. The above procedure is then repeated. It will be found that the resultant value of $r_{11\cdot9}$ will come fairly close to that assumed. This process of choosing $r_{11\cdot9}$ and drawing is repeated a few times; after the fourth attempt, it is customary to check the value of $r_{11\cdot9}$ last chosen.

Assume that the correct $r_{11\cdot9}$, with the attendant deformation contours round Joints (12), (11), and (10), are now known. Draw the perpendicular from the end of $r_{10\cdot8}$ to cut the vertex line of ΔI; through this point, draw a line parallel to the base line of ΔI. Use this parallel to determine $r_{9\cdot11}$ from $r_{11\cdot9}$. Complete the deformation contour of Joint (9) giving $r_{9\cdot7}$. This is now the reference contour for ΔH and ΔG. Using it, determine $r_{8\cdot10}$ from $r_{10\cdot8}$, and complete the deformation contour for Joint (8). This process is continued for all the joints. ΔA will have been reached and all the deformation contours of all the joints will have been drawn, without having made use of the base and vertex lines of ΔA, Vertex (3). Their usefulness lies in the following: Draw a perpendicular to the member, 1-2, from the

end of r_{31}, this being the reference contour. This line cuts the vertex line in a point through which a line is now drawn parallel to the base line. Draw lines at 45° to the member, 1-2, from the ends of r_{21} and r_{12}. If the work has been accurate, these 45° lines must intersect each other on the parallel line just drawn.

Consider, for example, Joint (6). The r's of all the members entering Joint (6) have been found. Lay off from Joint (6) the quantities, $(2\ r_{68} + r_{86})$, $(2\ r_{69} + r_{96})$, $(2\ r_{67} + r_{76})$, $(2\ r_{65} + r_{56})$, and $(2\ r_{64} + r_{46})$, along the members, 6-8, 6-9, 6-7, 6-5, and 6-4, respectively. These are now the 3 f's of the members. The resultant, R_6, of the forces, S_{68}, S_{69}, S_{67}, S_{65}, and S_{64}, is known in magnitude and direction from the force polygon, while its point of application is Joint (6). Draw the equilibrium polygon, considering everything known but the point of application of S_{64}. This will give the quantity, 3 f_{64}, which, together with the quantities, 3 f_{68}, 3 f_{69}, 3 f_{67}, and 3 f_{65}, will cause equilibrium round Joint (6), that is, $(\Sigma\ S\ .\ 3\ f)_6 = 0$. If the work has been exact this value, 3 f_{64}, will correspond exactly with 3 f_{64} found from the deformation contours, namely, $2\ r_{64} + r_{46}$; the closeness in agreement will be a measure of the accuracy of the drawing. Repeat this for all the joints. If the agreement is good in all cases, the values of the 3 f's may be considered to be correct. A third of each of these values gives the corresponding secondary stress.

Analytical Solution by the Scientific Arrangement of Computations

Introduction.—This method is simply the application of Equations (81) to (85), together with the equations found at the joints, namely, $\Sigma\ M = 0$. Its underlying principles, therefore, are easy. The difficulty is that, in common with the methods of deformation contour and successive deduction, it needs a long explanation, accompanied by a diagram. The method is indeed very much like that of Müller-Breslau; Mao also assumes two quantities known at the start, and using these throughout the process, finally arrives at two simultaneous equations the solution of which determines the two assumed quantities. The assumed quantities in this method are the secondary stresses of the first member of the truss.

Mao uses two truss diagrams. He has an ingenious device for determining the changes of the angle, $\Delta\ \alpha$, on the first diagram. On the second diagram, he places a number of skeleton tables which must be filled out in a certain definite way; each place in every table has a definite significance and seldom is any quantity repeated.

A point that must be emphasized again is this: If the two assumed secondary stresses of the first member are x and y, with reference to Equations (81) to (85) and the $\Sigma\ M = 0$ equations, that is, $\Sigma\ S\ .\ f = 0$ equations (see Equation (79)), it is to be noted that every relation to be established during the analysis will be one of the first degree in x and y.

Application.—As the theory has been proved, the method will be illustrated by actually applying it to a simple case. Consider a truss composed of two triangles; the solution of this truss embodies all the elements found in one with

a greater number of triangular elements. Further, as most trusses are composed of right-angled triangles, Mao's method for finding the changes of angle will be given for right-angled triangles only. It is really Ritter's method (see page 9).

In Fig. 48 are two diagrams, representing a truss composed of two triangles, A and B. On Fig. 48 (a), write out the lengths, L, in feet, moments of inertia, I, in inch units, and the extreme fiber distances, y, in inches, as shown. These are obtained from the dimensions of the truss. Now imagine the truss to be loaded, giving the unit stresses, P, in pounds per square inch, which are also written on the members. Obtain also, from the dimensions, the half values of the co-tangents of α and β.

In each of the triangles, A and B, Fig. 48(a), make a table as shown, observing the proper positions for the various values. The quantities at the three ends are K_{A1}, K_{A2}, and K_{A3}. These K's are the half values of the product of the modulus of elasticity and the changes of the angle, $\Delta\alpha$. (Equation (81); see, also, "Preliminary Considerations," page 42.) A comparison between this method, and "The Changes of Angle" as discussed on page 44, will show that the method is legitimate. K_{A1} is the change of Angle (1) in Triangle (A), and is found at the end of the A-table, nearest to Joint (1). Similarly, for all the other K's.

In Fig. 48(b), directly below Joints (1), (3), and (4), and directly above Joint (2), place the ΣM- and the ΣK-tables as shown. In the ΣK-table, include all the members that meet in the joint except the first. This comprises the members, 3-2 and 3-4, for Joint (3). The first member of a joint in the bottom chord is that which is first met on entering the truss in a clockwise direction round the joint; the first member of a joint in the top chord is that which is first met on entering the truss in a counterclockwise direction round the joint.

In the ΣM-tables all the members are included and again the clockwise direction for the bottom chord joints and the counterclockwise direction for the top chord joints are observed.

On the end chord, 1-2, a U-table is constructed, as shown; r_{21} the top letter, refers to the top end of the member, and the bottom letter, r_{12}, to the bottom end. Recalling the method of the deformation contour, these r's are the "contours" of the respective ends of the member.

On each of the two bottom chords, 1-3 and 3-4, a table is constructed, one for U_{13} and S_{13}, the other for U_{34} and S_{34}; these quantities are again obtained from Fig. 48(a). Note that the top r and f refer to the left end of the member, while the bottom r and f refer to the right end. For top chord members, tables identical with these U-tables would be used.

On each of the remaining members, Fig. 48(b), namely, 2-3 and 2-4, place two tables, as shown. It will be seen that the f and the r at the top refer to the top end of the member, while the lower r and f refer to the lower end of the member. The letters on the U_{24}-tables need no explanation; the remaining tables, namely, one on each end of every member, will be explained

later, as they come into use only after the problem of solving for the x and the y have been completed.

The first columns of the ΣM-tables are now filled with the values of the S for the members denoted at the sides. Thus, insert S_{31}, S_{32}, and S_{34} for the ΣM_3-table.

Consider one of the bottom ΣK-tables, say, the ΣK_3-table. Opposite K_{A3} and below the member, 3-2, place the value of K_{A3}, found from Fig. 48(a). In the space directly below, fill in the value of K_{B3} already found, sum the two quantities, and place the result in the adjacent space. This value will now refer to the member, 3-4. If there were more members at the joint, the table would be extended in the following manner: Suppose there were a K_{x3} and a member, 3-n: Directly below the value last found, would be placed the value of K_{x3}; add the two and put the sum in the adjacent space, and so on, for more members.

The ΣK-tables for the upper joints differ in that the rotation round the lower joints is clockwise, while that round the upper joints is counterclockwise; hence, use the negative values of the K's in the top K-tables. Otherwise, they are exactly similar to the bottom K-tables.

Recalling the method of deformation contour, the member to the left of a joint is the reference member. Therefore, the set of ΣK-tables is just a neat method of tabulating the changes of angle between the reference member and the other members of a joint.

The next step is to fill in the spaces shown extending out from the main tables, Fig. 48(b). As is evident from Equations (87) to (90), the r of a member at a joint is connected linearly with the reference, r, of that joint, and the constant term in such an equation is the quantity obtained by dividing the sum of the K's of all the angles between the member in question and the reference member by the U of the member considered. Hence, in the space of the U_{13}-table, put the value, $\frac{K_{A1}}{U_{13}}$, and in the space of the U_{34}-table, the value, $\frac{K_3}{U_{34}}$, that is, $\frac{K_{A3} + K_{B3}}{U_{34}}$. In the spaces of the center row of U-tables, however, put the third values, thus, for the lower space in the U_{23}-table, insert $\frac{K_{A3}}{3\ U_{23}}$, and for the upper space, $\frac{-K_{A2}}{3\ U_{23}}$. All the spaces that have been filled up to this stage are marked by crosses.

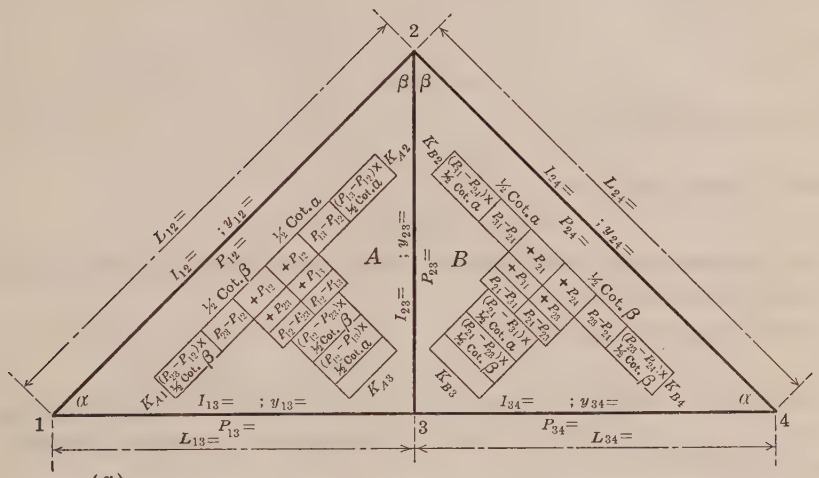
To proceed with the solution, let the secondary stresses in the member, 1-2, have the values, x and y, that is, let

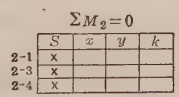
$$f_{12} = x \qquad (98)$$

and,

$$f_{21} = y \qquad (99)$$

The spaces of the row opposite f_{12} in the U_{12}-table, are filled in with the numbers, $+1, 0, 0$, while those opposite f_{21} are filled in with the number, $0, +1, 0$. These are the coefficients of the x and y and the constant term in the expres-





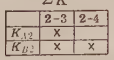


ILLUSTRATION OF
MAO'S ANALYTICAL METHOD

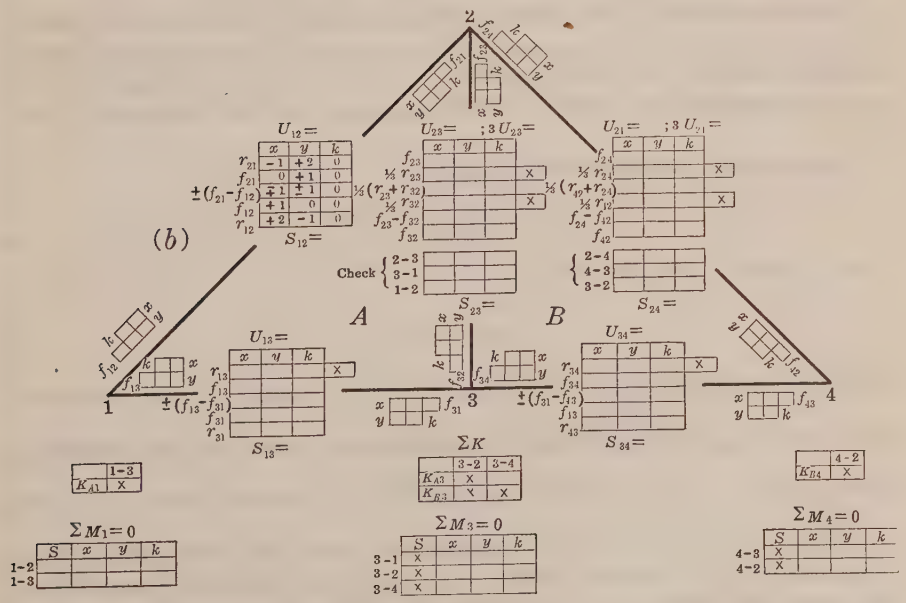


FIG. 48.

sions for the f's in terms of x and y, for, evidently, from Equations (98) and (99),

$$f_{12} = +1.x + 0.y + 0$$

and,

$$f_{21} = 0.x + 1.y + 0$$

Subtracting f_{12} from f_{21},

$$+(f_{21} - f_{12}) = -1.x + 1.y + 0$$

and by taking the negative value,

$$-(f_{21} - f_{12}) = +x - y + 0$$

Thus are obtained the coefficients, ∓ 1, ± 1, and 0, for the row opposite $\pm (f_{21} - f_{12})$ in the U_{12}-table.

Now, from Equation (82),

$$r_{21} = 2f_{21} - f_{12} = f_{21} + (f_{21} - f_{12}).$$

Adding the coefficients opposite f_{21} in the U_{12}-table to those belonging to $+(f_{21} - f_{12})$, gives -1, $+2$, and 0. Similarly, the coefficients opposite r_{12} are $+2$, -1, and 0.

As f_{12} is known, and f_{13} is the only unknown quantity left for Joint (1), the spaces opposite the member, 1-2, in the ΣM_1-table may now be filled. Multiply S_{12} by the coefficients of x, y, and k, opposite f_{12} in the U_{12}-table, that is, $+1$, 0, and 0, and insert the values in the corresponding spaces of the 1-2 row, ΣM_1-table. Now, as $\Sigma M_1 = 0$, the coefficients in the 1-3 row are the negative values of those of the 1-2 row. Hence, the ΣM_1-table is filled.

It is now possible to fill in the coefficients in the f_{12} row of the U_{13}-table, as the coefficients of the x, y, and k, of the 1-3 row, ΣM_1-table, are really a statement that M_{13} is a linear expression in x and y, and, if divided by S_{13}, give the coefficients of x and y and the constant term, to be inserted in the U_{13}-table.

Now, turn to Equation (83), namely,

$$r_{lm} = \frac{U_{ln}}{U_{lm}} \cdot r_{ln} + \frac{K_{mln}}{U_{lm}}$$

For r_{13} and r_{12}, this becomes:

$$r_{13} = \frac{U_{12}}{U_{13}} \cdot r_{12} + \frac{K_{A1}}{U_{13}}$$

The $\frac{K_{A1}}{U_{13}}$ has already been determined, and is the quantity in the extended space of the U_{13}-table. Multiply the coefficient of the x in the r_{12}-expression in the U_{12}-table, by $\frac{U_{12}}{U_{13}}$, and insert the result in the x-space, opposite r_{13} of the U_{13}-table. Repeat this with the y-coefficient, but place the quantity thus obtained from the constant term of r_{12}, above the extended space of the U_{13}-table. On adding this to the quantity in the extended space, the result is the quantity to be inserted in the k-space of r_{13}. It must be noted that, as soon as two rows of any U-table are filled in, the remainder follow, from

Equations (98) and (99). Hence, subtracting the f_{13}-coefficients from the r_{13}-coefficients, gives the $+(f_{13}-f_{31})$-coefficients; the correct signs to take care of the $\pm(f_{13}-f_{31})$-coefficients are placed on the side. Adding these coefficients, with the lower signs, to the coefficients in the row immediately above, gives the coefficients of x, y and the constant term for f_{31}. Finally, adding the set of quantities in the f_{31}-row to that immediately above, gives r_{31}.

Turning now to the U_{23}-table, note that Equation (83) becomes:

$$r_{32} = \frac{U_{31}}{U_{32}} \cdot r_{31} + \frac{K_{A3}}{U_{32}}$$

or,

$$\frac{1}{3} r_{32} = \frac{U_{31}}{3\,U_{32}} \cdot r_{31} + \frac{K_{A3}}{3\,U_{32}}$$

from which to fill in the spaces for $\frac{1}{3} r_{32}$. The constant term of this expression is already in the extended space opposite $\frac{1}{3} r_{32}$. The factor with which to multiply the various quantities in the r_{31}-row of the U_{13}-table is, therefore, $\frac{U_{31}}{3\,U_{32}}$. Note, here again, the k-term resulting from this operation is placed immediately above the extended space of $\frac{1}{3} r_{32}$. From the fact that,

$$\frac{1}{3} r_{23} = \frac{U_{21}}{3\,U_{23}} \cdot r_{21} + \frac{-K_{A2}}{3\,U_{23}}$$

proceed in exactly a similar way to fill the spaces of the $\frac{1}{3} r_{23}$-row of the U_{23}-table. With two rows completed the remaining spaces are readily filled in. Adding the $\frac{1}{3} r_{23}$-row to the $\frac{1}{3} r_{32}$-row fills the spaces that lie between these rows. Adding the $\frac{1}{3} r_{23}$-row to the $\frac{1}{3}(r_{23} + r_{32})$-row gives the f_{23}-row; a reference to Equation (85) proves this. The remainder follows. The U-tables of the three members of the triangle, A, are now complete.

To fill in the check-table on the member, 2-3, turn to Equation (81) and express it in terms of the quantities of the ΔA. Thus,

$$\left.\begin{aligned} K_{A1} &= U_{13}(2f_{13}-f_{31}) - U_{12}(2f_{12}-f_{21}) \\ K_{A2} &= U_{21}(2f_{21}-f_{12}) - U_{23}(2f_{23}-f_{32}) \\ K_{A3} &= U_{32}(2f_{32}-f_{23}) - U_{31}(2f_{31}-f_{13}) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (100)$$

However, as the sum of the three angles of a triangle is constant,

$$K_{A1} + K_{A2} + K_{A3} = 0$$

Note that $U_{13} = U_{31}$, $U_{21} = U_{12}$, and $U_{32} = U_{23}$. Adding Equations (100) and dividing by 3,

$$U_{13}(f_{13}-f_{31}) + U_{21}(f_{21}-f_{12}) + U_{32}(f_{32}-f_{23}) = 0$$

that is,

$$-U_{13}(f_{13}-f_{31}) - U_{21}(f_{21}-f_{12}) + U_{32}(f_{23}-f_{32}) = 0.$$

Hence, multiply the $(f_{23} - f_{32})$-coefficients in the U_{23}-table by U_{23}, to obtain the coefficients in the 2-3 row of the check-table. Corresponding quantities are found from the U_{13} and U_{12}-tables, noting that the lower signs of the coefficients are used in each case. If the sum of the x-coefficients in the check-table, of the y-coefficients, and also the sum of the constant terms, is zero, the check is complete. In filling all the remaining tables, it will be found that the relation, $\Sigma M = 0$, is not used at all in completing the ΣM-tables for the last two joints—in this case, the ΣM_2 and ΣM_4-tables.

Adding the three columns in each, an expression is obtained from the M_2-table of the form $(a x + b y + c)$; but as $\Sigma M_2 = 0$,

$$a x + b y + c = 0$$

Similarly, from $\Sigma M_4 = 0$, results an equation such as,

$$a' x + b' y + c' = 0$$

By solving for x and y, from these equations are obtained what were assumed to be known at the outset, namely, f_{12} and f_{21}. Next fill in the small tables around the joints (Fig. 48(b)); these tables have been mentioned, but not explained. Take the table for f_{31}. Transfer the k-value, found in the f_{31}-row of the U_{13}-table, to the space next to k in the f_{31}-table. Multiply the x-coefficient of f_{31} in the U_{13}-table by the value of x just found, and insert the result in the space next to x in the f_{31}-table, and, similarly, for the y-space. Now, add these two quantities, and insert the result in the space above that containing the k-value. Add the k-value to the quantity above it, and place the result in the last space of the f_{31}-table. The quantity thus obtained is the secondary stress of the member, 1-3, for End 3. The computer should proceed in the same manner for all the f-tables and thus determine all the secondary stresses.

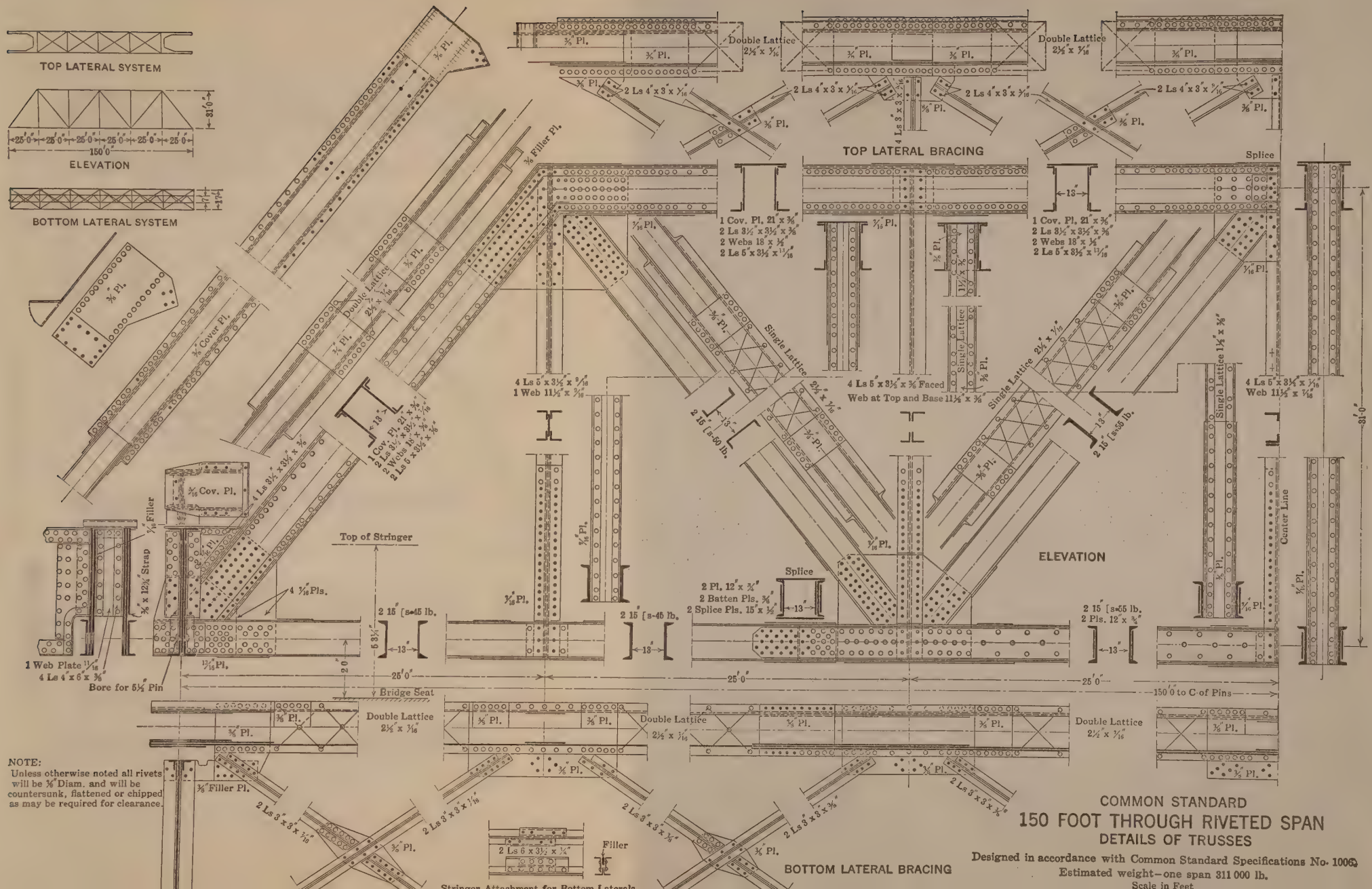
PART III.—A CRITICAL COMPARISON BETWEEN THE METHODS OF SOLVING SECONDARY STRESSES IN BRIDGE TRUSSES

1.—Introduction

Many methods of computing the secondary stresses in the trusses of a bridge have been described. At once the question arises: "Which one is the best in its practical application"? In order to arrive at a definite conclusion, it is proposed to apply each method in turn to the same bridge under a load, identical for all the methods. Many comparisons have been made, generally for a bridge loaded symmetrically. This comparison will be for an unsymmetrical load, by which each method will be tested in the severest manner; the one best enduring the test will undoubtedly be the best method in practice.

Plate I and Figs. 49 and 50 show the design of a 150-ft., common standard through riveted Warren truss bridge in present use. The trusses are composed entirely of triangular elements and if they had frictionless hinges would be regarded as statically determinate. Therefore, each of the methods may be applied without adding any further explanations to the theoretical investigations already given.

PLATE I.
TRANS., AM. SOC. CIV. ENGRS.
VOL. 89, PAPER No. 1574.
VON ABO ON
SECONDARY STRESSES IN BRIDGES.



NOTE:
Unless otherwise noted all rivets will be ⅞" Diam. and will be countersunk, flattened or chipped as may be required for clearance.

COMMON STANDARD
150 FOOT THROUGH RIVETED SPAN
DETAILS OF TRUSSES

Designed in accordance with Common Standard Specifications No. 1006.
Estimated weight—one span 311 000 lb.
Scale in Feet

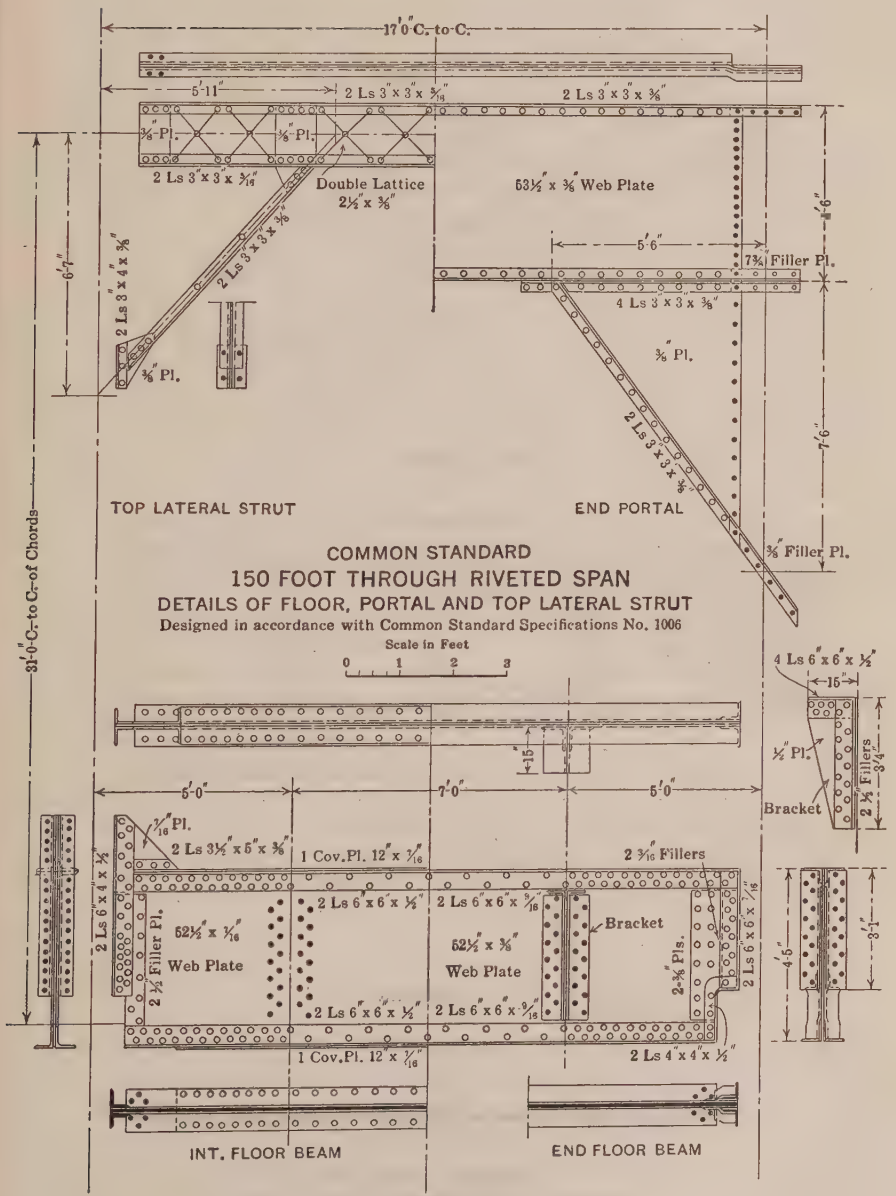
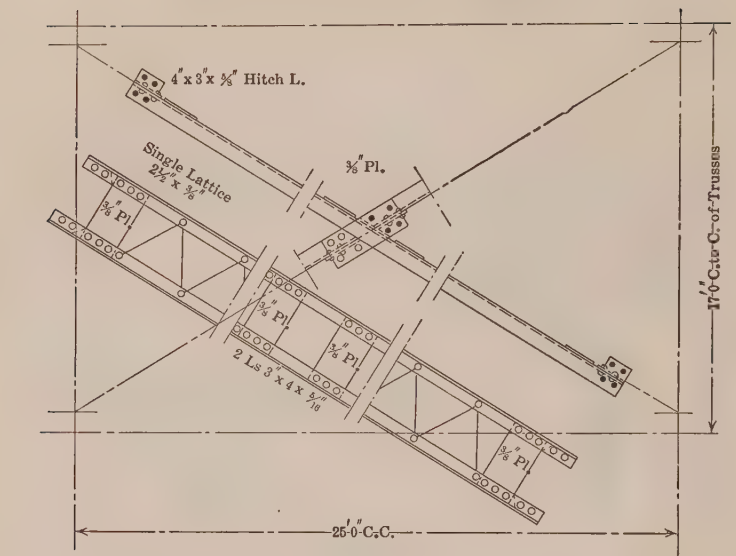
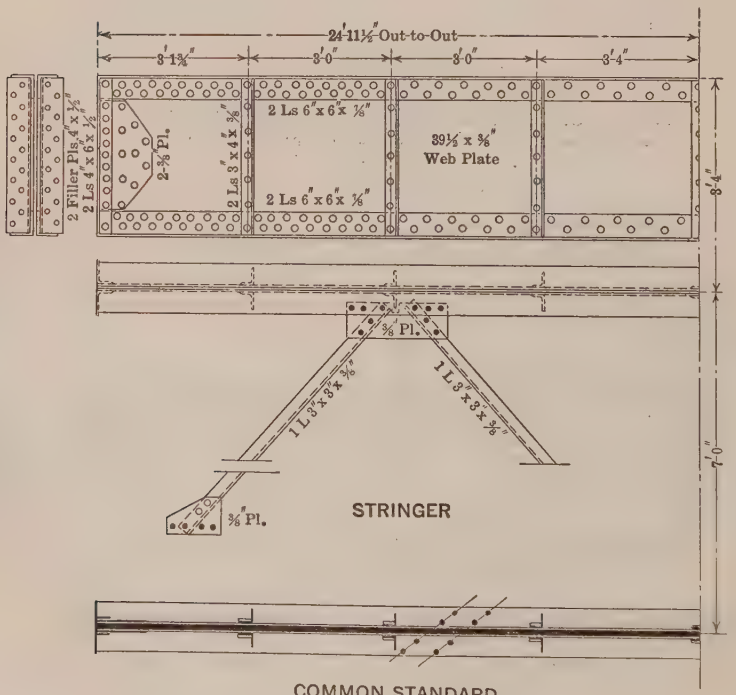

17'0" C. to C.
5'11"
2 Ls 3" x 3" x 5/16"
2 Ls 3" x 3" x 3/8"
3/8" Pl.
1/8" Pl.
2 Ls 3" x 3" x 5/16"
Double Lattice 2 1/2" x 3/8"
53 1/2" x 3/8" Web Plate
6'6"
5'6"
7 3/4" Filler Pl.
4 Ls 3" x 3" x 3/8"
2 Ls 3" x 3" x 3/8"
3/8" Pl.
6'7"
2 Ls 3" x 4" x 3/8"
3/8" Pl.
2 Ls 3" x 3" x 3/8"
7'6"
TOP LATERAL STRUT
END PORTAL
3/8" Filler Pl.
31'0" C. to C. of Chords
COMMON STANDARD
150 FOOT THROUGH RIVETED SPAN
DETAILS OF FLOOR, PORTAL AND TOP LATERAL STRUT
Designed in accordance with Common Standard Specifications No. 1006
Scale in Feet
0 1 2 3
4 Ls 6" x 6" x 1/2"
15"
1/2" Pl.
2 1/2" Fillers
3 3/4"
Bracket
5'0"
7'0"
5'0"
7/16" Pl.
2 Ls 3 1/2" x 5" x 3/8"
1 Cov. Pl. 12" x 7/16"
2 3/16" Fillers
2 Ls 6" x 4" x 1/2"
2 Ls 6" x 6" x 1/2"
2 Ls 6" x 6" x 5/16"
Bracket
52 1/2" x 7/16"
Web Plate
52 1/2" x 3/8"
Web Plate
2 1/2" Filler Pl.
2-3/8" Pls.
2 Ls 6" x 6" x 1/2"
2 Ls 6" x 6" x 1/2"
2 Ls 6" x 6" x 5/16"
4'5"
3'1"
1 Cov. Pl. 12" x 7/16"
2 Ls 4" x 4" x 1/2"
INT. FLOOR BEAM
END FLOOR BEAM

FIG. 49.



TOP LATERALS



STRINGER

COMMON STANDARD
150 FOOT THROUGH RIVETED SPAN
DETAILS OF FLOOR AND LATERALS
Designed in accordance with
Common Standard Specifications No. 1006
Scale in Feet
0 1 2 3

FIG. 50.

Because influence lines are to be used extensively in this study, a single load of definite value is chosen. Influence lines deal with units of load; such a unit may be 1 lb., 1 kip, or 1 000 lb., 1 ton, etc. Here the kip will be adopted as the unit. When the truss has a load of 1 kip at the central panel point of the bottom chord, the left-hand reaction is ½ kip. If the load were 1½ kips placed at the first panel point to the right of the center, the left-hand reaction would again be ½ kip. Finally, if it were 3 kips placed at the panel point to the right, two panel distances from the center, the left-hand reaction would still be ½ kip. It will be found that several members on the left side of the truss are stressed equally under the 1-kip and 1½-kip loads, while an even greater number are stressed equally under the 1½-kip and the 3-kip loads. This method of loading a bridge for the determination of its secondary stresses is due to Professor Turneaure; its advantages will be shown later. In this study, the load will be taken as 1½ kips, placed at the panel point just to the right of the center; this is Panel Point (9).

From Plate I and Figs. 49 and 50 are found the moments of inertia, the centers of gravity of sections, the distances to the extreme fibers, the areas of cross-section and the lengths of the members. None of the calculations is reproduced, only the results are given. It should be remembered that E, the modulus of elasticity, may be regarded as unity throughout the investigation.

The work of calculating the secondary stresses in the trusses when the bridge is under a large number of concentrated loads, will not be appreciably greater than when a single load is considered. The secondary stresses depend on the set of elongations of all the members considering the bridge as statically determinate. The actual work of determining the secondary stresses from the set of elongations under many loads has, therefore, exactly the same weight as for a single load; there is, of course, a little more preliminary work necessary in calculating the elongations.

As was stated previously, the exact method of Manderla (see page 9) is one that involves a great amount of work, and gives results that differ little from those obtained by the modified method. This modified method is generally known as Manderla's method. No solution is made by the exact method; the method, designated "Manderla" in this comparison, is the modified form, due to Winkler. The whole investigation was facilitated by the use of a calculating machine.

2.—Application of Methods

Manderla's Method.—In order to avoid repetition, only one computation was prepared to illustrate the two methods of setting about a solution. The first method would contain the solution of a set of simultaneous linear equations by means of the method of substitution, while the second would solve a set of simultaneous linear equations by means of Gauss' theory of normal equations. Otherwise, the first method would be similar to the second. Only the results of the first method are given in Table 24.

Manderla-Gauss Method.—The formation of the equations for the truss and loading shown in Fig. 51 and Plate I, is given in Table 1. The re-

TABLE 1.—Formation of Equations.

1.—$-15.930\,\tau_1 + 5.464\,\tau_2 + 2.501\,\tau_3 + 80.58 - 52.62 = 0$

	52.62
	27.96

2.—$-30.806\,\tau_2 + 7.951\,\tau_4 + 1.685\,\tau_5 + 0.303\,\tau_8 + 5.464\,\tau_1 - 216.08 + 270.89 = 0$

		45.61
		11.78
		273.47
		270.89
		2.58

3.—$-10.610\,\tau_3 + 2.501\,\tau_1 + 0.303\,\tau_2 + 2.501\,\tau_5 - 55.98 + 40.29 = 0$

		9.81
		65.79
		40.29
		25.50

4.—$-32.184\,\tau_4 + 7.951\,\tau_6 + 0.190\,\tau_5 + 7.951\,\tau_2 + 532.64 - 11.34 = 0$

		308.02
		840.06
		11.34
		828.72

5.—$-19.528\,\tau_5 + 2.501\,\tau_3 + 1.685\,\tau_2 + 0.190\,\tau_4 + 1.800\,\tau_6 + 3.588\,\tau_7 - 569.62 + 25.43 = 0$

			16.21
			45.61
			4.87
			636.31
			25.43
			610.88

6.—$-39.610\,\tau_6 + 7.951\,\tau_8 + 1.800\,\tau_9 + 0.303\,\tau_7 + 1.800\,\tau_5 + 7.951\,\tau_4 + 565.42 - 44.30 = 0$

		1 510.93	18.18
		2 076.35	105.62
		168.10	168.10
		1 908.25	

7.—$-14.958\,\tau_7 + 3.588\,\tau_5 + 0.303\,\tau_6 + 3.588\,\tau_9 - 314.36 = 0$

		122.24
		6.44
		443.04

8.—$-19.528\,\tau_9 + 3.588\,\tau_7 + 1.800\,\tau_6 + 0.190\,\tau_8 + 1.685\,\tau_{10} + 2.501\,\tau_{11} - 1\,523.90 + 22.68 = 0$

		139.00	58.76
		44.30	81.44
		1 707.20	
		81.44	
		1 625.76	

9.—$-32.184\,\tau_8 + 7.951\,\tau_{10} + 0.190\,\tau_9 + 7.951\,\tau_6 + 3\,067.22 - 153.80 = 0$

	166.72	13.42
2 900.50	166.72	

TABLE 1.—(*Continued*).

$$10.—10.610\,\tau_{11} + 2.501\,\tau_9 + 0.303\,\tau_{10} + 2.501\,\tau_{12} - 25.46 + 13.79 = 0$$

$$\begin{array}{r} 475.27 \\ \hline 500.73 \\ 13.79 \\ \hline 486.94 \end{array}$$

$$11.—30.806\,\tau_{10} + 5.464\,\tau_{12} + 0.303\,\tau_{11} + 1.685\,\tau_9 + 7.951\,\tau_8 - 161.50 + 176.05 = 0$$

$$\begin{array}{rr} 228.96 & 19.63 \\ \hline 390.46 & 195.68 \\ 195.68 & \\ \hline 194.78 & \end{array}$$

$$12.—15.930\,\tau_{12} + 2.501\,\tau_{11} + 5.464\,\tau_{10} + 352.10 - 32.36 = 0$$

$$\begin{array}{r} 32.36 \\ \hline 319.74 \end{array}$$

mainder of the solution of the secondary stresses according to Gauss' theory of normal equations is given in Tables 2 to 7.

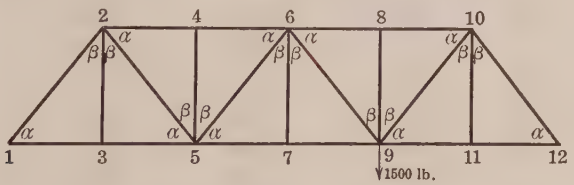


FIG. 51.

TABLE 2.—MANDERLA-GAUSS METHOD. ELEMENT OF MEMBERS.

Member.	Sectional area, in square inches.	Length, l, in inches.	Primary stress, in pounds.	Unit stress, in pounds.	I, in inches^4.	$K = \frac{I}{l}$.	Make-up of member.
1- 3	26.48	300	+ 403.2	+15.23	750 2	2.501	Two 15-in. angles, 45 lb.
11-12			+ 806.5	+30.46			
3- 5	26.48	300	+ 403.2	+15.23	750.2	2.501	Two 15-in. angles, 45 lb.
9-11			+ 806.5	+30.46			
5- 7	50.36	300	+1 209.7	+24.02	1 076.4	3.588	Two 15-in. angles, 55 lb.
7- 9			+1 209.7	+24.02			Two plates. 12 in. × ¾ in.
1- 2	49.45	477.9	— 642.3	—12.99	2 611.5	5.464	One cover plate, 21 in. × ⅜ in.; Two webs 18 in. × ⅝ in.
10-12			−1 284.7	−25.98			Two angles 3½ × 3½ × 7/16 in.; Two angles 5 × 3½ × ⅞ in.
2- 4	43.33	300	— 806.5	—18.61	2 385.4	7.951	One cover plate, 21 in. × ⅜ in.; Two webs 18 in. × ½ in.
8-10			−1 612.9	−37.22			Two angles 3½ × 3½ × ⅜ in.; Two angles 5 × 3½ × 13/16 in.
4- 6	43.33	300	— 806.5	—18.61	2 385.4	7.951	One cover plate 21 in. × ⅜ in.; Two webs 18 in. × ½ in.
6- 8			−1 612.9	−37.22			Two angles 3½ × 3½ × ⅜ in.; Two angles 5 × 3½ × 13/16 in.
2- 3	24.35	372	.....	.....	112.86	0.303	Four angles 5 × 3½ × 9/16 in.
6- 7			.....	.....			One web 11½ × 9/16 in.
10-11			.....	.....			
4- 5	12.20	372	.....	.....	70.62	0.190	Four angles 5 in. × 3½ × ⅜ in.
8- 9			.....	.....			
2- 5	29.42	477.9	+ 642.3	+21.83	805.4	1.685	Two 15-in. angles, 50 lb.
9-10			+1 284.7	+43.67			
5- 6	32.36	477.9	— 642.3	—19.85	860.4	1.800	Two 15-in. angles, 55 lb.
6- 9			+ 642.3	+19.85			

Table 2 is obtained from the dimensions of the truss and is self-explanatory. Use is then made of Equations (2), namely,

$$E \cdot \Delta \alpha = (s_1 - s_2) \cot \gamma + (s_1 - s_3) \cot \beta$$
$$E \cdot \Delta \beta = (s_2 - s_3) \cot \alpha + (s_2 - s_1) \cot \gamma$$
$$E \cdot \Delta \gamma = (s_3 - s_1) \cot \beta + (s_3 - s_2) \cot \alpha$$

to obtain Table 3.

The method of constructing Tables 3 and 4 and hence of forming the expression for the equality of the moments acting at each joint to zero are given in standard works on structural design.*

A little practice will enable the engineer to perform the latter operation mechanically.

TABLE 3.—MANDERLA-GAUSS METHOD. ANGLES AND FUNCTIONS.

Triangle.	Angle.	cot. α = 0.8065	cot. β = 1.24	$\frac{E\,\delta\ \text{Angle}}{10^6}$
2.1.3	2.1.3	.....	+16.11	+16.11
	1.3.2	−22.76	−16.11	−38.87
	3.2.1	+22.76	.....	+22.76
2.3.5	2.3.5	+ 5.32	+27.07	+32.39
	3.5.2	.....	−27.07	−27.07
	5.2.3	− 5.32	.....	− 5.32
2.5.4	2.5.4	−32.61	.....	−32.61
	5.4.2	+32.61	+27.07	+59.68
	4.2.5	.....	−27.07	−27.07
4.5.6	4.5.6	+ 1.00	.....	+ 1.00
	5.6.4	.....	+24.61	+24.61
	6.4.5	− 1.00	−24.61	−25.61
5.7.6	5.7.6	−35.38	−24.61	−59.99
	7.6.5	+35.38	.....	+35.38
	6.5.7	.....	+24.61	+24.61
6.7.9	6.7.9	− 3.36	+24.61	+21.25
	7.9.6	.....	− 24.61	−24.61
	9.6.7	+ 3.36	.....	+ 3.36
6.9.8	6.9.8	−46.03	.....	−46.03
	9.8.6	+46.03	+24.61	+70.64
	8.6.9	.....	−24.61	−24.61
8.9.10	8.9.10	−65.24	.....	−65.24
	9.10.8	.....	−54.15	−54.15
	10.8.9	+65.24	+54.15	+119.39
9.11.10	9.11.10	+10.65	+54.15	+64.80
	11.10.9	−10.65	.....	−10.65
	10.9.11	.....	−54.15	−54.15
10.11.12	10.11.12	−45.52	−32.22	−77.74
	11.12.10	.....	+32.22	+32.22
	12.10.11	+45.52	.....	+45.52

Table 5 is then constructed. It represents the solution of the twelve equations already formed; Gauss' method of normal equations is used. Textbooks on least squares explain the theory; the remarks in the second column ought, however, to be sufficient to indicate the reasoning. The check term in the last column is the sum of all the coefficients of the τ's in an equation, together with its absolute term. Thus, + 15.930 + 5.464 + 2.501 + 2.796 = + 26.691 for Equation (1). All the absolute terms are divided by 10, in order to obtain the three decimal places of the τ-coefficients. The resulting τ's are, therefore, to be multiplied by 10, before being used in Table 5.

* See Johnson, Bryan, and Turneaure, "Theory of Framed Structures."

TABLE 4.—MANDERLA-GAUSS METHOD. CHANGES OF ANGLE AND SUMMATIONS.

Joint.	Member.	$K = \frac{I}{l}$.	Angle.	δ Angle.	Σ δ Angle.	$K\Sigma\delta$ Angle.
1	1-2	5.464	.....	.....	.....	.....
	1-3	2.501	2.1.3	+16.11	+16.11	+40.29
	.....	7.965	.....	.....	.....	+40.29
2	2-4	7.951	.....	.....	.....	.....
	2-5	1.685	4.2.5	−27.07	−27.07	−45.61
	2-3	0.303	5.2.3	− 5.32	−32.39	− 9.81
	2-1	5.464	3.2.1	+22.76	− 9.63	−52.62
	.....	15.403	.....	.....	.....	−108.04
3	3-1	2.501	.....	.....	.....	.....
	3-2	0.303	1.3.2	−38.87	−38.87	−11.78
	3-5	2.501	2.3.5	+32.39	− 6.48	−16.21
	.....	5.305	.....	.....	.....	−27.99
4	4-6	7.951	.....	.....	.....	.....
	4-5	0.190	6.4.5	−25.61	−25.61	− 4.87
	4-2	7.951	5.4.2	+59.68	+34.07	+270.89
	.....	16.092	.....	.....	.....	+266.02
5	5-3	2.501	.....	.....	.....	.....
	5-2	1.685	3.5.2	−27.07	−27.07	−45.61
	5-4	0.190	2.5.4	−32.61	−59.68	−11.34
	5-6	1.800	4.5.6	+ 1.00	−58.68	−105.62
	5-7	3.588	6.5.7	+24.61	−34.07	−122.24
	.....	9.764	.....	.....	.....	−284.81
6	6-8	7.951	.....	.....	.....	.....
	6-9	1.800	8.6.9	−24.61	−24.61	−44.30
	6-7	0.303	9.6.7	+ 3.36	−21.25	− 6.44
	6-5	1.800	7.6.5	+35.38	+14.13	+25.43
	6-4	7.951	5.6.4	+24.61	+38.74	+308.02
	.....	19.805	.....	.....	.....	+282.71
7	7-5	3.588	.....	.....	.....	.....
	7-6	0.303	5.7.6	−59.99	−59.99	−18.18
	7-9	3.588	6.7.9	+21.25	−38.74	−139.00
	.....	7.479	.....	.....	.....	−157.18
8	8-10	7.951	.....	.....	.....	.....
	8-9	0.190	10.8.9	+119.39	+119.39	+22.68
	8-6	7.951	9.8.6	+70.64	+190.03	+1510.93
	.....	16.092	.....	.....	.....	+1533.61
9	9-7	3.588	.....	.....	.....	.....
	9-6	1.800	7.9.6	−24.61	−24.61	−44.30
	9-8	0.190	6.9.8	−46.03	−70.64	−13.42
	9-10	1.685	8.9.10	−65.24	−135.88	−228.96
	9-11	2.501	10.9.11	−54.15	−190.03	−475.27
	.....	9.764	.....	.....	.....	−761.95
10	10-12	5.464	.....	.....	.....	.....
	19-11	0.303	12.10.11	+45.52	+45.52	+13.79
	10-9	1.685	11.10.9	−10.65	+34.87	+58.76
	10-8	7.951	9.10.8	−54.15	−19.28	−153.30
	.....	15.403	.....	.....	.....	−80.75
11	11-9	2.501	.....	.....	.....	.....
	11-10	0.303	9.11.10	+64.80	+64.80	+19.63
	11-12	2.501	10.11.12	−77.74	−12.94	−32.36
	.....	5.305	.....	.....	.....	−12.73
12	12-11	2.501	.....	.....	.....	.....
	12-10	5.464	11.12.10	+32.22	+32.22	+176.05
	.....	7.965	.....	.....	.....	+176.05

When the operations on Equations (1) and (2), Table 5, have been performed, Equation I is obtained by adding the two equations above. The check terms also are added. The final check for Equation I is that the sum of the coefficients of its τ's, together with its absolute term, must give the check term already found by adding -9.155 and $+45.951$, namely, $+36.796$. This check is applied for Equations II, III, IV, etc., until finally Equation XI is reached, which gives the value for τ_{12}. Going back to Equations X, IX, VIII, etc., τ_{10}, τ_{11}, τ_8, etc., are solved in turn.

These values are now inserted in Table 6, and by use of Equation (27), the τ's of all the members are obtained. For example, $\tau_{12} = -2.16$. In order to get the τ of the next member, namely, 1-3, it is necessary to add the δ-angle of

TABLE 5.—MANDERLA-GAUSS METHOD. VALUES FOR EQUATIONS.

Number of equation.	Remarks.	τ_1.	τ_2.	τ_3.	τ_4.	τ_5.	τ_6.	τ_7.	τ_9.	τ_8.	τ_{11}.	τ_{10}.	τ_{12}.	Absolute term.	Check.
1	.....	+15.930	+5.464	+2.501	.....	.....	.....	.....	.....	.....	.....	.....	.....	+2.796	+26.691
1′	$1 \times -\frac{5.464}{15.930}$, *i. e.*, 0.34300	[−5.464]	−1.874	−0.858	.....	.....	.....	.....	.....	.....	.....	.....	.....	−0.959	−9.155
2	.....	[+5.464]	+30.806	+0.303	+7.951	+1.685	.....	.....	.....	.....	.....	.....	.....	−0.258	+45.951
I	$1' + 2$	.....	+28.932	−0.555	+7.951	+1.685	.....	.....	.....	.....	.....	.....	.....	−1.217	+36.796
1″	$1 \times -\frac{2.501}{15.930}$, *i. e.*, 0.15700	[−2.501]	[−0.858]	−0.393	.....	.....	.....	.....	.....	.....	.....	.....	.....	−0.439	−4.190
I′	$I \times +\frac{0.555}{28.932}$, *i. e.*, 0.01918	.....	[+0.555]	−0.011	+0.153	+0.032	.....	.....	.....	.....	.....	.....	.....	−0.023	+0.706
3	.....	[+2.501]	[+0.303]	+10.610	.....	+2.501	.....	.....	.....	.....	.....	.....	.....	−2.550	+13.365
II	$1'' + I' + 3$	.....	.....	+10.206	+0.153	+2.533	.....	.....	.....	.....	.....	.....	.....	−3.012	+9.881
I″	$I \times -\frac{7.951}{28.932}$, *i. e.*, 0.27482	.....	[−7.951]	[+0.153]	−2.185	−0.463	.....	.....	.....	.....	.....	.....	.....	+0.334	−10.112
II′	$II \times -\frac{0.153}{10.206}$, *i. e.*, 0.01499	.....	.....	[−0.153]	−0.002	−0.038	.....	.....	.....	.....	.....	.....	.....	+0.045	−0.148
4	.....	.....	[+7.951]	.....	+32.184	+0.190	+7.951	.....	.....	.....	.....	.....	.....	+82.872	+131.148
III	$I'' + II' + 4$	.....	.....	.....	+29.997	−0.311	+7.951	.....	.....	.....	.....	.....	.....	+83.251	+120.888
I‴	$I \times -\frac{1.685}{28.932}$, *i. e.*, 0.05824	.....	[−1.685]	[+0.032]	[−0.463]	−0.098	.....	.....	.....	.....	.....	.....	.....	+0.071	−2.143
II″	$II \times -\frac{2.533}{10.206}$, *i. e.*, 0.24819	.....	.....	[−2.533]	[−0.038]	−0.629	.....	.....	.....	.....	.....	.....	.....	+0.748	−2.452
III′	$III \times +\frac{0.311}{29.997}$, *i. e.*, 0.01037	.....	.....	.....	[+0.311]	−0.003	+0.082	.....	.....	.....	.....	.....	.....	+0.863	+1.253
5	.....	.....	[+1.685]	[+2.501]	[+0.190]	+19.528	+1.800	+3.588	.....	.....	.....	.....	.....	−61.088	−31.796
IV	$I'' + II'' + III' + 5$	.....	.....	.....	.....	+18.798	+1.882	+3.588	.....	.....	.....	.....	.....	−59.406	−35.138

TABLE 5.—(*Continued*).

Number of equation.	Remarks.	τ_1.	τ_2.	τ_3.	τ_4.	τ_5.	τ_6.	τ_7.	τ_9.	τ_8.	τ_{11}.	τ_{10}.	τ_{12}.	Absolute term.	Check.
III''	III $\times -\frac{7.951}{29.997}$, *i. e.*, 0.26506	.....	.....	.....	[−7.951]	[+0.082]	−2.107	.....	.....	.....	.....	.....	.....	−22.067	−32.043
IV'	IV $\times -\frac{1.882}{18.798}$, *i. e.*, 0.10012	.....	.....	.....	.....	[−1.882]	−0.188	−0.359	.....	.....	.....	.....	.....	+5.947	+3.518
6	.....	.....	.....	.....	[+7.951]	[+1.800]	+39.610	+0.303	+1.800	+7.951	.....	.....	.....	+190.825	+250.240
V	III'' + IV' + 6	.....	.....	.....	.....	.....	+37.315	−0.056	+1.800	+7.951	.....	.....	.....	+174.705	+221.715
IV''	IV $\times -\frac{3.588}{18.798}$, *i. e.*, 0.19087	.....	.....	.....	.....	[−3.588]	[−0.359]	−0.685	.....	.....	.....	.....	.....	+11.339	+6.707
V'	V $\times +\frac{0.056}{37.315}$, *i. e.*, 0.00150	.....	.....	.....	.....	.....	[+0.056]	−0.000	+0.003	+0.012	.....	.....	.....	+0.262	+0.333
7	.....	.....	.....	.....	.....	[+3.588]	[+0.303]	+14.958	+3.588	.....	.....	.....	.....	−44.304	−21.867
VI	IV'' + V' + 7	.....	.....	.....	.....	.....	.....	+14.273	+3.591	+0.012	.....	.....	.....	−32.703	−14.827
V''	V $\times -\frac{1.800}{37.315}$, *i. e.*, 0.04824	.....	.....	.....	.....	.....	[−1.800]	[+0.003]	−0.087	−0.384	.....	.....	.....	−8.427	−10.695
VI'	VI $\times -\frac{3.591}{14.273}$, *i. e.*, 0.25159	.....	.....	.....	.....	.....	.....	[−3.591]	−0.903	−0.003	.....	.....	.....	+8.228	+3.730
8	.....	.....	.....	.....	.....	.....	[+1.800]	[+3.588]	+19.528	+0.190	+2.501	+1.685	.....	−162.576	−133.284
VII	V'' + VI' + 8	.....	.....	.....	.....	.....	.....	.....	+18.538	−0.197	+2.501	+1.685	.....	−162.775	−140.249
V'''	V $\times -\frac{7.951}{37.315}$, *i. e.*, 0.21308	.....	.....	.....	.....	.....	[−7.951]	[+0.012]	[−0.384]	−1.694	.....	.....	.....	−37.226	−47.243
VI''	VI $\times -\frac{0.012}{14.273}$, *i. e.*, 0.00084	.....	.....	.....	.....	.....	.....	[−0.012]	[−0.003]	−0.000	.....	.....	.....	+0.027	+0.012
VII'	VII $\times +\frac{0.197}{18.538}$, *i. e.*, 0.01063	.....	.....	.....	.....	.....	.....	.....	[+0.197]	−0.012	+0.027	+0.018	.....	−1.730	−1.490
9	.....	.....	.....	.....	.....	.....	[+7.951]	.....	[+0.190]	+32.184	.....	+7.951	.....	+290.050	+338.326
VIII	V''' + VI'' + VII' + 9	.....	.....	.....	.....	.....	.....	.....	.....	+30.488	+0.027	+7.969	.....	+251.121	+289.605

TABLE 5.—(*Continued*).

Number of equation.	Remarks.	τ_1.	τ_2.	τ_3.	τ_4.	τ_5.	τ_6.	τ_7.	τ_9.	τ_8.	τ_{11}.	τ_{10}.	τ_{12}.	Absolute term.	Check.
VII''	VII $\times -\frac{2.501}{18.538}$, *i. e.*, 0.13491	.....	.....	.....	.....	.....	.....	.....	[−2.501]	[+0.027]	−0.337	−0.227	.....	+21.960	+18.921
VIII'	VIII $\times -\frac{0.027}{30.488}$, *i. e.*, 0.00089	.....	.....	.....	.....	.....	.....	.....	.....	[−0.027]	−0.000	−0.007	.....	−0.222	−0.256
10	.....	.....	.....	.....	.....	.....	.....	.....	[+2.501]	.....	+10.610	+0.303	+2.501	−48.694	−32.779
IX	VII'' + VIII' + 10	.....	.....	.....	.....	.....	.....	.....	.....	.....	+10.273	+0.069	+2.501	−26.956	−14.114
VII'''	VII $\times -\frac{1.685}{18.538}$, *i. e.*, 0.09089	.....	.....	.....	.....	.....	.....	.....	[−1.685]	[+0.018]	[−0.227]	−0.153	.....	+14.795	+12.748
VIII''	VIII $\times -\frac{7.969}{30.488}$, *i. e.*, 0.26138	.....	.....	.....	.....	.....	.....	.....	.....	[−7.969]	[−0.007]	−2.083	.....	−65.638	−75.697
IX'	IX $\times -\frac{0.069}{10.273}$, *i. e.*, 0.00672	.....	.....	.....	.....	.....	.....	.....	.....	.....	[−0.069]	−0.000	−0.017	+0.181	+0.095
11	.....	.....	.....	.....	.....	.....	.....	.....	[+1.685]	[+7.951]	[+0.303]	+30.806	+5.464	−19.478	+26.731
X	VII''' + VIII'' + IX' + 11	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+28.570	+5.447	−70.140	−36.123
IX''	IX $\times -\frac{2.501}{10.273}$, *i. e.*, 0.24345	.....	.....	.....	.....	.....	.....	.....	.....	.....	[−2.501]	[−0.017]	−0.609	+6.563	+3.436
X'	X $\times -\frac{5.447}{28.570}$, *i. e.*, 0.19065	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	[−5.447]	−1.038	+13.372	+6.887
12	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	[+2.501]	[+5.464]	+15.930	+31.974	+55.869
XI	IX'' + X' + 12	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+14.283	+51.909	+66.192
		−0.21599	+0.35784	−0.52401	−1.90951	+3.41579	−3.13281	+0.30432	+7.92766	−9.06261	+3.48761	+3.14792	−3.63432	.....	.....

TABLE 6.—MANDERLA-GAUSS METHOD. SOLUTION FOR TERMS, T AND M.

1	τ_{12}	$= -2.16 + 16.11$	$M_{12} = 5.464\ (-4.32 - 6.05) \times 2 = -113.32$ in.-lb.
	τ_{13}	$= +13.95$	$M_{13} = 2.501\ (+27.90 - 5.24) \times 2 = +113.35$ in.-lb.
2	τ_{24}	$= +3.58 - 27.07$	$M_{24} = 7.951\ (+7.16 + 14.97) \times 2 = +351.91$ in.-lb.
	τ_{25}	$= -23.49 - 5.32$	$M_{25} = 1.685\ (-46.98 + 7.09) \times 2 = -134.43$ in.-lb
	τ_{23}	$= -28.81 + 22.76$	$M_{23} = 0.303\ (-57.62 - 44.11) \times 2 = -61.65$ in.-lb
	τ_{21}	$= -6.05$	$M_{21} = 5.464\ (-12.10 - 2.16) \times 2 = -155.83$ in.-lb
3	τ_{31}	$= -5.24 - 38.87$	$M_{31} = 2.501\ (-10.48 + 13.95) \times 2 = +17.36$ in.-lb
	τ_{32}	$= -44.11 + 32.39$	$M_{32} = 0.303\ (-88.22 - 28.81) \times 2 = -70.92$ in.-lb.
	τ_{35}	$= -11.72$	$M_{35} = 2.501\ (-23.44 + 34.16) \times 2 = +53.62$ in.-lb.
4	τ_{46}	$= -19.10 - 25.61$	$M_{46} = 7.951\ (-38.20 + 7.41) \times 2 = -489.62$ in.-lb.
	τ_{45}	$= -44.71 + 59.68$	$M_{45} = 0.190\ (-89.42 - 25.52) \times 2 = -43.68$ in.-lb.
	τ_{42}	$= +14.97$	$M_{42} = 7.951\ (+29.94 + 3.58) \times 2 = +533.04$ in.-lb.
5	τ_{53}	$= +34.16 - 27.07$	$M_{53} = 2.501\ (+68.32 - 11.72) \times 2 = +283.11$ in.-lb.
	τ_{52}	$= +7.09 - 32.61$	$M_{52} = 1.685\ (+14.18 - 23.49) \times 2 = -31.37$ in.-lb.
	τ_{54}	$= -25.52 + 1.00$	$M_{54} = 0.190\ (-51.04 - 44.71) \times 2 = -36.38$ in.-lb.
	τ_{56}	$= -24.52 + 24.61$	$M_{56} = 1.800\ (-49.04 - 17.20) \times 2 = -238.46$ in.-lb.
	τ_{57}	$= +0.09$	$M_{57} = 3.588\ (+0.18 + 3.04) \times 2 = +23.11$ in.-lb.
6	τ_{68}	$= -31.33 - 24.61$	$M_{68} = 7.951\ (-62.66 + 99.40) \times 2 = +584.24$ in.-lb.
	τ_{69}	$= -55.94 + 3.36$	$M_{69} = 1.800\ (-111.88 + 54.67) \times 2 = -205.96$ in.-lb.
	τ_{67}	$= -52.58 + 35.38$	$M_{67} = 0.303\ (-105.16 - 56.95) \times 2 = -98.24$ in.-lb.
	τ_{65}	$= -17.20 + 24.61$	$M_{65} = 1.800\ (-34.40 - 24.52) \times 2 = -212.11$ in.-lb.
	τ_{64}	$= +7.41$	$M_{64} = 7.951\ (+14.82 - 19.10) \times 2 = -68.06$ in.-lb.
7	τ_{75}	$= +3.04 - 59.99$	$M_{75} = 3.588\ (+6.08 + 0.09) \times 2 = +44.28$ in.-lb.
	τ_{76}	$= -56.95 + 21.25$	$M_{76} = 0.303\ (-113.90 - 52.58) \times 2 = -100.89$ in.-lb.
	τ_{79}	$= -35.70$	$M_{79} = 3.588\ (-71.40 + 79.28) \times 2 = +56.55$ in.-lb.
8	$\tau_{8.10}$	$= -90.63 + 119.39$	$M_{8.10} = 7.951\ (-181.26 + 12.20) \times 2 = -2\,688.39$ in.-lb.
	τ_{89}	$= +28.76 + 70.64$	$M_{89} = 0.190\ (+57.52 + 8.64) \times 2 = +25.14$ in.-lb.
	τ_{86}	$= +99.40$	$M_{8.6} = 7.951\ (+198.80 - 31.33) \times 2 = +2\,668.11$ in.-lb.

TABLE 6.—(*Continued*).

9	$\tau_{9.7} = +79.28 - 24.61$		$M_{9.7} = 3.588\ (+158.56 - 35.70) \times 2 = +881.64$ in.-lb.
	$\tau_{9.6} = +54.67 - 46.03$		$M_{9.6} = 1.800\ (+109.34 - 55.94) \times 2 = +192.24$ in.-lb.
	$\tau_{9.8} = +8.64 - 65.24$		$M_{9.8} = 0.190\ (+17.28 + 28.76) \times 2 = +17.50$ in.-lb.
	$\tau_{9.10} = -56.60 - 54.15$		$M_{9.10} = 1.685\ (-113.20 + 66.35) \times 2 = -157.88$ in.-lb.
	$\tau_{9.11} = -110.75$		$M_{9.11} = 2.501\ (-221.50 + 34.88) \times 2 = -933.47$ in.-lb.
10	$\tau_{10.12} = +31.48 + 45.52$		$M_{10.12} = 5.464\ (+62.96 - 4.12) \times 2 = +643.00$ in.-lb.
	$\tau_{10.11} = +77.00 - 10.65$		$M_{10.11} = 0.303\ (+154.00 + 99.68) \times 2 = +153.73$ in.-lb.
	$\tau_{10.9} = +66.35 - 54.15$		$M_{10.9} = 1.685\ (+132.70 - 56.60) \times 2 = +256.46$ in.-lb.
	$\tau_{10.8} = +12.20$		$M_{10.8} = 7.951\ (+24.40 - 90.63) \times 2 = -1\,053.19$ in.-lb.
11	$\tau_{11.9} = +34.88 + 64.80$		$M_{11.9} = 2.501\ (+69.76 - 110.75) \times 2 = -205.03$ in.-lb.
	$\tau_{11.10} = +99.68 - 77.74$		$M_{11.10} = 0.303\ (+199.36 + 77.00) \times 2 = +167.47$ in.-lb.
	$\tau_{11.12} = +21.94$		$M_{11.12} = 2.501\ (+43.88 - 36.34) \times 2 = +37.72$ in.-lb.
12	$\tau_{12.11} = -36.34 + 32.22$		$M_{12.11} = 2.501\ (-72.68 + 21.94) \times 2 = -253.80$ in.-lb.
	$\tau_{12.10} = -4.12$		$M_{12.10} = 5.464\ (-8.24 + 31.48) \times 2 = +253.97$ in.-lb.

the angle between the members; this is found from Table 4 to be + 16.11. The bending moments are then obtained by making use of Equations (39). A check is applied at this juncture; the sum of the bending moments around a joint must be zero.

Table 7 then follows directly. It is to be noted that when a member like 1-3 is subjected to the bending moment, + 113.35 in-lb., positive in a counter-clockwise direction, the top set of fibers of Member 1-3, at End (1) is in tension and the bottom set is in compression. In other words, the quantities, — 0.41, + 0.41, ± 1.13, in the last column of Table 7, Joint (1), are the secondary stresses of the top set of Member 1-2, the bottom set of Member 1-2, the top set of Member 1-3, and the bottom set of Member 1-3, in the order named, which is also the order due to a clockwise movement around Joint (1). The quantities, — 0.41 and + 0.41, are separated because the distance of the top set of fibers from the neutral plane, through the center of gravity, differs from that of the bottom set.

Müller-Breslau's Method.—The work of solving the problem is given in Tables 8 to 10, inclusive, and Fig. 52. For this solution the K's and the changes of angle were just taken from Table 4. The ρ's, which are the $\frac{Ml}{I}$-quantities, are written at both ends of all the members. Thus, ρ_1 to ρ_{42}, inclusive, are obtained. Six times the changes of angle are the quantities found in the corners.

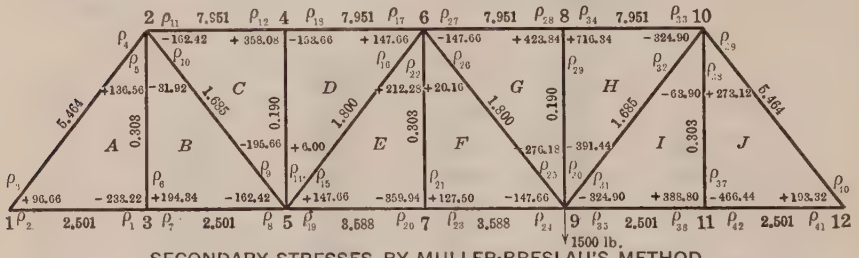
TABLE 7.—MANDERLA-GAUSS METHOD. SECONDARY STRESSES AND MEMBERS.

Joint.	Member.	Extreme fiber distance, C, in inches.	I, in inches^4.	$\frac{C}{I}$	M, in pounds.	$f = \frac{C}{I} \times M$, in pounds per square inch.
1	1-2	9.38	2 611.5	0.003592	− 113.32	− 0.41
		9.49		0.003634		+ 0.41
	1-3	7.5	750.2	0.009997	+ 113.35	± 1.13
	2-4	9.33	2 385.4	0.003911	+ 351.91	+ 1.38
		9.54		0.003999		− 1.41
2	2-5	7.5	805.4	0.009312	− 134.43	∓ 1.25
	2-3	5-28	112.86	0.046784	− 61.65	∓ 2.88
	2-1	9.49	2 611.5	0.003634	− 155.83	− 0.57
		9.38		0.003592		+ 0.56
	3-1	7.5	750.2	0.009997	+ 17.36	± 0.17
3	3-2	5.28	112.86	0.046784	− 70.92	∓ 3.32
	3-5	7.5	750.2	0.009997	+ 53.62	± 0.56
	4-6	9.33	2 385.4	0.003911	− 489.62	− 1.92
		9.54		0.003999		+ 1.96
4	4-5	5.19	70.62	0.073492	− 43.68	∓ 3.21
	4-2	9.54	2 385.4	0.003999	+ 533.04	+ 2.13
		9.33		0.003911		− 2.08
	5-3	7.5	750.2	0.009997	+ 283.11	± 2.83
	5-2	7.5	805.4	0.009312	− 31.37	∓ 0.29
5	5-4	5.19	70.62	0.073492	− 36.88	∓ 2.70
	5-6	7.5	860.4	0.008717	− 238.46	∓ 2.08
	5-7	7.5	1 076.4	0-006968	+ 23.11	± 0.16
	6-8	9.33	2 385.4	0.003911	+ 584.24	+ 2.28
		9.54		0.003999		− 2.34
	6-9	7.5	860.4	0.008717	− 205.96	∓ 1.80
6	6-7	5.28	112.86	0.046784	− 98.24	∓ 4.60
	6-5	7.5	860.4	0.008717	− 212.11	∓ 1.85
	6-4	9.54	2 385.4	0.003999	− 68.06	− 0.27
		9.33		0.003911		+ 0.27
	7-5	7.5	1 076.4	0.006968	+ 44.28	± 0.31
7	7-6	5.28	112.86	0.046784	− 100.89	∓ 4.72
	7-9	7.5	1 076.4	0.006968	+ 56.55	± 0.39
	8-10	9.33	2 385.4	0.003911	−2 688.39	−10.52
		9.54		0.003999		+10.75
8	8-9	5.19	70.62	0.073492	+ 25.14	± 1.85
	8-6	9.54	2 385.4	0.003999	+2 663.11	+10.65
		9.33		0.003911		−10.42
	9-7	7.5	1 076.4	0.006968	+ 881.64	± 6.14
	9-6	7.5	860.4	0.008717	+ 192.24	± 1.68
9	9-8	5.19	70.62	0.073492	+ 17.50	± 1.29
	9-10	7.5	805.4	0.009312	− 157.88	∓ 1.47
	9-11	7.5	750.2	0.009997	− 933.47	∓ 9.33
	10-12	9.38	2 611.5	0.003592	+ 643.00	+ 2.31
		9.49		0.003634		− 2.34
	10-11	5.28	112.86	0.046784	+ 153.73	± 7.19
10	10-9	7.5	805.4	0.009312	+ 256.46	± 2.39
	10-8	9.54	2 385.4	0.003999	−1 053.19	− 4.21
		9.33		0.003911		+ 4.12
	11-9	7.5	750.2	0.009997	− 205.03	∓ 2.05
11	11-10	5.28	112.86	0.046784	+ 167.47	± 7.83
	11-12	7.5	750.2	0.009997	+ 37.72	± 0.38
	12-11	7.5	750.2	0.009997	− 253.80	∓ 2.54
12	12-10	9.49	2 611.5	0.003634	+ 253.97	+ 0.92
		9.38		0.003592		− 0.91

It is assumed that the two quantities, ρ_1 and ρ_2, on Ends (3) and (1) of the member, 1-3, respectively, are known. $\Sigma M = 0$ for Joint (1), that is, $K_{12} \cdot \rho_3 + K_{13} \cdot \rho_2 = 0$, expressing ρ_3 in terms of ρ_2. Three of the six quantities, ρ_1, ρ_2, ρ_3, ρ_4, ρ_5, and ρ_6, therefore, are known. Applying Equations (47), (48), and (49), ρ_4, ρ_5, and ρ_6 are given in terms of ρ_1 and ρ_2 as shown.

$\Sigma M = 0$ for Joint (3) gives an expression for ρ_7 in terms of ρ_1 and ρ_2. This process is continued until the $\Sigma M = 0$ equations for Joints (10) and (12) give the solution of ρ_1 and ρ_2. On substituting these values, ρ_3, ρ_4, ρ_5, * * * ρ_{42}, are obtained as shown in Table 9. Table 10 is self-explanatory. Note that a clockwise moment was assumed to be positive at the start (see

page 19). This means that the top fiber of the member, 1-3, End (1), is + 1.14, the result of the ρ_2 being — 45.60. This agrees with the signs of the fiber stresses found in Table 7.



SECONDARY STRESSES BY MULLER-BRESLAU'S METHOD

REMARKS.

The numbers written on the members are the $K = {}^{I}/_{l}$ - Values;
e. g. 5.464 on $1-2 = K_{12}$

The numbers written in the corners are the Changes of Angle, multiplied by 6E:
e. g. +96.66 in corner of Triangle $A = 6E.\ \delta A1.$

FIG. 52.

Ritter's Method.—In Fig. 53 and Tables 11 and 12 are given the results of applying Ritter's graphical method to a solution of the truss. The unit stresses are again assumed to be known, and with these the changes of angle (see page 9) are determined graphically as indicated in Fig. 53 and Table 11.

The quantities, $\frac{I}{l}$, are the forces of the force and equilibrium polygons. As already described (page 24), these forces are laid out, spaced by horizontal distances equal to the $E \cdot \Delta\theta$'s of Table 11. This gives 3, 5, 1, 4, in order from left to right for Joint (2). The scale used is of importance. Thus (Fig. 53 (2)), 5 is at a distance corresponding to 2.72 to the left of 4, representing the change of angle, $C\,2 = -27.2$, in Table 11. When all these forces have been laid out, draw the force polygons at the sides, with the forces in the order met in going from left to right along the horizontal. Thus, the force polygon of Joint (2) has the forces, 3, 5, 1, 4, in the order named, Force 3 meaning $\frac{I_{23}}{l_{23}}$, etc. The scale of the force polygons is arbitrary.

Draw the equilibrium polygons for the undisplaced forces, obtaining the positions of their resultants. By means of Equations (53), are found the distances to which all these forces have to be displaced, and then the resultant of these forces by means of a second set of equilibrium polygons. The resultants just found will be displaced from the previous resultants. Again, use Equations (53) to obtain a fresh set of resultants, and so on, until it is found that every resultant after a certain number of trials coincides with the corresponding resultant of the preceding trial. The diagrams of Fig. 53 represent the result of the sixth trial, when the variation between every resultant of displaced forces and the corresponding resultant belonging to the fifth trial was found to be negligible.

TABLE 8.—Müller-Breslau Method. Equations Involving ρ_1 to ρ_{42}

$\Sigma M = 0$ gives :

$$
\left\{\begin{array}{l}
5.464\,\rho_3 + 2.501\,\rho_2 = 0 \quad (1)\\
7.951\,\rho_{11} + 1.685\,\rho_{10} + 0.303\,\rho_5 + 5.464\,\rho_4 = 0 \quad (2)\\
2.501\,\rho_1 + 0.303\,\rho_6 + 2.501\,\rho_7 = 0 \quad (3)\\
7.951\,\rho_{18} + 0.190\,\rho_{13} + 7.951\,\rho_{12} = 0 \quad (4)\\
2.501\,\rho_8 + 1.685\,\rho_9 + 0.190\,\rho_{14} + 1.800\,\rho_{15} + 3.588\,\rho_{19} = 0 \quad (5)\\
7.951\,\rho_{27} + 1.800\,\rho_{26} + 0.303\,\rho_{22} + 1.800\,\rho_{16} + 7.951\,\rho_{17} = 0 \quad (6)\\
3.588\,\rho_{20} + 0.303\,\rho_{21} + 3.588\,\rho_{23} = 0 \quad (7)\\
7.951\,\rho_{34} + 0.190\,\rho_{29} + 7.951\,\rho_{28} = 0 \quad (8)\\
3.588\,\rho_{24} + 1.800\,\rho_{25} + 0.190\,\rho_{30} + 1.685\,\rho_{31} + 2.501\,\rho_{35} = 0 \quad (9)\\
5.464\,\rho_{39} + 0.303\,\rho_{38} + 1.685\,\rho_{32} + 7.951\,\rho_{33} = 0 \quad (10)\\
2.501\,\rho_{36} + 0.303\,\rho_{37} + 2.501\,\rho_{42} = 0 \quad (11)\\
2.501\,\rho_{41} + 5.464\,\rho_{40} = 0 \quad (12)
\end{array}\right.
$$

Joint (1) :

$$
\begin{aligned}
&5.464\,\rho_3 + 2.501\,\rho_2 = 0\\
&\text{or,}\quad \rho_3 = -0.4577\,\rho_2.
\end{aligned}
$$

Triangle A :

$$
\left\{\begin{array}{rl}
(2\,\rho_3 - \rho_4) &= +\ 96.66 + 2\,\rho_2 - \rho_1\\
\rho_4 &= -\ 96.66 - 2\,\rho_2 + \rho_1 + 2\,\rho_3\\
&= -\ 96.66 + \rho_1 - 2.9154\,\rho_2\\
(\rho_5 - \rho_4) &= +136.56 + (\rho_1 - \rho_2)\\
\rho_5 &= +136.56 + \rho_1 - \rho_2 + \rho_4\\
&= +\ 39.90 + 2\,\rho_1 - 3.9154\,\rho_2\\
(\rho_1 - \rho_6) &= -233.22 + \rho_3 - \rho_4\\
\rho_6 &= +233.22 - \rho_3 + \rho_4 + \rho_1\\
&= +136.56 + 2\,\rho_1 - 2.4577\,\rho_2
\end{array}\right.
$$

Joint (3) :

$$
\begin{aligned}
&2.501\,\rho_1 + 0.303\,\rho_6 + 2.501\,\rho_7 = 0\\
&\therefore\ \rho_7 = -\,\rho_1 - 0.1212\,\rho_6\\
&\qquad = -16.55 - 1.2424\,\rho_1 + 0.2979\,\rho_2
\end{aligned}
$$

Triangle B :

$$
\left\{\begin{array}{rl}
2\,\rho_6 - \rho_5 &= +194.34 + 2\,\rho_7 - \rho_8\\
\rho_8 &= +194.34 + 2\,\rho_7 - 2\,\rho_6 + \rho_5\\
&= -\ 71.98 - 4.4848\,\rho_1 + 1.5958\,\rho_2\\
\rho_8 - \rho_9 &= -162.42 + \rho_6 - \rho_5\\
\rho_9 &= +162.42 - \rho_6 + \rho_5 + \rho_8\\
&= -\ \ 6.22 - 4.4848\,\rho_1 + 0.1381\,\rho_2\\
\rho_{10} - \rho_5 &= -\ 31.92 + \rho_8 - \rho_7\\
\rho_{10} &= -\ 31.92 + \rho_8 - \rho_7 + \rho_5\\
&= -\ 47.45 - 1.2424\,\rho_1 - 2.6175\,\rho_2
\end{array}\right.
$$

Joint (2) :

$$
\begin{aligned}
&7.951\,\rho_{11} + 1.685\,\rho_{10} + 0.303\,\rho_5 + 5.464\,\rho_4 = 0\\
&\therefore\ \rho_{11} = -\,0.2119\,\rho_{10} - 0.0381\,\rho_5 - 0.6872\,\rho_4\\
&\qquad = +74.93 - 0.5001\,\rho_1 + 2.7073\,\rho_2
\end{aligned}
$$

Triangle C :

$$
\left\{\begin{array}{rl}
2\,\rho_{11} - \rho_{12} &= -162.42 + 2\,\rho_{10} - \rho_9\\
\rho_{12} &= +162.42 - 2\,\rho_{10} + \rho_9 + 2\,\rho_{11}\\
&= +400.96 - 3.0002\,\rho_1 + 10.7877\,\rho_2\\
\rho_{13} - \rho_{12} &= +358.08 + \rho_9 - \rho_{10}\\
\rho_{13} &= +358.08 + \rho_9 - \rho_{10} + \rho_{12}\\
&= +800.27 - 6.2426\,\rho_1 + 13\ 5433\,\rho_2\\
\rho_9 - \rho_{14} &= -195.66 + \rho_{11} - \rho_{12}\\
\rho_{14} &= +195.66 - \rho_{11} - \rho_{12} + \rho_9\\
&= +515.47 - 6.9849\,\rho_1 + 8.2185\,\rho_2
\end{array}\right.
$$

Joint (4) :

$$
\begin{aligned}
&7.951\,\rho_{18} + 0.190\,\rho_{13} + 7.951\,\rho_{12} = 0\\
&\therefore\ \rho_{18} = -\,0.0239\,\rho_{13} - \rho_{12}\\
&\qquad = -420.09 + 3.1494\,\rho_1 - 10.9841\,\rho_2
\end{aligned}
$$

Triangle D :

$$
\left\{\begin{array}{rl}
2\,\rho_{18} - \rho_{17} &= -153.66 + 2\,\rho_{13} - \rho_{14}\\
\rho_{17} &= +153.66 - 2\,\rho_{13} - \rho_{14} + 2\,\rho_{18}\\
&= -1\ 771.59 + 11.7991\,\rho_1 - 40.8363\,\rho_2\\
\rho_{16} - \rho_{17} &= +147.66 + \rho_{14} - \rho_{13}\\
\rho_{16} &= +147.66 + \rho_{14} - \rho_{13} + \rho_{17}\\
&= -1\ 908.73 + 11.0568\,\rho_1 - 46.1611\,\rho_2\\
\rho_{14} - \rho_{15} &= +6.00 + \rho_{13} - \rho_{17}\\
\rho_{15} &= -\ 6.00 - \rho_{13} + \rho_{17} + \rho_{14}\\
&= -842.03 + 1.6488\,\rho_1 - 21.6337\,\rho_2
\end{array}\right.
$$

Joint (5) :

$$
\begin{aligned}
&2.501\,\rho_8 + 1.685\,\rho_9 + 0.190\,\rho_{14} + 1.800\,\rho_{15} + 3.588\,\rho_{19} = 0\\
&\therefore\ \rho_{19} = -\,0.6970\,\rho_8 - 0.4696\,\rho_9 - 0.0530\,\rho_{14} - 0.5017\,\rho_{15}\\
&\qquad = +448.22 + 4.7670\,\rho_1 + 9.2408\,\rho_2
\end{aligned}
$$

Triangle E :

$$
\left\{\begin{array}{rl}
2\,\rho_{15} - \rho_{16} &= +147.66 + 2\,\rho_{19} - \rho_{20}\\
\rho_{20} &= +147.66 + 2\,\rho_{19} + \rho_{16} - 2\,\rho_{15}\\
&= +819.43 + 17.2612\,\rho_1 + 15.5879\,\rho_2\\
\rho_{20} - \rho_{21} &= -359.94 + \rho_{15} - \rho_{16}\\
\rho_{21} &= +359.94 - \rho_{15} + \rho_{16} + \rho_{20}\\
&= +112.67 + 26.6532\,\rho_1 - 8.9395\,\rho_2\\
\rho_{22} - \rho_{16} &= +212.28 + \rho_{20} - \rho_{19}\\
\rho_{22} &= +212.28 + \rho_{20} - \rho_{19} + \rho_{16}\\
&= -1\ 325.24 + 23.5510\,\rho_1 - 39.8140\,\rho_2
\end{array}\right.
$$

TABLE 8.—(*Continued*).

Joint (7):

$$3.588\,\rho_{20} + 0.303\,\rho_{21} + 3.588\,\rho_{23} = 0$$
$$\rho_{23} = -\rho_{20} - 0.0844\,\rho_{21}$$
$$= -828.94 - 19.5107\,\rho_1 - 14.8334\,\rho_2$$

Triangle F:

$$\begin{aligned}
2\,\rho_{21} - \rho_{22} &= +127.50 + 2\,\rho_{23} - \rho_{24} \\
\rho_{24} &= +127.50 + 2\,\rho_{23} + \rho_{22} - 2\,\rho_{21} \\
&= -3\,080.96 - 68.7768\,\rho_1 - 51.6018\,\rho_2 \\
\rho_{24} - \rho_{25} &= -147.66 + \rho_{21} - \rho_{22} \\
\rho_{25} &= +147.66 - \rho_{21} + \rho_{22} + \rho_{24} \\
&= -4\,371.21 - 71.8790\,\rho_1 - 82.4763\,\rho_2 \\
\rho_{26} - \rho_{22} &= +20.16 + \rho_{24} - \rho_{23} \\
\rho_{26} &= +20.16 + \rho_{24} - \rho_{23} + \rho_{22} \\
&= -3\,557.10 - 25.7151\,\rho_1 - 76.5824\,\rho_2
\end{aligned}$$

Joint (6):

$$7.951\,\rho_{27} + 1.800\,\rho_{26} + 0.303\,\rho_{22} + 1.800\,\rho_{16} + 7.951\,\rho_{17} = 0$$
$$\rho_{27} = -0.2264\,\rho_{26} - 0.0381\,\rho_{22} - 0.2264\,\rho_{16} - \rho_{17}$$
$$= +3\,059.55 - 9.3778\,\rho_1 + 70.1424\,\rho_2$$

Triangle G:

$$\begin{aligned}
2\,\rho_{27} - \rho_{28} &= -147.66 + 2\,\rho_{26} - \rho_{25} \\
\rho_{28} &= +147.66 - 2\,\rho_{26} + \rho_{25} + 2\,\rho_{27} \\
&= +9\,009.75 - 39.2044\,\rho_1 + 210.9733\,\rho_2 \\
\rho_{29} - \rho_{28} &= +423.84 + \rho_{25} - \rho_{26} \\
\rho_{29} &= +423.84 + \rho_{28} + \rho_{25} - \rho_{26} \\
&= +8\,619.48 - 85.3683\,\rho_1 + 205.0794\,\rho_2 \\
\rho_{25} - \rho_{30} &= -276.18 + \rho_{27} - \rho_{28} \\
\rho_{30} &= +276.18 - \rho_{27} + \rho_{28} + \rho_{25} \\
&= +1\,855.17 - 101.7056\,\rho_1 + 58.3546\,\rho_2
\end{aligned}$$

Joint (8):

$$7.951\,\rho_{34} + 0.190\,\rho_{29} + 7.951\,\rho_{28} = 0$$
$$\rho_{34} = -0.0239\,\rho_{29} - \rho_{28}$$
$$= -9\,215.76 + 41.2447\,\rho_1 - 215.8747\,\rho_2$$

Triangle H:

$$\begin{aligned}
2\,\rho_{34} - \rho_{36} &= +716.34 + 2\,\rho_{29} - \rho_{30} \\
\rho_{33} &= -716.34 - 2\,\rho_{29} + \rho_{30} + 2\,\rho_{34} \\
&= -34\,531.65 + 151.5204\,\rho_1 - 783.5536\,\rho_2 \\
\rho_{32} - \rho_{33} &= -324.90 + \rho_{30} - \rho_{29} \\
\rho_{32} &= -324.90 + \rho_{33} + \rho_{30} - \rho_{29} \\
&= -41\,620.86 + 136.1831\,\rho_1 - 930.2784\,\rho_2 \\
\rho_{30} - \rho_{31} &= -391.44 + \rho_{34} - \rho_{33} \\
\rho_{31} &= +391.44 - \rho_{34} + \rho_{33} + \rho_{30} \\
&= -23\,069.28 + 8.5701\,\rho_1 - 509.3243\,\rho_2
\end{aligned}$$

Joint (9):

$$3.588\,\rho_{24} + 1.800\,\rho_{25} + 0.190\,\rho_{30} + 1.685\,\rho_{31} + 2.501\,\rho_{35} = 0$$
$$\rho_{35} = -1.4316\,\rho_{24} - 0.7197\,\rho_{25} - 0.0760\,\rho_{30} - 0.6737\,\rho_{31}$$
$$= +22\,966.69 + 152.3544\,\rho_1 + 472.0830\,\rho_2$$

Triangle I:

$$\begin{aligned}
2\,\rho_{31} - \rho_{32} &= -324.90 + 2\,\rho_{35} - \rho_{36} \\
\rho_{36} &= -324.90 + 2\,\rho_{35} + \rho_{32} - \rho_{31} \\
&= +50\,126.18 + 423.7517\,\rho_1 + 1\,032.5362\,\rho_2 \\
\rho_{38} - \rho_{32} &= -63.90 + \rho_{36} - \rho_{35} \\
\rho_{38} &= -63.90 + \rho_{36} - \rho_{35} + \rho_{32} \\
&= -14\,525.27 + 407.5804\,\rho_1 - 369.8252\,\rho_2 \\
\rho_{36} - \rho_{37} &= +388.80 + \rho_{31} - \rho_{32} \\
\rho_{37} &= -388.80 - \rho_{31} + \rho_{32} + \rho_{36} \\
&= +31\,185.80 + 551.3647\,\rho_1 + 611.5821\,\rho_2
\end{aligned}$$

Joint (11):

$$2.501\,\rho_{36} + 0.303\,\rho_{37} + 2.501\,\rho_{42} = 0$$
$$\rho_{42} = -\rho_{36} - 0.1212\,\rho_{37}$$
$$= -53\,905.90 - 490.5771\,\rho_1 - 1\,106.6600\,\rho_2$$

Triangle J:

$$\begin{aligned}
2\,\rho_{37} - \rho_{38} &= -466.44 + 2\,\rho_{42} - \rho_{41} \\
\rho_{41} &= -466.44 + 2\,\rho_{42} + \rho_{38} - 2\,\rho_{37} \\
&= -185\,175.11 - 1\,676.3032\,\rho_1 - 3\,806.3094\,\rho_2 \\
\rho_{39} + \rho_{38} &= +273.12 + \rho_{41} + \rho_{42} \\
\rho_{39} &= +273.12 + \rho_{41} + \rho_{42} + \rho_{38} \\
&= -145\,521.36 - 777.7228\,\rho_1 - 3\,069.4746\,\rho_2 \\
\rho_{41} - \rho_{40} &= +193.32 + \rho_{37} + \rho_{38} \\
\rho_{40} &= -193.32 - \rho_{37} + \rho_{38} + \rho_{41} \\
&= -231\,079.50 - 1\,820.0875\,\rho_1 - 4\,787.7167\,\rho_2
\end{aligned}$$

Joints (10) and (12) give the two simultaneous equations:

$$5.464\,\rho_{39} + 0.303\,\rho_{38} + 1\,685\,\rho_{32} + 7.951\,\rho_{33} = 0$$
$$2.501\,\rho_{41} + 5.464\,\rho_{40} = 0$$

In terms of ρ_1 and ρ_2, these become:

$$-1\,144\,222.17 - 2\,691.7733\,\rho_1 - 24\,681.2200\,\rho_2 = 0,$$
$$-1\,725\,741.34 - 14\,137.3924\,\rho_1 - 35\,679.6638\,\rho_2 = 0.$$

that is,

$$+425.081 + \rho_1 + 9.16913\,\rho_2 = 0$$
$$+122.069 + \rho_1 + 2.52378\,\rho_2 = 0.$$
$$\therefore +303.012 \qquad + 6.64585\,\rho_2 = 0$$
$$\therefore \rho_2 = -45.5976$$
$$\rho_1 = -6.9907$$

Substituting these values in the ρ-expressions, Table 9 is obtained.

TABLE 9.—MÜLLER-BRESLAU METHOD. SOLUTION OF VALUES OF ρ_3 TO ρ_{42}.

$\rho_3 = +20.8700$
$\rho_4 = -96.66 - 6.9907 + 132.9352 = +29.2845$
$\rho_5 = +39.90 - 13.9814 + 178.5328 = +204.4514$
$\rho_6 = +136.56 - 13.9814 + 112.0652 = +234.6438$
$\rho_7 = -16.55 + 8.6852 - 13.5835 = -21.4483$
$\rho_8 = -71.98 + 31.3519 - 72.7647 = -113.3928$
$\rho_9 = -6.22 + 31.3519 - 6.2970 = +18.8340$
$\rho_{10} = -47.45 + 8.6852 + 119.3517 = +80.5869$
$\rho_{11} = +74.93 + 3.4960 - 123.4464 = -45.0204$
$\rho_{12} = +400.96 + 20.9735 - 491.8932 = -69.9597$
$\rho_{13} = +800.27 + 43.6401 - 617.5420 = +226.3681$
$\rho_{14} = +515.47 + 48.8293 - 374.7439 = +189.5554$
$\rho_{15} = -842.03 - 11.6381 + 986.4448 = +132.7767$
$\rho_{16} = -1\,908.73 - 77.2948 + 2\,104.8354 = +118.8106$
$\rho_{17} = -1\,771.59 - 82.4840 + 1\,862.0373 = +7.9633$
$\rho_{18} = -420.09 - 22.0165 + 500.8486 = +58.7421$
$\rho_{19} = +448.22 - 33.3247 - 421.3583 = -6.4630$
$\rho_{20} = +819.43 - 120.6679 - 710.7708 = -12.0087$
$\rho_{21} = +112.67 - 186.3245 + 407.6197 = +333.9652$
$\rho_{22} = -1\,325.24 - 164.6380 + 1\,815.4228 = +325.5448$
$\rho_{23} = -828.94 + 136.3935 + 676.3674 = -16.1791$
$\rho_{24} = -3\,080.96 + 480.7980 + 2\,352.9182 = -247.2438$
$\rho_{25} = -4\,371.21 + 502.4845 + 3\,760.7213 = -108.0042$
$\rho_{26} = -3\,557.10 + 179.7665 + 3\,491.9736 = +114.6401$
$\rho_{27} = +3\,059.55 + 65.5574 - 3\,198.3251 = -73.2177$
$\rho_{28} = +9\,009.75 + 274.0662 - 9\,619.8761 = -336.0599$
$\rho_{29} = +8\,619.48 + 596.7842 - 9\,351.1284 = -134.8642$
$\rho_{30} = +1\,855.17 + 710.9933 + 2\,660.8297 = -94.6664$
$\rho_{31} = -23\,069.28 - 59.9110 + 23\,223.9657 = +94.7747$
$\rho_{32} = -41\,620.86 - 952.0152 + 42\,418.4624 = -154.4128$
$\rho_{33} = -84\,531.65 - 1\,059.2337 + 85\,728.1636 = +137.2799$
$\rho_{34} = -9\,215.76 - 288.3293 + 9\,843.3682 = +339.2789$
$\rho_{35} = +22\,966.69 - 1\,065.0639 - 21\,525.8518 = +375.7743$
$\rho_{36} = +50\,126.18 - 2\,962.3210 - 4\,7081.1726 = +82.6864$
$\rho_{37} = +31\,185.80 - 3\,854.4252 - 27\,886.6760 = -555.3012$
$\rho_{38} = -14\,525.27 - 2\,849.2723 + 16\,863.1415 = +511.4008$
$\rho_{39} = -145\,521.36 + 5\,436.8268 + 139\,960.6750 = -123.8582$
$\rho_{40} = -231\,079.50 + 12\,723.6857 + 218\,308.3910 = +47.4233$
$\rho_{41} = -185\,175.11 + 11\,718.5328 + 173\,558.5735 = +101.9963$
$\rho_{42} = -53\,905.90 + 3\,429.4773 + 50\,461.0400 = -15.3827$

TABLE 10.—MÜLLER-BRESLAU METHOD. SECONDARY STRESSES IN MEMBERS.

Joint.	Member.	Extreme fiber distance, c, in inches.	l, in inches.	$\frac{2c}{l}$.	ρ.	$f=\frac{c}{l}\times\rho$ in pounds per square inch.
1	1-2	9.38	477.9	0.03925	+ 20.87	− 0.41
		9.49		0.03971		+ 0.41
	1-3	7.5	300	0.05	− 45.60	± 1.14
2	2-4	9.33	300	0.0622	− 45.03	+ 1.40
		9.54		0.0636		− 1.43
	2-5	7.5	477.9	0.03139	+ 80.59	∓ 1.26
	2-3	5.28	372	0.02839	+204.45	∓ 2.90
3	2-1	9.49	477.9	0.03971	+ 29.28	− 0.58
		9.38		0.03925		+ 0.57
	3-1	7.5	300	0.05	− 6.99	± 0.18
	3-2	5.28	372	0.02839	+234.64	∓ 3.33
	3-5	7.5	300	0.05	− 21.45	± 0.54
4	4-6	9.33	300	0.0622	+ 58.74	− 1.83
		9.54		0.0636		+ 1.87
	4-5	5.19	372	0.02790	+226.37	∓ 3.16
	4-2	9.54	300	0.0636	− 69.96	+ 2.22
		9.33		0.0622		− 2.17
5	5-3	7.5	300	0.05	−113.39	± 2.83
	5-2	7.5	477.9	0.03139	+ 18.83	∓ 0.30
	5-4	5.19	372	0.02790	+189.56	∓ 2.64
	5-6	7.5	477.9	0.03139	+132.78	∓ 2.08
	5-7	7.5	300	0.05	− 6.46	± 0.16
6	6-8	9.33	300	0.0622	− 73.22	+ 2.28
		9.54		0.0636		− 2.33
	6-9	7.5	477.9	0.03139	+114.64	∓ 1.80
	6-7	5.28	372	0.02839	+325.54	∓ 4.62
	6-5	7.5	477.9	0.03139	+118.81	∓ 1.86
7	6-4	9.54	300	0.0636	+ 7.96	− 0.25
		9.33		0.0622		+ 0.25
	7-5	7.5	300	0.05	− 12.01	± 0.30
	7-6	5.28	372	0.02839	+333.97	∓ 4.74
	7-9	7.5	300	0.05	− 16.18	± 0.40
8	8-10	9.33	300	0.0622	+339.28	−10.55
		9.54		0.0636		+10.79
	8-9	5.19	372	0.02790	−134.86	± 1.88
9	8-6	9.54	300	0.0636	−336.06	∓10.69
		9.33		0.0622		−10.45
	9-7	7.5	300	0.05	−247.24	± 6.18
	9-6	7.5	477.9	0.03139	−108.00	± 1.69
	9-8	5.19	372	0.02790	− 94.67	± 1.32
	9-10	7.5	477.9	0.03139	+ 94.77	∓ 1.49
	9-11	7.5	300	0.05	+375.77	∓ 9.39
10	10-12	9.38	477.9	0.03925	−123.86	+ 2.43
		9.49		0.03971		− 2.44
	10-11	5.28	372	0.02839	−511.40	± 7.26
	10-9	7.5	477.9	0.03139	−154.41	± 2.42
	10-8	9.54	300	0.0636	+137.28	− 4.36
		9.33		0.0622		+ 4.27
11	11-9	7.5	300	0.05	+ 82.69	∓ 2.07
	11-10	5.28	372	0.02839	−555.30	± 7.88
	11-12	7.5	300	0.05	− 15.38	± 0.38
12	12-11	7.5	300	0.05	+101.99	∓ 2.55
	12-10	9.49	477.9	0.03971	− 47.42	+ 0.94
		9.38		0.03925		− 0.93

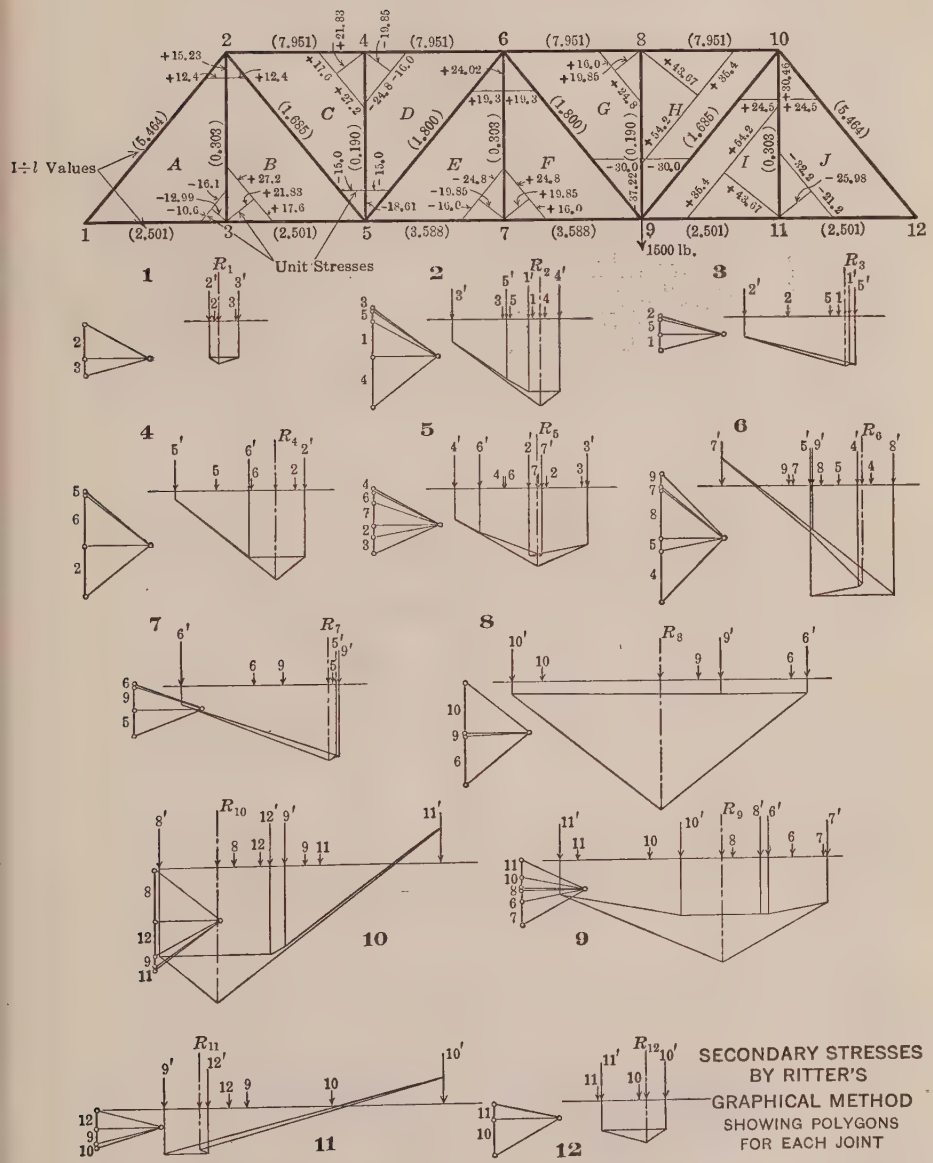
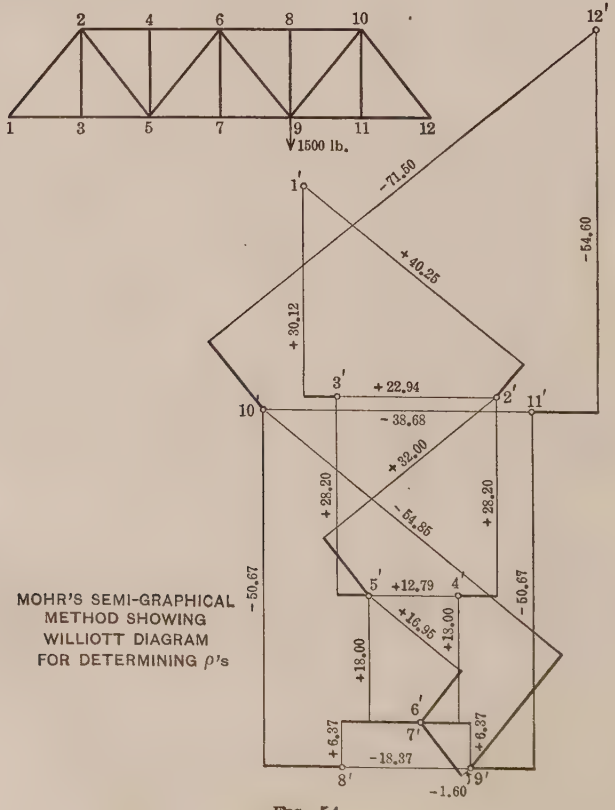

I÷l Values
Unit Stresses
1600 lb.
SECONDARY STRESSES
BY RITTER'S
GRAPHICAL METHOD
SHOWING POLYGONS
FOR EACH JOINT

FIG. 53.

TABLE 11.—Ritter's Graphical Method. Values of $\Delta\theta$.

Triangle.	Angle.	$\Delta\theta$.	Triangle.	Angle.	$\Delta\theta$.
A.	1	$\pm 0 + 16.1 \pm 0 = + 16.1$	*F.*	6	$+ 19.3 - 16.0 \pm 0 = + 3.3$
	3	$- 26.7 - 12.4 \pm 0 = - 39.1$		7	$+ 40.8 \pm 0 - 19.3 = + 21.5$
	2	$+ 12.4 \pm 0 + 10.6 = + 23.0$		9	$\pm 0 \pm 0 - 24.8 = - 24.8$
B.	2	$+ 12.4 \pm 0 - 17.6 = - 5.2$	*G.*	9	$- 30.0 - 16.0 \pm 0 = - 46.0$
	3	$+ 44.8 \pm 0 - 12.4 = + 32.4$		8	$+ 40.8 \pm 0 + 30.0 = + 70.8$
	5	$\pm 0 \pm 0 - 27.2 = - 27.2$		6	$\pm 0 \pm 0 - 24.8 = - 24.8$
C.	5	$- 15.0 - 17.6 \pm 0 = - 32.6$	*H.*	10	$\pm 0 - 54.2 \pm 0 = - 54.2$
	4	$+ 44.8 \pm 0 + 15.0 = + 59.8$		8	$+ 89.6 + 30.0 \pm 0 = + 119.6$
	2	$\pm 0 \pm 0 - 27.2 = - 27.2$		9	$- 30.0 \pm 0 - 35.4 = - 65.4$
D.	6	$\pm 0 + 24.8 \pm 0 = + 24.8$	*I.*	9	$\pm 0 - 54.2 \pm 0 = - 54.2$
	4	$- 40.8 + 15.0 \pm 0 = - 25.8$		11	$+ 89.6 - 24.5 \pm 0 = + 65.1$
	5	$- 15.0 \pm 0 + 16.0 = + 1.0$		10	$+ 24.5 \pm 0 - 35.4 = - 10.9$
E.	5	$\pm 0 + 24.8 \pm 0 = + 24.8$	*J.*	10	$+ 24.5 + 21.2 \pm 0 = + 45.7$
	7	$- 40.8 - 19.3 \pm 0 = - 60.1$		11	$- 53.4 \pm 0 - 24.5 = - 77.9$
	6	$+ 19.3 \pm 0 + 16.0 = + 35.3$		12	$\pm 0 \pm 0 + 32.2 = + 32.2$



MOHR'S SEMI-GRAPHICAL METHOD SHOWING WILLIOTT DIAGRAM FOR DETERMINING ρ's

Fig. 54.

TABLE 12.—RITTER'S GRAPHICAL METHOD. SECONDARY STRESSES IN MEMBERS.

Joint.	2μ.	C.	l.	$\frac{2C}{l}$	$f = \frac{6C}{l}\mu$, in pounds per square inch.
1	$2\mu_2 = +7.0$	9.38	477.9	0.03926	− 0.41
		9.49		0.03972	+ 0.42
	$2\mu_3 = -15.5$	7.5	300	0.05	± 1.16
	$2\mu_1 = +9.6$	9.49	477.9	0.03972	− 0.57
		9.38		0.03926	+ 0.57
2	$2\mu_3 = +68.0$	5.28	372	0.02839	∓ 2.90
		9.33	300	0.0622	+ 1.36
	$2\mu_4 = -14.6$	9.54		0.0636	− 1.39
	$2\mu_5 = +26.6$	7.5	477.9	0.03139	∓ 1.25
	$2\mu_1 = -3.3$	7.5	300	0.05	± 0.25
3	$2\mu_2 = +78.0$	5.28	372	0.02839	∓ 3.32
	$2\mu_5 = -7.8$	7.5	300	0.05	± 0.58
	$2\mu_2 = -22.5$	9.54	300	0.0636	+ 2.15
		9.33		0.0622	− 2.10
4	$2\mu_5 = +76.6$	5.19	372	0.02790	∓ 3.21
	$2\mu_6 = +20.2$	9.33	300	0.622	− 1.88
		9.54		0.636	+ 1.93
	$2\mu_2 = +5.9$	7.5	477.9	0.03139	∓ 0.28
	$2\mu_3 = -88.5$	7.5	300	0.05	± 2.89
5	$2\mu_4 = +63.6$	5.19	372	0.02790	∓ 2.66
	$2\mu_6 = +44.0$	7.5	477.9	0.03139	∓ 2.07
	$2\mu_7 = -2.9$	7.5	300	0.05	± 0.22
	$2\mu_4 = +2.6$	9.54	300	0.0636	− 0.25
		9.33		0.0622	+ 0.24
	$2\mu_5 = +39.3$	7.5	477.9	0.03139	∓ 1.85
6	$2\mu_7 = +108.5$	5.28	372	0.02839	∓ 4.62
	$2\mu_8 = -24.8$	9.33	300	0.0622	+ 2.31
		9.54		0.0636	− 2.37
	$2\mu_9 = +38.1$	7.5	477.9	0.03139	∓ 1.79
	$2\mu_5 = -4.3$	7.5	300	0.05	± 0.32
7	$2\mu_6 = +111.4$	5.28	372	0.02839	∓ 4.74
	$2\mu_9 = -5.8$	7.5	300	0.05	± 0.43
	$2\mu_6 = -112.1$	9.54	300	0.0636	+10.64
		9.33		0.0622	−10.41
8	$2\mu_9 = -44.0$	5.19	372	0.02790	± 1.84
	$2\mu_{10} = +113.1$	9.33	300	0.0622	−10.55
		9.54		0.0636	+10.79
	$2\mu_6 = -35.8$	7.5	477.9	0.03139	± 1.69
	$2\mu_7 = -82.5$	7.5	300	0.05	± 6.19
9	$2\mu_8 = -30.8$	5.19	372	0.02790	± 1.29
	$2\mu_{10} = +31.1$	7.5	477.9	0.03139	∓ 1.46
	$2\mu_{11} = +124.4$	7.5	300	0.05	∓ 9.33
	$2\mu_8 = +44.4$	9.54	300	0.0636	− 4.24
		9.33		0.0622	+ 4.14
	$2\mu_9 = -50.9$	7.5	477.9	0.03139	± 2.40
10	$2\mu_{11} = -169.9$	5.28	372	0.02839	± 7.23
	$2\mu_{12} = -39.7$	9.38	477.9	0.03926	+ 2.34
		9.49		0.03972	− 2.37
	$2\mu_9 = +27.2$	7.5	300	0.05	∓ 2.04
11	$2\mu_{10} = -185.2$	5.28	372	0.02839	± 7.89
	$2\mu_{12} = -5.4$	7.5	300	0.05	± 0.40
	$2\mu_{10} = -15.5$	9.49	477.9	0.03972	+ 0.92
12		9.38		0.03926	− 0.91
	$2\mu_{11} = +34.0$	7.5	300	0.05	∓ 2.55

The quantities, 2 μ, were then listed in Table 12. For example, in the figure for Joint (1), 2′ lies at a distance of 0.7 to the left of R_1, the resultant of the displaced forces. Hence, 2 $\mu_2 = +7.0$ and this is tabulated. Equation (54) is then used to determine the secondary stresses. The moments were to be positive when acting in a clockwise direction. The signs of the secondary stresses, therefore, are justified.

Mohr's Semi-Graphical Method.—The result of applying Mohr's semi-graphical method to the solution of the problem is shown in Fig. 54 and Tables 13 to 16, inclusive. Table 13 is partly filled (the first four columns), when the Williot diagram is drawn. The Δ l's are laid out to a convenient scale; this is on the supposition that E is unity. From the completed diagram are then obtained the ρ's to be entered in Column (5) of Table 13, which table can then be completed.

Table 14 is then constructed, by the use of which the equations for the joints may be built (see Equation (59)). Table 15 is the solution of these equations by Gauss' method of normal equations.

Table 16 is then completed, making use of Equation (60). Note, again, that a clockwise bending moment is reckoned to be positive in this analysis. Hence, it is that the + 0.105 of Member 1-2 gives a compressive stress for the top fiber and a tensile stress for the lower fiber of Member 1-2, End (1).

Mohr's Elastic Weight Method.—The solution of the problem by means of Mohr's elastic weight method is given in Fig. 55 and Tables 17 to 22, inclusive. Table 17 is filled out first. Then, the truss is separated into triangles (Fig. 55), and on each is indicated by arrows the direction of following around the chain. Thus, it is a clockwise rotation for Δ A followed by a counter-clockwise rotation for Δ B, etc. At the vertices of each triangle, place the ψ's belonging to that triangle. Thus, in going around Joint (2) of Δ A, the direction is from Member 1-2 to Member 2-3; ψ_2 is, therefore,

$$\lambda_{23} - \lambda_{12} = 0 - (-12.99) = +12.99.$$

Then, place the elastic weights at the third points of the members. Thus, $\phi_{12} = +0.091\,M_{12}$ at the first "third" point on the way from Joint (1) to Joint (2), and $\phi_{21} = -0.091 M_{21}$ at the second, etc.

Assume that M_{12} and M_{21} are known. The $\Sigma M = 0$ for Joint (1) gives Equation I-1 of Table 18. Taking moments of the ϕ's about the u_1-axis in Δ A and also moments of the ψ's about the v_1-axis, marked in Δ A, as the u_1-axis coincides with the v_1-axis on a clockwise rotation of 90°, the sum of the ϕ-moments, together with the sum of the ψ-moments, must be zero. (Refer to Equations (72)). In taking these axes, only one fresh unknown has been introduced, namely, M_{31}; Equation II-1 of Table 18 is the result. By taking moments about the axes, u_2 and v_2, it will be found that Equation II-2 contains only one fresh unknown, namely, M_{23}. Equation II-3 is the result of taking moments about u_3 and v_3.

Then Equation I-3 is the equation representing the fact that $\Sigma M_3 = 0$. The remainder of the equations in Table 18 follow in the same manner. The axes are marked 1, 2, 3, in the order and place, to make sure that only one new unknown is added with every additional moment equation. The height

TABLE 13.—MOHR'S SEMI-GRAPHICAL METHOD. ELEMENTS OF MEMBERS.

Member. (1)	Length, l, in inches. (2)	Unit stress, s, in pounds. (3)	$\Delta l = \frac{s \cdot l}{1\,000}$ (4)	ρ, in kips per square inch. (5)	$\psi = \frac{\rho}{l}$, in 100 lb. per sq. in. (6)	k, in cubic inches. (7)	$k\,\psi$, in 100 in-lb. (8)
1- 2	477.9	−12.99	− 6.208	+40.25	+0.842	5.464	+ 4.601
2- 4	300	−18.61	− 5.583	+28.20	+0.940	7.951	+ 7.474
4- 6	300	−18.61	− 5.583	+18.00	+0.600	7.951	+ 4.771
6- 8	300	−37.22	−11.166	+ 6.37	+0.212	7.951	+ 1.686
8-10	300	−37.22	−11.166	−50.67	−1.689	7.951	−13.429
10-12	477.9	−25.98	−12.416	−71.50	−1.496	5.464	− 8.174
1- 3	300	+15.23	+ 4.569	+30.12	+1.004	2.501	+ 2.511
3- 5	300	+15.23	+ 4.569	+28.20	+0.940	2.501	+ 2.351
5- 7	300	+24.02	+ 7.206	+18.00	+0.600	3.588	+ 2.153
7- 9	300	+24.02	+ 7.206	+ 6.37	+0.212	3.588	+ 0.761
9-11	300	+30.46	+ 9.138	−50.67	−1.689	2.501	− 4.224
11-12	300	+30.46	+ 9.138	−54.60	−1.820	2.501	− 4.552
2- 3	372	.....	.....	+22.94	+0.617	0.303	+ 0.187
4- 5	372	.....	.....	+12.79	+0.344	0.190	+ 0.065
6- 7	372	.....	.....	± 0	.....	0.303	.....
8- 9	372	.....	.....	−18.37	−0.494	0.190	− 0.094
10-11	372	.....	.....	−38.68	−1.040	0.303	− 0.315
2- 5	477.9	+21.83	+10.433	+32.00	+0.670	1.685	+ 1.129
5- 6	477.9	−19.85	− 9.486	+16.95	+0.355	1.800	+ 0.639
6- 9	477.9	+19.85	+ 9.486	− 1.60	−0.033	1.800	− 0.059
9-10	477.9	+43.67	+20.870	−54.85	−1.148	1.685	− 1.934

TABLE 14.—MOHR'S SEMI-GRAPHICAL METHOD. VALUES OF k AND $k\,\psi$.

Joint.	Member.	k, in cubic inches.	$k\,\psi$, in 100 in-lb.	Joint.	Member.	k, in cubic inches.	$k\,\psi$, in 100 in-lb.
1	1-2	5.464	+ 4.601	7	7-5	3.588	+ 2.153
	1-3	2.501	+ 2.511		7-6	0.303	.....
	Σ	7.965	+ 7.112		7-9	3.588	+ 0.761
	2-4	7.951	+ 7.474		Σ	7.479	+ 2.914
	2-5	1.685	+ 1.129	8	8-10	7.951	−13.429
2	2-3	0.303	+ 0.187		8-9	0.190	− 0.094
	2-1	5.464	+ 4.601		8-6	7.951	+ 1.686
	Σ	15.403	+13.391		Σ	16.092	−11.837
	3-1	2.501	+ 2.511		9-7	3.588	+ 0.761
3	3-2	0.303	+ 0.187		9-6	1.800	− 0.059
	3-5	2.501	+ 2.351	9	9-8	0.190	− 0.094
	Σ	5.305	+ 5.049		9-10	1.685	− 1.934
	4-6	7.951	+ 4.771		9-11	2.501	− 4.224
4	4-5	0.190	+ 0.065		Σ	9.764	− 5.550
	4-2	7.951	+ 7.474		10-12	5.464	− 8.174
	Σ	16.092	+12.310		10-11	0.303	− 0.315
	5-3	2.501	+ 2.351	10	10-9	1.685	− 1.934
	5-2	1.685	+ 1.129		10-8	7.951	−13.429
5	5-4	0.190	+ 0.065		Σ	15.403	−23.852
	5-6	1.800	+ 0.639		11-9	2.501	− 4.224
	5-7	3.588	+ 2.153	11	11-10	0.303	− 0.315
	Σ	9.764	+ 6.337		11-12	2.501	− 4.552
	6-8	7.951	+ 1.686		Σ	5.305	− 9.091
	6-9	1.800	− 0.059	12	12-11	2.501	− 4.552
	6-7	0.303	.....		12-10	5.464	− 8.174
6	6-5	1.800	+ 0.639		Σ	7.965	−12.726
	6-4	7.951	+ 4.771				
	Σ	19.805	+ 7.037				

TABLE 15.—MOHR'S SEMI-GRAPHICAL METHOD. SOLUTION OF EQUATIONS.

Number of equation.	Remarks.	ϕ_1.	ϕ_2.	ϕ_3.	ϕ_4.	ϕ_5.	ϕ_6.	ϕ_7.	ϕ_9.	ϕ_8.	ϕ_{11}.	ϕ_{10}.	ϕ_{12}.	Absolute term.	Check.
1	.....	+15.930	+5.464	+2.501	.....	.....	.....	.....	.....	.....	.....	.....	.....	−21.386	+2.559
1′	$1 \times -\frac{5.464}{15.930}$, *i. e.*, 0.34300	[−5.464]	−1.874	−0.858	.....	.....	.....	.....	.....	.....	.....	.....	.....	+7.318	−0.878
2	.....	[+5.464]	+30.806	+0.303	+7.951	+1.685	.....	.....	.....	.....	.....	.....	.....	−40.173	+6.036
I	1′ + 2	.....	+28.932	−0.555	+7.951	+1.685	.....	.....	.....	.....	.....	.....	.....	−32.855	+5.158
1″	$1 \times -\frac{2.501}{15.930}$, *i. e.*, 0.15700	[−2.501]	[−0.858]	−0.393	.....	.....	.....	.....	.....	.....	.....	.....	.....	+3.350	−0.402
I′	$I \times +\frac{0.555}{28.932}$, *i. e.*, 0.01918	.....	[+0.555]	−0.011	+0.153	+0.032	.....	.....	.....	.....	.....	.....	.....	−0.630	+0.099
3	.....	[+2.501]	[+0.303]	+10.610	.....	+2.501	.....	.....	.....	.....	.....	.....	.....	−15.147	+0.768
II	1″ + I′ + 3	.....	.....	+10.206	+0.153	+2.533	.....	.....	.....	.....	.....	.....	.....	−12.427	+0.465
I″	$I \times -\frac{7.951}{28.932}$, *i. e.*, 0.27482	.....	[−7.951]	[+0.153]	−2.185	−0.463	.....	.....	.....	.....	.....	.....	.....	+9.029	−1.418
II′	$II \times -\frac{0.153}{10.206}$, *i. e.*, 0.01499	.....	.....	[−0.153]	−0.002	−0.038	.....	.....	.....	.....	.....	.....	.....	+0.186	−0.007
4	.....	.....	[+7.951]	.....	+32.184	+0.190	+7.951	.....	.....	.....	.....	.....	.....	−36.930	+11.346
III	I″ + II′ + 4	.....	.....	.....	+29.997	−0.311	+7.951	.....	.....	.....	.....	.....	.....	−27.715	+9.921
I‴	$I \times -\frac{1.685}{28.932}$, *i. e.*, 0.05824	.....	[−1.685]	[+0.032]	[−0.463]	−0.098	.....	.....	.....	.....	.....	.....	.....	+1.913	−0.300
II″	$II \times -\frac{2.533}{10.206}$, *i. e.*, 0.24819	.....	.....	[−2.533]	[−0.038]	−0.629	.....	.....	.....	.....	.....	.....	.....	+3.084	−0.115
III′	$III \times +\frac{0.311}{9.997}$, *i. e.*, 0.01037	.....	.....	.....	[+0.311]	−0.003	+0.082	.....	.....	.....	.....	.....	.....	−0.287	+0.103
5	.....	.....	[+1.685]	[+2.501]	[+0.190]	+19.528	+1.800	+3.588	.....	.....	.....	.....	.....	−19.011	+10.281
IV	I‴ + II″ + III′ + 5	.....	.....	.....	.....	+18.798	+1.882	+3.588	.....	.....	.....	.....	.....	−14.301	+9.969

TABLE 15.—(*Continued*).

Number of equation.	Remarks.	ϕ_1.	ϕ_2.	ϕ_3.	ϕ_4.	ϕ_5.	ϕ_6.	ϕ_7.	ϕ_9.	ϕ_8.	ϕ_{11}.	ϕ_{10}.	ϕ_{12}.	Absolute term.	Check.
III''	III $\times -\frac{7.951}{29.997}$, *i. e.*, 0.26506	.....	.....	.....	[−7.951]	[+0.082]	−2.107	.....	.....	.....	.....	.....	.....	+7.346	−2.630
IV'	IV $\times -\frac{1.882}{18.798}$, *i. e.*, 0.10012	.....	.....	.....	.....	[−1.882]	−0.188	−0.359	.....	.....	.....	.....	.....	+1.432	−0.998
6	.....	.....	.....	.....	[+7.951]	[+1.800]	+39.610	+0.303	+1.800	+7.951	.....	.....	.....	−21.111	+38.304
V	III'' + IV' + 6	.....	.....	.....	.....	.....	+37.315	−0.056	+1.800	+7.951	.....	.....	.....	−12.333	+34.676
IV''	IV $\times -\frac{3.588}{18.798}$, *i. e.*, 0.19087	.....	.....	.....	.....	[−3.588]	[−0.359]	−0.685	.....	.....	.....	.....	.....	+2.730	−1.903
V'	V $\times +\frac{0.056}{37.315}$, *i. e.*, 0.00150	.....	.....	.....	.....	.....	[+0.056]	−0.000	+0.003	+0.012	.....	.....	.....	−0.019	+0.052
7	.....	.....	.....	.....	.....	[+3.588]	[+0.303]	+14.958	+3.588	.....	.....	.....	.....	−8.742	+13.695
VI	IV'' + V' + 7	.....	.....	.....	.....	.....	.....	+14.273	+3.591	+0.012	.....	.....	.....	−6.031	+11.844
V''	V $\times -\frac{1.800}{37.315}$, *i. e.*, 0.04824	.....	.....	.....	.....	.....	[−1.800]	[+0.003]	−0.087	−0.384	.....	.....	.....	+0.595	−1.673
VI'	VI $\times -\frac{3.591}{14.273}$, *i. e.*, 0.25159	.....	.....	.....	.....	.....	.....	[−3.591]	−0.903	−0.003	.....	.....	.....	+1.517	−2.980
8	.....	.....	.....	.....	.....	.....	[+1.800]	[+3.588]	+19.528	+0.190	+2.501	+1.685	.....	+16.650	+45.942
VII	V'' + VI' + 8	.....	.....	.....	.....	.....	.....	.....	+18.538	−0.197	+2.501	+1.685	.....	+18.762	+41.289
V'''	V $\times -\frac{7.951}{37.315}$, *i. e.*, 0.21308	.....	.....	.....	.....	.....	[−7.951]	[+0.012]	[−0.384]	−1.694	.....	.....	.....	+2.628	−7.388
VI''	VI $\times -\frac{0.012}{14.273}$, *i. e.*, 0.00084	.....	.....	.....	.....	.....	.....	[−0.012]	[−0.003]	−0.000	.....	.....	.....	+0.005	−0.010
VII'	VII $\times +\frac{0.197}{18.538}$, *i. e.*, 0.01063	.....	.....	.....	.....	.....	.....	.....	[+0.197]	−0.012	+0.027	+0.018	.....	+0.199	+0.439
9	.....	.....	.....	.....	.....	.....	[+7.951]	.....	[+0.190]	+32.184	.....	+7.951	.....	+35.511	+83.787
VIII	V''' + VI'' + VII' + 9	.....	.....	.....	.....	.....	.....	.....	.....	+30.488	+0.027	+7.969	.....	+38.343	+76.828

TABLE 15.—(*Continued*).

Number of equation.	Remarks.	ϕ_1.	ϕ_2.	ϕ_3.	ϕ_4.	ϕ_5.	ϕ_6.	ϕ_7.	ϕ_9.	ϕ_8.	ϕ_{11}.	ϕ_{10}	ϕ_{12}.	Absolute term.	Check.
VII''	VII $\times -\frac{2.501}{18.538}$, *i. e.*, 0.13491	.....	.....	.....	.....	.....	.....	.....	[−2.501]	[+0.027]	−0.337	−0.227	.....	−2.531	−5.570
VIII'	VIII $\times -\frac{0.027}{30.488}$, *i. e.*, 0.00089	.....	.....	.....	.....	.....	.....	.....	.....	[−0.027]	−0.000	−0.007	.....	−0.034	−0.068
10	.....	.....	.....	.....	.....	.....	.....	.....	[+2.501]	.....	+10.610	+0.303	+2.501	+27.273	+43.188
IX	VII'' + VIII' + 10	.....	.....	.....	.....	.....	.....	.....	.....	.....	+10.273	+0.069	+2.501	+24.708	+37.550
VII'''	VII $\times -\frac{1.685}{18.538}$, *i. e.*, 0.09089	.....	.....	.....	.....	.....	.....	.....	[−1.685]	[+0.018]	[−0.227]	−0.153	.....	−1.705	−3.753
VIII''	VIII $\times -\frac{7.969}{30.488}$, *i. e.*, 0.26138	.....	.....	.....	.....	.....	.....	.....	.....	[−7.969]	[−0.007]	−2.083	.....	−10.022	−20.081
IX'	IX $\times -\frac{0.069}{10.273}$, *i. e.*, 0.00672	.....	.....	.....	.....	.....	.....	.....	.....	.....	[−0.069]	−0.000	−0.017	−0.166	−0.252
11	.....	.....	.....	.....	.....	.....	.....	.....	[+1.685]	[+7.951]	[+0.303]	+30.806	+5.464	+71.556	+117.765
X	VII''' + VIII'' + IX' + 11	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+28.570	+5.447	+59.663	+93.679
IX''	IX $\times -\frac{2.501}{10.273}$, *i. e.*, 0.24345	.....	.....	.....	.....	.....	.....	.....	.....	.....	[−2.501]	[−0.017]	−0.609	−6.015	−9.142
X'	X $\times -\frac{5.447}{28.570}$, *i. e.*, 0.19065	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	[−5.447]	−1.038	−11.375	−17.860
12	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	[+2.501]	[+5.464]	+15.930	+38.178	+62.073
XI	IX'' + X' + 12	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	+14.283	+20.788	+35.071
		+0.86345	+0.90366	+1.05699	+0.79069	+0.59943	+0.52612	+0.56932	−0.58077	−0.78252	−2.03864	−1.81082	−1.45544	.....	.....

TABLE 16.—MOHR'S SEMI-GRAPHICAL METHOD. SECONDARY STRESSES IN MEMBERS.

Joint. (1)	Member. (2)	$\frac{2c}{l}$. (3)	ϕ. (4)	-3ψ. (5)	$(2\phi_{an}+\phi_n-3\psi_{an})$. (6)	f(Column (3) × Column (6)) in pounds per square inch. (7)
1	1- 2	0.03926	+0 863	−2.526	+0.105	− 0.41
	1- 3	0.03972		−3.012	−0.228	+ 0.42
		0.05				± 1.14
	2- 4	0.0622		−2.820	−0.222	+ 1.38
		0.0636				− 1.41
2	2- 5	0.03139	+0.904	−2.010	+0.397	± 1.25
	2- 3	0.02839		−1.851	+1.013	∓ 2.88
	2- 1	0.03972		−2.526	+0.145	− 0.58
		0.03926				+ 0.57
	3- 1	0.05		−3.012	−0.035	± 0.18
3	3- 2	0.02839	+1.0570	−1.851	+1.167	∓ 3.31
	3- 5	0.05		−2.820	−0.107	± 0.54
	4- 6	0.0622		−1.800	+0.308	− 1.92
		0.0636				+ 1.96
4	4- 5	0.02790	+0.791	−1.032	+1.149	∓ 3.21
	4- 2	0.0636		−2.820	−0.335	+ 2.13
		0.0622				− 2.08
	5- 3	0.05		−2.820	−0.564	± 2.82
	5- 2	0.03139		−2.010	+0.093	∓ 0.29
5	5- 4	0.02790	+0.599	−1.032	+0.958	∓ 2.67
	5- 6	0.03139		−1.065	+0.660	∓ 2.07
	5- 7	0.05		−1.800	−0.032	± 0.16
	6- 8	0.0622		−0.636	−0.366	+ 2.28
		0.0636				− 2.33
	6- 9	0.03139		+0.099	+0.569	∓ 1.79
6	6- 7	0.02839	+0.526	.....	+1.622	∓ 4.60
	6- 5	0.03139		−1.065	+0.587	∓ 1.84
	6- 4	0.0636		−1.800	+0.043	− 0.27
		0.0622				+ 0.27
	7- 5	0.05		−1.800	−0.062	± 0.31
7	7- 6	0.02839	+0.569	.....	+1.665	∓ 4.73
	7- 9	0.05		−0.636	−0.079	± 0.39
	8-10	0.0622		+5.067	+1.691	−10.52
		0.0636				+10.75
8	8- 9	0.02790	−0.783	+1.482	−0.665	± 1.86
	8- 6	0.0636		−0.636	−1.675	+10.65
		0.0622				−10.42
	9- 7	0.05		−0 636	−1.230	± 6.15
	9- 6	0.03139		+0.099	−0.538	± 1.69
9	9- 8	0.02790	−0.582	+1.482	−0.464	± 1.29
	9-10	0.03139		+3.444	+0.470	∓ 1.47
	9-11	0.05		+5.067	+1.865	∓ 9.33
	10-12	0.03926		+1.488	−0.589	+ 2.31
		0.03972				− 2.34
	10-11	0.02839		+3.120	−2 540	± 7.21
10	10- 9	0.03139	−1.811	+3.444	−0.759	± 2.38
	10- 8	0.0636		+5.067	+0.663	− 4.22
		0.0622				+ 4.12
	11- 9	0.05		+5.067	+0.408	∓ 2.04
11	11-10	0.02839	−2.039	+3.120	−2.768	± 7.86
	11-12	0.05		+5.460	−0.073	± 0.36
	12-11	0.05		+5.460	+0.510	∓ 2.55
12	12-10	0.03972	−1.455	+4.488	−0.234	+ 0.93
		0.03926				− 0.92

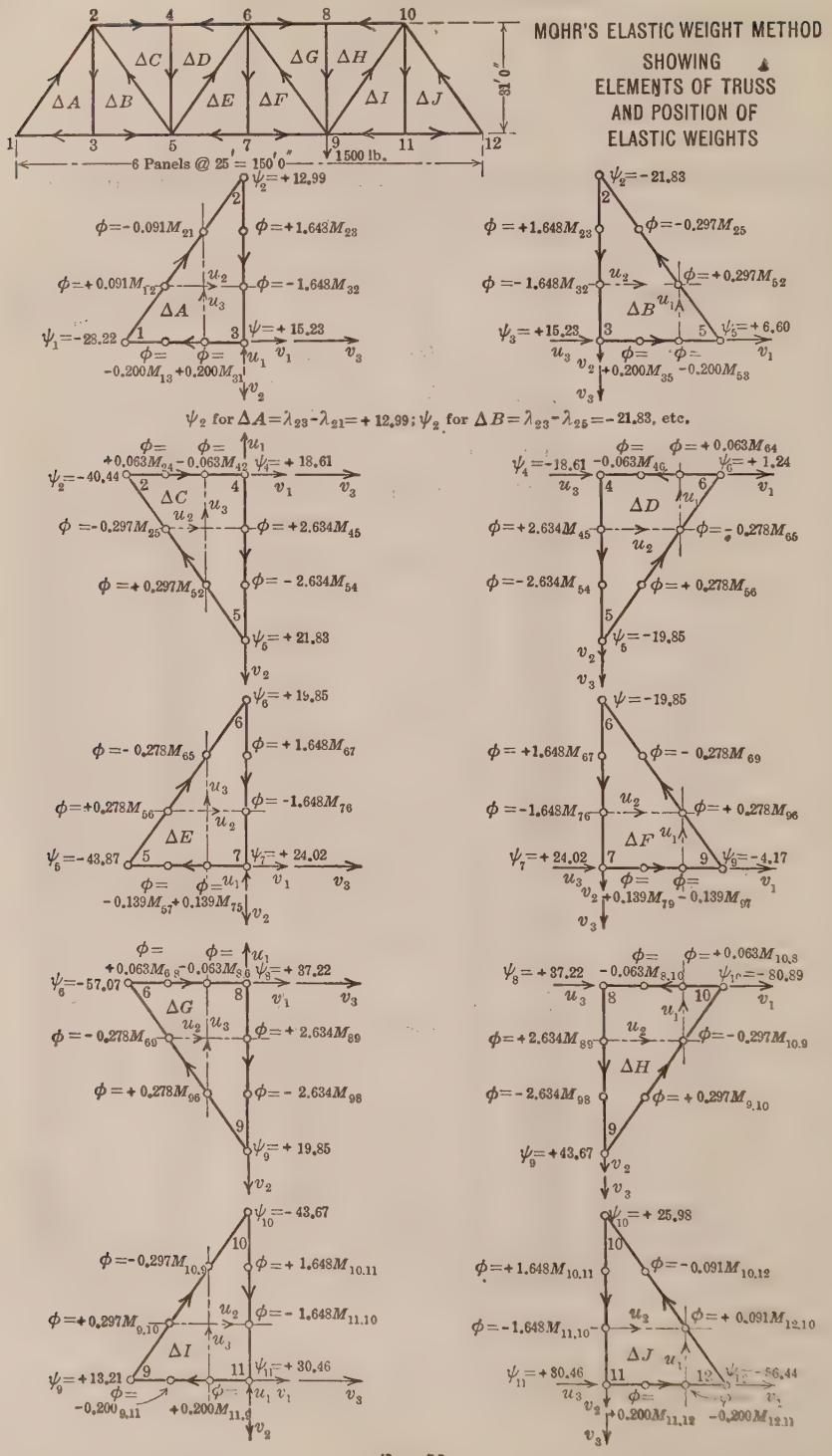

MOHR'S ELASTIC WEIGHT METHOD
SHOWING
ELEMENTS OF TRUSS
AND POSITION OF
ELASTIC WEIGHTS
ΔA ΔB ΔC ΔD ΔE ΔF ΔG ΔH ΔI ΔJ
31' 0"
6 Panels @ 25' = 150' 0"
1500 lb.
ψ2 for ΔA = λ23 - λ21 = + 12.99; ψ2 for ΔB = λ23 - λ25 = - 21.83, etc.

FIG. 55.

TABLE 17.—Mohr's Elastic Weight Method. Elements of Members.

Member.	l.	A.	S.	I.	λ.	β.
1-3	300	26.48	+ 403.25	750.2	+15.23	0.200
3-5	"	"	"	"	"	"
5-7	"	50.36	+1209.7	1076.4	+24.02	0.139
7-9	"	"	"	"	"	"
9-11	"	26.48	+ 806.45	750.2	+30.46	0.200
11-12	"	"	"	"	"	"
1-2	477.9	49.45	— 642.35	2611.53	—12.99	0.091
2-4	300	43.33	— 806.45	2385.38	—18.61	0.063
4-6	"	"	"	"	"	"
6-8	"	"	—1612.9	"	—37.22	"
8-10	"	"	"	"	"	"
10-12	477.9	49.45	—1284.65	2611.53	—25.98	0.091
2-3	372	24.35	.....	112.86	.....	1.648
4-5	"	12.20	.....	70.62	.....	2.634
6-7	"	24.35	.....	112.86	.....	1.648
8-9	"	12.20	.....	70.62	.....	2.634
10-11	"	24.35	.....	112.86	.....	1.648
2-5	477.9	29.42	+ 642.35	805.4	+21.83	0.297
5-6	"	32.36	— 642.35	860.4	—19.85	0.278
6-9	"	"	+ 642.35	"	+19.85	"
9-10	"	29.42	+1284.65	805.4	+43.67	0.297

E = Modulus of elasticity, taken as unity.
A = Area of cross-section of member, in square inches.
S = Total load in member, in pounds.
l = Length of member, in inches.
I = Moment of inertia of member, in inch units.

$\lambda = \frac{S}{EA}$, strain (*Dehnung*).

$\beta = \frac{l}{2EI}$, elastic weight (*Masse der Biegsamkeit*).

of the triangles is 31 ft., while the base is 25 ft.; hence, it is that the factors, $\frac{25}{31}$ and $\frac{31}{25}$, appear in the ψ-terms of Table 18.

Table 19 is only a re-arrangement of the equations of Table 18. Table 20 is the result of expressing every M in terms of M_{12} and M_{21}, which is possible by starting at the beginning of Table 19.

It will be found that $\Sigma M_{10} = \Sigma M_{12} = 0$ are two equations that have not been used at all. After solving these, for they are simultaneous linear equations in M_{12} and M_{21}, Table 21 and, finally, Table 22 follow.

It must be noted that positive moments are counterclockwise, hence, the signs are as shown in front of the various secondary stresses.

The results in Table 19 and the following tables have been computed each time to five places of decimals. The writer believed that a good basis for accuracy in all the methods would be to carry out Mohr's elastic weight method to ten places of decimals, the computing machine making this quite possible. It goes without saying that no engineer demands such accuracy; the only justification in the present case is that the method which gave results closest to what may be termed the standard in this investigation was the best for purposes of accuracy combined with speed in obtaining the secondary stresses.

Mao's Graphical Method of Deformation Contour.—Figs. 56 and 57 and Table 23 represent Mao's graphical method applied to the problem. On the truss diagram, Fig. 56 (*a*), the moments of inertia, I, and the unit stresses, P,

TABLE 18.—MOHR'S ELASTIC WEIGHT METHOD. MOMENT EQUATIONS, ASSUMING M_{12} AND M_{21} TO BE KNOWN.

I 1	$0 = M_{12} + M_{81}$
II 1	$0 = -0.200\ M_{81} + 2 \times 0.200\ M_8 - 2 \times 0.091\ M_{12} + 0.091\ M_{21} - 12.99 \times 3 \times \frac{31}{25}$
2	$0 = +0.091\ M_{21} - 1.648\ M_{23} + 0.200\ M_{31} - 0.200\ M_{13} - 28.22 \times 3 \times \frac{25}{31}$
3	$0 = +0.200\ M_{13} - 0.091\ M_{12} + 1.648\ M_{23} - 1.648\ M_{32} - 12.99 \times 3 \times \frac{31}{25}$
I 3	$0 = M_{31} + M_{32} + M_{35}$
II 4	$0 = -0.200\ M_{35} + 2 \times 1.648\ M_{32} - 2 \times 1.648\ M_{23} + 0.297\ M_{25} + 21.85 \times 3 \times \frac{31}{25}$
5	$0 = -1.648\ M_{23} + 0.297\ M_{25} - 0.200\ M_{53} + 0.200\ M_{35} - 6.60 \times 3 \times \frac{25}{31}$
6	$0 = +1.648\ M_{32} - 2 \times 1.648\ M_{23} + 2 \times 0.297\ M_{25} - 0.297\ M_{52} - 6.60 \times 3 \times \frac{25}{31}$
I 2	$0 = M_{24} + M_{25} + M_{23} + M_{21}$
II	$0 = -0.297\ M_{52} + 2 \times 0.297\ M_{25} - 2 \times 0.063\ M_{24} + 0.063\ M_{42} + 21.83 \times 3 \times \frac{31}{25}$
8	$0 = -0.063\ M_{24} + 0.063\ M_{42} - 2.634\ M_{54} + 0.297\ M_{52} + 40.44 \times 3 \times \frac{25}{31}$
9	$0 = +0.297\ M_{25} - 0.063\ M_{24} + 2.634\ M_{45} - 2.634\ M_{54} + 21.83 \times 3 \times \frac{31}{25}$
I 4	$0 = M_{46} + M_{45} + M_{42}$
II 10	$0 = -0.278\ M_{56} + 2 \times 2.634\ M_{54} - 2 \times 2.634\ M_{45} + 0.063\ M_{46} - 19.85 \times 3 \times \frac{31}{25}$
11	$0 = +0.063\ M_{46} - 0.063\ M_{64} + 0.278\ M_{56} - 2.634\ M_{54} - 1.24 \times 3 \times \frac{25}{31}$
12	$0 = -0.278\ M_{65} + 2 \times 0.278\ M_{56} - 2 \times 2.634\ M_{54} + 2.634\ M_{45} - 1.24 \times 3 \times \frac{25}{31}$
I 5	$0 = M_{53} + M_{52} + M_{54} + M_{56} + M_{57}$
II 13	$0 = -0.139\ M_{75} + 2 \times 0.139\ M_{57} - 2 \times 0.278\ M_{56} + 0.278\ M_{65} - 19.85 \times 3 \times \frac{31}{25}$
14	$0 = +0.278\ M_{65} - 1.648\ M_{67} + 0.139\ M_{75} - 0.139\ M_{57} - 43.87 \times 3 \times \frac{25}{31}$
15	$0 = +0.139\ M_{57} - 0.278\ M_{56} + 1.648\ M_{67} - 1.648\ M_{76} - 19.85 \times 3 \times \frac{31}{25}$
I 7	$0 = M_{75} + M_{76} + M_{79}$
II 16	$0 = -0.139\ M_{79} + 2 \times 1.648\ M_{76} - 2 \times 1.648\ M_{67} + 0.278\ M_{69} + 19.85 \times 3 \times \frac{31}{25}$
17	$0 = -1.648\ M_{67} + 0.278\ M_{69} - 0.139\ M_{97} + 0.139\ M_{79} + 4.17 \times 3 \times \frac{25}{31}$
18	$0 = +1.648\ M_{76} - 2 \times 1.648\ M_{67} + 2 \times 0.278\ M_{69} - 0.278\ M_{96} + 4.17 \times 3 \times \frac{25}{31}$
I 6	$0 = M_{68} + M_{69} + M_{67} + M_{65} + M_{64}$
II 19	$0 = -0.278\ M_{96} + 2 \times 0.278\ M_{69} - 2 \times 0.063\ M_{68} + 0.063\ M_{86} + 19.85 \times 3 \times \frac{31}{25}$
20	$0 = -0.063\ M_{68} + 0.063\ M_{86} - 2.634\ M_{98} + 0.278\ M_{96} - 57.07 \times 3 \times \frac{25}{31}$
21	$0 = +0.278\ M_{69} - 0.063\ M_{68} + 2.634\ M_{89} - 2.634\ M_{98} + 19.85 \times 3 \times \frac{25}{31}$
I 8	$0 = M_{8.10} + M_{89} + M_{86}$
II 22	$0 = -0.297\ M_{9.10} + 2 \times 2.634\ M_{98} - 2 \times 2.634\ M_{89} + 0.063\ M_{8.10} + 43.67 \times 3 \times \frac{31}{25}$
23	$0 = +0.063\ M_{8.10} - 0.063\ M_{10.8} + 0.297\ M_{9.10} - 2.634\ M_{98} + 80.89 \times 3 \times \frac{25}{31}$
24	$0 = -0.297\ M_{10.9} + 2 \times 0.297\ M_{9.10} - 2 \times 2.634\ M_{98} + 2.634\ M_{89} + 80.89 \times 3 \times \frac{25}{31}$

TABLE 18.—(*Continued.*)

I 9	$0 = M_{97} + M_{96} + M_{98} + M_{9.10} + 9.11$
II 25	$0 = -0.200\,M_{11.9} + 2 \times 0.200\,M_{9.11} - 2 \times 0.297\,M_{9.10} + 0.297\,M_{10.9} + 43.67 \times 3 \times \frac{31}{25}$
26	$0 = +0.297\,M_{10.9} - 1.648\,M_{10.11} + 0.200\,M_{11.9} - 0.200\,M_{9.11} + 13.21 \times 3 \times \frac{25}{31}$
27	$0 = +0.200\,M_{9.11} - 0.297\,M_{9.10} + 1.648\,M_{10.11} - 1.648\,M_{11.10} + 43.67 \times 3 \times \frac{31}{25}$
I 10	$0 = M_{10.12}\,M_{10.11} + M_{10.9} + M_{10.8}$
II 28	$0 = -0.200\,M_{11.12} + 3 \times 1.648\,M_{11.10} - 2 \times 1.648\,M_{10.11} + 0.091\,M_{10.12} - 25.98 \times 3 \times \frac{31}{25}$
29	$0 = -1.648\,M_{10.11} + 0.091\,M_{10.12} - 0.200\,M_{12.11} + 0.200\,M_{11.12} + 56.44 \times 3 \times \frac{25}{31}$
30	$0 = +1.648\,M_{11.10} - 2 \times 1.648\,M_{10.11} + 2 \times 0.091\,M_{10.12} - 0.091\,M_{12.10} + 56.44 \times 3 \times \frac{25}{31}$
I 11	$0 = M_{11.9} + M_{11.10} + M_{11.12}$
12	$0 = M_{12.11} + M_{12.10}$

TABLE 19.—Mohr's Elastic Weight Method. Moment Equations.

I 1	$M_{18} = -M_{12}$
II 1	$M_{31} = +2M_{18} - 0.91M_{12} + 0.455M_{21} - 241.614$
2	$M_{23} = +0.05522M_{21} + 0.12136M_{31} - 0.12136M_{18} - 41.42852$
3	$M_{32} = +0.12136M_{18} - 0.05522M_{12} + M_{23} - 29.32208$
I 3	$M_{35} = -M_{31} - M_{32}$
II 4	$M_{25} = +0.67340M_{35} - 11.09764M_{32} + 11.09764M_{23} - 273.42626$
5	$M_{53} = -8.24M_{23} + 1.485M_{25} + M_{35} - 79.83871$
6	$M_{52} = +5.54882M_{32} - 11.09764M_{23} + 2M_{25} - 53.76344$
I 2	$M_{24} = -M_{25} - M_{23} - M_{21}$
II 7	$M_{42} = +4.71429M_{52} - 9.42857M_{25} + 2M_{24} - 1289.00952$
8	$M_{54} = -0.02392M_{24} + 0.02392M_{42} + 0.11276M_{52} - 37.14454$
9	$M_{45} = -0.11276M_{25} + 0.02392M_{24} + M_{54} - 30.83053$
I 4	$M_{46} = -M_{45} - M_{42}$
II 10	$M_{56} = +18.94964M_{54} - 18.94964M_{45} + 0.22662M_{46} - 265.61870$
11	$M_{64} = +M_{46} + 4.41270M_{56} - 41.80952M_{54} - 47.61905$
12	$M_{65} = +2M_{56} - 18.94964M_{54} + 9.47482M_{45} - 10.79136$
I 5	$M_{57} = -M_{53} - M_{52} - M_{54} - M_{56}$
II 13	$M_{75} = +2M_{57} - 4M_{56} + 2M_{65} - 531.23741$
14	$M_{67} = +0.16869M_{65} + 0.08434M_{75} - 0.08434M_{57} - 64.40358$
15	$M_{76} = +0.08434M_{57} - 0.16869M_{56} + M_{67} - 44.80704$
I 7	$M_{79} = -M_{75} - M_{76}$
II 16	$M_{69} = +0.5M_{79} - 11.85612M_{76} + 11.85612M_{67} - 265.61870$
17	$M_{97} = -11.85612M_{67} + 2M_{69} + M_{79} + 72.58065$
18	$M_{96} = +5.92806M_{76} - 11.85612M_{67} + 2M_{69} + 36.29032$
I 6	$M_{68} = -M_{69} - M_{67} - M_{65} - M_{64}$
II 19	$M_{86} = +4.41270M_{96} - 8.82540M_{69} + 2M_{68} - 1172.09524$
20	$M_{98} = -0.02392M_{68} + 0.02392M_{86} + 0.10554M_{96} - 52.41935$
21	$M_{89} = -0.10554M_{69} + 0.02392M_{68} + M_{98} - 28.03417$
I 8	$M_{8.10} = -M_{89} - M_{86}$
II 22	$M_{9.10} = +17.73737M_{98} - 17.73737M_{89} + 0.21212M_{8.10} + 546.97778$
23	$M_{10.8} = +M_{8.10} + 4.71429M_{9.10} - 41.80952M_{98} + 3106.37481$
24	$M_{10.9} = +2M_{9.10} - 17.73737M_{98} + 8.86869M_{89} + 658.92799$
I 9	$M_{9.11} = -M_{97} - M_{96} - M_{98} - M_{9.10}$
II 25	$M_{11.9} = +2M_{9.11} - 2.97M_{9.10} + 1.485M_{10.9} + 812.262$
26	$M_{10.11} = +0.18022M_{10.9} + 0.12136M_{11.9} - 0.12136M_{9.11} + 19.39301$
27	$M_{11.10} = +0.12136M_{9.11} - 0.18022M_{9.10} + M_{10.11} + 98.57549$
I 10	$M_{10.12} = -M_{10.11} - M_{10.9} - M_{10.8}$
II 28	$M_{11.12} = +16.48M_{11.10} - 16.48M_{10.11} + 0.455M_{10.12} - 483.228$
29	$M_{12.11} = -8.24M_{10.11} + 0.455M_{10.12} + M_{11.12} + 682.74194$
30	$M_{12.10} = +18.10989M_{11.10} - 36.21978M_{10.11} + 2M_{10.12} + 1500.53173$

are given, also, the U-lines, which are drawn from one end of a member tangent to the circle the center of which is at the other end, and the radius of which is equal to the extreme fiber distance for the member. For example, there is such a line from Joint (1) to the circle of radius, $y = 9.38$ in. at Joint (2). Finally, by Ritter's method the half values of the changes of angle are graphically determined on this diagram. It has been shown (see page 42) that the scales call for the half values of the changes of angle when the lengths of the members are expressed in feet and the fiber distances in inches.

TABLE 20.—Mohr's Elastic Weight Method. Moments in Terms of M_{12} and M_{21}.

I	1	M_{13}	$=-$	M_{12}			
II	1	M_{31}	$=-$	$2.91M_{12}$	$+$	$0.455M_{21}$	$-$ 241.614
	2	M_{23}	$=-$	$0.23180M_{12}$	$+$	$0.11044M_{21}$	$-$ 70.75080
	3	M_{32}	$=-$	$0.40888M_{12}$	$+$	$0.11044M_{21}$	$-$ 100.07288
II	3	M_{35}	$=+$	$3.31838M_{12}$	$-$	$0.56544M_{21}$	$+$ 341.68688
I	4	M_{25}	$=+$	$4.19422M_{12}$	$-$	$0.38077M_{21}$	$+$ 282.05736
	5	M_{53}	$=+$	$11.45683M_{12}$	$-$	$2.04091M_{21}$	$+$ 1 263.68994
	6	M_{52}	$=+$	$8.69485M_{12}$	$-$	$1.37435M_{21}$	$+$ 740.23179
I	2	M_{24}	$=-$	$3.96242M_{12}$	$-$	$0.72967M_{21}$	$-$ 211.30656
II	7	M_{42}	$=-$	$6.48030M_{12}$	$-$	$4.34830M_{21}$	$-$ 881.35287
	8	M_{54}	$=+$	$0.92020M_{12}$	$-$	$0.24153M_{21}$	$+$ 30.29649
	9	M_{45}	$=+$	$0.35248M_{12}$	$-$	$0.21604M_{21}$	$-$ 37.39328
I	4	M_{46}	$=+$	$6.12782M_{12}$	$+$	$4.56434M_{21}$	$+$ 918.74615
II	10	M_{56}	$=+$	$12.14678M_{12}$	$+$	$0.55134M_{21}$	$+$ 1 225.28432
	11	M_{64}	$=+$	$21.25480M_{12}$	$+$	$17.09549M_{21}$	$+$ 5 011.25752
	12	M_{65}	$=+$	$10.19578M_{12}$	$+$	$3.63265M_{21}$	$+$ 1 511.37510
I	5	M_{57}	$=-$	$33.21866M_{12}$	$+$	$3.10545M_{21}$	$-$ 3 259.50254
II	13	M_{75}	$=-$	$94.63288M_{12}$	$+$	$11.27084M_{21}$	$-$ 8 928.62957
	14	M_{67}	$=-$	$3.45975M_{12}$	$+$	$1.30146M_{21}$	$-$ 287.58888
	15	M_{76}	$=-$	$8.31045M_{12}$	$+$	$1.47036M_{21}$	$-$ 813.99057
I	7	M_{79}	$=+$	$102.94333M_{12}$	$-$	$12.74120M_{21}$	$+$ 9 742.62014
II	16	M_{89}	$=+$	$108.98214M_{12}$	$-$	$8.37310M_{21}$	$+$ 10 846.83226
	17	M_{97}	$=+$	$361.92682M_{12}$	$-$	$44.91767M_{21}$	$+$ 34 918.49430
	18	M_{98}	$=+$	$209.71864M_{12}$	$-$	$23.46009M_{21}$	$+$ 20 314.19889
I	6	M_{68}	$=-$	$136.97297M_{12}$	$-$	$13.65650M_{21}$	$-$ 17 081.88100
II	19	M_{86}	$=-$	$310.33148M_{12}$	$-$	$56.93938M_{21}$	$-$ 41 423.02523
	20	M_{98}	$=+$	$17.98697M_{12}$	$-$	$3.51131M_{21}$	$+$ 1 509.30103
	21	M_{89}	$=+$	$3.20860M_{12}$	$-$	$2.95427M_{21}$	72.10640
I	8	$M_{8\cdot10}$	$=+$	$307.12288M_{12}$	$+$	$59.89365M_{21}$	$+$ 41 495.13163
II	22	$M_{9\cdot10}$	$=+$	$327.27633M_{12}$	$+$	$2.82422M_{21}$	$+$ 37 398.93381
	23	$M_{10\cdot8}$	$=+$	$1\,097.97183M_{12}$	$+$	$220.01408M_{21}$	$+$157 807.77451
	24	$M_{10\cdot9}$	$=+$	$363.96720M_{12}$	$+$	$41.72934M_{21}$	$+$ 48 046.27519
I	9	$M_{9\cdot11}$	$=-$	$916.90876M_{12}$	$+$	$69.06485M_{21}$	$-$ 94 140.92803
II	25	$M_{11\cdot9}$	$=-$	$2\,265.77909M_{12}$	$+$	$191.70984M_{21}$	$-$ 227 195.70838
	26	$M_{10\cdot11}$	$=-$	$98.10473M_{12}$	$+$	$22.40466M_{21}$	$-$ 7 469.23536
	27	$M_{11\cdot10}$	$=-$	$268.36252M_{12}$	$+$	$30.27739M_{21}$	$-$ 25 535.63875
I	10	$M_{10\cdot12}$	$=-$	$1\,363.83430M_{12}$	$-$	$284.14803M_{21}$	$-$198 384.81464
II	28	$M_{11\cdot12}$	$=-$	$3\,426.39299M_{12}$	$+$	$0.45524M_{21}$	$-$388 482.64653
	29	$M_{12\cdot11}$	$=-$	$3\,238.55460M_{12}$	$-$	$313.44651M_{21}$	$-$416 518.49588
	30	$M_{12\cdot10}$	$=-$	$4\,034.35258M_{12}$	$-$	$831.46771M_{21}$	$-$ 587 182.64488

Equations I (11) and I (12) give:

$$0 = -5\,960.53460M_{12} + 222.44247M_{21} - 641\,213.99366$$
$$0 = -7\,272.90718M_{12} - 1\,144.91422M_{21} - 1\,003\,701.14076,$$

that is:

$$0 = -26.795848M_{12} + M_{21} - 2\,882.605977$$
$$0 = -6.352360M_{12} - M_{21} - 876.660560,$$

that is:

$$M_{12} = -113.407836,$$
$$M_{21} = -156.253159.$$

In Fig. 56(b) is shown the force diagram of the section moduli, these being the forces the distances of which from the joints will be the secondary stresses. These S-forces are drawn in directions perpendicular to their various members. The broken lines represent the directions that the R-forces take at the various joints, in order to preserve equilibrium with the S-forces.

The property and deformation lines are then constructed (Fig. 57). All the sets of property lines but one are omitted because any more lines than those given would make the diagram rather confusing. The lines marked K_{A1}, K_{A2}, etc., are the deformation lines in position (see page 45). Then, all the base and vertex lines of all the triangles are located, the full lines being the base lines and the broken lines the vertex lines (see page 51).

TABLE 21.—Mohr's Elastic Weight Method. Values of Moments.

I	1	M_{13}	= +	113.408			= +	113.408
II	1	M_{31}	= +	330.01680	— 71.09519	— 241.614	= +	17.308
	2	M_{23}	= +	26.28794	— 17.25660	— 70.75080	= —	61.719
	3	M_{32}	= +	46.31349	— 17.25660	— 100.07288	= —	71.016
I	3	M_{35}	= —	376.33029	+ 88.35179	+ 341.68686	= +	53.708
II	4	M_{25}	= —	475.65741	+ 59.49652	+ 282.05736	= —	134.104
	5	M_{53}	= —	1 299.29430	+ 318.89862	+ 1 263.68994	= +	283.294
	6	M_{52}	= —	986.06412	+ 214.74653	+ 740.23179	= —	31.086
I	2	M_{24}	= +	449.36948	+ 114.01324	— 211.30656	= +	352.076
II	7	M_{42}	= +	734.91680	+ 679.43561	— 881.35287	= +	533.000
	8	M_{54}	= —	104.35789	+ 37.73983	+ 30.29649	= —	36.322
	9	M_{45}	= —	39.97399	+ 33.75693	— 37.39328	= —	43.610
I	4	M_{46}	= —	694.94281	— 713.19254	+ 918.74615	= —	489.389
II	10	M_{56}	= —	1 377.54003	— 86.14862	+ 1 225.28432	= —	238.404
	11	M_{64}	= —	2 410.46087	— 2 671.22432	+ 5 011.25752	= —	70.428
	12	M_{65}	= —	1 156.28135	— 567.61304	+ 1 511.37510	= —	212.519
I	5	M_{57}	= +	3 767.25635	— 485.23637	— 3 259.50254	= +	22.517
II	13	M_{75}	= +	10 732.11014	— 1 761.10435	— 8 928.62957	= +	42.376
	14	M_{67}	= +	392.36276	— 203.35724	— 287.58388	= —	98.578
	15	M_{76}	= +	942.46815	— 229.74839	— 813.99057	= —	101.271
I	7	M_{79}	= —	11 674.58029	+ 1 990.85275	+ 9 742.62014	= +	58.893
II	16	M_{69}	= —	12 359.42866	+ 1 308.32333	+ 10 846.83226	= —	204.273
	17	M_{97}	= —	41 045.31530	+ 7 018.52783	+ 34 918.49480	= +	891.707
	18	M_{96}	= —	23 783.73713	+ 3 665.71817	+ 20 314.19889	= +	196.175
I	6	M_{68}	= +	15 533.80812	+ 2 133.87127	— 17 081.88100	= +	585.798
II	19	M_{86}	= +	35 194.02159	+ 8 896.95800	— 41 423.02523	= +	2 667.954
	20	M_{98}	= —	2 039.86334	+ 548.65328	+ 1 509.30103	= +	18.091
	21	M_{89}	= —	363.88038	+ 461.61402	— 72.10640	= +	25.627
I	8	$M_{8\cdot10}$	= —	34 830.14121	— 9 358.57202	+ 41 495.18163	= —	2 693.582
II	22	$M_{9\cdot10}$	= —	37 115.70036	— 441.29830	+ 37 398.93381	= —	158.060
	23	$M_{10\cdot8}$	= —	124 518.60923	— 34 377.88721	+ 157 807.77451	= —	1 088.722
	24	$M_{10\cdot9}$	= —	41 276.73253	— 6 520.34120	+ 48 046.27549	= +	249.202
I	9	$M_{9\cdot11}$	= +	103 984.63828	— 10 791.60099	— 94 140.92803	= —	947.892
II	25	$M_{11\cdot9}$	= +	256 957.10345	— 29 955.26811	— 227 195.70838	= —	193.873
	26	$M_{10\cdot11}$	= +	11 125.84513	— 3 500.79890	— 7 469.23536	= +	155.811
	27	$M_{11\cdot10}$	= +	30 434.41266	— 4 730.93783	— 25 585.63875	= +	167.836
I	10	$M_{10\cdot12}$	= +	154 669.49663	+ 41 399.02731	— 198 384.81464	= +	683.709
II	28	$M_{11\cdot12}$	= +	388 579.81428	— 71.13269	— 388 482.64653	= +	26.035
	29	$M_{12\cdot11}$	= +	367 277.46895	+ 48 977.00737	— 416 518.49588	= —	264.020
	30	$M_{12\cdot10}$	= +	457 527.19576	+ 129 919.45629	— 587 182.64488	= +	261.007

Following the instructions to determine the reference contour, $r_{11\cdot9}$ by means of the base and vertex lines of ΔJ, Vertex (11), *via* Vertex (12), and the base and vertex lines of ΔJ, Vertex (11), *via* Vertex (10), it was found that the fourth attempt gave an $r_{11\cdot9}$, that coincided with the value of $r_{11\cdot9}$, assumed before that trial was made. The shaded area around Joint (11) is the resulting deformation contour, while those around Joint (10) and Joint (12) had already been drawn in the final trial of determining $r_{11\cdot9}$.

A vertical from the end (*a*) of $r_{10\cdot8}$ to the base of ΔI, at (*b*), determines the point on the vertex line of ΔI, through which a parallel, (*b-d*), to the base line is drawn. A 45° line to this newly drawn line from the end (*c*) of $r_{11\cdot9}$ determines the point (*d*) from which the second 45° line is drawn to determine $r_{9\cdot11}$ (from (9) to (*e*)), which is negative in this case. The deformation contour is then drawn; it is shown by shading the edges. All the deformation contours are determined in a similar way. Finally, from the ends of r_{12}

TABLE 22.—MOHR'S ELASTIC WEIGHT METHOD. SECONDARY STRESSES IN MEMBERS.

Joint.	Member.	Extreme fiber distance, C.	I, in inches^4.	$\frac{C}{I}$.	M.	$f = \frac{C}{I} \times M$, in pounds per square inch.
1	1-2	9.38	2 611.5	0.003592	— 113.408	— 0.41
		9.49		0.003634		+ 0.41
	1-3	7.5	750.2	0.009997	+ 113.408	± 1.13
2	2-4	9.33	2 385.4	0.003911	+ 352.076	+ 1.38
		9.54		0.003999		— 1.41
	2-5	7.5	805.4	0.009312	— 134.104	∓ 1.25
	2-3	5.28	112.86	0.046784	— 61.719	∓ 2.89
3	2-1	9.49	2 611.5	0.003634	— 156.253	— 0.57
		9.38		0.003592		+ 0.56
	3-1	7.5	750.2	0.009997	+ 17.308	± 0.17
	3-2	5.28	112.86	0.046784	— 71.016	∓ 3.32
	3-5	7.5	750.2	0.009997	+ 53.708	± 0.54
4	4-6	9.33	2 385.4	0.003911	— 489.389	— 1.91
		9.54		0.003999		+ 1.96
	4-5	5.19	70.62	0.073492	— 43.610	∓ 3.20
5	4-2	9.54	2 385.4	0.003999	+ 533.000	+ 2.13
		9.33		0.003911		— 2.08
	5-3	7.5	750.2	0.009997	+ 283.294	± 2.83
	5-2	7.5	805.4	0.009312	— 31.086	∓ 0.29
	5-4	5.19	70.62	0.073492	— 36.322	∓ 2.67
	5-6	7.5	860.4	0.008717	— 238.404	∓ 2.08
	5-7	7.5	1 076.4	0.006968	+ 22.517	± 0.16
6	6-8	9.33	2 385.4	0.003911	+ 585.798	+ 2.29
		9.54		0.003999		— 2.34
	6-9	7.5	860.4	0.008717	— 204.273	∓ 1.78
	6-7	5.28	112.86	0.046784	— 98.578	∓ 4.61
	6-5	7.5	860.4	0.008717	— 212.519	∓ 1.85
7	6-4	9.54	2 385.4	0.003999	— 70.428	— 0.28
		9.33		0.003911		+ 0.28
	7-5	7.5	1 076.4	0.006968	+ 42.376	± 0.30
	7-6	5.28	112.86	0.046784	— 101.271	∓ 4.74
	7-9	7.5	1 076.4	0.006968	+ 58.893	± 0.41
8	8-10	9.33	2 385.4	0.003911	—2693.582	—10.53
		9.54		0.003999		+10.77
	8-9	5.19	70.62	0.073492	+ 25.627	± 1.88
9	8-6	9.54	2 385.4	0.003999	+2667.954	+10.67
		9.33		0.003911		— 10.43
	9-7	7.5	1 076.4	0.006968	+ 891.707	± 6.21
	9-6	7.5	860.4	0.008717	+ 196.175	± 1.71
	9-8	5.19	70.62	0.073492	+ 18.091	± 1.33
	9-10	7.5	805.4	0.009312	— 158.061	∓ 1.47
	9-11	7.5	750.2	0.009997	— 947.892	∓ 9.48
10	10-12	9.38	2 611.5	0.003592	+ 683.709	+ 2.46
		9.49		0.003634		— 2.48
	10-11	5.28	112.86	0.046784	+ 155.811	± 7.29
	10-9	7.5	805.4	0.009312	+ 249.202	± 2.32
	10-8	9.54	2 385.4	0.003999	—1088.722	— 4.35
		9.33		0.003911		+ 4.26
11	11-9	7.5	750.2	0.009997	— 193.873	∓ 1.94
	11-10	5.28	112.86	0.046784	+ 167.836	± 7.85
	11-12	7.5	750.2	0.009997	+ 26.035	± 0.26
	12 11	7.5	750.2	0.009997	— 264.020	∓ 2.64
12	12-10	9.49	2 611.5	0.003634	+ 264.007	+ 0.96
		9.38		0.003592		— 0.95

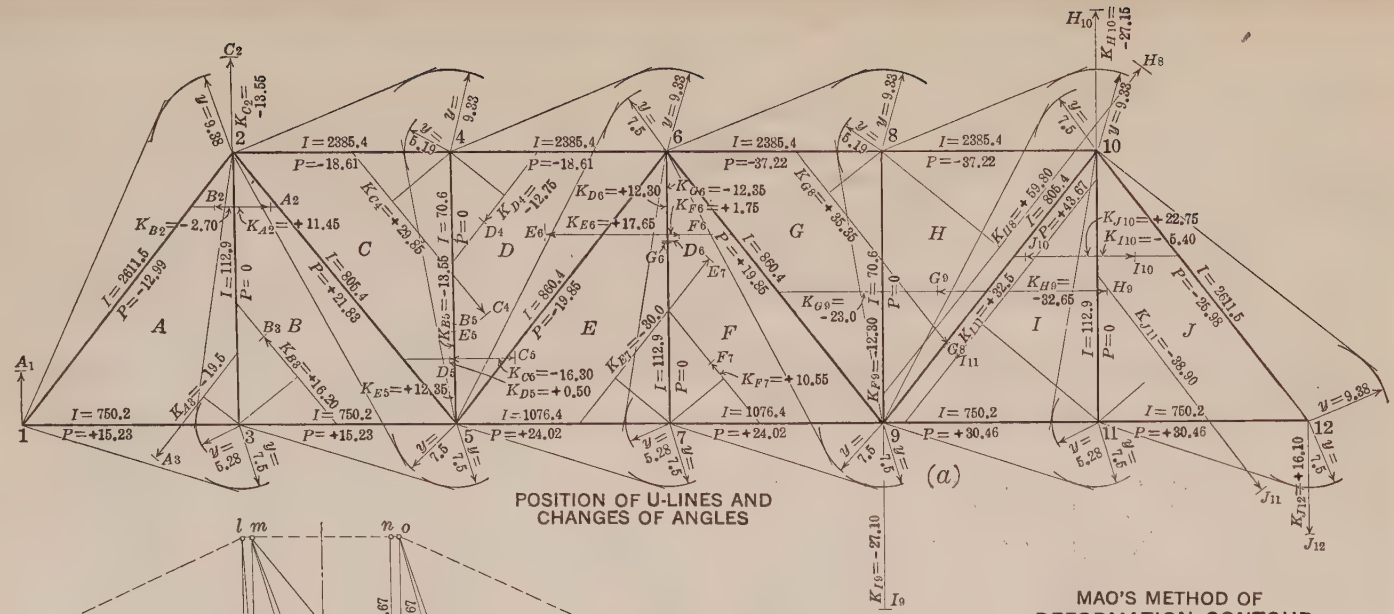

POSITION OF U-LINES AND CHANGES OF ANGLES
(a)
(b)
S-FORCES AND DIRECTION OF R'S
MAO'S METHOD OF
DEFORMATION CONTOUR
GRAPHICAL DETERMINATION
OF ELEMENTS
Scale for Truss Diagram
0 5 10
Scale for Stresses
0 5 10 15
Scale for Section Moduli
0 50 100 150
NOTES
I = Moment of Inertia in in.⁴
y = Extreme Fibre Distance in in.
S = Section Modulus = I/y.
P = Primary Fibre Stress in lb. per sq. in.
r = Value of Deformation Contour
K_A1 = Value of Deformation Line for Joint I in Triangle A = ½E.δα
All Positive Moments Counter Clockwise
For Details of Truss see Plate VI, Figs. 49 and 50
Loading:—1500 lb. at Panel Point No. 9.

FIG. 56.

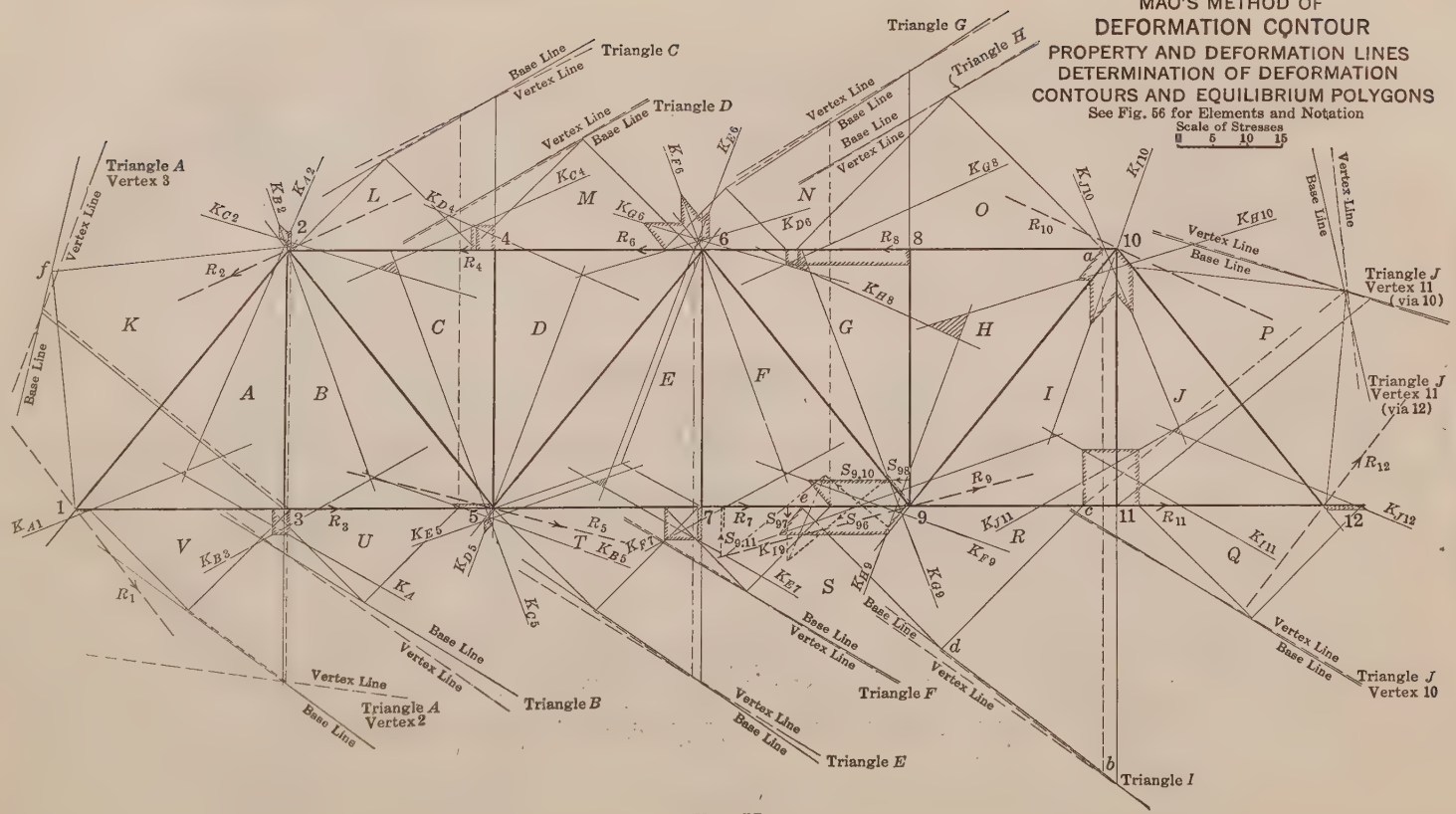

MAO'S METHOD OF
DEFORMATION CONTOUR
PROPERTY AND DEFORMATION LINES
DETERMINATION OF DEFORMATION
CONTOURS AND EQUILIBRIUM POLYGONS
See Fig. 56 for Elements and Notation
Scale of Stresses
0 5 10 15
Triangle A Vertex 3
Triangle A Vertex 2
Triangle B
Triangle C
Triangle D
Triangle E
Triangle F
Triangle G
Triangle H
Triangle I
Triangle J Vertex 10
Triangle J Vertex 11 (via 10)
Triangle J Vertex 11 (via 12)
Base Line
Vertex Line
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
S
T
U
V
1
2
3
4
5
6
7
8
9
10
11
12
a
b
c
d
e
f

FIG. 57.

and r_{21}, 45° lines to Member 1-2 are drawn, and it is found that their intersection, (f), practically falls on the parallel line to the base line of ΔA, Vertex (3), determined by drawing a perpendicular to Member 1-2 from the end of r_{31} and thus locating the point of intersection on the vertex line. This was a check on the work.

TABLE 23.—MAO'S METHOD OF DEFORMATION CONTOUR. SECONDARY STRESSES IN MEMBERS.

Joint.	Member.	r's, in pounds per square inch, from Fig. 51.	Secondary stress, in pounds per square inch.	Joint.	Member.	r's, in pounds per square inch, from Fig. 51.	Secondary stress, in pounds per square inch.
1	1- 2	— 0.25	— 0.42	7	7- 5	+ 0.55	+ 0.42
	1- 3	+ 2.15	+ 0.15		7- 6	— 4.75	— 4.67
2	2- 4	+ 0.70	+ 1.40		7- 9	— 5.60	+ 0.25
	2- 5	— 2.20	— 1.20	8	8- 6	+18.65	+10.53
	2- 3	— 2.50	— 2.98		8- 9	+ 2.45	+ 1.92
	2- 1	— 0.75	— 0.58		8-10	—17.00	—10.62
3	3- 1	— 0.85	+ 0.15	9	9- 7	+11.95	+ 6.10
	3- 2	— 3.80	— 3.37		9- 6	+ 5.30	+ 1.80
	3- 5	— 2.30	+ 0.23		9- 8	+ 0.85	+ 1.38
4	4- 6	— 3.60	— 1.98		9-10	— 5.30	— 1.45
	4- 5	— 3.75	— 3.23		9-11	—16.45	— 9.27
	4- 2	+ 2.80	+ 2.10	10	10-12	+ 3.60	+ 2.18
5	5- 3	+ 5.30	+ 2.77		10-11	+ 6.95	+ 7.47
	5- 2	+ 0.80	— 0.20		10- 9	+ 6.25	+ 2.40
	5- 4	— 2.20	— 2.72		10- 8	+ 2.15	— 4.23
	5- 6	— 2.30	— 2.10	11	11- 9	+ 5.10	— 2.08
	5- 7	+ 0.15	+ 0.28		11-10	+ 8.50	+ 7.98
6	6- 8	— 5.70	+ 2.41		11-12	+ 3.35	+ 0.35
	6- 9	— 5.20	— 1.70	12	12-11	— 5.65	— 2.65
	6- 7	— 4.50	— 4.58		12-10	— 0.65	+ 0.77
	6- 5	— 1.70	— 1.90				
	6- 4	+ 1.25	— 0.37				

As a final check, equilibrium polygons must be drawn around every joint. Only one, namely, at Joint (9), has been constructed. All the points of application of the S's, except $S_{9\cdot11}$, were found from the deformation contour of Joint (9) and those of the surrounding joints. These S's together with R_9 gave an $S_{9\cdot11}$ that lies only a short distance to the left of the $S_{9\cdot11}$, determined by $r_{9\cdot11}$ and $r_{11\cdot9}$. For the sake of clarity in the drawing the other equilibrium polygons are not shown.

There is a departure from Mao's method in that the f's are determined from a table of r's; Mao completes all the equilibrium polygons.

Mao's Analytical Method.—Plate II gives the complete solution of the problem by means of Mao's analytical method with the rational arrangement of computations.

The explanations given previously (see page 58 and Fig. 48) should serve to clarify the method of procedure here.

There is one departure from the original method; in the ΣM-table for Joint (6), there is an extra row, termed Σ, inserted between Row 6-7 and Row 6-9. The reason for this is to avoid errors. All the coefficients of the x and y and the constant terms for Rows 6-4, 6-5, and 6-7, are added. When Row 6-9 is completed, the negative of the sum of the x's of the Σ and Row 6-9 give the x-coefficient of Row 6-8. In the same manner, the y-coefficient and the k of Row 6-8 are obtained.

In the ΣM_{10} and ΣM_{12}-tables, the Σ is in the last row. An addition of the various columns gives the coefficients for the two simultaneous equations for finding x and y, that is, f_{12} and f_{21}, respectively. The little f-tables around the joints are then filled in, giving the secondary stresses.

Numerical Values.—For purposes of comparison, the amounts of secondary stresses by the foregoing methods have been collected in Table 24.

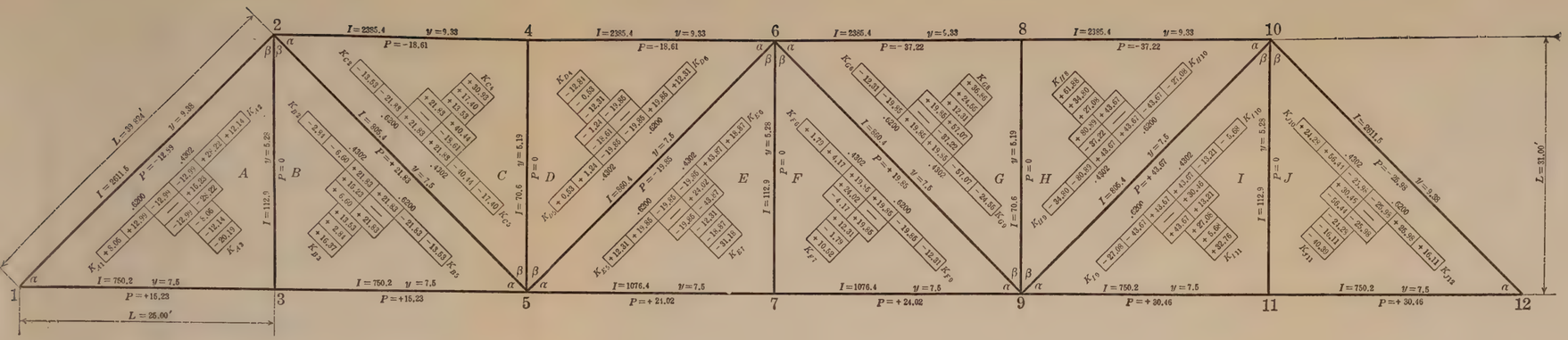
In determining the nature of the stress of an extreme fiber a clockwise rotation around each joint is made; a bending moment reckoned positive if counterclockwise then produces a tensile stress on the set of fibers first met and a compressive stress on those crossed last. Thus, at Joint (2), Member 2-4 is crossed over from top to bottom. It has a positive bending moment. The top fiber, therefore, has a secondary stress of —1.35 lb. per sq. in. and the bottom fiber one of —1.38 lb. per sq. in. (The difference in the numerical values is due to the respective fibers being 9.33 and 9.54 from the center of gravity of the member.) The resultant stresses of the top and bottom sets of fibers, therefore, would be (— 18.61 — 1.35) lb. per sq. in. and (— 18.61 — 1.38) lb. per sq. in., respectively.

For Member 2-3 at Joint (2) the set of fibers on the right-hand side (side first met) has a secondary stress of — 2.89 lb. per sq. in. and the left-hand side set has one of + 2.89 lb. per sq. in. The primary stress is zero and, therefore, the resultant stresses of the right and left-hand sets are — 2.89 and + 2.89 lb. per sq. in. The primary stresses are also given in Table 24 for information.

3.—Criticism of Methods

This criticism is a process of elimination, starting with Dr. Mao's methods which are believed to be very unsatisfactory.

Mao's Analytical Method.—Dr. Mao worked out the case for a symmetrical load. He was thereby able to solve the problem by using only half the number of triangles in the truss. The solution of the unsymmetrical case (Plate II) makes possible conclusive statements with regard to certain features of this method. It shows how rapidly the coefficients of the x and y and the constant terms, k, increase, as triangle is added to triangle. The coefficients in the U-group on Member 1-2 are of the magnitude 0, 1, or 2, while those in the ΣM_{12}-group range from 213 155.93 to 8 779 438. The question at once presents itself: "What is going to happen in a truss with twice the number of triangles"? These coefficients increase so rapidly, that it is certain they would amount to billions at least. A change of scale might be helpful. At the start the coefficients might be correct to three places of decimals as far as the quarter-point, where all decimals are discarded as far as the half-way point; between the middle and three-quarter points, the last three digits may be neglected, while the remainder of the work may only retain the significant figures up to and including the seventh digit. Now the solution depends on the differences of large numbers, and even if three decimal places were carefully maintained throughout the work, the f-values obtained did not give nearly as fine a check for the $\Sigma M = 0$ equations around the points as some of the other methods. The changes in scale to bring Mao's method within the bounds



SOLUTION OF EQUATIONS

From $\Sigma M_{10} = 0$ and $\Sigma M_{12} = 0$.

$+3\,639\,544\,x - 137\,223\,y = -1\,406\,877$

$+8\,579\,438\,x - 718\,960\,y = -3\,106\,013,$

i.e., $+26.522842\,x - \quad y = -10.2524868$

$+11.933123\,x - \quad y = -4.3201472,$

i.e., $+14.589719\,x \quad = -5.9323396$

$x = -0.40661,$

$y = -4.85214 + 4.32015$

$= -0.53199$

LEGEND

I = Moment of Inertia in in.⁴

y = Extreme Fibre Distance in ins.

L = Length of Member in ft.

$U = L/y$

S = Section Modulus = I/y

P = Primary Stress in lbs. per sq. in.

f_{mn} = Secondary Stress In Member mn, at end m, in lbs. per sq. in.

$2K_{Nn}$ = Change of Angle in Triangle N at Vertex n

ΣM_m = Sum of Moments around Joint m. Positive Moments Counter Clockwise

$x = f_{12}$

$y = f_{21}$

Loading: 1500 lbs. at Joint 9.

FORMULAE

$K_{Nm} = (P_{nq} - P_{mn}) \cdot \text{Cot. } \beta/2$

$K_{Nq} = (P_{mn} - P_{nq}) \text{ Cot. } \beta/2 + (P_{mn} - P_{mq}) \text{ Cot. } \alpha/2$

$f_{mn} = a_{mn}\,x + b_{mn}\,y + k_{mn}$, where a, b and k are Constants

$r_{mn} = 2f_{mn} - f_{nm}$

$= \frac{U_{mp}}{U_{mn}}\, r_{mp} + \frac{\Sigma^{m}_{mp} K_m}{U_{mn}}$, where mp is the Reference Member for Joint m and ΣK is Negative for Top Chord Joints

$r_{nm} = 3f_{mn} - 2r_{mn}$

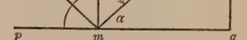
$f_{mn} = \frac{1}{2} r_{mn} + \frac{1}{2}(r_{mn} + r_{nm})$

$f_{mn} - f_{nm} = \frac{1}{2} r_{mn} - \frac{1}{2} r_{nm} = r_{mn} - f_{mn}$

$\Sigma M_m = \Sigma S_{mn} \cdot f_{mn} = 0$

Check for Every Triangle N:

$U_{mq}(f_{qm} - f_{mq}) + U_{nq}(f_{nq} - f_{qn}) + U_{mn}(f_{mn} - f_{nm}) = 0$



$\Sigma M_2 = 0$

	S	x	y	k
2–1	278.41	——	+ 278.41	——
2–3	21.38	− 64.70	+ 30.92	− 73.50
	Σ	− 64.70	+ 309.33	− 73.50
2–5	107.39	+1170.55	− 106.75	+ 284.06
2–4	255.67	−1166.85	− 202.58	− 210.56

ΣK

	2–3	2–5	2–4
K_{A2}	− 12.14		
K_{B2}	+ 2.84	− 9.30	
K_{C2}		+ 13.53	+ 4.23

$\Sigma M_4 = 0$

	S	x	y	k
4–2	255.67	− 1806.56	− 1215.20	—
4–5	13.60	+ 98.34	− 60.34	—
4–6	255.67	+ 1708.22	+ 1275.54	—

ΣK

	4–5	4–6
K_{C4}	− 30.93	
K_{D4}	+ 12.84	− 18.09

$U = 2.680$

x	y	k	
− 1.534	+ 3.169	+ 1.578	+ 1.578
− 4.325	− 0.792	− 0.824	
± 2.741	± 3.961	± 2.402	
− 7.066	− 4.753	− 3.226	
− 9.807	− 5.714	− 5.628	

$S = 255.67$

$U = 4.246$

x	y	k
− 1	+ 2	0
0	+ 1	0
∓ 1	± 1	0
+ 1	− 0	0
+ 2	− 1	0

$S = 278.41$

$U = 5.871$ $3U = 17.614$

x	y	k	
− 3.026	+ 1.446	− 3.488	——
− 0.241	+ 0.482	− 0.689	− 0.689
− 2.785	+ 0.964	− 2.749	− 0.914
− 2.544	+ 0.482	− 2.060	− 1.146
+ 2.303	——	+ 1.371	
− 5.329	+ 1.446	− 4.809	
+ 13.521	——	+ 8.049	
− 17.768	+ 4.246	− 8.049	
+ 4.246	− 4.246	——	

Check

$S = 21.38$

$U = 5.810$ $3U = 15.990$

x	y	k	
+ 10.900	− 0.994	+ 2.645	——
− 0.267	+ 0.533	− 0.584	− 0.584
+ 11.167	− 1.527	+ 3.229	+ 4.602
+ 11.434	− 2.060	+ 3.813	− 0.549
− 11.701	+ 2.593	− 4.397	
+ 22.601	− 3.587	+ 7.042	
− 62.132	+ 13.769	− 23.348	
+ 75.652	− 13.769	+ 31.394	
− 13.521	——	− 8.049	

$S = 107.39$

$U = 6.973$ $3U = 17.919$

x	y	k
+ 7.231	− 4.437	− 2.718
− 1.467	+ 1.863	− 2.566
+ 8.698	− 3.134	− 0.150
− 10.165	+ 1.531	+ 2.418
− 11.632	+ 0.528	− 4.386
+ 13.863	− 4.965	+ 2.268
− 69.478	+ 8.154	− 29.781
+ 62.132	− 13.769	+ 23.348
+ 7.346	− 16.615	+ 6.437

$S = 13.60$

ΣK

	5–2	5–4	5–6
K_{B5}	− 13.53		
K_{C5}	− 17.40	− 30.93	
K_{D5}		+ 0.53	− 30.40
K_{E5}			+ 12.31

$\Sigma M_1 = 0$

	S	x	y	k
1–2	278.41	+ 278.41	——	——
1–3	100.03	− 278.41	——	——

ΣK

	1–3
K_{A1}	+ 8.05

$U = 3.333$

x	y	k	
+ 2.548	− 1.274	+ 2.415	+ 2.415
− 2.783	——	——	
± 5.331	∓ 1.274	± 2.415	
− 8.114	+ 1.274	− 2.415	
− 13.445	+ 2.548	− 4.830	

$S = 100.03$

$\Sigma M_3 = 0$

	S	x	y	k
3–1	100.03	− 811.64	+ 127.44	− 241.57
3–2	21.38	− 113.93	+ 30.92	− 102.92
3–5	100.03	+ 925.57	− 158.36	+ 344.49

ΣK

	3–2	3–5
K_{A3}	− 20.19	
K_{B3}	+ 16.37	− 3.82

$U = 3.333$

x	y	k	
− 13.445	+ 2.548	− 5.976	− 4.830
+ 9.253	− 1.583	+ 3.443	− 1.146
− 22.698	+ 4.131	− 9.419	
+ 31.951	− 5.714	+ 12.862	
− 54.649	+ 9.845	− 22.281	

$S = 100.03$

$\Sigma M_5 = 0$

	S	x	y	k
5–3	100.03	+ 3196.06	− 571.57	+ 128
5–2	107.39	+ 2427.12	− 385.21	+ 73
5–4	13.60	+ 256.54	− 67.52	+ 3
	Σ	+5879.72	− 1024.30	+ 207
5–6	114.72	+ 3388.88	+ 152.58	+ 121

PLATE II.
TRANS., AM. SOC. CIV. ENGRS.
VOL. 89, PAPER No. 1574.
VON ABO ON
SECONDARY STRESSES IN BRIDGES.

SECONDARY STRESSES BY MAO'S ANALYTICAL METHOD

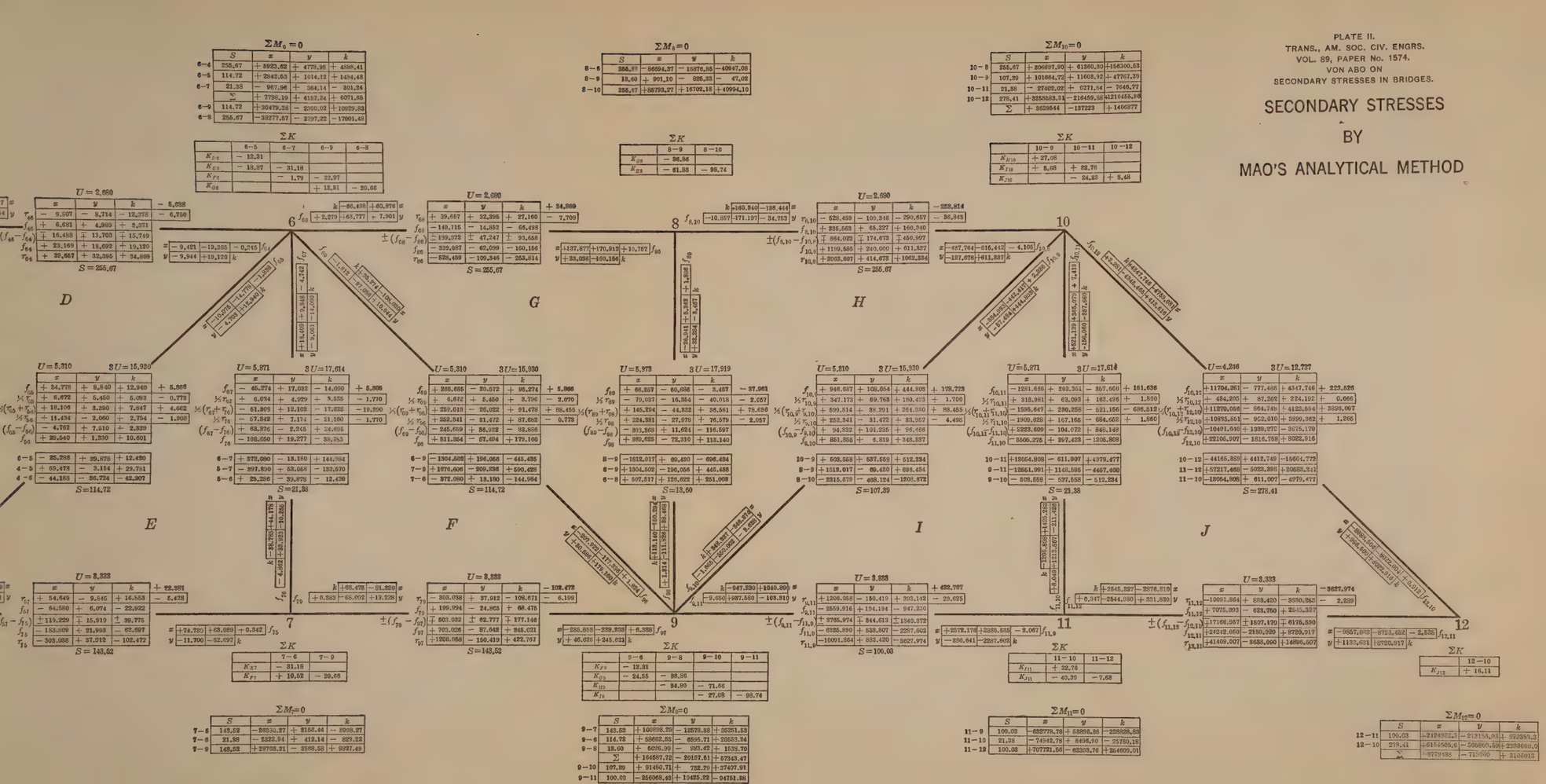


TABLE 24.—COMPARISON OF SECONDARY STRESSES DERIVED BY VARIOUS METHODS, TRUSS OF PLATE I; LOADING, 1 500 LB. AT JOINT (9).

Joint.	Member.	Primary stress, in pounds per square inch.	Secondary Stress, in Pounds per Square Inch.								
			Manderla.	Manderla-Gauss.	Muller-Breslau.	Ritter.	Mohr (Semi-Graphical).	Mohr (Elastic Weight).		Mao Analytical.	Mao Graphical.
								5 Decimal Places.	10 Decimal Places.		
1	1-2	−12.99	− 0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.41	0.42
			+ 0.42	0.41	0.41	0.42	0.42	0.41	0.41	0.41	0.43
	1-3	+15.23	∓ 1.12	1.13	1.14	1.16	1.14	1.13	1.13	1.13	1.15
	2-4	−18.61	∓ 1.35	1.38	1.40	1.36	1.38	1.38	1.38	1.36	1.40
			− 1.38	1.41	1.43	1.39	1.41	1.41	1.41	1.39	1.43
2	2-5	+21.83	∓ 1.26	1.25	1.26	1.25	1.25	1.25	1.25	1.26	1.20
	2-3	.....	∓ 2.89	2.88	2.90	2.90	2.88	2.89	2.89	2.98	2.93
	2-1	−12.99	− 0.58	0.57	0·58	0.57	0.58	0.57	0.57	0.54	0.59
			+ 0.57	0.56	0.57	0.57	0.57	0.56	0.56	0.53	0.58
3	3-1	+15.23	∓ 0.16	0.17	0.18	0·25	0.18	0.17	0.17	0.21	0.15
	3-2	.....	∓ 3.33	3.32	3.33	3.32	3.31	3.32	3.32	3.41	3.37
	3-5	+15.23	± 0.53	0.56	0.54	0.58	0.54	0.54	0.54	0.52	0.23
4	4-6	−18.61	− 1.95	1.92	1.83	1.88	1.92	1.91	1.91	2.00	1.98
			∓ 1.99	1.96	1.87	1.93	1.96	1.96	1.95	2.04	2.02
	4-5	.....	∓ 3.22	3.21	3.16	3.21	3.21	3.20	3.21	3.30	3.23
	4-2	−18.61	+ 2.10	2.13	2.22	2.15	2.13	2.13	2.13	2.23	2.15
			− 2.05	2.08	2.17	2.10	2.08	2.08	2.08	2.18	2.10
5	5-3	+15.23	∓ 2.82	2.83	2.83	2.89	2.82	2.83	2.83	2.91	2.77
	5-2	+21.83	∓ 0.30	0.29	0.30	0.28	0.29	0.29	0.29	0.24	0.20
	5-4	.....	∓ 2.69	2.70	2.64	2.66	2.67	2.67	2.67	2.76	2.72
	5-6	−19.85	∓ 2.09	2.08	2.08	2.07	2.07	2.08	2.07	2.12	2.10
	5-7	+24.02	± 0.15	0.16	0.16	0.22	0.16	0.16	0.15	0.11	0.28
6	6-8	−37.22	+ 2.27	2.28	2.28	2.31	2.28	2.29	2.26	2.28	2.41
			− 2.32	2.34	2.33	2.37	2.33	2.34	2.31	2.33	2.46
	6-9	+19.85	∓ 1.80	1.80	1·80	1.79	1.79	1.78	1.77	1.81	1.70
	6-7	.....	∓ 4.60	4.60	4.62	4.62	4.60	4.61	4.60	4.74	4.58
	6-5	−19.85	∓ 1.86	1.85	1.86	1.85	1.84	1.85	1.84	1.84	1.90
	6-4	−18.61	− 0.31	0.27	0.25	0.25	0.27	0.28	0.26	0.25	0.38
			+ 0.30	0.27	0.25	0.24	0.27	0.28	0.25	0.25	0.37

TABLE 24.—(*Continued*).

Joint.	Member.	Primary stress, in pounds per square inch.	Secondary Stress, in Pounds per Square Inch.								
			Manderla.	Manderla-Gauss.	Muller-Breslau.	Ritter.	Mohr (Semi-Graphical).	Mohr (Elastic Weight).		Mao Analytical.	Mao Graphical.
								5 Decimal Places.	10 Decimal Places.		
7	7–5	+24.02	± 0.30	0.31	0.30	0.32	0.31	0.30	0.29	0.34	0.42
	7–6	.....	∓ 4.78	4.72	4.74	4.74	4.73	4.74	4.73	4.86	4.67
	7–9	+24.02	± 0.40	0.39	0.40	0.43	0.39	0.41	0.41	0.38	0.25
8	8–10	−37.22	−10.52	10.52	10.55	10.55	10.52	10.53	10.58	10.86	10.62
			+10.75	10.75	10.79	10.79	10.75	10.77	10.82	11.09	10.85
	8–9	.....	± 1.85	1.85	1.88	1.84	1.86	1.88	1.82	1.89	1.92
	8–6	−37.22	+10.64	10.65	10.69	10.64	10.65	10.67	10.72	10.99	10.76
			−10.41	10.42	10.45	10.41	10.42	10.43	10.49	10.76	10.53
9	9–7	+24.02	± 6.15	6.14	6.18	6.19	6·15	6.21	6.14	6.39	6.10
	9–6	+19.85	± 1.68	1.68	1.69	1.69	1.69	1.71	1.62	1.82	1.80
	9–8	.....	± 1.30	1.29	1.32	1.29	1.29	1.33	1.29	1.31	1.38
	9–10	+43.67	∓ 1.47	1.47	1.49	1.46	1.47	1.47	1.44	1.66	1.45
	9–11	+30.46	∓ 9.32	9.33	9.39	9.33	9.33	9.48	9.29	9.65	9.27
10	10–12	−25.98	+ 2.30	2.31	2.43	2.34	2.31	2.46	2.33	2.28	2.18
			− 2.33	2.34	2.44	2.37	2.34	2.48	2.35	2.31	2.21
	10–11	.....	± 7.19	7.19	7.26	7.23	7.21	7.29	7.20	7.42	7.47
	10–9	+43.67	± 2.39	2.39	2.42	2.40	2.38	2.32	2.41	2.39	2.40
	10–8	− 37.22	− 4.22	4.21	4.36	4.24	4.22	4.35	4.24	4.20	4.32
			+ 4.13	4.12	4.27	4.14	4.12	4.26	4.15	4.10	4.23
11	11–9	+30.46	∓ 2.05	2.05	2.07	2.04	2.04	1.94	2.04	2.07	2.08
	11–10	.....	± 7.83	7.83	7.88	7.89	7.86	7.85	7.84	8.05	7.98
	11–12	+30.46	∓ 0.37	0.38	0.38	0.40	0.36	0.26	0.36	0.35	0.35
12	12–11	+30.46	∓ 2.54	2.54	2.55	2.55	2.55	2.64	2.55	2.53	2.65
	12–10	−25.98	+ 0.92	0.92	0.94	0.92	0.93	0.96	0.93	0.92	0.78
			− 0.91	0.91	0.93	0.91	0.92	0.95	0.92	0.91	0.77

of practice in solving the secondary stresses for a truss with only a few more triangles than that used in Plate II, are, therefore, certain to introduce large discrepancies.

Dr. Mao claims that the method is self-checking, that is, that if a computer makes an error, he is almost immediately aware of it. The writer's experience proved the contrary. An error which he made near the beginning of his computations was not discovered until a considerable time afterward and then only by comparing the results with those of the other methods. If there had been no other solution, the error would only have been detected when checking the $\Sigma M = 0$ equations.

This problem concerns a Warren truss with triangular elements. Suppose it had been a truss with subdivided panels, or even the top lateral system of the bridge, shown in Fig. 50. In the former case, some elements other than triangles appear, while all the triangles of the latter case have a vertex in the interior, instead of on the boundary of the frame. To adopt Mao's method to such cases would make an already weak method of smaller value still.

Mao's Graphical Method.—In this case, too, the unsymmetrical loading brings out certain features that were not so pronounced in the symmetrical case treated by Dr. Mao. It must be remembered that Figs. 56 and 57 give only those lines and points that are essential in the final solution. In carrying out the solution, the number of lines that must be drawn is enormous. Consider the problem of finding the base line for Member 5-7. There are required at least three points that must prove to be collinear, before the required line is accurately determined.

An explanation will be given of the steps necessary to arrive at one of these three points. (The process is repeated in its entirety for each of the other two, while each of the many base and vertex lines call for similar determinations.) At the stage where the base line for Member 5-7 is the next line to be determined, the base and vertex lines for the triangles, *A, B, C,* and *D,* have already been determined.

First, assume the reference contour for ΔE, namely, r_{64}, to be zero, and with this value finish a portion of the deformation contour to obtain the corresponding value for r_{65}.

Next, assume some value for r_{56}, and proceed to complete the corresponding deformation contour for Joint (5). Now, draw the vertical line through the end of r_{53} to cut the vertex line of ΔD, and also that of ΔC, as r_{53} is the reference contour for both these triangles. Through each of the two points thus found, draw a parallel to the base line of the corresponding triangle. Returning to the deformation contour at Joint (6), draw a line at 45° from the point, (6) (it being also the end of r_{64} as r_{64} is zero), until it meets the parallel to the base line of ΔD. From this point a line downward at 45° to meet Member 4-6, gives the value of r_{46}. To complete the corresponding deformation contour round Joint (4), draw a line from the end of r_{42}, at 45° upward to the left to cut the parallel to the base line of ΔC, and from this point, a line downward to cut Member 2-4, giving the value of r_{24}. With this the deformation contour around Joint (2) may be completed.

The work is now at the point where the value of the reference contour for ΔB, namely, r_{21}, is known. Draw a line vertically downward through the end of r_{21} to cut the vertex line of ΔB; through this point draw a line parallel to the base line of ΔB. Returning to the deformation contour of Joint (5), draw a line through the end of r_{53} downward at 45° to cut the parallel just found, and from the point of intersection draw a line at 45° upward to the left to cut Member 3-5, giving r_{35}.

This gives all the r's of all the members entering Joint (5) except r_{75}. Making use of the fact that $3\ f_{5n} = 2\ r_{5n} + r_{n5}$, determine the points on Members 5-3, 5-2, 5-4, and 5-6, that are distant $3\ f_{53}$, $3\ f_{52}$, $3\ f_{54}$, and $3\ f_{56}$, respectively, from Joint (5). Draw the S's through these points and complete the equilibrium polygon. S_{57} is regarded as the resultant of R_5, S_{53}, S_{52}, S_{54}, and S_{56}. Its distance from Joint (5) along Member 5-7 is $3\ f_{57}$.

Now, $r_{75} = 3\ f_{57} - 2\ r_{57}$, and as r_{57} was assumed at the start, the value of r_{75} is known. Draw a line through the end of r_{57} downward to the right and at 45°; draw another line through the end of r_{75} downward to the left and at 45 degrees. The point of intersection is the first point on the required base line.

It is at once evident that the amount of labor is prodigious, while the likelihood of error in the process may be the cause of many hours being wasted. Of course, where two or three members enter a joint, the process of determining the base and vertex lines, although complicated, is not as extended as that just explained.

In applying this graphical method, the number of lines becomes so great that the draftsman becomes lost. The natural thing to do would be to erase those lines that had been used to find a point and would not be used again. No erasures should be made, however, until the base line or the vertex line has been definitely found, otherwise the three points may be non-collinear and the lines may be lost from which these points were found; this would necessitate starting the problem of finding that particular base or vertex line from the very beginning.

There is no doubt that if all the checks are carefully carried out, Mao's graphical method will be correct, as the drawing proceeds, and will give results that are surprisingly accurate. However, it is not to be recommended. In the first place, it requires a large scale for reasonable accuracy; the result is an unwieldy drawing. Again, it requires ingenuity to adapt it to the solution of trusses with subdivided panels, for, as it is now, it cannot be so used. Thirdly, the labor involved to arrive at the "base" and "vertex" lines is excessive, and because of the many repetitions of the same process, is bound to become extremely monotonous. Finally, any graphical method is objectionable in the drafting-room, as a complete and thorough check by a second man requires an entirely new solution of the whole problem.

Ritter's Graphical Method.—This method entails much more than appears in Fig. 53. In this case, as in Mao's graphical method, the great number of lines that were required to arrive at the final solution shown, have been erased. The method gives reasonably accurate results. Bridge engineers, however,

would prefer an analytical method, which can be carried to any degree of accuracy. It may be said in favor of Ritter's method, that it may be checked by drawing a fresh set of equilibrium polygons. The writer, however, has an objection that eliminates Ritter's method. When the influence lines for the secondary stresses are required, and several conditions of loading are analyzed, Ritter's method cannot be used as advantageously as that of Manderla.

Müller-Breslau's and Mohr's "Elastic Weight" Methods.—Both these methods are to be discarded for the simple reason that the derivation of the $(4n\text{-}6)$ unknown quantities from a set of $(4n\text{-}6)$ simultaneous equations requires many places of decimals for reasonably accurate results. During the process of elimination, errors are likely to enter and cannot be detected until the $\Sigma M = 0$ checks for the joints are performed. Engineers have been bitterly opposed to methods that do not contain some checking features; how any one, therefore, can advocate these methods is difficult to understand.

Mohr's Semi-Graphical Method.—This method is better than all the preceding methods, when the solution of the simultaneous equations is performed by the method of Gauss' normal equations. Its accuracy depends on the accuracy with which the Williot diagram is drawn. Errors are liable to enter in the determination of the signs of the ρ's in the diagram, but, as the diagram is clear and contains few lines, it may easily be checked independently. It has the advantage over Manderla's method in that the changes of slope of the various members are at once found and, therefore, can be readily tested in practice.

Manderla's Method.—On comparison with the standard (Mohr's elastic weight method, carried to ten places of decimals) Mohr's semi-graphical and Manderla's methods give results that are close (Table 24). The calculations of these methods involve only three decimal places. In the matter of checking the computations, analytical methods are generally preferred to graphical. The writer personally much prefers Manderla's method to that of Mohr and has found that it could be used advantageously in the determination of influence lines.

In the case of plane frames that are not entirely composed of triangular elements Manderla's method shows itself more advantageous than any other. There is one exception, with rectangular frames that suffer bending but no change in the length of their members, Mohr's elastic weight method may be used to advantage.

Although Manderla's method still retains the features that made scientists anxious to improve on it, the writer's investigations have convinced him that none of the later ones is superior. There is a point of great importance that seems to be overlooked by many; the very fact that Manderla was able to obtain a set of relations between a portion of the unknown quantities (the number being the smallest possible, namely, the number of the joints) counts heavily in favor of that method.

The writer would urge that Gauss' theory of normal equations, or Turneaure's method of elimination, be used in every solution. With regard to the former, the instructions are put in such a manner that an inexperienced

engineer can carry out the solution of the unknown τ's to any desired degree of accuracy by means of a calculating machine; if he assures himself of the check at every stage of the solution, he cannot go wrong. The method is superior to that by substitution, where many trials are performed before any reasonable result is found, and where the additions and subtractions that are continually carried out on papers at the side are most annoying. The very directness of Gauss' method, where the object is definitely in view, ought to appeal to every one. Finally, the process of multiplying the equations by quantities far less than unity never allows the coefficients to become large. The solution may be made correct to five decimal places almost as easily as to three.

PART IV.—SOLUTION OF SECONDARY STRESSES BY INFLUENCE LINES

1.—Introduction

The stresses that occur in a bridge, subjected to (1) dead load and (2) dead load plus live load, will now be investigated. The bridge shown in Plate I and Figs. 49 and 50 will again be used. As symmetry is maintained about the longitudinal central vertical plane under both conditions of loading, it will be possible to use the same set of influence lines in each case. This set will tell when a certain member is taxed most heavily, what fiber suffers most, and what is the value of the stress in it at the instant.

To make the problem complete, it would be necessary to analyze the bridge for wind load and traction also, and combine the resulting solutions with those found in this paper to get the absolute maximum.

A certain amount of work has been omitted in order to save space. For example, only the equations in Section 2 resulting from 1 000 lb. at Panel Point (7) and 3 000 lb. at Panel Point (11), are reproduced, the work of obtaining these being exactly similar to that found in Tables 1 to 5, inclusive.

2.—Influence Lines for the Bridge Trusses

Load of 1 Kip at Panel Point (7).—Tables are prepared corresponding to Tables 2, 3, and 4, for use in forming the equations. As there is symmetry about the central vertical member, 6-7, no bending moments are found in it, while certain relations between the τ's reduce the number of equations to be solved to five. If the equations for Joints (1), (2), (3), (4), and (5) are written, it will be found that τ_6 and τ_7 enter into the last two equations. As there can be no bending in Member 6-7,

$$\tau_{67} = \tau_{76} = 0$$

Table 3 gives:

$$\tau_{67} = \tau_6 + 24.61 + 35.38$$

and,

$$\tau_{76} = \tau_7 - 110.92$$

whence,

$$\tau_6 = 59.99 \quad \text{and} \quad \tau_7 = +110.92$$

Finally, substituting these values in Equations (4) and (5) the equations for joints:

Joint (1)... $+15.930\,\tau_1 + 5.464\,\tau_2 + 2.501\,\tau_3 + 27.96 = 0$

Joint (2)... $+30.806\,\tau_2 + 7.951\,\tau_4 + 1.685\,\tau_5 + 0.303\,\tau_3 + 5.464\,\tau_1 - 2.58 = 0$

Joint (3)... $+10.610\,\tau_3 + 2.501\,\tau_1 + 0.303\,\tau_2 + 2.501\,\tau_5 - 25.50 = 0$

Joint (4)... $+32.184\,\tau_4 + 0.190\,\tau_5 + 7.951\,\tau_2 + 997.68 = 0$

Joint (5)... $+19.528\,\tau_5 + 2.501\,\tau_3 + 1.685\,\tau_2 + 0.190\,\tau_4 + 190.82 = 0$

Load of 1½ Kips at Panel Point (9).—This is the work completed on Fig. 51 and Tables 1 to 7.

Load of 3 Kips at Panel Point (11).—A process similar to that followed in the preceding case gives:

Joint (1)... $+15.930\,\tau_1 + 5.464\,\tau_2 + 2.501\,\tau_3 + 27.96 = 0$

Joint (2)... $+30.806\,\tau_2 + 7.951\,\tau_4 + 1.685\,\tau_5 + 0.303\,\tau_3 + 5.464\,\tau_1 - 2.58 = 0$

Joint (3)... $+10.610\,\tau_3 + 2.501\,\tau_1 + 0.303\,\tau_2 + 2.501\,\tau_5 - 25.50 = 0$

Joint (4)... $+32.184\,\tau_4 + 7.951\,\tau_6 + 0.190\,\tau_5 + 7.951\,\tau_2 + 828.72 = 0$

Joint (5)... $+19.528\,\tau_5 + 2.501\,\tau_3 + 1.685\,\tau_2 + 0.190\,\tau_4 + 1.800\,\tau_6 + 3.588\,\tau_7 - 610.88 = 0$

Joint (6)... $+39.610\,\tau_6 + 7.951\,\tau_8 + 1.800\,\tau_9 + 0.303\,\tau_7 + 1.800\,\tau_5 + 7.951\tau_4 + 842.42 = 0$

Joint (7)... $+14.958\,\tau_7 + 3.588\,\tau_5 + 0.303\,\tau_6 + 3.588\,\tau_9 - 443.04 = 0$

Joint (8)... $+32.184\,\tau_8 + 7.951\,\tau_{10} + 0.190\,\tau_9 + 7.951\,\tau_6 + 2\,619.39 = 0$

Joint (9)... $+19.528\,\tau_9 + 3.588\,\tau_7 + 1.800\,\tau_6 + 0.190\,\tau_8 + 1.685\,\tau_{10} + 2.501\,\tau_{11} + 227.60 = 0$

Joint (10).. $+30.806\,\tau_{10} + 5.464\,\tau_{12} + 0.303\,\tau_{11} + 1.685\,\tau_9 + 7.951\,\tau_8 + 5271.52 = 0$

Joint (11).. $+10.610\,\tau_{11} + 2.501\,\tau_9 + 0.303\,\tau_{10} + 2.501\,\tau_{12} - 2\,911.27 = 0$

Joint (12).. $+15.930\,\tau_{12} + 2.501\,\tau_{11} + 5.464\,\tau_{10} + 1\,034.14 = 0$

Simultaneous Equations.—It will be noticed that the first three equations of all the sets are identical, while the equations for Joints (1), (2), (3), (4), (5), and (7) are the same for the last two sets. These facts are used in the solutions, Table 25. The blank spaces could be filled with the quantities in the corresponding spaces of the preceding cases; it is evident that a certain amount of work for Case II was already done in solving Case I, while an even larger amount is duplicated in Cases II and III. The solution of such sets of equations is exceedingly easy and once all the coefficients in the τ-columns are found, any extra solution offers little trouble. The checks that occur all through the work make it impossible for a computer to go wrong in that part of the problem.

TABLE 25.—SOLUTION OF SIMULTANEOUS

Number of equation.	Remarks.	τ_1.	τ_2.	τ_3.	τ_4.	τ_5.	τ_6.	τ_7.
1	.....	15.930	5.464	2.501	.....	.....	.....	.....
1′	$1 \times -\frac{5.464}{15.930}$ *i. e.*, 0.34300	[−5.464]	−1.874	−0.858	.....	.....	.....	.....
2	.....	[+5.464]	+30.806	+3.303	+7.951	+1.685	.....	.....
I	1′ + 2	.....	+28.932	−0.555	+7.951	+1.685	.....	.....
1″	$1 \times -\frac{2.501}{15.930}$ *i. e.*, 0.15700	[−2.501]	[−0.858]	−0.393	.....	.....	.....	.....
I′	$I \times +\frac{0.555}{28.932}$ *i. e.*, 0.01918	.....	[+0.555]	−0.011	+0.153	+0.032	.....	.....
3	.....	[+2.501]	[+0.303]	+10.610	.....	+2.501	.....	.....
II	I″ + I′ + 3	.....	.....	+10.206	+0.153	+2.533	.....	.....
I″	$I \times -\frac{7.951}{28.932}$ *i. e.*, 0.27482	.....	[−7.951]	[+0.153]	−2.185	−0.463	.....	.....
II′	$II \times -\frac{0.153}{10.206}$ *i. e.*, 0.01499	.....	.....	[−0.153]	−0.002	−0.038	.....	.....
4	.....	.....	[+7.951]	.....	+32.184	+0.190	+7.951	.....
III	I′ + II′ + 4	.....	.....	.....	+29.997	+0.311	+7.951	.....
I‴	$I \times -\frac{1.685}{28.932}$ *i. e.*, 0.05824	.....	[−1.685]	[+0.032]	[−0.463]	−0.098	.....	.....
II″	$II \times -\frac{2.533}{10.206}$ *i. e.*, 0.24819	.....	.....	[−2.533]	[−0.038]	−0.629	.....	.....
III′	$III \times +\frac{0.311}{29.997}$ *i. e.*, 0.01037	.....	.....	.....	[+0.311]	−0.003	+0.082	.....
5	.....	.....	[+1.685]	[+2.501]	[+0.190]	+19.528	+1.800	+3.588
IV	I‴ + II″ + III′ + 5	.....	.....	.....	.....	+18.798	+1.882	+3.588
III″	$III \times -\frac{7.951}{29.997}$ *i. e.*, 0.26506	.....	.....	.....	[−7.951]	[+0.082]	−2.107	.....
IV′	$IV \times -\frac{1.882}{18.798}$ *i. e.*, 0.10012	.....	.....	.....	.....	[−1.882]	−0.188	−0.359
	.....	.....	.....	.....	[+7.951]	[+1.800]	+39.610	+0.303
V	III″ + IV′ + 6	.....	.....	.....	.....	.....	+37.315	−0.056
IV″	$IV \times -\frac{3.588}{18.798}$ *i. e.*, 0.19087	.....	.....	.....	.....	[−3.588]	[−0.359]	−0.685
V′	$V \times +\frac{0.056}{37.315}$ *i. e.*, 0.00150	.....	.....	.....	.....	.....	[+0.056]	+0.000
7	.....	.....	.....	.....	.....	[+3.588]	[+0.303]	+14.958
VI	IV″ + V′ + 7	.....	.....	.....	.....	.....	.....	+14.273
V″	$V \times -\frac{1.800}{37.315}$ *i. e.*, 0.04824	.....	.....	.....	.....	.....	[−1.800]	[+0.003]
VI′	$VI \times -\frac{3.591}{14.273}$ *i. e.*, 0.25159	.....	.....	.....	.....	.....	.....	[−3.591]
8	.....	.....	.....	.....	.....	.....	[+1.800]	[+3.588]
VII	V″ + VI′ + 8	.....	.....	.....	.....	.....	.....	.....
V‴	$V \times -\frac{7.951}{37.315}$ *i. e.*, 0.21308	.....	.....	.....	.....	.....	[−7.951]	[+0.012]
VI″	$VI \times -\frac{0.012}{14.273}$ *i. e.*, 0.00084	.....	.....	.....	.....	.....	.....	[−0.012]
VII′	$VII \times +\frac{0.197}{18.538}$ *i. e.*, 0.01063	.....	.....	.....	.....	.....	.....	.....
9	.....	.....	.....	.....	.....	.....	[+7.951]	.....
VIII	V‴ + VI″ + VII′ + 9	.....	.....	.....	.....	.....	.....	.....
VII″	$VII \times -\frac{2.501}{18.538}$ *i. e.*, 0.13491	.....	.....	.....	.....	.....	.....	.....

Equations for Influence Lines.

r_9.	r_8.	r_{11}.	r_{10}.	r_{12}.	1 000 lb. at Joint (7).		1 500 lb. at Joint (9).		3 000 lb. at Joint (11).	
					Absolute term.	Check.	Absolute term.	Check.	Absolute term.	Check.
.....	.....	.....	.....	.....	+2.796	+26.691	.....	.....	.....	.....
.....	.....	.....	.....	.....	−0.959	−9.155	.....	.....	.....	.....
.....	.....	.....	.....	.....	−0.258	+45.951	.....	.....	.....	.....
.....	.....	.....	.....	.....	−1.217	+36.796	.....	.....	.....	.....
.....	.....	.....	.....	.....	−0.439	−4.190	.....	.....	.....	.....
.....	.....	.....	.....	.....	−0.023	+0.706	.....	.....	.....	.....
.....	.....	.....	.....	.....	−2.550	+13.365	.....	.....	.....	.....
.....	.....	.....	.....	.....	−3.012	+9.881	.....	.....	.....	.....
.....	.....	.....	.....	.....	+0.334	−10.112	+0.334	−10.112	.....	.....
.....	.....	.....	.....	.....	+0.045	−0.148	+0.045	−0.148	.....	.....
.....	.....	.....	.....	.....	+99.768	+140.093	+82.872	+131.148	.....	.....
.....	.....	.....	.....	.....	+100.147	+129.833	+83.251	+120.888	.....	.....
.....	.....	.....	.....	.....	+0.071	−2.143	+0.071	−2.143	.....	.....
.....	.....	.....	.....	.....	+0.748	−2.452	+0.748	−2.452	.....	.....
.....	.....	.....	.....	.....	+1.038	+1.346	+0.863	+1.253	.....	.....
.....	.....	.....	.....	.....	+19.082	+42.986	−61.088	−31.796	.....	.....
.....	.....	.....	.....	.....	+20.939	+39.737	−59.406	−35.138	.....	.....
.....	.....	.....	.....	.....	.....	.....	−22.067	−32.043	−22.067	−32.043
.....	.....	.....	.....	.....	.....	.....	+5.947	+3.518	+5.947	+3.518
+1.800	+7.951	.....	.....	.....	.....	.....	+190.825	+250.240	+84.242	+143.657
+1.800	+7.951	.....	.....	.....	.....	.....	+174.705	+221.715	+68.122	+115.132
.....	.....	.....	.....	.....	.....	.....	+11.339	+6.707	+11.339	+6.707
+0.003	+0.012	.....	.....	.....	.....	.....	+0.262	+0.333	+0.102	+0.173
+3.588	.....	.....	.....	.....	.....	.....	−44.304	−21.867	−44.304	−21.867
+3.591	+0.012	.....	.....	.....	.....	.....	−32.703	−14.327	−32.863	−14.987
−0.087	−0.384	.....	.....	.....	.....	.....	−8.427	−10.695	−3.286	−5.554
−0.903	−0.003	.....	.....	.....	.....	.....	+8.228	+3.730	+8.268	+3.771
+19.528	+0.190	+2.501	+1.685	.....	.....	.....	−162.576	−133.284	+22.760	+52.052
+18.538	−0.197	+2.501	+1.685	.....	.....	.....	−162.775	−140.249	+27.742	+50.269
[−0.384]	−1.694	.....	.....	.....	.....	.....	−37.226	−47.243	−14.515	−24.532
[−0.003]	−0.000	.....	.....	.....	.....	.....	+0.027	+0.012	+0.028	+0.013
[+0.197]	−0.012	+0.027	+0.018	.....	.....	.....	−1.730	−1.490	+0.295	+0.534
[+0.190]	+32.184	.....	+7.951	.....	.....	.....	+290.050	+338.326	+261.939	+310.215
.....	+30.488	+0.027	+7.969	.....	.....	.....	+251.121	+289.605	+247.747	+286.230
[−2.501]	[+0.027]	−0.337	−0.227	.....	.....	.....	+21.960	+18.921	−3.743	−6.782

TABLE 25.—

Number of equation.	Remarks.	τ_1	τ_2	τ_3	τ_4	τ_5	τ_6	τ_7
VIII′	VIII $\times -\frac{0.027}{30.488}$ *i. e.*, 0.00089	.....	.....	.....	.....	.....	.....	.....
10	.....	.....	.....	.....	.....	.....	.....	.....
IX	VII″ + VIII′ + 10	.....	.....	.....	.....	.....	.....	.....
VII‴	VII $\times -\frac{1.685}{18.538}$ *i. e.*, 0.09089	.....	.....	.....	.....	.....	.....	.....
VIII″	VIII $\times -\frac{7.969}{30.488}$ *i. e.*, 0.26138	.....	.....	.....	.....	.....	.....	.....
IX′	IX $\times -\frac{0.069}{10.273}$ *i. e.*, 0.00672	.....	.....	.....	.....	.....	.....	.....
11	.....	.....	.....	.....	.....	.....	.....	.....
X	VII‴ + VIII″ + IX′ + 11	.....	.....	.....	.....	.....	.....	.....
IX″	IX $\times -\frac{2.501}{10.273}$ *i. e.*, 0.24345	.....	.....	.....	.....	.....	.....	.....
X′	X $\times -\frac{5.447}{28.570}$ *i. e.*, 0.19065	.....	.....	.....	.....	.....	.....	.....
12	.....	.....	.....	.....	.....	.....	.....	.....
XI	IX″ + X′ + 12	.....	.....	.....	.....	.....	.....	.....
	1 000 lb. at Joint (7)	−0.6297	+1.0395	+0.6218	−3.3501	−1.1139	.....	.....
	1 500 lb. " Joint (9)	−0.2160	+0.3578	−0.5240	−1.9095	+3.4158	−3.1328	+0.8043
	3 000 lb. " Joint (11)	−0.3291	+0.5889	−0.3084	−2.5595	+2.5865	−0.7130	+3.3796

Table of τ-Values and Bending Moments.—A table similar to Table 6 can now be drawn up for the three cases, after which the $\Sigma M = 0$ test must be fulfilled for every joint.

Table of Secondary Stresses.—The bending moments found for Cases II and III are for 1 500 lb. and 3 000 lb., respectively. To obtain influence lines, the two sets are divided by 1.5 and 3, respectively. The resulting quantities are found in Table 26. The secondary stresses are found by multiplying the bending moments by $\frac{c}{I}$, in which c is the extreme fiber distance, and I the moment of inertia of the member under consideration.

Influence Lines.—Figs. 58 to 68 contain the influence lines for the primary stresses and those secondary stresses that are due to the rigidity of the joints in the plane of the truss.

Consider the end chord, 1-2 (Fig. 58). Compressive primary stresses are drawn downward from a base line equal in length to the span of the bridge, while tensile primary stresses are drawn upward. The secondary stresses are drawn in just the reverse way, that is, compressive upward and tensile downward. To determine end effects, plot a set of lines for each end. The primary stress influence line, therefore, is drawn twice. Turning to Table 26, note that the primary stress in Member 1-2, is — 12.99 lb. per sq. in. for a load of 1 000 lb. at Joint (7), — 8.66 lb. per sq. in. for a load of 1 000 lb. at Joint (9), and — 4.33 lb. per sq. in. for a load of 1 000 lb. at Joint (11).

Continued).

τ_9	τ_8	τ_{11}	τ_{10}	τ_{12}	1 000 LB. AT JOINT (7).		1 500 LB. AT JOINT (9).		3 000 LB. AT JOINT (11).	
					Absolute term.	Check.	Absolute term.	Check.	Absolute term.	Check.
.....	[−0.027]	−0.000	−0.007	.....	.....	.....	−0.222	−0.256	−0.219	−0.253
[+2.501]	.....	+10.610	+0.303	+2.501	.....	.....	−48.694	−32.779	−291.127	−275.212
.....	.....	+10.273	+0.069	+2.501	.....	.....	−26.956	−14.114	−295.089	−282.247
[−1.685]	[+0.018]	[−0.227]	−0.153	.....	.....	.....	+14.795	+12.748	−2.522	−4.569
.....	[−7.969]	[−0.007]	−2.083	.....	.....	.....	−65.638	−75.697	−64.756	−74.815
.....	.....	[−0.069]	−0.000	−0.017	.....	.....	+0.181	+0.095	+1.982	+1.896
[+1.685]	[+7.951]	[+0.303]	+30.806	+5.464	.....	.....	−19.478	+26.731	+527.152	+573.361
.....	.....	.....	+28.570	+5.447	.....	.....	−70.140	−36.123	+461.856	+495.873
.....	.....	[−2.501]	[−0.017]	−0.609	.....	.....	+6.563	+3.496	+71.841	+68.714
.....	.....	.....	[−5.447]	+15.930	.....	.....	+13.372	+6.887	−88.055	−94.540
.....	.....	[+2.501]	[+5.464]	+15.930	.....	.....	+31.974	+55.869	+103.414	+127.309
.....	.....	.....	.....	+14.283	.....	.....	+51.909	+66.192	+87.200	+101.483
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
+7.9277	−9.0626	+3.4876	+3.1479	−3.6343	.....	.....	.....	.....	.....	.....
−4.2673	−4.2317	+30.3118	−15.0018	−6.1052	.....	.....	.....	.....	.....	.....

Now, when 1 000 lb. act at Joint (5), the effect on Member 1-2 is exactly the same as the effect of 1 000 lb. at Joint (9) on Member 10-12. From Table 26, this is — 17.32 lb. per sq. in. Finally, when 1 000 lb. act at Joint (3), the effect on Member 1-2 is exactly the same as the effect of 1 000 lb. at Joint (11) on Member 10-12, or — 21.65 lb. per sq. in.

The secondary stresses in Member 1-2 for End (1), are — 0.46, — 0.27, and — 0.13 lb. per sq. in., respectively, for the outside fiber stress, when the 1 000-lb. load is at Joints (7), (9), and (11), respectively, and + 0.47, + 0.28, and + 0.14 lb., respectively, for the inside fibers.

Again, 1 000 lb. acting at Joint (5) has exactly the same effect on End (1) as 1 000 lb. acting at Joint (9) has on End (12) of Member 12-10. It should be remembered that all the tabulations are for clockwise rotations around the joints, hence, the first set of fibers of Member 1-2, at End (1) corresponds to the last set of fibers of Member 12-10, at End (12). Therefore, take — 0.608 lb. per sq. in., found in Row 12-10, as the secondary stress in the outside fibers of End (1), when the 1 000 lb. act at Joint (5). The remainder follows in exactly the same way.

The reason for the reverse manner of representing the two kinds of stress is now evident; a glance at the four numbers that lie between the primary and secondary stress lines shows that the very maximum may happen to Member 1-2, when the bridge is fully loaded, and that it is the outside fibers of End (1), that are taxed more heavily than any of the three remaining sets.

TABLE 26.—Primary and Secondary Stresses for Influence Lines of Truss Members.

Joint.	Member.	$\frac{C}{I}$, in inch units.	1 000 lb. at Joint (7).			1 000 lb. at Joint (9).			1 000 lb. at Joint (11).		
			Primary stress, in pounds per square inch.	Bending moment, in inch-pound units.	Secondary stress, in pounds per square inch.	Primary stress, in pounds per square inch.	Bending moment, in inch-pound units.	Secondary stress, in pounds per square inch.	Primary stress, in pounds per square inch.	Bending moment, in inch-pound units.	Secondary stress, in pounds per square inch.
1	1-2	0.003592	−12.99	−129.27	−0.464	−8.66	−75.55	−0.271	−4.33	−37.59	−0.135
		0.003634			+0.470			+0.275			+0.137
	1-3	0.009997	+15.23	+129.27	±1.292	+10.15	+75.57	±0.755	+5.08	+37.62	±0.376
2	2-4	0.003911	−18.61	+339.65	+1.328	−12.41	+234.61	+0.918	−6.20	+107.34	+0.420
		0.003999			−1.358			−0.938			−0.429
	2-5	0.009312	+21.83	−241.15	∓2.246	+14.55	−89.62	∓0.835	+7.28	−48.93	∓0.456
	2-3	0.046784	.....	−46.44	∓2.173	.....	−41.10	∓1.923	.....	−19.18	∓0.897
	2-1	0.003634	−12.99	−52.09	−0.189	−8.66	−103.89	−0.378	−4.33	−39.23	−0.143
		0.003592			+0.187			+0.373			+0.141
3	3-1	0.009997	+15.23	+111.29	±1.113	+10.15	+11.57	±0.116	+5.08	+11.10	±0.111
	3-2	0.046784	.....	−52.90	∓2.475	.....	−47.28	±2.212	.....	−22.30	∓1.043
	3-5	0.009997	+15.23	−58.34	∓0.583	+10.15	+35.75	±0.357	+5.08	+11.25	±0.112
4	4-6	0.003911	−18.61	−111.50	−0.436	−12.41	−326.41	−1.277	−6.20	−103.84	−0.406
		0.003999			+0.446			+1.305			+0.415
	4-5	0.073492	.....	−71.84	∓5.279	.....	−29.12	∓2.140	.....	−17.26	∓1.268
	4-2	0.003999	−18.61	+183.40	+0.733	−12.41	+355.36	+1.421	−6.20	+121.01	+0.484
		0.003911			−0.717			−1.390			−0.473
5	5-3	0.009997	+15.23	−112.74	∓1.127	+10.15	+188.74	±1.887	+5.08	+70.33	±0.703
	5-2	0.009312	+21.83	−313.72	∓2.921	+14.55	−20.90	∓0.195	+7.28	−26.49	∓0.247
	5-4	0.073492	.....	−76.28	∓5.606	.....	−24.51	∓1.801	.....	−15.05	∓1.106
	5-6	0.008717	−19.85	−375.33	∓3.272	−13.23	−158.97	∓1.386	−6.62	−70.34	∓0.613
	5-7	0.006968	+24.02	−878.07	±6.118	+16.01	+15.41	±0.107	+8.01	+41.62	±0.290
6	6-8	0.003911	−18.61	−1 375.189	−5.378	−24.81	+389.49	+1.523	−12.41	−3.18	−0.012
		0.003999			+5.499			−1.558			+0.013
	6-9	0.008717	−19.85	−3.39	∓0.030	+13.23	−137.31	∓1.197	+6.62	−156.91	∓1.368
	6-7	0.046784	+41.07	.....	.....	.....	−65.49	∓3.064	.....	−16.76	∓0.784
	6-5	0.008717	−19.85	+3.39	±0.030	−13.23	−141.41	∓1.233	−6.62	−22.57	∓0.197
	6-4	0.003999	−18.61	+1 375.19	+5.499	−12.41	−45.37	−0.181	−6.20	+199.41	+0.797
		0.003911			−5.378			+0.177			−0.780

TABLE 26—(*Continued*).

Joint.	Member.	$\frac{C}{I}$, in inch units.	1 000 LB. AT JOINT (7). Primary stress, in pounds per square inch.	Bending moment, in inch-pound units.	Secondary stress, in pounds per square inch.	1 000 LB. AT JOINT (9). Primary stress, in pounds per square inch.	Bending moment, in inch-pound units.	Secondary stress, in pounds per square inch.	1 000 LB. AT JOINT (11). Primary stress, in pounds per square inch.	Bending moment, in inch-pound units.	Secondary stress, in pounds per square inch.
8	7–5	0.006968	+ 24.02	+1 632.98	± 11.379	+ 16.01	+ 29.52	± 0.206	+ 8.01	+ 142.08	± 0.990
	7–6	0.046784	+ 41.07	.....	.....	.....	− 67.26	∓ 3.147	.....	− 16.31	∓ 0.763
	7–9	0.006968	+ 24.02	−1 632.98	∓ 11.379	+ 16.01	+ 37.70	± 0.263	+ 8.01	− 125.70	∓ 0.876
	8–10	0.003911	− 18.61	− 183.40	− 0.717	− 24.81	−1 792.26	− 7.010	− 12.41	− 78.40	− 0.307
		0.003999			+ 0.733			+ 7.167			+ 0.314
	8–9	0.073492	.....	+ 71.84	± 5.279	.....	+ 16.76	± 1.232	.....	− 28.79	∓ 2.116
	8–6	0.003999	− 18.61	+ 111.50	+ 0.446	− 24.81	+1 775.41	+ 7.100	− 12.41	+ 107.02	+ 0.428
		0.003911			− 0.436			− 6.944			− 0.419
9	9–7	0.006968	+ 24.02	− 878.07	∓ 6.118	+ 16.01	+ 587.76	± 4.096	+ 8.01	− 215.95	∓ 1.505
	9–6	0.008717	− 19.85	+ 375.33	± 3.272	+ 13.23	+ 128.16	± 1.117	+ 6.62	− 199.56	∓ 1.740
	9–8	0.073492	.....	+ 76.28	± 5.606	.....	+ 11.67	± 0.858	.....	− 35.92	∓ 2.640
	9–10	0.009312	+ 21.83	+ 313.72	± 2.921	+ 29.11	− 105.25	∓ 0.980	− 7.28	− 234.39	∓ 2.183
	9–11	0.009997	+ 15.23	+ 112.74	± 1.127	+ 20.31	− 622.31	∓ 6.221	+ 25.38	+ 685.87	± 6.857
10	10–12	0.003592	− 12.99	+ 52.09	+ 0.187	− 17.32	+ 428.67	+ 1.540	− 21.65	− 465.46	− 1.672
		0.003634			− 0.189			− 1.558			+ 1.691
	10–11	0.046784	.....	+ 46.44	± 2.173	.....	+ 102.49	± 4.795	+ 41.07	− 5.69	∓ 0.266
	10–9	0.009312	+ 21.83	+ 241.15	± 2.246	+ 29.11	+ 170.97	± 1.592	− 7.28	− 45.11	∓ 0.420
11	10–8	0.003999	− 18.61	− 339.65	− 1.358	− 24.81	− 702.13	− 2.808	− 12.41	+ 516.18	+ 2.064
		0.003911			+ 1.328			+ 2.746			− 2.019
	11–9	0.009997	+ 15.23	+ 58.34	± 0.583	+ 20.31	− 136.69	∓ 1.336	+ 25.38	+1 101.04	± 11.007
	11–10	0.046784	.....	+ 52.90	∓ 2.475	.....	+ 111.65	± 5.223	+ 41.07	+ 10.57	∓ 0.495
	11–12	0.009997	+ 15.23	− 111.29	∓ 1.113	+ 20.31	+ 25.15	± 0.251	+ 25.38	−1 111.63	∓ 11.113
12	12–11	0.009997	+ 15.23	− 129.27	∓ 1.292	+ 20.31	− 169.20	∓ 1.691	+ 25.38	− 708.50	∓ 7.083
	12–10	0.003634	− 12.99	+ 129.27	+ 0.470	− 17.32	+ 169.31	+ 0.615	− 21.65	+ 708.50	+ 2.575
		0.003592			− 0.464			− 0.608			− 2.545

FIG. 58.

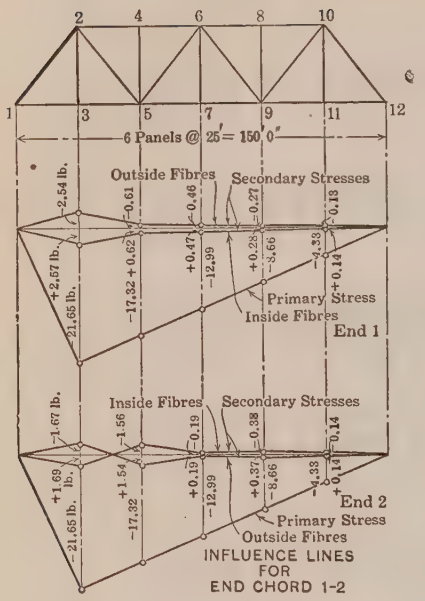
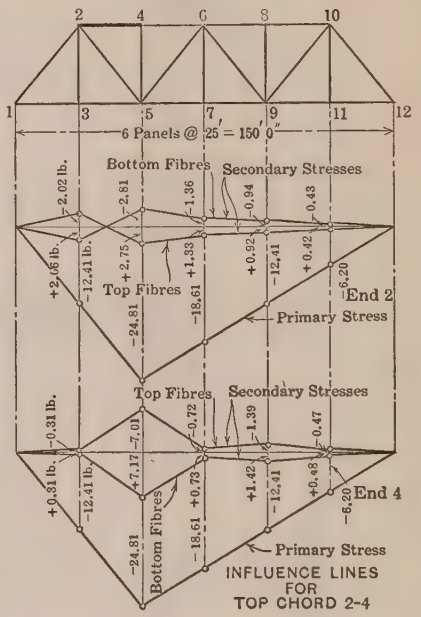


FIG. 59.



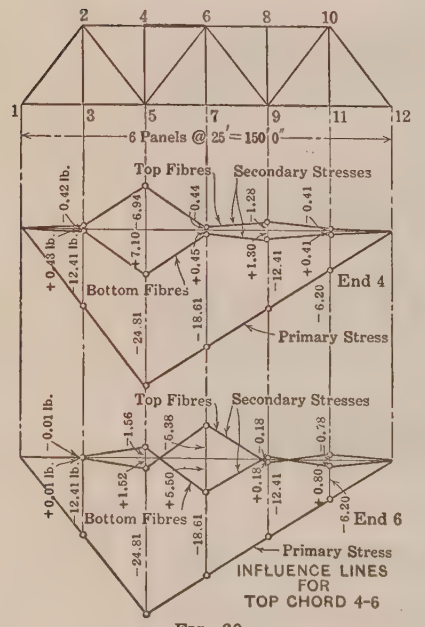


FIG. 60.

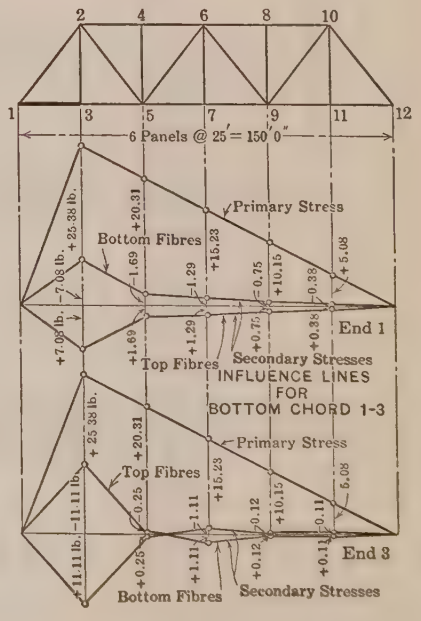


FIG. 61.

FIG. 62.

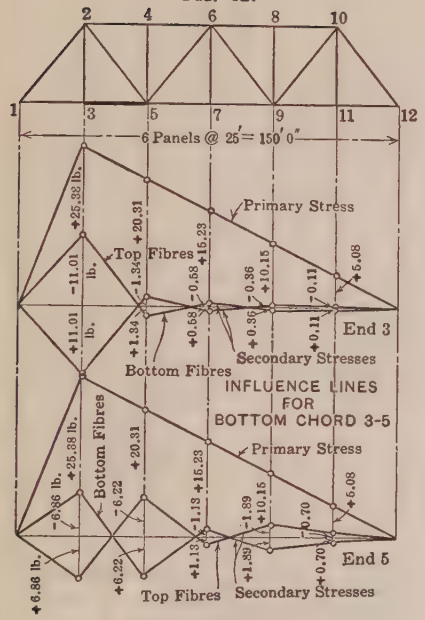
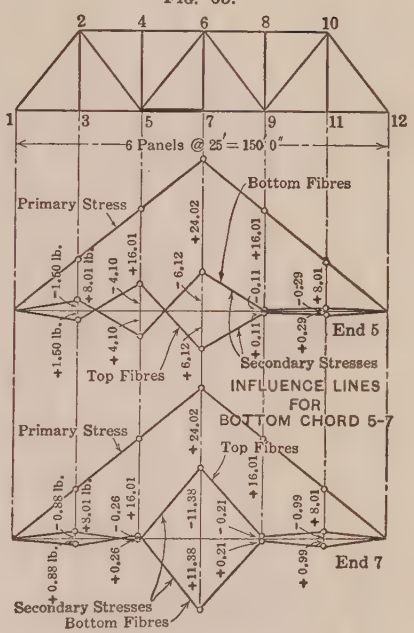


FIG. 63.



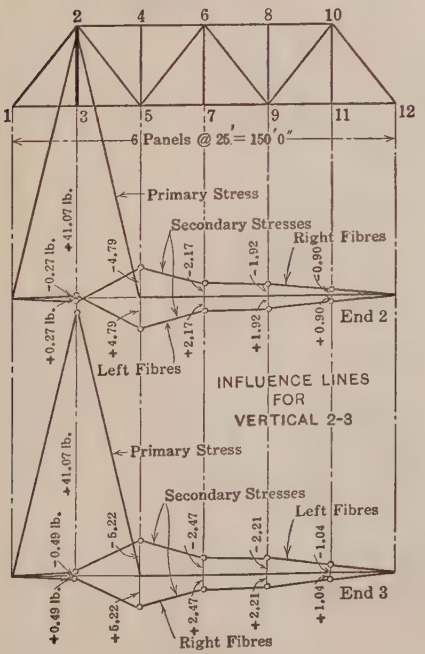


FIG. 64.

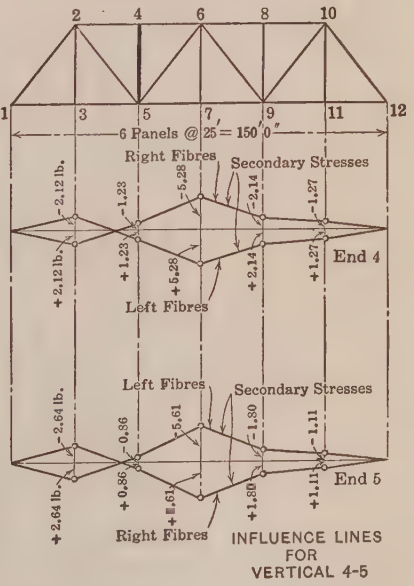


FIG. 65.

FIG. 66.

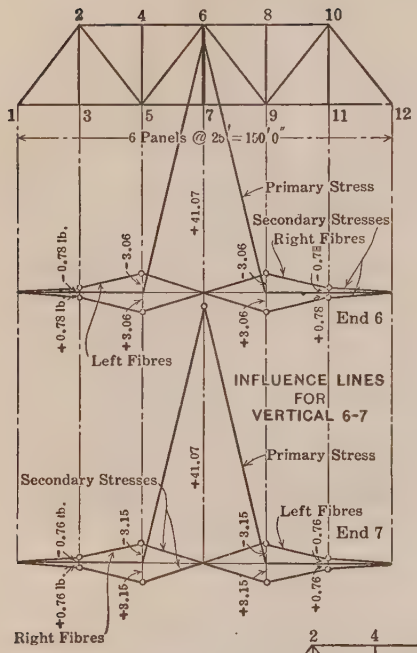
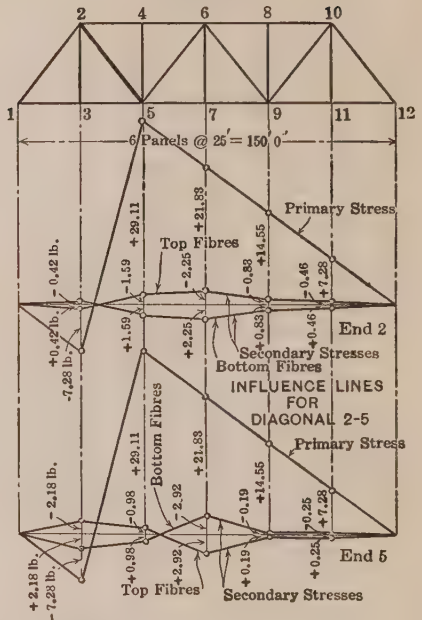


FIG. 67.



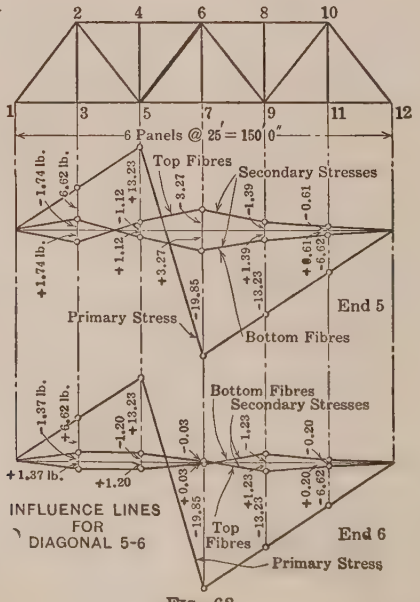


FIG. 68.

3.—Influence Lines for the Top Lateral System

The top lateral system forms a plane framework that has secondary stresses induced under either dead load or a combination of dead and live loads. The frame is considered complete from Joint (2) to Joint (10) of the bridge. Theoretically, the end portals contribute to the secondary stresses; their effect on the top lateral system, however, differs little from that found by reckoning the web-plates of the end portals as the closing members of the frame.

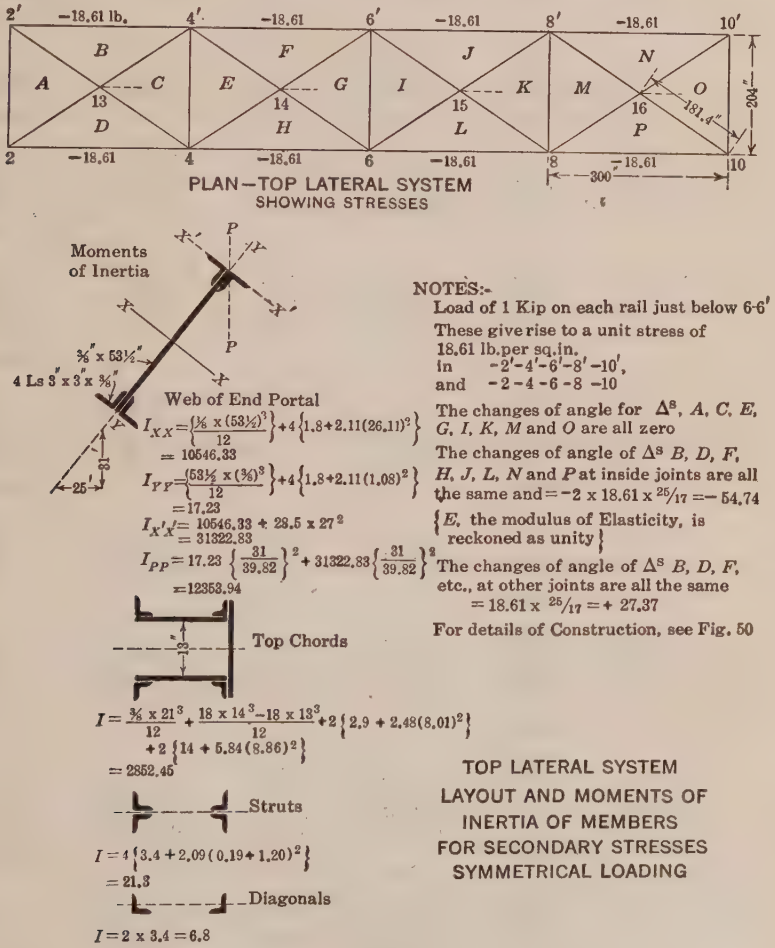


Fig. 69.

When there are two 1 000-lb. loads directly below Member 6-6′, Fig. 69, and symmetrical about the longitudinal center line, the unit stresses induced in the top chords are uniform and equal to — 18.61 lb. per sq. in. Fig. 69 and Tables 27 to 29 represent the solution for this case. The determination of the new set of moments of inertia, characteristic of a horizontal frame, is to be noted.

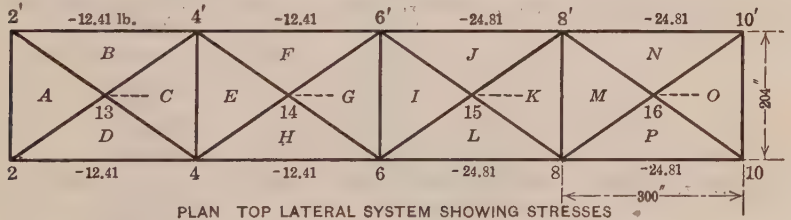
TABLE 27.—Secondary Stresses in Top Lateral System Due to Dead and Symmetrical Live Loads. Elements of Members.

Joint.	Member.	$K = \frac{I}{l}$	Angle.	δ Angle.	$\Sigma\,\delta$ Angle.	$K\,\Sigma\,\delta$ Angle.
2′	2′– 4′	9.508	.....	.....	.....	.....
	2′–13	0.037	4′– 2′–13	+27.37	+ 27.37	+ 1.013
	2′– 2	60.559	13 – 2′– 2	.....	+ 27.37	+1 657.500
		70.104				+1 658.513
2	2 – 2′	60.559	.....	.....	.....	.....
	2 –13	0.037	2′– 2 –13	.....	.....	.....
	2 – 4	9.508	13 – 2 – 4	+27.37	+ 27.37	+ 260.234
		70.104				+ 260.234
13	13 – 4	0.037	.....	.....	.....	.....
	13 – 2	0.037	4 –13 – 2	–54.74	– 54.74	– 2.025
	13 – 2′	0.037	2 –13 – 2′	.....	– 54.74	– 2.025
	13 – 4′	0.037	2′–13 – 4′	–54.74	–109.48	– 4.051
		0.148				– 8.102
4′	4′– 6′	9.508	.....	.....	.....	.....
	4′–14	0.037	6′– 4′–14	+27.37	+ 27.37	+ 1.013
	4′– 4	0.104	14′– 4′– 4	.....	+ 27.37	+ 2.846
	4′–13	0.037	4 – 4′–13	.....	+ 27.37	+ 1.013
	4′– 2′	9.508	13 – 4′– 2′	+27.37	+ 54.74	+ 520.468
		19.194				+ 525.340
4	4 – 2	9.508	.....	.....	.....	.....
	4 –13	0.037	2 – 4 –13	+27.37	+ 27.37	+ 1.013
	4 – 4′	0.104	13 – 4 – 4′	.....	+ 27.37	+ 2.846
	4 –14	0.037	4′– 4 –14	.....	+ 27.37	+ 1.013
	4 – 6	9.508	14 – 4 – 6	+27.37	+ 54.74	+ 520.468
		19.194				+ 525.340
14	14 – 6	0.037	.....	.....	.....	.....
	14 – 4	0.037	6 –14 – 4	–54.74	– 54.74	– 2.025
	14 – 4′	0.037	4 –14 – 4′	.....	– 54.74	– 2.025
	14 – 6′	0.037	4′–14 – 6′	–54.74	–109.48	– 4.051
		0.148				– 8.102
6′	6′– 8′	9.508	.....	.....	.....	.....
	6′–15	0.037	8′– 6′–15	+27.37	+ 27.37	+ 1.013
	6′– 6	0.104	15 – 6′– 6	.....	+ 27.37	+ 2.846
	6′–14	0.037	6 – 6′–14	.....	+ 27.37	+ 1.013
	6′– 4′	9.508	14 – 6′– 4′	+27.37	+ 54.74	+ 520.468
		19.194				+ 525.340
6	6 – 4	9.508	.....	.....	.....	.....
	6 –14	0.037	4 – 6 –14	+27.37	+ 27.37	+ 1.013
	6 – 6′	0.104	14 – 6 – 6′	.....	+ 27.37	+ 2.846
	6 –15	0.037	6′– 6 –15	.....	+ 27.37	+ 1.013
	6 – 8	9.508	15 – 6 – 8	+27.37	+ 54.74	+ 520.468
		19.194				+ 525.340
15	15 – 8	0.037	.....	.....	.....	.....
	15 – 6	0.037	8 –15 – 6	–54.74	– 54.74	– 2.025
	15 – 6′	0.037	6 –15 – 6′	.....	– 54.74	– 2.025
	15 – 8′	0.037	6′–15 – 8′	–54.74	–109.48	– 4.051
		0.148				– 8.102
8′	8′–10′	9.508	.....	.....	.....	.....
	8′–16	0.037	10′– 8′–16	+27.37	+ 27.37	+ 1.013
	8′– 8	0.104	16 – 8′– 8	.....	+ 27.37	+ 2.846
	8′–15	0.037	8 – 8′–15	.....	+ 27.37	+ 1.013
	8′– 6′	9.508	15 – 8′– 6′	+27.37	+ 54.74	+ 520.468
		19.194				+ 525.340

TABLE 27.—(*Continued*).

Joint.	Member.	$K = \frac{I}{l}$	Angle.	δ Angle.	Σ δ Angle.	K Σ δ Angle.
8	8 - 6	9.508	.....	.....	.....	.....
	8 -15	0.037	6 - 8 -15	+27.37	+ 27.37	+ 1.013
	8 - 8′	0.104	15 - 8 - 8′	.....	+ 27.37	+ 2.846
	8 -16	0.037	8′- 8 -16	.....	+ 27.37	+ 1.013
	8 -10	9.508	16 - 8 -10	+27.37	+ 54.74	+ 520.468
		19.194				+ 525.340

When there are two 1 000-lb. loads directly below Member 8-8′, Fig. 70, symmetrical about the longitudinal center line, the unit stresses in Members 2-4, 4-6, 2′-4′, and 4′-6′, are the same and equal to — 12.41 lb. per sq. in., while the unit stresses in Members 6-8, 8-10, 6′-8′, and 8′-10′, are the same and equal to — 24.81 lb. per sq. in. Fig. 70 and Tables 30 to 33 represent the solution for this case.



PLAN TOP LATERAL SYSTEM SHOWING STRESSES

NOTES:

Load of 1 Kip on each rail just below 8 - 8′.

The changes of angle in all the Δs A, C, E, G, I, K, M and O are all zero.

The changes of angle of B_2', B_4', F_4', F_6', D_2, D_4, H_4 and H_6 are each + 18.25.

The changes of angle of J_6', J_8', N_8', N_{10}', L_6, L_8, P_8 and P_{10} are each + 36.49.

The changes of angle of B_{13}, D_{13}, F_{14} and H_{14} are each - 36.50; those of J_{15}, L_{15}, N_{16}, P_{16} are each - 72.98.

For details of construction see Fig. 50.

For Moments of Inertia of Members see Fig. 69.

TOP LATERAL SYSTEM
LAYOUT FOR OBTAINING
SECONDARY STRESSES
UNSYMMETRICAL LOADING

FIG. 70.

When there are two 1 000-lb. loads directly below Member 10-10′, symmetrical about the longitudinal center line, the unit stresses in the top chords are exactly one-half those of the former case (the 1 000-lb. loads directly below Member 8-8′). The previous solution therefore determines the effect of loads directly below Members 2-2′ and 10-10′, as the bending moments and secondary stresses are just one-half what they are when the loads are directly below Member 4-4′ and Member 8-8′, respectively.

Therefore, only two sets of solutions are required, which are extremely easy, as may be noted. The ease with which Manderla's method of solution is used when there are interior joints is worthy of note.

In this analysis for the top lateral system, the influence lines for Members 2-4 and 4-6 are the only ones required for the chord members, as the loading

TABLE 28.—Secondary Stresses in Top Lateral System Due to Dead and Symmetrical Live Loads. Computation of T's.

$2'.-140.208\,\tau_2' + 9.508\,\tau_4' + 0.037\,\tau_{13} + 60.559\,\tau_2 + 3\,317.025 - 2.025 = 0$

	520.468
	3 837.493
	2 025
	+3 835 468

$2.-140.208\,\tau_2 + 60.559\,\tau_2' + 0.037\,\tau_{13} + 9.508\,\tau_4 + 520.468 - 2.025 = 0$

	1 657.500
	2 177.968
	2.025
	+2 175.943

$13.-0.296\,\tau_{13} + 0.037\,\tau_4 + 0.037\,\tau_2 + 0.037\,\tau_2' + 0.037\,\tau_4' - 16.203 + 1.013 = 0$

	3.038	1.013
	−13.165	1.013
		3.038

$4'.-38.388\,\tau_4' + 9.508\,\tau_6' + 0.037\,\tau_{14} + 0.104\,\tau_4 + 0.037\,\tau_{13} + 9.508\,\tau_2' + 1\,050.680 - 2.025 = 0$

	520.468	4.051
	2.846	6 076
	1 573.994	
	6.076	
	+1 567.918	

$4.-38.388\,\tau_4 + 9.508\,\tau_2 + 0.037\,\tau_{13} + 0.104\,\tau_4' + 0.037\,\tau_{14} + 9.508\,\tau_6 + 1\,050.680 - 2.025 = 0$

	260.234
	2.846
	1 313.760
	2.025
	+1 311.735

$14.-0.296\,\tau_{14} + 0.037\,\tau_6 + 0.037\,\tau_4 + 0.037\,\tau_4' + 0.037\,\tau_6' - 16.203 + 1.013 = 0$

	4.051	1.013
	−12.152	1.013
		1.013
		4.051

$6'.-38.388\,\tau_6' + 9.508\,\tau_8' + 0.037\,\tau_{15} + 0.104\,\tau_6 + 0.037\,\tau_{14} + 9.508\,\tau_4' + 1\,050.680 - 2.025 = 0$

	520.468	4.051
	2.846	6.076
	+1 573.994	
	6 076	
	+1 567.918	

$6.-38.388\,\tau_6 + 9.508\,\tau_4 + 0.037\,\tau_{14} + 0.104\,\tau_6' + 0.037\,\tau_{15} + 9.508\,\tau_8 + 1\,050.680 - 2.025 = 0$

	520 468
	2.846
	+1 573.994
	2.025
	+1 571.969

TABLE 28.—(*Continued*).

FROM SYMMETRY:

$$\begin{aligned}
\tau_{8'} &= \tau_{8'\cdot 10'} = -\tau_{4'2'} \\
&= -(\tau_{4'} + 54.74) \\
&= -\tau_{4'} - 54.74, \\
\tau_8 &= \tau_{86} = -\tau_{46} \\
&= -\tau_4 - 54.74, \\
\tau_{15} &= \tau_{15\cdot 8} = -\tau_{14\cdot 4} = -\tau_{14} + 54.74, \\
\tau_{24} &= -\tau_{2'4'} \\
&\text{Therefore } \tau_{2'} = -\tau_2 - 27.37, \\
\tau_{42} &= -\tau_{4'2'} \\
&\text{Therefore } \tau_4 = -\tau_{4'} - 54.74, \\
\tau_{6\cdot 4} &= -\tau_{6'4'} \\
&\text{Therefore } \tau_6 = -\tau_{6'} - 54.74.
\end{aligned}$$

SOLUTION:

$$\begin{aligned}
2.&-140.208\,\tau_2 + 60.559\,(-\tau_2 - 27.37) + 0.037\,\tau_{13} + 9.508\,\tau_4 + 2\,175.943 = 0 \\
13.&-0.296\,\tau_{13} + 0.037\,\tau_4 + 0.037\,\tau_2 + 0.037\,(-\tau_2 - 27.37) + 0.037\,(-\tau_4 - 54.74) - 13.165 = 0 \\
4.&-38.388\,\tau_4 + 9.508\,\tau_2 + 0.037\,\tau_{13} + 0.104\,(-\tau_4 - 54.74) + 0.037\,\tau_{14} + 9.508\,\tau_6 + 1\,311.735 = 0 \\
14.&-0.296\,\tau_{14} + 0.037\,\tau_6 + 0.037\,\tau_4 + 0.037\,(-\tau_4 - 54.74) + 0.037\,(-\tau_6 - 54.74) - 12.152 = 0 \\
6.&-38.388\,\tau_6 + 9.508\,\tau_4 + 0.037\,\tau_{14} + 0.104\,(-\tau_6 - 57.74) + 0.037\,(-\tau_{14} + 54.74) \\
&\quad + 9.508\,(-\tau_4 - 54.74) + 1\,571.969 = 0
\end{aligned}$$

that is,

$$\begin{aligned}
2.&+79.649\,\tau_2 + 0.037\,\tau_{13} + 9.508\,\tau_4 + 518.443 = 0 \\
13.&+0.296\,\tau_{13} - 16.203 = 0 \\
4.&+38.284\,\tau_4 + 9.508\,\tau_2 + 0.037\,\tau_{13} + 0.037\,\tau_{14} + 9.508\,\tau_6 + 1\,306.042 = 0 \\
14.&+0.296\,\tau_{14} - 16.203 = 0 \\
6.&+38.284\,\tau_6 + 1\,047.834 = 0
\end{aligned}$$

$$\begin{aligned}
&\text{Therefore } \tau_{13} = \tau_{14} = +54.74, \\
&\qquad \tau_6 = -27.37, \\
&\text{and } +79.649\,\tau_2 + 9.508\,\tau_4 + 520.468 = 0 \\
&\qquad +9.508\,\tau_2 + 38.284\,\tau_4 + 1\,049.859 = 0, \\
&\text{Therefore } \tau_4 = -26.588, \\
&\qquad \tau_2 = -3.361.
\end{aligned}$$

$$\text{Therefore } \begin{cases}
\tau_2 = -3.361 \\
\tau_4 = -26.588 \\
\tau_6 = -27.370 \\
\tau_{13} = +54.740 \\
\tau_{14} = +54.740 \\
\tau_{2'} = -24.009 \\
\tau_{4'} = -28.152 \\
\tau_{6'} = -27.370 \\
\tau_{15} = \ldots\ldots \\
\tau_8 = -28.152 \\
\tau_{8'} = -26.588
\end{cases}$$

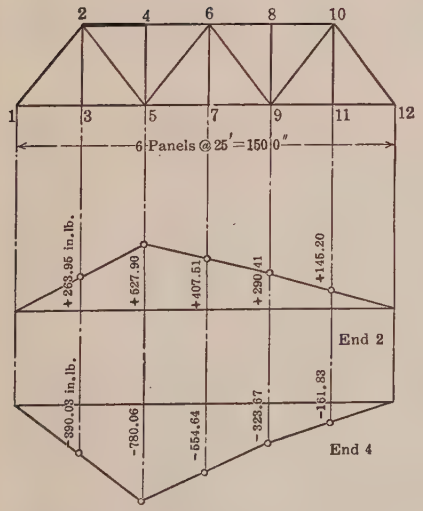
is symmetrical about the longitudinal center line. If necessary, the influence lines for the struts and diagonals could easily be drawn. The influence lines for Members 2-4 and 4-6 are given in Figs. 71 and 72. They have been purposely drawn for bending moments, and not for secondary stresses. From Fig. 69 it is seen that the extreme fiber distances of the cover-plate and the bottom angles of the chords are, respectively, 10.5 in. and 12 in. from the neutral plane under consideration. As the moment at the end of a member is numerically the same, no matter which extreme fiber distance is considered, an influence line for bending moment can be converted at once into the secondary stress influence line for a particular extreme fiber by suitably changing the scale. Hence, by multiplying the quantities for End (4) by $\pm \frac{10.5}{2\,852.45}$, the secondary stress influence lines for the extreme fibers of the cover-plate are obtained, and if by $\pm \frac{12}{2\,852.45}$, those for the extreme fibers of the bottom angles.

TABLE 29.—Secondary Stresses in Top Lateral, etc. Values of Moments.

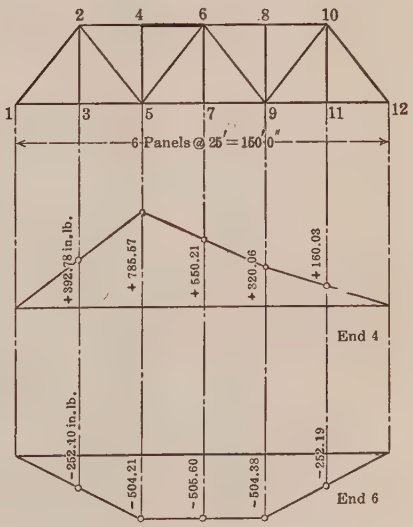
2′	$\tau_{2'4'}$	= − 24.009 + 27.370	$M_{2'4'}$	= 19.016 (− 48.018 + 26.588) = − 407.51
	$\tau_{2'.13}$	= + 3.361	$M_{2'.13}$	= 0.074 (+ 6.724 + 0) = + 0.50
	$\tau_{2'2}$	= + 3.361	$M_{2'2}$	= 121.118 (+ 6.724 − 3.361) = + 407.08
2	$\tau_{22'}$	= − 3.361	$M_{22'}$	= 121.118 (− 6.724 + 3.361) = − 407.08
	$\tau_{2.13}$	= − 3.361 + 27.370	$M_{2.13}$	= 0.074 (− 6.724 + 0) = − 0.50
	τ_{24}	= + 24.009	M_{24}	= 19.016 (+ 48.018 − 26.588) = + 407.51
13	$\tau_{13.4}$	= + 54.740 − 54.740	$M_{13.4}$	= 0.074 (+ 109.48 + 0.782) = + 8.16
	$\tau_{13.2}$	=	$M_{13.2}$	= 0.074 (0 − 3.361) = − 0.25
	$\tau_{13.2'}$	= ..... − 54.740	$M_{13.2'}$	= 0.074 (0 + 3.361) = + 0.25
	$\tau_{13.4'}$	= − 54.740	$M_{13.4'}$	= 0.074 (− 109.48 − 0.782) = − 8.16
4′	$\tau_{4'6'}$	= − 28.152 + 27.370	$M_{4'6'}$	= 19.016 (− 56.304 + 27.370) = − 550.21
	$\tau_{4'.14}$	= − 0.782	$M_{4'.14}$	= 0.074 (− 1.564 + 0) = − 0.12
	$\tau_{4'.4}$	= − 0.782	$M_{4'.4}$	= 0.208 (− 1.564 + 0.782) = − 0.16
	$\tau_{4'.13}$	= − 0.782 + 27.370	$M_{4'.13}$	= 0.074 (− 1.564 − 54.740) = − 4.17
	$\tau_{4'.2'}$	= + 26.588	$M_{4'.2'}$	= 19.016 (+ 53.176 − 24.009) = + 554.64
4	$\tau_{4.2}$	= − 26.588 + 27.370	M_{42}	= − 554.64
	$\tau_{4.13}$	= + 0.782	$M_{4.13}$	= + 4.17
	$\tau_{44'}$	= + 0.782	$M_{44'}$	= + 0.16
	$\tau_{4.14}$	= + 0.782 + 27.370	$M_{4.14}$	= + 0.12
	$\tau_{4.6}$	= + 28.152	$M_{4.6}$	= + 550.21
14	$\tau_{14.6}$	= + 54.74 − 54.74	$M_{14.6}$	= 0.074 (+ 109.48 + 0) = + 8.10
	$\tau_{14.4}$	=	$M_{14.4}$	= 0.074 (0 + 0.782) = + 0.06
	$\tau_{14.4'}$	= ..... − 54.74	$M_{14.4'}$	= − 0.06
	$\tau_{14.6'}$	= − 54.74	$M_{14.6'}$	= − 8.10
6′	$\tau_{6'8'}$	= − 27.37 + 27.37	$M_{6'.8'}$	= 19.016 (− 54.74 + 28.152) = − 505.60
	$\tau_{6'.15}$	=	$M_{6'.15}$	= 0.074 (0 + 54.74) = + 4.05
	$\tau_{6'.6}$	=	$M_{6'.6}$	= 0.208 (0 + 0) = ...
	$\tau_{6'.14}$	= ..... + 27.37	$M_{6'.14}$	= − 4.05
	$\tau_{6'4'}$	= + 27.37	$M_{6'4'}$	= + 505.60

TABLE 29.—(*Continued*).

6	$\tau_{6.4}$ =	− 27.37 + 27.37	M_{64} =	− 505.60
	$\tau_{6.14}$ =	....	$M_{6.14}$ =	+ 4.05
	$\tau_{66'}$ =	....	$M_{66'}$ =	.....
	$\tau_{6.15}$ =	..... + 27.37	$M_{6.15}$ =	− 4.05
	$\tau_{6.8}$ =	+ 27.37	$M_{6.8}$ =	+ 505.60



INFLUENCE LINES OF BENDING MOMENTS
INDUCED IN 2-4
AS PART OF TOP LATERAL SYSTEM
FIG. 71.



INFLUENCE LINES OF BENDING MOMENTS
INDUCED IN 4-6
AS PART OF TOP LATERAL SYSTEM
FIG. 72.

TABLE 30.—Secondary Stresses in Top Lateral System Due to Dead and Unsymmetrical Live Loads. Elements of Members.

Joint.	Member.	$K = \frac{I}{l}$	Angle.	δ Angle.	Σ δ Angle.	K Σ δ Angle.
2′	2′– 4′	9.508	.....	.....	.....	.....
	2′–13	0.037	4′– 2′–13	+18.25	+ 18.25	+ 0.675
	2′– 2	60.559	13 – 2′– 2	.....	+ 18.25	+1105.202
		70.104				+1105.877
2	2 – 2′	60.559	.....	.....	.....	.....
	2 –13	0.037	2′– 2 –13	.....	.....	.....
	2 – 4	9.508	13 – 2 – 4	+18.25	+ 18.25	+ 173.521
		70.104				+ 173.521
13	13 – 4	0.037	.....	.....	.....	.....
	13 – 2	0.037	4 –13 – 2	–36.50	– 36.50	– 1.350
	13 – 2′	0.037	2 –13 – 2′	.....	– 36.50	– 1.350
	13 – 4′	0.037	2′–13 – 4′	–36.50	– 73.00	– 2.701
		0.148				– 5.402
4′	4′– 6′	9.508	.....	.....	.....	.....
	4′–14	0.037	6′– 4′–14	+18.25	+ 18.25	+ 0.675
	4′– 4	0.104	14 – 4′– 4	.....	+ 18.25	+ 1.898
	4′–13	0.037	4 – 4′–13	.....	+ 18.25	+ 0.675
	4′– 2′	9.508	13 – 4′– 2′	+18.25	+ 36.50	+ 347.042
		19.194				+ 350.290
4	4 – 2	9.508	.....	.....	.....	.....
	4 –13	0.037	2 – 4 –13	+18.25	+ 18.25	+ 0.675
	4 – 4′	0.104	13 – 4 – 4′	.....	+ 18.25	+ 1.898
	4 –14	0.037	4′– 4 –14	.....	+ 18.25	+ 0.675
	4 – 6	9.508	14 – 4 – 6	+18.25	+ 36.50	+ 347.042
		19.194				+ 350.290
14	14 – 6	0.037	.....	.....	.....	.....
	14 – 4	0.037	6 –14 – 4	–36.50	– 36.50	– 1.350
	14 – 4′	0.037	4 –14 – 4′	.....	– 36.50	– 1.350
	14 – 6′	0.037	4′–14 – 6′	–36.50	– 73.00	– 2.701
		0.148				– 5.402
6′	6′– 8′	9.508	.....	.....	.....	.....
	6′–15	0.037	8′– 6′–15	+36.49	+ 36.49	+ 1.350
	6′– 6	0.104	15 – 6′– 6	.....	+ 36.49	+ 3.795
	6′–14	0.037	6 – 6′–14	.....	+ 36.49	+ 1.350
	6′– 4′	9.508	14 – 6– 4′	+18.25	+ 54.74	+ 520.468
		19.194				+ 526.963
6	6 – 4	9.508	.....	.....	.....	.....
	6 –14	0.037	4 – 6 –14	+18.25	+ 18.25	+ 0.675
	6 – 6′	0.104	14 – 6 – 6′	.....	+ 18.25	+ 1.898
	6 –15	0.037	6′– 6 –15	.....	+ 18.25	+ 0.675
	6 – 8	9.508	15 – 6 – 8	+36.49	+ 54.74	+ 520.468
		19.194				+ 523.716
15	15 – 8	0.037	.....	.....	.....	.....
	15 – 6	0.037	8 –15 – 6	–72.98	– 72.98	– 2.700
	15 – 6′	0.037	6 –15 – 6′	.....	– 72.98	– 2.700
	15 – 8′	0.037	6′–15 – 8′	–72.98	–145.96	– 5.401
		0.148				– 10.801

TABLE 30.—(*Continued.*)

Joint.	Member.	$K = \frac{I}{l}$	Angle.	δ Angle.	Σ δ Angle.	K Σ δ Angle.
8′	8′-10′	9.508	.....	.....	.....	.....
	8′-16	0.037	10′- 8′-16	+36.49	+ 36.49	+ 1.350
	8′- 8	0.104	16 - 8′- 8	.....	+ 36.49	+ 3.795
	8′-15	0.037	8 - 8′-15	.....	+ 36.49	+ 1.350
	8′- 6′	9.508	15 - 8′- 6′	+36.49	+ 72.98	+ 693.894
		19.194				+ 700.389
8	8 - 6	9.508	.....	.....	.....	.....
	8 -15	0.037	6 - 8 -15	+36.49	+ 36.49	+ 1.350
	8 - 8′	0.104	15 - 8 - 8′	.....	+ 36.49	+ 3.795
	8 -16	0.037	8′- 8 -16	.....	+ 36.49	+ 1.350
	8 -10	9.508	16 - 8 -10	+36.49	+ 72.98	+ 698.894
		19.194				+ 700.389
16	16 -10	0.037	.....	.....	.....	.....
	16 - 8	0.037	10 -16 - 8	−72.98	− 72.98	− 2.700
	16 - 8′	0.037	8 -16 - 8′	.....	− 72.98	− 2.700
	16 -10′	0.037	8′-16 -10′	−72.98	−145.96	− 5.401
		0.148				− 10.801
10′	10′-10	60.559	.....	.....	.....	.....
	10′-16	0.037	10 -10′-16	.....	.....	.....
	10′- 8′	9.508	16 -10′- 8′	+36.49	+ 36.49	+ 346.947
		70.104				+ 346.947
10	10 - 8	9.508	.....	.....	.....	.....
	10 -16	0.037	8 -10 -16	+36.49	+ 36.49	+ 1.350
	10 -10′	60.559	16 -10 -10′	.....	+ 36.49	+2209.798
		70.104				+2211.148

Tables 29 and 33 permit the construction of the influence lines as follows: When the loads are below Member 6-6′, $M_{46} = +550.21$ and $M_{64} = -505.60$ in-lb. When the loads are below Member 8-8′, $M_{46} = +320.06$ and $M_{64} = -504.38$ in-lb. When the loads are below Member 10-10′, these last two quantities are halved, giving +160.03 and −252.19, respectively. When the loads are below Member 4-4′, the effects on End (4) and End (6) of Member 4-6 are the same as the effects on End (8) and End (6), respectively, of Member 8-6 when the loads are situated below Member 8-8′. Here, there is a slight variation, depending on the convention of signs. M_{86} is given as −785.57 in-lb., Table 33, that is, a bending moment that is acting in a clockwise direction around Joint (8); the corresponding one around Joint (4), therefore, must be counterclockwise and reckoned as + 785.57 in-lb. The remaining influence lines can be easily deduced.

4.—Influence Lines for Cross-Frame 6-6′-7′-7

The effect of the slender angles that extend from the top lateral strut to the vertical posts is neglected; the secondary stresses due to them are extremely small. Therefore, the problem concerns a rectangular frame, as shown in Fig. 73. Consider the unsymmetrical case of a load on one rail only; not only will this determine the best method under the worst conditions, but it may easily be extended to take into account wind forces.

TABLE 31.—Secondary Stresses in Top Lateral System Due to Dead and Unsymmetrical Live Loads. Formation of Equations.

$2'.\quad +140.208\,\tau_2' + 9.508\,\tau_4' + 0.037\,\tau_{13} + 60.559\,\tau_2 + 2\,211.754 - 1.350 = 0$

	347.042
	2 558.796
	1.350
	+2 557.446

$2.\quad +140.208\,\tau_2 + 60.559\,\tau_2' + 0.037\,\tau_{13} + 9.508\,\tau_4 + 347.042 - 1.350 = 0$

	1 105.202
	1 452.244
	1.350
	+1 450.894

$13.\quad +0.296\,\tau_{13} + 0.037\,\tau_4 + 0.037\,\tau_2 + 0.037\,\tau_2' + 0.037\,\tau_4' - 10.804 + 0.675 = 0$

	2.025
−8.779	0.675
	2.025

$4'.\quad +38.388\,\tau_4' + 9.508\,\tau_6' + 0.037\,\tau_{14} + 0.104\,\tau_4 + 0.037\,\tau_{13} + 9.508\,\tau_2' + 700.580 - 1.350 = 0$

	520.468
	2.701
1.898	4.051
	+1 222.946
	4.051
	+1 218.895

$4.\quad +38.388\,\tau_4 + 9.508\,\tau_6 + 0.037\,\tau_{14} + 0.104\,\tau_4' + 0.037\,\tau_{13} + 9.508\,\tau_2 + 700.580 - 1.350 = 0$

	173.521
	1.898
	875.999
	1.350
	+874.649

$14.\quad +0.296\,\tau_{14} + 0.037\,\tau_6 + 0.037\,\tau_4 + 0.037\,\tau_4' + 0.037\,\tau_6' - 10.804 + 0.675 = 0$

	3.376
−7.428	0.675
	1.350
	3 376

$6'.\quad +38.388\,\tau_6' + 9.508\,\tau_8' + 0.037\,\tau_{15} + 0\,104\,\tau_6 + 0.037\,\tau_{14} + 9.508\,\tau_4' + 1\,053.926 - 2.700 = 0$

	693.894
	2.701
1.898	5.401
	1 749.718
	5.401
	+1 744.317

$6.\quad +38.388\,\tau_6 + 9.508\,\tau_4 + 0.037\,\tau_{14} + 0.104\,\tau_6' + 0.037\,\tau_{15} + 9.508\,\tau_8 + 1\,047.432 - 2.700 = 0$

	347.042
	3.795
	+1 398.269
	2.700
	+1 395.569

TABLE 31.—(*Continued*).

15. $+0.296\,\tau_{15}+0.037\,\tau_8+0.037\,\tau_6+0.037\,\tau_6'+0.037\,\tau_8'-21.602+0.675=0$
$4.725 \quad 1.350$
$-16.877 \quad 1.350$
1.350
4.725

8′. $+38.388\,\tau_8'+9.508\,\tau_{10}'+0.037\,\tau_{16}+0.104\,\tau_8+0.037\,\tau_{15}+9.508\,\tau_6'+1\,400.778-2.700=0$
$346.947 \quad 5.401$
$3.795 \quad 8.101$
$1\,751.520$
8.101
$+1\,743.419$

8. $+38.388\,\tau_8+9.508\,\tau_6+0.037\,\tau_{15}+0.104\,\tau_8'+0.037\,\tau_{16}+9.508\,\tau_{10}+1\,400.778-2.700=0$
520.468
3.795
$1\,925.041$
2.700
$+1\,922.341$

16. $+0.296\,\tau_{16}+0.037\,\tau_{10}+0.037\,\tau_8+0.037\,\tau_8'+0.037\,\tau_{10}'-21.602+1.350=0$
$4.050 \quad 1.350$
$-17.552 \quad 1.350$
4.050

10′. $+140.208\,\tau_{10}'+60.559\,\tau_{10}+0.037\,\tau_{16}+9.508\,\tau_8'+693.894-5.401=0$
$2\,209.798$
$2\,903.692$
5.401
$+2\,898.291$

10. $+140.208\,\tau_{10}+60.559\,\tau_{10}'+0.037\,\tau_{16}+9.508\,\tau_8+4\,422.296=0$
693.894
$+5\,116.190$

From Symmetry:

$$\left.\begin{aligned}\tau_2' &= -\tau_2 - 18.25\\ \tau_4' &= -\tau_4 - 36.50\\ \tau_6' &= -\tau_6 - 54.74\\ \tau_8' &= -\tau_8 - 72.98\\ \tau_{10}' &= -\tau_{10} - 36.49\end{aligned}\right\}$$

Equation (2′) becomes:
$+140.208\,(-\tau_2-18.25)+9.508\,(-\tau_4-36.50)+0.037\,\tau_{13}+60.559\,\tau_2+2\,557.446=0,$
that is, $+79.649\,\tau_2+9.508\,\tau_4-0.037\,\tau_{13}+348.392=0.$

Equation (2) becomes:
$+79.649\,\tau_2+9.508\,\tau_4+0.037\,\tau_{13}+345.692=0.$

Combined:
$+79.649\,\tau_2+9.508\,\tau_4+347.042=0.$

Similarly, Equations (4′) and (4) give:
$+38.284\,\tau_4+9.508\,\tau_6+9.508\,\tau_2+873.551=0,$

Equations (6′) and (6) give:
$+38.284\,\tau_6+9.508\,\tau_8+9.508\,\tau_4+1\,393.927=0,$

Equations (8′) and (8) give:
$+38.284\,\tau_8+9.508\,\tau_{10}+9.508\,\tau_6+1\,920.151=0,$

Equations (10′) and (10) give:
$+79.649\,\tau_{10}+9.508\,\tau_8+2\,909.092=0.$

TABLE 32.—SECONDARY STRESSES IN TOP LATERAL SYSTEM DUE TO DEAD AND UNSYMMETRICAL LIVE LOADS. COMPUTATION OF T'S.

No. of equation.	Remarks.	τ_2.	τ_4.	τ_6.	τ_8.	τ_{10}.	Absolute term.	Check
2	.....	+79.649	+ 9.508	.....	.....	.....	+ 347.042	+ 436.199
2′	$2 \times -\frac{9.508}{79.649}$ *i. e.*, — 0.119374	[— 9.508]	— 1.135	.....	.....	.....	— 41.428	— 52.071
4	.....	[+ 9.508]	+38.284	+ 9.508	.....	.....	+ 873.554	+ 930.854
I	2′ + 4	.....	+37.149	+ 9.508	.....	.....	+ 832.126	+ 878.783
I′	$I \times -\frac{9.508}{37.149}$ *i. e.*, — 0.255942	.....	[— 9.508]	— 2.433	.....	.....	— 212.976	— 224.918
6	.....	.....	[+ 9.508]	+38.284	+ 9.508	.....	+1 393.927	+1 451.227
II	I′ + 6	.....	.....	+35.851	+ 9.508	.....	+1 180.951	+1 226.309
II′	$II \times -\frac{9.508}{35.851}$ *i. e.*, — 0.265209	.....	.....	[— 9.508]	— 2.522	.....	— 313.199	— 325.228
8	.....	.....	.....	[+ 9.508]	+38.284	+ 9.508	+1 920.151	+1 977.451
III	II′ + 8	.....	.....	.....	+35.762	+ 9.508	+1 606.952	+1 652.223
III′	$III \times -\frac{9.508}{35.762}$ *i. e.*, — 0.265869	.....	.....	.....	[— 9.508]	— 2.528	— 427.238	— 439.275
10	.....	.....	.....	.....	[+ 9.508]	+79.649	+2 909.092	+2 998.249
IV	III′ + 10	.....	.....	.....	.....	+77.121	+2 481.854	+2 558.974
		—2.3948	—16.43813	—23.29260	—36.37862	—32.18130		

Equation (13) gives:

$$+0.296\,\tau_{13} - 8.779 - 0.037 \times 18.25 - 0.037 \times 36.50 = 0,$$

that is,

$$+0.296\,\tau_{13} - 10.805 = 0,$$

$$\tau_{13} = +36.50.$$

Equation (14) gives:

$$+0.296\,\tau_{14} - 7.428 - 0.037 \times 91.24 = 0,$$

$$\tau_{14} = +36.50.$$

Equation (15) gives:

$$+0.296\,\tau_{15} - 16.877 - 0.037 \times 127.72 = 0,$$

$$\tau_{15} = +72.98.$$

Equation (16) gives:

$$+0.296\,\tau_{16} - 17.552 - 0.037 \times 109.47 = 0,$$

$$\tau_{16} = +72.98.$$

TABLE 33.—Secondary Stresses in Top Lateral System Due to Dead and Unsymmetrical Live Loads. Values of Moments.

2	$\tau_{22'}$	= − 2.395	$M_{22'}$	= 121.118 (− 4.790 + 2.395)	= − 290.08 in.-lb.
		.....			
	$\tau_{2.13}$	= − 2.395	$M_{2.13}$	= 0.074 (− 4.790 + 0)	= − 0.35
		+ 18.250			
	τ_{24}	= + 15.855	$M_{2.4}$	= 19.016 (+ 31.710 − 16.438)	= + 290.41
13	$\tau_{13.4}$	= + 36.50	$M_{13.4}$	= 0.074 (+ 73.00 + 1.812)	= + 5.54 in.-lb.
		− 36.50			
	$\tau_{13.2}$	=	$M_{13.2}$	= 0.074 (0 − 2.395)	= − 0.18
		.....			
	$\tau_{13.2'}$	=	$M_{13.2'}$	=	+ 0.18
		− 36.50			
	$\tau_{13.4'}$	= − 36.50	$M_{13.4'}$	=	− 5.54
4	$\tau_{4.2}$	= − 16.438	M_{42}	= 19.016 (− 32.876 + 15.855)	= − 323.67 in.-lb.
		+ 18.250			
	$\tau_{4.13}$	+ 1.812	$M_{4.13}$	= 0.074 (+ 3.624 + 36.50)	= + 2.97
		.....			
	$\tau_{44'}$	= + 1.812	$M_{44'}$	= 0.208 (+ 3.624 − 1.812)	= + 0.38
		.....			
	$\tau_{4.14}$	= + 1.812	$M_{4.14}$	= 0.074 (+ 3.624 + ...)	= + 0.27
		+ 18.250			
	τ_{46}	= + 20.062	$M_{4.6}$	= 19.016 (+ 40.124 − 23.293)	= + 320.06
14	$\tau_{14.6}$	= + 36.50	$M_{14.6}$	= 0.074 (+ 73.00 − 5.043)	= + 5.03 in.-lb.
		− 36.50			
	$\tau_{14.4}$	=	$M_{14.4}$	= 0.074 (.... + 1.812)	= + 0.13
		.....			
	$\tau_{14.4'}$	=	$M_{14.4'}$	=	= − 0.13
		− 36.50			
	$\tau_{14.6'}$	= − 36.50	$M_{14.6'}$	=	= − 5.03
6	τ_{64}	= − 23.293	M_{64}	= 19.016 (− 46.586 + 20.062)	= − 504.38 in.-lb.
		+ 18.250			
	$\tau_{6.14}$	= − 5.043	$M_{6.14}$	= 0.074 (− 10.086 + 36.50)	= + 1.95
		.....			
	$\tau_{6.6'}$	= − 5.043	$M_{6.6'}$	= 0.208 (− 10.086 + 5.043)	= − 1.05
		.....			
	$\tau_{6.15}$	= − 5.043	$M_{6.15}$	= 0.074 (− 10.086 + ...)	= − 0.75
		+ 36.490			
	$\tau_{6.8}$	= + 31.447	$M_{6.8}$	= 19.016 (+ 62.894 − 36.379)	= + 504.21
15	$\tau_{15.8}$	= + 72.98	$M_{15.8}$	= 0.074 (+ 145.96 + 0.111)	= + 10.81 in.-lb.
		− 72.98			
	$\tau_{15.6}$	=	$M_{15.6}$	= 0.074 (.... − 5.043)	= − 0.37
		.....			
	$\tau_{15.6'}$	=	$M_{15.6'}$	=	+ 0.37
		− 72.98			
	$\tau_{15.8'}$	= − 72.98	$M_{15.8'}$	=	− 10.81
8	τ_{86}	= − 36.379	$M_{8.6}$	= 19.016 (− 72.758 + 31.447)	= − 785.57 in.-lb.
		+ 36.490			
	$\tau_{8.15}$	= + 0.111	$M_{8.15}$	= 0.074 (+ 0.222 + 72.98)	= + 5.42
		.....			
	$\tau_{88'}$	= + 0.111	$M_{8.8'}$	= 0.208 (+ 0.222 − 0.111)	= + 0.02
		.....			
	$\tau_{8.16}$	= + 0.111	$M_{8.16}$	= 0.074 (+ 0.222 + ...)	= + 0.02
		+ 36.490			
	$\tau_{8.10}$	= + 36.601	$M_{8.10}$	= 19.016 (+ 73.202 − 32.181)	= + 780.06
16	$\tau_{16.10}$	= + 72.98	$M_{16.10}$	= 0.074 (+ 145.96 + 4.309)	= + 11.12 in.-lb.
		− 72.98			
	$\tau_{16.8}$	=	$M_{16.8}$	= 0.074 (.... + 0.111)	= + 0.01
		.....			
	$\tau_{16.8'}$	=	$M_{16.8'}$	=	− 0.01
		− 72.98			
	$\tau_{16.10'}$	= − 72.98	$M_{16.10'}$	=	− 11.12
10	$\tau_{10.8}$	= − 32.181	$M_{10.8}$	= 19.016 (− 64.362 + 36.601)	= − 527.90 in.-lb
		+ 36.490			
	$\tau_{10.16}$	= + 4.309	$M_{10.16}$	= 0.074 (+ 8.618 + 72.980)	= + 6.04
		.....			
	$\tau_{10.10}$	= + 4.309	$M_{10.10'}$	= 121.118 (+ 8.618 − 4.309)	= + 521.90

As the angles at the joints remain right angles at all times, because of the rigidity of the joints in the Plane 6-6′-7′-7, the posts suffer bending at their lower ends when the floor-beam is loaded. The shape of the frame is then somewhat like the dotted part of Fig. 74.

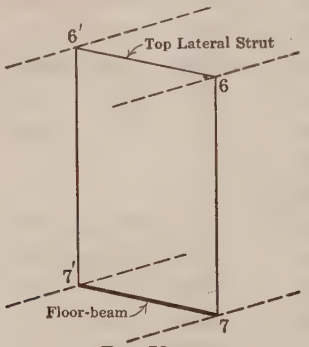


FIG. 73.

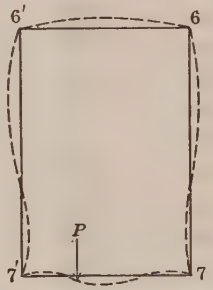


FIG. 74.

It must be remembered that no direct stresses enter the frame, for all the loads are taken care of by the diagonals and bottom chords that meet in Joints (7) and (7′). The analyses of Table 2 show that the primary stresses in Members 6-7 and 6′-7′ for dead and live loads are always zero.

There are three ways of attacking the problem, namely, the method of least work, Mohr's elastic weight method, and Batho's slope-deflection method. The first method will be omitted as it is unnecessarily long.

Mohr's Method.—The principles underlying this method are exactly those of the method for trusses, already described (see page 31), and, therefore, need no explanation. E is again regarded as unity.

From the dimensions of the rectangular frame, Fig. 75,

$$I_{6'\text{-}6} = 529 \text{ in.}^4$$

and

$$l_{6'\text{-}6} = 204 \text{ in.} \quad \therefore \beta_{6'\text{-}6} = \frac{204}{2 \times 529} = 0.1928. \text{ (See Equation (78))}$$

$$I_{6'\text{-}7'} = I_{6\text{-}7} = 548 \text{ in.}^4$$

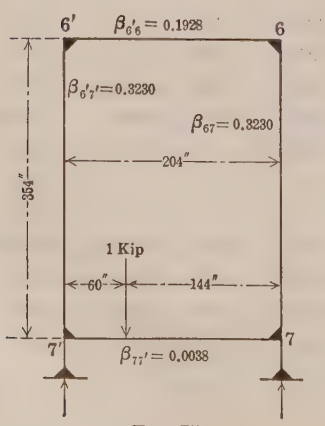


FIG. 75.

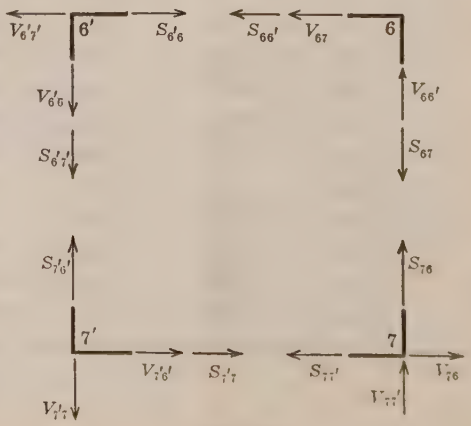


FIG. 76.

and,

$$l_{6'\text{-}7'} = l_{6\text{-}7} = 354 \text{ in.} \therefore \beta_{6'\text{-}7'} = \beta_{6\text{-}7} = \frac{354}{2 \times 548} = 0.3230$$

$$I_{7\text{-}7'} = 27\ 020 \text{ in.}^4$$

and,

$$l_{7\text{-}7'} = 204 \text{ in.} \therefore \beta_{7\text{-}7'} = \frac{204}{2 \times 27\ 020} = 0.0038$$

If the equilibrium of the joints is considered, as represented in Fig. 76, certain shearing forces, V, are induced at the joints due to the end bending moments resulting from the rigidity of the joints. These bending moments, eight in all, are the unknowns to be determined. $M_{6'\,7'}$ and $M_{7'\,6'}$ of Member 6′-7′ give rise to the two equal and opposite shearing forces, $V_{6'\,7'}$ and $V_{7'\,6'}$ at Joint (6′) and Joint (7′), respectively. In the sense that they are drawn, positive bending moments are calculated clockwise (see Fig. 5). All the other V's are inserted in the same manner. Finally, internal direct stresses, S, are induced.

For equilibrium of joints under direct stresses:

$$\left.\begin{array}{llll} (a)\ S_{6'\text{-}7'} = +\ V_{7\text{-}7'} & ; & (b)\ S_{6'\text{-}7'} = -\ V_{6'\text{-}6} \\ (c)\ S_{6'\text{-}6} = +\ V_{7'\text{-}6'} & ; & (d)\ S_{6'\text{-}6} = -\ V_{6\text{-}7} \\ (e)\ S_{6\text{-}7} = +\ V_{6'\text{-}6} & ; & (f)\ S_{6\text{-}7} = -\ V_{7\text{-}7'} \\ (g)\ S_{7\text{-}7'} = +\ V_{6\text{-}7} & ; & (h)\ S_{7\text{-}7'} = -\ V_{7'\text{-}6'} \end{array}\right\} \quad \ldots\ldots\ldots (101)$$

For equilibrium of the joints under bending moments:

$$\left.\begin{array}{ll} (a)\ M_{7'\text{-}7} = -\ M_{7'\text{-}6'} \\ (b)\ M_{6'\text{-}7'} = -\ M_{6'\text{-}6} \\ (c)\ M_{6\text{-}6'} = -\ M_{6\text{-}7} \\ (d)\ M_{7\text{-}6} = -\ M_{7\text{-}7'} \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (102)$$

Also,

$$\left.\begin{array}{ll} (a)\ V_{7'\text{-}6'} = V_{6'\text{-}7'} = \dfrac{M_{7'\text{-}6'} + M_{6'\text{-}7'}}{354} \\ (b)\ V_{6'\text{-}6} = V_{6\text{-}6'} = \dfrac{M_{6'\text{-}6} + M_{6\text{-}6'}}{204} \\ (c)\ V_{6\text{-}7} = V_{7\text{-}6} = \dfrac{M_{6\text{-}7} + M_{7\text{-}6}}{354} \\ (d)\ V_{7\text{-}7'} = V_{7'\text{-}7} = \dfrac{M_{7\text{-}7'} + M_{7'\text{-}7}}{204} \end{array}\right\} \quad \ldots\ldots\ldots\ldots (103)$$

This is easily proved as follows: Let two end moments, M_{ab}, M_{ba}, act on Member a-b, Fig. 77, and let the two end shears be V_{ab} and V_{ba}.

For equilibrium in a vertical direction, $V_{ab} = V_{ba}$. Also, $M_x = M_{ab} - V_{ab} \cdot x$.

When $x = l$,

$$M_{ba} + M_{ab} - V_{ab} \cdot l = 0$$

or,

$$V_{ab} = V_{ba} = \frac{M_{ab} + M_{ba}}{l}$$

Consider now the elastic weights that are involved in the problem (see Fig. 79):

$$\left.\begin{aligned}
&(a)\ \phi_{7'\text{-}6'} = +\beta_{7'\text{-}6'} \cdot M_{7'\text{-}6'} = +0.3230\ M_{7'\text{-}6'}\\
&(b)\ \phi_{6'\text{-}7'} = -\beta_{7'\text{-}6'} \cdot M_{6'\,7'} = -0.3230\ M_{6'\text{-}7'}\\
&(c)\ \phi_{6'\text{-}6} = +0.1928\ M_{6'\text{-}6}\\
&(d)\ \phi_{6\text{-}6'} = -0.1928\ M_{6\text{-}6'}\\
&(e)\ \phi_{6\text{-}7} = +0.3230\ M_{6\text{-}7}\\
&(f)\ \phi_{7\text{-}6} = -0.3230\ M_{7\text{-}6}\\
&(g)\ \phi_{7\text{-}7'} = +0.0038\ M_{7\text{-}7'}\\
&(h)\ \phi_{7'\text{-}7} = -0.0038\ M_{7'\text{-}7}
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (104)$$

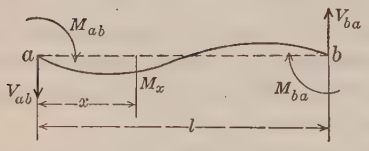


FIG. 77.

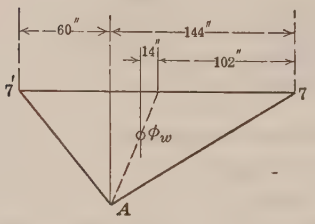


FIG. 78.

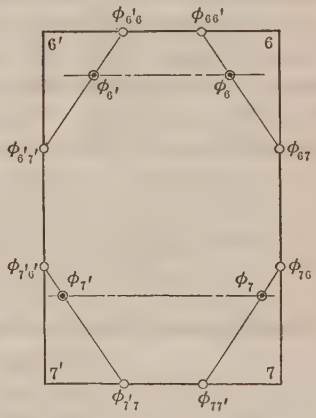


FIG. 79.

These eight quantities may be replaced by four elastic weights, the positions of which are determined by:

$$\left.\begin{aligned}
&\phi_{7'\text{-}6'}\ \phi_{7'} : \phi_{7'}\ \phi_{7'\text{-}7} = 0.0038 : 0.3230\\
&\phi_{6'\text{-}7'}\ \phi_{6'} : \phi_{6'}\ \phi_{6'\text{-}6} = 0.1928 : 0.3230\\
&\phi_{6\text{-}6'}\ \phi_{6} : \phi_{6}\ \phi_{6\text{-}7} = 0.3230 : 0.1928\\
&\phi_{7\text{-}6}\ \phi_{7} : \phi_{7}\ \phi_{7\text{-}7'} = 0.0038 : 0.3230
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (105)$$

and the weights of which are:

$$\left.\begin{aligned}
\phi_{7'} &= +0.3268\ M_{7'\text{-}6'}\\
\phi_{6'} &= +0.5158\ M_{6'\text{-}6}\\
\phi_{6} &= +0.5158\ M_{6\text{-}7}\\
\phi_{7} &= +0.3268\ M_{7\text{-}7'}
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (106)$$

The four quantities are at the corners of a trapezoid the parallel sides of which, $\phi_{6'}$-ϕ_6 and ϕ_{7}-$\phi_{7'}$, have the lengths.

$$\phi_{6'}\text{-}\phi_6 = 68 + \frac{3230}{5158} \times 136 = 153.16 \text{ in.}$$

and,

$$\phi_{7'}\text{-}\phi_7 = 68 + \frac{3230}{3268} \times 136 = 202.42 \text{ in.}$$

also, the height of which is:

$$+354 = \frac{3230}{3268} \times 118 - \frac{3230}{5158} \times 118 = 163.48 \text{ in.}$$

To account for the elastic weight due to the bending moment of 1 kip on Member 7-7′, 60 in. from Point (7′) (see Fig. 78), the bending moment area is a triangle, the height of which is $\frac{1\,000 \times 60 \times 144}{204}$ in-lb.

The area, therefore, is,

$$\frac{1}{2} \times 204 \times \frac{1\,000 \times 60 \times 144}{204} = (12\,000 \times 360) \text{ in-sq-lb.}$$

As,

$$I_{7\text{-}7'} = 27\,020 \text{ in.}$$

$$\phi_w = + \frac{12\,000 \times 360}{27\,020} = + 159.88$$

The point of application of ϕ_w is in Member 7′-7, distant 88 in. from Point (7′), this being the distance of the center of gravity of $\Delta\, 7'\, A\, 7$ from Point (7′).

The elastic chain then is replaced by the system of weights shown in Fig. 80.

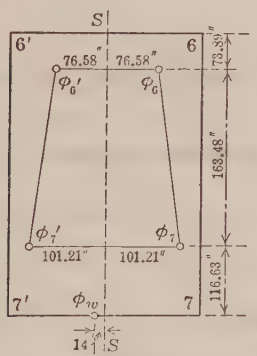


Fig. 80.

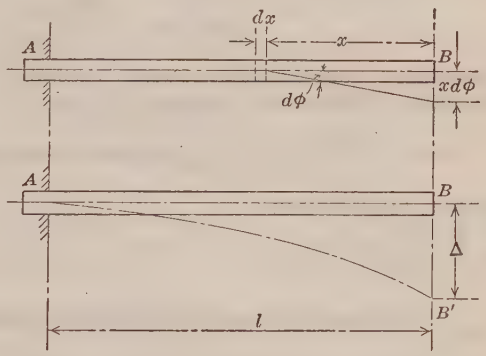


Fig. 81.

Taking moments about $\phi_{6'}$-ϕ_6, ϕ_7-$\phi_{7'}$, s-s;

$$+ (0.3268\, M_{7'\text{-}6'} \times 163.48) + (0.3268\, M_{7\text{-}7'} \times 163.48) + (159.88 \times 280.11) = 0 \quad \ldots\ldots (107)$$

$$+ (0.5158\, M_{6'\text{-}6} \times 163.48) + (5\,158\, M_{6\text{-}7} \times 163.48) - (159.88 \times 116.63) = 0 \quad \ldots\ldots (108)$$

$$+ (0.5158\, M_{6'\text{-}6} \times 76.58) - (0.5158\, M_{6\text{-}7} \times 76.58) + (0.3268\, M_{7'\text{-}6'} \times 101.21) - (0.3268\, M_{7\text{-}7'} \times 101.21) + 159.88 \times 14 = 0 \quad \ldots\ldots (109)$$

Also, Equations (101) and (102) give

$$V_{7'\text{-}7} + V_{6'\text{-}6} = 0$$

or,

$$\frac{M_{7\text{-}7'} + M_{7'\text{-}7}}{204} + \frac{M_{6'\text{-}6} + M_{6\text{-}6'}}{204} = 0$$

that is,

$$M_{7\text{-}7'} + M_{7'\text{-}7} + M_{6'\text{-}6} + M_{6\text{-}6'} = 0$$

that is,

$$M_{7\text{-}7'} - M_{7'\text{-}6} + M_{6'\text{-}6} - M_{6\text{-}7} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (110)$$

The four equations, (107), (108), (109), and (110), contain the four unknowns, $M_{7'\text{-}6}$, $M_{6'\text{-}6}$, $M_{6\text{-}7}$, and $M_{7\,7'}$ only. When these equations are solved:

$$\left.\begin{aligned} M_{7'\text{-}6'} &= -\ 434.55 \text{ in-lb.} \\ M_{6'\text{-}6} &= +\ 95.15 \text{ in-lb.} \\ M_{6\text{-}7} &= +\ 125.99 \text{ in-lb.} \\ M_{7\text{-}7'} &= -\ 403.71 \text{ in-lb.} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (111)$$

The four remaining bending moments are the reciprocals of these, respectively.

Batho's Method.—This is a very elegant method, depending on two well-known properties of a beam subjected to bending. By referring to page 31, the relationship between the "elastic weight" idea and the following will be evident: Let $A\,B$ be a beam (Fig. 81), subjected to bending. As before, Equation (76),

$$d\,\phi = \frac{M_x \cdot d_x}{E\,I}$$

while the contribution to the deflection from B is,

$$x \cdot d\,\phi = \frac{M_x \cdot x\,d\,x}{E\,I}$$

Integrating over the whole length, the total change in slope, that is, the angle between $A\,B$ and the tangent at B', is:

$$\phi = \int_0^l \frac{M_x \cdot d\,x}{E\,I} = \frac{1}{E\,I} \times \text{(Area of bending moment diagram)} \ldots (112)$$

while the total deflection is:

$$\begin{aligned} \Delta &= \int_0^l \frac{M_x \cdot x\,d\,x}{E\,I} \\ &= \frac{1}{E\,I} \times \text{(Moment of bending moment area about } B) \\ &= \frac{1}{E\,I} \times \text{(Area of bending moment} \times \text{distance of its center of gravity from } B) \quad \ldots\ldots\ldots\ldots\ldots\ldots (113) \end{aligned}$$

Consider a complete rectangular frame, Fig. 82. Imagine the top member, $A\,B$, to be broken at its mid-point, O, and let a bending moment, M, a horizontal thrust, H, and a vertical shear, V, act on the end, O, of $A\,O$, while the reverse of these act on the end, O, of $B\,O$.

Now, consider the effect of the M, H and V, on End O of $A\,O$ separately.

The bending moment diagram for the M is as shown in Fig. 83. Imagine the frame to be resting on two hinges at D and C. The bending moment is constant over the portion, O-A-D, but between D and C, it varies uniformly from $+M$ to zero, while no bending occurs in $C\,B$ or $B\,O$. Now, determine the amount of movement, x, to the left that the point, O, undergoes. The bending moment area over $A\,O$ contributes a quantity of the second order of smallness. From Fig. 81, it is clear that a small displacement occurs perpendicularly to the member, while the movement in the direction of the

member is negligible. Now, the bending moment along $A\ D$ causes A to deflect to the left from its initial position by an amount equal to (see Equation (113)):

$$\frac{1}{E\,I_2} \times M\,h \times \frac{h}{2} = \frac{M\,h^2}{2\,E\,I_2}$$

but in doing so, it carries $A\ O$, and, therefore, the point, O, to the left by this amount.

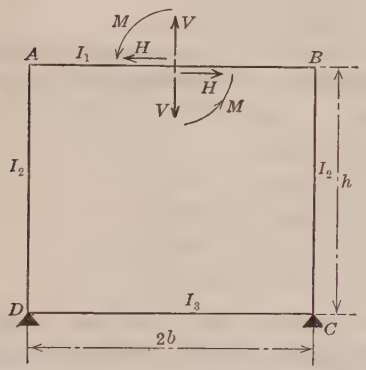


FIG. 82.

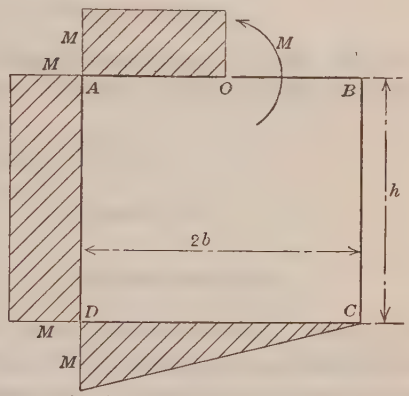


FIG. 83.

The bending moments along $D\ C$ cause a change in slope between the end tangents at D and C equal to (see Equation (112)):

$$\frac{1}{E\,I_3} \times \frac{1}{2}\,M \,.\, 2\,b = \frac{M\,b}{E\,I_3}$$

but as the angle between the tangents to $D\ A$ and $D\ C$ is constant, $D\ A$ is deflected to the left by an amount equal to $\frac{M\,b}{E\,I_3}$. In doing so, A, and, therefore, O, is moved to the left by an amount equal to:

$$\frac{M\,b}{E\,I_3} \times h = \frac{M\,b\,h}{E\,I_3}$$

The total contribution of M to the horizontal movement of O, to the left is, therefore,

$$\frac{M\,h^2}{2\,E\,I_2} + \frac{M\,b\,h}{E\,I_3}$$

It is possible to find similar expressions for the upward (y) displacement of O from its initial position, and for the total rotation between the end tangent at O of $A\ O$ and the initial position, $A\ O$ (counterclockwise). These expressions are:

$$\left.\begin{aligned} x\ \text{(left)} &= \frac{M\,h^2}{2\,E\,I_2} + \frac{M\,b\,h}{E\,I_3} \\ y\ \text{(upward)} &= \frac{M\,b^2}{2\,E\,I_1} + \frac{M\,h\,b}{E\,I_2} + \frac{M\,b^2}{E\,I_3} \\ \alpha\ \text{(counterclockwise)} &= \frac{M\,b}{E\,I_1} + \frac{M\,h}{E\,I_2} + \frac{M\,b}{E\,I_3} \end{aligned}\right\} \quad \ldots\ldots\ldots (114)$$

Similarly, the bending moment diagram for H is as shown in Fig. 84, while the contributions to x (left), y (upward), and α (counterclockwise) are:

$$\left.\begin{aligned} x \text{ (left)} &= \frac{H\,h^3}{3\,E\,I_2} + \frac{H\,h^2\,b}{E\,I_3} \\ y \text{ (upward)} &= \frac{H\,h^2\,b}{2\,E\,I_2} + \frac{H\,h\,b^2}{E\,I_3} \\ \alpha \text{ (counterclockwise)} &= \frac{H\,h^2}{2\,E\,I_2} + \frac{H\,h\,b}{E\,I_3} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (115)$$

Finally, the bending moment diagram for the V is as shown in Fig. 85, while the contributions to x (left), y (upward), and α (counterclockwise) are:

$$\left.\begin{aligned} x \text{ (left)} &= \frac{V\,b\,h^2}{2\,E\,I_2} + \frac{V\,b^2\,h}{2\,E\,I_3} \\ y \text{ (upward)} &= \frac{V\,b^3}{3\,E\,I_1} + \frac{V\,b^2\,h}{E\,I_2} + \frac{V\,b^3}{2\,E\,I_3} \\ \alpha \text{ (counterclockwise)} &= \frac{V\,b^2}{2\,E\,I_1} + \frac{V\,b\,h}{E\,I_2} + \frac{V\,b^2}{2\,E\,I_3} \end{aligned}\right\} \quad \ldots\ldots\ldots (116)$$

(The bending moment diagram for the V must be noted. To the x (left), y (upward), and α (counterclockwise) contributions of the M, H, and V, on AO, already noted, the x (right), y (downward), and α (clockwise) contributions

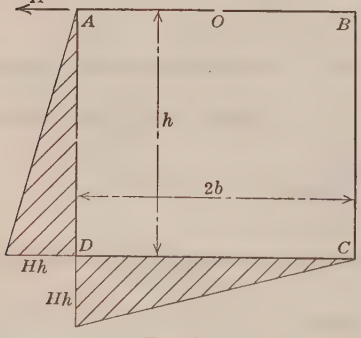


FIG. 84.

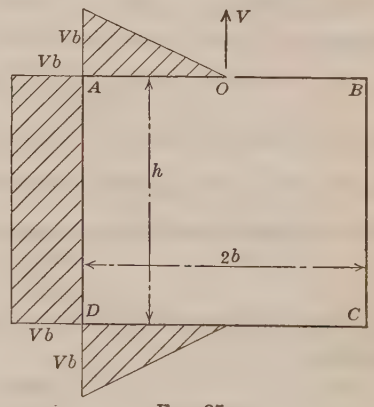


FIG. 85.

of the M, H, and V, on BO must be added. As the two V's act in opposite directions, the bending moment at the center point of DC due to the V's will be zero, and the portion shown on DC of Fig. 85 may be regarded as that due to the V on AO.) Consider now the combined effects of the two sets of M, H, and V. Notice that the M on AO, together with the M on BO, doubles the x-distance between the ends at O, and also doubles the α-angle between the end tangents at O, the respective ends being moved an equal vertical amount; in other words, the relative vertical displacement is zero. Similarly, the two H's double the x-contribution, and also the α-contribution, but the y-contribution becomes zero.

On the other hand, the two V's give zero contributions to the x and the α, but double the y-contribution. The combination of these is indicated as follows:

	x	y	α
M	2	0	2
H	2	0	2
V	0	2	0

Hence, if the top member is cut, the ends become separated under the two sets of M, H, and V, so that:

$$\left.\begin{aligned} x &= \frac{M h^2}{E I_2} + \frac{2 M b h}{E I_3} + \frac{2 H h^3}{3 E I_2} + \frac{2 H h^2 b}{E I_3} \\ y &= \frac{2 V b^3}{3 E I_1} + \frac{2 V b^2 h}{E I_2} + \frac{V b^3}{E I_3} \\ \alpha &= \frac{2 M b}{E I_1} + \frac{2 M h}{E I_2} + \frac{2 M b}{E I_3} + \frac{H h^2}{E I_2} + \frac{2 H h b}{E I_3} \end{aligned}\right\} \quad \ldots\ldots\ldots (117)$$

The x is the amount by which the O's of $A\,O$ and $B\,O$ are separated horizontally, the O of $A\,O$ lying to the left and the O of $B\,O$ to the right. The y is the vertical gap, the O of $A\,O$ lying above the O of $B\,O$. The α is the angle between the end tangents, the $A\,O$ being turned counterclockwise and the $B\,O$, clockwise.

Next, determine the effect on the two ends of the cut frame under the actual load, instead of the M, H, and V. The x must now be the horizontal distance between the ends, but with this difference that $A\,O$ and $B\,O$ overlap. The y must be the vertical distance between the ends, but the $B\,O$ must now lie above $A\,O$. The α must be the angle between the end tangents, but the $A\,O$ is now considered as being turned clockwise while the $B\,O$ is turned counterclockwise.

Equating the corresponding x's, y's, and α's, of the M-H-V-set and the load-set, gives three linear equations for the three unknowns M, H, and V. The values of M, H, and V are then those that actually exist at the center of the top member of the uncut frame.

Before applying the theory to the cross-frame, Members 6-6′-7′-7, the formulas for the deflection angles at the ends of a simply supported beam, when it is subjected to a single concentrated load, will be reproduced.

The deflection angles, α_1 and α_2*, of the beam shown in Fig. 86 are given by:

$$\left.\begin{aligned} \alpha_1 &= \frac{P}{12\,E\,I\,b}\,(p^3 - 6\,p^2\,b + 8\,p\,b^2) \\ \alpha_2 &= \frac{P}{12\,E\,I\,b}\,(4\,p\,b^2 - p^3) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (118)$$

* Church's "Mechanics", Article (234), Equation (7).

Turning now to the problem of the cross-frame, 6-6′-7′-7, shown again in Fig. 88, the α, x, and y-terms from the kip load are:

$$\alpha = \alpha_1 + \alpha_2 = \frac{P}{12\, I_3\, b}\,(12\, p\, b^2 - 6\, p^2\, b)$$

$$= \frac{1\,000}{12 \times 27\,020 \times 102}\,(12 \times 60 \times 102^2 - 6 \times 60^2 \times 102)$$

$$x = \alpha_1\, h + \alpha_2\, h = \frac{P}{121_3\, b}\,(12\, p\, b^2 - 6\, p^2\, b)\, h$$

$$= \frac{1\,000}{12 \times 27\,020 \times 102}\,(12 \times 60 \times 102^2 - 6 \times 60^2 \times 102) \times 354$$

and,

$$y = \alpha_1\, b - \alpha_2\, b = \frac{P}{121_3}\,(2\, p^3 - 6\, p^2\, b + 4\, p\, b^2)$$

$$= \frac{1\,000}{12 \times 27\,020}\,(2 \times 60^3 - 6 \times 60^2 \times 102 + 4 \times 60 \times 102^2).$$

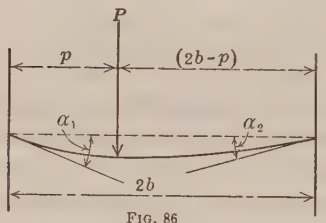


FIG. 86

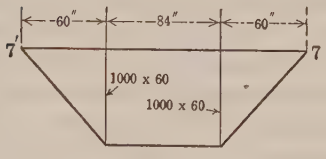


FIG. 87.

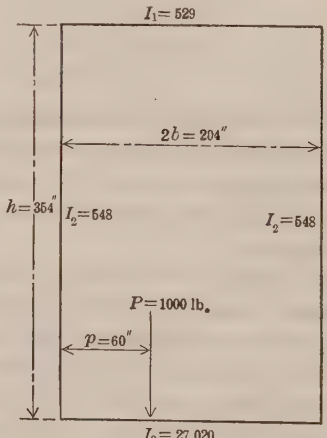


FIG. 88.

These reduce to:

$$\alpha = \frac{36 \times 12 \times 1\,000}{2\,702}$$

$$x = \frac{354 \times 36 \times 12 \times 1\,000}{2\,702}$$

and,

$$y = \frac{6\,048 \times 1\,000}{2\,702}$$

Making use of Equations (117):

$$\frac{M \times 354^2}{548} + \frac{2\, M \times 102 \times 354}{27\,020} + \frac{2\, H \times 354^3}{3 \times 548}$$

$$+ \frac{2\, H \times 354^2 \times 102}{27\,020} = \frac{354 \times 36 \times 12 \times 1\,000}{2\,702}$$

$$\frac{2\, V \times 102^2 \times 354}{548} + \frac{2\, V \times 102^3}{3 \times 529} + \frac{V \times 102^3}{27\,020} = \frac{6\,048 \times 1\,000}{2\,702}$$

and,

$$\frac{2\,M \times 354}{548} + \frac{2\,M \times 102}{529} + \frac{2\,M \times 102}{27\,020} + \frac{H \times 354^2}{548} + \frac{2\,H \times 354 \times 102}{27\,020} = \frac{36 \times 12 \times 1\,000}{2\,702}$$

giving,

$$V = +0.151 \text{ lb.}$$
$$H = +1.497 \text{ lb.}$$

and,

$$M = -110.579 \text{ in-lb.}$$

The bending moment diagram is, therefore, as shown in Fig. 89. Note how closely the values found by the two methods correspond.

	Mohr.	Batho.	Sense.
$M_{7'\text{-}6'}$	434.55	437.32	Counterclockwise
$M_{6'\text{-}6}$	95.15	92.45	Clockwise
$M_{6\text{-}7}$	125.99	128.71	Clockwise
$M_{7\text{-}7'}$	408.71	401.06	Counterclockwise

Thus far, the problem has been for an unsymmetrical load. The method will be useful in any subsequent work, where wind loads, causing an unsymmetrical load on the rails, are considered.

The present investigation, however, aims at symmetrical loads, that is, at the effect of a kip load on each rail, in the plane of the frame. Turning to Mohr's method, the bending moment area, Fig. 87, of the loads now becomes:

$$(60 \times 1\,000 \times 60) + (84 + 1\,000 \times 60) = 144 \times 60 \times 1\,000$$

$$\phi_w = \frac{144 \times 60 \times 1\,000}{27\,020} = +319.76$$

and acts at the center of the floor-beam, from symmetry.

Taking moments about the three axes used previously:

$$+ (0.3268\ M_{7'\text{-}6'} \times 163.48) + (0.3268\ M_{7\text{-}7'} \times 163.48) + 319.76 \times 280.11 = 0$$
$$+ (0.5158\ M_{6'\text{-}6} \times 163.48) + (0.5158\ M_{6\text{-}7} \times 163.48) - 319.76 \times 116.63 = 0$$
$$+ (0.5158\ M_{6'\text{-}6} \times 76.58) - (0.5158\ M_{6\text{-}7} \times 76.58) + (0.3268\ M_{7'\text{-}6'} \times 101.21) - (0.3268\ M_{7\text{-}7'} \times 101.21) = 0$$

and, as before:

$$M_{7\text{-}7'} - M_{7'\text{-}6'} + M_{6'\text{-}6} - M_{6\text{-}7} = 0$$

The last two equations give:

$$33.075\ (M_{7'\text{-}6'} - M_{7\text{-}7'}) + 39.4999\ (M_{6'\text{-}6} - M_{6\text{-}7}) = 0$$

and,

$$(M_{7'\text{-}6'} - M_{7\text{-}7'}) - (M_{6'\text{-}6} - M_{6\text{-}7}) = 0$$

hence,

$$M_{7'\text{-}6'} = M_{7\text{-}7'} \text{ and } M_{6'\text{-}6} = M_{6\text{-}7}$$

which is to be expected from symmetry.

Hence,

$$M_{7'\text{-}6'} = -838.25 \text{ in-lb.}$$

and,

$$M_{6'\text{-}6} = +221.14 \text{ in-lb.}$$

(Note that the effect of a kip load on the rail near Joint (7) is just the reverse of that of a kip load on the rail near Joint (7'). The two values just found could easily have been reached by compounding $M_{7'\text{-}6'}$ and $M_{7\text{-}7'}$ of the unsymmetrical case, giving,

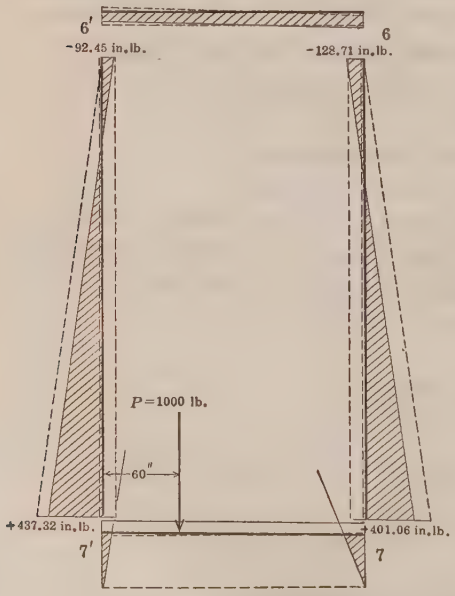
$$M_{7'\text{-}6'} = M_{7\text{-}7'} = -838.25 \text{ in-lb.}$$

and, similarly, $M_{6'\text{-}6}$ and $M_{6\text{-}7'}$, giving,

$$M_{6'\,6} = M_{6\,7} = +221.14 \text{ in-lb.}$$

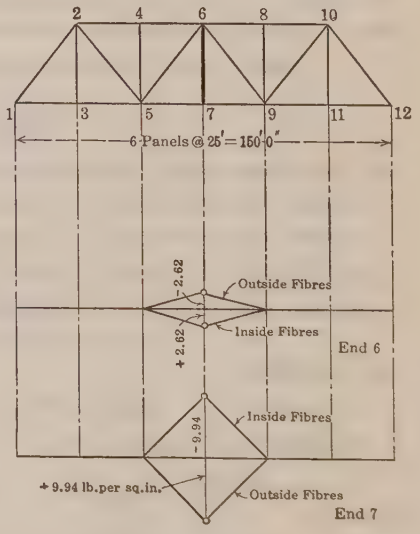
when the joint effect of the two kips is considered.)

Dr. Batho's method has the advantage over Mohr's method in that, when the x, y, and α-expressions for M, H, and V, are once established, they can be used for any frame whatever, making any new solution comparatively short. Great accuracy, however, is not certain in dealing with the differences of very large quantities. Again, the drawing of the bending moment diagram, which is a combination of four diagrams, is a source of error. Although Mohr's method is longer, its accuracy depends only on the labor expended, while the certainty of arriving at the correct signs of the bending moments is a large factor in its favor.



MOMENTS IN CROSS-FRAME BY BATHO'S METHOD.

FIG. 89.



INFLUENCE LINES OF SECONDARY STRESSES INDUCED IN 6-7 AS PART OF CROSS-FRAME.

FIG. 90.

In drawing the influence line for the vertical post, 6-7, as part of the cross-frame, note that it is zero over the whole span, except the two adjacent panels. As the member is such that all the extreme fibers are 6.5 in. away from the neutral plane, the influence lines could best be drawn for secondary stresses, and not for bending moments.

The outside fibers at End (7) of Member 6-7 are in tension, while the outside fibers at End (6) are in compression. The factor by which the bending moments are to be multiplied is $\frac{C}{I} = \frac{6.5}{548}$, giving the quantities found in Fig. 90.

5.—Application of the Influence Lines to the Solution of the Bridge Under Actual Loads

The total dead load of the bridge is 360 000 lb., or 30 000 lb. per panel point. It is assumed that the live load is a train of Cooper's *E*-50 class. Further, the impact allowance for primary stresses will be $\frac{300}{300 + L}$ of the live load contribution, in which L is the length of the span that is loaded. All the secondary stresses that arise from the live load are to be increased by 50% to allow for impact and vibration. Two typical cases will be computed.

Member 6-7.—The influence lines (Figs. 66 and 90), together with Cooper's loading diagram give the stresses. The dead load effects are the following:

The primary stress $= +41.07 \times 30 = +1\,232.10$ lb. per sq. in.

The secondary stresses, due to the rigidity of the truss joints, are zero.

The secondary stresses, due to the rigidity of the cross-frame, give:

For End (6), outside fibers.... $-2.62 \times 30 = -78.60$ lb. per sq. in.
For End (6), inside fibers.... $+2.62 \times 30 = +78.60$ " " " "
For End (7), outside fibers.... $+9.94 \times 30 = +298.20$ " " " "
For End (7), inside fibers.... $-9.94 \times 30 = -298.20$ " " " "

The maximum primary stress due to live load occurs when Wheel (5) is opposite Panel Point (7). Then,

Primary stress $= +(12.5 \times 4.928) + 25\,(16.428 + 24.642 + 32.856 + 41.070) + 16.25\,(26.285 + 18.071 + 8.214) = +3\,790.76$ lb. per sq. in.

$$\text{Impact} = +3\,790.76 \times \left(\frac{300}{300 + 50}\right) = +3\,249.22 \text{ lb. per sq. in.}$$

The secondary stresses, due to rigidity of truss joints, are:

For End (6), right fibers.... $-1.5\,(147.22) = -220.83$ lb. per sq. in.
For End (6), left fibers.... $+220.83$ " " " "
For End (7), right fibers.... $+221.22$ " " " "
For End (7), left fibers.... -221.22 " " " "

The secondary stresses, due to rigidity of cross-frame, are:

For End (6), outside fibers.... $-1.5\,(243.14) = -364.71$ lb. per sq. in.
For End (6), inside fibers.... $+364.71$ " " " "
For End (7), outside fibers.... $+1\,383.67$ " " " "
For End (7), inside fibers.... $-1\,383.67$ " " " "

Hence, under the severest conditions for dead and live load combined, the total stresses at the eight end corner fibers, (Fig. 91) become:

$$f_{6a} = +(1\,232 + 3\,791 + 3\,249) + (-78.60 - 220.83 - 364.71)$$
$$= +7\,608 \text{ lb. per sq. in.}$$

and, similarly,

$f_{6b} = +\ 8\,050$ lb. per sq in.
$f_{6c} = +\ 8\,936$ " " " "
$f_{6d} = +\ 8\,494$ " " " "
$f_{7a} = +10\,175$ " " " "
$f_{7b} = +\ 9\,733$ " " " "
$f_{7c} = +\ 6\,369$ " " " "
$f_{7d} = +\ 6\,811$ " " " "

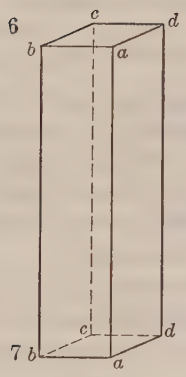


FIG. 91.

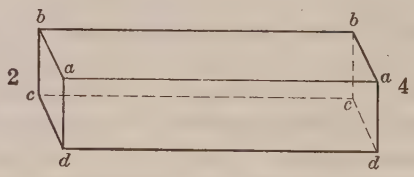


FIG. 92.

Member 2-4.—Referring to the influence lines (Figs. 59 and 71), the dead load gives:

Primary stress = — 2 233.20 lb. per sq. in.

The secondary stresses, due to rigidity of truss joints, give:

For End (2), top fibers..... + 102.00 lb. per sq. in.
For End (2), bottom fibers..... — 104.40 " " " "
For End (4), top fibers..... — 297.00 " " " "
For End (4), bottom fibers..... + 303.30 " " " "

The secondary stresses, due to the rigidity of the top lateral system, give:

For End (2), a bending moment area = + 49 049.25
For End (4), a bending moment area = — 66 307.05

These are to be multiplied by $\pm \frac{10.5}{2\,852.45}$ and $\pm \frac{12}{2\,852.45}$, giving,

For End (2), cover-plate outside fiber..... — 270.83 lb. per sq. in.
For End (2), cover-plate inside fiber..... + 270.83 " " " "
For End (2), bottom angles outside fiber.... — 308.02 " " " "
For End (2), bottom angles inside fiber.... + 308.02 " " " "

and,

For End (4), cover-plate outside fiber..... — 366.12 lb. per sq. in.
For End (4), cover-plate inside fiber..... + 366.12 " " " "
For End (4), bottom angle outside fiber.... — 418.42 " " " "
For End (4), bottom angle inside fiber.... + 418.42 " " " "

The maximum primary stress due to live load occurs when the bridge is almost fully loaded, namely, when Wheel (9) is at Panel Point (5). Then:

Primary stress = — 5 729.41 lb. per sq. in.,
Impact = — 3 819.61 " " " ".

The secondary stresses, due to the rigidity of the truss joints, become:

For End (2), bottom fibers.... — 1.5 (209.57) = — 314.35 lb. per sq. in.
For End (2), top fibers..... + 307.43 " " " "
For End (4), bottom fibers.... + 1 038.95 " " " "
For End (4), top fibers..... — 1 016.09 " " " "

The secondary stresses, due to the rigidity of the top lateral system, become:

For End (2), cover-plate outside fiber..... — 693.39 lb. per sq. in.
For End (2), cover-plate inside fiber..... + 693.39 " " " "
For End (2), bottom angle outside fiber..... — 792.44 " " " "
For End (2), bottom angle inside fiber..... + 792.44 " " " "
For End (4), cover-plate outside fiber..... — 943.89 " " " "
For End (4), cover-plate inside fiber..... + 943.89 " " " "
For End (4), bottom angle outside fiber..... — 1 078.74 " " " "
For End (4), bottom angle inside fiber..... + 1 078.74 " " " "

Hence, under the severest conditions, the total stresses at the eight corner fibers, Fig. (92), become:

$$f_{2a} = -(2\,233.20 + 5\,729.41 + 3\,819.61) + (102.00 - 270.83 + 307.43 - 693.39)$$
$$= -12\,337 \text{ lb. per sq. in.}$$
$$f_{2b} = -10\,409 \text{ " " " "}$$
$$f_{2c} = -11\,101 \text{ " " " "}$$
$$f_{2d} = -13\,301 \text{ " " " "}$$
$$f_{4a} = -14\,406 \text{ " " " "}$$
$$f_{4b} = -11\,785 \text{ " " " "}$$
$$f_{4c} = -8\,943 \text{ " " " "}$$
$$f_{4d} = -11\,937 \text{ " " " "}$$

6.—Conclusion

By means of the set of influence lines for the primary and secondary stresses in the truss members, due to the rigidity of the joints in the plane of the truss, and the sets of influence lines for the top lateral system and the cross-frames, it has been possible to determine the maximum stresses that occur at the end corners of all the members except the bottom chords. In order to obtain the maximum stresses in the bottom chords, it is first necessary to obtain the influence lines for the floor system. With such a complete set available, superimpose the secondary stresses due to dead load together with those brought

into play by the axial loads. At this point it would be well to use Westergaard's analysis (35). It must be remembered that the variation between two end moments in a certain plane follows the straight-line law, consequently, it is possible to obtain influence lines for secondary stresses due to the rigidity of the joints at any section of the member. All the influence lines that have been found thus far, are for sections at the joints. As end connections prevent the determination of end effects, and, also, as these connections result in stresses quite different from what they would be if members were supposed to be rigidly held at the very ends, observations are taken beyond the gusset-plates, etc. It is possible, however, to make an analysis for the secondary stresses at some such section and then verify them experimentally. Discrepancies are to be expected, but just as tension specimens have under the clamps a distribution of stress which is not uniform, and to all intents and purposes a uniform distribution a little beyond the clamps, so it may be inferred that, in practice, the theoretical stresses at the corner fibers of a member of a bridge will exist at sections beyond the end connections.

So much for the dead and the live load; consider the wind load separately. Its presence means certain horizontal loads at the panel points. An analysis of the top and bottom lateral systems will indicate the extra primary stresses in the chords, together with the additional secondary stresses in the two horizontal planes. The primary stresses induced by the wind in the top and bottom chords of each truss cause additional secondary stresses in these trusses. A set of influence lines for these must be drawn. Again, the overturning effect of the wind on the train throws more weight on the leeward rail than on the windward. Not only will the cross-frame secondary stresses due to the live load be effected, but, also, the secondary stresses in the two trusses caused by the live load. Imagine that the wind causes the load of 1 kip on the leeward rail to become 1.05 kips and that on the windward, 0.95 kip. With little effort the cross-frame secondary stresses may be determined from the analysis given on pages 127 to 143, while the secondary stress quantities found on pages 108 to 118 are to be multiplied by 1.05 to obtain those in the leeward truss and by 0.95 to obtain those in the windward truss. Finally, the top lateral system needs revision.

Traction effects should now be considered in more or less the same way. When all the influence lines have been determined, they should be combined in such a manner as to arrive at the maximum corner stress that a member can possibly have. It will generally be found to occur when the primary stress is a maximum. It seems justifiable, therefore, to compound the secondary stresses arising from axial and transverse loads at that moment with those already found from the end moments, and thus obtain a fairly good estimate of what happens along the length of the member.

The analysis given on pages 143 to 145, clearly shows how widely the four corner stresses at the end of a member may differ. Any device that lessens this difference is to be encouraged. In practice, the top members of the cross-frames are often longer than their theoretical length by an amount that introduces secondary stresses opposite in effect to those given on pages 127 to 143.

and equal to the dead load plus one-half the live load contribution. In large structures, too, either the top chords are intentionally made longer or the bottom chords shorter than their theoretical unstressed lengths, so that when the dead load acts, the top chords become straight; the axial effect, which is intensified with bent chords, is, therefore, minimized.

In conclusion, the writer believes that he has demonstrated conclusively, which methods the practical engineer must adopt in order best to obtain proper results. His idea of compounding secondary stresses as indicated is probably far from the truth, but it is an endeavor to explain, by means of the present limited knowledge of mathematics and mechanics, the true state of affairs in a bridge. He has indicated how this study can be pursued to a conclusion and, although he frankly acknowledges that it may never be adopted in its complete form in practice, he sincerely hopes that all engineers will regard such a study not so much as a possible means of material benefit but as a search for truth.

Acknowledgments

The writer wishes to express his thanks to H. M. MacKay, M. Am. Soc. C. E., and Dr. Cyril Batho, of McGill University, and to Henry S. Jacoby, M. Am. Soc. C. E., and Dr. E. W. Rettger, of Cornell University, for their instruction, advice, and encouragement.

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DISCUSSION

O. H. AMMANN,* M. AM. SOC. C. E. (by letter).—The author deserves credit for his painstaking and elaborate investigation. For practical purposes the outstanding merit of this paper lies in its critical comparison of various methods of calculating secondary stresses.

Contrary to Mr. von Abo the writer does not expect that engineers will avail themselves extensively of a systematic determination of secondary stresses in bridges, nor does he believe that, in general, such practice would produce bridges of greater strength and economy than those built with what the author calls "the factor of ignorance".

An experienced engineer, of course, must be familiar with the character of secondary stresses, but he will not trouble himself with their determination in each ordinary case. For an unusual structure he will generally know where large secondary stresses are to be expected and confine himself to such critical determinations. Nor will it be a question of extreme refinement for he knows that many "practical factors" affect the true values and that even the true secondary stresses are not at all a criterion or measure for the safety or ultimate carrying capacity of a bridge. He will prefer therefore such methods as will determine these stresses, where desirable, in as simple a manner as is compatible with a fair degree of accuracy.

The writer has calculated the secondary stresses in a number of trusses and has invariably used the semi-graphical method of Mohr with which he was most familiar and which appeared to him to have the advantages of simplicity and lucidity.

A modification of this method which appears to possess its advantages to an even greater degree has since been developed by M. Ros and is contained in a report by the Technical Commission of the Swiss Association of Structural Steel Manufacturers on a comprehensive investigation of secondary stresses. A brief résumé of that report is given in a recent article by the writer.†

The author mentions only casually the effect of gusset-plates on the true secondary stresses, yet this is most important and, more than anything else, suggests the futility of extreme accuracy in the theoretical determination of secondary stresses for the purpose of design. He assumes that the theoretical secondary stresses exist as sections beyond the end connections. Although that arbitrary assumption is often made, and is, perhaps, as good as any other in many practical cases, it is far from correct where gussets are rigid and comparatively large. The true state is extremely difficult or almost impracticable of determination by theory, but from practical observations, especially those previously mentioned, it appears that the maximum stresses beyond the end connections may be found approximately by multiplying the secondary moments at the theoretical ends of a member by the ratio of the theoretical to the actual length of the member (between connections) and assuming these increased moments to be applied at the points of connection with the gussets.

* Bridge Engr., Port of New York Authority, New York, N. Y.

† *Engineering News-Record*, October 23, 1924.

It is evident that in many cases this correction means considerably greater secondary stresses. It remains an open question, however, as to whether rigid gussets, producing large secondary stresses, are not a source of strength rather than of weakness. The writer is inclined to the view that the former is the case, although he feels that this question offers a field for further investigation.

William Cain,* M. Am. Soc. C. E. (by letter).—This valuable paper on secondary stresses in bridges is unique, in that it gives, not only the theory of various methods pertaining to secondary stresses, but likewise their application to a railway bridge, with the numerical or graphical work required for complete solutions, the main object being a critical comparison of the methods with a view to selecting the best of them, considering the labor involved and the accuracy. This task the author has accomplished successfully, and the full numerical solutions show the comparative merits, from a practical point of view, of the several methods. The author finally concludes that, for the railroad truss, Manderla's method is to be preferred; the writer is inclined to agree with him, perhaps using as a check the Mohr semi-graphical method. The computations by either of these methods involve only three decimal figures, whereas Müller-Breslau's or Mohr's elastic weight methods introduce a much greater number of equations, which thus require many more decimal places for an accurate solution.

The writer having used the Gauss method of elimination considerably, warmly commends it. By this method, each derived equation is checked at once, so that the computer has the great satisfaction of being assured of correct results at every stage of the elimination. Professor Turneaure† has given a complete derivation of the required formulas by Manderla's method, together with a systematic method of procedure for their solution, which is to be commended. Tentative methods, such as those of Ritter and Mao, are not very attractive when direct solutions that require no more labor are available.

‡In connection with the development of Mohr's elastic weight method, the author states that, after the axes are turned 90°, "$u_{4\psi}$ of Equations (69) may be regarded as the negative of the sum of the moments of the ψ's about the new u'-axis, while $v_{4\phi}$ is the sum of the moments of the ϕ's about the new v'-axis." Evidently, if the ψ's are turned to a vertical position, they can have no moments about a horizontal line (the new u'-axis); and, if the ϕ's are turned to a horizontal position, they can have no moments about a vertical line (the new v'-axis). (See Fig. 93.)

From Equations (68):

$$u_{4\psi} = (u_4 - u_1)\,\psi_1 + (u_4 - u_2)\,\psi_2 + \ldots;$$
$$-v_{4\phi} = -[(v_4 - v_1)\,\phi_1 + (v_4 - v_2)\,\phi_2 + \ldots]$$

The distances $(u_4 - u_1), \ldots, (v_4 - v_1), \ldots$ (Fig. 93), are unchanged by the rotation of the axes; but if the ϕ's are regarded as forces acting horizontally

* Prof. Emeritus, Univ. of North Carolina, Chapel Hill, N. C.

† Johnson, Bryan and Turneaure, "Modern Framed Structures," Vol. II.

‡ For a discussion of this point, see p. 208 of the author's closing discussion.

and the ψ's as forces acting vertically, then taking counterclockwise moments as positive:

$$du_4 = (u_{4\psi} - v_{4\phi})$$
$$= \Sigma \text{ moments of } \psi\text{'s about the new } v'\text{-axis}$$
$$+ \Sigma \text{ moments of } \phi\text{'s about the new } u'\text{-axis}$$

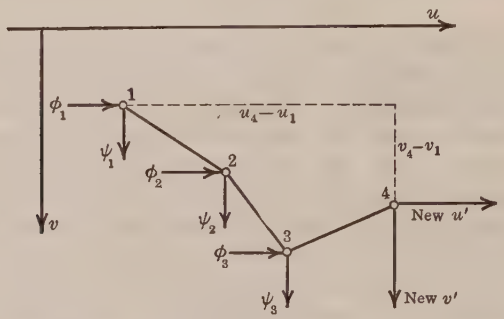


FIG. 93.

Further, the author shows, when $du_4 = 0$, that the origin of the new axes can be taken anywhere, as illustrated in the triangles of Fig. 55, where u_2, v_2, correspond to the new u'-axis and the new v'-axis, respectively. Formulas such as II 2 in Table 18 are at once derived by the formula just stated. The author will doubtless clear up the ambiguity mentioned. He is requested to explain (1) why the co-ordinates, u_1, v_1, etc., are not drawn parallel to the axes, u, v, respectively, as is usual, and (2) as to the signs of the co-ordinates.

This method of elastic weights is readily adapted to the rectangular frame, as the author illustrates with an example. He also gives Batho's interesting analysis, leading to Equations (117), which apply to any rectangular frame, and thus makes possible a solution by mere substitution. The author has failed to note two important papers on statically indeterminate frames, one by Mikishi Abe,* M. Am. Soc. C. E., the other by Messrs. Wilson, Richart, and Weiss.* These papers are especially valuable to the designer, in that they give readily applied formulas for numerous statically indeterminate structures.

In conclusion, the writer desires to express his appreciation of the author's notable contribution, which has evidently involved great labor and which leads to practical conclusions that will prove of material service to engineers.

ALMON H. FULLER,† M. AM. SOC. C. E. (by letter).—Although this paper may not contain much that is new in the sense of theoretical or experimental research, it is a valuable contribution to the subject of secondary stresses. The procedure of computation and the comparison of methods are of value to all structural engineers, especially to those, still in the majority, who have not given personal attention to this phase of analysis. The emphasis on the

* Univ. of Illinois *Bulletin No. 107*, October 21, 1918, "Analysis and Tests of Rigidly Connected Reinforced Concrete Frames," by Mikishi Abe, M. Am. Soc. C. E.; and Univ. of Illinois *Bulletin No. 108*, November 4, 1918, "Analysis of Statically Indeterminate Structures by the Slope-Deflection Method," by W. M. Wilson, F. E. Richart, Assoc. M. Am. Soc. C. E., and Camillo Weiss.

† Prof., Civ. Eng., and Head of Dept., Iowa State Coll., Ames, Iowa.

effect of lateral and sway-bracing and on the general subject of stress distribution helps greatly in producing a well rounded investigation.

The value of the paper is enhanced by references to, and brief comments on, the field measurements of stresses which were available. Other similar investigations to which the author refers as not being available may be many and, if brought to light, may be co-ordinated so as to be of considerable value. To this end, the writer contributes his mite by giving some significant data which are yet only published in part.

The results for 1922* and for 1923† of a four-year program in stress measurements on bridges have appeared since the paper was written. While the object has been primarily to investigate impact in highway bridges, the study of stress distribution under static and dynamic loads has naturally followed, and although the greater part of the work relates to floor systems, considerable data on truss members have been obtained. For instance, the 1923 results for the hip vertical of the Skunk River Bridge show a unit stress of 6 200 lb. per sq. in. on the inside flanges (the ones to which the floor-beams are attached) and 2 400 lb. on the outside flanges, under an impact of about 25% (live load stresses with the load in motion on a smooth floor exceed by 25% the static stresses for the same load). Under high impact (above 100%), the respective values are 13 000 and 4 800 lb. per sq. in. Corresponding stresses for the hip vertical of the Rowland Bridge are 4 100, 1 700, 8 500, and 5 000 lb. per sq. in. A tendency will be noted in the Rowland Bridge for a smaller proportionate range of stress under the more severe loading. The bending in these members is almost entirely in planes perpendicular to the trusses; there is little indication of secondary stresses due to bending in the planes of the trusses.

The results of some of the writer's strain-gauge readings for secondary stresses on the Niagara Arch‡ have already been published. They indicate the bending in horizontal and in vertical planes and are averages of two or more readings. The individual readings, not given in that paper, indicate even a greater range of stress; they show that for the upper panel point, U_2, for a given static live load, G_2, the range of the stress in the upper chord to the left is from 5 300 lb. per sq. in. compression to 2 800 lb. tension; in the upper chord to the right, from 100 to 4 800 lb. compression; in the vertical, 1 000 to 5 200 lb. compression; and in the diagonal, from 2 800 to 7 000 lb. tension. For the corresponding members at the point, U_6, for a given static live load, D_1, the values are, respectively, 3 000 lb. per sq. in. compression to 800 lb. tension, 1 400 to 3 600 lb. compression, 400 lb. tension to 6 200 lb. compression and 2 400 to 4 200 lb. tension.

The results of strain-gauge readings at sixteen points on each of three sections of four columns during the course of erection of an eighteen-story building, have been published§ by the writer. In general, the distribution of

* "Preliminary Impact Studies—Skunk River Bridge on the Lincoln Highway near Ames, Iowa," *Bulletin No. 63*, Eng. Experiment Station, Iowa State Coll., Ames, Iowa, and Progress Report of the Special Committee on Impact in Highway Bridges, *Proceedings*, Am. Soc. C. E., March, 1923, Papers and Discussions, p. 457.

† "Impact Tests on Highway Bridges," *Public Roads*, September, 1924.

‡ "Revision of the Niagara Railway Arch Bridge," by Charles Evan Fowler, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1936.

§ "Measurements of Stresses in Four Steel Columns of the Equitable Building, Des Moines, Iowa", *Bulletin No. 71*, Eng. Experiment Station, Iowa State Coll., Ames, Iowa.

stress was excellent. There was one exception, however, of sufficient range to merit attention, although not to cause alarm. Just above the base of one column under a load of about 700 000 lb., the average stress was about 10 000 lb. per sq. in. and the range from less than 7 000 to more than 14 000 lb. The average of the four lowest points on one side of the column was 7 400 lb. per sq. in., and the average of the four highest corresponding points on the other side of the column was 13 600 lb. This shows an unmistakable bending about an axis very nearly parallel to two sides and suggests something which should be known, and which, if anticipated, might be prevented.

As yet, none of these values has been checked against computed secondary stresses, but, regardless of what the comparison may be, they show such a range as to give encouragement for further investigations from both theoretical and experimental standpoints. It would seem fortunate if field measurements could be taken on the structure for which the author has made such satisfactory computations.

K. P. Bahjejian,* Jun. Am. Soc. C. E. (by letter).—On comparing the methods of Mao and Müller-Breslau it is noted that their fundamental equations are similar in form. If Equation (80) is applied to the angle at A in Figs. 7 and 8, and the same notation is used as indicated in these diagrams, remembering that f and M have the same subscripts, there is obtained:

$$6\,E\,.\,\Delta\,\alpha = \frac{l_1}{y_1}\,(2\,f_2 - f_1) - \frac{l_2}{y_2}\,(2\,f_3 - f_4)$$

The first form of Equation (43) is:

$$6\,E\,.\,\Delta\,\alpha = (2\,\rho_3 - \rho_4) - (2\,\rho_2 - \rho_1)$$

in which,

$$\rho_3 = \frac{M_3\,l_2}{I_2} \text{ and } M_3 = \frac{f_3\,I_2}{y_2}, \text{ whence } \rho_3 = \frac{f_3\,l_2}{y_2}$$

Similarly,

$$\rho_4 = \frac{f_4\,l_2}{y_2};\ \rho_2 = \frac{f_2\,l_1}{y_1};\ \text{and } \rho_1 = \frac{f_1\,l_1}{y_1}$$

Making these substitutions, the equation becomes:

$$6\,E\,.\,\Delta\,\alpha = \frac{l_2}{y_2}\,(2\,f_3 - f_4) - \frac{l_1}{y_1}\,(2\,f_2 - f_1).$$

As bending moments in a clockwise direction in the latter method are regarded as positive, and in the former as negative, these equations are really identical.

The author has called attention to the fact that in applying the fundamental equations to a given truss, Mao "assumes two quantities known at the start, and using these throughout the process, finally arrives at two simultaneous equations the solution of which determines the two assumed quantities", just as described in connection with Fig. 9 for Müller-Breslau's procedure modified by Páez. Mao assumes the secondary stresses, f_1 and f_2, in the first member, 1-2, of the truss, instead of ρ_1 and ρ_2 for the same member.

* In Chg., Valuation Dept., East Penn Elec. Co., Pottsville, Pa.

C. A. P. Turner,* M. Am. Soc. C. E. (by letter).—The author defines secondary stress in a truss member as equal to one-half the difference between the extreme fiber stresses at any section.

Stress variation in a compression member, however, may largely result from column action of the primary stresses independent of truss distortion (the cause of so-called additional stress), but depending on the buckling or torsional tendency incident to the make-up of the built section, which is explicitly disregarded in the proposed analysis.

Secondary stress theories were first developed to explain the collapse of certain German bridges designed in accordance with the letter of commonly used formulas and customary working stresses but lacking in engineering judgment as to the make-up of the sections and the application of formulas not theoretically perfect to provide for the primary stresses in the compression member. Between the behavior of the serviceable and the unserviceable design there exists a wide difference unless the foregoing ingredient of good practice is taken into consideration. Even then the scope of assumption necessary to bridge the gap needs to be wide indeed, unconfined by the narrow limitations of Hooke's law on which elastic theory is founded.

According to the author:

"In 1877, the Technical University of Munich offered a prize for the solution of a problem, worded by Professor Asimont as follows: What stresses arise in the members of a bridge truss owing to the fact that the angles of the truss triangles suffer no change?"

In the published theory of secondary stress is found no experimental measurement proving that the angular distortion of the gusset-plates is of such a low order, in comparison with the minute angular change in the truss triangles, that it may be disregarded. Computation covering this pertinent question is likewise lacking, notwithstanding that the accuracy and utility of the analysis depend on these relative magnitudes.

Engineering opinion may be divided into two schools of thought on the question of the value of the stereotyped computation of secondary bending stresses. The old school regards such secondary stresses as an element of additional strength in the well proportioned frame—one free from radically acute angles each of the elements of which presents a small departure only from the equilateral triangle. Under such practical limitations the elastic stiffness of gusset-plates at the joints reduces the vibration under rolling load and the bending resulting from the weight of the members. In the case of compression members the restraint of the ends caused by the gusset-plates increases the column strength. In the top chords distortion of the truss triangles particularly in a camel-back simple span results in a favorable upward shifting of the center of compression from the lowered position caused by the bending due to the weight of the member. The trend of present practice in the progressive substitution of stiff riveted construction for pin-connected spans strongly supports this viewpoint.

The younger school of theorists, on the other hand, scent danger in all secondary bending stresses regardless of their direction or sign and treat

* Cons. Engr., Minneapolis, Minn.

them as an invariable source of weakness, to be combined numerically instead of algebraically with the primary stresses. Contrary to the elastic theory the strength is computed as proportional to the principal stress and independent of the greatest displacement. Now, displacement is the joint product of the principal stress and the shear that is explicitly ignored in the premise on which the theory rests.

In the ordinary built-up steel column, twisting or buckling of webs or outstanding flanges is the predominant distortion preceding the plastic bending incident to the follow-up motion of the machine after the elastic resistance has been overcome. This rotary or buckling tendency of the compression member made up of thin plates and angles is neglected in the ordinary column theory of primary stress. This deficiency in proportioning metal for the primary stress, although unrelated to truss distortion, is assumed to be provided for in the secondary stress computations advanced to explain the collapse of defective bridge designs.

The value of secondary stress theory as a useful explanation of these failures narrows down to a question of the relative magnitude of the torsional weakness of the column under primary stress (disregarded in column theory) and of the bending weakness under truss deformation. Torsional weakness varies with the form and make-up of the section, as shown by tests of angle and pipe struts of equal slenderness ratios, having pin and ball-and-socket bearings. Round pipe columns develop the same strength whether the ends are hinged on a pin or fitted with a spherical bearing. On the contrary, angle and T-sections, the slenderness ratios of which are greater than 140, develop only 57% of the strength with round ball-and-socket bearings that they develop with pin-bearings; and the pin-bearing, angle-iron strut, develops only about 85% of the resistance of the pipe strut. Nevertheless, according to the common (but defective) theory of primary stress resistance of simple compression members, these divergent types and details would be computed as possessing identically the same strength. If such a major deficiency exists in providing against the weakness of members under primary stress with divergent forms of section, the cause should be investigated before endeavoring to compute the magnitude of secondary sources of weakness.

As the strength of the entire section is the sum of the resistance of its parts plus their combined action as co-ordinated in the form or make-up of the cross-section, the column section may be divided lengthwise into narrow parallel strips. The slenderness ratio of each strip of the hollow column is less than that of the whole, and therefore the strength of the sum of the strips would be less than the whole by the amount of their combined action in supporting each other. In the round pipe the strips support each other so perfectly that pure torsion would simply increase the length of each as in the case of a solid section; but with an angle, T, or star-shaped section, twist shortens the section and a tendency develops to twist out as well as to bend out from under the load. The tendency of an outstanding flange to bend is restrained by the metal normal to its least dimension; hence, if several flanges have like tendencies to buckle as in a Z, T, or star-shaped section, rotary

distortion results, the tendency varying as the square of the projecting length of flange divided by the distance between the centers of gravity of this and the opposing part which resists the buckling.

This matter has been discussed at length elsewhere* by the writer, the conclusion being that the compressive resistance of sections such as a T-section composed of one 30-in. by ½-in. web and two 5 by 3½-in. by ⅜-in. angles, or star-shaped sections composed of one 20-in. by ⅜-in. plate, two angles, 4 by 4 in. by ⅜ in., and two plates, 10 in. by ⅜ in. (similar to the make-up of the defective German designs) is so much lower than the apparent strength computed by ordinary formulas, that their collapse under primary stresses should have caused no surprise or comment. As the collapse of the defective German designs may thus be accounted for without resorting to the theories of secondary stresses evolved by Manderla, Müller-Breslau, Mao, and Ritter, it is obvious that the bending stresses computed by these methods are greatly exaggerated or largely imaginary. For a given loading it is evident also that two unrelated causes cannot account *in toto* for the same phenomena.

The assumptions underlying the conventional secondary stress theories may be considered as follows:

(*a*) That the secondary effects of primary stress are confined to truss distortion or bending of the member in the plane of the truss without torsional strain.

(*b*) That the gusset connections at the joints may be assumed as absolutely rigid, whereas the members connected by the joint plates are treated as elastic.

(*c*) That safety from failure is measured by extreme fiber stresses independent of shear strain.

(*d*) That the moment of inertia of a member under predominant axial stress accompanied by minor transverse stress is of the same magnitude as if the member were subject to pure flexure.

(*e*) That the stress mechanism of columns and beams may be treated as similar in nature for practical purposes of computation.

Assumption (*a*) is inadmissible, for torsion of the column or its elements resulting in the buckling or the rotary distortion of an unsymmetrical section is the greatest and most dangerous of indirect stresses; Assumption (*b*) is inconsistent with Hooke's law on which elastic theory is founded; Assumption (*c*) is discredited by experiment; Assumption (*d*) is at least fourfold in error; and Assumption (*e*) would upset fundamental conceptions of the theory of internal stress and is moreover discredited by experience, which teaches that the short column cannot be logically proportioned by Euler's formula for the bending of long columns.

The kind of accuracy so laboriously attained under these assumptions recalls the case of a shop drawing once checked by the writer. The lettering and figures were as neat as copy-plate, but each different rivet space was given in odd $\frac{1}{32}$ in. or $\frac{1}{64}$ in. Pressed as to his reason for such spacing, the draftsman stated that it made the drawing look accurate. On further questioning regarding field connections that did not match and main dimensions

* "Elasticity and Strength of Materials," Section IV.

that were 1½ ft. in error, the man assured the writer he would "get a looking glass" to read his scale closer.

Looking with a similar instrument into the German assumption of identity as to stress mechanism of columns and beams in flexure, it would appear that whereas, for the beam, the deflection is closely measured by the extreme fiber stress, in the case of the column it is not; this latter is because, unlike the deflection of the beam, it is determined largely by shear distortion which is regarded as negligible in the beam. To locate the axis of the compression member about which the shear distortions are balanced, there is found the locus of the center of pressure instead of the center of gravity of the section so that the moment and the resistance to moment differ radically from beam action. Thus, in the beam, the greatest tensions and compressions are parallel to the axis and their values on a cross-section of severest intensity reduce to zero at the center of gravity of the section. In the column, on the contrary, the greatest tensions are at right angles to the axis instead of parallel or slightly inclined while the compressive stress is a minimum at one outer fiber, a maximum at the other, and a mean at the center of the section instead of zero as in the beam. Accordingly, the same conditions of bending and the same moments of inertia cannot apply to two conditions of stress so different from each other.

In tests it is observed that when the column commences to bend appreciably its equilibrium is unstable and it is practically on the point of failure. In reaching this point the state of stress within the column must have been changing radically with very little deflection. Stated in another way, the bending of the column does not take place in proportion to the applied force; nevertheless, as the writer has endeavored to show elsewhere,* if the divergent state of stress from that of the beam is given proper consideration it does follow Hooke's law.

Any effort by which the magnitude of subsidiary stresses may be rationally determined is a matter of scientific interest. It becomes of practical value only as these stresses attain substantial magnitude. Ordinarily, torsional stresses and shearing stresses in the compression member (which are neglected and ignored in the German theory of secondary stresses) greatly overshadow the longitudinal secondary stresses produced by truss distortion which, therefore, need to be given consideration only in the case of improper design.

By avoiding acute-angled triangles which cause the deformations to become excessive, the secondary bending stresses are reduced to a negligible amount. Nevertheless, their precise determination will be of scientific interest when developed in a manner consistent with Hooke's law, with the elastic bending of the compression member, and with the known characteristics of the state of stress. These have been disregarded in the theories heretofore proposed.

P. L. Pratley,† Esq. (by letter).—The writer believes that the collection and criticism of the various methods discussed by the author, by which the distortional stresses in a frame may be determined, is a really valuable work.

* "Elasticity and Strength of Materials," Section IV.

† Montreal, Que., Canada.

A paper of this nature, however, is not one that lends itself readily to discussion, as so much of it is demonstration, either of theory or methods of calculation. The author is certainly to be congratulated on the patience displayed in the prosecution of such an amount of tedious labor, and also on the general clearness with which he has presented both the methods of the various investigators and his own conclusions.

For about fifteen years the writer has been accustomed to calculate these distortional stresses for bridge trusses wherever either the magnitude, the layout, or the duty of the structure suggested, and although he claims some acquaintance with most of the methods presented, he must confess total ignorance of that attributed to Dr. Mao. From the study made in the paper he is decidedly of the opinion that this new method cannot be considered as of practical value, and agrees with the author that the solutions of Manderla and Mohr are much to be preferred. On the other hand, from his own experience, the writer would reverse the order in which the author classifies these two methods, preferring the semi-graphical method of Mohr and Williot. Having begun with Manderla, he has come to accept the other as more quickly solvable, and more "visual". To his mind the Mohr method deals with actualities, rather than with imaginary or artificial functions such as are introduced by Mao and other investigators. The ease with which the angles and moments can be plotted, makes the graphical method a realistic means of portraying to the eye the actual conditions in the framework, and making the calculator familiar with the comparative values. Furthermore, the functions used in the Mohr analysis are susceptible to experimental checking, as has been demonstrated by Swiss engineers. The writer solves Equations (55) by a simple method of successive approximation; four or five repetitions always result in sufficiently accurate results for $\Sigma M = 0$ to be established at all panel points. The author points out in his Tables 16 and 24 that the Mohr method is practically as accurate as any, giving results almost identical to ten decimal places with the elastic weight and the improved Manderla methods.

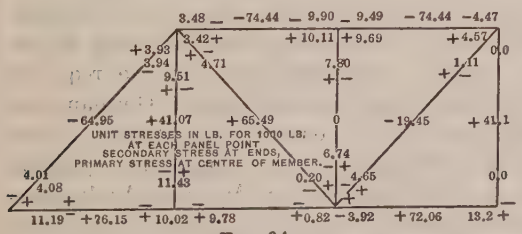


FIG. 94.

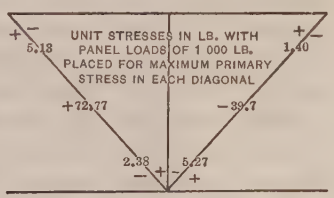


FIG. 95.

The influence line diagrams Figs. 58 to 68, inclusive, are also of value, and the thanks of the profession are due the author for compiling them. The writer has collected from these diagrams the primary and secondary stresses due to full load conditions (which is the case usually adopted for calculating purposes) and for comparison has added the corresponding diagrams for the two diagonals under the loading which produces their maximum primary stress (Figs. 94 and 95). From these it appears that the full load condition

does give satisfactorily close approximations to the maximum combinations, for even at the bottom of the first diagonal where the variation is the greatest, the fiber stress amounts only to about 270 lb. per sq. in., taking a panel load of 125 000 lb. (approximately E-50). In Fig. 96 is shown the total resultant fiber stress on gross areas, the truss being fully loaded with 125 000 lb. at each panel

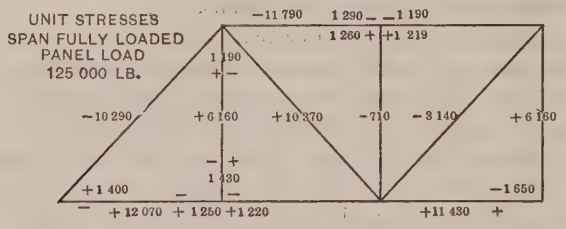


FIG. 96.

point. The secondary stress is given only where it exceeds 1 000 lb. per sq. in., and it will be seen that it never reaches 11% of the common unit, 16 000 lb.; so that for this type of truss it can be safely neglected. An initial adjustment in the length of the vertical members is sometimes resorted to, as a means of minimizing the secondary stresses induced in the chords in this type of truss.

HENRY S. JACOBY,* M. AM. SOC. C. E.—It may be of interest to the members of the Society to know some items of human interest connected with the work on secondary stresses in the Graduate School of Cornell University, which culminated in this paper. In the fall of 1912, Mr. José Páez came to the University for a year's graduate study before returning to the Philippine Islands, his native land. He had completed his secondary education in Germany, had been graduated in Civil Engineering at the Zurich, Switzerland, Polytechnicum, and had been employed in the Engineering Department of a bridgeworks in Berlin, Germany, for a year, while additional practical experience had been secured during one or more summer vacation periods. When asked whether he had considered the choice of a thesis subject he replied that he would like to write a thesis on secondary stresses. This was a delightful surprise, since this subject had been in the speaker's mind for some years, awaiting a student who had the requisite qualifications. Mr. Páez had been accustomed to use French, German, and English, in addition to his native language. In order to secure some knowledge of the relative advantages of the different methods of computing secondary stresses, he applied most of the methods described in Mr. von Abo's paper to a single bridge truss. He called special attention to the time and labor saved by the use of Gauss' theory of normal equations, thus also checking the solution step by step. He was led to use this method throughout because he had chosen Geodesy as the minor subject in his graduate work. He also developed an additional approximate method.

Mr. Thomson E. Mao, a Chinese student, came to Cornell University for a year's graduate work in 1916. His thesis was on the design of a two-hinged spandrel-braced arch with cantilever extensions on each end. He studied the

* Prof. of Bridge Eng., Emeritus, Cornell Univ., Bethlehem, Pa.

thesis of Mr. Páez and voluntarily extended his own thesis beyond its original plan by computing the secondary stresses in the arch truss for a load at each panel point separately, which required the solution of ten sets of simultaneous equations with thirty-five in each set.

Mr. Mao's interest in secondary stresses due to this experience did not diminish after leaving the University to secure practical experience with a large bridge company in Pittsburgh, Pa. He thought that there ought to be some method which would enable the necessary computations to be made with less labor. He devoted much study to the subject for several years, and finally both the analytic and graphic methods which he discovered were presented in a thesis for the degree of Doctor of Philosophy at the Carnegie Institute of Technology. Time did not permit Mr. Mao to apply his methods beyond the case of full live loading. When Mr. von Abo applied both of Mao's methods to a bridge truss with an unsymmetrical load, it was a disappointment to learn that the promise of advantages previously indicated could not be realized in practice.

In 1921, a student—Mr. K. P. Bahjejian—devoted some time to an examination of Mao's methods as given in the published abstract of his thesis. He discovered the interesting fact that although the deduction of the fundamental equations is based on radically different ideas, the form of certain equations is the same as the corresponding ones in Müller-Breslau's method.

This leads to an explanation of how Mr. von Abo's paper came to be prepared in its original form. The most decisive element of the proceedings was the action of the Committee in charge of Mr. von Abo's graduate work at McGill University, of which Henry M. MacKay, M. Am. Soc. C. E., was Chairman; after completing two years of his work at that institution, the Committee recommended that Mr. von Abo spend the third year in residence at Cornell University and write his thesis there.

Since the Committee made no suggestion regarding the thesis subject and Mr. von Abo had not found any subject which made an effective appeal, he was ready to consider and then to adopt a suggestion to write on secondary stresses. This subject was suggested because of his exceptional educational preparation for the task. He had previously received four degrees: Bachelor of Arts, Bachelor of Science in Engineering, Master of Arts in Pure Mathematics, and Master of Arts in Applied Mathematics. In no case, either before or after coming to America, did he ask for advanced credit and, therefore, each degree represents full-time work in residence. It was found that his courses of study had given him excellent facility in handling advanced mathematics, a working knowledge of theories of elasticity, methods of least work or of deformation, advanced graphic statics, etc. Accordingly, he had command of all the tools necessary to handle this difficult technical subject. That a student of such character, ability, and maturity, having previously completed four years of graduate work, should be advised to take his last year in a sister university, represents a generous spirit of co-operation unparalleled in higher education.

The speaker believes that the Society deserves great credit for the publication of this paper, thus making available to the Engineering Profession in the

English language the present status of theoretical knowledge on secondary stresses, the importance of which will undoubtedly increase with the years, especially to bridge engineers.

CHARLES EVAN FOWLER,* M. AM. SOC. C. E.—The purpose in this discussion is to explain what was done at the Niagara Arch Bridge with reference to accounting for the secondary stresses. The secondary stress in a member is an increment of stress in excess of the calculated axial stress. An engineer usually has not the time to make a complete calculation of secondary stresses, and seldom has the funds with which to establish them by calculation or strain-gauge measurement. It is desirable, therefore, to have some means of approximating secondary stresses, that is, to determine what the maximum values are.

A great many secondary stress readings were made on the Niagara Bridge, of which only a few were published in the speaker's paper.† It is hoped that the complete records of those measurements may be placed on file at Society Headquarters so that all engineers may have a chance to study them.

The speaker does not deem it proper to compare measured secondary stresses at the ends of a member with the calculated stresses at the center. At Niagara, the stresses for certain members were read at the ends and also at the center under the same conditions of loading, thus giving a direct experimental comparison. The results were plotted and compared with those made on the Hell Gate Bridge, to determine a safe maximum to apply to all members.

The real purpose of computing secondary stress is to justify the use of higher unit stresses; for instance, at Niagara, instead of a unit stress of 16 000 lb., 20 000 lb. was used; and had the funds and the time sufficed to determine more closely all the stresses, that value would probably have been exceeded. The time is coming, it is hoped, when a locomotive loading such as *E*-90 will be used and when, knowing all the stresses in the bridge, the elastic limit will be used as the maximum allowable unit stress; then the exact limits to which a bridge can be loaded will be known.

According to the records the Niagara Arch as originally constructed did not close at the center of the top chord, so it was jacked apart with a small toggle at the center; actually it was jacked partly with the small toggle, and fully with the erection toggles at the end, and then a shim 1 in. thick was inserted. This introduced what might be termed a secondary axial stress in all the members of the bridge. This was very fortunate as the investigation in 1918, by jacking the top chord apart with a heavy toggle which exerted a pressure of about 700 tons, made it possible to replace the shim with a thin one, thus making the bridge almost a three-hinged arch and reduced the stresses to such an extent in the top chords and web members that the bridge would carry heavy modern loads.

A modification of this toggle was used in 1919 to take up the dead load stresses of the intermediate posts in the side spans so that the dead load in the members was entirely relieved while new reinforcing angles were being

* Cons. Civ. Engr., New York, N. Y.

† "Revision of the Niagara River Arch Bridge," *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1919.

riveted to these posts. Under full load all parts then took stress uniformly, and the new parts did not have to wait until the old parts were over-stressed before they assumed their load. Strain-gauge readings were made that showed this was actually the case.

The strain gauges used were of three types; the Berry gauge which is a lever instrument; the Fuller-West, an improved lever gauge; and the Howard gauge which is a micrometer instrument and cannot be used for measuring stress under a moving load, but only for dead load, or a live load standing on the bridge. This practically disqualifies the Howard gauge for future live load stress measurements. The Berry gauge, on account of the delicate lever mechanism, was found to vibrate so much in taking impact measurements that one could only guess at the maximum readings for impact.

After some consultation with the speaker, A. H. Fuller, M. Am. Soc. C. E., proceeded to have his mechanician, Mr. West, at Lafayette College, devise an improved instrument, the Fuller-West gauge. Quite a number of these instruments have since been used, and it has been found that this gauge because of its more practical design and dial gives steady readings; in other words, when a train is running across the bridge, the dial needle will crawl up gradually to the maximum without vibration, and as the load goes off, will drop back again gradually so that it is possible to read the maximum accurately.

All engineers who are engaged in strain-gauge work will find that this gauge (although not so accurate as the mirror extensometer) will give satisfactory results for ordinary practical work. Arrangements are being made with one of the large manufacturers of instruments to put this gauge on the market, so that it will be available for general use.

The secondary stresses at Niagara were represented in the speaker's paper by diagrams* showing the percentages on the members at the panel points. No dangerous secondary stresses were found, probably on account of the fact, as Mr. Ammann has stated, that there was a re-distribution of stress when it approached the elastic limit. On the inside of the posts, curved brackets were connected with the railway floor-beams, which greatly increased the secondary stresses; it seems practically impossible to calculate such secondary stresses accurately.

Recently, in a large arch bridge, angles were provided to reinforce the edges of large gussets against secondary stresses. Readings made at the Niagara Arch on the edges of the large gussets disclosed no secondary stresses; evidently, the truss members rotate uniformly and carry the gussets with them. By taking account of secondary stresses, an engineer can feel perfectly safe, regardless of the distribution of stress, as to the total unit stresses in all members.

The late Paul Wölfel, M. Am. Soc. C. E., related an interesting case covered by some German experimenters who measured the stresses in a roof truss, in which the knee-brace ran from the post, through the bottom chord of the truss, up to the top chord. They varied the sections of these members in different experimental trusses, making each of the members in turn too small to take

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1936.

its stress; and measuring the stresses in the various members described, it was found that the stresses were re-distributed, so that none of the members was over-stressed. That is another corroboration of the fact that secondary stresses will distribute themselves throughout the cross-section.

An engineer always desires to know the magnitude of the secondary stresses. At Niagara, there being neither the funds nor the time to make a complete calculation for every point in the truss, these stresses were determined by comparison with the Hell Gate curves superimposed on the Niagara panel points to the same relative scale, and with curves plotted from the Niagara secondary stress readings; in every case secondary stresses were used so as to be on the safe side.

One stress not considered at Niagara was the traction or braking stress from a suddenly stopped train. Account was taken of all the other stresses; but an approximation of the traction and braking stresses showed them to be relatively so small, that they were not included in the total stresses. The unit stresses were fixed at 20 000 lb. in tension, for steel with an elastic limit of more than 30 000 lb. per sq. in.

It is hoped that sometime the Society will issue a specification that will represent the superior limit of engineering knowledge. Engineers are conservative, but progress demands that in bridge specifications all factors should be included, and the unit stresses fixed accordingly.

H. M. MacKay,* M. Am. Soc. C. E.—The speaker considers this paper the most useful treatise on secondary stresses that has yet been published. The various methods of calculation are so clearly explained and so frankly compared that the reader can easily choose for himself the one best suited to his problem. It is not, however, the speaker's purpose to discuss the paper, but to deal with another phase of the question. Given the fundamental assumptions, the theory of secondary stress calculations may be regarded as well established; but there is still some doubt as to the extent to which the assumptions are valid and to which actual stresses agree with those calculated.

In all the methods of calculation explained in the paper two assumptions are tacitly made:

1.—That the members meeting at a joint are so rigidly held that the angles between their axes at the extreme ends remain constant.

2.—That between its extreme ends each member is quite free to bend and that its moment of inertia remains constant.

The first of these assumptions is dependent on the relative rigidity of the gusset-plates, connecting rivets, and members themselves. Relative rotation of the ends of members may sometimes occur. This, of course, would reduce the secondary stresses. The second assumption is never true in riveted frames. The members are under a measure of restraint, or, to put it differently, have an increased moment of inertia for some distance from their ends. This tends to increase the secondary stresses. Many other factors affect the stresses at critical sections, so that although a number of valuable measure-

* Dean, Faculty of Applied Science, McGill Univ., Montreal, Que., Canada.

ments have been made, present knowledge of actual stresses is far from complete, and any addition to the slowly accumulating data may be pertinent.

In the course of an examination of some 104 ft. 7-in., Pony truss spans for a Canadian railway (Fig. 97), the speaker made a fairly thorough survey of one-half of one truss, with a 10-in. Howard gauge, for the purpose of ascertaining the maximum stresses under a heavy type of Mikado engine then recently put into service. Gauge points were first established as close as possible to the ends of the members selected for examination. From eight to ten gauge lines were placed at each section as shown in Fig. 97, and their

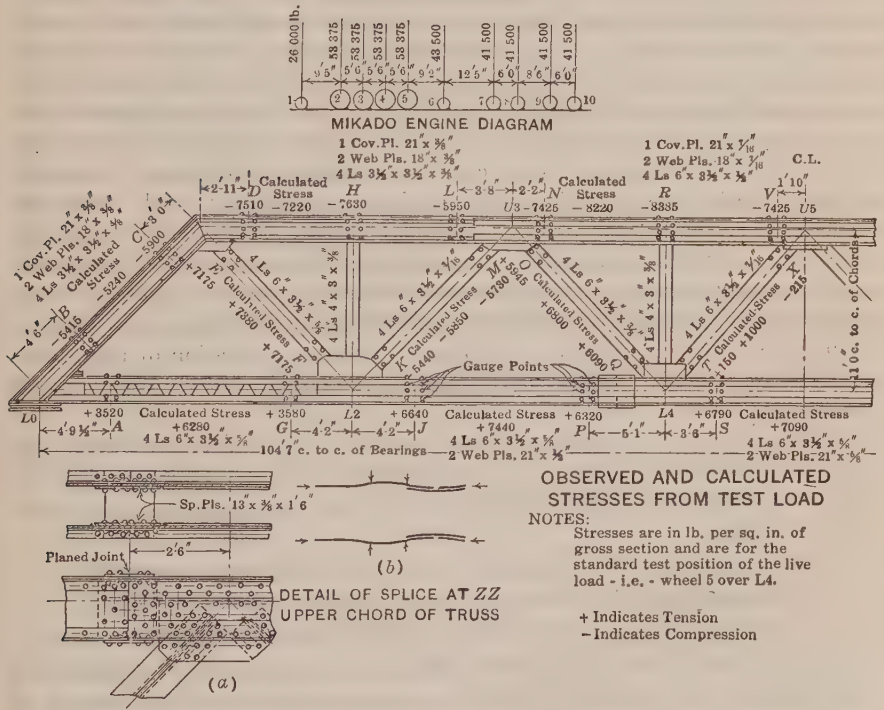


FIG. 97.

lengths measured with the span unloaded. A test train consisting of a 225-ton Mikado engine followed by two 50-ton coal cars fully loaded (Fig. 97) was then placed on the span in such a position as to give the maximum stress in the central section of the top chord, which was the principal source of apprehension. The stresses in the other important members for this position of the load were, however, not far below the maximum. The established gauge lengths were again measured and the stresses due to the static live load thus determined. Afterward, impact tests were carried out, using recording deflectometers and extensometers, but these do not concern this discussion.

Although these impact measurements brought out nothing new, they confirmed very well the principal conclusions reached by the Committee on Impact of the American Railway Engineering Association.

The speaker believes that the method here described whereby the measurement of static and dynamic effects is made separately, has much to commend it. The determination of the highest stresses existing at critical sections required the use of short gauge lengths. It is desirable to have numerous points of measurement across the section, since a planar variation of stress is a purely imaginary condition in most built-up members. Even if the recording and other extensometers and gauges available for measuring dynamic effects were entirely reliable (which is perhaps not yet fully established), it is hardly practicable to use them with such short gauge lengths, and in such numbers, simultaneously, as to give satisfactory results. On the other hand, considerable time and expense are involved in making a number of consecutive measurements by shifting the instruments and running the test train over the span repeatedly for each member measured; and it is impossible to ensure the duplication of the same conditions under repeated runs. Measurements of static stress can be made quickly, inexpensively, and, above all, reliably. A comparatively small number of determinations of the dynamic effects at convenient points will then give all the data required. The speaker would strongly urge that at least some check measurements of static stress should be made a part of every program for stress determinations in bridges. He believes that this procedure would prevent the accumulation of much data of doubtful accuracy which may becloud rather than throw light on the questions at issue.

The static live load primary stresses as calculated, and as measured, taking E as 29 000 000 lb. per sq. in., are shown on Fig. 97. The average differences between the measured stresses at the two ends of the same member were, as follows:

Bottom chords, 190 lb. per sq. in., or 3.6% of stresses measured.
Web members, 145 lb. per sq. in., or 2.3% of stresses measured.
Top chords, 840 lb. per sq. in., or 12% of stresses measured.

At the point, L, in the top chord, the stress, although the measurements were repeated and carefully checked, was abnormally low. The section of measurement was at the end of a splice plate as shown in Fig. 97(a), and it is suggested that the webs and flanges of the angles may have been affected as in Fig. 97(b). If the webs were thus bent (the angle flanges would probably be similarly affected), the fact that the gauge measurements were taken on the convex side of the bend would account for the low stresses observed. Doubtless, similar effects would occur in some degree at other points. The speaker has often noted effects of this kind in extensometer work.

Comparing the measured and calculated stresses, it appears that in the bottom chords all the measured stresses were less than those calculated. The average difference was 1 530 lb. per sq. in., or 22% of the calculated stress. The difference was greatest in the end section, 2 930 lb. per sq. in., and least

in the center section, 300 lb. per sq. in. In the web members, too, all the measured stresses were less, the average difference between calculated and measured stresses being 310 lb. per sq. in., or 5% of the average measured stress. In the top chords and end post, the measured stress was in some cases greater and in some cases less, the average difference being 570 lb. per sq. in., or 8% of the average measured stress. The mean measured stress in these members was, however, only 145 lb. per sq. in., or 2% less than the mean calculated stress.

In explanation of the great discrepancy between the observed and calculated stresses in the bottom chord, it is to be noted:

1.—That the roller bearings were inoperative.

2.—That four lines of stringers were used, the outer line being about 3 ft. 2 in. from the ends of the floor-beams.

3.—That the floor-beams were provided with 15 in. by $\frac{5}{8}$-in. cover-plates extending about 10 in. beyond the outer line of stringers. The floor-beams were, therefore, unusually rigid laterally. Analysis of the conditions thus set up hardly seems worth while, particularly in view of the character of the end connections of the end floor-beams. Qualitatively, however, they explain reasonably the observed stresses existing in the bottom chords. Discrepancies elsewhere may be attributed partly to local conditions, partly to uncertainties regarding the distribution of wheel loads, and partly to errors of measurement.

In calculating the secondary stresses the following adjustments were made:

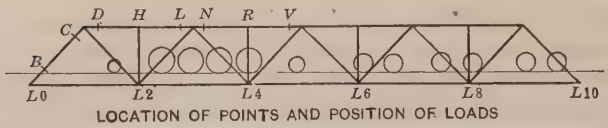
1.—Measured primary stresses were used in preference to those calculated where there was a material difference between them.

2.—Inasmuch as the observed deflection at the center of the span was only 94% of that calculated on the basis of the measured stresses, all primary stresses were reduced 6% for the purpose of computing angular distortions. This it was hoped would make a reasonable allowance for the effect of details and end connections in diminishing the axial distortion of the members.

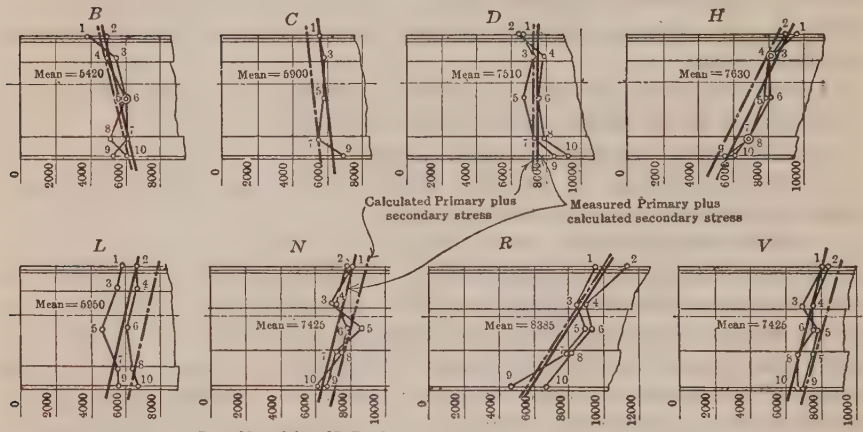
3.—Although the stresses in the verticals were not measured, the Howard gauge being unsuitable for measuring small stresses, an estimate of these stresses was made by considering the deflection produced in the top chords, and the values thus obtained, amounting to a few hundred pounds per square inch, were inserted.

4.—The end of the span rested on hardwood blocking without any hinge. An approximate analysis showed that, as might be expected, there was no close approach to fixity at this joint. In the absence at the time of any suitable means for measuring the rotation of the joint, it was impossible to calculate independently the value of the external moment. The speaker, therefore, "loaded the dice" to a certain extent by introducing the value of this moment derived from the measured secondary stresses in the adjacent members. The measured secondary stresses at this point are, therefore, in no way a check on the calculated values, except as regards the relative bending moments in the two members meeting there.

The calculated and observed stresses are shown in Figs. 98 and 99 on which the observed stresses at all gauge points are plotted on an elevation of



LOCATION OF POINTS AND POSITION OF LOADS



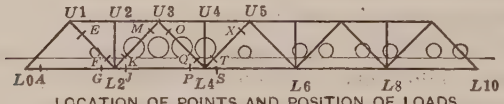
Material at *B, C, D, H, L*
1 Cover Pl. 21″ x ⅜″
2 Web Pls. 18″ x ⅜″
4 Angles 3½″ x 3½″ x ⅜″

Material at *N, R, V*
1 Cover Pl. 21″ x 1/16″
2 Web Pls., 18″ x 1/16″
4 Angles 6″ x 3½″ x ½″

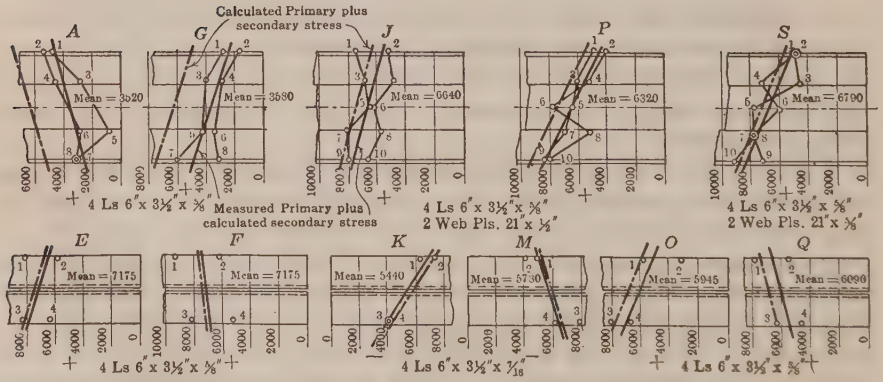
NOTES:
All stresses are given in lb. per sq. in.
Readings with odd numbers are on inside of truss.
Readings with even numbers are on outside of truss.

DISTRIBUTION OF LIVE LOAD STRESSES END POST AND TOP CHORD

FIG. 98.



LOCATION OF POINTS AND POSITION OF LOADS



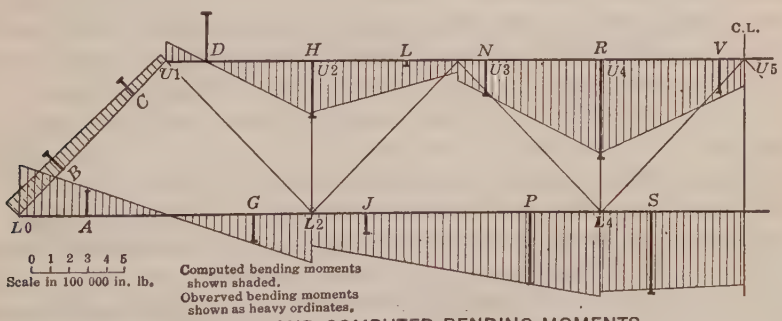
NOTES:
All stresses are given in lb. per sq. in.
Readings with odd numbers are on inside of truss.
Readings with even numbers are on outside of truss.
— for Compression Stresses
+ for Tension Stresses

DISTRIBUTION OF LIVE LOAD STRESSES BOTTOM CHORD AND DIAGONALS

FIG. 99.

the member. The full inclined lines drawn across the diagrams show the average measured primary stress plus the calculated secondary stress; the broken lines show the calculated primary plus the secondary stress. In Fig. 100, the calculated and observed bending moments for the chord members are shown. The "observed bending moments" were deduced solely from the measured stresses in fibers near the upper and lower edges of the members, all intermediate measurements being neglected for this purpose.

Figs. 98 and 99 show that in most cases the agreement between the calculated secondary stresses is very good. A notable exception is the point, *D*, (Fig. 98) near the end of the top chord, which, according to calculations based on the usual assumptions, would be close to a point of inflection. Actually, the effect of the adjacent gusset-plate is to produce a considerable bending moment at the section of measurement. In addition, the stress distribution across the face of the member, as shown in Fig. 98, indicates that the stresses in the extreme fibers (which were used for calculating the "measured" bending moment) were affected by local bending and do not follow the general trend. At the point, *L*, where there is also a relatively large discrepancy between the calculated and measured values, the measurements were probably affected by the adjacent splice, as suggested previously in connection with the discrepancy between the measured and calculated primary stresses at this point. It may be observed from Figs. 98 and 99 that there is considerable secondary stress due to lateral bending, particularly in the diagonals. In the top chords, contrary to expectation, the lateral bending is not great, at least under a full load on the span.



OBSERVED AND COMPUTED BENDING MOMENTS

FIG. 100.

The somewhat erratic distribution of stress indicated by these measurements need not be ascribed entirely or even mainly to errors of measurement. In the somewhat rough measurements made with ordinary strain gauges reading only to 0.0001 in. considerable irregularity is indeed inevitable. It is, however, the speaker's experience that even when using instruments of precision under laboratory conditions, where every precaution is taken to preclude the possibility of serious instrumental error, the stress distribution in built-up members is, in general, far from planar across the section, and often apparently erratic, although, on the whole, checking satisfactorily with the applied loads. Even in such a simple element as an apparently straight

plate in tension, this effect is very noticeable, while in built-up members kinks in the component parts—latticing, splices, tie-plates, diaphragms, and end connections—all have their effect. The increments of stress arising in this manner may often amount to from 10 to 20% of the axial stress, and, in compression members, at least, are possibly a greater source of weakness than secondary stresses of similar magnitude, since their largest values are not necessarily confined to the ends of the members.

In the truss discussed, the chords were relatively rigid as compared with the gusset-plates, and the latter had apparently little effect on the secondary stresses. The secondary stresses in an experimental truss with relatively slender members and rigid gusset-plates were measured by the speaker's colleague, Professor H. M. Lamb, in the testing laboratory of McGill University. Sensitive mirror extensometers were used, and the measurements were made with a high degree of precision. In this case the observed and calculated primary stresses agreed very closely. The observations also showed a linear variation of secondary stress from one end of a member to the other in perfect agreement with theory. The observed values of the secondary stresses were, in general, higher than the calculated values; but an analysis taking into account the influence of the gusset-plates gave a considerably closer agreement. These two trusses were probably near the opposite extremes as regards relative rigidity of members and gusset-plates, and the data obtained from them suggest that actual secondary stresses, where the elastic limit is not exceeded, are likely to lie between the values as ordinarily calculated and those calculated by some method which takes the rigidity of the gusset-plates into account. The speaker thinks it is incorrect to conclude that, as is sometimes stated, actual secondary stresses due to rigidity of joints are necessarily less than those calculated by the usual methods.

The proper provision to make for secondary stresses, after they have been determined, is by no means an easy matter to decide. There are probably few structures that, if subjected to the full loads for which they were designed, would not show local stresses approaching or even exceeding the elastic limit of the material. In fully loaded eye-bars, for example, it is well known that such is the case. In old and overloaded structures excessive local stress must be quite common; and yet these structures may function safely for long periods. The speaker suggests that a thorough strain-gauge survey of some of these old structures would give information of great value in guiding engineering judgment as to the proper provision to make for secondary stresses generally. Tests to destruction of specially designed structures have been advocated and would be highly interesting. In such tests, however, it is difficult, if not impossible, to ascertain the effects of repeated loading, while in old structures judiciously selected the information required would be, in a measure, at hand.

George E. Beggs,* M. Am. Soc. C. E.—It is possible to determine the secondary stresses in the member of a truss by a laboratory study of an

* Associate Prof., Civ. Eng., Princeton Univ., Princeton, N. J.

elastic model. Such a model should simulate the elastic action of the truss as nearly as possible. If the model is cut from sheet material, as celluloid or cardboard, the width of the model members should be proportional to the cube root of the moment of inertia of the actual cross-section of the truss members. The effect of gusset-plates may be closely represented.

If such a model is supported on a flat drawing board, with small steel balls between the model and the board, and with hinge and expansion bearings, as indicated in Fig. 101, the influence values for the shear and moment in any member, as 5-7, may be obtained by attaching a "deformeter gauge" to this member, by introducing in such member rotative and shear deformations and by measuring with a microscope the deflections of the load points of the structure. A full description of this method of mechanical solution is given in a discussion of the paper by A. C. Janni, M. Am. Soc. C. E., entitled "The Design of a Multiple-Arch System and Permissible Simplifications".*

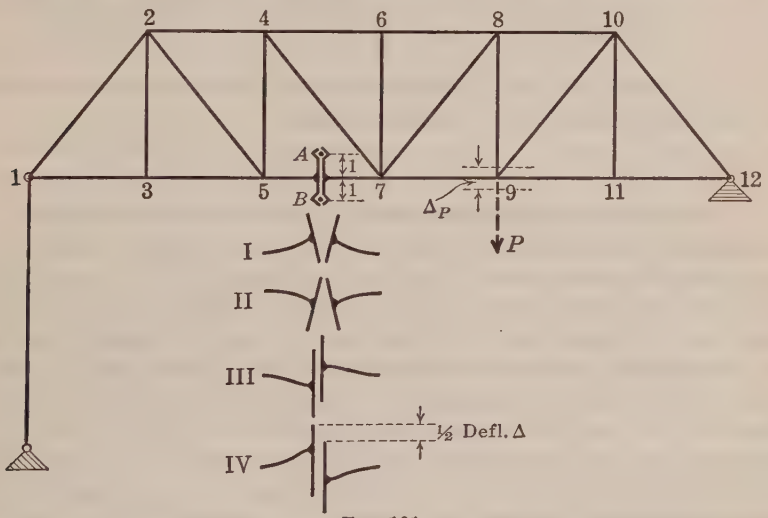


FIG. 101.

Suppose, for example, it is desired to find the bending moment at Section AB in the member, 5-7 (Fig. 101) caused by a load, P, at a panel point, 9. Introduce into the deformeter gauge plugs of unequal diameter so as to produce in the member, 5-7, at Section AB, the angular deformation indicated by Fig. 101 (I). With a measuring microscope set over a point, 9, and oriented in the direction of P, read the first deflected position of Point 9. Reserve the unequal plugs in the gauge so as to produce in the member, 5-7, the angular deformation indicated by Fig. 101 (II). Read with the microscope the new deflected position of 9. It follows from Maxwell's theorem that the bending moment in AB is given by:

$$M \text{ (foot-pounds)} = \frac{\Delta_P}{\Delta_G} \cdot P\,(m)$$

in which, Δ_P equals the component of the deflection of the load point, 9, in the assumed direction of P lb., m equals the scale of the model such that 1 in.

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 1208.

equals m ft., and Δ_G is the total angular deformation in radians introduced at Section AB.

To obtain the shear at the same section, introduce into the gauge suitable plugs to produce the shearing deformation indicated by Fig. 101 (III) and (IV). Read the corresponding component deflection, Δ_P, of the load point, P. If Δ_G now represents the shearing deformation at Section AB to the same scale as that used in measuring Δ_P, it again follows by Maxwell's theorem that the shear, S, at Section AB is given by:

$$S = \frac{\Delta_P}{\Delta_G} \cdot P$$

The direct, or normal, stress in the member is most easily computed by Mechanics. The bending moment at any other section of this member, 5-7, readily follows from the known moment and shear at Section AB. For example, at a section distance, x, to the right of AB, the bending moment is:

$$M_x = M + S\,x$$

It is apparent, therefore, that the resultant fiber stresses at any section may now be determined by the familiar formula:

$$S = \frac{P}{A} \pm \frac{M\,c}{I}$$

= unit stress due to direct stress ± unit stress due to bending.

In reply to the comments of Professor Cross, a considerable number of laboratory tests of model structures has led the speaker to the following conclusion: If the deflection of an assumed load point of an elastic model, made about one-thirtieth of the full size, is so small that it cannot be measured accurately with a precise filar micrometer microscope detecting 0.00005 in. of motion of the load point, when a correspondingly accurate deformation is introduced at the section of a member by a deformeter gauge in which precise moment and shear plugs are used, the conclusion to be drawn is that, in the corresponding full-sized structure, there is a very delicate, uncertain, and probably rather unstable relationship between the value of the applied load and the value of the moment and shear which it produces. Where the model fails to register a small effect accurately here, also, does the seemingly accurate mathematics fail, and when this happens, the stress sought for is of no great practical importance.

In regard to Professor Cross' criticism about proportioning the widths of members according to the moment of inertia, the speaker has found by a considerable number of experiments with models that this is a satisfactory approximation. The manner in which the bending effects are distributed through a rigidly connected framework is dependent for the most part on the moment-of-inertia relationship.

D. B. Steinman,* M. Am. Soc. C. E.—Bridge engineers are indebted to Mr. von Abo for his excellent treatise on secondary stresses—a valuable contribution to technical literature.

* Cons. Engr., New York, N. Y.

The comparison of different methods applied to the same problem is instructive. The author's conclusion that Winkler's method (modification of Manderla's method) is to be preferred over the others for practical application agrees with the previously published conclusion of the speaker.*

For the solution of the series of simultaneous equations involved in the secondary stress computations, Mr. von Abo expresses a preference for Gauss' method of "normal equations." The speaker has always preferred that of "successive substitutions", which he believes to be more attractive to the practicing engineer. It is a slide-rule method, continuously self-checking, and self-correcting; it dispenses with formal rules of procedure; it permits curtailment of calculations after one or two substitutions if the last degree of precision is not required; and it reduces the tediousness of secondary stress computations. Actual comparisons show that the method of "successive substitutions" is more expeditious in practice and proves an effective check against errors; on the other hand incorrect results have been obtained by the method of "normal equations" without the computers becoming cognizant of their errors.†

Several of the discussors have referred to the effect of the gussets at the ends of the truss members in producing a difference between the calculated and the measured secondary stresses. A logical method for including the influence of the gussets in the analysis is to abandon the conventional initial assumption of constant moment of inertia throughout each member and to substitute the assumption of varying moment of inertia. The value of I may properly be taken as constant over the intermediate length of each member between gussets and as increasing uniformly from these points to the theoretical ends of the member. This revised assumption affects only the numerical constants in the basic moment equation; otherwise, the computation of secondary stresses is unaltered. If this modification is used, the error due to the effect of the gusset is practically eliminated. This procedure was used in the calculations for the new railway arch bridge over the Niagara River for the Michigan Central Railroad.

It has been stated that actual secondary stresses may sometimes exceed the calculated values; this will not be the case if the secondary stresses are calculated correctly. The assumption of a varying I near the ends of members is a simple correction that has been overlooked by past writers on secondary stresses.

The standard methods of secondary stress analysis described in the paper are, at best, long-winded and tedious. The need of the bridge engineer is for practical short-cuts that will quickly give approximate values of the secondary stresses at any selected points of a structure. Published treatments of secondary stresses appear to be confined to the longer, academic methods which yield a degree of precision not warranted by the accuracy of the underlying assumptions nor by practical requirements. The speaker has developed simple graphic methods which quickly give approximate values of the secondary stresses in any member without requiring a lengthy analysis covering the

* "Stress Measurements on the Hell Gate Arch Bridge," *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1073.

† *Loc. cit.*, pp. 1130–1131.

entire structure. Actual comparison of these values with the results obtained by the standard methods shows an agreement within a small percentage. This is certainly sufficiently accurate for practical purposes, and the time required is a matter of minutes instead of hours.

THOMAS C. SHEDD,* M. AM. SOC. C. E. (by letter).—Although this paper admittedly presents nothing new in the theory of secondary stress calculations it brings together under one heading, and in one language, most of the methods already proposed for determining such stresses. As a result it will be of great value to one interested in a critical study of the various methods.

In his Introduction the author states that:

"In recent specifications the tendency is to recognize the importance of secondary stresses. Higher unit stresses are properly permitted for combinations of secondary and primary stresses. This undoubtedly leads to a better distribution of metal and to a more uniform strength throughout the structure."

It will be interesting to examine these statements somewhat further. Among the most recent specifications are those of the American Railway Engineering Association, issued in 1920 and revised under date of May, 1922. These specifications have had a remarkably wide acceptance in the short time since their publication. To the end of 1922, according to a report† made to the Association, 79 railways with a combined mileage of 170 320 miles "have either adopted the A. R. E. A. Specifications in complete form or incorporated their provisions in their own specifications, or have signified their intentions of following one or the other of these courses as early as practicable." Of these 79 roads, 75 with a combined mileage of 169 690 are listed by the Interstate Commerce Commission with the 262 541 miles of Class I railroads. Thus, these specifications within a period of three years after their issuance have been accepted as authoritative for the design of bridges on 65% of the first-class railway mileage in the United States. It is not unreasonable to assume that there have been further acceptances since the preparation of the report here mentioned.

In view of this widespread use the ruling of these specifications regarding secondary stresses may fairly be taken as representing current practice in the United States. The ruling is found under "Secondary Stress",‡ Paragraph 47, and reads as follows:

"Designing and detailing shall be done so as to avoid secondary stresses as far as possible. In ordinary trusses without sub-paneling, no account usually need be taken of the secondary stresses in any member whose width measured in the plane of the truss is less than one-tenth of its length. Where this ratio is exceeded, or where sub-paneling is used, secondary stresses due to deflection of the truss shall be computed. The unit stresses specified in Article 38 (16 000 lb. per sq. in. basic) may be increased one-third for a combination of the secondary stresses with the other stresses, but the section shall not be less than that required when secondary stresses are not considered."

* Asst. Prof., Structural Eng., Civ. Eng. Dept., Univ. of Illinois, Urbana, Ill.

† *Bulletin*, Am. Ry. Eng. Assoc., October, 1923, p. 4.

‡ "General Specifications for Steel Railway Bridges 1920," Am. Ry. Eng. Assoc., Second Edition.

A clause identical in meaning and nearly identical in wording is included in the Final Report on Specifications for Design and Construction of Steel Railway Bridge Superstructure,* presented by the Special Committee on Specifications for Bridge Design and Construction of the Society, and also in the Final Report on Specifications for Design and Construction of Steel Highway Bridge Superstructure† by the same Committee. As further evidence of the recent tendency to recognize secondary stresses, it may be mentioned that the Standard Specifications for Steel Highway Bridges adopted by the American Association of State Highway Officials and published by the U. S. Department of Agriculture,‡ contains a clause similar to that of the reports of the Special Committee of the Society except that the allowable increase in unit stress when secondary stresses are included with other stresses is 30% instead of 33⅓ per cent.

In contrast with the specifications mentioned—which require that secondary stresses be computed in certain cases—are the 1917 Specifications of the New York Central Lines which, without qualification, definitely require that secondary stresses be computed; Paragraph 48 reads:

"The secondary stresses in truss members due to rigidity of joints and the deflection of the truss shall be computed. The unit stresses in Paragraph 41 (18 000 lb. per sq. in. basic) may be increased one-third for a combination of the axial stresses and the above mentioned secondary stresses. Stresses due to eccentricity of transverse bending shall be computed. * * * For a combination of axial stresses and stresses due to eccentricity and transverse bending the unit stresses in Paragraph 41 shall not be exceeded."

Somewhat less definite than the others, but nevertheless showing the present tendency, are the 1916 Specifications of the Pennsylvania Railroad which, in Paragraph 24, require that: "All bending and other secondary stresses in addition to direct stress are to be provided for."

These clauses, cited from various specifications, not only clearly support the author's statement regarding the tendency of recent specifications to recognize the importance of secondary stresses, but seem to indicate that the majority of railroad bridges in the United States are now designed under specifications definitely requiring more or less consideration of such stresses.

The other statements by the author, namely, that "higher unit stresses are properly permitted for combinations of secondary and primary stresses", and that "this undoubtedly leads to a better distribution of metal and to a more uniform strength throughout the structure", should be considered together.

It does not seem that current specifications offer much support to this thesis. It is true that all the specifications cited—with one exception—permit an increase in the allowable unit stress of one-third (30% in one case), but—with the same exception—they include a qualifying clause which in substance requires that "the section shall not be less than that required when secondary stresses are not considered."§ The result is that unless the secondary

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 471.

† *Loc. cit.*, Vol. LXXXVII (1924), p. 1273.

‡ *Bulletin No. 1259*, U. S. Dept. of Agriculture.

§ "General Specifications for Steel Railway Bridges, 1920," Am. Ry. Eng. Assoc., Second Edition.

stress exceeds 33⅓%, there is no change whatever in the distribution of the metal and the strength of the structure is no more uniform than if secondary stresses had not been considered.

It is this feature of current specifications which seems objectionable to the writer. In the case where secondary stresses have been computed, the tendency will be to assume that the structure has a better distribution of metal and a more uniform strength than in the opposite case—an assumption which is not supported by the facts. It is true, of course, that some metal may have been added at the points of high secondary stress, but the fact remains that some members having little or no secondary stress will be working at the basic unit stress while others will have a unit stress varying from the basic value to a value one-third higher. The result is a structure which does not have uniform strength; some members are wastefully proportioned, or else other members are over-stressed, depending on whether or not the basic stress be considered too high. This is not necessarily a dangerous condition (if it is, most of the bridges now in operation are in a dangerous condition). The danger is that engineers will convince themselves by the multiplicity of their calculations and the complexity of their analyses that they have greatly improved their structures. They may know more about the actual stress conditions, which is very desirable, but it is certain that by complying with specifications as at present written they will have done little toward securing greater uniformity of strength.

In the 1916 Specifications of the Pennsylvania Railroad, the clause dealing with secondary stresses (Paragraph 24 quoted previously) is not definite, but the writer understands that where secondary stresses are included with primary stresses the allowable intensity of stress is not increased. This is the correct method of design. If secondary stresses are calculated and included with primary stresses in an effort to secure a better distribution of metal the same unit stress must be used in all members whatever the amount of secondary stress. Whether that unit stress should be the one now in common use or a higher one is entirely another question. The writer believes that when the problem of secondary stress has been thoroughly analyzed higher unit stresses may properly be permitted.

Emphasis should be placed on the matter of thorough analysis. If secondary stresses are to be added to primary stresses both must be simultaneous; this involves the question of whether the maximum primary stress combined with the simultaneous secondary stress will be greater than the sum of some other greater secondary stress combined with its simultaneous primary stress. The problem does not have an obvious answer, and its solution requires extensive calculations.

Assuming, however, that the necessary computations have been made, there are still many problems. All the methods thus far proposed for the determination of secondary stresses assume, for a given member, a constant moment of inertia from intersection to intersection. This is obviously not the case; for heavy trusses with large, thick, or multiple gussets and outside splice-plates at the connections, the error becomes considerable. Correcting for this condi-

tion by the method suggested in an interesting paper by O. H. Ammann,* M. Am. Soc. C. E., there still exists the uncertainty of the distribution of stress (secondary and primary) in the joints—about which practically nothing is known at present. Many engineers are of the opinion that this question is one which cannot be solved.

It is not apparent that the inclusion of secondary effects increases the uncertainty already existing in a discussion of joints with respect to primary effects. If the methods of joint analysis (crude though they are) are extended to include secondary effects, it would seem justifiable to assume that at least as much is known as though secondary effects had been disregarded. If that is correct then this uncertainty, serious though it is, should not be considered an impassable barrier to improvement in design by a consideration of secondary stresses.

At present little is known as to how the consideration of secondary stresses will affect the economics of design and construction. Should such stresses be reduced by placing restrictions on the ratio of width to length of members, and if so, should the length be defined as the distance between intersection points or the clear distance between joint plates? Is it proper to ignore the secondary stresses if the limiting ratio is met, and if not should they be eliminated, either partly or entirely, by adjustments of lengths of members and angles between them, or should additional metal be provided to keep the maximum stresses within the prescribed basic stress? Which method leads to the structure having the greatest strength and wearing qualities for the quantity of metal used, and which to the greatest ultimate economy?

The writer is a firm believer in thorough-going studies of secondary stresses, but holds that investigation and discussion of the problems just mentioned is the immediate need rather than the further elaboration of the mathematics of analysis.

Hardy Cross,† M. Am. Soc. C. E. (by letter).—The author has done a very careful piece of work and has succeeded in stating with admirable clearness the standard methods for the computation of secondary stress. It is well that so comprehensive a statement is now available in American literature. The author's opinions as to the relative advantages of the different methods are of value, moreover, on account of his familiarity with all of them, for the writer has found that current statements as to the advantages of any one method are too often supported by complete ignorance as to all others.

In such computations, most engineers will combine from the standard methods those features which appeal to their individual tastes. The writer prefers algebraic methods of analysis; for structures with a large number of panels they are almost necessary as checks on graphic solutions. In using Mohr's method, therefore, he computes the rotation angles for the bars of the statically determinate truss. These may be easily found from the angle changes as computed by Manderla. It is to be noted that the true rotations are not required, but only relative rotations with reference to any bar, such as the

* *Engineering News-Record*, October 23, 1924, p. 666.

† Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

center vertical, which is assumed to stand fast. If the changes of angle have been independently computed, this gives a complete check on the values of ψ.

The writer thinks the author does not give due prominence to the methods of successive approximation. For large trusses with many panels these seem decidedly the most practical of all, if not the only possible ones for office use. In this connection the author has omitted from his bibliography two interesting *Bulletins* of the University of Minnesota,* which deal with successive approximations by Mohr's method. A method apparently almost identical with this has resulted from Swiss studies.† Although this method is said to be new, in its essentials it does not appear to be so. It is self-checking, converges with a reasonable number of approximations, and has a decided advantage in that the work can be picked up and carried on in spite of frequent interruptions. A method which requires uninterrupted attention is difficult to use in an office. The convergence has been sometimes misrepresented, the variations in successive values of ϕ at any point being shown, whereas it is successive values of $(\phi - \psi)$, which are really significant.

Mao's analytical method is essentially the same as Müller-Breslau's, but complicated by the use of an unnecessary unwieldy and unfamiliar paraphernalia; the author is correct in discarding it. Müller-Breslau's method, however, is quite comparable, in simplicity and ease of application, with those of Manderla or Mohr. The modification of considering all clockwise moments as positive adds to its practicalness. The writer has used it for some years with modifications as explained herewith.

The author's Equations (47), (48), and (49) may be rewritten, as follows:

$$\rho_4 = 2(\rho_1 - \rho_2 + \rho_3) - \rho_1 \pm (-6E\Delta\alpha) \ldots\ldots\ldots\ldots (47a)$$

$$\rho_5 = 2(\rho_1 - \rho_2 + \rho_3) - \rho_2 \pm (-6E\Delta\alpha + 6E\Delta\beta) \ldots\ldots\ldots (48a)$$

$$\rho_6 = 2(\rho_1 - \rho_2 + \rho_3) - \rho_3 \pm (+6E\Delta\beta) \ldots\ldots\ldots\ldots (49a)$$

The alternative signs before the angle changes will be taken as positive in those triangles in which the succession, ρ_1, ρ_2, ρ_3, etc., is clockwise, but reversed in those triangles in which the succession is anti-clockwise.

Now this re-arrangement makes possible a systematic computation of successive values in terms of ρ_1 and ρ_2. Moreover,

$$\rho_1 - \rho_2 + \rho_3 = \rho_4 - \rho_5 + \rho_6 \ldots\ldots\ldots\ldots\ldots\ldots (119)$$

which gives a repeated check on the additions.

Again, in a symmetrical truss, all the ρ's on the left may be expressed in terms of ρ_1 and ρ_2 and all the ρ's on the right in terms of ρ_{42} and ρ_{41} (Fig. 52), the corresponding values of ρ at the other end of the truss. In corresponding ρ's, all but the constants will be identical on the two sides of the truss. The values of ρ_1, ρ_2, ρ_{42}, and ρ_{41}, will then be determined by four equations at the center, still using the notation of Fig. 52 namely:

$$\rho_{21} \text{ left} = \rho_{21} \text{ right}$$

$$\rho_{22} \text{ left} = \rho_{22} \text{ right}$$

* *Bulletins 2 and 4*, Engineering Studies, Univ. of Minnesota.

† "Secondary Stresses in Steel Riveted Bridges," O. H. Ammann, M. Am. Soc. C. E., *Engineering News-Record*, October 23, 1924, p. 666.

$$M_{20} + M_{21} + M_{23} = 0$$
$$M_{17} + M_{16} + M_{22} + M_{26} + M_{27} = 0$$

The first two will be found to be equations in $(\rho_1 - \rho_{42})$ and $(\rho_2 - \rho_{41})$ and the last two, equations in $(\rho_1 + \rho_{42})$ and $(\rho_2 + \rho_{41})$, thus furnishing two pairs of simultaneous equations which are easily solved.

When the values of the end ρ's have been determined, they may be substituted in the expressions already found, as the author himself has done (page 81). It is, however, quicker to use again Equations (47*a*), (48*a*), and (49*a*), and carry these values through from each end to an absolute check by the four equations indicated at the center.

The writer finds convenient a form as indicated in Fig. 102 and Table 34. The values are taken in many cases from the author's computations; the checks are not always absolute, although the discrepancies are small.

Fig. 102 shows the position of the ρ's and thus the order in which they are taken up in Table 34. It will be noted that they are slightly different from the author's Fig. 52. ρ_{13} is repeated after ρ_{14}, as the computation here "backtracks" to get to ρ_{15}, and so around the triangle. The signs in Fig. 102 indicate whether the signs of the angle changes are to be taken as computed or are to be reversed.

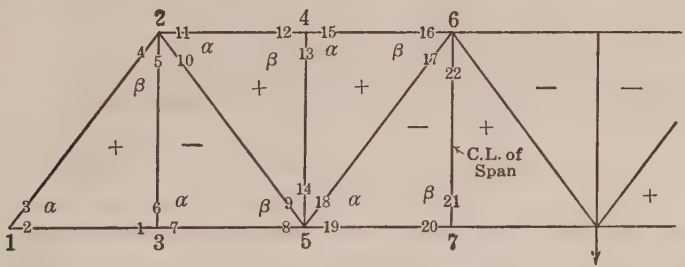


FIG. 102.

The procedure here indicated presents the following advantages:

1.—It systematizes both the writing and the solving of the equations in Table 8.

2.—It adapts the work to the use of a calculating machine.

3.—It gives as the work proceeds an absolute check on all equations.

4.—It reduces the number of equations, and lessens the labor of their solution more than one-third.

5.—It simplifies the labor of evaluation shown in Table 9, if the values found are "carried through" by the routine indicated. This also presents an absolute check on the work.

In sub-divided panels and similar cases it is necessary to introduce temporarily a third unknown which is evaluated by balanced moments in that panel. Four-sided figures such as occur in sub-divided trusses and in *K*-trusses may be solved by introducing a false member or by writing special equations. Müller-Breslau's method, however, is not so convenient in this very important group of cases as that of Mohr or Manderla.

TABLE 34.—Analysis of Secondary Stresses in a 150-Ft. Truss.

	K	ρ	Coefficients of: ρ_1.	Coefficients of: ρ_2.	Constant: Left.	Constant: Right.	$6E\Delta$ Angle: Left.	$6E\Delta$ Angle: Right.	Final Values: Left.	Final Values: Right.
	2.501	1	+ 1	.....	.....	.....	.....	.....	— 6.9907	— 15.3827
		2	.....	+ 1	.....	.....	.....	.....	— 45.5976	+101.9963
		* 3	.....	+ 0.4577	.....	.....	.....	.....	+ 20.8700	— 47.4233
	5.464	† Σ	+ 1	— 1.4577	.....	.....	.....	.....	+ 59.4769	—164.8023
		† 2 Σ	+ 2	— 2.9154	.....	.....	.....	.....	+118.9533	—329.6046
		4	+ 1	— 2.9154	— 96.66	+ 193.32	— 96.66	+ 193.32	+ 29.2845	—123.8582
	0.303	5	+ 2	— 3.9154	+ 39.90	— 79.80	+ 39.90	— 79.80	+204.4514	—511.4008
‡ Check		6	+ 2	— 2.4577	+ 136.56	— 273.12	+136.56	— 273.12		
		* 7	— 1.2424	+ 0.2979	— 16.55	+ 33.10	.....	.....		
	2.501	† Σ	— 1.2424	— 1.1598	— 113.21	+ 226.42	.....	.....		
		† 2 Σ	— 2.4848	— 2.3196	— 226.42	+ 452.84	.....	.....		
		8	— 4.4848	+ 1.5958	— 71.98	+ 143.84	+194.34	— 388.80		
	1.685	9	— 4.4848	+ 0.1381	— 6.22	+ 12.26	+356.76	— 713.70		
‡ Check		10	— 1.2424	— 2.6175	— 47.45	+ 94.84	+162.42	— 324.90		
		*11	— 0.5001	+ 2.7073	+ 74.93	— 149.91	.....	.....		
	7.951	† Σ	— 3.7425	+ 5.4629	+ 116.16	— 232.49	.....	.....		
		† 2 Σ	— 7.4850	+10.9258	+ 232.32	— 464.98	.....	.....		
		12	— 3.0002	+10.7877	+ 400.96	— 802.14	+162.42	— 324.90		
	0.190	13	— 6.2426	+13.5433	+ 800.27	—1601.06	+520.50	—1041.24		
‡ Check		14	— 6.9849	+ 8.2185	+ 515.47	—1031.41	+358.08	— 716.34		
		13	— 6.2426	+13.5433	+ 800.27	—1601.06	.....	.....		
		*15	+ 3.1494	—10.9841	— 420.09	+ 840.40	.....	.....		
	7.951	† Σ	+ 2.4071	—16.3089	— 704.89	+1410.05	.....	.....		
		† 2 Σ	+ 4.8142	—32.6178	—1409.78	+2820.10	.....	.....		
		16	+11.7991	—40.8363	—1771.59	+4275.35	+153.66	+ 423.84		
	1.800	17	+11.0568	—46.1611	—1908.73	+4992.66	+301.32	+ 571.50		
‡ Check		18	+ 1.6488	—21.6337	— 842.03	+2127.36	+147.66	+ 147.66		
		*19	+ 4.7670	+ 9.2408	+ 448.22	—1118.40	.....	.....		
	3.588	† Σ	+14.1750	—15.2866	— 618.48	+1746.90	.....	.....		
		† 2 Σ	+28.3500	—30.5732	—1236.96	+3493.80	.....	.....		
		20	+17.2932	+15.5879	+ 819.43	—1351.20	+147.66	+ 147.66		
	0.303	21	+26.7012	— 8.9395	+ 112.67	+1641.60	+507.60	+ 275.16		
‡ Check		22	+23.5830	—39.8140	—1325.24	+4739.70	+359.94	+ 127.56		

Continued to a check at the center line on all these values.

$$\rho_{21} = \rho_{21}' + 26.7012\,(\rho_1 - \rho_1') - 8.9395\,(\rho_2 - \rho_2') = + 1528.93$$
$$\rho_{22} = \rho_{22}' + 23.5830\,(\rho_1 - \rho_1') - 39.8140\,(\rho_2 - \rho_2') = + 6064.94$$

$$\rho_1 - \rho_1' = + 8.3920;\quad \rho_2 - \rho_2' = - 147.5939$$

$$M_{20} + \frac{1}{2} M_{21} + M_{20}' + \frac{1}{2} M_{21}' = 0 \qquad + 18.4012\,(\rho_1 + \rho_1') + 15.2104\,(\rho_2 + \rho_2') = + 457.27$$

$$M_{16} + M_{17} + \frac{1}{2} M_{22} + M_{16}' + M_{17}' + \frac{1}{2} M_{22}' = 0 \quad - 14.7509\,(\rho_1 + \rho_1') - 52.0536\,(\rho_2 + \rho_2') = - 3266.82$$

$$(\rho_1 + \rho_1') = - 22.3734\,\rho_2 + \rho_2 \\ = + 56.3987$$

$$\rho_1 - \rho_1' = + 8.3920 \qquad \rho_2 - \rho_2' = - 147.5939$$
$$\rho_1 + \rho_1' = - 22.3734 \qquad \rho_2 + \rho_2' = + 56.3987$$

$$\rho_1 = - 6.9907;\ \rho_1' = - 15.3827;\ \rho_2 = - 45.5976;\ \rho_2' = + 101.9963$$

*These values of ρ are obtained from equations of balanced moments at the joints, these being the first five and last five equations indicated by the author (see p. 79).

† These values are the sum of the first and last less the middle one of the three values just preceding.

‡ The word, "check", indicates that a check is here applied to the summation, as indicated in Equation (119). This check is absolute on the continued summation, but not on the solution by moments for the values of ρ. The latter may be checked by assuming any convenient values, such as 10 for ρ_1 and ρ_2, and then testing the equation, $\Sigma M = 0$.

Further simplification is also possible by rounding off the values of K and of the angle changes. In this particular case, the K-values for Members 2-4, 4-6, 6-8, 8-10, may well be taken as unity and the others in proportion, these ratios being rounded off to two places of decimals. Evidently, it is the K-ratios and not their absolute values which are significant in the equations. It is also evident that these ratios and the angular changes need not be computed with great precision provided exactly the same values are always used for

the same terms. If this were not the case a slight difference from the computed value of the area or moment of inertia of a bar would upset the results to such an extent as to make all secondary stress computations futile.

This indicates another field of investigation. Neither the length of member to be used in calculating the distortions of the truss, nor the length involved in the secondary distortion, is known with accuracy. It remains to be determined to what extent these uncertainties affect the values of the secondary stresses; it may be that only a roughly approximate solution is justifiable, in which case the method of successive approximations has even greater advantages. While, therefore, the mathematical theory of secondary stresses is quite complete and little more is to be expected from its elaboration, there is still room for systematizing the computations and especially for simplifying them by approximations and for determining what degree of approximation is desirable or permissible. The writer believes that, in general, any relative advantages of the methods of Manderla, Mohr, and Müller-Breslau, when these methods are carried out in full, are largely illusory, but it does appear that Müller-Breslau's method is very readily systematized while the first two—which are practically identical—are more readily adapted to successive approximation.

Whatever is done, the computation of secondary stresses will remain a strenuous pursuit and at the end of the chase one faces the problem of what to do with the quarry. The cases will probably not be numerous in which forcible measures of elimination are justified. In other cases, evaluation gives some idea of possible points of weakness. High secondary stresses may then be reduced by revising the make-up of the members or by such devices as adjusting hangers and idle struts. The writer, however, believes that it is not practicable at present to effect a better distribution of metal by designing for the total stress (using generally increased basic stresses) rather than for the primary stress. The following uncertainties bear on the question:

(*a*) Secondary stresses in tension members are probably less serious than in compression members;

(*b*) Secondary stresses in compression members are less serious when there is double flexure (with a point of contraflexure in the member) than otherwise and they may even be beneficial in such cases in that they retard buckling;

(*c*) Secondary stresses are often less dangerous than primary stresses because of their localization, this being especially true when secondary moments in two planes are combined;

(*d*) Present knowledge of impact effects on secondary stresses is much less than in regard to primary impact, inadequate as are the data on the latter; perhaps secondary stresses tend to dampen vibrations and an unformulated knowledge of this supports the structural engineer's preference for "stiff" structures in spite of their secondary stresses.

These statements, however, should not be taken to indicate any doubt as to the growing importance of a thorough understanding by bridge engineers of the nature, approximate theoretical value, and general theory of secondary stresses in bridge trusses; but the same importance attaches to the study of

secondary stresses due to the deformation in the frames of steel buildings and in other cases. There is, at present, a decided swing toward more elaborate analyses in both steel and concrete structures; the requirement in the new specifications for concrete of the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete* that bending stresses shall be computed in the columns of concrete buildings, even in the face of uncertainties more pronounced than those outlined herein, is an instance in point. All this is very well as long as it is understood that such analyses merely indicate danger points. Elaborate analyses alone, however, will not justify a general increase in fiber stress; the relation of secondary stresses to the factor of safety is a much more intricate problem than their mathematical formularization or evaluation.

The writer is surprised that Professor Beggs is able, by the use of his models, to easily and accurately determine secondary stresses. As secondary stress depends on the area as well as the stiffness of the member, it would seem that it could not, in general, be found by use of a uni-planar model. Moreover, the direct stresses produced in a model loaded as he suggests must be extremely small, and, therefore, the truss deflections, which are a result of the direct stresses alone, must be difficult to measure at all.

If, by rotation of the cut ends of a member, the ends of an outer fiber are displaced one unit relative to each other, the deflections will be influence ordinates for the fiber stress. If the rotation is around the neutral point of the section, the influence line is for the stress due to the moment only; if about the kern point, it is for the total stress. Since in this instance the direct stresses are relatively large, the centers of rotation must be located with surprising precision; and, for similar reasons, the shear must have been applied along a line very precisely normal to the axis.

W. M. Wilson,† M. Am. Soc. C. E. (by letter).—The Engineering Profession is indebted to the author for presenting, in a form for comparison, the various methods that have been developed for computing the secondary stresses in the members of a truss due to axial deformations. The selection of a method usually embodies an accidental element of considerable importance for, with most engineers, the first method that has been mastered and systematized seems better than a new one and is likely to be retained, even if the new method may be superior.

The writer prefers the method that the author has designated as Mohr's semi-graphical method. This is certainly the easiest to understand and, as indicated by the author, is as short as any. Moreover, each operation can be checked as it is performed, thus eliminating costly errors that might otherwise be undetected until revealed by equating the sum of the moments at a panel point to zero. Furthermore, the extensions can be made with sufficient accuracy by a 20-in. slide-rule. Nor will the writer concede that the graphical work (Williot diagram) introduces errors large enough to be objectionable; he has used the method extensively and has supervised its use on work where a high degree of accuracy was necessary, and has not found it wanting.

* *Proceedings*, Am. Soc. C. E., October, 1924, Papers and Discussions, pp. 1204–1206.
† Research Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

In introducing his graduate class in advanced stress analysis to the subject of secondary stresses, for the first problem the writer required the nine students to determine the secondary stresses at both ends of all members of a structure consisting of a single triangle, *ABC*. The moments computed for the ends of the members are listed in Table 35. It is realized that the problem, as problems in secondary stresses go, is simple. Moreover, the results obtained by the various students do not agree very closely in some cases; but, considering that, in each case, the student was working his first problem in secondary stresses and using his own Williot diagram, the discrepancies are not large. Moreover, the students who obtained answers differing most from the average also failed to obtain summations of moments at the panel points equal to zero. Inasmuch as a failure to obtain equilibrium is due entirely to inaccuracies in the arithmetical work rather than in the graphical part of the solution, the latter cannot be charged with all the discrepancies in Table 35.

Recently, the writer was engaged by a firm of consulting engineers to compute secondary stresses. Knowing that he used the semi-graphical method, the firm, unknown to him, submitted the same structure for analysis to another engineer who uses an algebraic method; the firm redeemed itself for whatever breach of ethics it may have committed by reporting to the writer that the secondary stresses, as computed by the algebraic method, agreed almost exactly with those obtained by the writer using the Mohr semi-graphical method.

TABLE 35.—Secondary Moments in Triangular Frame.

Moment.	Student No.								
	1.	2.	3.	4.	5.	6.	7.	8.	9.
M_{AB}	−285 000	−290 000	−288 000	−288 000	−284 000	−300 000	−286 000	−286 200	−291 600
M_{AC}	+293 400	+291 000	+291 000	+291 600	+302 400	+284 000	+283 200	+288 300	+290 700
M_{BA}	−642 000	−651 000	−648 000	−648 000	−649 200	−654 000	−656 000	+645 800	−648 600
M_{BC}	+655 600	+651 000	+653 000	+648 000	+652 500	+655 200	+652 000	−646 200	+650 400
M_{CB}	+169 200	+175 000	+176 500	+172 800	+174 000	+190 800	+168 000	+174 400	+175 000
M_{CB}	−169 000	−175 000	−174 500	−172 800	−178 800	−174 600	−171 000	−174 600	−174 600

As another example cited to establish the accuracy of Mohr's semi-graphical method, the writer had the secondary stresses in a five-panel Pratt truss computed by two methods, as follows: Method 1, by determining the secondary stress at a certain point due to the primary stress in some one member, all other members being unstressed; likewise determining the secondary stress at the same point due to the primary stress in each of the other members considered separately; and Method 2, by determining the secondary stress at the specified point when all members are stressed simultaneously. Barring inaccuracies in the work, the sum of the separate secondary stresses obtained by Method 1 will equal the secondary stresses obtained by Method 2.

The secondary stresses were computed for thirty-six points, one at each end of all the eighteen members, with the following results: The difference between

the secondary stresses obtained by the two methods in no case exceeds 3% of the total; at two points, the difference exceeded 2% and was less than 3% of the total; at four points, the difference exceeded 1% and was less than 2% of the total; at thirty points, the difference was less than 1 per cent. The differences exceeding 2% occurred at points where the secondary stress was very small, being less than 1½% of the maximum secondary stress; the differences exceeding 1% occurred at points where the secondary stress is less than 20% of the maximum secondary stress. The degree of accuracy attained is thus much greater than is necessary. The writer's experience, not only in the cases cited, but in many others as well, has convinced him that the method is accurate enough for all practical purposes. The Williot diagrams, however, must be carefully drawn.

The cross-frame treated by the author* is a special case of the rectangular frame and may be solved by the equations that have been developed for rectangular frames in general.†

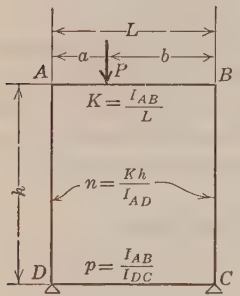


Fig. 103.

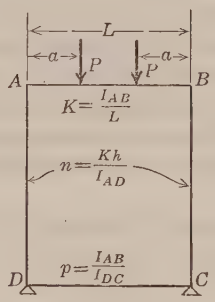


Fig. 104.

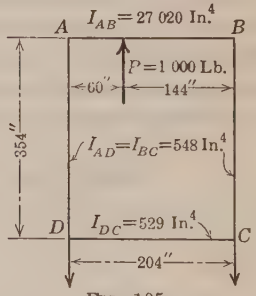


Fig. 105.

For the frame represented by Fig. 103, symmetrical about a vertical center line, but loaded unsymmetrically:‡

$$M_{AB} = -\frac{1}{2}\left\{ C_{BA}\left[\frac{2n+3p}{\alpha} - \frac{1}{\beta}\right] + C_{AB}\left[\frac{2n+3p}{\alpha} + \frac{1}{\beta}\right]\right\}$$

$$M_{BC} = -\frac{1}{2}\left\{ C_{BA}\left[\frac{2n+3p}{\alpha} + \frac{1}{\beta}\right] + C_{AB}\left[\frac{2n+3p}{\alpha} - \frac{1}{\beta}\right]\right\}$$

$$M_{CD} = \frac{1}{2}\left\{ C_{BA}\left[\frac{n}{\alpha} - \frac{1}{\beta}\right] + C_{AB}\left[\frac{n}{\alpha} + \frac{1}{\beta}\right]\right\}$$

$$M_{DA} = \frac{1}{2}\left\{ C_{BA}\left[\frac{n}{\alpha} + \frac{1}{\beta}\right] + C_{AB}\left[\frac{n}{\alpha} - \frac{1}{\beta}\right]\right\}$$

in which,

$$\alpha = n^2 + 2pn + 2n + 3p$$

$$\beta = 6n + p + 1$$

* See pp. 127–143.

† See *Bulletin No. 108*, Eng. Experiment Station, Univ. of Illinois, "Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by W. M. Wilson, F. E. Richart, and Camillo Weiss, p. 105.

‡ *Loc. cit.*, p. 107.

n = ratio of K (moment of inertia divided by length) of top member to K of left-hand column, for a rectangular frame.

p = ratio of K of top member to K of bottom member for a rectangular frame.

M_{AB} = moment at End A of Member A-B.

M_{BC} = moment at End B of Member B-C.

M_{CD} = moment at End C of Member C-D.

M_{DA} = moment at End D of Member D-A.

C_{BA} = moment at End B of Member A-B, due to any system of loads, when A and B are on the same level and when the tangents to the elastic curve at A and B are horizontal.*

C_{AB} = moment at End A of Member A-B.

For a frame symmetrical about a vertical center line and supporting loads symmetrical about the same center line, the condition usually found in practice, the moments are given by the equations:

$$M_{AB} = M_{BC} = -C_{AB}\frac{2n + 3p}{\alpha}$$

$$M_{CD} = M_{DA} = C_{AB}\frac{n}{\alpha}$$

In these equations, C_{AB} and C_{BA} are functions of the loads. For the single load shown in Fig. 103, $C_{AB} = \frac{P a b^2}{L^2}$† and $C_{BA} = \frac{P a^2 b}{L^2}$. For the two symmetrically spaced equal loads of Fig. 104,

$$C_{AB} = C_{BA} = \frac{P a}{L}(L - a).$$

For the problem solved by the author, illustrated by Fig. 105:

$I_{DC} = 529$ in.^{4} $\quad L = 204$ in.

$I_{AB} = 27\ 020$ in.^{4} $\quad h = 354$ in.

$I_{BC} = I_{AD} = 548$ in.^{4} $\quad a = 60$ in.

$P = -1\ 000$ lb. $\quad b = 144$ in.

$$K = \frac{27\ 020}{204} = 132.451 \text{ in.}^3$$

$$n = 132.451 \times \frac{354}{548} = 85.561$$

$$p = \frac{27\ 020}{529} = 51.078$$

$$C_{AB} = \frac{-1\ 000 \times 60 \times 144 \times 144}{204 \times 204} = -29\ 896 \text{ in-lb.}$$

$$C_{BA} = \frac{-1\ 000 \times 60 \times 60 \times 144}{204 \times 204} = -12\ 456 \text{ in-lb.}$$

$$\alpha = 85.561^2 + 2 \times 51.078 \times 85.561 + 2 \times 85.561 + 3 \times 51.078$$
$$= 16\ 380$$

* Values of C for various loads are given in *Bulletin No. 108*, Eng. Experiment Station, Univ. of Illinois, pp. 24, 26.

† See *Bulletin No. 108*, Eng. Experiment Station, Univ. of Illinois, pp. 24, 26.

$$\beta = 6 \times 85.561 + 51.078 + 1 = 565.444$$

$$\frac{2n + 3p}{\alpha} = \frac{2 \times 85.561 + 3 \times 51.078}{16\,380} = 0.01980$$

$$\frac{1}{\beta} = \frac{1}{565.444} = 0.001769$$

$$\frac{n}{\alpha} = \frac{85.561}{16\,380} = 0.005223$$

$$M_{AB} = -\frac{1}{2}\Big[-12\,456\,(0.019802 - 0.001769) - 29\,896\,(0.019802 + 0.001769)\Big] = 434.6 \text{ in-lb.}$$

$$M_{BC} = -\frac{1}{2}\Big[-12\,456\,(0.019802 + 0.001769) - 29\,896\,(0.019802 - 0.001769)\Big] = 403.8 \text{ in-lb.}$$

$$M_{CD} = \frac{1}{2}\Big[-12\,456\,(0.005223 - 0.001769) - 29\,896\,(0.005223 + 0.001769)\Big] = -126.0 \text{ in-lb.}$$

$$M_{DA} = \frac{1}{2}\Big[-12\,456\,(0.005223 + 0.001769) - 29\,896\,(0.005223 - 0.001769)\Big] = -95.2 \text{ in-lb.}$$

The writer made the extensions with a 20-in. slide-rule and the results are in almost exact agreement with those obtained by the author using Mohr's method.

For a cross-frame having two equal symmetrically spaced loads (the usual condition), the computations are very short and simple. Using the same frame as before, but considering two loads, each equal to 1 000 lb., the computations are as follows:

$$M_{AB} = M_{BC} = -C_{AB}\frac{2n + 3p}{\alpha}$$

$$M_{CD} = M_{DA} = C_{AB}\frac{n}{\alpha}$$

in which,

$$C_{AB} = \frac{Pa}{L}(L - a) = \frac{-1\,000 \times 60}{204} \times 144 = -42\,350 \text{ in-lb.}$$

As previously determined:

$$\frac{2n + 3p}{\alpha} = 0.01980$$

$$\frac{n}{\alpha} = 0.005223$$

Substituting these quantities in the equations for the moments, gives:

$$M_{AB} = M_{BC} = 42\,350 \times 0.01980 = 838.3 \text{ in-lb.}$$

$$M_{CD} = M_{DA} = -42\,350 \times 0.005223 = -221.2 \text{ in-lb.}$$

These computations are so short and simple and the secondary stresses in the cross-frame are so important that the omission of the calculations for secondary stresses from the design seems hard to justify even in structures of moderate size.

The equations used by the writer and by the author are equivalent; the writer believes, however, that his form* is the more convenient for the designer.

THOMSON E. MAO,† ESQ. (by letter).—The writer desires to pay his highest respects to the author for the exhaustive treatment of the subject and the splendid presentation of the writer's graphic and analytic methods for the solution of secondary stresses. As far as the writer knows the paper is the most complete study of secondary stresses in the English language, and yet is not too mathematical for perusal by busy engineers in practice. The author is to be congratulated for this painstaking work, for unless a person has studied the subject deeply and actually solved some examples he cannot really appreciate the amount of time and patience required to prepare such a comprehensive paper. This work appeals particularly to the writer, who was, for some time, completely "immersed" in the subject of secondary stresses and has longed to find a companion who would have exactly the same taste and interest in such an unpopular and "lonely" subject. From the fact that secondary stresses are bound to be a factor in design, the author also deserves the appreciation of the Engineering Profession at large, as he has again proved at great expense of time and labor that, after all, this important branch of stress computation could not well be taken care of by the "factor of ignorance."

This discussion will largely deal with the two methods introduced by the writer in 1919 and described in the paper. Quoting from the preface to the writer's printed thesis:

"The methods for computing secondary stresses have been greatly improved in recent years. Two objections, however, still remain: First, the amount of time involved is often excessive, and, second, the lack of a systematic checking device by which the correctness of the various steps of procedure may be ensured. While there are numerous other defects, these two alone are generally sufficient to reduce their practical utility. Ever since the beginning of 1917, when the writer undertook the analysis of secondary stresses in a two-hinged arch, the results of which have been published in the *Transactions* of the American Society of Civil Engineers, Vol. 82, p. 1104, it always occurred to him that there must be some method which is not only shorter and less cumbersome than the current ones but which also admits of a check. For two and a half years he had worked on the subject almost incessantly, striving to find some new method that will accomplish both the above mentioned results. At last, much to his satisfaction, the graphic method of deformation contour was obtained, which not only takes less time and checks itself, but also gives remarkably accurate results. Along with this method, almost contemporaneously, the analytic method was evolved."

* Presented in *Bulletin No. 108*, Eng. Experiment Station, Univ. of Illinois.

† Pres., Conservancy Eng. Coll., Nanking, China.

It may be helpful to enumerate here the important advantages of these methods as compared with others.

Features common to both methods are:

(1) No simultaneous equations involving more than two unknowns are required.

(2) Both methods deal with a quantity which is of the same dimensional degree as the secondary stress itself. This gives more accurate results than when using a quantity involving large multiples of f.

(3) They take much less time than the ordinary methods.

(4) There are a number of checks available, especially for the graphic method.

(5) They can be easily applied to influence lines of secondary stresses.

(6) The complete information is contained in only one sheet of paper, including the rough computations, calculations, data, and formula.

Special features of the graphic method are:

(*a*) It is strictly graphical. From beginning to end no computations are required.

(*b*) It gives a complete representation of secondary stresses in every member of the truss.

(*c*) It not only shows secondary stresses at the two ends of a member, but also the variation of the stress along its whole length.

(*d*) It gives the point of inflection in a member and also the form in bending.

(*e*) With the aid of the force diagram for secondary stress lines it shows how the secondary stresses are affected by changing the section moduli of the members.

(*f*) It is more accurate than the ordinary analytic methods if the latter be computed by the use of a slide-rule. This is due to the fact that in analytic expressions the secondary stress is often found from the difference of two large numbers which must be very accurate, since the difference is small.

(*g*) The errors made in locating the base lines and vertex lines are not cumulative, as the effect of one line on the other is very small.

(*h*) It has a sufficient number of checks at every step of the procedure.

Characteristics of the analytic method are:

(1) In bridge offices where a large number of standardized trusses are to be analyzed for secondary stresses, the truss diagram, together with the small tables, could be blue-printed.

(2) This method is semi-graphical in that it shows the stresses at the proper places and their effects on each other.

(3) The entire process is mechanical. Every figure has a definite space; every procedure has a definite order.

(4) No figure is to be recorded twice and no necessary figures are omitted.

(5) All rough computations are shown on the diagram, thus rendering possible a check at any future time.

(6) The figures on the diagram are arranged so that all the additions and subtractions do not involve more than two lines of figures and can be accomplished without the use of extra paper, as when three columns of figures are to be added, etc. All the other computations can be made on a slide-rule.
(7) The method is capable of affording any degree of accuracy which may be desired.
(8) Practically all back references are eliminated.
(9) It has a unique check of secondary stresses as soon as they are found.

As explained in the paper both of the writer's graphic and analytic methods depend on two fundamental equations. The derivation of the deformation equation, as given in the writer's thesis, is criticized by the author as being unnecessarily complicated, but the writer's aim was to derive it from Mohr's work equation with the hope that it might convey more physical meaning than a pure mathematical analysis. It appears complicated because it starts from the very beginning of the theory, but the principle involved is believed to be very simple, as the author agreed.

The author also complains about the descriptions of the methods as being inevitably too long. This is due to the fact that many novel processes and features are introduced. Should some of the processes be already known to the engineers, there would not be so much explanation necessary. It can be easily seen that in Mohr's method, if one is not familiar with the Williot diagram or the Gauss solution, much more description would have been necessary to complete the paper. The writer has aimed in the methods to systematize the procedure as much as possible, and this necessitates a detailed and minute description not only of the general principles involved, but also of the art of manipulation. This is deemed necessary to promote efficiency in the work and thus reduce the time and labor required. In his book on "Shop Management", Mr. Frederick W. Taylor gives a minute description of the most simple work of handling pig iron which, to most people, would seem to need no explanation. As it is acknowledged that secondary stresses require an enormous amount of time, any seemingly unnecessary yet important explanation would not be out of place in presenting a method that is considered perfectly new.

With reference to the writer's graphic method it is indeed gratifying to know that the author found it to give results that are "surprisingly accurate". This is in agreement with what the writer himself proved when testing the method and later with the results obtained by his students at the Tangshan Engineering College. The author, however, expressed his unwillingness to recommend the method on the basis that all graphic methods are undesirable; his objections therefore will virtually exclude every graphic solution from practical use. First, he seemed to dislike the use of a large-sized drawing paper on the ground that it might become unwieldy. It is true that to solve accurately a long-span truss under unsymmetrical loading a sheet of unusually large size is indispensable; but this is not a feature peculiar to the writer's

method alone—all graphic methods, without exception, are subject to this disadvantage. When a large-sized drawing paper is compared with ten or more pages of tabulated computations as used in other methods (the author used seven tables in the Manderla-Gauss method, requiring fully ten sheets of 8 by 12-in. paper), it is only a question of individual taste as to which is more adaptable to use. Personally, the writer much prefers the drawing to the tabulated sheets for the evident reason that it conveys a complete mental picture of the problem and thus enables the computer to see the effect of one stress on the other. This is important from the standpoint of design, as it will help to change the proportioning of the members so as to arrive at a more balanced design. When the designer is faced with a large mass of numbers, whether tabulated or not, he is really solving a mathematical problem and is at a loss to realize the significance of the result obtained in every step of the procedure. Inasmuch as secondary stresses are important on account of their practical bearing on the design, rather than of their mere mathematical interest, every method proposed for their solution should take this factor of illustration into account.

The author further objects to the graphic method because of the difficulty of adapting it to trusses with subdivided panels. This is true for the method presented in the author's form, but the writer believes that it can be modified to accomplish the purpose, as it is certainly susceptible to improvements.

It is rather surprising that the author complains of the labor required to arrive at the "base" and "vertex" lines as "excessive". He did not give the actual time consumed in getting these sets of lines; the writer, from personal experience in solving for a number of trusses, did not find it unbearable. To construct those two sets of lines would require some time, but it must be remembered that this corresponds to the time required to form and solve the large set of simultaneous equations demanded by other methods like those of Manderla or Mohr. To determine whether or not the time or labor required is excessive, an accurate record of the actual time consumed for each method should have been kept, thus rendering possible a conclusive comparison. The writer hopes that the author has made such a record in his work, as the time element is an important factor in the discussion of methods proposed for the analysis of secondary stresses.

In connection with the "base" and "vertex" lines, the author remarks that because of the many repetitions of the same process, the labor involved "is bound to become extremely monotonous". In the writer's opinion, however, this feature of the method is something that deserves merit instead of criticism, as it is well known that to cultivate speed and accuracy "repetition of the same process" is highly desirable. It must be remembered that the subject of secondary stresses is important because of its practical bearing on the design; if it appealed only to the mathematician it would not have interested the bridge engineer. Most bridge engineers object to the consideration of secondary stresses largely because of the excessive labor required for their complete solution. Any means proposed to facilitate the solution is, therefore, in urgent demand. Such psychological effect as monotony, even if it may

exist and be unpleasant, is indeed too trivial to mention if the method be adaptable to practical use.

The author seems to judge the writer's graphic method from his experience with other graphic solutions, for in discussing the question of checking devices, he states that "any graphical method is objectionable in the drafting-room as a complete and thorough check by a second man requires an entirely new solution of the whole problem." It is true that if the second man's check is to make sure that every line in the drawing has its correct position, he will have to repeat exactly the same process as performed in the first solution; but if his purpose is to determine the correctness of the result, there are in the writer's method two neat and simple checks. In the first place (Fig. 57), there are two sets of "base" and "vertex" lines for the first triangle of the truss (Triangle *A*) each using one end of the hip vertical as the vertex. This gives the lines as marked "Triangle *A*, Vertex (2)" and "Triangle *A*, Vertex (3)." Now only one of these two sets is required in the solution of the problem while the other is not used until all the "correct deformation contours, *r*," are found. If the contours for the vertices of the first triangle satisfy the other set of base and vertex lines, not used in the solution, the correctness of every step in the procedure is thereby insured. (This is similar, in a way, to the check of an ordinary Maxwell stress diagram.) Another check of the solution is obtained by drawing equilibrium polygons around every joint of the truss. If the secondary stresses produce no moment around the joints, the correctness of the secondary stresses themselves is also proved (or the writer's graphic method of successive deduction may be used instead). It is evident, therefore, that the writer's graphic method is self-checking and there is no necessity of going over the whole process, even if it has to be checked by a second man.

The author questions the use of two 45° lines with which to close the deformation contour. Although these lines are not essential to the solution of the problem, they are introduced for the purpose of furnishing an additional check, as their intersection point must lie on the deformation line constructed for the outside angle of the truss around the joint.

The value of the writer's graphic method will be greatly enhanced by using a drafting machine. The time saved may be compared with that in the analytic solution by using an adding machine.

The writer's analytic method was evolved for a twofold purpose: First, to afford a practical solution of the problem, reducing much of the labor and time required in comparison with the existing methods; second, to introduce a new arrangement of computation believed to be more rational and useful than the ordinary tabulated form. The principles involved in this analytic method are simple and are believed to be more understandable than most of the existing methods. Were it not for the fact that a novel form of computation has been evolved at the same time, intended to promote efficiency and reduce the chances of making mistakes, the description of the method would have been materially shortened.

The writer is much disappointed that the author has failed to mention some of the exclusive features of the analytic method, especially in regard to the amount of time required for a solution. The writer has found from repeated

experience that under symmetrical loading the analytic method would require not more than one-half the time necessary in other solutions, whether the span is long or short. As full loading is probably the only type that will be investigated for secondary stresses in practice, in view of the fact that those stresses are now rarely investigated for any kind of loading, the writer's analytic method would supply a practical working tool for the modern bridge office. It is more rational and systematic than the others and requires only a few hours' work for a complete solution. When the bridge office has to design a large number of standard trusses the writer's method is even more useful, on account of the fact that the "computation diagrams" may then be blueprinted for permanent use.

Under unsymmetrical loading the author finds that the coefficients of the unknowns increase very rapidly as triangle is added to triangle. The writer is not unaware of this shortcoming of the method, as he has also tested the method for unsymmetrical loading, but on a closer study of the various elements involved, he does not consider it to be very serious. The reason why those coefficients seem to follow on an ever-increasing trend, is due to the peculiar way in which they are formed. Almost all the coefficients in the U-tables are obtained by adding numbers derived from the previous coefficients. The major part of the arithmetic operations required, therefore, is addition, which is the easiest form of calculation even with large numbers. Subtraction of numbers is found in rare cases, while multiplication and division are necessary only in the moment equations around the joints and the checking equations in each triangle. With most of the arithmetic operations in addition, the writer cannot see that the practical value of the analytic method would be materially reduced even in a long-span truss.

The analytic method has a unique check which must be satisfied as soon as the stresses in a triangle are completely known; but, as the writer noted in the printed abstract of his thesis, "particular attention should be paid to the solution of equilibrium equations and also to the correct substitution of the values of U, as the checking equation does not apply in either case." In criticizing the checking feature of the analytic method, the author has probably overlooked that statement.

To those who are familiar with the writer's paper, it is probably not necessary to remark that the analytic method is also adaptable to trusses with subdivided panels. Whether the triangles of the truss have vertices in the interior or on the boundary of the frame, does not seem to have any effect on the utility of the method. A fair comparison can be made only by applying all methods to the same kind of structure.

It will be noted that in the analytic method the computations are arranged so that every operation has a predetermined order and all necessary calculations are clearly shown in the diagram, thus eliminating the personal equation and facilitating a thorough check at any future time. Another exclusive feature of the method is found in the fact that the unknowns of the problem are reduced to the secondary stresses themselves, which enables the computer to obtain some idea of their relative importance as soon as they are found. It is

sincerely hoped that these features may provoke new ideas in the subsequent investigations of secondary stresses.

In 1922, two of the writer's students at the Tangshan Engineering College, Messrs. T. S. Yen and T. H. Kê, applied his process in a general way to Mohr's method, thereby arriving at two modified forms, graphic and analytic, which enabled them to solve the problem in much less time than with the original method. These modified forms were patterned very much after the writer's methods except that the angular rotation of the joints were used as unknowns instead of the deformation contours. This shows that a continued interest in this subject will broaden the scope for investigation and any step tending to promote the interest cannot be taken in vain.

The writer wishes to express his indebtedness to the Society for the publication of this valuable paper of Dr. von Abo, especially as nearly one-quarter of the printed matter is devoted to the writer's two methods. This worthy task undertaken by the Society will surely be deeply appreciated by the Engineering Profession at large.

Edward Godfrey,* M. Am. Soc. C. E. (by letter).—In discussing the subject of secondary stresses of course the first thing to establish is a definition of the term. There is a sharp difference of opinion as to the importance of secondary stresses, and little can be accomplished in analyzing this difference of opinion unless the term "secondary stress" is first defined and understood.

Engineering terms must be defined by engineers. The dictionary is of little or no use except to give the general meanings of the words. The engineering application of these words is a matter for engineers to agree upon or to develop by custom and usage, guided somewhat by authoritative utterances. Many engineering terms are not found in the dictionary, although they are in common use among engineers and have definite meaning.

In general, primary stresses are well understood. They include axial tension or compression in members ordinarily designed for such stresses. They include bending in any member subject to transverse loading, whether this loading be an extraneous load or the weight of the member itself. In other words, a compression or tension member, as the chord of a truss which carries floor ties, is not subject to secondary stress by reason of this transverse loading, for both the direct axial load and the transverse load are of primary importance in the design of these members. In the same manner, a long horizontal member must be designed to carry its own weight, as a primary consideration, and it is only when custom and experience have proven that members are so short as to give bending stresses, due to their own weight, of no consequence in the design, that these stresses may be neglected.

Stresses that are produced by wind loads, whether direct axial stresses in truss members or bending stresses created in these members by wind shears that must be carried by the truss members to an adequate resisting medium, are not secondary. They are primary, and when of considerable magnitude must be accounted for in the design. Only when they are small in comparison

* Structural Engr., Pittsburgh, Pa.

with the main truss stresses or floor system stresses is there any warrant for neglecting them.

Stresses that are produced by an eccentric application of the direct load of a tension or compression member are not secondary. Specifications require that axial stresses be applied at the center of gravity of the cross-section of a member and imply or express the requirement that if axial stresses are not so applied, the bending moments produced must be provided for.

When a girder or beam rests on a bracket on a column without web connection and without connection of both flanges to the column, there is a definitely calculable bending moment in the column due to this bracket load, and the column must take all this bending moment—the girder or beam cannot take any part of it. The bending moment is not a secondary stress in the column. It is conceivable that such a load could wreck the column, the stress being greater in amount than the direct axial load on the column.

In all the foregoing cases there can be no dispute, in proper designing, as to whether and when the stresses referred to should be accounted as worthy of consideration. When by ordinary calculation these additional stresses amount to a considerable fraction of the axial stresses, they must be governing factors in proportioning the members. Safety in structures demands this.

Some of the cases that will doubtless be conceded by all to be classed as secondary stresses are the following:

(*a*) In a column to which a girder is rigidly attached—the bending stresses due to the deflection of the girder. It is assumed that the plane of the web of the girder coincides with the axis of the column.

(*b*) In a truss rigidly riveted at all joints—the bending stresses due to the deflection of the truss.

(*c*) In a pin-connected truss—the bending stresses due to the tendency of the members to rotate as the truss deflects, the friction on the pins supplying the force.

All these have the common characteristic that they are almost universally ignored by designers.

In designing, a difference of opinion as to the precise use of the words "secondary stresses" is of little importance. As intimated in the beginning of this discussion, however, the sharp difference of opinion as to whether secondary stresses are worthy of consideration in design is of great importance and the line of demarcation in this difference of opinion is that outlined in the foregoing classification. It is pertinent, in considering a subject of this nature, to inquire into the basic assumptions and to establish their soundness. If extremely complex mathematics must be used in the design, it is particularly appropriate to establish the necessity for that method of design; otherwise engineering is burdened with unnecessary complications.

One of the simplest structural problems is that of designing, for a given loading, two columns and the girder connected to and supported by them. If this girder rests on brackets on the sides of the columns, with no web connection and no top flange connection, the span is the distance center to center of these brackets, and the bending moment on the column is simple to cal-

culate. This is not good construction, however, and would not ordinarily be considered good engineering, because lacking in rigidity.

Suppose, then, the girder rests across the top of the column or has a riveted web connection to the side of the column or has top and bottom flange connections to the column. Any one of these accepted and proper details of construction creates a condition that, theoretically, demands consideration of bending moments in the columns due to the deflection of the girder. The same condition theoretically would permit the design of the girder for a span less than the distance between the columns. No practical designer (that is, no person actually engaged in the practical design of structures) ever designs columns for these bending moments, nor does a practical designer who is careful ever use less than the distance center to center of the supporting posts for the span of a girder, where the posts support no other load than that transmitted by this girder.

The burden laid on the practical designer would be great if he were required to apply the method of least work in the solution of this simple problem. It would be enormous, in fact, unbearable, if he were compelled to follow this method throughout in the design of a small building.

General acceptance of a system of design has no weight if logic or tests can prove it erroneous, but logic and tests must show a pretty good case before they can displace a generally accepted standard of design.

The best tests are the practical tests of existing structures. The writer believes he can state without fear of contradiction that no steel structures have failed because the designers have neglected to take into account the secondary stresses due to deflection of the girders. He will go a step further and state that he does not believe it is possible for a structure to fail, if the ordinary rules of design are followed, from such cause as the deflection of the girders.

The argument of logic is in favor of the standard method of designing and against the ultra-theoretical. If steel were brittle and perfectly elastic, like glass, this would not be the case—all calculable stresses would have to be accounted for; but steel has a property that makes it safe for structures, the property of toughness or malleability. Theory can no more ignore this property than it can ignore any other known property of a structural material.

Consider the following two cases where the elastic theory shows exactly equal stress in the columns. Each case consists of a pair of columns and a girder supported by them. All four columns are identical in cross-section; but one girder is connected rigidly to the columns, and the other simply rests on brackets, the distance from center of column to bracket support being 3 ft.

The girders are ample in section for their loads, but the columns are weak to the extent that the unit stress, computed theoretically, is the ultimate for the columns, although the columns are capable of taking the direct stress due to the girder load. The theory by which the unit stress in the rigidly connected system is computed is that which considers relative rigidities of the columns and girders and utilizes the method of least work or some other involved process, and finds imaginary points of inflection 3 ft. from the centers

of the columns. Thus the unit stresses, theoretically, are identical when the systems are loaded.

The system in which the girder rests on the brackets will fail. The other system will not fail.

The first system will fail because there is nothing whatever in it to come to the aid of the columns when they deflect in the act of failing. The second system will not fail because the girder prevents the columns from deflecting to the point where they will crush or buckle.

These facts can be reasoned out by analysis, starting with the known properties of steel. They can be demonstrated by test. Every engineer who has examined many existing bridges is familiar with the fact that there are cases without number where the theoretical stresses, if realized, would wreck the structures in which they occur. These facts provide a sound basis for the general disregard of secondary stresses as defined in the present discussion. Unfortunately, some designers do not discriminate between computed stresses that may with impunity be ignored, and that are among those classed as secondary in this discussion, and the other stresses which are herein defined as not secondary.

What is said of the effect on a structure of secondary stresses due to the deflection of a girder is equally true of secondary stresses in truss members due to the deflection of a riveted truss or a pin-connected truss, either because of the rigid joints or because of rotation on the pins. The point of distinction between the case that needs consideration and the one that does not is the possibility of continued deflection. If a member can deflect only so far and is positively prevented from deflecting any farther, and if reverse deflection (that is, bending back and forth) is not a possibility, it is known from the properties of steel that, in general, failure is impossible.

A bar, rod, or wire can be bent over a corner or around a curve into such a shape that calculations, based on the perfect elasticity of steel, would show ultimate stresses in the piece, and yet in this shape it will carry a load practically if not fully equal to its capacity as a straight member. Has theory any right or warrant to say this member is unsafe, when countless members in this identical position are doing full service without any distress whatever? This bent member will carry the load imposed on it indefinitely—forever, in fact—barring deterioration. What standing has the theory which declares the member to be dangerously stressed?

On the other hand, if the piece were bent back and forth repeatedly, even through a much smaller angle than would be safe for the first bend, it would soon, or eventually, break. This behavior, as well as the other, can be reasoned from the known properties of steel—its toughness and malleability; and these properties are just as real and, when understood, just as dependable as the elasticity of steel, which is the only recognized property of the metal that the average theorist uses in his theorizing.

There are grades of steel, very high in carbon and very brittle, that would not stand safely even one bend that other steels would stand with perfect safety; but structures are not made of the hard, brittle grades of steel. It

is not proper to introduce into the theory of structures the properties of a steel that is not used in the fabrication of structures. Economic, safe, and proper design is not advanced thereby.

Consider the two pairs of posts in Fig. 106. The posts of one pair, *A*, have been jacked apart and a ball placed between them. The posts of the other pair, *B*, are bowed in their fabrication. All posts are of the same cross-section. The unit stress from direct load in each of the four posts is the same, namely, 10 000 lb. per sq. in. The unit stress from bending is, theoretically, exactly the same, namely, 12 000 lb.; but the bending moment in one pair is due to the wedging or springing action of the ball forced between the columns, while that in the other pair is due to the bowed shape of the columns and the eccentric application of the load.

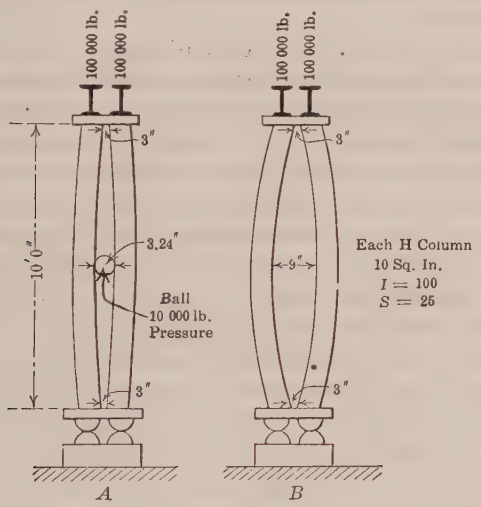


Fig. 106.

In spite of the identical stresses in these two pairs of columns, theoretically determined, they are by no means equally unsafe. The writer would not hesitate to condemn the pair of columns shown at *B*, if he found such a condition in a structure. He would not be greatly concerned for the safety of the structure, if he found the condition at *A* existing.

The condition at *B* is a menace to the integrity of the columns, and failure, once started, would continue without hindrance. Part of the deflection of 3 in. (about $\frac{1}{8}$ in.) is produced by the load which the columns are supporting. There is, therefore, the start of failure in the columns as shown. The load will not cause any deflection in the columns at *A*, but will relieve some of the pressure on the ball.

In the fabrication of "fish-belly" compression members the parts are bent cold. This puts into the metal initial stresses that correspond to, in fact, are much greater than, those found by the theorist in members of a truss as due to secondary stress. The lattice-bars in these members with bowed parts, however, hold these parts against deflection, and no weakness is developed in the

members. In exactly the same manner the theoretical unit stresses found in the members of a truss due to the secondary stress caused by deflection of the truss mean nothing, because their effect ceases at their inception and cannot be augmented. In fact, it is very doubtful if they actually exist. The tests on the Hell Gate Bridge showed far from consistent and expected results. Malleable metal, such as structural steel, can take up and absorb such stresses. They mean little or nothing as regards safety.

D. B. Steinman, M. Am. Soc. C. E., who made the tests referred to, states:* "For the highest percentages of secondary stresses, the measured values are only a small fraction of the calculated values". Again, "The actual secondary stresses will generally be lower than the calculated values. There is an automatic readjustment of strains within a structure in such direction as to relieve the secondary stresses".

Referring to measurements made on buildings it has been stated† that: "At certain points of heavy columns where one might reasonably expect considerable bending to be developed by distortion or secondary effect, there is generally speaking no bending". As a result of tests of secondary stresses, O. H. Ammann, M. Am. Soc. C. E., states:‡ "Eccentric connections increase the secondary stresses almost invariably and should therefore be avoided". Such stresses are not true secondary stresses. He also states, "Rigid riveted connections, on the other hand, always act favorably, distributing the moments and reducing the stresses from eccentric connections". It is rigid connections which, theoretically, cause the greatest secondary stresses. He states again: "The results of the investigation indicate that rigidity of members and connections tends to prevent localization of high secondary stresses and may therefore constitute an element of safety rather than of weakness".

Imperfections in workmanship can be shown theoretically to produce far greater stresses in structural members than are produced by deflections of these members in the structure. Suppose, for example, that a girder is milled out of true a small fraction of an inch. When this girder is riveted to the column, the column will be bent to a far greater extent than the deflection of the girder would bend it, yet a practical engineer, understanding the properties of steel, would not concern himself in the least if this condition were found to exist in his structure.

Again, suppose a column or truss chord were milled out of true. The calculated stress could be enormous in intensity—sufficient to break any material that is brittle like glass or concrete. In steel the member may show no distress whatever. Of course, careless workmanship is not a thing to be desired or encouraged, but any engineer who, like the writer, has examined in detail a hundred or more existing structures and observed these imperfections, and has seen metal peened down by reason of its malleability (under high stress), by stresses that, theoretically, would wreck a structure, is aware that malleability is a property of steel that can be relied upon under certain conditions.

* *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1071.
† *Engineering News-Record*, October 23, 1924, p. 655.
‡ *Loc cit.*, p. 668.

The outstanding example of a structure that has many varieties of secondary stresses due to its design—stresses of theoretically great intensity—is the Firth of Forth Bridge in Scotland, but none the less this bridge is relatively one of the safest in existence, because of its design and because of the very rigidity of the joints that gives rise to the theoretical secondary stresses.

If the theory of secondary stresses and the perfect elasticity of steel is worthy of consideration in design, there are other cases where it would have application. One of these is a simple web splice in a girder. The writer knows of an eminent engineer, now deceased, who insisted that every web splice should be proportioned for bending or flange stress as well as shear, for the reason that the bottom edge of the web plate elongates and the top edge shortens as the girder deflects. As designers know, it is customary to splice the web plate for its shear only, unless it is desired to take advantage of the aid a web plate supplies in taking up flange stress. In many girders, particularly light ones, there is no need of including the bending value of the web, and a single row of rivets on each side of the cut is sufficient for the shear. To splice the web plate for bending would require four or more vertical rows of rivets, or extra horizontal plates—an expensive detail merely to satisfy a theory; for consider what could happen if a web plate is spliced for shear only, say, with a single row of rivets on each side of the cut. Suppose the extreme rivets should slip an insignificant trifle (the stretch of the flange angle in 3 in.); this would not impair their value for taking shear one iota. With the flanges proportioned for the full flange stress, there is absolutely no weakening of any kind in the girder.

In a triple riveted splice in a boiler or tank seam (Fig. 107), if the elastic theory be applied in a manner similar to the way it is applied in computing secondary stresses, it will be seen that there cannot be equal distribution of stress among the three rivet lines. In one plate the intensity of stress is twice as great as in the other plate, where the two plates lie side by side. Ignoring malleability, this joint would not be safe, for there is no way to work out equal shear on the three rows of rivets and simultaneously equal stress on the contiguous layers of plate. Quadruple riveted joints and, in fact, almost all splices offer greater theoretical difficulties than these.

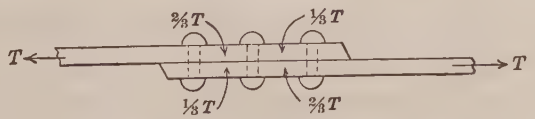


FIG. 107.

As the whole fabric of the secondary stress theory rests on the soundness of its fundamental assumptions, and as it is proven by the cases presented that these fundamental assumptions are without warrant in fact, or experience, or test, it is clear that there is no justification for consideration of secondary stresses nor for adding the complexities of that theory to the literature of engineering.

In the last few years two bascule bridges have exhibited failure in the members that supported the heavy counterweights. Statements have been

published to the effect that lack of consideration of secondary stresses in the design of these counterweight supports was responsible for the failures, and engineers who advocate the ignoring of secondary stresses are held accountable for these failures and any others of a similar character that may occur.

Little is given in the way of details of the supports of these counterweights. However, one thing is made clear, namely, that the counterweights are of concrete cast around built-up truss members. The raising and lowering of the counterweight causes these structural members to bend back and forth as one would bend a wire to break it. It is not to be expected that truss members would withstand this bending back and forth due to a heavy counterweight moving through a large angle in changing its position. If loaded bridges were to be turned upside down and all stresses and moments reversed many times a day, this feature would have to be considered in their design, just as a revolving axle must be designed with a larger factor of safety than is necessary for a stationary member in bending.

The writer has yet to find any case where secondary stress is worthy of computation and consideration in design.

Cyrus C. Fishburn,* Esq. (by letter).—In using Mohr's semi-graphical solution for the moments at the ends of the members of a truss due to the change in length of the members, the following check is useful in verifying the equations before solving for the unknowns, thus insuring against arithmetical errors in the original equations.

The Mohr semi-graphical method first requires the deflections of each member of the truss, which are obtained by means of the Williot diagram. An equation is then written for each joint of the truss, the unknowns being the rotation or angular change at the joints. These equations are derived from the basic equation:

$$M_{AB} = 2\,E\,K_{AB}\,(2\,\theta_A + \theta_B - 3\,R)$$

in which, M_{AB} is the moment at the end, A, of the member, $A\,B$, K_{AB} is equal to $\frac{I}{L}$ (movement of inertia divided by length) for the member, $A\,B$, θ_A and θ_B are the rotations at the ends, A and B, respectively, and R is equal to $\frac{d}{L}$, d being the deflection of $A\,B$. From this fundamental equation, a series of other equations is derived by equating the sum of all the moments at any joint to zero. The equation for Joint A (Fig. 108) is:

$$\begin{aligned}&[2\,\theta_A\,K_{AB} + \theta_B\,K_{AB} - 3\,(R\,K)_{AB}]\\ &+ [2\,\theta_A\,K_{AC} + \theta_C\,K_{AC} - 3\,(R\,K)_{AC}]\\ &+ [2\,\theta_A\,K_{AD} + \theta_D\,K_{AD} - 3\,(R\,K)_{AD}]\\ &+ [2\,\theta_A\,K_{AE} + \theta_E\,K_{AE} - 3\,(R\,K)_{AE}] = 0 \quad \ldots\ldots\ldots\ldots (120)\end{aligned}$$

which may be written in the form:

$$2\,\theta_A\,(K_{AB} + K_{AC} + K_{AD} + K_{AE}) + \theta_B\,K_{AB} + \theta_C\,K_{AC} + \theta_D\,K_{AD} + \theta_E\,K_{AE} = 3\,[(R\,K)_{AB} + (R\,K)_{AC} + (R\,K)_{AD} + (R\,K)_{AE}] \ldots\ldots (121)$$

* Graduate Student in Civ. Eng., Univ. of Illinois, Urbana, Ill.

It is to be noted that the left-hand member of Equation (121) contains the rotation, θ, of both ends of all members entering the joint, having as coefficients the K's of the members. The right-hand member of Equation (121) contains $3\,R\,K$ for all members.

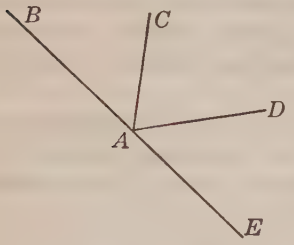


FIG. 108.

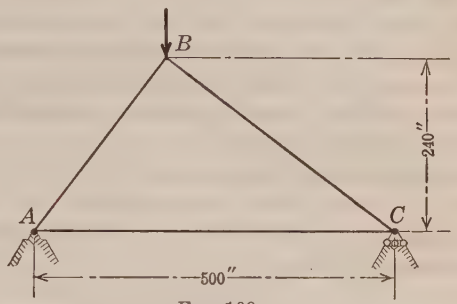


FIG. 109.

The equations for all joints in the structure of Fig. 109 are presented in tabular form in Table 36.

TABLE 36.—FUNCTIONS OF JOINTS.

	Equation.	θ_A.	θ_B.	θ_C.	Right-hand member of equation.
Joint A	(1)	$2(K_{AB}+K_{AC})$	K_{AB}	K_{AC}	$=3\,[(RK)_{AB}+(RK)_{AC}]$
Joint B	(2)	K_{AB}	$2\,(K_{AB}+K_{BC})$	K_{BC}	$=3\,[(RK)_{AB}+(RK)_{BC}]$
Joint C	(3)	K_{AC}	K_{BC}	$2\,(K_{AC}+K_{BC})$	$=3\,[(RK)_{AC}+(RK)_{BC}]$

Before setting up Equations (120) and (121), it is convenient to tabulate the geometrical properties of the members. Such a tabulation for the simple truss in Fig. 109 is presented in Table 37.

TABLE 37.—PROPERTIES OF MEMBERS.

Member.	Length, L, in inches.	Area, A, in square inches.	I, in inches^4.	$K\,\frac{I}{L}$.	Stress, P, in pounds.	$\frac{PL}{AE}$, in inches.	Deflection, d, in inches.	$R\,\frac{d}{L}$.	$\frac{3\,R\,K}{3\,I}\,\frac{d}{L^2}$.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
AB	300	32	3 000	10	− 500 000	− 0.1562	+ 0.408	+0.00136	+0.0408
BC	400	24	1 600	4	− 375 000	− 0.208	− 0.307	−0.000768	−0.009216
AC	500	20	1 500	3	+ 300 000	+ 0.250	0	0	0
Totals...	...	..	.....	17	.....	.....	.....	.....	+0.03158

It may be seen by inspection of the equations in Table 36 that the sum of the coefficients of θ in all three equations equals 6 $(K_{AB} + K_{AC} + K_{BC})$, or six times the sum of all the K's in Column (5) of Table 37. Similarly, the

sum of the right-hand members of these equations equals 6 $[(RK)_{AB} + (RK)_{AC} + (RK)_{BC}]$, or twice the algebraic sum of the quantities in Column (10) of Table 37. This check applies to any truss, irrespective of the number of joints.

The numerical value of the quantities of Table 36, are substituted from Table 37, in Table 38. The sum of the arithmetical coefficients of the θ's in Equations (1), (2) and (3) of Table 38 equals 102, or six times the total, 17, of Column (5), Table 37. Likewise, the algebraic sum of the numerical values of $3RK$ in these equations, equals twice the algebraic total, 0.03158, of Column (10), Table 37.

After checking the accuracy of the equations in the manner shown, the work of solving them simultaneously may be continually verified by the well known method of carrying a check column. Such a check column is illustrated in Table 38.

TABLE 38.—Numerical Check.

Equation.	θ_A	θ_B	θ_C	Right-hand member of equation.	Check column.
(1)	+ 26	+ 10	+ 3	+ 0.0408	+ 79.8
(2)	+ 10	+ 28	+ 4	+ 0.03158	+ 73.58
(3)	+ 3	+ 4	+ 14	— 0.00922	+ 11.78
(4)	+ 1	+ 0.385	+ 0.115	+ 0.001569	+ 3.069
(5)	+ 1	+ 2.8	+ 0.4	+ 0.003158	+ 7.358
(6)	+ 1	+ 1.333	+ 4.667	+ 0.003073	+ 3.927
(5) — (4) = (7)	.....	+ 2.415	+ 0.285	+ 0.001589	+ 4.289
(5) — (6) = (8)	.....	+ 1.467	— 4.267	+ 0.006231	+ 3.431
(7)	.....	+ 1	+ 0.116	+ 0.000658	+ 1.774
(8)	.....	+ 1	— 2.903	+ 0.00424	+ 2.337
(7) — (8) = (9)	.....	.....	+ 3.019	— 0.003582	— 0.563
			$\therefore \theta_C = -0.001187$		

The check column contains the algebraic sum of all the factors in each equation, and it is treated in the same manner as the quantities in the equations. It may be found convenient to multiply the right-hand members of the equations by a constant factor, 100 or 1 000, before adding it to the check column. The factor used in Table 38 was 1 000.

By checking the arithmetical quantities in the original equations, as written and by checking the successive steps in the solution of these equations, the chance of arithmetical error in the result is reduced to a minimum.

F. P. Shearwood,* M. Am. Soc. C. E. (by letter).—This investigation must have involved a great amount of painstaking work, and it is very useful for correctly indicating how the members of a frame will be strained under load. The labor involved in working out these stresses is immense, and it is quite impossible, under present commercial conditions, to apply it to the ordinary structure.

The real aim of designing is to produce the safest, most rigid, and lasting structure with the least amount of material. Theoretical fiber stresses should not be permitted to befog the real issue, but should be used in conjunction with

* Chf. Engr., Dominion Bridge Co., Ltd., Montreal, Que., Canada.

experience on existing structures to temper engineering judgment in designing and proportioning new structures or in determining the safety of old ones.

The term "secondary stress" is generally understood to include only those stresses which would not occur if the structure had perfectly articulated joints. This ideal condition is impossible, because of certain physical facts which cannot be eliminated; actually, secondary stresses exist in every framed structure because there is restriction to some degree in every joint.

Secondary stresses, unlike other stresses, are relieved by deformation or distortion of the material, and, provided the range of repeated deformation is not sufficient to cause fatigue, ordinary secondary stresses will not reduce the safety of the structure.

It was with great interest that the writer read Mr. Godfrey's discussion, with whom he agrees most thoroughly. Mr. Godfrey truly states that no steel structure has failed through the neglect of providing for secondary stresses. The failure of the counterweight truss members of the bascule bridges referred to by Mr. Godfrey was attributed by some to secondary stresses; but no conclusive proof of this was given. A careful study of these designs would point to an equal probability of the failures being due to the tearing effect of the embedded trusses on the main members, because the combination of concrete and embedded steel had ceased to function. The breaking of the bond of the concrete was due to the frequent reversal of the stress.

Any engineer familiar with the fabrication and erection of steelwork must realize that even in the hands of the best bridge companies, the steel is subjected to greater strains than it can ever receive under its normal loading.

It seems peculiar that, although steel is the most ductile of all materials, the design of steel structures is practically the only one in which it is customary to neglect this quality deliberately. Indeterminateness is avoided and any form of continuity is unfavorably considered, because the supports might move and cause strains beyond the elastic limit, but, in practice, many examples of steel structures are found which are safely performing their duty in spite of some injury to their members that has distorted them far beyond the ultimate strength of the material, if calculated according to the elastic theory. On the other hand, material, such as concrete, is used for all kinds of construction where continuity and other forms of indeterminateness exist in spite of the fact that these materials lack ductility.

To determine the effect of secondary stress from rigid joints on the safety of structures, it is necessary to know by experiment or practical experience whether or not restricted straining (secondary stress) really weakens members stressed in direct tension or compression in proportion to the increase of the theoretical fiber stress. Experience seems to indicate that it does not, in fact, there is much to show that secondary stress is more often a source of increased strength than of weakness.

LAURENCE J. WALLER,* ASSOC. M. AM. SOC. C. E. (by letter).—The author's work in compiling and comparing the various types of secondary

* Civ. Engr., Austin & Ashley, Los Angeles, Calif.

stress analysis has placed before the profession an invaluable compendium of scientific information.

It has seemed to the writer that, in the case of the average engineer, the selection of a method of secondary stress calculation has been almost entirely fortuitous. The sources of information are not extensive; the subject itself is not in common usage, and is relatively difficult. Faced with a choice between a number of highly technical systems and having no basis of comparison other than actual trial, engineers have largely used the system nearest at hand, with the varying degrees of labor required and accuracy attained. The author's critique makes it possible for the engineer promptly to discard complicated or inherently faulty systems in favor of the simpler and more accurate types of analysis.

The writer considers Mohr's semi-graphical method to be the most practical thus far devised for the usual type of bridge truss. The precise details of this method, however, are subject to considerable variation. Personally, the writer uses an arrangement of Mohr's method so far not touched on in this discussion, namely, that of Mr. G. A. Maney.* The nomenclature and the arrangement are lucid and concise, and possess to an eminent degree the property of being readily visualized. Contrary to the author's preference, the solution by successive substitutions will probably commend itself to the profession rather than elimination by Gauss' normal equations, particularly in the case of long span bridges. Instead of the usual substitutions, a more rapid convergence of values is effected by using in a given equation the last found values in that series. By this process, the third substitution will almost invariably give values sufficiently precise.

Extreme precision in secondary stress determination is not only unwarranted by reason of failure to secure the basic assumptions of analysis, but is also rendered futile by the modifying effect of certain factors which are not considered in the usual analyses. Under the first heading come such considerations as:

(1) The assumption of perfect fixity at the truss intersections;
(2) The assumption of perfect elastic deformation of the members between the intersections; and
(3) The assumption of constant moment of inertia.

According to the elastic theory there is bound to be considerable variation in the fixity because the moment of inertia of some gussets may be as great as that of the truss member itself, whereas the moment of inertia of other gussets may be only a small fraction of that of the main member. Indeed, German and Swiss tests, as well as tests at the University of Illinois, have indicated the lack of perfect fixity in gusset-plate connections. This lack of fixity is due to (*a*) rivet slip and (*b*) elastic deformation. The variations from the assumption of perfect fixity have been characterized† as "not introducing serious errors into the results." This finding is to be expected, but it would indicate the uselessness of highly refined secondary stress calculations.

* "Studies in Engineering No. 1," Univ. of Minnesota.
† *Bulletin No. 104*, Eng. Experiment Station, Univ. of Illinois.

Due to the variant moment of inertia and length of the gussets, there is a variant and appreciable zone of restraint past the intersections, with a resultant effect on the secondary stress.

The assumption of constant moment of inertia is, of course, never realized in practice. Deep and heavy gussets and splice members will modify materially the moment of inertia of the section and, consequently, the secondary stresses calculated on the premise mentioned.

In his discussion Mr. Steinman mentions a variation in the usual secondary stress procedure which involves the recognition of a variable moment of inertia in the members. This procedure should largely compensate for the two latter considerations.

The total stresses, primary plus secondary, as determined by the usual methods, are somewhat modified by the existence of certain concomitant conditions not usually analyzed or considered. The more obvious of these are:

(1) The effect on the unit primary axial stress of increased section due to gussets and splice members.

(2) The effect on the unit primary flexural stress of increased moment of inertia due to gussets and splice members.

(3) Faulty fabrication, including such items as incorrect rivet spacing, inaccurate facing of compression surfaces, sections that are in wind, incorrect shop lengths, etc.

(4) Initial stress in sections due to the effect of the straightening rolls.

(5) Temperature variation between different members of the truss.

(6) Eccentricity produced by flexure in the member between the panel points.

(7) Bending in the verticals caused by deflection of floor-beams.

(8) Torsion in chord members due to deflection of the floor-beams extending over part of the span.

(9) Longitudinal stress caused by friction of roller bearings at a movable reaction.

(10) Decrease in axial stress due to the fact that the floor system assumes load.

(11) Non-uniform stress intensity in outstanding legs of angles, splice members, etc.

These various factors produce stress variations which, in general, are small individually. The design should be arranged so as to correct for them as far as possible, but the calculation of their precise values is commonly disregarded by engineers. This procedure is justified by reason, for their complex nature and erratic occurrence preclude the possibility of a precise analysis without an expenditure of time out of all proportion to the small additional accuracy attained.

These considerations would indicate the futility of striving after great precision in secondary stress calculations through the medium of rigorous mathematical analyses following the so-called "precise methods". As mentioned, Mohr's semi-graphical method with the solution of equations by successive substitutions provides a lucid and expeditious analysis, and one which

possesses ample accuracy considering the modifying effect of the indeterminate factors alluded to.

Apropos of the moot questions of relative fixity of members at panel points, some recent German strain-gauge tests may be of interest.* They were taken on the center bottom chord gusset and connecting members in a truss 6.0 m. in span and 1.5 m. in height. The truss was loaded at its center upper chord panel point with a load of 50 metric tons. In every instance, a relatively close agreement was found between the actual stresses and those premised from the assumption of fixed ends. The results of this test, as of other similar tests, emphasize the statement that the fundamental theory of fixity of ends as applied to secondary stress analysis is sound; but that highly refined methods of calculation are unwarranted, because of inevitable and indeterminate modifying influences.

R. McC. Beanfield,† Assoc. M. Am. Soc. C. E. (by letter).—The author's comparative analyses of various methods and conclusions involving a graphical and analytical solution for computing secondary stresses constitute valuable additional data on a technical subject lacking in comprehensive information of a practical nature.

Most of the accepted standard methods used to compute secondary stresses involve theoretical assumptions that do not correspond to conditions in actual practice. A few of these erroneous assumptions are as follows:

1.—The use of a constant moment of inertia throughout the theoretical length of the member, whereas in most instances the moment of inertia varies greatly, particularly at the end gusset connections, which in some cases is further aggravated by the use of clip angles and filler plates.

2.—The assumption that the length of the member is the theoretical length between intersection points. The theoretical length of the member as taken between axial intersection points may become considerably shortened by the use of large gusset-plates or other end conditions. If values ranging between the theoretical length and the free length (edge of gusset-plates) are substituted in the formula, a considerable variation in results is obtained.

3.—The restraining moment at the joint: In riveted connections considerable distortion may take place by reason of the elastic deformation of gusset-plates, the initial slip and distortion of rivets, and the frictional resistance between plates, all of which make the action of the joint under load rather uncertain and produce a complex condition of stresses, wherein it is practically impossible to determine accurately the restraining moment of the joint.‡

4.—The unknown frictional resistance of pin joints: Secondary stresses for pin connections are calculated in practically the same manner as for riveted connections. If the moment at the pin joint caused by the distortion of the truss exceeds the frictional resisting moment of the pin, then intersecting members may be assumed to rotate and to offer no resistance to moment. In other words, the secondary stresses in a pin joint are measured

* *Schweizerische Bauzeitung*, September 23, 1923.

† Structural and Mech. Engr., Los Angeles, Calif.

‡ "Nickel Steel Riveted Joints," *Bulletin No. 49*, Univ. of Illinois; *Schweizerische Bauzeitung*, September 15, 1923.

by the stresses caused by the frictional resistance of the pin. Thus, the accuracy of secondary stresses for pin-connected joints depends largely on the correctness of the coefficient of friction between the pin and its bearing. There are very meager data covering the frictional resistance of pins, therefore computed secondary stresses for pin-connected joints are based on highly theoretical assumptions which may give results varying greatly from actual conditions.

The writer has had occasion to investigate the secondary stresses in a 168-ft. main balcony truss weighing approximately 250 tons, for the Shrine Auditorium at Los Angeles, Calif. A careful investigation of the secondary stresses was made because it was evident that the large, rigid, gusset connections would involve considerable restraining moments which were further complicated by reason of pin connections; some of the members had pin connections at one end and riveted connections at the other. The value of the frictional resistance between the nickel steel pins and the ordinary structural steel bearing was an unknown factor and had to be assumed.

Prior to making secondary stress computations for this truss, considerable study was given to the various accepted methods. An adaptation from Mohr's semi-graphical analytical system, as described by Mr. G. A. Maney,* was utilized and gave a clear and concise analysis, materially shorter than any of the methods suggested by the author.

The writer found some helpful data in a paper by the late F. C. Kunz, M. Am. Soc. C. E., entitled "Secondary Stresses".† Those interested in secondary stresses should read this paper carefully.

The secondary stresses in some of the members for the truss in the Shrine Auditorium were found to be too great and had to be reduced. All the secondary stresses were practically eliminated by erection methods that involved introducing initial stresses in the members by lengthening or shortening them as the deformation demanded.

In computing the secondary stresses of a number of large structural steel trusses in buildings the writer has found that the so-called approximate methods of solution, as proposed by Mohr and later suggested by Maney, give sufficiently accurate results for practical purposes and involve considerably less labor than the method proposed by the author.

In many specifications, and in some building codes, secondary stresses are not clearly defined. Some authorities allow an increase in secondary stresses ranging from 10 to 33⅓ per cent. In other words, in complicated structures in which the stresses are difficult to analyze, relatively higher unit stresses are permitted, whereas in simple frame structures which have practically no secondary stresses, a much smaller unit stress is permitted.

Little data are available relative to secondary stresses in reinforced concrete frames, particularly long span trusses. It would be interesting to know

* "Secondary Stresses and Other Problems in Rigid Frames," *Bulletin No. 1*, Univ. of Minnesota.

† *Engineering News-Record*, October 5, 1911.

what consideration was given to the secondary stresses in a number of long-span reinforced concrete trusses that have been erected.

The writer much prefers the reduction or elimination of secondary stresses in structural steel trusses by erection methods rather than by altering the design of the structure. The elimination of secondary stresses by proper erection methods does not involve complicated or expensive construction operations. As a matter of fact, the introduction of initial stresses by shortening or lengthening the members as the deformation demands and forcing them into position by jacks, or otherwise, proved to be a simple and comparatively inexpensive operation for the large balcony truss in the Shrine Auditorium in Los Angeles.

Secondary stresses should be investigated for all structural frames involving rigid or restrained joints, for even if the results may be approximate, they indicate the seriousness of neglecting them. The most efficient and economical method for reducing or eliminating secondary stresses requires considerable ingenuity on the part of the designing engineer who must be experienced and able to visualize as well as theorize.

Cecil Vivian von Abo,* Jun. Am. Soc. C. E. (by letter).—The writer is grateful to the Society for publishing his paper and to the many members who have contributed such useful information and criticism. The original idea was to investigate Dr. Mao's methods, but when the writer found them too cumbersome, he was advised by Professor Jacoby to make a thorough comparison of all existing methods. This in itself entailed a tremendous amount of work, and, therefore, he decided to confine his attention to what may be termed the more exact methods.

It was with great reluctance that, through lack of time, the writer had to forego the investigation of column action in compression members for publication in the paper, but the latter part of this discussion contains some interesting results.

Mr. Ammann raised the point that, in order to take the gusset-plates into account, each bending moment calculated on the assumption of constant moments of inertia throughout the theoretical lengths of the members should be multiplied by the ratio of its theoretical length to the clear length between gussets. On the other hand, Mr. Steinman allows for an increasing moment of inertia toward the end of each member and modifies the fundamental equations. The writer is inclined to favor Mr. Steinman's method, as it seems to be the more logical.

The writer is indebted to Professor Cain for pointing out an error in the paragraph following Equation (69). Fixing attention on the second part of Equation (69), that is, $d\,v_4 = v_4\,\psi + u_4\,\phi$ and on Fig. 21, dv_4 equals the sum of the counterclockwise moments of the ϕ's about u' plus the sum of the counterclockwise moments of the ψ's about v'. (Counterclockwise is negative in Mohr's "elastic weight" method). Not only are the axes, u, v, u', and v', turned clock-

* Care, Chf. Civ. Engr., South African Rivers and Harbors, Johannesburg, Transvaal, South Africa.

wise through 90°, but also the ϕ's and the ψ's. This gives Professor Cain's Fig. 93. Only the quantities, $(u_4 - u_1)$, $(v_4 - v_1)$, etc., remain unchanged. The first part of Equation (69), that is, $d\,u_4 = u_{4\,\psi} - v_{4\,\phi}$, interpreted by means of Fig. 93, then becomes:

$d\,u_4$ = the sum of the clockwise moments of the new ψ's about the new v'-axis — the sum of the counterclockwise moments of the new ϕ's about the new u'-axis;

that is,

$d\,u_4$ = — (the sum of the counterclockwise moments of the new ϕ's about the new u'-axis + the sum of the counterclockwise moments of the new ψ's about the new v'-axis).

This change in sign would be troublesome if open chains were to be considered, but as the theory is extended to closed chains for which both these expressions then become zero (see Equation (72)), the two parts of the equation mean one and the same thing. Any point on the closed chain is chosen as origin and any direction as that of the u-axis, which then fixes the direction of the v-axis as being at 90° clockwise to the u-axis. Equation (72), therefore, shows that the sum of the moments of the ϕ's (regarded as acting parallel to the u-axis) about the u-axis plus the sum of the moments of the ψ's (regarded as acting parallel to the v-axis) about the v-axis, is zero.

Professor Fuller calls attention to the variations of stress across a section of a member recorded by him from observations in the field. It seems desirable that a theoretical investigation should be undertaken to determine all the principal variations of stress across sections of members of an existing bridge, due to a stationary locomotive, to be followed by a practical study when the more accurate extensometers could be used. If theory and practice agree, it would go far toward proving the validity of initial assumptions. Impact investigations could then very well follow, in order to get an idea of the factor for secondary and column-action stresses. The work would be stupendous, but if the outcome was a justification for the general acceptance of higher allowable unit stresses, the saving of metal in the bridges of the future would be considerable.

Messrs. Turner and Godfrey both question the definition of "secondary stress". They have also raised some very difficult points which cannot be solved with absolute certainty mathematically. The end bending moments of a particular member are known fairly accurately and the writer demonstrates the importance to be attached to them.

It was very gratifying to note the research contributed by Dean MacKay. It is hoped that the following investigation will account for some of the discrepancies that he found in the field. It is shown, for example, that the variation in the bending moments along the length of a member is not linear. In tensile members, the bending moment curve will differ but little from a straight line, whereas in compression members it will differ widely. This would be still more marked if the unit compressive stress were increased.

Professor Beggs is to be congratulated for his laboratory studies. It would save much calculation if one could obtain a close estimate of the secondary stresses in the manner he suggests.

Professor Shedd has provided a lucid explanation of the present status of secondary stresses, and Professor Cross has given a modification of Mohr's semi-graphical method, which enhances its value. It is quite evident that a student of secondary stresses can fix his attention on the method of either Manderla or Mohr and know that he cannot do better. The writer found little to choose between them; in the light of the discussions, he would be quite prepared to give Mohr's method the premier position.

Professor Wilson has a remarkably fine method of solving the problem of cross-frames. It shows how small the range of individual research is with regard to the reading of all literature on a particular subject.

Many discussors have referred to the subject of column action and the following investigation may be of interest.

Column Action

Most of the work on the theory of column action involves complicated mathematics which makes the subject difficult to comprehend. The approximate theories are better known. They, however, are based on one of two assumptions: (1) That the ends of a column are entirely free to turn; and (2) that the ends are rigidly held. Neither assumption is true in practice. It would be interesting to consider the problem which accounts for some definite end bending moments.

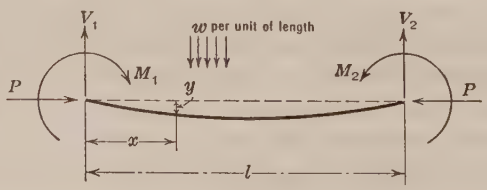


Fig. 110.

Section A

Consider a member which is subjected to an axial thrust, two end bending moments, and a uniform load over its whole length. Assume the member to be straight initially and not to be bent much at any time, so that the radius of curvature at a point will be closely given by $R = \pm \dfrac{1}{\left(\dfrac{d^2 y}{d x^2}\right)}$. Let the modulus of elasticity and the moment of inertia be denoted by E and I, and regarded as constant. Fig. 110 shows the left-hand bending moment, M_1, acting clockwise and the right-hand bending moment, M_2, acting counterclockwise.

The two end shears are given by:

$$\left.\begin{aligned} V_1 &= \frac{w\,l}{2} - \frac{M_1 - M_2}{l} \\ V_2 &= \frac{w\,l}{2} + \frac{M_1 - M_2}{l} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots (122)$$

The bending moment at the point, x, is:

$$M_x = M_1 + P\,y + V_1\,x - \frac{w\,x^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (123)$$

in which, y is the downward deflection of the member at x. Therefore,

$$E\,I\,\frac{d^2\,y}{d\,x^2} = -P\,y + \frac{w\,x^2}{2} - V_1\,x - M_1$$

that is,

$$E\,I\,\frac{d^2\,y}{d\,x^2} + P\,y = \frac{w\,x^2}{2} - V_1\,x - M_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (124)$$

The general solution of the differential equation is:

$$y = A\,\sin\,Q\,x + B\,\cos\,Q\,x + \frac{w\,x^2}{2\,P} - \frac{V_1\,x}{P} - \frac{M_1}{P} - \frac{w\,E\,I}{P^2} \quad \ldots\ldots (125)$$

in which,

$$Q = \sqrt{\frac{P}{E\,I}}.$$

Now $y = 0$ when $x = 0$ and also when $x = l$. Therefore:

$$B = \frac{w\,E\,I}{P^2} + \frac{M_1}{P}$$

and,

$$A = \frac{1}{P\,\sin\,Q\,l}\left\{\left(-\frac{w\,l^2}{2} + V_1\,l + M_1 + \frac{w\,E\,I}{P}\right) - \left(\frac{w\,E\,I}{P} + M_1\right)\cos\,Q\,l\right\}$$

$$= \frac{1}{P\,\sin\,Q\,l}\left\{\left(\frac{w\,E\,I}{P} + M_2\right) - \left(\frac{w\,E\,I}{P} + M_1\right)\cos\,Q\,l\right\}$$

using Equation (122). Hence, the solution is:

$$y = \frac{\sin\,Q\,x}{P\,\sin\,Q\,l}\left\{\left(\frac{w\,E\,I}{P} + M_2\right) - \left(\frac{w\,E\,I}{P} + M_1\right)\cos\,Q\,l\right\}$$

$$+ \frac{\cos\,Q\,x}{P}\left\{\frac{w\,E\,I}{P} + M_1\right\} + \frac{1}{P}\left(\frac{w\,x^2}{2} - V_1\,x - M_1 - \frac{w\,E\,I}{P}\right) \quad \ldots (126)$$

Using Equation (123), the bending moment at x is:

$$M_x = \frac{\sin\,Q\,x}{\sin\,Q\,l}\left\{\left(\frac{w\,E\,I}{P} + M_2\right) - \left(\frac{w\,E\,I}{P} + M_1\right)\cos\,Q\,l\right\}$$

$$+ \left(\frac{w\,E\,I}{P} + M_1\right)\cos\,Q\,x - \frac{w\,E\,I}{P} \quad \ldots\ldots\ldots\ldots\ldots (127)$$

The maxima and minima points are given by:

$$\tan Q\,x = \frac{\left\{\left(\frac{w\,E\,I}{P} + M_2\right) - \left(\frac{w\,E\,I}{P} + M_1\right)\cos Q\,l\right\}}{\left(\frac{w\,E\,I}{P} + M_1\right)\sin Q\,l} \quad \ldots\ldots(128)$$

Consider a few typical cases, as follows:

Case 1.—Consider a horizontal strut of uniform cross-section, under a compression of P lb. and assumed to have zero end moments. This is similar to Euler's assumptions of freely hinged ends. Equation (122) becomes,

$$V_1 = V_2 = \frac{w\,l^2}{2}$$

and Equation (128) becomes,

$$\tan Q\,x = \frac{1 - \cos Q\,l}{\sin Q\,l} = \tan\frac{Q\,l}{2} \quad \ldots\ldots(129)$$

therefore,

$$Q\,x = \frac{Q\,l}{2} \pm n\,\pi,$$

in which n may be any integer. Therefore,

$$x = \frac{l}{2} \pm \frac{n\,\pi}{Q}. \quad \ldots\ldots(130)$$

provided $0 < x < l$.

Equation (127) becomes,

$$M_x = \frac{w\,E\,I}{P}\left(\tan\frac{Q\,l}{2} \times \sin Q\,x + \cos Q\,x - 1\right) \quad \ldots\ldots(131)$$

As an example, take a 3 by 3 in. by ½-in. angle-iron, with an area of cross-section of 2.70 sq. in., a least moment of inertia of 0.90 in.^{4}, and a length of 60 in., giving the ratio of l to r of somewhat more than 100. Let the strut be in the position of least strength, as shown in Fig. 111. If $Q\,l = \pi$, Equation (130) gives,

$$x = \frac{l}{2} \pm n\,l$$

and, therefore, n must be zero. For all values of Q making $Q\,l < \pi$, n will be found to be zero because Equation (130) will give,

$$x = \frac{l}{2} \pm \frac{n\,\pi\,l}{(\pi - \text{something})}$$

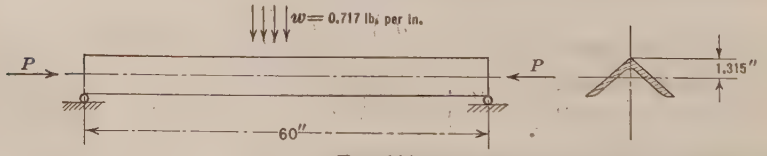


FIG. 111.

and x must lie between 0 and l. Hence, there is only one section to consider, namely the central one. $Q\,l = \pi$ was chosen both because it gives Euler's critical load and also because it is the value that makes M_x of Equation (131) infinite.

This critical load is,

$$\frac{\pi^2 E I}{l^2} = \frac{\pi^2 \times 30 \times 10^6 \times 0.90}{60 \times 60} = 74\ 022 \text{ lb.}$$

taking $E = 30 \times 10^6$ lb. per sq. in.

Study the effect of various loads less than the critical: Starting with 50 000 lb., Equation (131) gives:

$$M_{\frac{l}{2}} = \frac{w E I}{P}\left(\tan \frac{Q l}{2} \cdot \sin \frac{Q l}{2} + \cos \frac{Q l}{2} - 1\right) = \frac{w E I}{P}\left(\sec \frac{Q l}{2} - 1\right)$$

$$Q = \sqrt{\frac{5 \times 10^4}{27 \times 10^6}}; \quad \frac{Q l}{2} = \sqrt{\frac{5}{3}} \text{ radians} = 73° 58';$$

$$\frac{w E I}{P} = \frac{717 \times 27 \times 10^6}{5 \times 10^4} = 387.18.$$

Therefore,

$$M_{\frac{l}{2}} = 387.18\ (3.6206 - 1) = 1\ 014.65 \text{ in-lb.}$$

Similarly, the results for loads of 55 000 lb., etc., are given in Table 39 and Fig. 112.

TABLE 39.

Total load, in pounds.	Primary stress, in pounds per square inch.	Maximum bending moment (at center), in inch-pounds.	Secondary stress (top fiber), in pounds per square inch.	Total stress (top fiber), in pounds per square inch.
50 000	18 520	1 015	1 480	20 000
55 000	20 370	1 285	1 880	22 250
60 000	22 220	1 747	2 550	24 470
65 000	24 070	2 719	3 970	28 040
70 000	25 930	6 106	8 920	34 850
72 500	26 850	16 125	23 560	50 410
74 022	27 420	.....	.....	.....

Case 2.—Suppose two end bending moments are introduced into the foregoing problem, both assisting the weight of the beam. Take $M_1 = +\ 1\ 000$ in-lb. and $M_2 = +\ 2\ 000$ in-lb. Assuming a load of 50 000 lb., Equation (128) gives:

$$\tan\left(\frac{\sqrt{5}}{30\sqrt{3}} x\right)^c = \frac{2387.18 - 1387.18 \cos\left(\frac{2\sqrt{5}}{\sqrt{3}}\right)}{1387.18 \sin\left(\frac{2\sqrt{5}}{\sqrt{3}}\right)}$$

$$= \frac{2387.18 - 1387.18 \cos 147° 56'}{1387.18 \sin 147° 56'} = \frac{2387.18 + 1175.54}{736.46} = \tan 78.3207°$$

Therefore,

$$x = \frac{78.3207\ \pi}{180} \times \frac{30\sqrt{3}}{\sqrt{5}} = 31.765 \text{ in.}$$

and,

$$M_{max.} = M_{31.765\text{ in.}} = \frac{2387.18 - 1387.18 \cos 147° 56'}{\sin 147° 56'} \times \sin 78° 19'$$

$$+ 1387.18 \cos 78° 19' - 387.18 = 6571.6 + 280.9 - 387.18 = 6465.3 \text{ in-lb.}$$

Similarly, the results for loads of 55 000 lb., etc., are given in Table 40 and Fig. 113.

TABLE 40.

Total load, in pounds.	Primary stress, in pounds per square inch.	Maximum bending moments, in inch-pounds.	Distance from left-hand end, in inches.	Secondary stress (top fiber), in pounds per square inch.	Total stress (top fiber), in pounds per square inch.
50 000	18 520	6 465	31 765	9 450	27 970
55 000	20 370	8 276	31 317	12 090	32 460
60 000	22 220	11 382	30 920	16 630	38 850
65 000	24 070	17 930	30 562	26 200	50 270
70 000	25 930	40 790	30 239	59 600	85 530

Case 3.—Assume two end bending moments, both in opposition to the dead weight, namely, $M_1 = -1\,000$ in-lb. and $M_2 = -2\,000$ in-lb. Considering a load of 50 000 lb., Equation (128) gives,

$$\tan\left(\frac{\sqrt{5}}{30\sqrt{3}}x\right)^c = \frac{-1612.82 + 612.82 \cos 147° 56'}{-612.82 \sin 147° 56'} = \frac{2132.14}{325.25} = \tan 81.324°.$$

Therefore,

$$x = \frac{81.324\,\pi}{180} \times \frac{30\sqrt{3}}{\sqrt{5}} = 32.983 \text{ in.}$$

and,

$$M_{max.} = M_{32.983\text{ in.}} = -\frac{2132.14}{\sin 147° 56'} \times \sin 81° 19' - 612.82 \cos 81° 19' - 387.18$$

$$= -4449.7 \text{ in-lb.}$$

The negative bending moments mean that the top fibers have their primary stress (compressive in nature) decreased, and the bottom fibers have their primary stress increased. The results are given in Table 41 and Fig. 114.

TABLE 41.

Total load, in pounds.	Primary stress, in pounds per square inch.	Maximum bending moment, in inch-pounds.	Distance from left-hand end, in inches.	Secondary stress (bottom fiber), in pounds per square inch.	Total stress, (bottom fiber), in pounds per square inch.
50 000	18 520	— 4 449.7	32.98	3 985	22 505
55 000	20 370	— 5 715.6	32.33	5 120	25 490
60 000	22 220	— 7 893.4	31.05	7 070	29 290
65 000	24 070	—12 492	30.84	11 190	35 260
70 000	25 930	—28 573	30.35	25 590	51 520

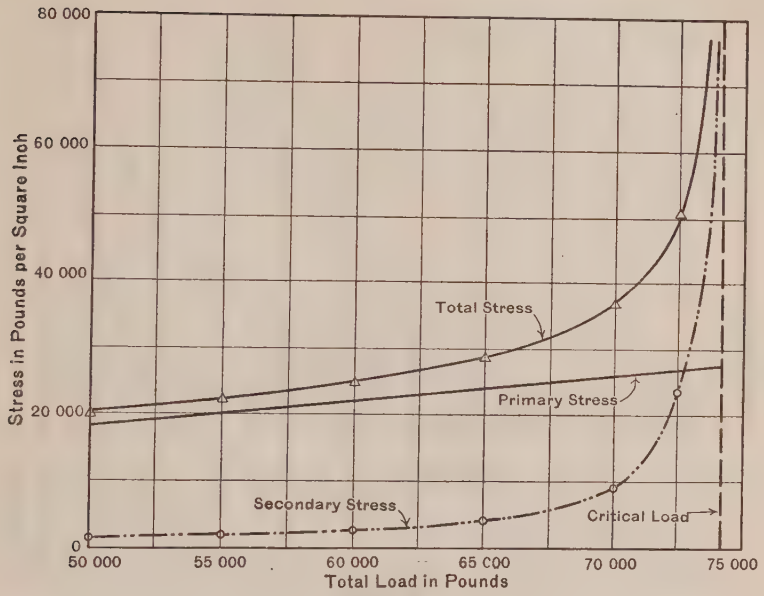


FIG. 112.

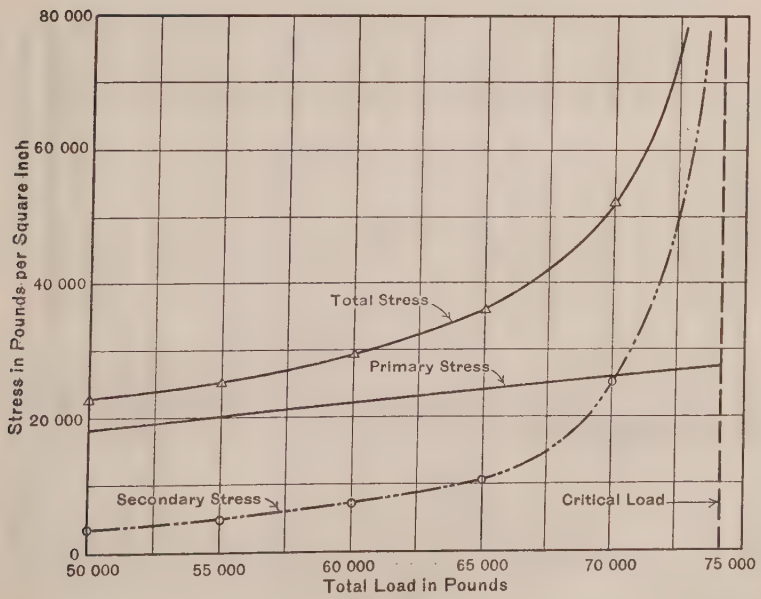


FIG. 113.

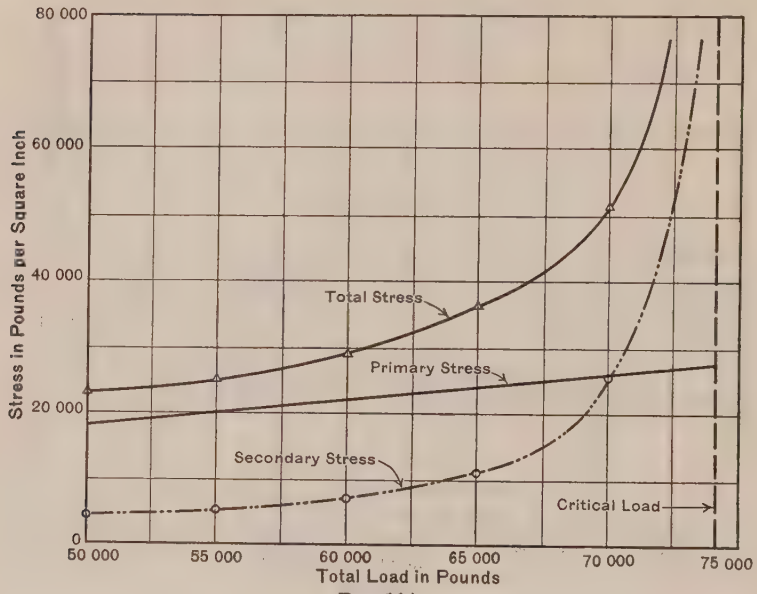


FIG. 114.

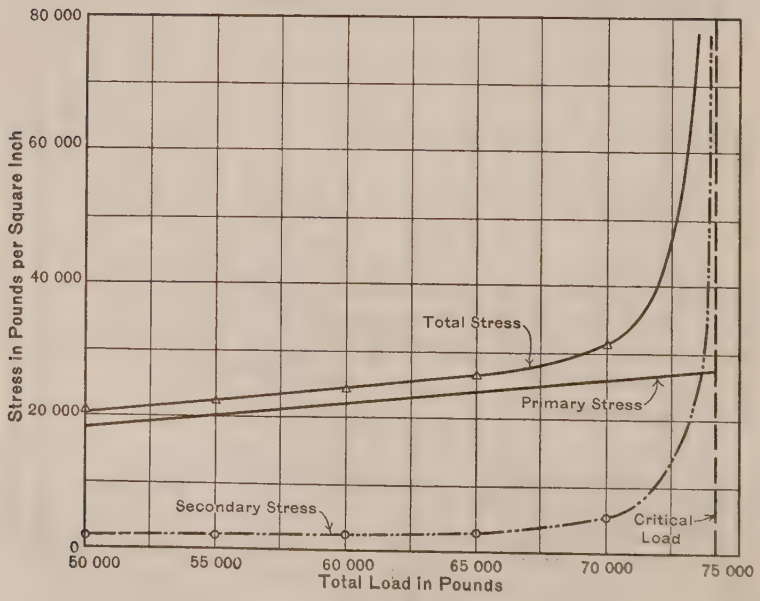


FIG. 115.

Case 4.—Consider the effect of end bending moments, opposite in character, say, $M_1 = +\ 1\,000$ in-lb. and $M_2 = -\ 2\,000$ in-lb. Considering a load of 50 000 lb., Equation (128) gives,

$$\tan\left(\frac{\sqrt{5}}{30\sqrt{3}}x\right) = \frac{(387.18 - 2\,000) - (387.18 + 1\,000)\cos 147° 56'}{1\,387.18 \sin 147° 56'}$$

$$= -\frac{437.28}{736.46}$$

giving,

$$\frac{\sqrt{5}\,x}{30\sqrt{3}} = -\frac{30.7}{180}\pi + \pi,$$

because, if anything, x must be positive. This gives $x = 60.553$ in., an amount greater than the length of the member. Hence, up to a load of a little more than 50 000 lb., the maximum bending moment occurs at one end. In this case it is — 2 000 in-lb. The bending moment at the center is:

$$M_{30} = -\frac{437.28 \sin 73° 58'}{\sin 147° 56'} + 1\,387.18 \cos 73° 58' - 387.18 = -795.66 \text{ in-lb.}$$

Considering a load of 55 000 lb., Equation (128) gives:

$$\tan\left(\frac{\sqrt{11}}{30\sqrt{6}}x\right) = \frac{(351.98 - 2\,000) - (351.98 + 1\,000)\cos 155° 10'}{1\,351.98 \sin 155° 10'}$$

$$= -\frac{421.04}{567.80} = \tan(-36° 33') = \tan 143° 27'$$

giving,

$$x = 55.470 \text{ in.}$$

The increase from 50 000 to 55 000 lb. has caused a bending moment within the length of the member, the amount of which will be numerically greater than M_2:

$$M_{55.470\text{ in.}} = -\frac{421.04 \sin 143° 27'}{\sin 155° 10'} + 1\,351.98 \cos 143° 27' - 351.98$$

$$= -2\,035.04 \text{ in-lb.}$$

Similarly, for 60 000 lb., etc., the results are given in Table 42 and Fig. 115.

TABLE 42.

Total load, in pounds.	Primary stress, in pounds per square inch.	Maximum bending moment, in inch-pounds.	Distance from left-hand end, in inches.	Secondary stress (bottom fiber), in pounds per square inch.	Total stress (bottom fiber), in pounds per square inch.
50 000	18 520	—2 000	60	1 790	20 310
55 000	20 370	—2 035	55.47	1 820	22 190
60 000	22 220	—2 220	49.68	2 000	24 220
65 000	24 070	—2 840	42.93	2 540	26 610
70 000	25 930	—5 808	35.42	5 200	31 130

It is evident that the secondary stress is reduced if there are end bending moments that produce contraflexure, and it will be still further reduced if the larger bending moment opposes the effect of the transverse load. The

analysis is also interesting in that it determines the axial load that must be subjected before bending moments larger than the end moments occur.

Case 5.—To illustrate the effect of changing the length of a compression member without changing the thrust, assume a built-up beam to be placed horizontally in a testing machine in the position of least strength, as shown on Fig. 116. Consider lengths of 120, 160, and 200 in., giving the ratios of l to r of 75, 100, and 120, respectively. As there will be some resisting moments

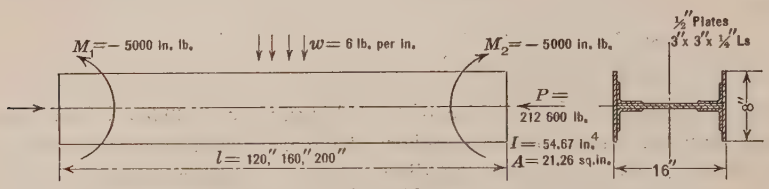


FIG. 116.

at the ends, assume them to be 5 000 in-lb. each. Up to the critical load, the maximum bending moment will occur at the center of the beam. Loads greater than the critical value are not considered. A thrust giving a primary stress of 10 000 lb. per sq. in., is applied to the three lengths, and the problem is repeated for a thrust giving a primary stress of 20 000 lb. per sq. in. Table 43 gives the results, obtained by means of Equation (127).

TABLE 43.

Primary stress, in pounds per square inch.	Length of member, in inches.	Bending moment, at center, in inch-pounds.	Maximum compressive fiber stress, in pounds per square inch.
10 000	120	6 503	10 465
10 000	160	21 052	11 540
10 000	200	52 264	13 825
20 000	120	26 364	21 930
20 000	160	77 764	25 690
20 000	200	Load greater than critical load.	

Section B.—Uniform Transverse Load Over Part of Span

Consider the problem of a compression member with an axial load, two end bending moments, and a uniform transverse load over part of the span, all acting in one plane, as shown in Fig. 117.

For $0 < x < a$,

$$M_x = M_1 + V_1 x + P y \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (132)$$

for $a < x < (a + b)$,

$$M_x = M_1 + V_1 x - \frac{w (x - a)^2}{2} + P y \quad \ldots\ldots\ldots\ldots \quad (133)$$

and for $(a + b) < x < l$,

$$M_x = M_1 + V_1 x - w b \left(x - a - \frac{b}{2}\right) + P y \quad \ldots\ldots\ldots \quad (134)$$

$$\left.\begin{aligned} V_1 &= \frac{w\,b\left(c+\frac{b}{2}\right)}{l} - \frac{M_1 - M_2}{l} \\ V_2 &= \frac{w\,b\left(a+\frac{b}{2}\right)}{l} + \frac{M_1 - M_2}{l} \end{aligned}\right\} \quad \ldots\ldots\ldots (135)$$

The differential equations are:

For $0 < x < a$,

$$E\,I\frac{d^2y}{dx^2} + P\,y = -V_1\,x - M_1 \quad \ldots\ldots\ldots (136)$$

for $a < x\ (a+b)$,

$$E\,I\frac{d^2y}{dx^2} + P\,y = \frac{w\,x^2}{2} - (V_1 + w\,a)\,x - \left(M_1 - \frac{w\,a^2}{2}\right) \ldots\ldots (137)$$

and for $(a+b) < x < l$,

$$E\,I\frac{d^2y}{dx^2} + P\,y = -(V_1 - w\,b)\,x - \left\{ M_1 + w\,b\left(a+\frac{b}{2}\right)\right\} \ldots (138)$$

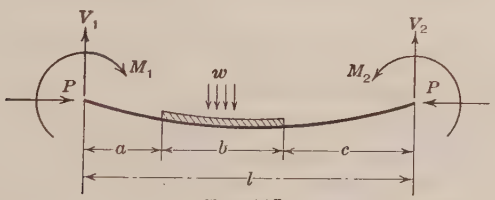


FIG. 117.

The general solutions are:

For $0 < x < a$,

$$y = A\sin Q\,x + B\cos Q\,x - \frac{V_1\,x}{P} - \frac{M_1}{P} \quad \ldots\ldots\ldots (139)$$

for $a < x < (a+b)$,

$$y = C\sin Q\,x + D\cos Q\,x + \frac{w\,x^2}{2\,P} - \frac{(V_1 + w\,a)\,x}{P} - \frac{\left(M_1 - \frac{w\,a^2}{2}\right)}{P} \ldots (140)$$

and for $(a+b) < x < l$,

$$y = E\sin Q\,x + F\cos Q\,x - \frac{(V_1 - w\,b)\,x}{P} - \frac{\left\{ M_1 + w\,b\left(a+\frac{b}{2}\right)\right\}}{P} \quad \ldots\ldots\ldots (141)$$

From the six conditions: — y from Equation (139) $= 0$, $x = 0$; y from Equation (139) $= y$ from Equation (140), $x = a$; $\frac{dy}{dx}$ from Equation (139), $= \frac{dy}{dx}$ from Equation (140), $x = a$; y from Equation (140) $= y$ from Equation

(141), $x = a + b$; $\frac{dy}{dx}$ from Equation (140) $= \frac{dy}{dx}$ from Equation (141), $x = a + b$; y from Equation (141) $= 0$, $x = l$; the six constants are determined. The final equations for the bending moments are:

For $0 < x < a$,

$$M_x = \frac{\sin Q x}{\sin Q l}\left[\frac{w E I}{P}\left\{\cos Q c - \cos Q (b + c)\right\} - M_1 \cos Q l + M_2\right] + M \cos Q x \quad \ldots\ldots (142)$$

for $a < x < a + b$,

$$M_x = \frac{\sin Q x}{\sin Q l}\left[\frac{w E I}{P}\left\{\cos Q c - \cos Q a \cdot \cos Q l\right\} - M_1 \cos Q l + M_2\right] + \left[M_1 + \frac{w E I}{P}\cos Q a\right]\cos Q x - \frac{w E I}{P} \quad \ldots\ldots (143)$$

and for $a + b < x < l$,

$$M_x = \frac{\sin Q x}{\sin Q l}\left[\frac{w E I}{P}\cos Q l\left\{\cos Q (a + b) - \cos Q a\right\} - M_1 \cos Q l + M_2\right] + \left[M_1 - \frac{w E I}{P}\left\{\cos Q (a + b) - \cos Q a\right\}\right]\cos Q x \quad \ldots\ldots (144)$$

Section C.—Concentrated Transverse Load

Mathematically, a concentrated load is considered to act at a point. If the problem of a concentrated transverse load is investigated, one arrives at a point of discontinuity for the bending moment just under the load. In practice, however, a concentrated load is always found to be spread over some definite distance. Equations (142), (143), and (144) are then used to determine the bending moments. A problem similar to the one given by Mr. Godfrey will be considered. The reference is to his Fig. 106 (*A*). By virtue of the fact that the post was initially straight at the top, the forcing of a ball between the two parts of the post will create a large bending moment at the top, and this bending moment will act in opposition to the pressure of the ball. A certain bending moment will be found at the base as well, due to friction, etc. This, too, will oppose the action of the ball.

Consider a built-up strut, held in a vertical position, with a load of 263 000 lb. on it and a horizontal pressure of 10 000 lb. at its center, acting in the direction of least strength of the strut and assumed to be acting over a space of 4 in. Fig. 118 shows the strut in a horizontal position.

Let,

$I = 106.93$ in.^{4};

Area $= 26.3$ sq. in.;

$w = 2\,500$ lb. per in.;

$$\frac{P}{E I} = Q^2 = \frac{263\,000}{30 \times 10^6 \times 106.93};$$

$Q l = 62° 15'$;

$Q c = Q a = 30° 5'$;

$Q (a + b) = Q (b + c) = 32° 10'$

Substituting in Equations (142) and (144) and differentiating, no maxima points are found within Spaces *a* and *c*, but Equation (143) gives $Q\,x = 31°\ 16'$ for the point of maximum bending moment. The center of the strut is given by $Q\,x = 31°\ 7\frac{1}{2}'$, so the maximum bending moment occurs just beyond the center.

$$\frac{w\,E\,I}{P} = 3.049335 \times 10^7$$

$$M_{max.} = \frac{\sin 31°\ 16'}{\sin 62°\ 15'}\left[3.049335 \times 10^7 \times \cos 30°\ 5'\ (1 - \cos 62°\ 15')\right.$$

$$\left. + 10^5 \cos 62°\ 15' - 2 \times 10^4\right] + \left[3.049335 \times 10^7 \times \cos 30°\ 5' - 10^5\right]$$

$$\cos 31°\ 16' - 30\,493\,350 = 259\,700 \text{ in-lb.}$$

$M_{60\text{ in.}} = 259\,600$ in-lb.

$M_{58\text{ in.}} = 253\,050$ in-lb.

Upon solving the problem under the supposition of freely-hinged ends:

$$M_{max.} = M_{\text{center}} = 295\,900 \text{ in-lb.}$$

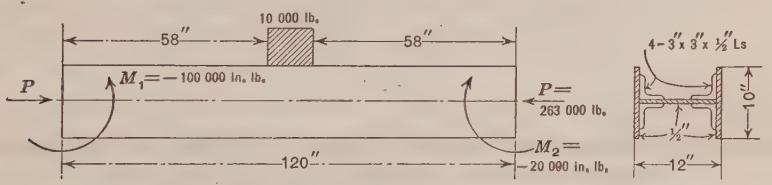


FIG. 118.

Section D.—Strut, Originally Straight, Subjected to an Eccentric Load

If *P* of Fig. 110 is off center, a positive distance, *e*, Equation (123) becomes:

$$M_x = M_1 + P\,(e + y) + V_1\,x - \frac{w\,x^2}{2},$$

resulting in changes of M_1 to $M_1 + Pe$ and M_2 to $M_2 + Pe$ in Equation (127). Hence, for eccentric loading,

$$M_x = \frac{\sin Q\,x}{\sin Q\,l}\left[\left(\frac{w\,E\,I}{P} + M_2 + P\,e\right) - \left(\frac{w\,E\,I}{P} + M_1 + P\,e\right)\cos Q\,l\right]$$

$$+ \left(\frac{w\,E\,I}{P} + M_1 + P\,e\right)\cos Q\,x - \frac{w\,E\,I}{P} \quad \ldots\ldots\ldots\ldots (145)$$

Similar changes occur in the Equations (142), (143), and (144).

Section E.—Tension Members

Consider the case of a tension member subjected to transverse loading, that is, assume *P* of Fig. 110 to act in the opposite direction. Equation (123) becomes:

$$M_x = M_1 + V_1\,x - P\,y - \frac{w\,x^2}{2} \quad \ldots\ldots\ldots\ldots (146)$$

Equation (124) becomes:

$$E\,I\,\frac{d^2\,y}{d\,x^2} - P\,y = \frac{w\,x^2}{2} - V_1\,x - M_1 \quad \ldots\ldots\ldots\ldots (147)$$

the general solution of which is:

$$y = A\,e^{Qx} + B\,e^{-Qx} - \frac{w\,x^2}{2\,P} + \frac{V_1\,x}{P} - \frac{w\,E\,I}{P^2} + \frac{M_1}{P} \ldots\ldots (148)$$

$y = 0$ when $x = 0$, and $x = l$ gives:

$$A = \frac{\left(\frac{w\,E\,I}{P^2} - \frac{M_2}{P}\right) - \left(\frac{w\,E\,I}{P^2} - \frac{M_1}{P}\right) e^{-Ql}}{(e^{Ql} - e^{-Ql})}$$

and,

$$B = \frac{\left(\frac{w\,E\,I}{P^2} - \frac{M_1}{P}\right) e^{Ql} - \left(\frac{w\,E\,I}{P^2} - \frac{M_2}{P}\right)}{(e^{Ql} - e^{-Ql})}$$

The equation for the bending moment at x is:

$$M_x = \frac{w\,E\,I}{P} - \frac{\left(\frac{w\,E\,I}{P} - M_2\right) - \left(\frac{w\,E\,I}{P} - M_1\right) e^{-Ql}}{(e^{Ql} - e^{-Ql})} \times e^{Qx}$$

$$- \frac{\left(\frac{w\,E\,I}{P} - M_1\right) e^{Ql} - \left(\frac{w\,E\,I}{P} - M_2\right)}{(e^{Ql} - e^{-Ql})} \times e^{-Qx} \ldots\ldots (149)$$

Section F.—Compression Members, Subjected to Pairs of End Bending Moments in Planes at Right Angles to Each Other.

This case is perhaps the most important. In considering the top chord of a bridge, end bending moments are found that act in a perpendicular plane and are due to the rigidity of the truss, and also those in the horizontal plane due to the rigidity of the lateral system. The resultant of the pair at one end is inclined at some angle to the vertical plane. If the resultant of the pair at the other end, the resultant of the uniform dead weight of the member, and the uniform wind load were to act in the same inclined plane, one would perhaps be justified in applying the theory of unsymmetrical bending.* This supposition is hardly ever true. It seems, however, that if the forces and moments in the vertical and horizontal planes are studied separately and the resulting bending moments at any particular section combined, a result would be determined which should be close to, if not actually, the true value.

As an illustration, investigate the chord member, 2–4, of the bridge studied by the writer in his paper. From the results given on pages 144 and 145 the maximum thrust is found to be 510 500 lb. and the vertical end moments are — 104 680 and + 335 800 in-lb. for Ends (2) and (4), respectively. The uniform dead load is estimated to be 16 lb. per in. and the moment of inertia is 2385.4 in.^{4} The horizontal end moments are + 275 040 and + 355 880 in-lb. for Ends (2) and (4), respectively, while the wind load, assumed to be acting on the outside of the chord, is taken as 6 lb. per lin. in., which is rather a high estimate. The moment of inertia to be used in this problem is 2852.45

* "Modern Framed Structures," Vol. III, Johnson, Bryan, and Turneaure.

in.⁴ The length of the member is 300 in. and E is taken as 30×10^6 lb. per sq. in. (See Fig. 119.)

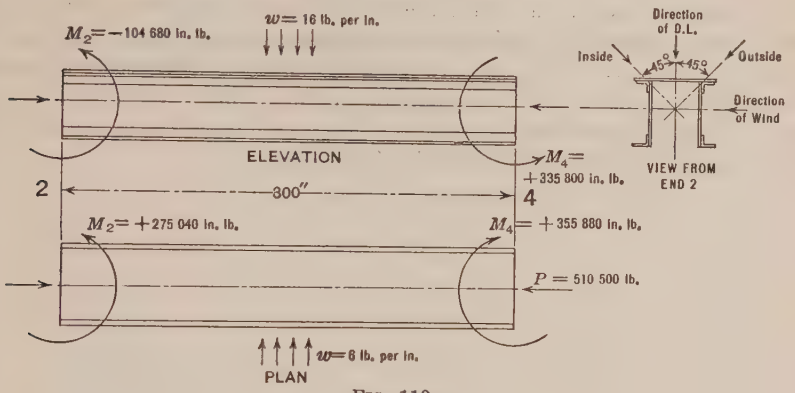


FIG. 119.

Consider the effect at the center: Equation (127) gives,

$M_c = +$ 318 360 in-lb. for the vertical plane.

and

$M_c = +$ 409 380 in-lb. for the horizontal plane.

The plus signs denote that the bending moments are increasing the compressive stresses in the top and the outside fibers, respectively.

By way of determining the effect in planes other than the vertical and horizontal, the two 45° planes were considered. For the plane inclined from the outside the combined end bending moments were + 120 460 and 489 090 in-lb. for Ends (2) and (4), respectively. The plane of bending is determined by $\tan^{-1}\left(\frac{I_x}{I_y}\cot 45°\right)$, that is, 39° 54¼′ to the horizontal, giving a moment of inertia of,

$$I_x \cos^2 39° 54\tfrac{1}{4}' + I_y \sin^2 39° 54\tfrac{1}{4}'$$

that is, 2 577.6 in.⁴ The bending moment at the center was found to be $M_c = +$ 515 460 in-lb. Similarly, the bending moment at the center for the plane inclined from the inside was found to be $M_c = -$ 67 740 in-lb. The results, $M_c = +$ 318 360 and $M_c = +$ 409 380, for the vertical and horizontal planes combine to give $\frac{318\,360 + 409\,380}{\sqrt{2}}$ and $\frac{318\,360 - 409\,380}{\sqrt{2}}$, that is + 514 590 and − 64 360 in-lb., for the respective inclined planes. There are, therefore, differences in the results which seem to point to further research in the theory of unsymmetrical bending.

Assuming, however, that combinations of the results obtained from the vertical and horizontal planes give true values, consider the effect at the section, 200 in. from End (2). Equation (127) gives:

$M_{200} = +$ 370 750 in-lb. for the vertical plane

and

$$M_{200} = +412\,590 \text{ in-lb. for the horizontal plane}$$

The particular section at which the most severe combined bending moment exists could be found by considering a few more sections. Assuming the 200-in. section as the critical one, + 370 750 in-lb. gives an additional compressive stress of 1 450 lb. per sq. in. to the outside top fiber, and + 412 590 in-lb. gives 1 446 lb. per sq. in. to the same fiber. The total stress, therefore, is 11 782 + 1 450 + 1 446 = 14 678 lb. per sq. in. in compression.

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Paper No. 1575

FIRE-BANKS FOR OIL STORAGE*

By H. H. Hall,† M. Am. Soc. C. E.

With Discussion by Messrs. H. L. Shoemaker, Frank A. Epps, Ralph J. Reed, and H. H. Hall.

Synopsis

During 1924 larger quantities of petroleum and petroleum products have been in storage in the United States than ever before. Considering the quantity of this oil in storage and its generally inflammable nature, the fire loss has been remarkably low.

Refined oils (which comprise by far the largest part of oil storage in or near cities and towns), when ignited, simply burn quietly until they are extinguished or burned dry; but crude petroleums, and occasionally refined fuel oils, are liable to boil over while burning. In order to make the storage of these latter products as safe as that of the refined oils, it is necessary to surround the container with so-called fire-walls.

The service to be rendered by these structures is a peculiar one from an hydraulic standpoint, as their primary function is to withstand the single wave of oil thrown out by the "boil-over". In an effort to find the best possible design, the series of model tests here described was conducted, the outstanding result being a demonstration of the value of some form of flash-board or coping which will curl the advancing wave back on itself.

Introduction

The discovery of a number of extremely prolific petroleum pools in both Oklahoma and California during 1922 and 1923 has resulted, in spite of a rapidly growing market, in a considerable over-production of all forms of petroleum products. In round figures, oil in storage has increased since

* Presented at the meeting of June 3, 1925.

† Chf. Engr., Standard Oil Co. (California), San Francisco, Calif.

1921 from 200 000 000 bbl. to more than 500 000 000 bbl. This phenomenal growth has fixed the attention of many petroleum engineers on the design and proper protection of oil storage, with the result that a number of advantageous changes have been made. One of the most obvious changes, brought about by this study, has been a material increase in the size of storage units—both of the reservoir and the steel-tank type. Concrete-lined reservoirs have been built to contain more than 3 000 000 bbl. (42 gal. each); and steel tanks holding 150 000 bbl. per unit are not uncommon.

Among the changes looking toward better protection of the storage there has been a considerable extension in the use of fire-proof and vapor-proof roofs, and the addition of numerous devices which are intended primarily to prevent evaporation loss, but which incidentally improve fire protection by lessening or eliminating the escape of inflammable vapors.

Although in such carefully protected storage fires are a rather rare occurrence (particularly in California where the danger from lightning is small) the study of methods for guarding against the spread of fire has not escaped some scrutiny in connection with this general study of oil storage. Unfortunately, some of these products have a disconcerting habit of boiling over after burning for varying lengths of time. Recent studies have shown that these boil-overs are confined to crude, and, in some cases, to fuel, oils. In order to prevent the possible spread of burning oil from structures containing either of these products, it is necessary to surround them with a fire-bank or wall of such dimensions that it will not be topped by the sudden wave-like advance of the oil which has boiled out. Aside from this requirement, the fire-walls need not, in general, be able to withstand much static pressure. The boil-overs, while sudden and violent, do not as a rule discharge any large part of a tank's contents; and although occasional tank leaks occur, the complete collapse of a full tank is so rare as to justify the designer in almost neglecting this possibility. The fire-wall problem is, therefore, primarily a question of stopping a single wave of high velocity.

Lack of knowledge as to which oils are likely to boil over, as well as lack of attention to this principal requirement of fire-wall design, has had two unfortunate results. Occasionally earth fire-walls have been built so low as to be topped by burning oil; and frequently city authorities and fire protection engineers, in the absence of exact knowledge as to what emergency to expect and how to meet it, have insisted on excessive clearance distances and fire-wall requirements around oil storage. With a view to clearing up the general question of fire-wall design, both of the permanent and temporary types, Standard Oil Company of California decided to undertake a series of model tests. Ultimately, these included more than 1 000 miniature boil-overs (using water for most of the tests) directed against 66 differently shaped walls and other obstructions.

Apparatus

The principal item of apparatus used was a wooden trough representing, at a scale of 1 to 14, a segment extending radially from the center of a tank

115 ft. in diameter. (See Figs. 1 and 2.) At the tank end of this trough was placed a box from which the boil-overs could be discharged by jerking open a gate; the area of this box was made considerably larger relatively than that of a segment of the tank, with the thought that when a boil-over occurs, the frothing oil coming out of the tank is generally driven by the wind, so that most of it is concentrated on one side of the tank. The box was made adjustable so that the height from which the "spills" occurred could be varied, corresponding to different heights of tanks and refinery receiving "pans".

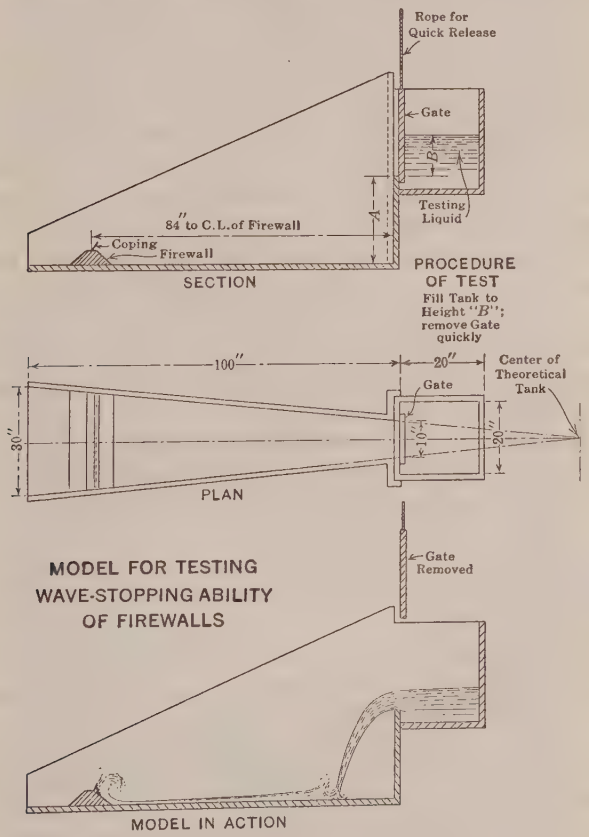


FIG. 1.

Model fire-walls were constructed, to the same scale, and placed at various locations in the trough. As shown in Fig. 1, the wall is at a distance corresponding to 100 ft. from the tank, this being common practice for walls surrounding tanks on comparatively level ground. A majority of the tests were made with the walls in this position; but data were also obtained for locations (to scale) 50 ft. and 20 ft. from the tank. Model walls of the following shapes were tested:

1.—Earth walls with a 1 on 1.5 side slope;

2.—Earth walls with a 1 on 1 side slope (Fig. 3, right);

3.—Vertical walls (Fig. 4, right); and
4.—Modifications of the foregoing, using copings of various sizes and shapes (see Figs. 3 and 4).

Procedure and Results

1.—Preliminary Tests.—Some preliminary experimenting was first done to determine (1) the way in which the gate releasing the water must be jerked open in order to obtain uniform results; (2) the criterion which should be used in judging the performance of different wall shapes; and (3) the effect of varying the height of the box from which the spill was discharged (Dimension *A*, Fig. 1). These tests are described in detail in the Appendix.

2.—Comparison of Standard Shapes.—The first main series of tests was run to compare the effectiveness of four of the most common shapes of wall now in use. The results are summarized by Curves 1 to 5, Fig. 5.

It will be noted that the curves for both shapes of earth wall, and for the plain vertical wall, are grouped close together, and have the same general shape and steep slope. Neither of these results is surprising, since all three shapes of wall depend primarily on height alone to stop the rush of water. This height, in turn, must depend approximately on the total energy of the oncoming stream, for the impact of the entire mass of water acts to push the topmost drops to a maximum height, with little loss in turbulence up to the time the spill takes place. As this total energy is approximately proportional to the square of the weir height, W,* it might be anticipated that the necessary height of wall, H, would vary approximately with the square of W. As a matter of fact, Curve 1 can be closely represented by the equation $H = 0.5\ W^{2.13}$.

In the case of the coped walls the situation is entirely different. Here, the principal part of the energy is expended in turbulence and shock when the coping turns the wave back on itself. The wall can be topped only when the energy of the oncoming stream is so great that all of it cannot be dissipated, and the remainder is sufficient to lift some of the water over the wall. Under these circumstances, as indicated by Curve 5, the height of the wall needs to be increased only approximately in direct proportion to the height of the weir. This result indicates that the efficacy of the coping as an energy absorber increases with the height of the wall and the violence of the wave—at least, over the range of heights covered by these tests.

Observation of the performance of the model coped walls during these tests showed that the effect of the coping is to turn all of the primary wave back inside the wall; in almost no case did any appreciable quantity pass over a coped wall in the first rush. The water turned back by the coping eventually piles up on that approaching from the tank until, after the velocity of approach is spent, there is for the instant a head causing flow over the wall. In any actual tank field a wave of oil would be free to spread out sideways, and it is probable that the secondary wave which tops the wall could not form

* The energy represented by the elevation of the bottom of the weir box above the floor board is not considered available, since it appears as vertical velocity and is largely dissipated in shock.

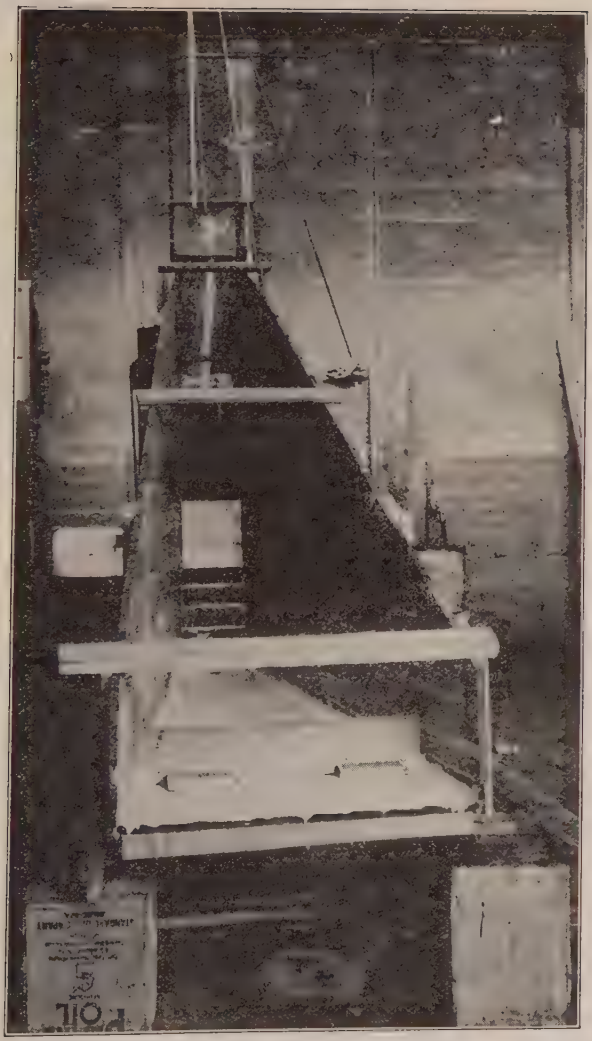


FIG. 2.—FRONT VIEW OF TEST APPARATUS, SHOWING MODEL COPED WALL IN PLACE. THE WALL WAS CONSIDERED TO HAVE STOPPED THE WAVE WHEN NOT ENOUGH WATER CAME OVER TO CAUSE ANY TO RUN OFF THE "TAIL-BOARD."

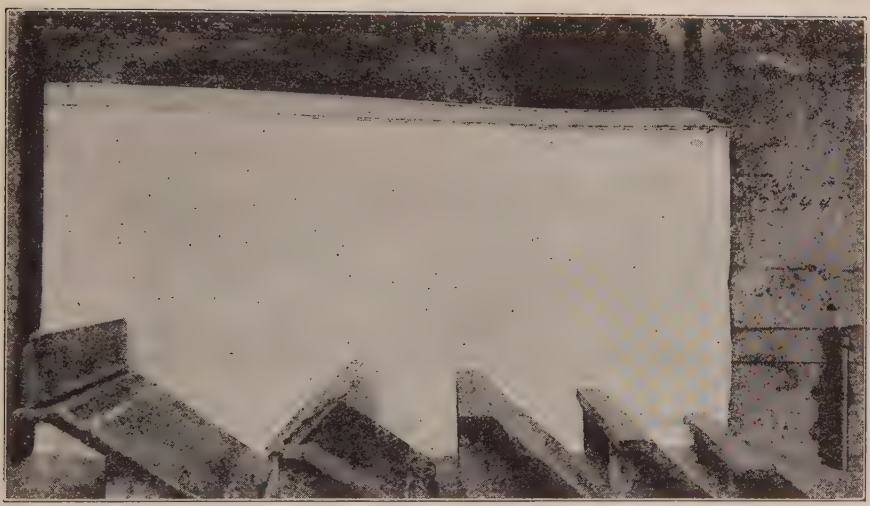


FIG. 3.—MODELS OF 1 ON 1 EARTH WALLS (PLAIN), AND 1 ON 1.5 EARTH WALLS WITH EXTENSIONS. ONLY THE FRONT HALF OF THE WALL WAS REPRODUCED, AS THE BACK HALF WOULD HAVE NO BEARING ON THE RESULTS.

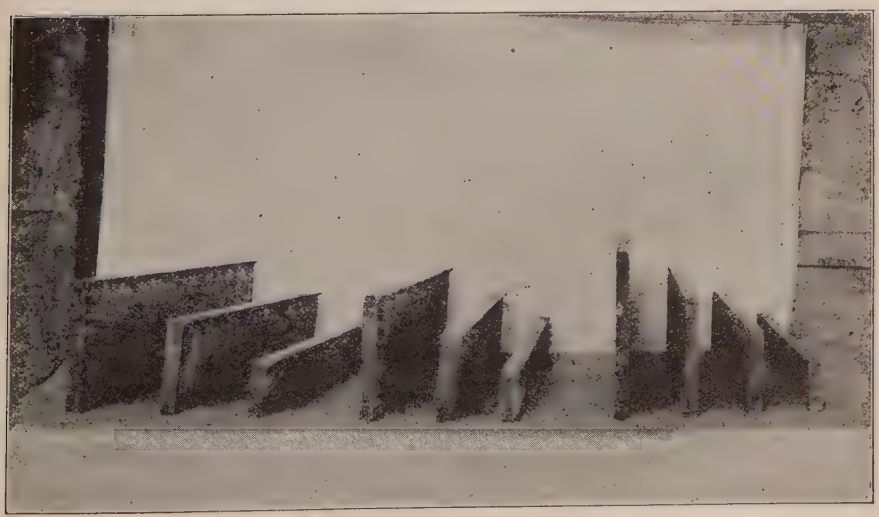
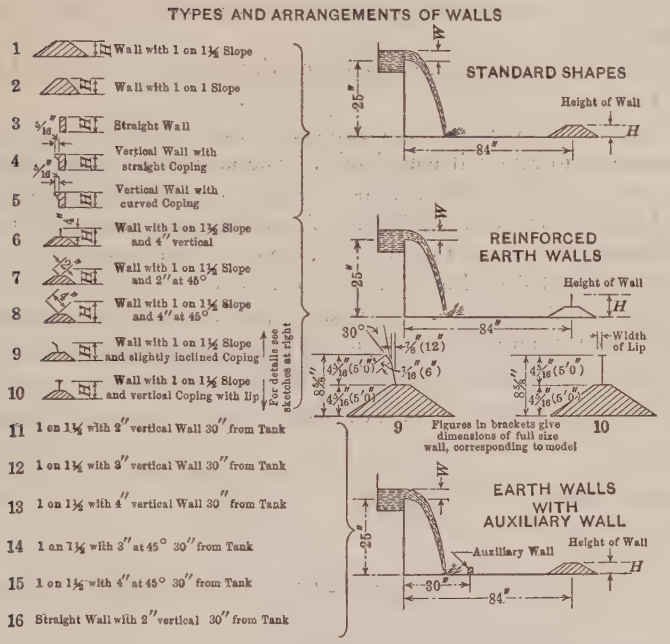


FIG. 4.—MODELS OF VERTICAL WALLS, PLAIN AND WITH COPINGS.



NOTE: Models were made to represent a Tank 30′ high with C.L. of firewall 100′ from the shell. Actual Scale used was 1:14; so that roughly 1″=1′+

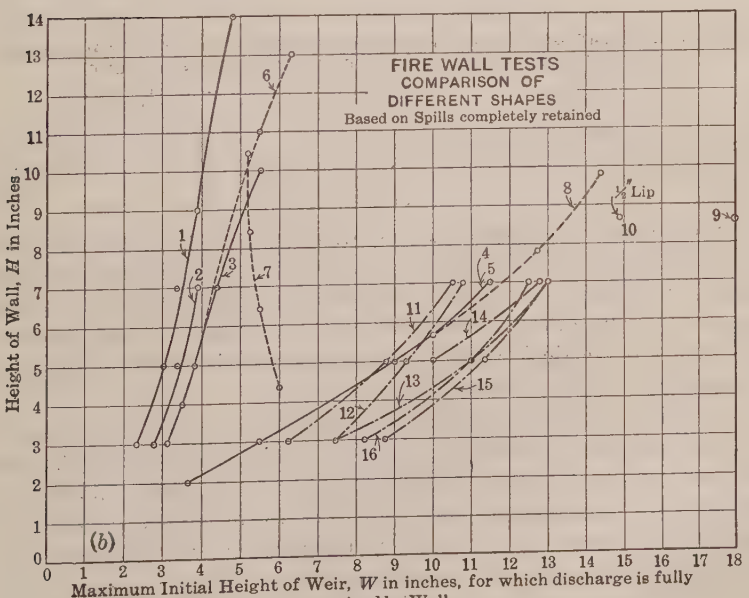


FIG. 5.

so easily as in the experimental model where the water is closely constrained by the side-walls.

The results indicated by these curves of Fig. 5 may be compared from two points of view:

(*a*) With a boil-over of given intensity, what height of each shape of wall is necessary to retain it completely? The answer is obtained by reading from the curves of Fig. 5 the heights of different walls that will stop a given "height of weir", for instance, 3½ in. A graphic comparison of the results is shown in the top row of Fig. 6. On account of the steepness of the slope of Curves, 1, 2, and 3 of Fig. 5, a very considerable difference in height is observed between the walls of different shapes necessary to stop this 3½-in. discharge. Furthermore, it may be noted that a coped wall of less than one-half the height of the straight wall will be equally effective.

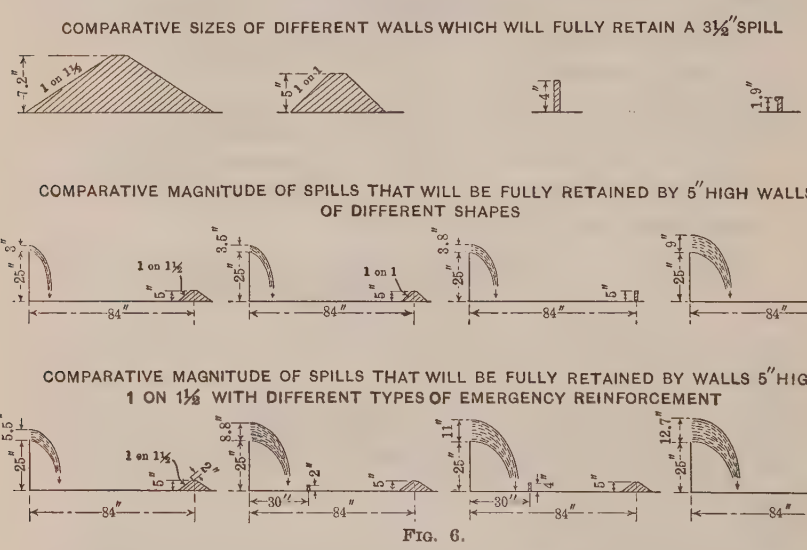


FIG. 6.

(*b*) With walls of the same height, what magnitude of boil-over will be stopped by different shapes? The middle row of Fig. 6 gives this comparison graphically for walls 5 in. high. When judged by this criterion, there is comparatively little difference between the effectiveness of the sloped walls and of the plain vertical walls; whereas, the coped vertical wall is almost three times as effective as any other.

The outstanding feature of these two comparisons is the effectiveness of the coped wall. This is further emphasized by the detailed curves for the different walls shown in the Appendix (Figs. 17 and 18). It will be noted that the curves for walls with sloping sides (Fig. 17) ascend rather steeply, whereas, the curves for the straight and coped walls (Fig. 18) are relatively flat. Considering, for example, the case of a 10-in. spill discharged against walls, 5 in. high, it will be noted that 40% of this spill passed over the 1 on 1.5 wall (Fig. 17), while under like conditions less than one-half of 1% passed over the vertical wall fitted with a coping (Fig. 18). Thus, while the actual "dry-points"

plotted on Fig. 5 are probably the most conservative criterion on which to judge the different shapes, it appears that a wall which permits not more than 1% of the spill striking it to pass over may be regarded as satisfactory under most conditions. The curves on Fig. 19 have been plotted to compare the different shapes on this basis. It will be noted that the coped walls make an even more favorable showing in comparison with the walls having sloping sides.

3.—Methods of Reinforcing Standard Walls.—

(*a*).—Extension on Top of Wall.—A method that has been used to some extent for increasing the effectiveness of earth walls has been to place on top of them a vertical extension consisting of scrap steel plates. The models of walls with a 1 on 1.5 slope were equipped with such extensions 4 in. high (representing an actual height of about 5 ft., see model at the left in Fig. 3). The results are plotted as Curve 6 of Fig. 5. It will be noted that this extension resulted in comparatively little increase in the effectiveness of the wall, judged on the basis of complete stoppage of the wave. However, in testing this model it was found that the extension markedly reduced the percentage of the larger spills that passed over the wall.

Observation of the behavior of this coped wall indicated that it would be desirable to tip the extension in toward the tank instead of leaving it vertical. This was accordingly done, with the results shown on Curve 8, Fig. 5. It will be noted that this greatly increased the effectiveness of the wall. (See, also, Figs. 7 and 8.) Tests were run using extensions sloped at both 30° and 45° from the vertical. No appreciable difference in the results could be observed. As the 30° slope would probably be more convenient in actual practice, it is to be recommended.

This 4-in. "coping" proved so effective it was felt that an extension of one-half the height (still tilted back toward the tank) would give satisfactory results. This was tried, with the results shown by Curve 7 of Fig. 5.

It will be noted that even this small reinforcement materially increased the effectiveness of the wall, particularly at lower heights. The shape of this curve, indicating as it does that high walls will not stop such large boil-overs as low ones, is peculiar; but it has been carefully checked and found to be correct. The explanation seems to be that, with such a small coping, the long slope of the higher walls gives the force of the wave such a strong component in an upward direction that the coping is less able to break it up than when placed on a lower wall with a shorter approach.

In discussing the practical application of these "copings", it was suggested that if they were sloped back toward the tank at an angle of 45°, or even 30°, there might be danger that burning oil discharged against the base of the fire-wall in a preliminary "slop-over" would so warp the sheets that they would not be in shape to function at the time of the main boil-over. Tests were accordingly run using the shape numbered "9" on Fig. 5, in which the coping was placed in an almost vertical position, with a short lip at the top turned back at a somewhat sharper angle. Only one height of this shape was tried, as shown by a single point on Fig. 5, but it will be noted that this point is well to the right of the others, indicating that this shape was the most effective of those tested. (See, also, Figs. 9, 10, and 11; the "wings"

shown in Fig. 11 were used in testing the other coped walls, as it was felt that the corner effect observed in these models would not obtain in practice and tended to obscure the real effectiveness of the different walls). From the results obtained with this model it is felt that a wall and coping of the dimensions shown in the detailed Sketch No. 9, Fig. 5, would completely turn back the wave from the most severe boil-over likely to occur in practice, provided the wall is 50 ft. or more from the tank.

It was next suggested that for reinforcing division walls between two tanks, a vertical coping with a horizontal lip on top, as shown in Sketch No. 10, Fig. 5, would be desirable. Tests were accordingly made on a model of this shape, using four different widths of lip. The result obtained with a ½-in. lip (corresponding, for the scale of model used, to a 7-in. extension on an actual coping) is plotted as the single point marked "10" on Fig. 5. It will be noted that this shape also proved very effective, so that a division wall of this shape with a 1-ft. plate at the top of the coping (giving a 6-in. lip on each side) can be counted on to turn back any boil-over, if the wall is at least 75 ft. from the tanks protected. Where conditions require that the wall lie closer to the tank, a wider lip would be desirable—perhaps with a maximum projection of 1 ft. on each side if the wall is within 40 or 50 ft.

(*b*).—Auxiliary Walls Close to Tank.—A study of the behavior of the waves created in those different tests suggested that by breaking the force of the wave just as it struck the ground the effectiveness of the fire-walls would be increased. Tests were accordingly made, first, with a small vertical wall placed 30 in. from the spill-box (corresponding in actual size to a vertical wall a little more than 2 ft. high, approximately 35 ft. from the tank shell). The results are shown in Curve 11, Fig. 5.

It will be noted that this small auxiliary wall had an important influence in increasing the effectiveness of the different types. Comparing Curves 11 and 7, (Fig. 5), it will be noted that in the case of the higher walls the same 2-in. strip used as a vertical wall near the tank was almost twice as effective as when placed as a coping on top of the wall. Since the auxiliary wall 35 ft. from the tank would be only about 60% as long as that 100 ft. away, the considerably smaller quantity of material needed for the closer wall is a further argument in its favor.

When 4-in. strips were used (corresponding approximately to a 5-ft. steel plate), this superiority of the closer wall was not so marked as far as stopping the wave was concerned (compare Curve 8 with Curves 13 and 15), but, of course, its advantage in requiring less material still remains.

Placing the auxiliary wall close to the tank either in a vertical position, or inclined back toward the tank, seemed to make comparatively little difference. Curve 12 applies to a 3-in. vertical wall, and Curve 14 to the same wall inclined at 45 degrees. Curves 13 and 15 give a corresponding comparison between 4-in. walls.

Curve 16 (Fig. 5) brings out the fact that the auxiliary wall close to the tank is just as much of a help to vertical fire-walls as it is to those inclined to a 1 on 1.5 slope. A 2-in. auxiliary wall close to the tank resulted in moving

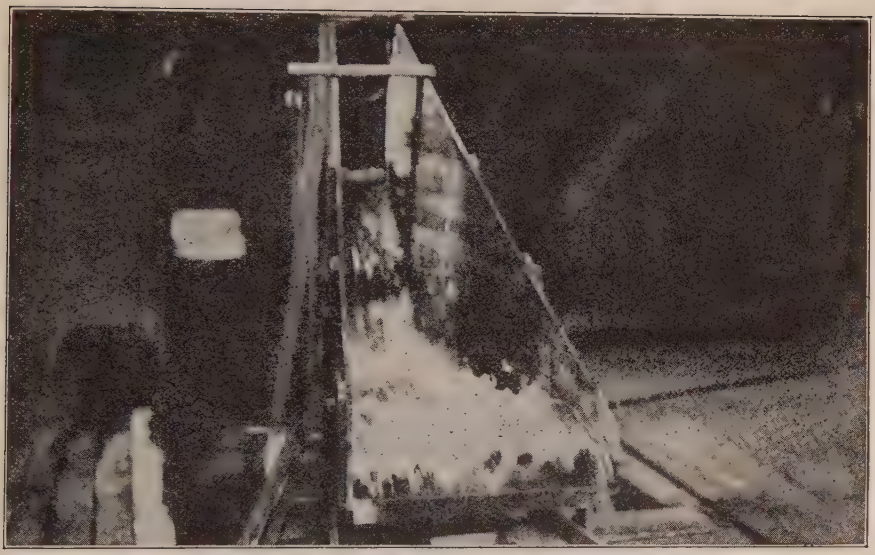


Fig. 7.—1 on 1.5 Wall; 5 Inches High; 12-Inch Boil-over, 45 Per Cent. Passing Over Wall.

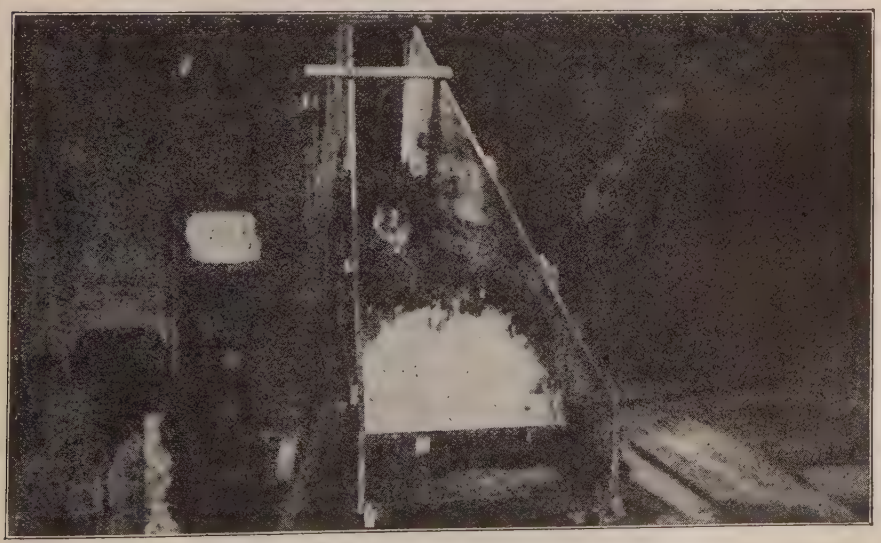


Fig. 8.—Same Wall as in Fig. 7, with Addition of 4-Inch Coping Sloped at 45 Degrees Toward Tank. 12-Inch Boil-over Stopped.

FIG. 9.—15-INCH BOIL-OVER COMPLETELY STOPPED.

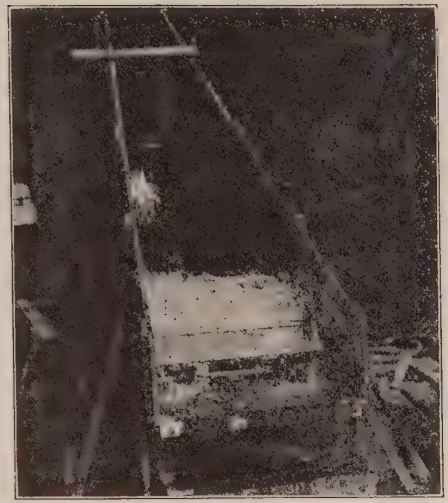


FIG. 10.—END OF 18-INCH BOIL-OVER; A SLIGHT QUANTITY PASSING OVER THE WALL AT THE CORNERS.

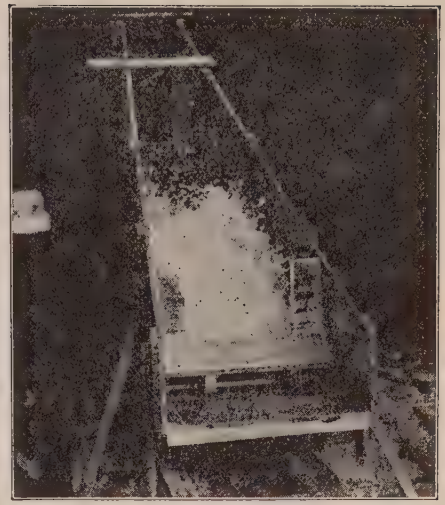
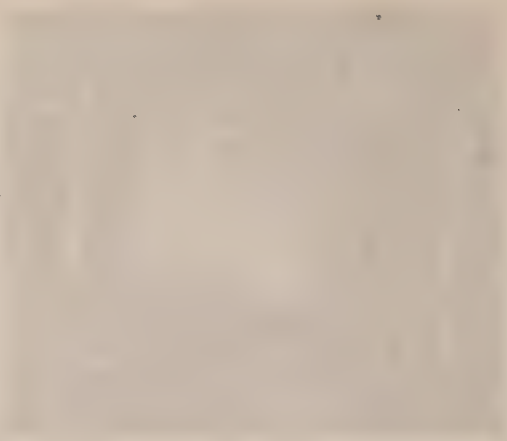


FIG. 11.—18-INCH BOIL-OVER COMPLETELY STOPPED, USING "WINGS" TO ELIMINATE CORNER EFFECT.

TESTS OF MODEL WALL No. 9 (FIG. 5).



Curve 3 (for the plain straight wall) out to the position of Curve 16, which shift strikingly emphasized the effectiveness of breaking the force of the wave by having an auxiliary wall close to the tank.

The bottom row of Fig. 6 gives a graphical comparison of the effectiveness of the different types of reinforcement discussed. As indicated, the main fire-wall in each of these tests was a 5-in., 1 on 1.5 wall. Comparison of the sizes of the spills mentioned, with that which the same wall without any reinforcement will stop (shown as the first diagram of the middle row) emphasizes the value of some type of reinforcement to ordinary 1 on 1.5 walls.

(*c*).—Borrow-Pits, Etc., Close to Tank.—It was next decided to see whether a pit dug around the base of the tank in such a way that the falling wave would strike the ground surface approximately normally (with the idea of creating maximum turbulence), would produce an effect similar to that of the vertical auxiliary walls. The floor of the model trough was modified to provide such a pit 3 in. deep; the results are shown as Curve 3 of Fig. 12. The plain 1 on 1.5 wall is also plotted for comparison. It is evident that the value of the plain wall has been only slightly increased by this arrangement.

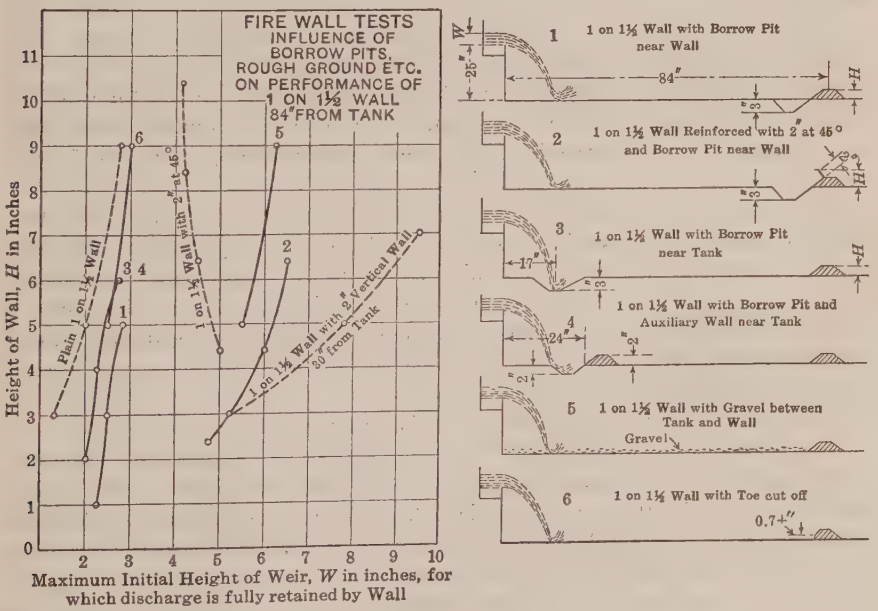


FIG. 12.

It was then suggested that a low earth wall built just outside the pit might increase its effectiveness. Curve 4 of Fig. 12 (which coincides with Curve 3) gives the results for a pit 2 in. deep with a 2-in. wall on the outside. This indicates that the wall on the outside makes a 2-in. pit as effective as a 3-in. pit without the wall; and as this would be the logical shape in which to place

the dirt, it is recommended. The slope toward the tank should be made as steep as soil conditions permit.

(*d*).—Borrow-Pit at Wall.—During the progress of the tests interest was expressed regarding the possible effectiveness of a borrow-pit on the tank side of an earth wall. The floor of the model trough was accordingly modified with results as indicated by Curves 1 and 2, Fig. 12. It will be noted that the pit next to the wall proved somewhat more effective than the one near the tank already described in Section (*c*) (compare Curve 1 with Curve 3), and considerably increased the effectiveness of a small coping at the top of the wall (compare Curve 2 with the dotted curve for "1 on 1.5 wall with 2 in. at 45°"). This is a further argument in favor of using a coping on top of 1 on 1.5 walls; for there is generally a borrow-pit on the tank side of such walls.

(*e*).—Rough Surface Between Tank and Wall.—In actual practice the space between the fire-walls and tank is sometimes cleared of weeds with a road scraper, which leaves the surface quite smooth; in other cases, the weeds are simply plowed under, leaving a rough surface.

With a view to determining the influence of such a rough surface on a boil-over wave, a strip of tarred paper was laid on the bottom of the model trough, into which was bedded gravel approximately ¼ in. in diameter and less. To scale, these bits of gravel simulated clods about 4 in. high. The results of tests with this equipment are shown as Curve 5 on Fig. 12. It will be noted that the gravel had a marked effect in breaking up the force of the wave, and this enabled a wall of given height to stop a much greater spill than it could if the intervening space were smooth (compare Curve 5 with the dotted curve for "Plain 1 on 1.5 Wall").

(*f*).—Toe Cut-Off, 1 on 1.5 Walls.—In cleaning up around certain fire-walls, it has been the practice to cut off the toe of the wall, on the tank side, leaving a vertical face about 1 ft. high at the base. Tests were requested to determine the influence of this change on the effectiveness of the wall. Curve 6 of Fig. 12 shows that, as in the case of borrow-pits, this expedient, while it does move the curve in the right direction, adds only slightly to the effectiveness of the plain wall.

(*4*).—*Effect of Placing Walls Closer to Tank.*—

(*a*).—Wall 50 Ft. from Tank.—The model walls were next set in a position corresponding to a distance of 50 ft. from the tank shell. The results are summarized by the curves on Fig. 13. It will be noted that the different shapes showed, in general, the same relative behavior as in the previous tests.

At this closer distance the effectiveness of the vertical wall, as compared with the 1 on 1.5 wall, was relatively greater than at the 100-ft. spacing. On the other hand, the coping on top of the vertical wall did not result in such a marked increase in its effectiveness as had been previously observed. This led to the suggestion that a heavier coping might be desirable; a comparison of Curves 3 and 4 of Fig. 13 indicates that doubling the width of the coping increased the effectiveness of the wall by about 20 per cent. The coping on top of the 1 on 1.5 wall showed the same marked effectiveness as in the previous tests (Curve 5 of Fig. 13).

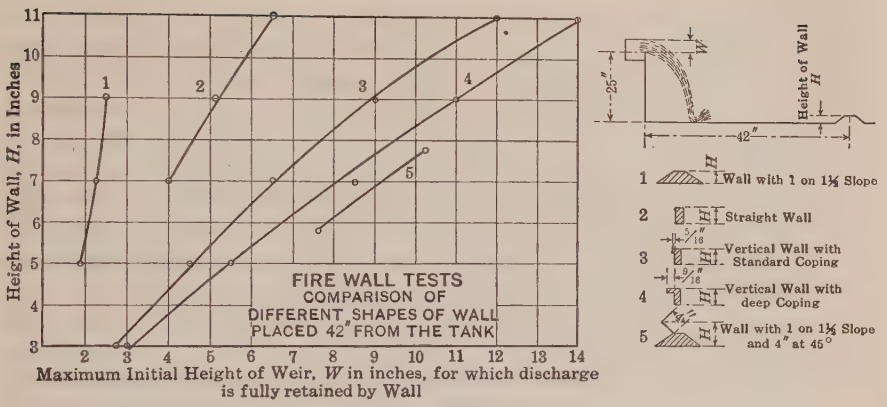


FIG. 13.

(*b*).—Wall 20 Ft. from Tank.—When the model walls were moved in to a distance (17 in.) corresponding to 20 ft. from the tank shell, the differences in the behavior of the various shapes were not so marked. The curves on the upper half of Fig. 14 show the results when the boil-overs were discharged from the 25-in. height used in previous tests (corresponding to a 30-ft. tank); and the data on the lower half summarize the results of a few tests in which boil-overs were discharged against low walls from a height of 8½ in., corresponding to the 10-ft. "receiving pans" commonly used in refineries.

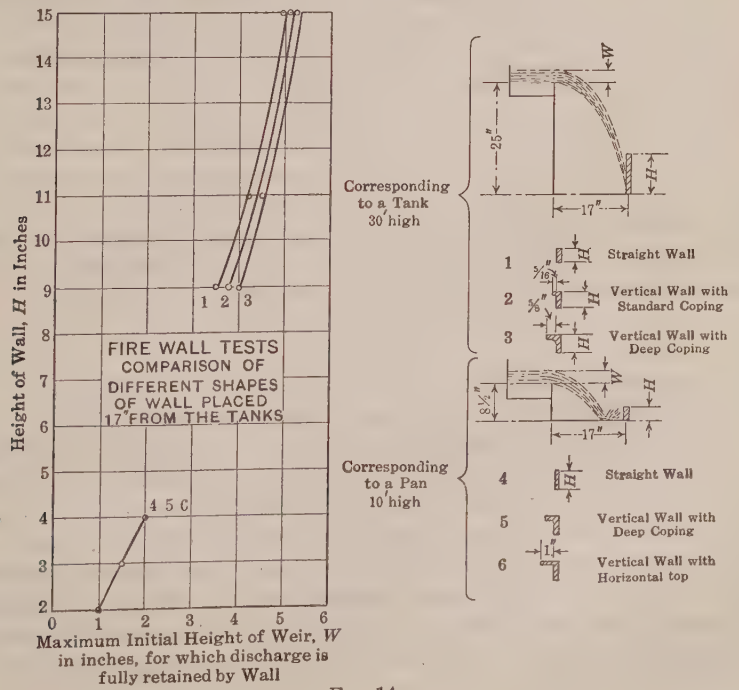


FIG. 14.

It is felt that the small differences indicated by these tests (Test 4(*b*)) are attributable mainly to the fact that, when the model walls were moved close to the tank, there was hardly room for wave action to occur. In practice, of course, these conditions would not hold. It seems likely that the advantages of copings observed in those previous tests where the wall was far enough away from the tank for wave action to develop, would make themselves felt to practically the same extent where actual walls are within 20 ft. of the tank.

5.—Reservoir Fire-Walls.—In connection with some contemplated strengthening of reservoir fire-banks, tests were requested to determine the influence of a small mound and borrow-pit at the toe of a reservoir slope.

As was the case with the vertical auxiliary walls near the tank, described under Test 4(*b*), the mound at the base of the slope served to break the force of the wave to such an extent as to enable a given height of main wall to stop a much larger wave; or, conversely, with a given intensity of boil-over to stop, the main wall can be made considerably lower if the auxiliary mound is built at the toe of the slope.

Since a relatively small quantity of earth is required to build this mound (both on account of its low height and small radius as compared with the main fire-wall), the total yardage of earth required to obtain a given degree of protection is 20 or 30% less when the mound is used, than when the main wall is built high enough to stop the wave without any auxiliary protection.

6.—Tests with Foam Solutions.—In the tests previously described, water was used on account of its convenience. However, in discussing some of the results, it was suggested that it would be interesting to use a liquid that froths and expands as it pours out of the tank, thus simulating conditions when burning oil boils over. To accomplish this, it was decided to use weak foam solutions.*

It was found that when these solutions were suitably diluted, the waves produced seemed to act in much the same way as a wave of frothing and steaming oil. The different shapes of wall were found to have essentially the same relative effectiveness as when water was used. It is felt, therefore, that the results of the water tests give a reliable indication of the relative value of different types and heights of wall for stopping waves of boiling oil.

Conclusions

The outstanding result of these tests (which comprised a total of more than 1 000 "boil-overs" discharged against 66 different model walls or combination of walls) is the value of some form of velocity or energy "killer" other than mere height of wall. Probably the most important of the experiments tried is the coping, although a rough surface between the tank and the wall and the auxiliary walls and pits will probably each have a place.

The practical application of this principle to typical oil-storage conditions is illustrated in Fig. 15. Several concrete walls of the recommended form have already been built. On most of these walls, however, the coping has

* Basic and acid liquids used to produce foam for extinguishing oil fires, see U. S. Bureau of Mines, *Bulletin No. 170*, p. 10.

been made only about 4 in. wide. The tests have indicated that a heavier coping—say, 8 in. wide—will add considerably to the effectiveness of walls which are fairly close to tanks; and as this increase in the width would add a negligible amount to the cost, it would seem desirable.

As indicated by the curves on Fig. 13, a wall 5 ft. high (model height, 4.3 in.) with an 8-in. coping is as effectual as a plain wall 9.6 ft. high (model height, 8.2 in.). A 5-ft. coped wall of standard T-shape will require about ¼ cu. yd. of concrete per lin. ft. at $30 per yd.; this would cost $7.50 per lin. ft. A wall, 9.6 ft. high, would contain about 0.6 cu. yd., costing about $18. From these figures it is apparent that a given degree of protection for turning back a wave will cost only about one-half as much with a coped as with a plain wall.

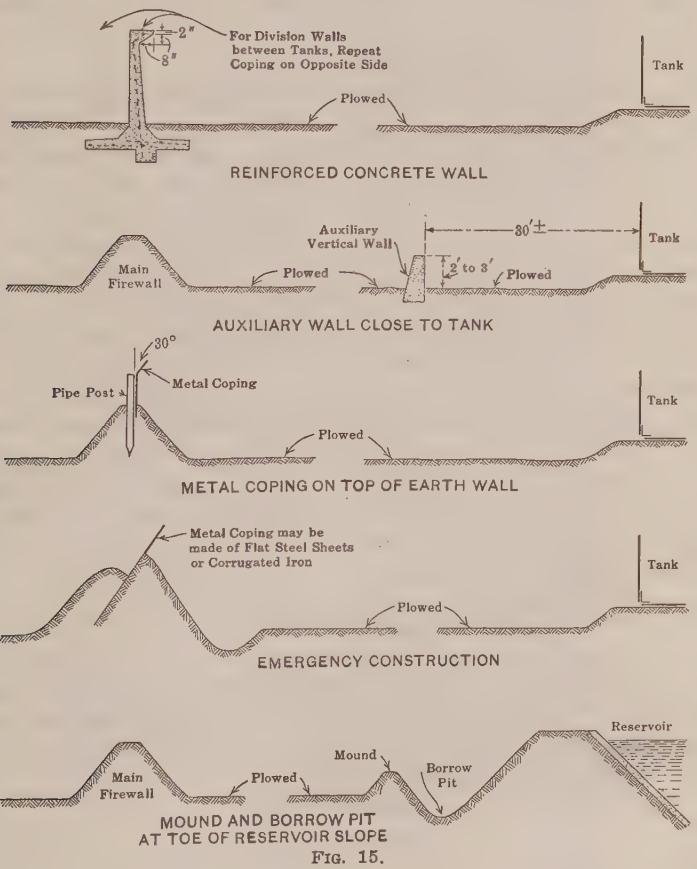


FIG. 15.

An even greater economy is effected by the use of copings on earth walls. Over these, on account of their sloping sides, waves have a tendency to pass readily; but as soon as a 3 or 4-ft. coping is placed at the top, a wall 5 ft. high becomes much more effective than an uncoped wall 15 ft. high. (See curves on Fig. 5.) Where scrap steel plates are available for building copings of the general form shown on Fig. 15, the cost will be very low. Even

where new galvanized-iron sheets have to be purchased, the cost figures indicate that a given effectiveness can be obtained with a coped wall at about one-quarter the cost of the high earth wall.

Where local conditions permit, a low vertical auxiliary wall about 30 ft. from the base of the tank is an even more economical method of increasing the effectiveness of the main wall. This auxiliary wall breaks the force of the wave and robs it of the energy required to carry it over a main wall that it could otherwise easily surmount. On account of the smaller radius of this auxiliary wall, only about 60% as much material is required to build it as for the coping on the main wall that would have equivalent effect.

A small mound 2 or 3 ft. high with a borrow-pit toward the tank, and the slope made as steep as soil conditions will permit, will produce an effect similar to these small auxiliary vertical walls; but on account of the relative ease with which the wave passes over a wall with sloping sides, the mound is not nearly so effective as the vertical wall.

The same principle of breaking the force of the wave just as it starts can be applied in the case of reservoirs by building a small mound just at the toe of the reservoir slope. Another application of the same general principle consists of roughly plowing the surface of the ground between the tank and the fire-wall. The tests indicated that this would serve to break up the energy of the waves sufficiently to enable a given wall to stop a considerably higher boil-over than if the surface between the tank and the wall were smooth.

A factor which adds materially to the value of this study has been the possibility of applying the principle to emergency construction. This was well illustrated in a recent crude-oil fire in the Midway Field in California. Four large tanks burned and boiled over, but the management was able to save the pump-house and other buildings distant only about 160 ft. from the nearest tanks, by throwing up emergency earth walls against the back of which were placed corrugated galvanized-iron sheets which projected several feet above the top of the wall and thus provided a coping tipped toward the fire. These walls proved entirely effective in stopping the boil-over and saving the buildings. (See Fig. 16.)

Personnel

The tests herein described were conducted by the Technical Staff of Standard Oil Company of California under the direct supervision of the writer. The equipment was designed by Mr. A. Kremser, the tests were actually made by Messrs. H. J. Berg and E. F. Post, and the results collated by Mr. G. O. Wilson.

APPENDIX

Preliminary Tests to Study Theoretical Applicability of Model and Performance of Apparatus

Theory of Model Application.—Although it was never intended to make quantitative use of these model experiments, it was hoped that in spite of the small scale they would show relative efficiencies among themselves. A number

Fig. 16.—Emergency Wall Used in Actual Fire.

of exhaustive studies of the application of models to various engineering problems have been presented in recent years. In their paper* describing the experiments with models of the Gilboa Dam, R. W. Gausmann, M. Am. Soc. C. E., and C. M. Madden, Assoc. M. Am. Soc. C. E., stated that "if a model of a weir or channel is geometrically and mechanically similar to its prototype, the velocity will vary as the square root of the scale ratio."

In the model experiments here considered, the criterion of comparison is the height of wall which will be topped by various spills. As this height will be approximately proportional to the square of the velocity, and as the velocity, in turn, will be in proportion to the square root of the scale ratio, it would appear that the results of these experiments will be approximately in direct proportion to the scale ratio; or what is more important, will show approximately true relative proportions between different tests.

Performance of Apparatus.—In the first trials of the apparatus it was observed that considerable difference in the behavior of the wave resulted from apparently minor variations in the method of opening the gate. A standard procedure therefore was adopted in which the operator stood in a certain place and jerked the rope hard enough to throw the gate up with considerable impact against the supporting beam.

The next point to be investigated was the proper criterion for judging the performance of the different shapes and heights of model walls to be tested. It was at first felt that the maximum height of spill which any given shape should be credited with retaining was that for which the top of the wall and the space in the end of the trough outside the wall (designated by *A* in Fig. 2, and spoken of hereafter as the "tail-board") remained absolutely dry. It was found, however, that this depended too much on accidental splashing when the wave hit the wall in some peculiar way; and it was finally decided that the criterion should be the condition where not enough water splashed over the wall to cause any measurable quantity to run off the tail-board.

To check the accuracy of the so-called "dry point" thus determined for each wall, the following procedure was adopted: Starting with a spill which would be large enough to throw a considerable percentage of water over the wall, a series of runs was made, gradually decreasing the size of spill until the dry point was reached. Several more runs were made after reaching this point with still smaller quantities, to make sure that these would be completely retained by the wall. By placing at the end of the tail-board a collector pan discharging into a 5-gal. can, the quantity of water passing over the wall in each test could be measured. These quantities were plotted as a percentage of the total quantity discharged in any given spill; and the intersection of the curve thus obtained with the base line gave a good check on the accuracy of the dry-point found. The curves of Figs. 17 and 18 are typical examples of these "percentage over" curves, whereas the final summarizing curves, Figs. 5, 12, 13, and 14, show only the dry points for the different sizes and shapes of wall. As explained (page 235), Fig. 19

* *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 300.

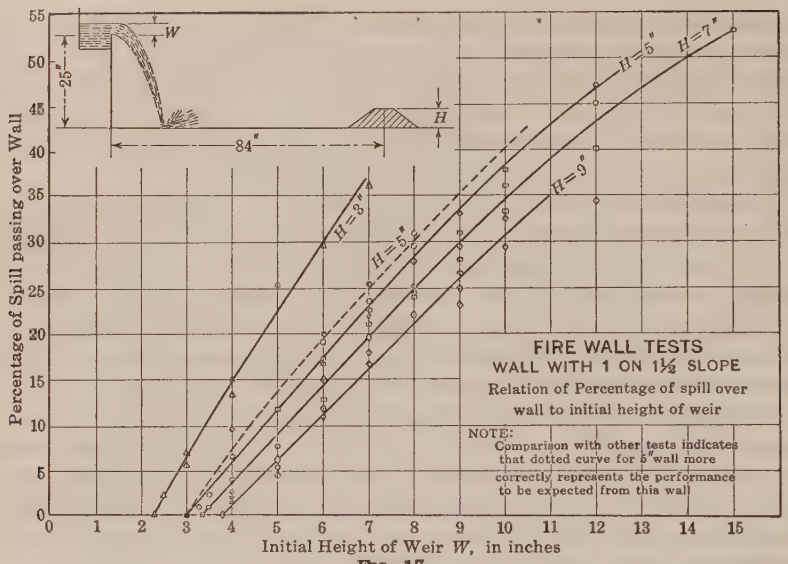


FIG. 17.

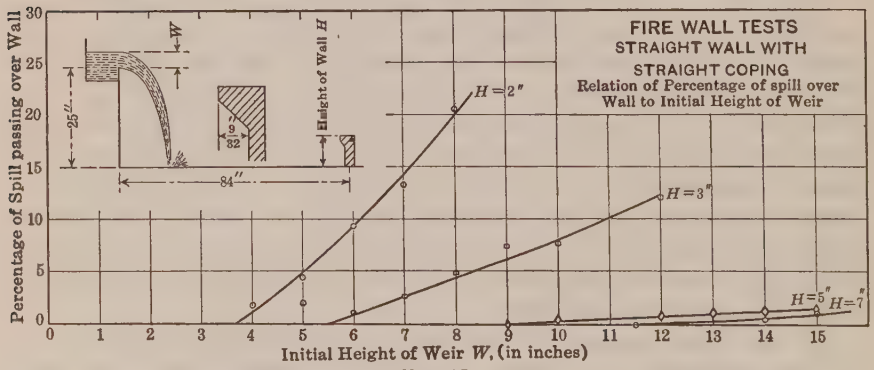


FIG. 18.

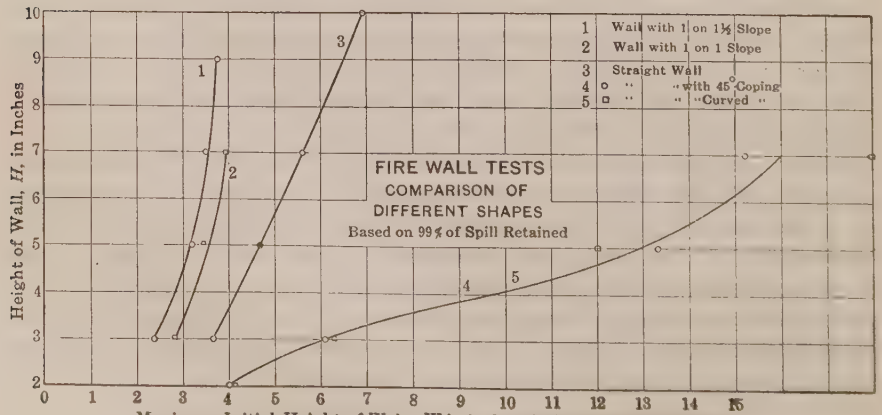


FIG. 19.

compares the different shapes on the basis of 99% of the spill being retained, instead of 100% as in Figs. 5, 12, etc.

One other preliminary step before starting the actual comparison of different sizes and shapes of wall was to investigate the effect of varying the height of the box from which the spill was discharged (Dimension *A* in Fig. 1). The relative effectiveness of different walls was found to be (within reasonable limits of experimental error) essentially the same regardless of whether the spill was delivered from a high or low box. Accordingly, it was decided to run most of the main tests with the box set at 25 in. above the floor, which corresponds to an actual tank height of 30 ft.—this being the height most commonly used by Standard Oil Company of California.

DISCUSSION

H. L. SHOEMAKER,* ESQ.—The practical results and conclusions at which the author has arrived, have not only been approved by the entire petroleum industry of the United States, but they have been used in a very practical way.

Mr. Hall's report was first presented at the Convention of the American Petroleum Institute in December, 1924, at Fort Worth, Tex., where his work was commented on very favorably by the oil industry. A committee of which the speaker was Chairman, collected and compiled data in regard to the hazards of petroleum products in storage, and found some astounding figures. The Committee was also able to use many of Mr. Hall's data, especially in relation to the problem of the spacing of above ground petroleum storage tanks.

Regarding the fire record of petroleum products in storage in the United States, extending over a period of ten years (1915-25), eighty-seven oil companies, including all the large companies operating in the country, submitted the following information covering their fire history: In this period, 143 refineries experienced 73 fires, or 1 fire per plant in 20 years. In 11 132 marketing stations (ordinary bulk distributing stations where petroleum products are received by tank car, unloaded into storage, and distributed by tank truck), which embrace the territory from the Pacific Coast to the Atlantic seaboard, inclusive, and many of which are within the corporate limits of cities and built-up communities, 19 fires were experienced in the 10-year period mentioned, or 1 fire per plant in 5 860 years.

Regarding large seaboard terminals and pipe-line terminals, 273 such properties reported 77 fires in the 10-year period, or 1 fire per plant in 35 years; and 713 tank farms, probably representing a storage capacity of 500 000 000 bbl. of crude oil, experienced 88 fires in the same period, or 1 fire per plant in 81 years.

The oil industry feels that this constitutes a very remarkable fire record. Expressing the statistics in other terms and establishing a fire frequency ratio, or the number of fires in tankage compared with the number of tanks in service, it is found that 15 873 all-steel gasoline tanks had 41 fires in the 10-year period, or a frequency ratio of 0.026% per year, and that 12 594 all-steel kerosene and refined oil tanks experienced 35 fires in the same period, or a fire frequency ratio of 0.028% per year.

It is interesting to compare these annual fire ratios, or percentage of tanks lost by fire per year, with similar figures covering dwelling houses. The number of dwelling houses in the United States in 1920† was approximately 20 697 204 The number of fires in such dwelling houses per year‡ was 131 035, or expressed as an annual fire loss ratio, or percentage of dwellings burned per year, equals

* Gen. Eng. Dept., Standard Oil Co. (New Jersey), Elizabeth, N. J.

† World Almanac, 1925.

‡ "Safeguarding America Against Fire", December, 1924.

0.63%, as compared with petroleum tank ratios mentioned of 0.026% and 0.028% for gasoline and refined oil products, respectively.

In view of the fact that the petroleum industry was vitally concerned with possible State and municipal regulations regarding the handling and storing of petroleum and its commodities, which might govern the construction of tanks, fire-walls and other requirements, the statistics given were of immense value. The Committee reporting on this work to the industry mentioned previously, in making recommendations regarding spacing of tanks, found Mr. Hall's work of immense practical value. To illustrate, in spacing crude oil tanks it was determined that a tank should be at least two diameters from shell of tank to adjacent property line in order to protect such property in case of fire. One of the essential reasons for recommending this distance was that Mr. Hall's data indicated that, to be fully effective, a fire-wall must be placed approximately one diameter from the shell of the tank. Further, most ordinances specify capacity as a requirement in fire-wall construction around oil tanks. As shown by Mr. Hall's paper, capacity of a fire-wall is only one factor. The horizontal distance of the fire-wall from the container, as well as the type of construction used (that is, the kind of coping or lack of coping), are of more practical importance in protecting life and property and in retaining "boil-over" waves than mere capacity inside the wall or dike.

Frank A. Epps,* Esq.—The oil industry has been intensely interested in the valuable data presented in this paper and also in a supplementary investigation conducted by Mr. Hall.† While this paper deals with fire-bank protection required for oils possessing "boil-over" characteristics, the fire-bank requirements for tanks containing oils not possessing "boil-over" characteristics are also of interest.

Mr. Hall has stated, and it is a well-known fact, that refined oils do not "boil over". Clearly then, for this type of oil storage, the fire-bank can serve only as a secondary container in case of spills, leakage, or rupture of the tank. Except under unusual conditions, the oil industry has very strongly questioned the necessity or advisability of fire-banks solely for such purpose. Fire-banks capable of retaining at least the entire tank contents cost a great deal in proportion to the remainder of the installation in which refined oils are usually stored. It is questionable whether the benefit of protection obtained from this expenditure is worth the cost. To answer this and other questions, a committee of the American Petroleum Institute was appointed to secure and analyze the complete tank-fire record of the oil industry in the United States for the 10-year period, 1915-25. The following is quoted from the report:

"* * * fire-walls surrounding gasoline and refined oil tanks were called upon to prevent the spread of fire in but 8 cases in 10 years. In 12 other cases fire spread because of the non-existence of fire-walls. Thus, fire-walls were effective or could have been effective in 10 years in only 20 instances. It is conservatively estimated that the fires reported represent the fire experience of not less than 50 000 steel-roofed tanks in service. Thus, fire-walls sur-

* Mgr., Fire Protection Dept., Tide Water Oil Co., New York, N. Y.

† *Mechanical Engineering*, July, 1925, p. 540.

rounding these 50 000 tanks could have been of practical benefit in only 20 instances—this in a 10-year period—or 2 per year.

"The Committee believes that this record of experience demonstrates the injustice of requiring elaborate fire-wall construction and the needless expense caused thereby, for the protection of this class of storage."

With respect to fuel oils, this Committee was able to obtain a record of only fourteen fires, in only one of which would a fire-wall have been of any practical benefit. The Committee felt, therefore, that the danger of fire in fuel-oil tankage is so extremely low as not to justify the cost of any fire-bank protection whatsoever.

Regarding fire-banks for crude oils, it is a peculiar fact that aside from the petroleum industry itself, fire-protection circles have regarded fire-banks merely as secondary containers, the function of which is to hold the contents of the tanks surrounded in case of spills, leakage, or rupture. Considerations affecting the fire-bank's capabilities of withholding a wave of burning oil thrown out of a tank by a "boil-over" have not received a great deal of attention; this, in spite of the fact that it has been generally recognized that the average fire-bank was not effective under this condition. Fire-protective legislation and insurance requirements to date have considered only the capacity of the enclosure and the material of which it is built, capacity, the chief specification, being usually 150% of the tank contents. The research work conducted by Standard Oil Company of California under the direction of the author has clearly demonstrated that capacity alone is not a measure of fire-wall effectiveness when the tank contains crude oil.

The Committee of the American Petroleum Institute reached the conclusion that the bank around a crude oil tank must function both to throw back the wave of burning oil and to contain the liquid ejected. The usual "boil-over" from a crude tank seldom exceeds 10% of the tank contents. On the other hand, there may be several "boil-overs" during the same fire, also records are available wherein "boil-overs" are reported to have ejected more than 10% of the tank contents. The fire-bank must contain or hold this oil, not to mention large quantities of water often thrown upon the sides of the burning tank to keep it cool. The Committee felt, therefore, that a definite capacity specification should be provided. After investigating the research work performed by Standard Oil Company of California and likewise considering the tank-fire record of the United States for the last ten years, the Committee reached the conclusion that:

"Above ground, gas-tight containers for crude oils should be surrounded by fire-walls or dikes of earth or other suitable material, and should be:

"(*a*) Of a capacity equal to that of the tank or tanks surrounded, unless adequate drainage to a sump of like capacity is provided;

"(*b*) Located not nearer to the shell of any of the enclosed tanks than the diameter of the largest tank enclosed, and in any case not less than 50 ft.;

"(*c*) Provided with a suitable coping projecting inward to turn back boil-overs."

Mr. Hall's research work had a great deal to do with the formation of that recommendation, of which a General Conference of Oil Industry Representatives has voiced approval. The industry has accordingly recommended to the National Fire Protection Association that its model ordinance for the regulation of oil storage within corporate limits be revised to include such provisions for fire-bank design. The oil industry is obviously very appreciative of this fine piece of research work by Standard Oil Company of California, and by Mr. Hall in particular.

Ralph J. Reed,* M. Am. Soc. C. E. (by letter).—The paper describes a practical and constructive piece of research which is of great and immediate value not only to the petroleum industry, but also to safety, fire prevention, and insurance interests. The studies made and the conclusions drawn form the groundwork for the proper and economic design for permanent fire-wall protection of containers in which crude or fuel oil is stored. They are particularly useful in outlining means for modifying existing protective structures either permanently, or temporarily in the event of fire, so as greatly to increase their effectiveness against boil-overs.

Well constructed oil storage works develop few leaks that must be retained by secondary structures; the careful operation of tanks and reservoirs results in few spills; and, where proper principles of design are followed, as the author states, one may practically disregard the possibility of the rupture or collapse of a full container. It follows that fire-wall protection is primarily required as a measure to prevent the spread of burning oil if a boil-over occurs during a fire involving crude or fuel oils.

Although some fire prevention authorities have insisted on greater fire-wall capacity, it has been the general custom to assume that a fire-wall structure so built as to retain the entire tankage which it is designed to protect will usually be adequate to prevent the spread of burning oil in case of fire. The sole criterion has been the quantity of fluid which the protective structure will safely hold, without regard to the form of barrier or its relative location with respect to the containers surrounded. Where sufficient gound has been available, low earthen embankments with side slopes varying from 1 on 1 to 1 on 2 are common fire-walls. The usual field practice of the oil industry has been to build such earthen walls to heights of 5 or 6 ft., and to place them from 75 to 100 ft. from the tank enclosed. In other locations in which tanks must be spaced more closely, either on account of operating conditions or because of the restricted areas available, vertical walls of masonry or concrete are used. Mr. Hall's experiments, a short series of which it was the writer's privilege to witness, show clearly the ineffective barrier to the progress of a boil-over wave of high velocity furnished by the ordinary earthen embankment or even by a plain vertical wall of moderate height. Experience with earthen embankments during tank fires, together with observations of some recent fires in large reservoirs, serves to emphasize this fact strongly and to point to the value of some device for destroying the energy of the wave.

* Chf. Engr., Union Oil Co. of California, Los Angeles, Calif.

All the types of reinforcement for ordinary earthen embankments on which the tests were run are valuable from a practical standpoint, although the greater effectiveness of the low auxiliary wall placed quite close to the container, as compared with a coping of the same height placed on the top of the standard embankment, and its relatively shorter length, would seem to make it most desirable. Such walls may be built of concrete, tile, or brick. It is a question whether lighter material than, say, $\frac{3}{16}$-in. steel plate, used as a coping, will successfully withstand the heat of burning oil, especially if repeated boil-overs occur; and in permanent installations even this material should be supported on posts, making a fairly expensive structure. Corrugated galvanized-iron sheets for temporary copings have been proven effective, but the writer questions their ability to resist the heat of successive boil-overs and is, therefore, of the opinion that their use should be confined to emergency installations as suggested by the author. Instead of steel plate, sheets of heavy asbestos board, properly supported, could be used.

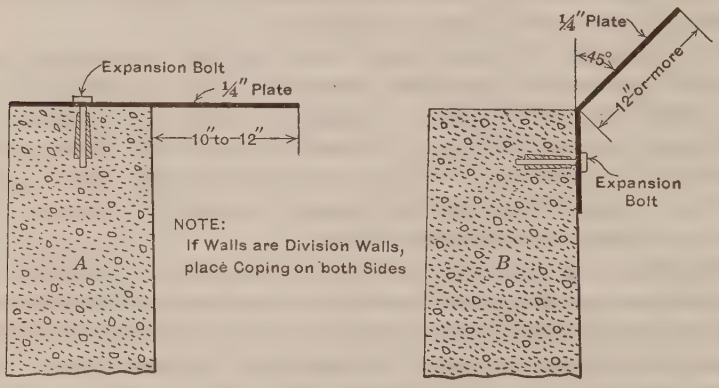


FIG. 20.—METAL COPING ATTACHED TO EXISTING FIRE-WALLS.

The striking value of even a relatively small coping on a vertical wall appeared in the few experiments witnessed by the writer. No fire-wall of this type should be built without incorporating a coping in its design. Existing fire-walls can be readily and quite inexpensively modified, it is believed, either by casting a concrete coping in place, or by the expedient of attaching a metal plate to the top of the wall, as shown in two forms, (*A*) and (*B*), Fig. 20.

H. H. HALL,* M. AM. SOC. C. E. (by letter).—The principles of fire-wall design developed in the paper are evidently proving of direct interest to the oil companies, several of which have carried the idea further and developed new methods for applying coping to their existing fire-walls. It is also interesting to note that in its recommendations to the National Fire Protection Association, the American Petroleum Institute has endorsed the coped type of wall.

* Chf. Engr., Standard Oil Co. (California), San Francisco, Calif.

Between January and November, 1925, the only oil fires in California of any magnitude have been in two large reservoirs; these were, incidentally, the only two reservoirs in the history of this industry on the Pacific Coast that have been fired by lightning. Although neither of these fires served to test the coped fire-wall idea completely, they both added to the knowledge of the general subject of fire-bank construction.

The first fire occurred at night in a reservoir about one-third full of heavy fuel oil. There was a substantial earth dike surrounding this structure, upon the top of which posts had been erected with the intention of establishing a permanent corrugated-iron coping somewhat similar to the metal coping on top of earth walls shown in Fig. 15. The coping had been delivered to the work, but none of it had as yet been put into place. After the fire started about 150 men placed 4 000 ft. of this coping in about 2 hours. At this same fire, another gang of about 150 men, with the aid of three tractors and fifteen teams, built about 3 000 ft. of temporary earth wall with metal coping. For this work the metal and posts had already been prepared; but otherwise everything had to be done after the fire started.

These figures serve to illustrate the speed which can reasonably be expected under stress of fire conditions, and at night. In a recent daylight fire drill, a gang of 18 men erected 150 lin. ft. of steel-plate coping on top of existing fire-walls in 10 min.

As a crude oil tank or reservoir always burns a number of hours before a boil-over occurs, and as there are generally a large number of men available for fire-fighting operations, it appears that there will practically always be ample time to place such temporary copings on fire-walls where they are needed. With this in view, the company with which the writer is connected is keeping on hand at various strategic locations a stock of plates, posts, tie-wires, and tools with which to erect emergency "steel fire-walls" as shown in Fig. 21. This plan obviates the heavy investment and maintenance charges that would be required for permanent copings on all walls, to which Mr. Reed refers.

Unfortunately for a study of coped fire-banks, the reservoir just mentioned did not boil over violently enough to test the coping. However, it did boil over; and an observation of the uniformity with which it spilled out practically all the way around the 1 600-ft. periphery suggests another form of fire-bank construction. This is the possibility, where reservoirs are concerned, of directing the boil-over to the least dangerous side of the container, by making the bank of the reservoir itself higher on the dangerous side. One reservoir embodying this principle was built four years ago; and there are now several reservoirs with auxiliary fire-banks built on top of their regular banks along the more dangerous side.

The other recent reservoir fire occurred in an isolated storage where there was practically no danger to other property. There was a substantial earth dike ranging from 8 to 16 ft. high, located about 200 ft. from the toe of the reservoir slope. The ground sloped gently toward the dike at a rate of about 5 ft. per 100 ft. This reservoir contained light crude oil, and boiled over with considerable violence. As there were no structures below it, no effort was made to erect a temporary coping; and this boil-over climbed the earth dike

with apparently little loss of energy—which evidence simply serves to confirm the test results and considerable previous experience with walls of this type.

Mr. Shoemaker's statement that "to be fully effective, the fire-wall must be placed one diameter from the shell of the tank", is in line with recommendations made by the Committee of the American Petroleum Institute, of which

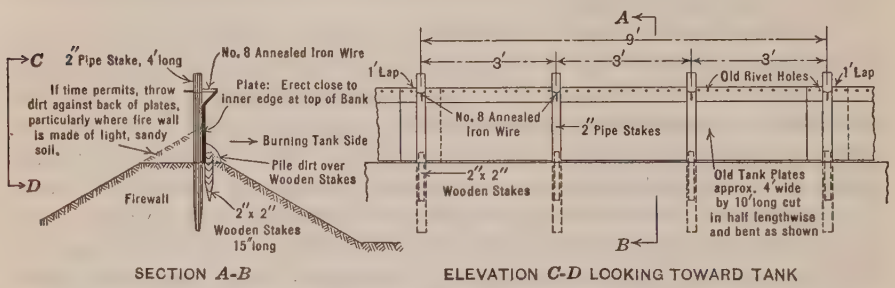


FIG. 21.—STEEL FIRE-WALL FOR REINFORCING FIRE-WALLS AROUND THAT PART OF A BURNING TANK OR RESERVOIR WHERE SPREAD OF BOIL-OVER WOULD BE ESPECIALLY DANGEROUS.

he was Chairman. This is the spacing which was considered ample to insure safety with all types of walls—particularly earthen dikes. In congested localities it is customary to use vertical concrete walls; and when such walls are provided with a substantial coping it will be noted from the comparison of Figs. 5 and 13 that at a distance of only 50 ft. from the tank they will stop substantially the same boil-over as at 90 or 100 ft. away—as would be required by strictly following the one diameter rule for large storage tanks. Hence, when space is limited, it would appear that almost as much reliance can be placed on heavily coped vertical walls located 50 ft. from the tank, as when such walls are farther away.

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Paper No. 1576

ELEMENTS GOVERNING THE DEVELOPMENT OF HIGHWAY TRAFFIC*

By A. N. Johnson,† M. Am. Soc. C. E.

With Discussion by Messrs. Arthur W. Dean and Van Alen Harris.

Synopsis

The purpose of this paper is to discuss the amount of future traffic upon American highways and to compare this with the traffic capacity of a two-lane road.

In order to do this, it is necessary to project what the progress is to be in the future, and this is always fraught with many uncertainties. Even if the physical facts on which to base a prediction are known, social forces may not continue to act as in the past, and thus new conditions arise that affect vitally the results. If the facts are known, and the forces, both social and economic, do not sensibly change, then one may predict with considerable assurance what the future may be expected to bring forth.

The physical facts upon which highway traffic rests are the number of vehicles using the road in a given time, the road mileage made by these vehicles during some definite period, and the mileage of the system of highways upon which the traffic moves. If, through a considerable period of time, these quantities have persisted at concordant rates, then future values may be predicted. In the instance of highway use, the values thus found will be approximately true, provided the causes that create a desire for the use of the motor vehicle do not change. For the present purpose, it is assumed that such will be the case; that people will not tire of the motor vehicle as a form of recreation; that, due to further congestion, increased traffic will not become the means that will strangle itself. All that is favorable to the continuance of travel by highway is to be assumed. With these conditions imposed, it is useful to endeavor to see what the future holds in store for the highways.

* Presented at the meeting of the Highway Division, New York, N. Y., January 22, 1925.
† Dean, Eng. Coll., Univ. of Maryland, College Park, Md.

As has been indicated, the density of traffic over a system of roads depends on the vehicles using the system in a given time, together with the mileage made by these vehicles in a limited period, and the number of miles of highway in the system. The total number of motor vehicles is known. The average mileage per year per vehicle is variously estimated at 4 000 to 6 000 miles. Based on the gasoline consumption of the vehicles in certain localities, the average travel is about 5 000 miles per year. For the present purpose, the larger figure of 6 000 miles per year will be used.

There are in use (1924) approximately 18 000 000 motor vehicles, which would yield, on the assumption noted, a total annual traffic of 108 000 000 000 vehicle-miles, or an average daily traffic of nearly 300 000 000 vehicle-miles. This traffic, as every one knows, is far from evenly distributed over the approximately 2 500 000 miles of highways throughout the country. The bulk of the traffic is confined to a comparatively small fraction of the total road mileage.

It is a fact that few State highway systems, as at present developed, exceed 10% of the total road mileage of the State. For this discussion, it will be assumed that 75% of all highway traffic is carried on 10% of the highways. From these figures it is found that there is imposed on the main road systems an average daily traffic of 900 vehicles per mile.

Since 1917, under the direction of John N. Mackall, Chairman and Chief Engineer of the State Roads Commission of Maryland, traffic counts have been made at regular intervals at approximately 200 counting stations on the State Highway System, which comprises very nearly 10% of the total road mileage. From these traffic counts there has been compiled the average number of vehicles per mile per day throughout each year since 1917.

TABLE 1.—Comparison of Traffic Density as Computed from Motor-Vehicle Registration, Applying 75% of the Traffic to 10% of the Total Road Mileage of the United States, with the Traffic Density on the State Road System of Maryland, as Found from Actual Traffic Counts.

Year.	Average Number of Vehicles Through the Year per Mile per Day.	
	As computed for the United States.	As counted on Maryland State Road System.
1917	250	258
1918	300	277
1919	375	355
1920	450	489
1921	500	537
1922	600	579
1923	740	726
1924	900	818

In Table 1 the results thus obtained from actual traffic counts are compared with the amount of traffic as computed from motor-vehicle registration as previously outlined. It is seen from an examination of this table that the computed results are not widely divergent from those obtained from actual traffic

counts and, at least, establish the fact that the amount of highway traffic is a linear function of motor-vehicle registration.

May limits be set to this highway traffic? May a point be reached where the increase in traffic would be at the same rate as the comparatively slow increase in population? As the number of motor vehicles is a prime factor in this calculation, an answer may be found if a study is made of the motor-vehicle registration curve.

It is logical to assume that eventually the limiting number of motor vehicles per population will be reached, and when such a point is reached, the rate of increase of the motor-vehicle curve will be equal to that for the population curve.

In Fig. 1 the motor-vehicle registration curve is shown, based on data furnished by the U. S. Bureau of Public Roads, as published by the National Automobile Chamber of Commerce. Likewise on this diagram is the population curve. These curves have been platted on a semi-logarithmic scale; thus, the same rates of increase for each curve will exist at those points at which the curves are parallel. When the motor-vehicle registration curve becomes parallel with the population curve, the saturation point in the market for motor vehicles has been reached. When does this occur?

To assist in projecting the motor-vehicle registration curve beyond the current year, the number of persons per automobile is perhaps the most direct method of arriving at an answer to this question.

In Fig. 2 a curve has been platted that shows the number of persons per motor vehicle for the past few years. It is noted that the number of persons per automobile in 1919 was 14; in 1924, 6.2. In California, in 1923, the number of automobiles per person, according to statistics published by the National Automobile Chamber of Commerce, reached 3, and in Iowa, 4, ranging from these values to 19 persons per motor vehicle in Alabama, the average for the United States for that year being 7.4. It is difficult to imagine that an average of less than 4 persons per automobile will be reached. A continuation of the curve in Fig. 2 indicates that probably not earlier than 1926, and more likely about 1928, this point may be reached. On this latter assumption, the registration curve in Fig. 1 has been drawn, indicating a registration in 1928 of 29 000 000 motor vehicles. After that date, the rate of increase is shown as directly proportional to the increase in population. It is to be kept in mind that the assumption that has been made is based on a continuation of the desire to use motor vehicles that exist to-day; and as it is the intention to outline maximum conditions it is not necessary to consider what may happen from causes that would tend to reduce the amount of traffic.

An estimate of the average daily traffic to be expected by 1928, from the figures just given, may be made on a basis of calculation similar to that given previously, that is, concentrating 75% of the traffic on 10% of the road mileage, which gives an average daily traffic of 1 500 motor vehicles per mile.

The mileage of improved roads is constantly increasing, many sections of roads now traveled by a comparatively small number of vehicles will experi-

ence, when improved, an increase in traffic. This traffic will have been drawn away from those roads now included in the 10%, that is, there will be a

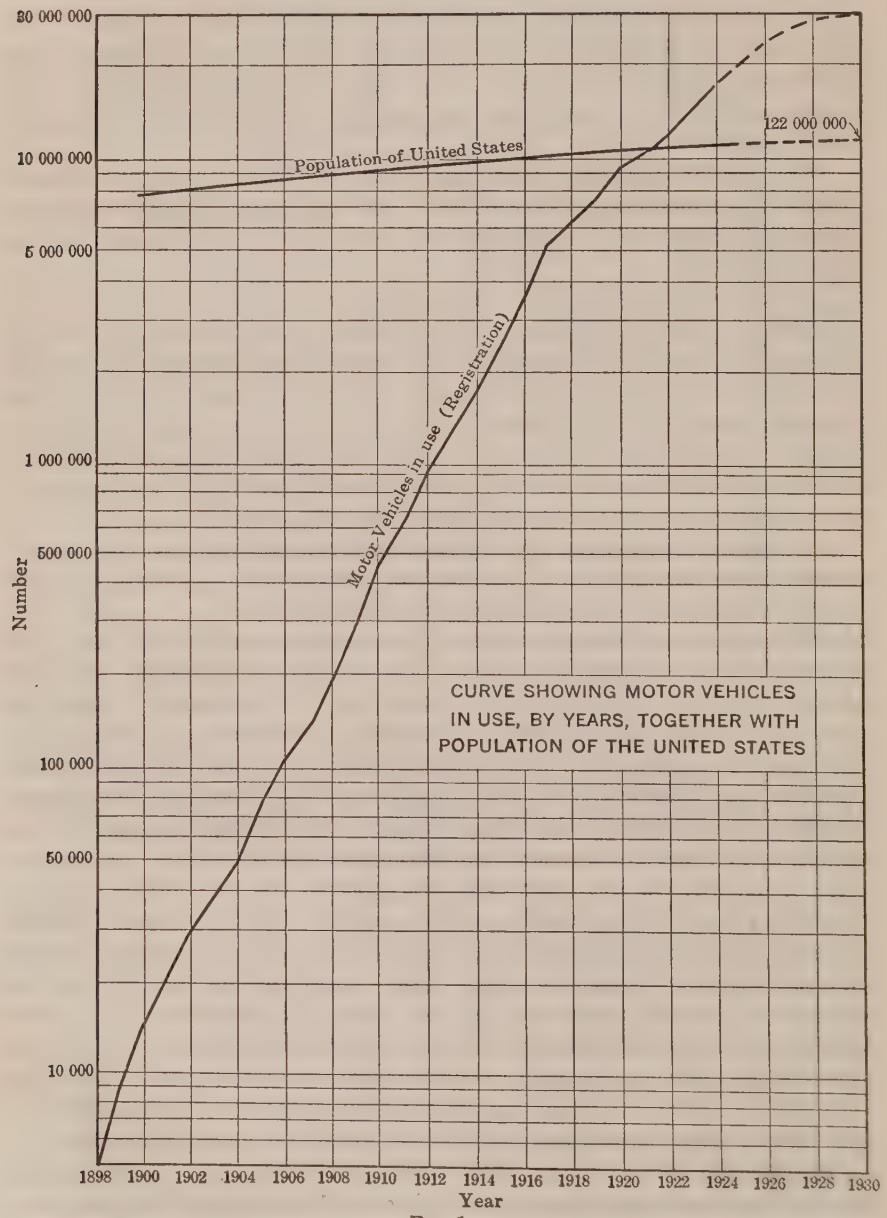

FIG. 1.

tendency to spread traffic over a larger proportion of the road mileage, all of which should have a tendency to reduce the density of traffic from the figure calculated—1 500 vehicles per day per mile.

From all the factors involved, and from the estimates made from these factors, it would seem to be safe to state that the present State highway systems, averaging about 10% of the total mileage, need not expect to be called on to carry in excess of an average daily traffic of 1 500 vehicles per mile. Every one can recall many evidences of traffic congestion and find it difficult to believe that much more traffic can be handled. This undoubtedly is true of some sections of certain highways where traffic concentrates. These particular sections of roads do carry nearly their capacity and frequently are taxed much beyond it; but the mileage of these sections of highways, so congested, forms only a small fraction of the total mileage of State highway systems.

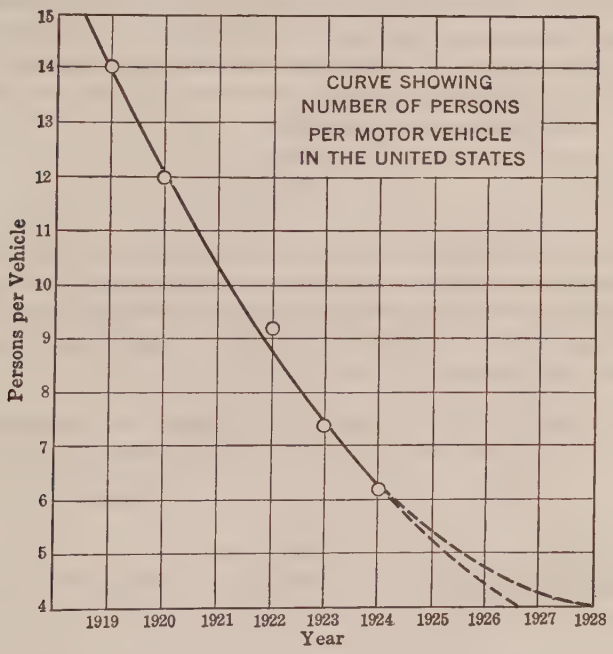

FIG. 2.

It is profitable, therefore, to make some comparison of the ultimate capacity of highways for the widths generally built at present with what has been set as the limit that may not be much exceeded for many years to come. The majority of highways on State highway systems provide for two lanes of traffic, and the traffic capacity of a two-lane road can be closely ascertained.

First, consider the question of the number of vehicles that may pass a given point in a single file, or what may be designated as the total discharge of a single lane of traffic. It is evident that this number is dependent on the velocity and the spacing or intervals between the vehicles; the vehicles to be considered as practically of a fixed length, which, for the present purpose, may be taken as 15 ft. Expressing the velocity, in miles per hour, as V,

and the clear distance between vehicles, in feet, as C, the total number of vehicles that will pass a given point in 1 hour is given by the relation:

$$N = \frac{5\,280\ V}{15 + C}$$

in which, 15 represents the average length, in feet.

It is known from observation that as the velocity of the vehicles increases, the spacing is varied. Whether this variation is directly as the velocity, or some other function of the velocity, is difficult to determine with any precision. There is some evidence, however, to indicate that the spacing between the vehicles varies approximately as the square of the velocity.

During special rush periods on the Washington-Baltimore Road, groups of vehicles moving 10 to 15 miles per hour, frequently were spaced as close as 15 ft., and other groups moving 25 to 30 miles per hour maintained an average spacing of 50 to 60 ft. Assuming that the average length of a vehicle is 15 ft., and that the clearance varies as the square of the velocity, the number of vehicles discharged per hour would be:

$$N = \frac{5\,280\ V}{15 + \frac{V^2}{15}}$$

If the first differential of this equation is equated to zero, it is found that N becomes a maximum when $V = 15$, that is, the interesting result is obtained that, if vehicles are spaced according to the square of the velocity, the maximum vehicle discharge will occur at 15 miles per hour, which, for a single line of traffic, equals 2 640 vehicles per hour.

TABLE 2.—Traffic Count, Baltimore-Washington Road, October 25, 1924.

Time, P. M.	Number of Vehicles.				Percentage of Vehicles.		Total vehicles, rate per hour.
	North.		South.				
	Automobiles.	Trucks.	Automobiles.	Trucks.	North.	South.	
4:30	23	1	15	1	60	40	480
4:35	19	1	15	..	57	43	420
4:40	18	..	17	1	50	50	432
4:45	54	..	19	1	73	27	888
4:50	52	..	22	..	70	30	888
4:55	47	1	11	1	80	20	720
5:00	44	2	7	..	86	14	636
5:05	49	..	20	1	70	30	840
5:10	104	1	8	1	92	8	1 368
5:15	111	1	12	1	90	10	1 500
5:20	126	..	7	..	95	5	1 596
5:25	93	2	15	..	86	14	1 320
5:30	110	1	6	..	95	5	1 404
5:35	112	..	10	..	91	9	1 464
5:40	...	..	..	..	..	..	.....

The traffic capacity of a two-track highway was observed at a number of different points on the Washington-Baltimore Road on different days during a special rush of traffic on the occasion of the Laurel Races in October, 1924. Vehicles were counted at 5-min. and sometimes 10-min. intervals, through the

peak of the traffic. Whenever the road became congested, a note was made. It was considered to be congested when a faster moving vehicle could not conveniently pass a slower one. As typical of these records that made on October 25, 1924, is given in Table 2.

This count began at 4:30 P. M., and the lowest number of vehicles recorded was at the rate of 420 per hour, at which time traffic was about evenly divided in each direction. No inconvenience was noted, that is, traffic in either direction could maintain its speed of about 30 miles per hour, and was not held back by the occasional slower moving trucks, there being ample opportunity to pass. This condition prevailed until about 5:10 P. M., when it was observed that the road began to be congested. From the stream of traffic north, which was moving at an average speed of 25 to 30 miles per hour, vehicles could not turn out readily and pass a slower moving vehicle in front. This condition occurred when the traffic was between 800 and 1 000 vehicles per hour.

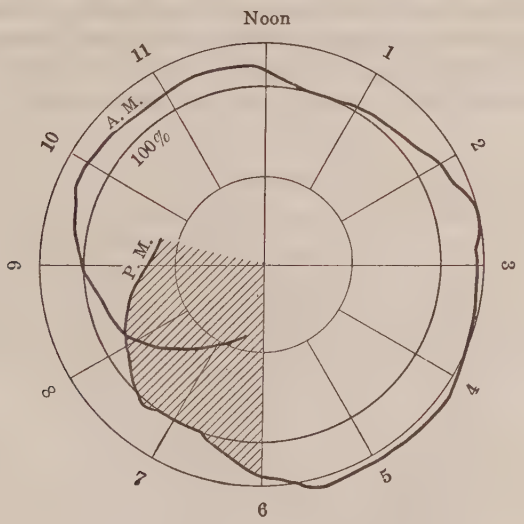

FIG. 3.—HOURLY VARIATION OF TRAFFIC.

It was noted that for a traffic of 1 368 vehicles per hour, of which approximately 90% were moving in one direction, the road was uncomfortably crowded. On another occasion when the traffic reached a rate of 1 296 vehicles per hour, the same notation was made, but, at 1 000 vehicles or less, this crowded condition was not recorded. It should be stated that the observations were made by a number of recorders.

At a speed of 30 miles per hour, based on the formula as previously stated, the distance between the vehicles varying as the square of the velocity, the number of vehicles passing in a single line per hour would be 2 100. The greatest number observed was 1 500, the average speed being noted as 30 miles per hour. Therefore, the actual spacing between the vehicles (185 ft.) was considerably greater than that provided by the formula which assumes the

spacing to vary as the square of the velocity, in which case vehicles would have a clearance of 60 ft. While the spacing was not uniform, yet the minimum clearance, as observed, varied from 50 to 60 ft.

From these conditions, the conclusion is reached that a two-lane road may carry traffic as great as 1 000 vehicles per hour without inconvenience to mixed traffic, the faster traffic having opportunity to overtake and pass the slower moving vehicles; and that for occasional peak traffic it will discharge readily, with traffic moving at an average speed of 25 miles per hour, at least to 1 600 vehicles per hour.

The next question is, what may be an average daily traffic if the recurrent high traffic load is not to exceed 1 000 vehicles per hour. This involves the diurnal variation of traffic. From results of various traffic studies, traffic in the mid-forenoon and late afternoon is 30 to 50% more than the average hourly traffic for the day. (See Fig. 3, which shows hourly variation of traffic.* The hourly average is shown as 100 per cent.) Thus, if 1 000 vehicles are to represent the recurrent daily peak traffic, the average hourly traffic should be taken at approximately 600 to 700 vehicles. For a 12-hour day, this would give an average daily traffic of 7 200 vehicles, which can be accommodated on a two-track road without congestion.

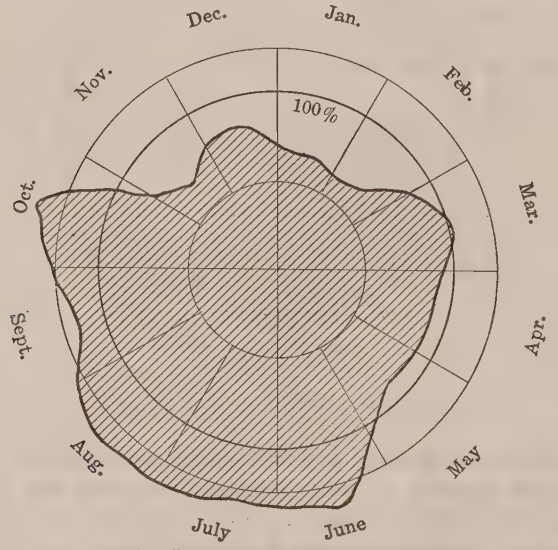

FIG. 4.—SEASONAL VARIATION OF TRAFFIC NEAR BALTIMORE, MD.

In order to determine the average daily traffic that may be maintained throughout the year, the seasonal variation of traffic must be studied. In the vicinity of Baltimore, which is perhaps illustrative of many sections, it is found that the average daily traffic, during the period from June through October, is about three-fourths greater than the average daily traffic for the

* Adapted from data on page 136 of California Highway System by the U. S. Bureau of Public Roads, 1920.

year. (See Fig. 4, which shows the seasonal variation of traffic near Baltimore, platted as percentage of daily average, shown as 100%, for 1917, 1918, 1919, and 1920.) If 7 200 vehicles per day is to represent the high seasonal average, then the yearly average should be taken at 4 000 vehicles per day, that is, there may be throughout the year an average daily traffic of 4 000 vehicles per day without causing congestion on a two-track road, allowance being made for the general increase in traffic due to seasonal variation and for the normal increase due to hourly variation throughout the day.

It was previously concluded that the largest probable expansion that may be reasonably expected in motor traffic, assuming 75% of the total traffic to be placed on only 10% of the roads, is an average daily traffic of 1 500 vehicles per mile, which is a little more than one-third the capacity of a two-track road.

With the highway systems built two lanes wide, supplemented by an increasing mileage of improved side roads, as well as extensions to the State roads themselves, it is not reasonable to expect that any general congestion of traffic will occur for many years to come. Congestion, however, will continue increasingly to occur at certain points where traffic tends to concentrate, until the forces which attract traffic to these centers cease to act. It may well be that the very congestion itself will cause such a point to be no longer a center of attraction. The traffic will stifle the very thing that attracts it. This is happening and will happen again unless means can be found to prevent continued congestion by additional roads, wider roads and streets, the dispersion of certain business activities, and, in general, the providing of a greater area for the traffic to be accommodated.

On the whole, however, engineers are proceeding wisely and sufficiently in the design of the larger proportion of the mileage of improved roads if a two-lane width is provided.

DISCUSSION

ARTHUR W. DEAN,* M. AM. SOC. C. E.—The prime origin of the extensive expansion of highways and the increase of highway traffic is the development of the internal combustion engine, just as the development of railroads and street railways in the Nineteenth Century originated with the improvement of the steam engine and the electric motor.

An outstanding factor of development in the United States, and one that is usually overlooked, is the general prosperity and aspiration of the people as a whole.

As in the past, various other elements will affect or govern the development of highways and the traffic over them, and some of these elements, although possibly mentioned by other writers, are so applicable that they will bear repetition:

1.—The gradual increase in efficiency and decrease in cost of motor vehicles leads directly to an increased use of such vehicles for the transportation of freight on what are commonly termed short hauls, and also toward an increase in the economical hauling distances.

2.—As the use and improvement of the motor vehicle progresses, the railway and railroad lines that are now being used mainly for short hauls may be outrivaled and their use discontinued, thereby increasing highway traffic.

3.—Many industries are now located at points somewhat remote from rail transportation lines, yet not so far from sources of raw material or from markets but that with improved highways and improved motor vehicles the highways may be profitably used for hauling their products. Such industries do and will tend to increase highway traffic.

4.—Opportunities exist for industrial and farming development in innumerable localities remote from transportation lines. Highway transportation will lead to development of the resources of these localities and, in turn, will re-act to increase further highway traffic.

5.—Passenger transportation by means of busses is increasing rapidly. Already there are several bus lines making regular daily trips between cities 100 miles apart, and many with hourly schedules between nearer points.

6.—There is always the inherent desire of the dwellers in congested areas to reach and enjoy the open country and for dwellers in the country to reach and enjoy the city scenes and activities. The average person enjoys a trip anywhere away from his own usual environment; there is also the professional man, the business man, and the artisan, who enjoy using, or may frequently of necessity use, the motor vehicle in preference to the old form of public or private conveyance. All these are large factors tending to increase highway traffic.

All these elements affect the development of highway traffic, but in order that such development may continue in healthy progress, the nations and subdivisions thereof must enact uniform and consistent laws and ordinances

* Chf. Engr., Div. of Highways, Dept. of Public Works, Boston, Mass.

of permissions and prohibitions, particularly as related to dimensions and speed of vehicles.

VAN ALEN HARRIS,* M. AM. SOC. C. E.—The viaducts across Biscayne Bay between Miami and Miami Beach, Fla., are illustrations of traffic saturation. The distance across the Bay is 3 miles, of which the larger part is a causeway with plenty of room for a very dense traffic. At each end of the causeway is a viaduct 2 000 ft. long and 20 ft. wide between curbs, allowing only one line of traffic in each direction. The speed limit is 25 miles per hour, with no passing of cars going in the same direction on the viaducts.

Miami Beach, with a building program of $4 000 000 per year, is a high-class residential section, and also the ocean bathing resort of Miami. Practically all labor working at Miami Beach comes from Miami across these viaducts and causeway. This flow of workmen returns to Miami between 4:00 and 6:00 P. M., which is the same time that the bathers return to Miami, with the consequent peak traffic between these hours.

In the summer of 1924, that is, out of the tourist season, 8 400 automobiles and trucks were counted crossing in 1 day, and between 1 200 and 1 300 in 1 hour, at the peak of the traffic, of which about 85% was going in one direction. No more could have passed, as there was a steady stream of cars which were urged to go as fast as possible, but, although the speed limit is 25 miles per hour, the average speed during the congestion was about 18 miles per hour. It may be assumed, therefore, that about 1 100 cars in each direction is the limit of a 20-ft. road under the most favorable conditions.

* Philadelphia, Pa.

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Paper No. 1577

THE EFFECT OF MOISTURE ON CONCRETE*

By W. K. Hatt,† M. Am. Soc. C. E.

With Discussion by Messrs. A. T. Goldbeck, G. E. P. Smith, and W. K. Hatt.

Synopsis

This paper reports investigations conducted by the Engineering Experiment Station of Purdue University in co-operation with the U. S. Bureau of Public Roads, and reviews the literature on the measurement of volume changes in Portland cements and concretes.

The primary purpose of the investigations was to measure the maximum warping and surface deformations produced in a concrete road slab as a result of a non-uniform distribution of moisture, to give a basis for estimating possible initial stresses. The tests also yield data of expansion and contraction of concrete, useful in the design of concrete structures.

The investigation was divided into two sections, namely: (1) Observations on a concrete road slab, 7 in. thick, 18 ft. wide, and 25 ft. long, resting on a porous sub-grade, covered with a tent and housed in a shed to control the distribution of moisture and the effects of temperature; (2) tests of strength and length changes of small beam units, some partly and some uniformly saturated.

Slab While Drying.—After curing under water the slab was dried by circulating air both above the slab and through the sub-grade. The more rapid drying of the top surface warped the slab to a maximum upward deflection of 0.12 in. at the corners and 0.04 in. at the unbroken edge during a period of 40 days, after which time upward movement ceased, the shortening of the diagonal being 0.0075 per cent. After 3 months of further drying, the slab had become approximately uniformly dry throughout its depth and substantially level, with a diagonal shortening of 0.0210 per cent.

* Presented at the meeting of the Highway Division, New York, N. Y., January 22, 1925.

† Prof. of Civ. Eng., School of Civ. Eng., Purdue Univ., Lafayette, Ind.

Slab Saturated on the Bottom.—In order to saturate the bottom of the slab, water was then introduced to the level of the top of the gravel sub-grade. During 110 days of saturation the slab again warped upward 0.20 in. at the corners and 0.066 in. at the unbroken edge. Simultaneously, a diagonal shortening of 0.0070% took place in the top face of the slab, of which 46% occurred in 2 hours and nearly all in 2 days. The shortening was caused by the expansive force of the saturated lower part of the slab, and occurred without a change in moisture content or temperature of the top layers. Assuming an elastic condition and a modulus of elasticity of 5 000 000 lb. per sq. in., a compressive stress of 350 lb. per sq. in. in the top is indicated. The level of the water was finally raised, but the concrete at the bottom apparently had become impervious and no further warping occurred. Then the surface was covered with saturated burlap to simulate the effect of prolonged rain.

Daily changes in the temperature of the slab resulted in differential warpings, depending on the degree of saturation and the temperature gradient. A typical daily change in air temperature above the slab of 42° Fahr. resulted in a variation of 3° Fahr. from top to bottom of the slab. The accompanying downward deflection of the corner due to this warping was 0.023 in., and the expansion of the surface, 0.0015 per cent. The movements due to this warping may be compared to a deflection of 0.20 in. and a shortening of 0.0057% due to the saturated sub-grade.

Measurements of the extensibility of concrete surfaces were reported by the writer to the American Concrete Institute, in February, 1926. The unit extensibility of 1 : 2 : 3 concrete before microscopic fissures appear was 0.00010, and at the time of cracks visible to the naked eye, 0.00018. Wet concrete was more flexible than dry concrete.

Loading tests were made on the corner of the slab for three conditions of warping.

The Beams.—Tests were conducted on concrete beams 4 in. by 7 in. by 4 ft.; some were uniformly wet, some uniformly dry, and some warped by non-uniform moisture distribution. The transverse strength was determined for dry, saturated, and partly saturated beams.

Beams uniformly saturated had a strength ranging from 80 to 85% that of dry beams, the average value being 82.0 per cent. Compression tests of wet concrete cylinders exhibited a strength of 75 to 83% that of dry cylinders, the average being 79.0 per cent.

Certain beams, previously dried from 3 to 4 months, were warped by saturating one face. In so doing the dry face was compressed for a small depth of saturation, and extended for medium and for maximum saturation. (See Appendix I.)

The dry face was tested as the tension face in a transverse flexure test. The variations in transverse strength for these various moisture conditions have been expressed in percentage values, 100% representing the completely dried specimens. The beams having a low depth of saturation and a shortened dry face exhibited a strength of from 109 to 127%, or an average of 118%; for medium saturation and slight extension of the dry face, the strength varied from 83.6 to 96.5%, or an average of 88.7%; for maximum saturation and greater extension of the dry face the strength was 80.7 per cent.

When the wet face was broken in transverse flexure these values became 83.5, 71.3, and 76.6%, respectively.

Volume Changes of Cements and Concretes.—Measurements, begun 2 days after moulding, of the volume changes in specimens of several brands of neat cements varied greatly. Expansion in water during 10 days was from 0.03 to 0.056 per cent. Contraction in air during 90 days subsequent to immersion was from 0.052 to 0.142 per cent.

Gravel concrete, 1 : 2 : 3.5 (Cement F), expanded 0.005% during 40 days in water, and contracted 0.041% during 90 days in air and 0.050% during 30 days in an oven. Gravel concrete, 1 : 1.5 : 3 (Cements A and D), expanded 0.005% in water, and contracted 0.015% in air and 0.0225% in the oven during the same periods.

Introduction

The investigation of the changes of shape of a concrete road slab caused by unequal moisture distribution, together with subsidiary tests described in this paper, was suggested by A. T. Goldbeck, Assoc. M. Am. Soc. C. E., at that time Chief of the Division of Tests and Investigations, of the U. S. Bureau of Roads, as a part of a co-operative work between the Bureau, T. H. MacDonald, Chief, and the Engineering Experiment Station of Purdue University, Dean A. A. Potter, Director.

The tests were made at Purdue University under the supervision of the writer and Professor R. B. Crepps. Mr. A. C. Benkelman was in direct charge, assisted by Mr. E. Gustavsen, both of the Staff of the U. S. Bureau of Roads. Mr. R. E. Mills, of the Engineering Experiment Station of Purdue University, assisted from time to time. Special acknowledgment should be made of the painstaking care in observations and the devotion to the tedious program of measurements contributed by Mr. Benkelman.

In these studies the following matters were examined:

1.—The maximum distortion (including deflection and fiber deformations) produced in a concrete road slab 18 ft. wide, 25 ft. long, and 7 in. thick, by a non-uniform distribution of moisture.

2.—The effect of moisture changes on the modulus of rupture and compressive strength of concrete, determined by subsidiary tests on (1) beams, 4 in. by 7 in. by 4 ft., and (2) cylinders, 6 in. in diameter by 12 in. long.

3.—The volume changes in the concrete beam specimens due to exposure, including warping due to partial saturation.

The approximate number of observations involved in the entire study was:

Type.	Number.
Deformation	40 000
Deflection	3 000
Strength	600
Weight	1 000
Temperature	1 500

NOTE.—When the word "dry" is used in this paper, it refers to condition of hardened concrete and not to consistency of fresh concrete.

Description of the Test Slab

The one-course concrete slab was 7 in. thick, 18 ft. 0 in. wide, and 25 ft. 0 in. long, with a longitudinal doweled center joint. A sub-grade, consisting of a 4-in. layer of coarse gravel, was held in an impervious wooden sub-base, the sides of which extended above the surface of the slab. The slab was housed in a shed and further protected under a tent. (Figs. 1 and 2.) It was cast on September 12, 1923. The subsequent treatment was as follows: (1) Cured under water for thirty days, until October 12, 1923; (2) the water removed and the slab dried for 188 days, until May 28, 1924; (3) the sub-grade saturated for 108 days, until October 20, 1924; and (4) the top covered with wet burlap.

Materials

A mixture of two brands of standard Portland cements, A and D, was used in one half-section of the slab and a single brand, F, in the other. The results of the usual standard tests of the cements are given in Table 1.

TABLE 1.—Test of Cement.

Brand.	Percentage, H_2O.		Percentage retained on 200-mesh sieve.	Time of Setting.				Soundness.	Tensile Strength in Pounds.	
				Initial.		Final.				
	Normal consistency.	Mortar.		Hours.	Minutes.	Hours.	Minutes.		7 days.	28 days.
A and D mixture, equal quantities..	24	10.5	17.17*	2	10	5	15	O. K.	297	373
D.....	23	10.3	20.8*	2	35	5	20	O. K.	242	326
A.....	24	10.5	22.6*	4	00	5	55	O. K.	255	327
F.....	25	10.7	19.1	3	35	5	15	O. K.	220	333

* Different samples.

The aggregates were washed sand and gravel from the Western Indiana Sand and Gravel Company at Lafayette, Ind.

The results of the sieve analysis and miscellaneous tests of the aggregates are given in Table 2.

The coarse aggregate was separated into three sizes: Between the 2-in. and 1-in. screens; between the 1-in. and the ½-in. screens; and between the ½-in. and the ¼-in. screens. It was then recombined in such proportions as to give maximum density according to Fuller's maximum density curve. The consistency of the mix was checked by the slump-cone method. All materials, including water, were weighed for each batch.

The properties of the concrete are listed in Table 3.

Expansion and Contraction of Cements.—The changes in length of six brands of cement and the concrete made from certain of the brands were measured using Berry extensometers. The specimens were 2 by 2 by 24 in. for neat cement and 4 in. by 7 in. by 4 ft. 0 in. for concrete. Measurements

began two days after moulding with previous moist-chamber storage. The results are shown in Table 4 and in Figs. 3, 4, and 5. Since the preparation of the paper in December, 1924, the measurements of expansion and contraction have been continued, and the curves in Fig. 3 extended. It appears that the individual behavior of the several brands of cement is confined to the first period of approximately sixty days, after which the movements of the several brands are alike. Fig. 5 represents a sampling of the market, six months later than Fig. 3. It appears that the several brands of cement maintain their relative properties.

TABLE 2.—Tests of Aggregates.

Coarse Aggregate, Unit Weight = 102.5 lb. per cu. ft.
Fine " " " = 106.8 " " " "
Fineness Modulus of Coarse Aggregate = 7.32.
Fineness Modulus of Fine Aggregate = 3.17.
Fineness Modulus of Combined Aggregate = 5.90 for a 1 : 1.5 : 3 mix.
= 5.77 for a 1 : 2 : 3.5 mix.

Sieve Analysis of Gravel.
(Tyler Standard Sieves.)

	Total Percentage Retained on Each Sieve.								
Sieve sizes	1½ in.	¾ in.	⅜ in.	No. 4	No. 8	No. 14	No. 28	No. 48	No. 100
Percentage	0	46	87	98	99	99	99	99	100

Sieve Analysis of Sand.
(Tyler Standard Sieves.)

	Total Percentage Retained on Each Sieve.						
Sieve sizes	⅜ in.	No. 4	No. 8	No. 14	No. 28	No. 48	No. 100
Percentage	0	0	3	38	77	97	99

TABLE 3.—Tests of Concrete.

Slump (Four 8 by 12-in. cones) = 1 to 1½ in.

$\frac{W}{C}$ = 0.809 for a 1 : 1.5 : 3 mix.

$\frac{W}{C}$ = 0.935 for a 1 : 2 : 3.5 mix.

Specific gravity = 2.68.
Density = 0.896.
Percentage of voids = 10.4.
Percentage of absorption by weight = 4.29.

	Average Weights of Both Concretes, in Pounds per Cubic Foot.		
	Oven-dried to constant weight.	Immersed in water to constant weight.	Air-dried in laboratory for 135 days.
Unit weight, in pounds per cubic foot	149.89	156.32	151.89

Fig. 1.—General View of Concrete Test Slab Showing Tent Covering and Means of Saturating Sub-Grade.

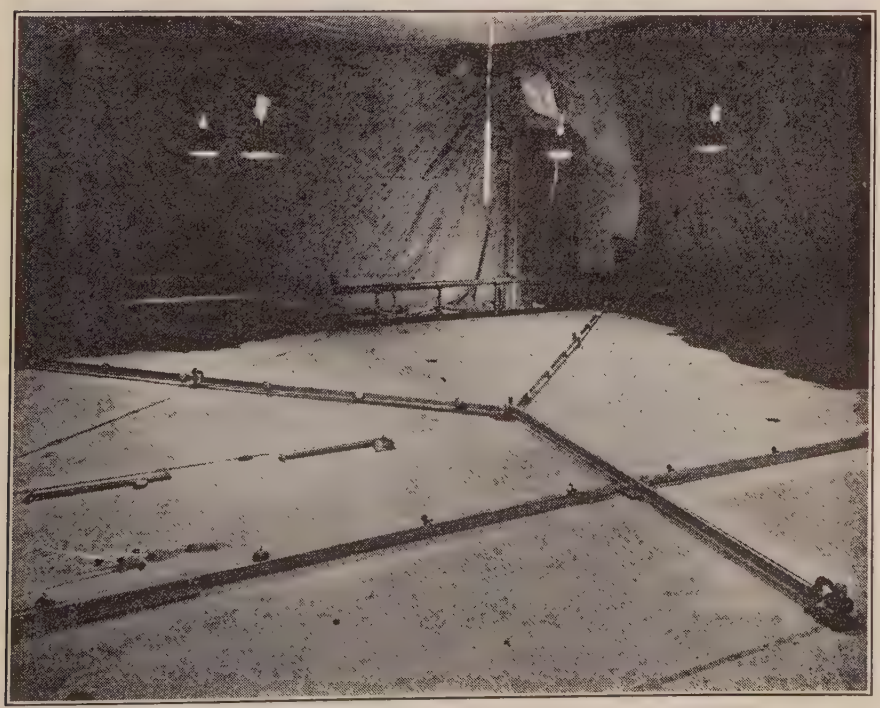


Fig. 2.—Details of Concrete Test Slab and Measuring Apparatus.

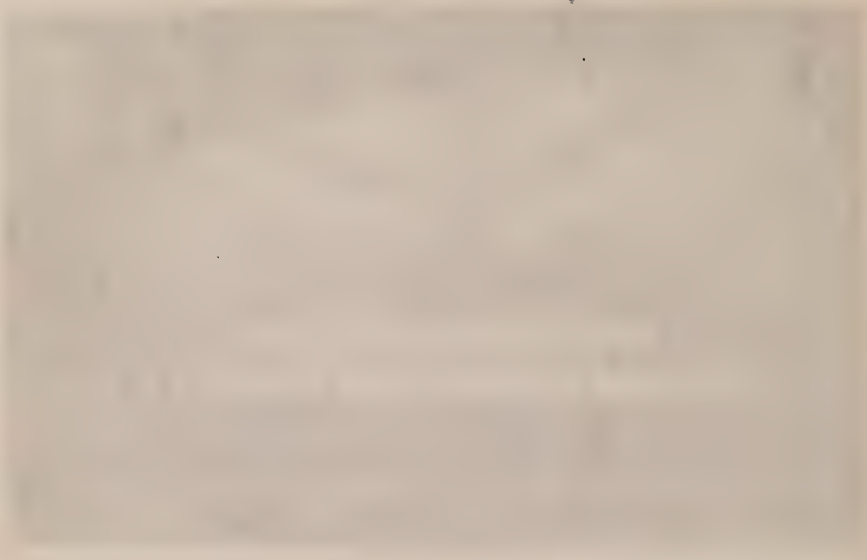


TABLE 4.—Expansion and Contraction of Neat Cements and Concretes.

Brand.	A.	B.	C.	D.	E.	F.	Average of A and D.
Neat Cement:							
Percentage of H_2O for normal consistency	24.00	24.00	22.00	23.00	23.00	25.00	.....
Percentage retained in 200-mesh sieve.	17.53	22.86	20.38	23.90	16.24	19.14	.....
Percentage of expansion in water, 10 days	+0.056	+0.044	+0.040	+0.036	+0.084	+0.080	+0.048
Percentage of contraction in air, 90 days (subsequent to immersion)	−0.052	−0.062	−0.080	−0.094	−0.114	−0.142	−0.073
Total percentage of change	0.108	0.106	0.120	0.130	0.148	0.172	0.121
Net percentage of change	+0.004	−0.018	−0.040	−0.058	−0.080	−0.112	−0.025
Concrete:							
Gravel concrete, 1:2:3.5:							
40 days in water	.....	.....	.....	.....	.....	+0.005	.....
90 days in air	.....	.....	.....	.....	.....	−0.035	.....
40 days in water, 90 days in air, 20 days in oven	.....	.....	.....	.....	.....	−0.040	.....
40 days in water, 90 days in air, 30 days in oven	.....	.....	.....	.....	.....	−0.0500	.....
Gravel concrete, 1:1.5:3:							
40 days in water	.....	.....	.....	.....	.....	.....	+0.0050
90 days in air	.....	.....	.....	.....	.....	.....	−0.0150
40 days in water, 90 days in air, 20 days in oven	.....	.....	.....	.....	.....	.....	−0.0200
40 days in water, 90 days in air, 30 days in oven	.....	.....	.....	.....	.....	.....	−0.0225

If the measurements had begun at the time the specimens were moulded, the comparisons might have been different. (See Appendix II.) A comparison of the basic water content and the voids in the cement paste at this basic water content would no doubt throw light on these differences in the changes of

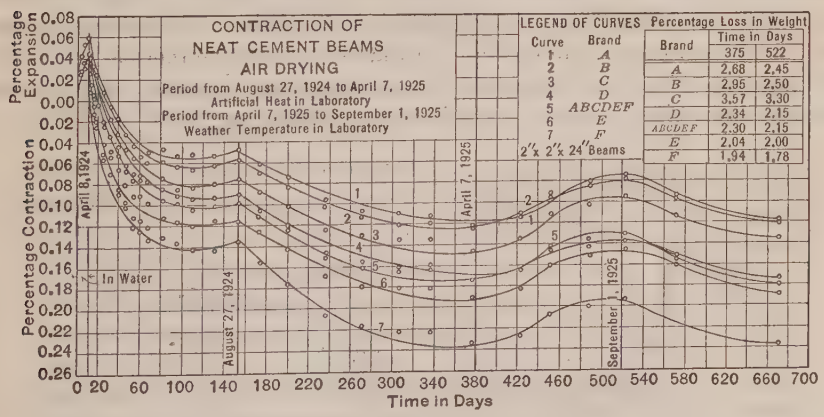


Fig. 3.

length, in the quantity of water required for standard consistency, and in the fineness or percentage retained on the 200-mesh screen as noted in Table 4. The basic water content has been defined by A. N. Talbot, Past-President, Am. Soc. C. E., as the percentage of water at which the voids are a minimum.

It appears that those cements which expand most in water, subsequently contract least in air. The neat cement prisms expanded in water nearly ten

times as much as the concrete beams; in air, they contracted nearly five times as much as the concrete.

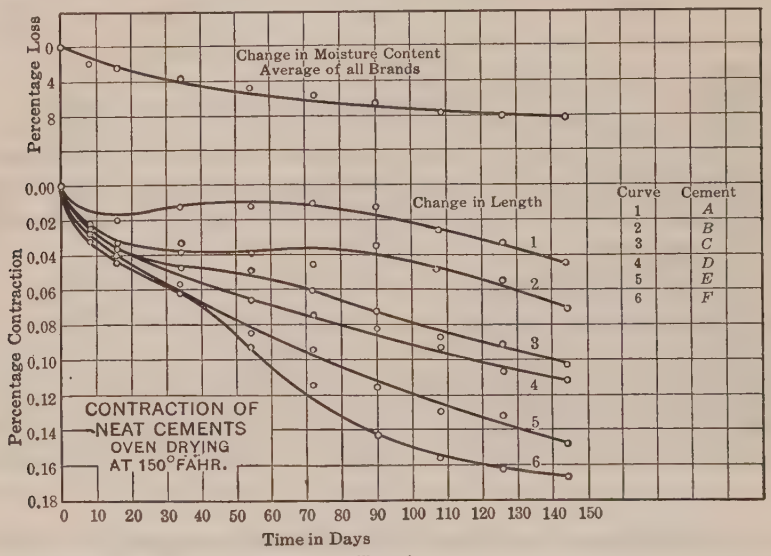


FIG. 4.

Expansion Due to Temperature.—The coefficient of temperature expansion of two concretes was measured, with the results shown in Fig. 6. The coefficient varies in amount depending on the temperature, ranging from 0.0000040 per 1° Fahr. to 0.0000065 at a temperature of 150° Fahr. The specimens were dried in an oven previous to testing. In Fig. 6 is shown the thermal coefficient of expansion of several concretes. The temperature of the concrete was obtained from blank beams in which were thermometer wells, consisting of

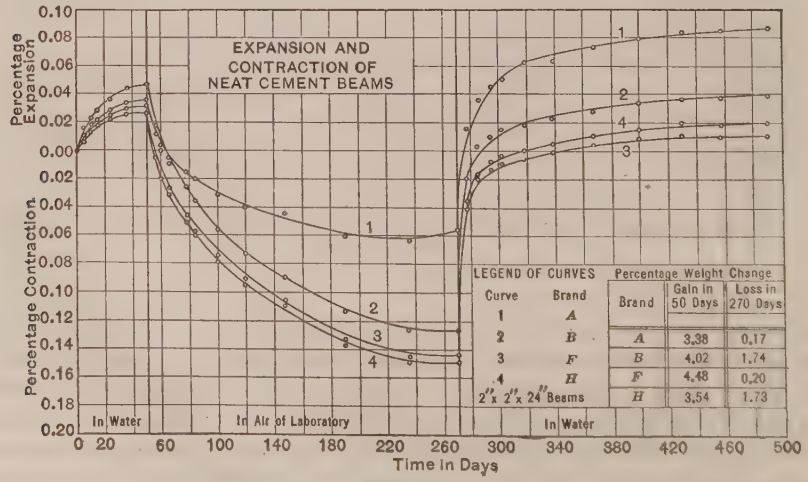


FIG. 5.

steel pipes filled with a light paraffin oil. The thermometers were graduated to one-fifth of a degree. Reading from the left, the chart covers: (1) Three beams, 6 in. by 8 in. by 4 ft. 0 in., 1 : 1.5 : 3 mix, age, 2 months, cured under wet burlap during 30 days, and dried in the oven at 150° Fahr. for 21 days. The beams were taken from the oven and readings taken at intervals of 20° Fahr. as the temperature dropped. Four cycles were read on as many days; (2) two beams, 7 in. by 4 in. by 4 ft. 0 in., 1 : 2 : 3 mix, age, 2 months, kept immersed in water the temperature of which was varied and held constant 2 hours before observation; (3) three beams, 4 in. by 7 in. by 4 ft. 0 in., 1 : 2 : 3.5 mix, age, 9 months, cured under wet burlap for 14 days, oven-dried for 1 month, and air-dried for 7½ months; the beams were moved from the warm laboratory air to the cold air of a shed; and (4) two beams, 4 in. by 6 in. by 4 ft. 0 in., 1 : 2 : 3 mix, age, 2 months, cured under wet burlap for 14 days, in air, 6 weeks.

These curves indicate that the variation of coefficient of thermal expansion varies with the temperature range under observation. The thermal coefficient of expansion for the several brands of cement tested neat was as given in Table 5.

TABLE 5.

Brand.	Age, 4 months Dried in air Range, 50°–73° Fahr.	Age, 2 months Under water Range, 52°–90° Fahr.
1	0.0000096	0.0000094
2	0.0000094	.....
3	0.0000092	0.0000087
4	0.0000090	.....
5	0.0000089	.....
6	0.0000083	.....
7	0.0000085	0.0000085

Beams and Cylinders—Uniform Moisture

A series of tests, subsidiary to the road slab, on beam units and compressive cylinders has included a study of the following:

Length changes resulting from:

1.—Continuous immersion in water.
2.—Continuous exposure to air in the laboratory.
3.—Continuous exposure to weather.
4.—Oven drying at 150° Fahr.
5.—Immersion of oven-dried and air-dried specimens.

Transverse strength determinations as to:

1.—Decrease due to uniform saturation.
2.—Changes due to age and treatment.
3.—Effect of freezing temperatures on saturated and dry concrete.

Compressive strength tests for:

1.—Increase due to age and treatment on compressive strength.

The size of the beam specimens was 4 in. by 7 in. by 4 ft. 0 in.

In the initial series of tests concrete similar to that of the road slab was investigated, namely, a 1 : 1.5 : 3 mix, using Cements A and D. In a later series of tests, Cement F was used, the mix being 1 : 2 : 3.5.

Length Changes of Beams.—Measurements of changes in lengths of the beams due to moisture were begun two days after moulding. A 20-in. Berry strain-gauge, used in the measurements, was compared to a standard Invar steel bar, maintained at a constant temperature. Six gauge lines were set in each beam, three on the top and three on the bottom. The results are the average of measurements on at least four beams. Corrections for the influence of temperature were made from observations of the temperature of two auxiliary beams, the thermal coefficient of which was previously measured.

Figs. 7 and 8 show the percentage changes in moisture content and the corresponding change in unit length of the concrete. The 1 : 2 : 3.5 concrete, Cement F (Fig. 7), expanded 0.0050% after a continuous curing under saturated burlap for 40 days and contracted 0.0035% after 90 days in air. The 1 : 1.5 : 3 concrete, Cements A and D (Fig. 8), expanded 0.0050% after a similar curing for 40 days and contracted 0.015% after 90 days in air. Thus, considerable variation is evident in the degree of length change of the two concretes as affected by the character of the cement.

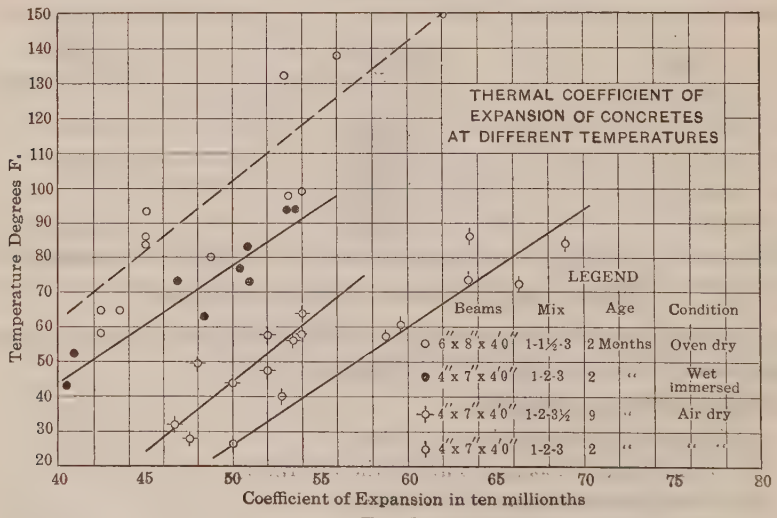


FIG. 6.

Additional beams of concrete, using Cement F, were exposed in the weather Changes in length and weight and the amount of rainfall were observed, as shown in Fig. 9. A maximum contraction of 0.033% occurred in 75 days. Beams of similar concrete exposed to the air of the laboratory for the same period of time contracted 0.033 per cent.

Tests at Purdue University in February, 1926, show the shrinkage of several aggregates in several mixes.

The beams were 4 in. by 7 in. by 4 ft., removed from the forms 3 days after pouring and cured under wet burlap for 14 days, following which they

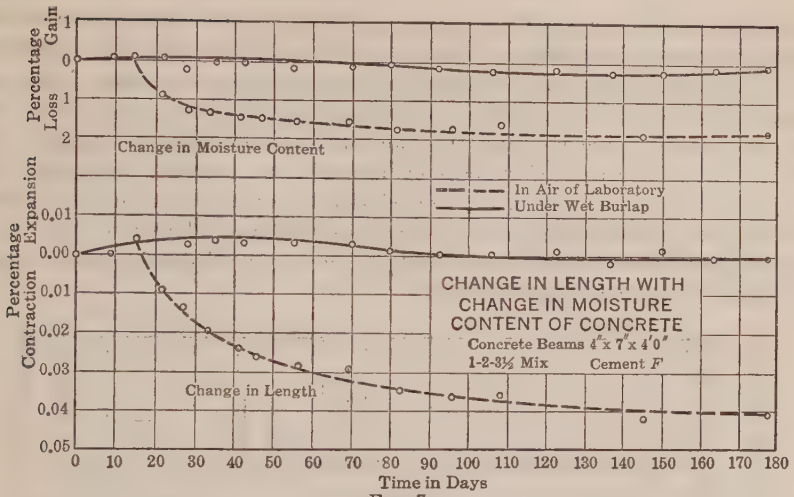


FIG. 7.

were exposed to the air of the laboratory. Corrections for temperatures varying by 5° were made. The aggregates were gravel, hard limestone, and slag. The fine aggregate was a well graded sand.

The contraction at the age of 80 days was, as follows:

	Contraction in percentage.
Neat cement	0.056
1 : 2 mortar	0.054

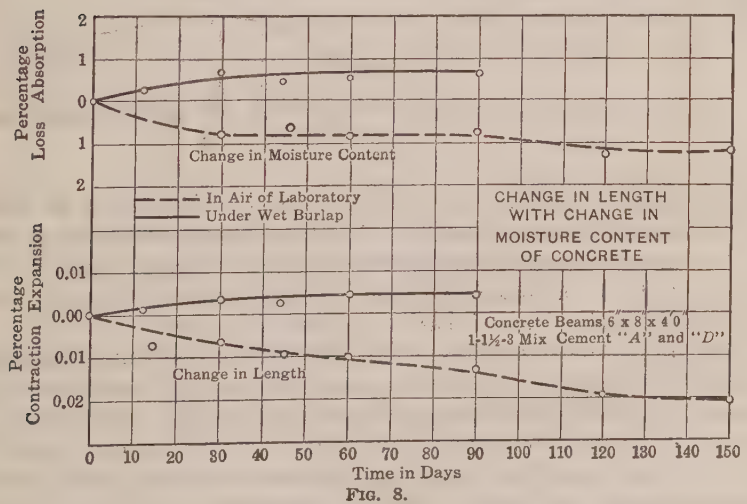


FIG. 8.

The average of the three aggregates shows the following contraction:

1 : 2 : 3	0.028
1 : 2.5 : 4	0.026
1 : 2.5 : 5	0.025
1 : 3 : 6	0.021

Transverse Strength.—The beams were tested (1) by loading at the third points, the span length being 42 in.; and (2) by loading them as cantilevers.

In the former case, the deformations of the top and bottom faces were measured by 10-in. Berry strain-gauges; in the latter, one end of the beam was fixed between the bed and the movable head of a testing machine and broken near the section of support by means of an attached lever arm 5 ft. in length. In this manner a number of sections of one beam may be broken consecutively. To load the testing machine shot was allowed to flow into a bucket. The halves of the beams broken under the third-point loading were also tested under cantilever loading.

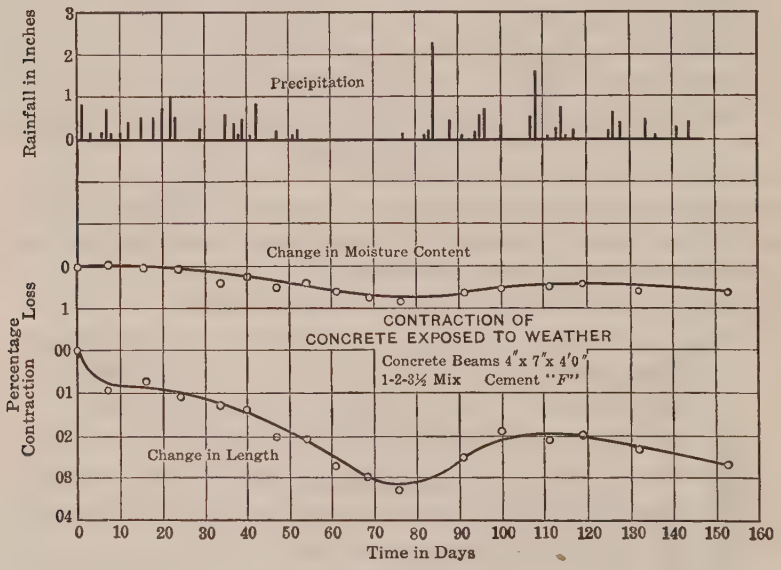


FIG. 9.

Fig. 10 shows the relative strength values of a series of tests on concrete beams made with Cements A and D. A 20% reduction results from saturation. An increase of approximately 100% in strength appears when saturated concrete is tested in a frozen condition. This is due to the strength of the ice. The percentage relation is based on air-dried concrete.

In Fig. 11 is shown the effect of age and moisture on transverse strength of beams. The specimens that were cured continuously under wet burlap and those air-dried, but subsequently saturated, contain approximately the same quantity of water by weight. The latter, however, are consistently 19% lower in strength on a direct basis than the former at an age of 90 days. All the specimens were made at the same time, and then exposed to varying conditions. Each plotted point in Fig. 11 represents the average of twelve tests.

No doubt the water is concentrated to some extent at the surface of the samples that are air-dried, but subsequently saturated. The appearance of the broken specimens, however, indicated uniform moisture distribution.

Compressive Strength.—Cylinders 6 in. in diameter by 12 in. long were capped with plaster of Paris and tested in compression. Deformations were measured to about one-half the ultimate strength by means of a ring com-

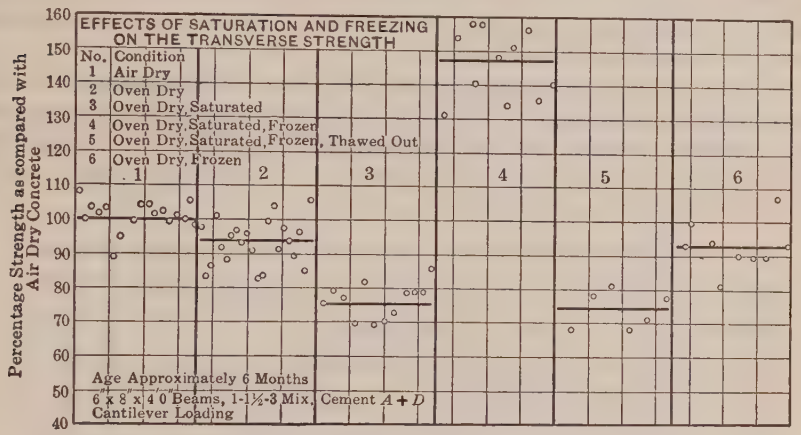


FIG. 10.

pressometer, with the results shown in Fig. 12, which checked the findings in the case of the beams. Each plotted point represents the average of three tests.

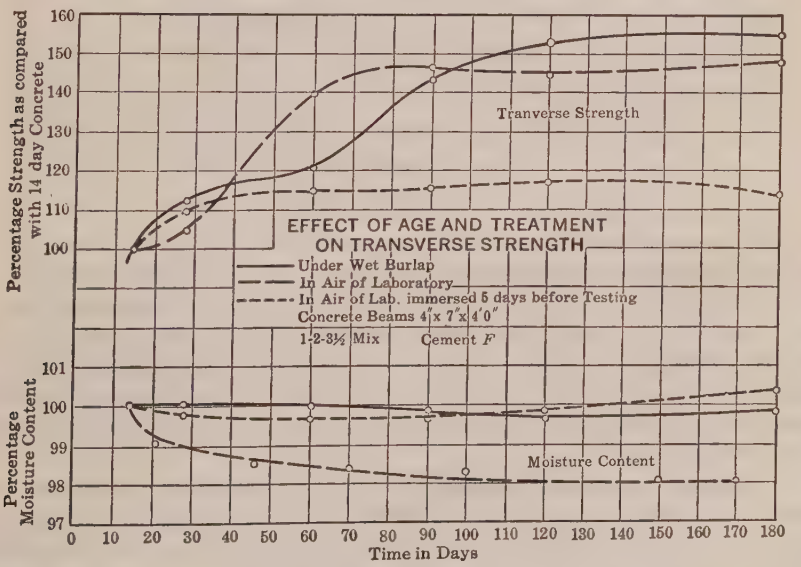


FIG. 11.

Aggregate failures occur with consistent regularity in the case of the air-dried or oven-dried specimens, likewise with specimens cured continuously under damp burlap; in the case of the saturated concrete, however, the failures are invariably due to bond between the mortar and the aggregate.

Results of Tests of Slab

Method of Tests.—By means of perforated pipes in the gravel fill and an air-pressure system, a dry sub-grade was finally obtained. Warm dry air was maintained above the slab in the tent enclosure. Water could also be introduced in the sub-grade when desired.

Previous to the construction of the slab, concrete bench-marks for the support of deflection beams were set 4 ft. 0 in. below the level of the ground. Rods extended upward from the bench-marks through pipes set in the sub-grade and slab. The slab and observation layout is shown in Fig. 1, and in Fig. 13.

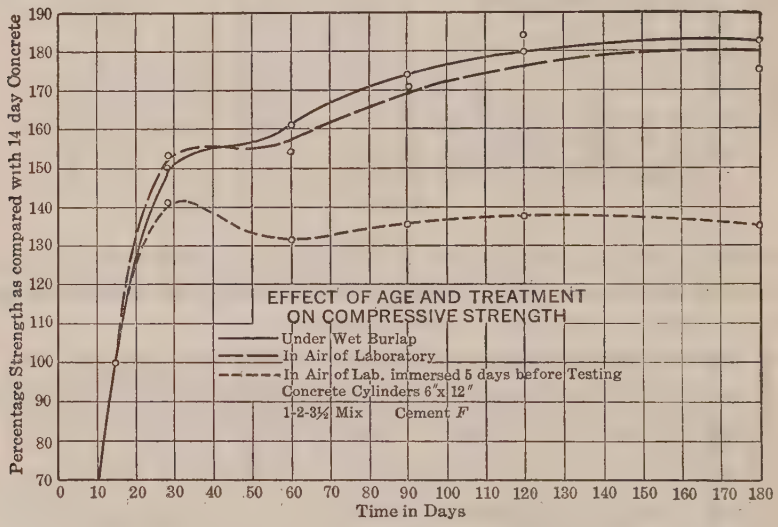


FIG. 12.

The deformation measurements were made with the 20-in. Berry strain-gauges. Deflections were measured with Ames dials reading to 0.001 in.

Non-Uniform Drying.—Following the curing period (immersion for 28 days) hot dry air was forced under the slab through the perforated pipes in the gravel fill by a pressure fan; simultaneously, a drying atmosphere was maintained above the slab in the tent enclosure. Observations of the subsequent deformations and deflections were taken at weekly intervals. (See Figs. 14, 15, 16, and 17.)

The more pronounced shrinkage occurring in the upper fibers was manifested by the fact that the edges and corners gradually curled up. A maximum uplift of 0.04 in. for the unbroken edges (at Point 2, Fig. 14), and 0.12 in. for the corners (at Point 1, Fig. 14), was attained at the end of 40 days. The subsequent return to a non-warped or uniformly dry condition required 5 months. A slight permanent distortion, as shown at Point 3, Fig. 15, representing the diagonal profile previous to the saturation of the sub-grade, is attributed to the more pronounced contraction of the richer or troweled surface than of the body of the slab. Observations were made of the longi-

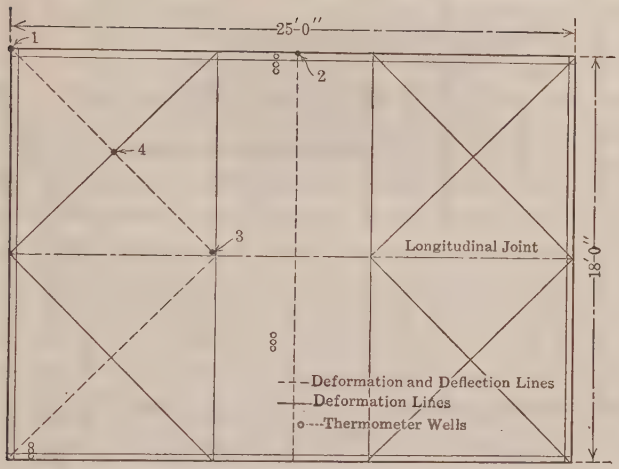


FIG. 13.—SLAB AND OBSERVATION LAYOUT.

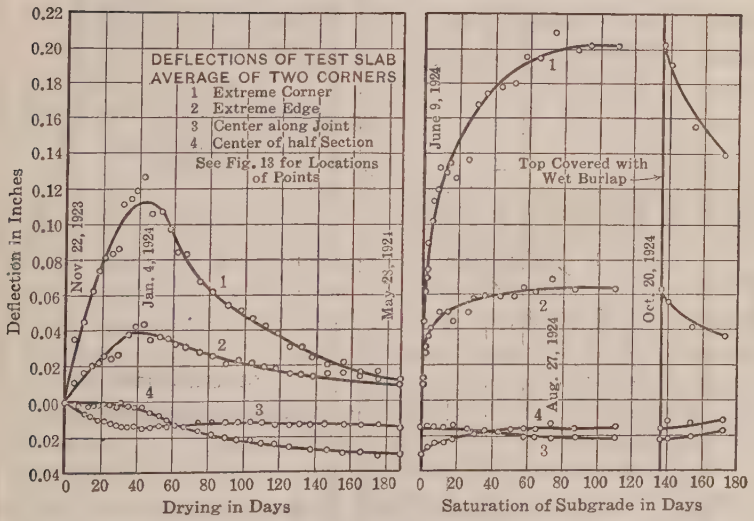


FIG. 14.

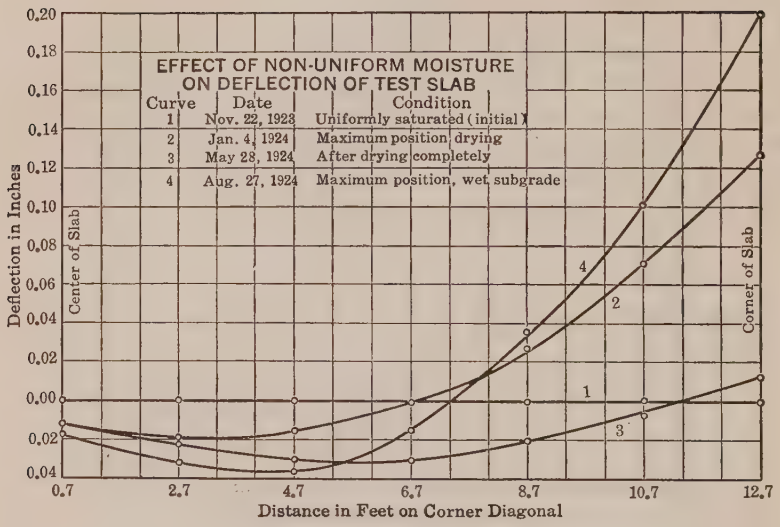

EFFECT OF NON-UNIFORM MOISTURE
ON DEFLECTION OF TEST SLAB
Curve Date Condition
1 Nov. 22, 1923 Uniformly saturated (initial)
2 Jan. 4, 1924 Maximum position, drying
3 May 28, 1924 After drying completely
4 Aug. 27, 1924 Maximum position, wet subgrade
Deflection in Inches
Center of Slab
Corner of Slab
Distance in Feet on Corner Diagonal
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.02
0.04
0.7
2.7
4.7
6.7
8.7
10.7
12.7

FIG. 15.

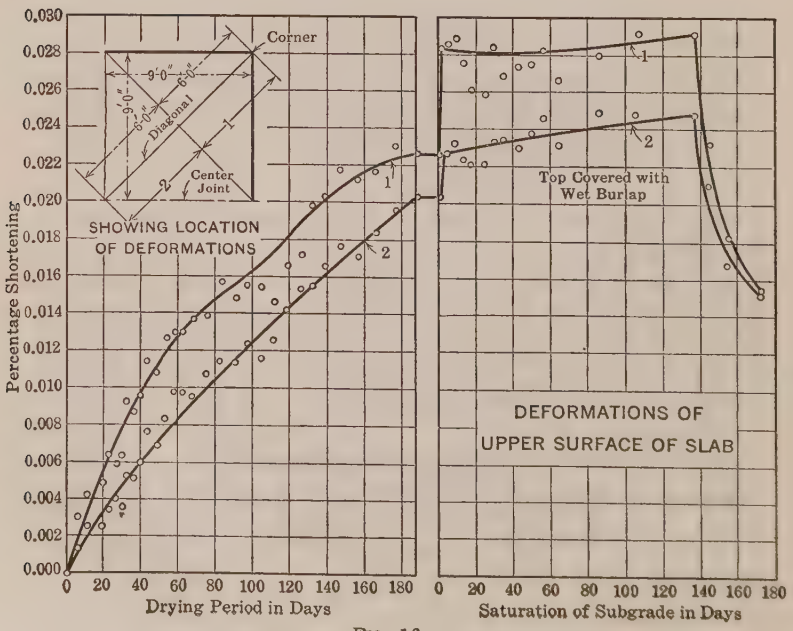

Corner
9'-0"
6'-0"
Diagonal
Center Joint
SHOWING LOCATION
OF DEFORMATIONS
Percentage Shortening
Drying Period in Days
Top Covered with
Wet Burlap
DEFORMATIONS OF
UPPER SURFACE OF SLAB
Saturation of Subgrade in Days
0.030
0.028
0.026
0.024
0.022
0.020
0.018
0.016
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
0 20 40 60 80 100 120 140 160 180

FIG. 16.

tudinal, transverse, and diagonal deformations in the upper surface of the slab. An average of two corner diagonals is shown in Figs. 16 and 17.

All measurements of surface deformations of the slab were corrected for these changes in temperature which took place uniformly throughout the concrete. That is, while the readings of warping, caused by the moisture gradient in the slab, were always taken when the temperature was uniform throughout the depth of the slab, there might be a general drop in temperature between the readings of two successive weeks. In this case, the deformation readings on top of the slab were corrected for the temperature on the basis of a coefficient of expansion appropriate to the temperature of the concrete. (See Fig. 6.) Therefore, the deformations shown in Fig. 16 are net after the corrections have been applied.

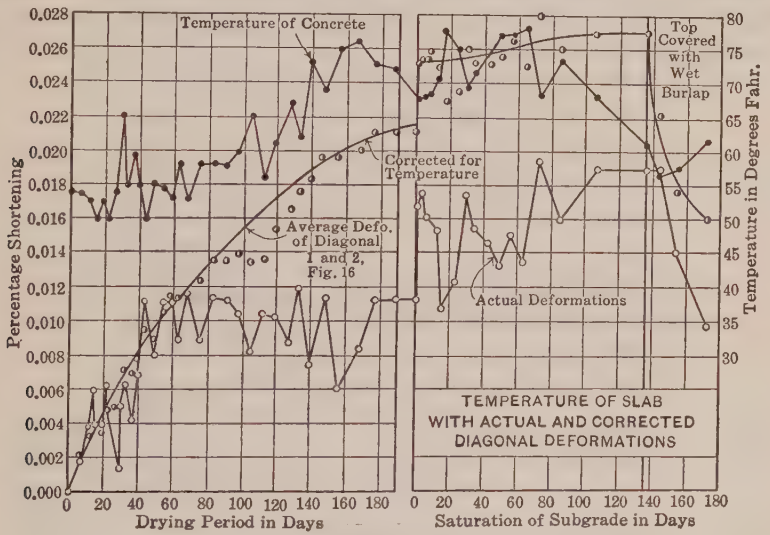


FIG. 17.

Fig. 17 shows the actual and the corrected deformations, together with the changes of temperature of the slab concrete (uniform throughout the depth of the concrete). It was not thought that the vertical deflections caused by the moisture gradient need be changed on account of these uniform changes.

The total diagonal contraction due to the moisture gradient between the condition of saturation (November 22, 1923) and that of completely dried concrete (May 28, 1924) was 0.021 per cent. This value, like all others, has been corrected for uniform temperature changes. Beams of 1 : 1.5 : 3 concrete contracted 0.020% in 160 days. The friction of the sub-grade apparently did not restrain these movements.

The fact that some of the points fall off the curve in Fig. 14 is attributed to a slight effect of temperature in the warping of the slab. In so far as possible, the observations were taken at times when the temperatures were uniform from the top to the bottom of the concrete.

Effect of Daily Variations of Temperature.—The effect of daily variations of temperature in the air above the slab (not the outdoor temperature) is shown in Fig. 18. A temperature rise of 40° Fahr. from 8:00 A. M. to 3:00 P. M.,

warped the corner downward 0.023 in., or 21.0% of the total warping (0.11 in.) at that period. The temperature of the concrete in relation to that of the air was as given in Table 6.

Non-Uniform Saturation.—Following the drying period, water was introduced into the gravel sub-base and maintained level with the lower surface of the slab. Differential expansion from bottom to top was again manifested in an immediate and continued gradual curling up of the edges and corners. A maximum distortion of 0.066 in. for the unbroken edges and 0.202 in. for the corners was attained at the end of 3 months. Typical results of the observations from June 2, 1924, the date of starting the saturation tests, until October, 1924, are shown in Figs. 14, 15, 16, and 17.

Subsequently after resaturating it the surface contracted diagonally 0.007% in 110 days; of this contraction, 46% took place in 2 hours and, approximately, 100% in 2 days.

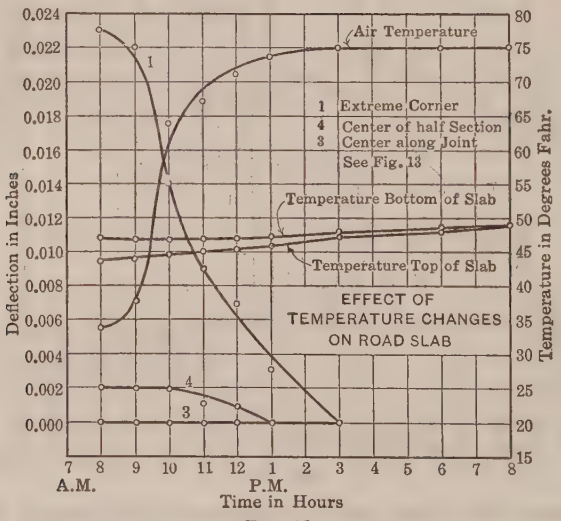


FIG. 18.

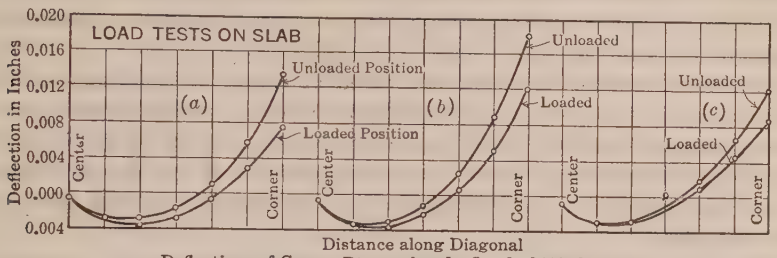
When the slab had reached a static condition with saturated sub-grade (on October 20, 1924), an attempt was made to expand the surface by applying saturated burlap. In 30 days the corner had dropped 0.060 in., to a deflection of 0.140 in.

TABLE 6.—TEMPERATURE VARIATIONS.

Time.	8:00 A.M.	3:00 P.M.	8:00 P.M.
Temperature of air, in degrees Fahrenheit	34	75	75
Temperature of top slab, in degrees Fahrenheit	44.0	47.5	49.0
Temperature of bottom slab, in degrees Fahrenheit	47.0	48	49.0

Load Tests on Slab.—Known loads were applied to one corner of the slab by jacking at three stages of the test: (*a*) On June 14, 1924, during the drying stage; (*b*) on September 23, 1924, during the saturation of the sub-grade; and (*c*) on November 14, 1924, when the top had been saturated 25

days with wet burlap. The profiles of the diagonal on these dates for a load of 3 000 lb., is shown in Fig. 19. The data are as given in Table 7.



Deflections of Corner Diagonal under Load of 3000 lb. at Corner

Figure	Condition	Date	Deformation along Corner Diag.	Stress from $S=\frac{MC}{I}$	Mod. Elas. in Tension
a	During Drying	June 14, '24	0.000037″ per inch	185.0 lb. per sq. in.	5 000 000
b	Bottom Wet	July 23, '24	0.000035″ " "	185.0 " " " "	5 300 000
c	Top Wet	Nov. 14, '24	0.000031″ " "	185.0 " " " "	6 000 000

FIG. 19.

As the warping of the slab increased (see Fig. 19(*b*)), the point of support must have moved in, increasing the deflection. Curiously enough, the deflection of the slab under load when the top was wet was less than in either of the other two cases.

TABLE 7.—DEFLECTION OF SLAB DUE TO CORNER LOAD OF 3 000 LB.*

	a.	*b.*	*c.*
Corner level, in inches :			
Before load	+0.0131	+0.0183	+0.0124
With load	+0.0077	+0.0120	+0.0091
Deflection due to load	0.0054	0.0063	0.0033
Set due to load, after 5 min.	+0.008	+0.009	+0.008
Percentage of elongation under stress (in distance of 48 in. adjacent to the corner) :			
Diagonal	0.0037	0.0035	0.0031
Edges	0.0025	0.0023	0.0017

* Sign + indicates that the elevation is above the original level of the surface of the slab after casting.

Reversed Stresses on Saturated Mortar.—The fatigue limit on 1 : 2 mortar tested dry at an age of 6 to 12 months was approximately 55% of the once applied load, as reported* by Professor R. B. Crepps, of Purdue University. Similar beams, 18 months old, were also tested in the fatigue machine, while saturated, with the following results:

The saturated beams showed 89% of the static strength of the dry beams.

Beam No. 162 failed at 200 reversals under a load equal to 55% of the once applied load on saturated concrete; duration of test, ⅓ hour.

Beam No. 161 failed at 8 400 reversals at 50% load; duration of test, ½ day.

Beam No. 165 failed at 17 200 reversals of 50%; duration of test, 1¼ days.

Beam No. 164 failed at 71 700 reversals of 40%; duration of test, 5 days.

These tests are being continued. It would appear that a little less than 55% of the static strength of dry concrete should be the working strength of the concrete that has been saturated 8 days.

* *Proceedings*, Am. Soc. for Testing Materials, 1923.

Observations on a Concrete Stadium.—Gauge points are set at a number of locations both in the open deck and in the bank revetment of the new Ross-Ade Stadium of Purdue University. The changes of length consequent upon moisture and temperature variations are under observation.

A parapet wall, 6 in. thick, reinforced at center of thickness with 0.3% of reinforcement each way, is integral with a heavy foundation wall. A drop of air temperature of 38° Fahr. appeared as a drop of 28° Fahr. in the wall. The coefficient of thermal contraction was 0.00000725 in. per degree Fahrenheit at the top of the wall and 0.00000305 in. at the junction of the parapet wall with the base.

The movement across a fine crack in the middle of the 42-ft. length of this parapet wall was 0.000471 in. per degree Fahrenheit; and at the bottom across the crack, 0.0000717 in.

Under a drop of air temperature of 38° Fahr., the concrete in the open deck section of the stadium was 37° Fahr.; and the concrete of the revetment, 24° Fahr. The coefficient of thermal expansion of the open deck was 0.00000590 and that of the revetment, 0.00000492.

The effects of a rainfall, followed by a drop in temperature of 10° Fahr., balanced each other and no change of length was recorded in the deck, corresponding to a thermal coefficient of 0.0000050 and a swelling of 0.005 per cent.

The top of the parapet wall between winter and summer expanded with a coefficient of 0.00000595 for 1° Fahr., and from summer to February, 1926, it had contracted with a coefficient of 0.00000432. The difference between these coefficients is no doubt due to the moisture in the concrete during the winter weather. The average of the two coefficients is 0.00000513.

These same coefficients for the reinforced open deck are 0.00000346 and 0.00000236 respectively, or an average of 0.00000291, which no doubt is a measure of the restraint of the columns and surrounding structure.

The coefficients for the deck on the fill, that is, the revetment, are 0.00000547 and 0.00000267, or an average of 0.00000407, again an expression of constraint.

Conclusions and Application

As a result of the studies described in this paper and its appendices the writer has reached the following conclusions:

1.—The strength of concrete, like that of wood, varies with its moisture content. Although the law relating to moisture content and to strength was not determined throughout various ranges of moisture content, it appears that saturated concrete will have 80 to 85% the strength of dry concrete.

2.—The thermal coefficient of expansion of concrete varies with the temperature of the concrete and ranges from 0.0000040 at 65° to 0.0000060 at 140 degrees.

3.—Concrete expands when immersed in water and contracts on drying. The degree of change of length varies with the characteristics of the brand of cement, with the richness of the mix, with the size of specimen, and with the conditions of exposure. Measurements begun after the initial hardening of the cement show that concrete may expand 0.01% due to the absorption of moisture, and may contract 0.06% when exposed to dry air. These

changes continue. To these values should be added the changes in length during the initial setting of the cement.

4.—Concrete road slabs warp upward at the corners and at the edges when the surface becomes dry, and also when the bottom absorbs moisture from the sub-grade. Drying the top surface of the slab under observation deflected the corners upward 0.02 in. When the bottom of the slab was saturated from a water-filled sub-grade the upward corner deflection reached 0.20 in.

5.—This curling upward of the corners of the road slab presents a cantilever beam to the load of a passing truck. The results of the tests indicate the presence of an initial stress in the surface of the road slab arising from the warping.

6.—A combination of shrinkage from drying and from a fall in temperature will produce maximum shrinkage strains. A combination of a drop in temperature and a rainfall may retain the concrete at its previous length. The friction of a road slab on its sub-grade is an element favoring the production of cracks.

7.—To be durable, concrete structures should be designed for the least favorable conditions. Stresses arising from the effect of temperature and moisture as well as those due to dead and live loads should be considered.

8.—Without careful curing concrete masses will undergo excessive surface shrinkage against their rigid cores, thereby favoring surface cracks. Therefore, steel reinforcement should be placed near enough to the surface to withstand these shrinkage strains, and at the same time far enough below the surface to be protected against corrosion.

9.—The foreign experiments cited show the benefit of early and careful curing of concrete in reducing final shrinkage. The greater part of the shrinkage of mortar specimens took place in the first ten hours. These foreign experiments also show the greatly increased shrinkage of mortar mixed with an excess of water.

10.—Since the results of measurements of expansion vary with several factors, such as the size of specimen, the time of initial measurement, and the exposure, the method of such tests should be standardized.

The writer believes that the following are the requirements for concrete structures exposed to the weather:

(*a*) A Portland cement of normal volume constancy and a clean, well graded aggregate.

(*b*) An impervious concrete of proportions not less than 1 : 2 : 4.

(*c*) A time of mixing of not less than 1 min. and preferably 1½ min. for the production of uniform concrete.

(*d*) A concrete mixed with a minimum quantity of water appropriate to the work at hand.

(*e*) Careful slicing of this dry concrete against the face of the forms so that the reinforcing steel may not be exposed nor the surface require patching.

(*f*) Early curing begun just after the initial set and continued for a time depending on local conditions.

(*g*) Design of reinforcing steel and contraction or expansion joints to prevent cracking and the entrance of water.

APPENDIX I

Study of Warped Beams

Description of Tests.—The beam specimens were dried to constant weight; some were then entirely immersed in water and others immersed to only a fraction of their depth. The changes in weight, length, and strength were determined for the different conditions. Five separate series of such experiments were conducted, varying the proportions of the mix, the brand of cement, the extent and nature of drying, and the depth of partial saturation. The results are shown in Table 8.

Results of Tests.—By reference to Table 8, a comparison of beams uniformly saturated shows that the expansion in water varies from 0.014 to 0.021%, with an average of 0.018 per cent. These beams were immersed for 5 days subsequent to the initial drying. The 1 : 2 : 3.5 concrete made with Cement F when air-dried for 2½ months contracted 0.041 per cent.

The 1 : 1.5 : 3 concrete made with Cements A and D mixed, after conditioning by 30 days, curing under wet burlap followed by complete oven-drying, contracted 0.022 per cent.

The reduction in strength from completely dried concrete to uniformly saturated concrete in the five cases ranges from 11.7 to 22.2%, an average of 15.4 per cent.

Certain beams were warped by saturation from the bottom upward to increasing heights (see Table 8), as determined by visual observations of the broken beams and by weighing.

Contractions from 0.0016 to 0.0065% were developed at the dry face, and expansions from 0.015 to 0.032% at the wet face, of the warped beams. When these warped beams were broken with the contracted dry side in tension, an increase from 9.5 to 27.0% in strength over the entirely dry beam was indicated, depending on the amount of initial contraction.

When saturation had proceeded to a greater height in the beams (approximately 2 in. from the top), the area of dry concrete became too small to hold back the expansion of the wet area, and, consequently, the dry face was actually elongated by the force of the expanding face. When the expanded dry face was tested in tension, the beams were only 80.7 to 96.5% as strong as the uniformly dry beams; when turned with the wet side in tension, the percentage was 71.3 to 83.5. The complete data are given in Table 8.

The writer accounts for this increase in strength of warped dry faces when they have been shortened and the decrease in strength when they have been lengthened due to the absorption of moisture on the opposite face by the presence of an initial compression in the former and of an initial tension in the latter case. However, an analysis of moduli of elasticity and the deformations of the top and bottom face of the beams, tested under the one-third point loading, fails to show any substantial difference between the extensibility, at rupture or 50% of rupture, of the beams with the dry face either shortened or lengthened by warping. The neutral axis in all these loaded warped beams

TABLE 8.—MODULUS OF RUPTURE DATA OF WARPED BEAMS.

Number of beams.	Initial contraction, in percentage.	Condition at time of test.	Visual height of water, in inches.	Percentage Deformation at Time of Test.		Face loaded in tension.	Modulus Rupture Cantilever Loading.			Modulus Rupture One-Third Point Loading.			Modulus rupture, percentage of grand average.
				Wet face.	Dry face.		Number of values.	Numerical values.	Percentage of reference values.	Number of values.	Numerical values.	Percentage of reference values.	
SERIES NO. 1. 6 IN. BY 8 IN. BY 4 FT. 0 IN. BEAMS. 1:1.5:3 MIX. CEMENTS A AND D. AGE, 4 MONTHS. OVEN DRY.													
5	0.022	Dry	....	.....	.....	....	20	720	100	....	....	....	100
2	0.022	Saturated	....	+0.020	.....	....	10	616	85	....	....	....	85
5	0.022	Saturated to 2 in. depth	3	+0.019	−0.0016	Dry	19	789	109	....	....	....	109
5	0.022	Saturated to 2 in. depth	3	+0.019	−0.0016	Wet	19	602	83	....	....	....	83
SERIES NO. 2. 7 IN. BY 4 IN. BY 4 FT. 0 IN. BEAMS. 1:2:3.5 MIX. CEMENT F. AGE, 3 MONTHS. AIR DRY.													
3	0.041	Dry	....	.....	.....	....	8	839	100	3	738	100	100
3	0.041	Saturated	....	+0.017	.....	....	8	672	80	2	506	68	77
4	0.041	Saturated to 2⅛ in. depth	3½	+0.024	+0.0035	Dry	12	750	89	4	614	83	87
4	0.041	Saturated to 2⅛ in. depth	3½	+0.024	+0.0035	Wet	12	618	73	4	472	64	71
SERIES NO. 3. 7 IN. BY 4 IN. BY 4 FT. 0 IN. BEAMS. 1:2:3.5 MIX. CEMENT F. AGE, 3 MONTHS. OVEN DRY.													
8	0.049	Dry	....	.....	.....	....	20	780	100	7	652	100	100
7	0.049	Saturated	....	+0.018	.....	....	19	671	85	5	544	83	85
3	0.049	Saturated to 2½ in. depth	4	+0.021	+0.016	Dry	10	637	81	3	511	78	80
3	0.049	Saturated to 2½ in. depth	4	+0.021	+0.016	Wet	10	609	78	3	478	72	76

TABLE 8.—(*Continued.*)

SERIES NO. 4. 7 IN. BY 4 IN. BY 4 FT. 0 IN. BEAMS. 1 : 2 : 3.5 MIX. CEMENT F. AGE, 7 MONTHS. OVEN DRY, 3 MONTHS. AIR DRY, 4 MONTHS.

Number of beams.	Initial contraction, in percentage.	Condition at time of test.	Visual height of water, in inches.	Percentage Deformation at Time of Test.		Face loaded in tension.	Modulus Rupture Cantilever Loading.			Modulus Rupture One-Third Point Loading.			Modulus rupture, percentage of grand average.
				Wet face.	Dry face.		Number of values.	Numerical values.	Percentage of reference values.	Number of values.	Numerical values.	Percentage of reference values.	
1	0.037	Dry	....	.....	.....	....	4	809	100	1	651	100	100
1	0.041	Saturated	....	+0.014	.....	....	3	699	86	1	553	85	86
1	0.043	Saturated to 1½ in. depth	3½	+0.017	−0.004	Dry	3	961	118	1	742	114	117
2	0.040	Saturated to 2¾ in. depth	4	+0.015	+0.0003	Dry	7	774	95	1	646	99	96
1	0.040	Saturated to 4 in. depth	5	+0.018	+0.0017	Dry	3	670	82	1	565	86	83

SERIES NO. 5. 7 IN. BY 4 IN. BY 4 FT. 0 IN. BEAMS. 1 : 2 : 3.5 MIX. CEMENT F. AGE, 7 MONTHS. AIR DRY.

2	0.051	Dry	....	.....	.....	....	6	825	100	2	731	100	100
1	0.047	Saturated	....	+0.021	.....	....	3	733	88	1	633	86	88
1	0.048	Bottom wet	2	+0.020	−0.0065	Dry	3	1066	129	1	884	121	127
2	0.052	Saturated to 1¼ in. depth	2½	+0.027	−0.0048	Dry	6	910	110	2	734	100	107
1	0.054	Saturated to 2½ in. depth	3½	+0.032	+0.0012	Dry	4	714	86	1	651	89	87

is nearer the compression than the tension face (wet or dry) at 50% of rupture and also at rupture. The maximum extensibility in tension was from 0.015 to 0.018 per cent. The simultaneous compression was from 0.012 to 0.016 per cent.

To assume an initial stress would presuppose an elastic concrete holding this stress throughout the five days previous to the test, and an unusual distribution of stresses throughout the cross-section. Inasmuch as these initial stresses must balance, it is difficult to reconcile an initial compression in the top with a compression in the saturated bottom. This matter is left open for discussion.

Deformations Throughout Cross-Section.—In order to interpret the deformations occurring in the bottom and top of the road slab, a study was made of similar deformations observed in the warped beam units. The size of these units, 4 in. by 7 in. by 4 ft. 0 in., was chosen to give a thickness equal to that of the road slab. After initial drying, the beams were placed with the bottom in contact with wet burlap and successive observations of deformation and corresponding deflection were taken. Deformations were recorded at seven different depths between two pairs of sections in each beam. The average results for three beams are shown in Fig. 20. During nine days the lower face expanded 0.0152% and the upper face shortened 0.0024 per cent. The data locate the position of the neutral axis of deformations and show the relative deformations occurring throughout the depth of the beam. The height to which the saturation, visually observed, proceeded at the end of nine days was approximately $2\frac{3}{8}$ in., or one-third the depth of the beams.

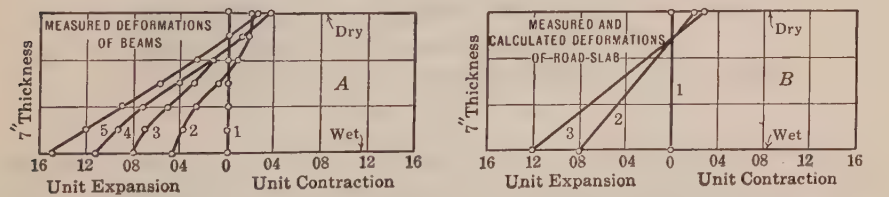


FIG. 20.

If the curve of distortion of a warped beam is assumed to be an arc of a circle, the following formula is applicable to the data:

$$d\,l = \frac{8\,h\,m}{L\,.\,L} \qquad (1)$$

in which,

$d\,l$ = algebraic summation of deformations in top and bottom of the beam;

h = depth of beam;

m = deflection;

L = length over which deflection occurs.

In checking any of the measured deflections against the deformations, or *vice versa,* by substitution in Equation (1), it is found that the calculated values (see Table 9) agree closely with the measured values (Fig. 20).

Application of Formula to Data of 'Road' Slab.—Table 9 shows the values of the deformations computed by formula as occurring in the bottom of the road slab and resulting from flooding the sub-grade, thus saturating the bottom concrete. Here the expansions of the under side of the slab are calculated by substitution of the known measurement of the deflection and deformation of the upper surface in Equation (1). Each step in the calculations is shown.

TABLE 9.

ILLUSTRATION A.

Curve.	Condition.	Measured deflection over 40-in. span.
1	Uniformly dry, unit contraction 0.0004	0.0000
2	Bottom wet, 24 hours	0.0020
3	" " 48 "	0.0030
4	" " 4 days	0.0041
5	" " 9 "	0.0048

ILLUSTRATION B.

Curve.	Condition.	Measured deflection over 40-in. span.
1	Uniformly dry, unit contraction 0.0002	0.0000
2	Bottom wet, 11 days (interior section)	0.0028
3	" " " " (exterior ")	0.0042

Substituting in Equation (1), $d\,l = \begin{cases} 0.00010 \text{ at Point 2} \\ 0.00015 \text{ at Point 3} \end{cases}$

Measured unit contraction, top $= \begin{cases} -0.000019 \text{ at Point 2} \\ -0.000028 \text{ at Point 3} \end{cases}$

Expansion bottom = algebraic sum, $d\,l$, top + bottom $= \begin{cases} +0.00008 \text{ at Point 2} \\ +0.00012 \text{ at Point 3} \end{cases}$

Fig. 20 indicates clearly the swelling of the bottom and the shortening of the top of the slab. A neutral axis of deformations separates the two, similar to that in the subsidiary beams. The greater swelling of the concrete at the unrestrained edge of the slab and the less swelling at the interior restrained portions are shown.

The translation of the measured deformations into stresses, and the variation of such stresses throughout the cross-section, is a field of discussion of considerable interest. The corners of the slab are held up against the dead weight of the overhanging cantilever, which has been raised by the swelling of the bottom concrete. What is the state of stress in the top, which has been shortened?

APPENDIX II

REVIEW OF LITERATURE ON EXPANSION AND CONTRACTION OF CEMENTS, MORTARS, AND CONCRETE*

I.—Volume Changes

The earlier measurements of volume changes in concrete cover a wide range as to size and age of specimen, quality of cement, kind of aggregate, richness of mix, duration of exposure, time of initial measurement, etc., and, consequently, a wide range of values. More recently the importance of volume changes occurring during the first ten hours have begun to be appreciated, the underlying influences have begun to be evaluated, the nature of the action understood, and, therefore, a possibility of remedy suggested. The necessity of protecting concrete from volume changes as soon as possible after pouring is evident.

Volume changes are due to: Shrinkage and expansion; temperature changes; and plastic flow under load.

The factors influencing the shrinkage and expansion, in part at least, are:

(1) Composition of cement.
(2) Aggregate (fineness and quality).
(3) Wetness of mix and proportioning.
(4) Size of specimen, and forms (whether absorptive or not).
(5) Exposure (humidity, temperature).
(6) Age of specimen and duration of test.
(7) The time of starting measurement (before or after initial or final set).

(*1*) *Quality of Cement.*—Various brands differ markedly in their volume constancy, depending on chemical composition and fineness.

Tests by the U. S. Bureau of Standards† were made on neat cement and on 1 : 3 mortar cured in water. At thirteen weeks the expansion was: Neat cement, 0.066% (0.057 to 0.120); 1 : 3 mortar, 0.024% (0.005 to 0.012). Using the same cements re-ground, the expansion increased to 0.085% and 0.13, respectively.

Mr. John Grant‡ showed that expansion was increased when cements contained gypsum.

Recent tests at Purdue University show the changes in six brands of cement and in concrete made from certain of the brands. (See Table 4 and Figs. 3 and 4.)

D. A. Abrams, M. Am. Soc. C. E., in 1919,§ in connection with an investigation of the effect of fineness of cement, found no appreciable differences in

* No attempt has been made to discuss the early experiments, such as the thorough investigation of Professor White of the University of Michigan.

† *Proceedings*, Am. Soc. for Testing Materials, Vol. 15, 1915, p. 141.

‡ *Minutes of Proceedings*, Inst. C. E., Vol. LXII (1879–80), Pt. IV., p. 108.

§ *Bulletin No. 4*, Structural Materials Laboratory, Lewis Inst., Chicago, Ill.

volume change due to fineness, but he states that the measurements were not precise.

A paper in *Le Ciment* (1923) quotes expansion tests with Le Chatelier apparatus as shown in Table 10.

TABLE 10.

	Residue.		Expansion: Le Chatelier apparatus.
	Sieve 900.	Sieve 4 900.	
Cement A.	3%	28%	6 mm.
	2%	20%	3 "
	1%	16%	1 "
Cement B.	2 %	25%	2 mm.
	1.5%	20%	2 "
	0.5%	14%	½ "

The author remarks that this expansion may be reduced by finer grinding, providing the cement is well burned. Methods such as airing of cement will also lessen expansion. Disaggregation of concrete is attributed to the influence of gypsum, which constituent, expressed as SO_3, varies from 1.75 to 3.00 per cent. Some countries allow only one-half as much SO_3 when the cement is to be used in sea water. Experiments in Germany on concrete exposed to sea water for 10 years are quoted to show that 2.5% of SO_3 is not detrimental either in fresh or salt water. The author believes that the Le Chatelier apparatus for which 6 mm. is specified will detect expansive cements, but will reject cements that are worthy; he recommends a specification of 10 mm. (See, also, tests by Otto Graf, discussed subsequently.)

Dr. H. Nitzsche* measured the volume change of several cements with the Bauschinger apparatus. Mortars approximately 1 : 4 and of consistencies appropriate in reinforced concrete were used. The Bauschinger prisms were conditioned as follows: 2 days in moulds, 7 days in water, 10 days in room temperature, and then dried to constant weight at 50° cent. The length of the "wassergesättigt" prisms in percentage of the dry mortars was as shown in Table 11 when the cements are listed in order of increasing fineness.

TABLE 11.

Cement.	Percentage, volume change, in water.
Finest slag cement	0.0275
" " "	0.0204
Finest Portland cement	0.0347
Coarsest slag cement	0.0224
Coarsest Portland cement	0.0255
Iron Portland cement	0.0204

Apparatus for Measuring Volume Change.—The British Standard Specifications for cement recommends the Le Chatelier method. The paste is 1.18 in.

* *Zement*, December, 1921.

in diameter and 1.18 in. high, moulded inside a split metal tube. Arms attached to each side of the split multiply the movement of the tube due to volume changes of the cement paste.

The paste is submerged in water at 58 to 64° Fahr. for 24 hours, and the position of the arms indicated. The test piece is again submerged, the water brought to a boiling temperature in 25 to 30 min., and kept boiling for 6 hours. The mould is then removed from the water and allowed to cool. The indicator arms are then read, the difference between the two measurements representing the expansion.

The Bauschinger apparatus is an arrangement for measuring the change of length of a short bar, 100 mm. in length and 5 sq. cm. in cross-section.

M. Deval* has discussed the method of measuring variations in the volume of hydraulic cements. He refers to the tests of Durand-Klaye, Debray, Bauschinger, Kleve, and Le Chatelier, and concludes that the Le Chatelier method gives the most concordant results under which cement of good quality will not show more than 0.2% of swelling. He remarks that cements differ greatly in expansion and in rapidity of hydration.

(*2*) *Aggregate.*—Mr. Cloyd Chapman† reported the contraction of 1 : 2 mortar specimens at 16 weeks to be 0.197% using screenings as aggregate and 0.083 with sand. Concrete (1 : 2 : 4 mix) with screenings contracted 0.062%, and with sand 0.042 per cent. The specimens were 2 by 2 by 24 in. in size; the measurements began 24 hours after moulding.

No doubt the aggregate has an important influence. In many cases the cause of disintegration of exposed structures is to be charged to defective aggregates, especially those containing a large proportion of fine limestone dust or clayey material. Porous concrete with changing moisture content must be subject to strains due to alternate expansion and contraction.

(*3*) *Proportioning.*—Leanness of mix diminishes the volume change. A number of investigations evidence this. Thus, a comparison of neat cement with 1 : 3 mortar gives the results shown (percentage) in Table 12. Evidently, as the mix becomes leaner the volume changes diminish.

Pearson's tests show the relative shrinkage of various mortars, the initial measurements being made while the specimens were plastic, as follows:

Mix.		Percentage of shrinkage.
Neat		0.217
1 : 1		0.123
1 : 2		0.110
1 : 3		0.109
1 : 4		0.094
1 : 5		0.086
1 : 6		0.084
Concrete	1 : 4	0.067
	1 : 6	0.050

* *Bulletin,* Société d'Encouragement pour l'Industrie Nationale, January 31, 1900, pp. 31–37.

† *Proceedings,* Am. Soc. for Testing Materials, 1924.

TABLE 12.—LEANNESS OF MIX AND VOLUME CHANGE IN PERCENTAGE.

Author.	Neat.		Mortar 1:3.		Concrete.		Duration.	Size of specimen.	Age at initial measurement.	Reference.
	Exp.	Cont.	Exp.	Cont.	Exp.	Cont.				
Gary	0.026	0.129	0.018	0.053	....	....	28 D	.....	.....	Mitt. a.d. kgl. tech. Versuchsanstalt zu Berlin, 1899 Erganzungsheft.
Bauschinger	0.046	0.214	0.009	0.132	....	....	16 W	4.7" cubes	.....	J. Mitt. tech. Lab. d tech. Hochschule in Munchen, Heft 8, 1879.
Schuman	0.082	....	0.021	.....	....	....	4 W	0.775 □"×3.9"	.....	Tonind. Zeit. 13-435, 1899.
Tornei	0.025	0.201	0.027	0.094	....	....	12 W	4.7" cubes	.....	Quoted by Gary A. S. C. E. 3 016, 1893.
Considere	0.079	0.132	0.028	0.050	....	....	9 W	.....	.....	Comptes Rendus 129-467, 1899.
Pearson	....	0.217	.....	0.109	....	0.050	9 W	1" × 4" × 24"	At time of moulding	A. C. I. Vol. 17, 1921.
Goldbeck	....	0.155	.....	0.078	....	0.054	6 M	8" □ × 5'0"	24 hours	Bulletin 532 U. S. Dept. Agric.
Chapman	....	....	.....	0.197	....	0.042	16 W	2" × 2" × 24"	" "	Proc. A. S. T. M. 1924.
Scofield	....	....	0.019	0.055	0.017	0.047	7 W	4" × 4" × 24"	" "	A. C. I. Vol. 13, 1917.
Matusomoto	....	....	0.0101	0.0503	0.005	0.036	60 D	2" × 3" × 24"	" "	Bulletin 126 Univ. of Illinois, 1921.
McMillan	....	....	.....	.....	....	0.084	13 M	4" × 5" × 4'0"	" "	Eng. Record Apr. 17, 1915.
Swain	....	....	.....	.....	....	....	12 W	5" cubes	" "	A.S.C.E. July 1887.
White	....	....	.....	.....	....	....	6 M	1" × 1" × 4"	" "	Proc. A. S. T. M. 1911.

In the tests of Goldbeck on 8 in. by 8 in. by 5-ft. beams* a wet or dry mix of 1 : 2 : 4 or 1 : 3 : 6 concrete varied but little in contraction when exposed to air for 1 year, the extreme ranges in percentage as scaled from a diagram being as shown in Table 13.

TABLE 13.

	30 days.	60 days.	1 year.
1 : 3 : 6 mixed very wet (10 to 12% water)	0.023	0.033	0.053
1 : 3 : 6 mixed very dry (8.5% water)	0.034	0.045	0.055
1 : 2 : 4 mixed very wet	0.023	0.034	0.058
1 : 2 : 4 mixed very dry	0.035	0.040	0.055

Tests at Purdue University on gravel concrete when measurements began 2 days after moulding showed for 1 : 2 : 3.5 and 1 : 1.5 : 3 concrete an expansion in water during 40 days of 0.005 per cent. During 150 days subsequent drying in air the contraction was 0.035 and 0.015%, respectively, the brands of cement varying in the two concretes.

Dr. Mörsch has recorded† tests of the influence of richness of mix and also of different varieties of sand and the volume changes due to swelling in water and shrinking in air. For one year the swelling in water continually increases

* *Bulletin No. 532*, U. S. Dept. of Agriculture, 1917.

† "Der Eisenbetonbau," Fifth Edition, Vol. I, pp. 32–33.

for neat cement, and for 1 : 3 and 1 : 5. mortars; the shrinkage in air reaches a maximum at 90 days. The relative values at the end of a year for swelling in water are:

Neat cement	180
1 : 5 mortar	45

For shrinking in air the values are:

Neat cement	150
1 : 5 mortar	52

The volume changes of mortar made with several sands seem to differ.

(4) *Size of Specimen and Forms.*—Evidently the size of the specimen, which affects the rapidity of loss and gain of moisture and the unequal distribution of moisture, will have a marked influence on shrinkage and expansion. Consequently, the data from many investigations are not comparable. Flat specimens will warp; there will be different frictional resistances due to the forms or to the bending on the supports. Moreover, the size of the specimen will control the early formation of surface cracks, which must affect the integral movements. Pearson has shown that absorptive forms will greatly reduce the shrinkage of mortar specimens. Graf has shown that shrinkage is lessened during early stages by forms with cast-iron ends. For these reasons the specimen itself and the technique of testing should be standardized for these tests which now seem to be of increasing importance. The specimens used range in nominal size from 5-in. cubes to prisms, 2 by 2 by 4 in., 2 by 2 by 24 in., 1 by 4 by 24 in., 1.5 by 1 by 4 in., 4 by 4 by 24 in., 8 by 4 by 2 in., 8 by 8 by 40 in., and 8 by 8 in. by 9 ft.

Mörsch* shows the effect on tensile strength of varying the size of the test piece from 5 sq. cm. to 400 sq. cm. (Table 14).

TABLE 14.

Cross-section, in square centimeters.	Tensile Strength, Neat.		Mortar.
	28 days moist.	7 days moist; 21 days dry.	
5	33.4	44.8	1:1.34
100	21.0	15.9	0.76
400	17.6	12.0	0.68

The effect of desiccation of briquettes following 1 day in air and 21 days in water is shown by a rapid decrease in strength and a gradual increase in strength following a longer period of desiccation, until at 12 days the strength of the briquettes exceeded the strength at the time they were taken from the water.

One aspect of surface drying and the production of fine surface checks is the consequent loss of strength in cement briquettes which have been allowed

* "Der Eisenbetonbau," 1920, Vol. I, p. 121.

to dry subsequent to immersion and previous to testing. In 1904, Van Ornum* determined the effect on tensile strength of taking briquettes from the storage tank for various periods before testing. The strength of the neat Portland cement decreased as much as 50% when exposed to air from 1 day to 3 days.

M. Anstett,† Chief of Laboratories of the City of Paris, France, made similar tests in 1922, and reported the decrease in strength of neat briquettes tested at 7 days, 37 days, and 90 days, after having been removed from the water 7, 20, and 26 hours, respectively. If the briquettes are allowed to dry for 7 hours before testing a loss of strength of 10% may be expected. This becomes an important matter in standardizing cement testing. In the case of 1 : 3 mortar briquettes, the loss is less marked, being approximately 5 per cent. The loss increases in proportion to the duration of the drying period.

No doubt the effect of drying will depend on the cement used, and the method of forming the briquette, on the rate of drying.

In a book by Foker and Barnes, on "Reinforced Concrete Design", published in 1912, they quote‡ tests on 1 : 3 mortar specimens that were cured in air 1 day and in water for 27 days, and afterward exposed to dry air for various periods up to 12 days. After a period of drying for 2 days the strength diminished 50%, following which it increased to approximately its value at the end of the water-curing period.

The surface shrinkage no doubt will affect the point in the extensibility of the tension side of beams at which hair cracks are observed. Beams carefully cured or saturated will probably show greater extensibility. A close analogy exists between concrete and wood that is case-hardened by defective kiln practice.

In the absence of reinforcement near the surface, it would appear that surface shrinkage might increase the number of hair cracks.

(*5*) *Exposure.*—Many early tests disclosed swelling in water and shrinkage in air. The wide ranges of values recorded were mystifying as the underlying controlling factors were not known. Exposures include immersion in water (cold or hot), a covering of wet burlap or damp sand, dry or moist air, all with various temperatures. The effects of humidity and temperature, while minor as compared to other factors, are not negligible.

The results of various measurements, therefore, are indicative of ranges rather than of definite application in specified cases.

Professor H. H. Scofield§ measured the change of length of mortars and concretes with and without an admixture of hydrated lime. The specimens were 4 by 4 by 24 in. He reports that the degree of volume change depends on the conjunction or opposition of temperature and moisture influences. A change of temperature in 1 : 5 mortar with fine sand screened out, age 8 weeks, from 33.8° to 180° Fahr., increased the length + 0.075%, or 0.00000535 per degree. In a change from 180° to 18° Fahr., the shrinkage was — 0.0664 per cent. From 22° in air to 107° Fahr. in water, the change was + 0.0377%; and from 107° Fahr. saturated to 18° Fahr. outside air, — 0.0467 per cent.

* *Engineering News*, Vol. 51, p. 24.

† *Revue de Matériels de Construction et Travaux Publiques*, 1922, Vol. 148, p. 1.

‡ *Mitteilungen über Forsch.* und *deut. Ing.*, Heft 72–74.

§ *Proceedings*, Am. Concrete Inst., Vol. 13, 1917.

The changes of length due to absorption of water were found to be as follows: From 71.5° Fahr. in air, to 71.5° Fahr. in water (24 hours) + 0.0192%; from 71.5° Fahr. in air to 100° Fahr. in water (72 hours), + 0.040%; from 100° Fahr. saturated to 10.4° Fahr. in air, — 0.0553 per cent.

For 1 : 5 gravel concrete giving a tensile strength of 2 314 lb. at the age of 7 weeks, these values become + 0.017%; + 0.036%; and — 0.0472 per cent.

In these tests a comparison is made between the shrinkage of wet and dry specimens under a sudden drop in temperature. The wet specimens shrank — 0.0553%, and the dry ones, — 0.0664 per cent. However, the sand used was different in the two cases.

Scofield's tests also show that admixtures of hydrated lime increase the volume and strength.

H. C. Berry,* M. Am. Soc. C. E., found that 1 : 2 : 4 concrete shrunk in air at 150 days, 0.023% and expanded in water, 0.020 per cent. This is less difference between shrinkage and expansion than in other experiments.

Mörsch† averages the results of tests on mortars, 1 cement and 3 sand, immersed in water for 7, 28,.90, and 196 days, and then allowed to dry in the air for periods ranging up to 4 years.‡ A set of test pieces was allowed to dry without immersion. His diagram is reproduced in English units in Fig. 21.

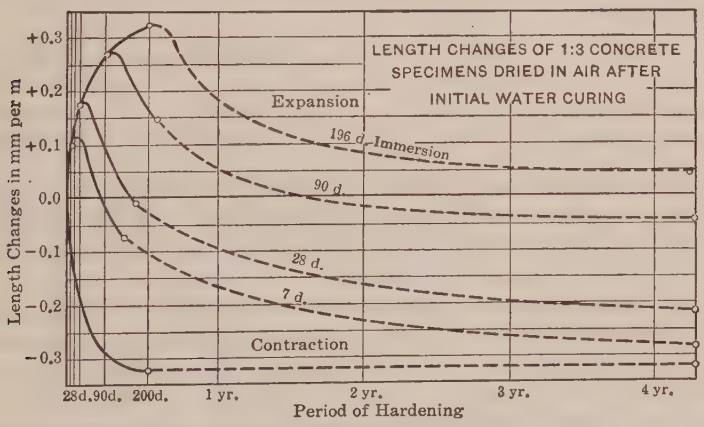


FIG. 21.

Professor F. R. McMillan,§ reviews tests of shrinkage specimens, 4 by 5 by 24 in., cured 2 weeks under wet burlap and afterward exposed to the ordinary air of the laboratory. The shrinkage was from — 0.055 to — 0.065 per cent. Initial readings were taken when the beams were 1 day or 2 days old. The shrinkage amounts to — 0.05% within the first 6 days. The effects of varying the proportions, quantity of water, or density were small. He states that the minimum shrinkage to be expected in concrete that has been allowed to dry out thoroughly is — 0.05 per cent. Values from 20 to 50% greater will usually result; the difference in early curing conditions has little effect on the final shrinkage.

* *Proceedings*, Am. Soc. for Testing Materials, Vol. 11, 1911, p. 563.

† "Der Eisenbetonbau", Fifth Edition, Vol. I, Pt. I, p. 136.

‡ *Mitteilungen* des Materialienprüfungsamts, Lichterfelde, 1916.

§ *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1378.

De Breuville* subjected reinforced plates to water pressure, off and on, to obtain the effect of moving water on the bond between mortar and steel. He concludes that moving water weakens the bond.

G. E. P. Smith, M. Am. Soc. C. E., explains† longitudinal cracks along the top of 20-in. concrete pipe at Tucson, Ariz., by the action of water swelling the bottom of the pipe. Experiments on dry pipe, partly submerged, appear to confirm this explanation. Tests of pipe, 28 days old, showed a strength of 2 624 lb. per lin. ft. for dry pipe and 1 790 lb. per lin. ft. for pipe that had been immersed for 5 hours.

Lagaard‡ tested 1:2:4 limestone concrete in compression. The specimens were cured for 7 days under wet sacks and approximately 450 days in air. One set showed a value of E of 3 700 000 and a compressive strength of 3 630. The other set subsequently immersed in water for 21 days showed a value of E of 3 150 000 and a compressive strength of 2 340, all in pounds per square inch.

Tests§ by G. M. Williams, Assoc. M. Am. Soc. C. E., showed no difference in the value of E, either for immersed or dry concrete, or for a low or high degree of consistency. In the discussion of this paper, Professor Walker, of the Lewis Institute, takes issue on this conclusion. His values, in pounds per square inch, are:

	Wet concrete. One year in wet sand.	Dry concrete. One year in wet sand. One month in air.
Compressive strength	4 950	5 920
Initial value of E	7 450 000	9 300 000

(*6*) *Age of Specimen and Duration of Test.*—Expansion and contraction continue for extended periods. The variation in duration of the tests accounts for differences in reported results. Rich specimens that have been saturated for extended periods become impervious.

(*7*) *The Time of Starting Measurements.*—In general, the initial observations of volume changes in the investigations reviewed, were taken subsequent to the setting periods. Pearson's investigations on plastic mortar show the marked volume changes that occur during the setting period. It is obvious from this standpoint, therefore, that the data of the many investigations are not to be compared.

In 1921,‖ Pearson investigated the development of change of length of mortars as time elapsed, especially during the first few hours. Up to 6 hours, while mortars were plastic, the shrinkage was moderate, but at the time of initial set, the volume changes were pronounced. They moderated after 10 hours. At the time of final set the material quiets down, and various cements then show about the same rate of shrinkage. When specimens were placed on non-absorptive bases the shrinkage at 64 days was: Neat cement, 0.21%; 1 : 3 mortar, 0.19%; 1 : 6 mortar, 0.084 per cent.

* *Annales des Ponts et Chaussées*, Sem. III, 1902.
† *Engineering News-Record*, 1919, v. 831.
‡ *Loc. cit.*, November 14, 1918.
§ *Proceedings*, Am. Soc. for Testing Materials, 1920, p. 8233.
‖ *Proceedings*, Am. Concrete Inst., 1921, p. 135.

For the first two or three hours while mortars are still plastic, either moderate expansion or contraction may occur due to manipulation, forms, and the quality of the cement. At the time of initial set there is a tendency toward pronounced contraction which moderates at 8 to 10 hours when final set is approaching. From 12 to 24 hours is a period of comparative quiet, with slight shrinkage. There is then a long continued shrinkage which persists for many months; at one year this may range from 0.15 to 0.34 per cent. The volume change, therefore, is in two phases as follows: (1) Plastic, 12 hours to, say, 24 hours; (2) rigid after 24 hours, changes here are independent of those in Phase (1).

Dr. Otto Graf,* of the Testing Laboratory of the Technical High School of Stuttgart, has made valuable investigations of the volume changes of mortars. The 1921 paper reports the progressive shrinkage of mortars from 1 hour to 28 days, and the effect of excess water. Four brands of Portland cement were used. The mortar was 1 : 1 by weight, and the water varied from 10% (firm consistency), 15% (soft consistency), and 19% (plastic consistency). The test piece was a disk, 15 mm. thick and 150 mm. in diameter, placed on oiled paper and moved under a microscope by a micrometer screw. His results are reproduced in Figs. 22 and 23. The importance of dry concrete and early curing to reduce shrinkage is evident. Damage due to shrinkage originates during the first ten hours.

Fig. 23 substantiates Abrams' results, showing the effect of water on strength, and emphasizes the value of dry concrete.

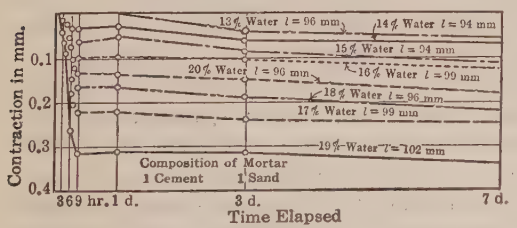


FIG. 22.—SHRINKAGE OF 1:1 MORTAR.

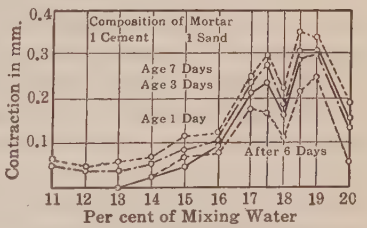


FIG. 23.—EFFECT OF WATER ON SHRINKAGE.

Graf also reports the marked effect of the cement content on the volume change. The shrinkage in a length of 100 mm. was as shown in Table 15.

TABLE 15.

Time.	9 hours.	7 days.	1 day to 28 days.
Neat cement	0.42 mm.	0.52 mm.	0.13 mm.
1 : 1 mortar	0.32 "	0.38	0.08 "
1 : 4 mortar	0.05 "	0.13	0.07 "

The four brands of cement used varied greatly as to the volume changes during the first 9 hours.

* *Beton und Eisen*, 1921–22.

In tests of prisms, 80 mm. square and 400 mm. long, made to detect the differential movements between the concrete and the embedded reinforcing bars, it was found that within the first day while considerable changes in volume took place in the mortar, no changes of length were recorded in the reinforcement. This condition lasted for three days.

In the 1921 tests of Graf, wax and oil were used to prevent the mortar from adhering to the sides of the moulds. In 1922, moulds of cast iron were used to study the effect of adherence of the concrete to these surfaces. Reinforced specimens were also studied. The concrete was 1 : 1.2 : 1.8, using 11.8% water.

It was shown that the contact with the ends of the cast-iron moulds lessened the natural shrinkage of the concrete. Reinforcement considerably reduced the shrinkage of concrete during the first day after moulding; plain concrete specimens show a reduction, in a length of 333 mm., of 0.004 cm. and reinforced concrete, of 0.0008 cm., after 9 hours. After 14 days, these values become 0.0024 and 0.0004 cm., respectively.

Volume Changes Due to Temperature.—The generally accepted value of the coefficient of expansion for concrete due to temperature is 0.0000065 for 1° Fahr. It is assumed to be constant throughout the ordinary ranges of temperature.

Professor Mills, of Cornell University, reports* a series of tests for the coefficient of temperature expansion using test pieces 8 by 4 by 2 in. The coefficient ranges from 0.00000783 for neat cement to 0.00000577 for 1 : 5 mortars; for concrete, 0.00000677 for 1 : 1.5 : 3 mix to 0.00000537 for 1 : 3 : 6 mix. In age the specimens ranged from 1 month to 8 months, and in temperature from 70° to 212° Fahr.

In 1901, at Purdue University,† W. D. Pence, M. Am. Soc. C. E., measured the expansion of 1 : 2 : 4 concrete specimen, 4 in. in diameter and 3 ft. long, placed inside a steel cylinder into which steam was introduced. The temperature range was 65° Fahr. Both stone and gravel aggregates were represented. The coefficient of expansion for 1° Fahr. ranged from 0.0000052 to 0.0000057 in stone concrete; a stone bar showed 0.0000056; gravel concrete averaged 0.0000056. These tests well represent average values. However, it was assumed that the coefficient is equal throughout the range of temperature, and no doubt there was also expansion due to the steam, if it was moist.

Fig. 6 shows the results of tests at Purdue University. The observations were conducted on completely dried specimens only, so that there is no error due to the change in length with fluctuations in the moisture content of the concrete.

Plastic Flow.—The yielding of concrete under continued loads was noticed at an early date, especially in laboratory tests to determine the modulus of elasticity. This plastic flow became of practical importance in specifying and measuring deflections of loaded floor panels. Later, it became apparent that it produced a readjustment of stresses between steel and concrete which was important from the standpoint of the design of floors and columns.

* "Concrete Engineer's Handbook", Hool and Johnson.

† *Journal*, Western Soc. of Engrs., 1901, Vol. 6, p. 547.

Several investigations have been made to determine the plastic flow or the time effect, a bibliography of which follows:

Hatt, W. K., *Proceedings* Am. Soc. for Testing Materials, 1907, p. 421.
Studies in Engineering No. 3, Univ. of Minnesota, 1915.
Janni, A., *Transactions* Am. Soc. C. E., Vol. LXXX (1916).
Fuller, A. H., and C. C. Moore, *Proceedings* Am. Concrete Inst., 1916.
Smith, E. B., *Proceedings* Am. Concrete Inst., 1916.
Goldbeck, A. T., and E. B. Smith, *Proceedings* Am. Concrete Inst., 1916.
McMillan, F. R., *Proceedings* Am. Concrete Inst., Vol. XVII, 1917.
Smith, E. B., *Proceedings* Am. Concrete Inst., 1917.
Hollister, S. C., *Proceedings* Am. Concrete Inst., 1919, p. 137.
Williams, G. M., *Proceedings* Am. Soc. for Testing Materials, 1920.
McMillan, F. R., *Proceedings* Am. Concrete Inst., 1921.

DISCUSSION

A. T. GOLDBECK,* ASSOC. M. AM. SOC. C. E. (by letter).—Those having to do with design are likely to consider the safety of concrete structure from the standpoint of its ability to carry loads. The life of many structures has been limited by factors other than ability to carry the loads for which they were designed. It is recognized that concrete highways are subjected to stresses of considerable intensity, which are produced by agencies other than the wheel loads of traffic, and a thoroughly rational procedure in the design of concrete pavements must give due weight to all the high stress-producing possibilities. It is known, of course, that temperature change brings about high stress in concrete pavements, and it is also quite apparent that moisture is another very active stress-producing agent.

The present investigation was made to obtain more exact information on the influence of moisture on concrete highway design. The author is to be commended on the careful and painstaking manner in which the present series of tests were conducted. The careful design of apparatus and attention to necessary detail in the conduct of the work are reflected in the character of the results obtained. The author has corroborated the general results obtained by other investigators in this field, namely, that Portland cement mortar or concrete shrinks on drying and expands when kept moist. It is significant, however, that the percentage of contraction on drying differs greatly in different brands of cement. Obviously, this must be an important element in the relative number of shrinkage cracks in concrete roads after the initial period of curing. It is desirable, of course, that the cement should shrink as little as possible. The variable coefficient of expansion with temperature shown by these tests is a fact that has not generally been recognized, for it is quite the custom to use a constant value for the coefficient of expansion irrespective of the temperature. Apparently, the value for the coefficient of expansion at ordinary air temperatures is about 0.00000425, which is somewhat lower than the value ordinarily accepted.

The values obtained by Professor Hatt might be well worth while corroborating through another series of investigations. Values obtained for a change in length of 1 : 2 : 3.5 concrete in general check with those obtained by the writer.† The decrease of contraction depends apparently not only on the character of the cement, but it has been the writer's experience that much also depends on the dryness of the atmosphere, for it has seemed that even a short period of high humidity could be detected through the slight expansion of a very dry specimen.

It is to be noted that the percentage of expansion of concrete kept moist is considerably lower than the percentage of contraction of concrete permitted to dry out. This conclusion likewise was borne out in the writer's experiments along this line. The effect of a period of wet weather on specimens exposed to the atmosphere is likewise clearly demonstrated. It is to be noted, however,

* Director, Bureau of Eng., National Crushed Stone Assoc., Washington, D. C.

† *Bulletin No. 532*, U. S. Dept. of Agriculture.

that the rate of change of length of the specimen is quite slow, not only in expansion, but also in contraction. It has often been noticed that the greatest percentage of "blow-ups" in concrete pavements occurs in the spring and that as the summer progresses the pavement settles back into position. This phenomenon is readily explained by the tests just reported by Professor Hatt, for in the spring the sub-grade is in its wettest condition. The concrete, therefore, is expanded not only by moisture, but by the high temperatures during that period. Gradually, the moisture evaporates and during the summer the concrete shrinks somewhat and is subjected then only to the expanding action of high temperature.

The lesson to be learned from Fig. 10 evidently is that engineers need not be afraid of the structural strength of concrete as long as it is in a frozen condition, but that its weakest condition is immediately after thawing or when it is saturated. The measurements on the movement of the corner of the road slab, depending on the extent of saturation from bottom to top, are what might be expected, and the deformations calculated from these deflections agree strikingly well with the measured deformations.

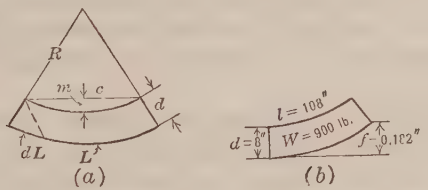


FIG. 24.

Perhaps it is not generally realized that under unfavorable conditions a rather high stress might be created transversely in a concrete slab due to a condition of high moisture at the bottom of the slab as compared with a dry condition at the top. For an 18-ft. slab, the following calculation shows that theoretically a stress produced by deflecting the end of an 18 ft. by 1 ft. by 8-in. concrete beam to its original shape after it had been warped because of difference of moisture content between top and bottom, may approximate the modulus of rupture of concrete.

From Fig. 24 (*a*):

$$R = \frac{c^2}{8\,m}; \quad R = \frac{d \times L}{d\,L}; \quad c = L; \quad d\,L = c'\,L$$

c_1 = unit change in length between top and bottom fibers = 0.00025.

By substitution:

$$m = \frac{c' \times L^2}{8d} = \frac{0.00025 \times 216 \times 216}{8 \times 8} = 0.182 \text{ in.}$$

Assuming the beam to be supported at the center, each half becomes a cantilever of length = 108 in. (Fig. 24 (*b*)).

The deflection due to weight of beam is:

$$f = \frac{W\,l^3}{8\,E\,I} = \frac{900 \times 108 \times 108 \times 108 \times 12}{8 \times 3\,000\,000 \times 8 \times 8 \times 8 \times 12} = 0.094 \text{ in.}$$

The stress produced by weight of beam is:

$$\frac{W\,l}{2} = \frac{S\,I}{C}; \quad S = \frac{W\,l\,c}{2\,I}; \quad S = 380 \text{ lb. per sq. in.}$$

The remaining deflection of 0.088 in. will be accompanied by a stress of 271 lb. per sq. in., and was found from the formula:

$$S = \frac{3\,f\,E\,d}{2\,l^2}$$

The total stress will be:

$$380 + 271 = 0.651 \text{ lb. per sq. in.}$$

This calculation assumes that the slab is bent back into a plane by passing vehicles after it has been warped by the unequal distribution of moisture from the bottom to the top of the slab. This condition is, of course, an extreme one and is generally not obtained. Nevertheless, the possibility of very high stress is quite apparent. It should be borne in mind that, in general, the extremes of moisture which can be obtained under laboratory conditions are not likely to occur in the field, for usually the evaporation and saturation are not carried to completion in the field, due to the intervening influence of periods of dry and hot weather and to the influence of capillary moisture from the sub-grade.

The study of the effect of moisture on concrete would not be complete without further study of the effect of repeated loading on wet concrete as well as on dry concrete. Professor Hatt and others have shown that repeated loads have telling effect when they produce a stress much in excess of the modulus of rupture of the concrete, the concrete being dry. It would be interesting to learn whether the effect of repeated loads might not be more severe on concrete in a saturated condition.

G. E. P. Smith,* M. Am. Soc. C. E. (by letter).—It has been realized for a long time that climatic effects on concrete are of great importance, and that quantitative data adequate for designing and for determining practice in light construction exposed to the weather, such as sidewalks, are lacking. This paper is, therefore, exceedingly welcome at the present time. Much still remains to be done, and it is to be hoped that in laboratories properly equipped for this work, investigations will be continued until the causes of change of volume and the factors which govern variation in the effects of temperature and moisture are well understood. In Southern Arizona and regions of similar climate, where the relative humidity may be less than 20% for many days in succession and where hot, dry forenoons are followed by rainy afternoons, the importance of such knowledge is evidenced by the prevalence of crazing marks and by the large percentage of concrete construction which is disfigured by cracks or which becomes disintegrated until replacement is necessary. Engineers speak of "concrete for permanence", and would be happy to feel assured that the motto is reliably true with respect to even severe weathering conditions.

* Prof. of Irrig. Eng., Univ. of Arizona, Tucson, Ariz.

In 1918, the writer called attention* to some important failures of concrete pipe lines at Continental, Ariz., and to similar failures in California. At Continental, cement pipe made on a McCracken revolving packer-head machine—pipe of unquestionably high-grade quality—failed soon after being put in service, by longitudinal cracks, in one case by a crack approximately 1 000 ft. long. The pipe had been allowed to stand in the stockyard until it had become thoroughly dry. The differential expansion in the shell due to absorption of water must have produced high tensile stresses, inasmuch as the pipe when tested withstood the stress produced by a pressure head of more than 100 ft. of water. The pipe was too "good", it broke itself; weak, porous, hand-tamped pipe never fails from this cause. In a study of the causes of these failures it was found that a joint of pipe immersed in water gained 5% in weight with an elongation of 0.056 per cent. A joint that had been broken in the internal-pressure testing machine and, therefore, had an open crack on one side, was tested by immersing one-half the pipe, laid flat, in water. With the absorption of water, mostly from the outside, the crack closed rapidly about 0.045 in. and then slowly opened again.

When the pipe-laying program at Continental was terminated, there was a considerable supply of 12-in. and 14-in. and some 16-in. pipes on hand, stacked three tiers high in the stockyard. These pipe have laid undisturbed for seven years exposed to sun and rain. When he examined them recently, the writer discovered that fully one-half the pipes were in bad condition and that about 10% of the top layer had cracked wide open. Fig. 25 shows the condition of the exterior and Fig. 26 of the interior of one of the pipes. Most of the cracks start at the tongue end. The hubs or grooves break off circumferentially. None of the pipe of this size failed underground, although similar 20-in. pipe failed by longitudinal cracks along the top of the pipe line.

The crazing on the exterior shows that the shrinkage has been both circumferential and lengthwise. On the interior, the circular cracks are more pronounced, and occur somewhat regularly beneath the corrugations or beads of the outer surface, due probably to the fact that the force of expansion is greater in the section beneath the beads than elsewhere.

Apparently the expansion of the outer part of the pipe wall produced the interior cracks and the rapid shrinkage of the outer part caused the crazing of the exterior surface. McCracken pipe has a very dense interior surface. Hence, most of the absorption of water and drying out is through the outer surface. The cracks tend to occur at those parts of the pipe that are relatively the most dense.

Whether temperature or moisture was the more active agent is uncertain. Professor Hatt has shown† that the coefficient of expansion is relatively high at high temperatures, such as occur at Continental during the summer, but with the free circulation of air through the open-ended pipe in the stack, it would seem unlikely that the difference in temperature between the outer and inner surfaces would be great.

* *Bulletin 86*, Arizona Agricultural Experiment Station, 1918; also, *Engineering News-Record*, Vol. 83, No. 3, p. 113, July 17, 1919.

† See p. 278.

Crazing cracks are common, also, on building floors that are continuously dry, and these cracks become progressively more pronounced year by year. Sidewalks, ornamental park benches, and building steps in Tucson, Ariz., suffer greatly from crazing cracks which develop into breaks and disintegration. Fully one-half the concrete sidewalks have a too limited life. Some of these results can be traced to poor workmanship or faulty curing, but by no means all of them.

The U. S. Bureau of Standards has investigated the properties of cement having a high magnesia content.* The effect of varying percentages of magnesia on the expansion when immersed and the contraction when dried indoors was studied by means of 5 by 5 by 45-in. bars. An inspection of the published results indicates that, neglecting two cements that were under-burned and unsound and three others, of the high alumina series, that were manifestly abnormal, the expansion from moist-chamber to air-dry condition varies directly with the percentage of magnesia. The cement used in the pipe at Continental had a high magnesia content, and there is a strong presumption that this had much to do with the failures of the pipe lines and the destruction of the pipe in the stockyard.

The shape of the pipe is such as to bring out to its fullest extent the injurious action of differential expansion. For massive construction and in humid climates, the effects may be inconsequential, but it is probable that for severe conditions the specifications of cement should contain a clause limiting the contraction of 1 : 3 mortar to, say, 0.02% in 1 month, or 0.03% in 3 months, and the methods of conducting tests should be standardized.

It is of great interest to note that Professor Hatt found wide differences in the effect of moisture on expansion and contraction in the six different brands of cement tested. The variation in the coefficient of expansion due to temperature was slight. It would be of value to know the percentages of magnesia in the six brands.

The thermal coefficient stated in the author's conclusions seems to be based on the oven-dry specimen reported in Fig. 6. The air-dry specimens have much higher coefficients.

In the case of the beams warped by the saturation of the under side, can it not be assumed that a part of the resulting compression on the bottom is relieved by warping, causing compression in the top side, and that tension exists in the beam between the line of saturation and the neutral axis to such an extent that the horizontal initial forces (stresses) are balanced? It may be remarked that, if the conditions shown in Fig. 20(*A*) are reversed, starting with a saturated beam, the effects on the beam will be much more injurious.

During saturation of specimens of cement pipe at Tucson, it was found that the expansion lags somewhat behind the absorption, suggesting that a change in crystallization is taking place, with the addition of water to the molecule.

* *Technologic Paper No. 102*, U. S. Bureau of Standards, 1918.

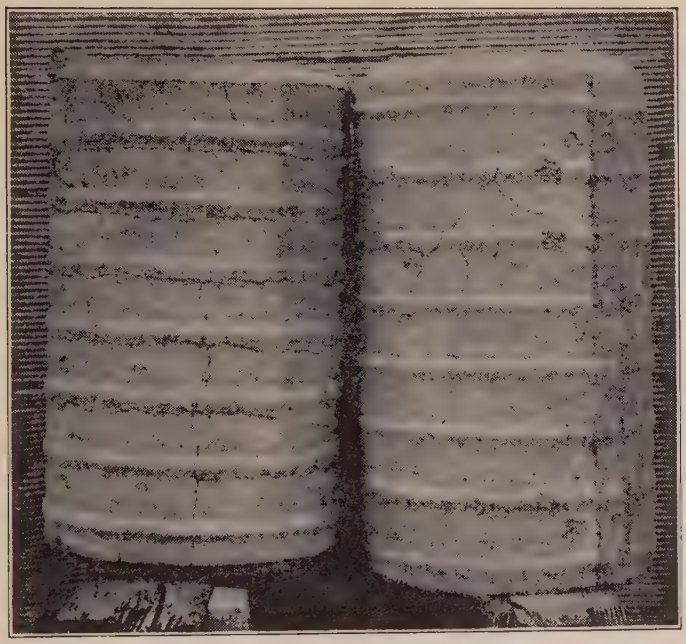


Fig. 25.—Crazing Marks and Open Cracks on 14-Inch McCracken Cement Pipe.

Fig. 26.—View of the Interior of a Cement Pipe, Showing Cracks. The Hubs or Grooves Broke Off in Segments.

W. K. Hatt,* M. Am. Soc. C. E. (by letter).—Experiences related by Professor Smith show the effects of expansion of the lower part of a concrete pipe under extreme conditions. It will be necessary to know something more about the manufacture of pipe in order to judge whether or not these examples are representative. In a paper published by the U. S. Bureau of Public Roads, entitled "Concrete Pipe for Use in Irrigation",† warning is given against laying such pipe in sandy soil that is hot and dry. It would appear that extra precaution should be taken in the curing of pipe in climates such as that of Arizona, because it is well known that surface crazes result from improper curing with extreme exposure. These conditions indicate the desirability of using mesh reinforcement.

During May, 1925, tests were made at Purdue University on two 30-in. concrete pipes, machine made, of 1 to 5 mix. The pipes were 3½ in. thick and one was reinforced with mesh. They had been steam-cured and air-dried for approximately 3 months. After submerging them in water to one-third of their depth for 5 days, measurements were made around the inner and outer surfaces with a Berry strain gauge. No cracks were apparent to the unaided eye. After this test the pipes were loaded in the testing machine and crushed; each pipe tested 2 500 lb. per lin. ft.

There is usually a difference in the behavior of new and old concrete under test. In old concrete small fissures appear but the pipe continues to take more load, whereas in newer concrete the pipe seems tougher and the first visible crack corresponds more nearly to the maximum load.‡

The behavior of concrete under moisture, temperature, and load has been discussed at greater length in a paper entitled "Researches in Concrete."‡

* Prof. of Civ. Eng., School of Civ. Eng., Purdue Univ., Lafayette, Ind.

† *Bulletin No. 852*, U. S. Bureau of Public Roads.

‡ *Bulletin No. 11*, Eng. Experiment Station, Purdue Univ., November, 1925.

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Paper No. 1578

THE ATMOSPHERE AND ITS RELATIONS TO HUMAN HEALTH AND COMFORT*

By C.-E. A. Winslow,† Esq.

With Discussion by George A. Soper, M. Am. Soc. C. E.

When the wise king of the Hebrews sought to express the fundamental unity of the problems which confront each human life he began with the atmosphere. "When I was born", he says, "I drew in the common air, and fell upon the earth, which is of like nature, and the first voice which I uttered was crying, as all others do." "The common air" is indeed the basic condition of human existence, controlling it in more ways than are yet fully realized. The atmosphere in its relations to human health and comfort seems, therefore, a topic peculiarly appropriate for consideration by a group of men whose objective is the application of the principles of engineering science in the special field of sanitation.

The chief business of science, as the speaker understands it, is to unify and simplify conceptions of the chaos of phenomena by relating those phenomena to underlying laws and principles. This task is more difficult in biology than in mechanics or hydraulics because the reactions of living matter are more complex than those of non-living substances. The basic conditions of life are, however, suspectible of relatively simple statement. From the amœba to man, from the bacterium to the oak tree, the life process depends upon at least five essential factors, each present to an extent above a certain minimum and in general below a certain maximum. The five positive essentials are:

1.—Food (including under this term regulative mineral salts).
2.—Oxygen.

* Address presented before the Sanitary Engineering Division, New York, N. Y., January 20, 1925.

† Prof. of Public Health, Yale School of Medicine, New Haven, Conn.

3.—Water.
4.—Temperature.
5.—Light.

In addition to these positive essentials, there must be freedom from:

1.—Mechanical injury.
2.—Toxic chemical substances.
3.—The attacks of hostile living beings, predatory or parasitic.

The list is obviously not a logically complete and perfect one. One factor, that of atmospheric pressure, which is omitted from the list will be discussed. The speaker is inclined, however, to believe that it includes all but a minor fraction of those influences which under actual conditions affect the vital processes of animals and plants. Taking it for what it may be worth, it will be noted that so far as man is concerned in all but one of the life factors noted, the atmosphere plays a more or less important rôle. We do not derive food directly from the air about us, although our supply of plant food depends largely upon atmospheric nitrogen, and the prevalence of goiter in certain regions is correlated with the presence or absence of iodine salts blown inland from the sea. Oxygen, temperature, and light as they affect the human body, and to a less extent the aqueous exchanges of the body, are directly related to atmospheric conditions; whereas the air may bear with it physically injurious dust particles, toxic gases, and, under certain conditions, the microbes of disease.

Composition and Characteristics of the Atmosphere

Before proceeding to discuss the more significant deviations from normal atmospheric conditions, as they affect health and comfort, it will be desirable to recall for a moment what these normal conditions are, to which the human body is presumably more or less perfectly adapted.

The atmosphere of the earth is a layer of gases, of continuously decreasing density as one passes outward from the surface of the earth itself, and of varying composition when one reaches a distance where the decrease of gravitational force is so great as to show a marked differential effect upon its various constituents. Thus, at a height of 20 km., it is estimated by Wegener that argon has practically disappeared and oxygen is reduced to 15%, whereas nitrogen has increased to 88 per cent. At 100 km., oxygen and nitrogen have practically disappeared, the atmosphere consisting of 67% hydrogen, 4% helium, and 29% of a hypothetical gas lighter than hydrogen, which Wegener called geocoronium. Whether or not these assumptions are sound is, however, of little practical moment to the sanitarian. The atmosphere as a whole has been estimated to be made up of nearly 4 000 000 000 tons of nitrogen (73% by weight), more than 1 000 000 000 tons of oxygen (23% by weight), nearly 160 000 tons of argon and other inert gases (3% by weight), and more than 50 000 tons of water vapor (1% by weight). As water vapor is so highly variable a constituent it is usually omitted from statements of the normal composition of the atmosphere; and this normal composition at or near the surface of the earth may be stated in the usual form as nitrogen, 78.1% by volume, oxygen,

20.9% by volume, and argon, 0.9% by volume. Carbon dioxide is normally present to the amount of 0.03% and the inert gases other than argon (neon, helium, krypton, xenon) in still smaller proportions.

The constancy of the relation maintained between the four principal gaseous constituents of the atmosphere is a striking phenomenon. Nitrogen and argon, being relatively inert, might be expected to show but slight variation. Oxygen on the other hand is constantly being removed by the respiratory action of plants and animals and by processes of combustion and is constantly being contributed by the process of photo-synthesis as carried out in the green parts of plants during the day. Carbon dioxide is being consumed in the process of photo-synthesis and in the secretion of calcareous skeletons by various animals; while it is produced by respiration, by combustion, and by volcanic action and other subterranean processes. The extent of these changes may be suggested by the calculation that 1 hectare of oat field or forest (about 2½ acres) consumes in an average summer season about 11 000 kg. of carbon dioxide and gives off about 8 000 kg. of oxygen. These changes produced by 1 hectare of agricultural land during the summer would be just about balanced by the respiratory activity of 32 human beings during the whole year.

With carbon dioxide, the waters of the ocean exert an important regulative influence, dissolving in the form of bi-carbonates any excess of the gas and yielding it up again as carbonic acid when the partial pressure in the air above decreases. As a net result of these complex processes the proportion of oxygen in winter or summer, in town or country, and even in the interiors of dwellings, rarely varies near the surface of the earth beyond the extreme limits of 20 and 21% of the atmosphere by volume; the variations in carbon dioxide as will be pointed out subsequently are somewhat wider.

Atmospheric moisture on the other hand varies widely from exceedingly small proportions to a point of approximate saturation. The saturation point itself depends directly upon the temperature of the atmosphere. Air at 0° Fahr. will hold only 0.5 grains of water vapor per cu. ft.; at 20° Fahr. the amount increases to 1.2 grains; at 32° Fahr. to 2.1 grains; at 68° Fahr. to 7.5 grains; and at 98° Fahr. to 18.7 grains. Thus, there is actually more moisture per cubic foot in air at 80° Fahr. with 20% relative humidity than in air at 35° Fahr. with 80% relative humidity; but the warmer air of lower relative humidity will be much more avid in absorbing moisture from the body or other moist surfaces and will thus be much more drying in its physiological effects. Desert air and the air of heated rooms in cold climates may fall as low as 10% of their potential saturation value.

In addition to its various gaseous constituents the atmosphere usually contains a greater or less proportion of minute suspended particles in the form of dust, the significance of which will be discussed in a subsequent paragraph; and, finally, in even the briefest review of the composition of the atmosphere, we must mention living micro-organisms, present only in very small numbers in pure outdoor air, as Pasteur demonstrated when he opened his flasks of putrescible liquid upon the Mer de Glace, but often rising to hundreds per cubic foot indoors and, when considerable contaminated dust is present, even to much higher figures.

The Physiological Effects of Atmospheric Pressure

The first problem which naturally suggests itself in connection with the relation of the atmosphere to human health and comfort is that of the influence of quantitative changes in concentration due to differences in atmospheric pressure. The pressure itself is apparently not a factor of primary importance although under certain circumstances sudden changes in pressure may produce serious and even fatal results. In the construction of tunnels and the laying of foundations the workers in caissons are exposed to pressures which may exceed three or four atmospheres. While under an excess pressure of even 50 lb. per sq. in. no marked discomfort is experienced, if the return to normal pressure is made too rapidly, caisson disease or compressed air illness frequently results. In the building of the tunnels under the East River, New York, N. Y., 3 692 attacks of compressed air illness with 20 deaths were recorded in a group of 10 000 employees. The pathology of this disease appears to be a relatively simple one, originating in the direct liberation within the blood vessels or tissues of bubbles of nitrogen gas with inevitable mechanical injury; and of course the condition can be controlled by regulated and gradual decompression.

Far more important than such direct variations in atmospheric pressure are the indirect effects due to changes in oxygen concentration. Oxygen is after all the most essential contribution of the atmosphere to human health and comfort; and the predecessors of Galen when they taught that the arteries were ducts for the transportation of air were after all very close to a physiological truth. As one passes upward from sea level the reduction in atmospheric pressure soon produces a reduction in partial pressure of oxygen which has real physiological significance. Thus, at 5 000 ft. above sea level the partial pressure of oxygen corresponds to 17.5% of the atmosphere at sea level (instead of the normal 21 per cent). Even at this point a few individuals begin to show an accelerated heart beat. At 10 000 ft. the concentration of oxygen is only 14.2% of a sea-level atmosphere and symptoms are more marked and more general. Beyond 13 000 ft., with concentrations below 12.7%, mountain sickness becomes a serious problem—a problem created entirely by deficiency in oxygen. The gallant struggle to conquer Mt. Everest, which has been followed with such eager interest during the past few years, is first and foremost a struggle against oxygen starvation and cannot be successful unless means are found for transporting oxygen apparatus for the use of the climbers who are to accomplish the last few hundred feet of rock which still tower unconquered into the sky. For the aviator too this problem is of the most pressing import. Bombing and observation planes are able to fly for hours at heights of 12 000 ft. or more and pursuit or battle planes not infrequently reach altitudes of 20 000 ft. The so-called "air staleness" among pilots is merely mountain sickness in a pronounced form, and the studies made by the Medical Research Laboratory of the Air Service at Mineola, N. Y., along lines outlined by the speaker's colleague, Professor Yandell Henderson, have shown how the worst effects of this malady may be avoided by the selection of pilots shown by specially devised tests to be peculiarly resistant to low oxygen pressure.

Chemical Problems of Ventilation

On mountain tops and in the vast spaces of the empyrean where the airman goes, the most serious atmospheric hazard is the lack of oxygen. At the other extreme of human pioneering—in the bowels of the earth—the same danger is met in a different form. In the deep parts of mines, and in old wells, the normal atmosphere may be replaced to so great an extent by carbon dioxide that men entering such places may perish of oxygen starvation. On the surface of the earth itself, however, in the normal haunts of man, the problem of oxygen content need give no concern. Even in the worst ventilated classroom or auditorium or workshop the oxygen in the atmosphere never falls below 20%, on account of leakage through cracks and the rapid diffusion which takes place even through solid walls and ceilings. Lavoisier, as early as 1777, showed by animal experiments that the symptoms experienced in a badly ventilated room could not be attributed to oxygen deficiency. It is only when the partial pressure of oxygen falls from its normal 21% of an atmosphere to a value of 16% or less that important physiological reactions are involved.

Lavoisier himself believed that the evil effects of vitiated air were due to the toxic effects of carbon dioxide, and this view was held by physiologists for about 100 years. It was Pettenkofer in 1863 who first clearly enunciated a third theory, that the influence of bad air was due to the presence of toxic organic substances excreted into the atmosphere from the human body. In his criticisms of the carbon dioxide theory, at least, Pettenkofer has been supported by all subsequent observers. The presence of carbon dioxide is indeed of fundamental importance in relation to human life, as it is the concentration of this gas in the blood which determines the activity of the respiratory mechanism. In the absence of carbon dioxide, breathing would cease and asphyxia result and the increased depth and rate of respiration after vigorous physical exercise is the direct result of increased carbon dioxide in the blood stream. An atmospheric concentration of 2% of this gas causes an automatic increase of about 50% in depth of breathing, just such as occurs with moderate exercise; and in breweries and mines such concentrations are actually borne with no appreciable discomfort. In the worst ventilated rooms carbon-dioxide content rises only from the normal 0.03% to about 0.5% of an atmosphere and such a change as this is well below the threshold of physiological significance.

On the other hand, Pettenkofer's alternative hypothesis of toxic organic substances in respired air has proved as untenable as the carbon-dioxide theory in explanation of the physiological phenomena characteristic of badly ventilated rooms. Revived again and again, by Brown-Sèquard, Weichardt, and other physiologists, the assumption of such organic poisons has repeatedly been disproved by more careful studies. Certain subtle psychological responses to the presence of atmospheric odors will be considered subsequently, but it is clear that the ordinary sensations experienced and the ordinary physiological responses manifested in a badly ventilated room are due neither to reduced oxygen nor to increased carbon dioxide nor to the presence of hypothetical

organic poisons. Human occupancy of a confined space produces five different changes in the atmosphere, namely, decrease of oxygen, increase of carbon dioxide, increase of products of organic decomposition (as evidenced by the "body odor"), increase of temperature, and increase of moisture. The first three having been shown to be inadequate to explain the effects of bad air, we must turn to the last two. As Professor Lee has summarized the situation in epigrammatic form, "the problem of ventilation is physical, not chemical, cutaneous not respiratory."

Thermal Problems of Ventilation

It was Hermans, of Amsterdam, Holland, who, in 1883, first clearly enunciated the theory that the evil effects of bad ventilation were primarily due to overheating and consequent interference with the elimination of heat from the surfaces of the body. This view was confirmed in 1905 and succeeding years by Flügge and his associates at Breslau, Germany, and, later, by Hill and Haldane in England and by Benedict and the New York State Commission on Ventilation in the United States. In one form or another these investigators have repeated a single crucial experiment which may be schematically described as follows: A group of subjects are placed in a closed experimental chamber and kept there while oxygen decreases and carbon dioxide, organic matter, temperature, and humidity increase. They manifest the usual symptoms characteristic of "vitiated" air—headache, general discomfort, lassitude, changes in blood pressure, pulse rate, and the like. The attempt is then made to relieve these symptoms in two alternative ways—by permitting the subjects to breathe chemically pure air (admitted, for example, through a tube from outside), or by lowering the effective temperature of the chamber by cooling coils or the operation of fans, without altering its chemical composition. The first procedure is ineffective, the second completely effective.

It will be noted that the vitally important point in all this is the rate of heat loss from the skin and that this rate of heat loss will depend on the combined influence of three distinct atmospheric factors, temperature, humidity, and air movement. When the temperature is low, heat loss from the body surface is effected chiefly by direct conduction and convection and as moist clothing conducts more readily than dry clothing, moist cool air feels colder than dry cool air. At high temperatures, on the other hand, the body loses heat chiefly by evaporation and as atmospheric moisture hinders this process, moist warm air feels hotter than dry warm air. At temperatures below that of the body, air movement accelerates heat loss and plays such an important rôle that still indoor air at 72° Fahr. may be really "warmer" from a physiological standpoint than a breezy outdoor atmosphere at 90° Fahr.

Recent investigations, particularly those carried on in New York, have made it clear that these thermal problems of ventilation are of the very first importance, and that even slight degrees of overheating may produce far-reaching results. In the first place the studies of the New York State Commission showed that even an atmospheric temperature of 75° Fahr. in still air caused a definite increase in body temperature and heart rate, a marked

decrease in general vasomotor tone, and an increased rate of respiration. In the second place this relatively slight overheating caused a decrease of 15% in the physical work performed by subjects not compelled to maximum effort but stimulated by a moderate cash bonus. The inferences from this fact in regard to the economic losses due to overheated factory workrooms are of far-reaching significance. Finally, the studies showed that in fan-ventilated schoolrooms, at an average temperature of 68.5° Fahr., there was an excess in respiratory illness amounting to 70% and an excess in absence due to such illnesses of 18%, as compared with window-ventilated schoolrooms at an average temperature of 66.5° Fahr.

The conclusions to be drawn from these investigations are of immediate practical importance to ventilating engineers, architects, schoolmen, and legislators. In a large number of States there are now laws which require the positive ventilation of schoolrooms by methods which effect an air change at the rate of 30 cu. ft. per min. These laws have been advocated by the American Society of Heating and Ventilating Engineers in complete good faith on the basis of the older physiological assumption that air vitiation was a chemical condition requiring a definite degree of dilution for its control. The New York State Commission results make it clear that the supply of 30 cu. ft. per min. of air in the schoolroom is not only needless but actually harmful to health and comfort as it almost inevitably leads to objectionable overheating. In the interest of health and of economy these laws should be modified so as to permit of the construction of school buildings ventilated by regulated window inlets and gravity exhaust which the New York studies have shown to be, under some conditions at least, distinctly preferable to the fan-pressure system.

Climate and Season and Disease

The new conceptions of ventilation here outlined make it clear that detrimental air conditions may exist outdoors as well as in and that in midsummer the whole world may be considered as badly ventilated. The prevalence of summer diarrhea of infants has been shown to bear a direct causal relation to summer temperature, and it would be well for architects and engineers to realize that August heat is as harmful as February cold and that provisions for specially cooled rooms and buildings in summer are as logical as those which are now made for heated buildings in winter. The speaker's colleague, Professor Ellsworth Huntington, has shown by exhaustive and carefully controlled statistical studies that the efficiency of factory labor reaches a maximum when the mean temperature for the 24 hours averages about 60° and that the death rate (based on figures for about 9 000 000 deaths) is at a minimum when the mean temperature for the 24 hours is 64° Fahr. and the mean relative humidity is 80 per cent. Professor Huntington points out that a climate which is essentially ideal from the standpoint of moderate temperature and humidity and stimulating variability exists only in Western Europe, in the Northeastern and North-Central United States, in Japan, and in parts of Australasia, and we may be prepared to agree with

him that a favorable climate has been one of the major factors in the development of the dominant civilizations of the world.

The Physiological Effects of Sunlight

The relations between the atmosphere and the human body with respect to oxygen, temperature, and moisture have been reviewed briefly. Of the four primary physiological relationships of the aerial envelope there remains only that of sunlight to be considered.

The value of sunlight as an adjunct to health has been stressed by hygienists since the days of the Greeks. In the medieval code of the School of Salernum prominent place is given to the admonition,

"Let air you breathe be sunny, clear and light,
Free from disease, or cesspool's fetid blight."

Soft coal was introduced into London about the end of the Thirteenth Century and almost at once an agitation against the "smoke nuisance" began, leading even in the reign of Edward I to a prohibition of the use of soft coal during the sessions of Parliament. English authorities have devoted much attention to this problem during the past few years and have demonstrated the astounding fact that between 80 and 400 tons of solid matter per annum is deposited from the air of English cities on every square mile of territory. In Manchester and Leeds, England, and Glasgow, Scotland, it has been shown that the average amount of winter sunlight is diminished about 40% as compared with adjacent rural districts and Besson reports that the smoke of Paris causes a reduction in sunlight of 15 to 25% as far as 10 km. from the city. In this country, the Cleveland, Ohio, Chamber of Commerce has estimated the financial loss due to smoke at $12 per capita per annum.

So far as the effects upon health are concerned, it must be remembered that the whole theory of the beneficial effects of sunlight has rested in the past on a purely empirical basis. Various authors reported statistical studies indicating that the death rate was higher in smoky cities than in rural districts and in the smoky winter season than in summer; but conclusions drawn from such analyses as to the harmful effect of a smoky atmosphere *per se* were clearly fallacious. Too many of what the late George C. Whipple, M. Am. Soc. C. E., has called "concealed classifications" were involved for any sound distinction between the influence of smoke and the influences of urban life or of the winter season as a whole. Even the revival of the cult of Apollo by Rollier and his Swiss colleagues in the treatment of tuberculosis by exposure to sunlight rested on purely clinical observations which were not altogether convincing, in view of the fact that tuberculous patients who live a hygienic life so generally recover whatever other special treatment they receive. Of sound and incontrovertible experimental evidence as to the hygienic and therapeutic value of sunlight there really was none until five years ago. The conscientious teacher of hygiene, when he came to such problems as that of the smoke nuisance, was forced to buttress his case by ingenious hypotheses as to the effect of soot in causing people to keep their windows closed, or in leading them to form habits of personal uncleanness; but he was never quite happy about the matter.

When Huldschinsky in 1919 showed that irradiation with sunlight or with the quartz mercury vapor lamp prevented and cured rickets, a new chapter in physiological science was opened. The work of Hess in New York, and of Park at Baltimore, Md., and New Haven, Conn., has made it clear that rickets can be controlled by two apparently distinct agents, the administration of cod liver oil and irradiation, neither obviously related to the abnormalities of calcium and phosphorus metabolism which characterize the disease in itself.

Now come Steenbock and Black and Hess with the demonstration that foods which do not in themselves possess anti-rachitic value may be endowed with such a value by exposure to ultra-violet light; and finally Kugelmass and McQuarrie with the really epoch-making discovery that a long series of substances curative of rickets actually possess the power, when oxidized, of blackening the photographic plate as ultra-violet rays would do, while substances which do not cure rickets fail to produce this effect. Furthermore, they find that anti-rachitic substances when irradiated give off oxygen, so that the apparently independent action of sunlight and cod liver oil turns out as Park predicted to be truly an interdependent one. When cod liver oil is given to cure rickets we are really administering "bottled sunlight". The possibilities for general physiological theory (in regard to such oxidative processes as those associated with hemoglobin and chlorophyll, for example) which are opened up by this discovery of the relation between oxidation and radiation are no less than astounding; and as light rays influence general body metabolism so profoundly in the case of rickets, it is altogether reasonable to suppose that they may do so in tuberculosis and other conditions.

For the practical hygienist the chief immediate point is that there is now a rational justification for the instinctive belief in the value of sunlight, a new ground for controlling the smoke nuisance, demanding sound building laws, and advocating the outdoor life. The superiority of scientific knowledge over empiricism is well illustrated, however, by the fact that sunlight admitted through glass is robbed of its most valuable radiations and, as Bovie has shown in his studies of diseases of fowls at the University of Maine, at Orono, this problem may perhaps be solved by the use of windows made of fused quartz. Our fathers were right in believing that sunlight was a good thing; how it really acts and how its benefits can be secured most practically we are just beginning to comprehend.

Bacteria in the Air

In addition to the normal and universal constituents of the atmosphere there are certain suspended materials generally present in greater or less amount, and among such suspended particles are living microbes of various types, to which much sanitary importance was attributed in the past. When Lister devised his first antiseptic spray in 1867, it was the microbes in the air which loomed largest in his mind as potential factors in wound infection, and the work of Tyndall and others demonstrated that as far as the decomposition of organic liquids was concerned the germs carried by air might indeed play a predominant rôle.

Since those microbes which are present in air, are largely carried on minute dust particles, it is of interest to note that the bacterial content of dust is a considerable one. Kligler and the speaker have found an average of about 50 000 000 bacteria per gramme in the dust of city streets and between 3 000 000 and 5 000 000 in indoor dusts. Most of these bacteria are of course of harmless saprophytic types, but about 1 in 1 000 of the street dust microbes and 1 in 4 000 of the indoor microbes were of an intestinal type, whereas 1 in 1 000 of the street organisms and 1 in 200 of the indoor organisms were members of the group of *Streptococci* characteristic of the human mouth. From 5 to 20 tubercle bacilli per gramme have been found in the dust from places occupied by tuberculous patients.

Of the dust which may be collected from floors and other surfaces, the amount which actually gets into the atmosphere is after all very small, except when the dust is stirred up by dry sweeping or violent winds. The actual bacterial content of the air is therefore generally low. In one of Pasteur's classic experiments, he opened sterile flasks of putrescible liquids (previously heated and sealed so as to contain a vacuum which would cause an inflow of air when they were opened), in various localities. Of twenty flasks thus exposed to normal country air at Arbois, eight showed the presence of micro-organisms; while of a similar series opened on the slopes of the Lower Jura Mountains only five spoiled; and of a third series exposed on the Mer de Glace one only became infected.

Browne and the speaker have found in country air an average of 56 microbes per cu. ft.; in city streets an average of 72; in the indoor air of offices and schools about 95; and in factories, 113. *Streptococci* characteristic of specific mouth pollution were present to the number of 11 to 12 per 100 cu. ft. in out-door air, 22 per 100 cu. ft. in offices, 30 in schools, and 43 in factories.

Such a bacterial content as this appears to have no important practical significance and as far as the transmission of disease microbes through the general medium of the atmosphere is concerned, epidemiological evidence is overwhelmingly negative. It has been made clear that even the widely distributed pus organisms get into wounds not from the air, but from the skin of the patients or the instrument which produced the wound or other objects which have come in contact with it; while the experience of well managed hospitals has shown that it is possible to care for various communicable diseases in a single open ward without serious danger of cross-infection, if aseptic technique is rigorously maintained.

Within a range of a few feet the direct discharge of mouth spray from an infected person is of course a real and important danger. Trillat, in Paris, has shown that bacteria suspended in moist droplets penetrate a cotton filter far more readily than when aspirated in a dry powder and that bacteria when inhaled by animals in droplets are drawn into the deepest parts of the lungs very quickly, while when borne by dry dust they are almost completely caught and held by the moist mucous membranes of the upper nose and throat, a phenomenon which the speaker suspects may be related to the electrical charges borne by the respective types of particles. The whole problem of droplet infection is, however, more nearly related to contact than to general

atmospheric pollution. The mouth spray is a fine rain which settles quickly to the ground, not a vapor which disseminates itself through the air, and Robinson and the speaker have found that the vast majority of the bacteria discharged in coughing, sneezing, and loud speaking, disappear from the air beyond a radius of a few feet.

Problems of Atmospheric Dust

The microbes of the atmosphere of course constitute only a minute fraction of the suspended matter which it normally carries. Lifeless organic particles and finely divided inorganic matter are present almost everywhere and are made manifest by the reflection of light as the dancing motes in a sunbeam. Such particles play an important part in promoting the formation of fog and rain, serving as nuclei for the condensation of atmospheric moisture; and to the reduction in the amount of aerial dust is due the clearness of the air after a heavy rain. As Tyndall showed in his classic experiments the larger of these particles will disappear so completely from the air in a closed box with glycerine-covered walls that a beam of light passed through the box is entirely invisible. Part of this dust is raised from dry surfaces by the wind like that borne into the City of Pekin, China, from the Gobi Desert. Part comes from volcanic eruptions. The dust discharged during the great eruption of Krakatoa remained for several years in the atmosphere and traveled many times around the globe. Another important fraction of aerial dust is of extra-terrestrial origin because in addition to the 20 000 000 meteorites per day which appear as "shooting stars", finer "mineral particles of foreign source are constantly pouring through our atmosphere in the form of a dry and invisible rain."

The number of dust particles which may be detected in the examination of air depends very largely upon the method of sampling and analysis. Optical methods which depend on the depth of fog produced by condensation of moisture on the dust particles in a vacuum tube may show billions of particles per cubic foot in ordinarily clear air. For practical purposes it seems probable that the more minute of the motes in the sunbeam are of little practical importance and most methods of examination depend on the collection of dust particles by water-spray or by impingement in water or on a vaselined plate and on the counting of the particles under the microscope, a procedure which gives in the main those particles with a diameter between 1 and 10 microns. Counts made with the Palmer water-spray apparatus show less than 10 000 particles per cubic foot (and less than 0.001 mg. per cu. ft.) in country air after a rain and 23 000 particles outside the fifty-eighth floor of the Woolworth Building; as compared with 118 000 particles per cubic foot in the street outside the Woolworth Building and about the same number (100 000) in ordinary indoor air. In dusty factories, on the other hand, the number of dust particles may rise to millions. In an abrasive factory at Niagara Falls the average of nine air samples gave the astounding result of 159 779 000 particles (7.805 mg.) of dust per cubic foot of air.

The influence of atmospheric dust upon health varies even more with the kind than with the gross amount of dust which may be present. As to the

effect of organic dusts little exact and conclusive information is available. Coal dust and cement dust when inhaled in considerable quantities appear quite definitely to favor acute respiratory infections, such as bronchitis and pneumonia, but if anything tend to diminish liability to tuberculosis. Silica dust, on the other hand, has a peculiar and characteristic influence in predisposing the tissues to the invasion of the tubercle bacillus. How far this may be due to the fact that silica particles are less readily eliminated from the lung tissues than carbon particles and, therefore, produce more far-reaching fibrotic changes through their irritating action; how far to some specific influence of silica in attracting or fixing tubercle bacilli in its vicinity; and how far to other factors as yet unrecognized, cannot be stated. The end result, however, is beyond dispute, as is clearly shown by the vital statistics of sand-blasters, granite cutters, polishers, and grinders, and other workers exposed to siliceous dusts. The City of Barre, Vt., in the center of the granite-cutting district, showed for a 10-year period a tuberculosis death rate more than 250% of that of the State of Vermont as a whole. In a certain axe factory in Connecticut, the men employed in grinding and polishing suffered from a tuberculosis mortality more than 1 180% that of their follow employees in the same factory not exposed to silica dust. Seventeen men out of every thousand in this group of grinders and polishers died every year as a direct result of industrial tuberculosis.

It is evident from these facts that the presence of fine particles of silica dust is one of the most deadly characteristics which the atmosphere can possess; and that definite measures for the protection of the worker in dusty trades, by the use of enclosed or moist processes, by the installation of local exhaust ventilation, or by the provision of masks and respirators, is of primary and essential importance.

Industrial Poisons

In addition to these siliceous dusts which so strikingly predispose to tuberculosis, the industrial hygienist must also contend, in certain crafts, with atmospheric dusts and with fumes and gases of a specifically toxic nature. In the State of Ohio, to take only one recently reported body of statistics, there were reported to the State Department of Health during a period of 10 years, 1 295 cases of industrial lead poisoning, and the vast majority of such poisonings are due to the inhalation of lead dust and fumes. In the hospitals of Detroit, Mich., during three years, there were treated 108 cases of lead poisoning, of which 67 occurred in a single factory among a group of men employed in the dry sand-papering of lead paint. The remedy here, as in many other processes, lies in the adoption of methods which will prevent the dissemination of atmospheric dusts of poisonous nature, but where this cannot be accomplished, the protection of the workers by masks and respirators should be enforced.

Lead is still the most important of the purely industrial poisons, but there are many others which affect smaller groups of workers in a more or less intensive degree. Mercury salts which cause the "hatter's shakes", arsenic, and other heavy metals, the oxides of nitrogen which produce acute and often

fatal injuries to the lungs of munition and chemical workers, the irritant halogens, and a whole host of organic poisons, such as anilin and benzine and their derivates, are all common and serious industrial poisons. The increasing use of benzol in industry has attracted special attention during the past few years; a Committee of the National Safety Council has been studying the problem, with the conclusion that the danger from this substance is a real one but that it can be largely controlled by efficient local ventilation.

Of more than purely industrial significance is the menace of carbon monoxide poisoning. Unlike carbon dioxide, this gas is an active poison attacking the red cells of the blood; and it is discharged into the atmosphere under a wide variety of conditions which involve the one common factor of imperfect combustion. In industry, employees about blast furnaces, electric ovens, and similar installations are stricken down in almost instantaneous death when exposed through some accident to a heavy dose of this gas. In the home, fatal cases of carbon monoxide poisoning are increasingly common, due to defective heating fixtures or to the use in small rooms of gas heaters not designed to secure perfect combustion when their radiating surfaces are cold. In the State of Ohio during the winter of 1922-23 there were 113 cases of partial asphyxiation and 64 deaths from carbon monoxide poisoning, 46 of the cases and 41 of the deaths being due to gas-fired room heaters.

Above all, however, it is in connection with the automobile that the danger of carbon monoxide poisoning has become most serious. One can scarcely pick up a paper in winter without reading of accidents due to the liberation of carbon monoxide from engines kept running in closed garages. Furthermore, it seems probable that aside from the acute and fatal cases of poisoning there is a widespread chronic intoxication among garage workers and others exposed to this gas. A recent study made by Ciampolini for the State Department of Labor of New York, showed that of 31 garages, 24 showed carbon monoxide in the air, 17 of them in an amount in excess of 0.1% which is set as a danger limit by competent authorities; while of 42 workers in these garages, who were examined physically, 29 showed evidence of carbon monoxide in their blood and many exhibited definite clinical symptoms of carbon monoxide poisoning. Henderson and Haggard believe that even outdoors dangerous concentration of carbon monoxide may be reached under conditions of crowded street traffic. The remedy suggested by these investigators is the equipment of all closed automobiles with a vertical exhaust, discharging above the roof so as to carry the poisonous gases away from the breathing zone outdoors and to discharge them into the range of roof ventilators in the garage. In view of the seriousness of the hazard involved, this policy deserves and is receiving the very serious consideration of health authorities throughout the country.

Odors and Health

The theory that miasmatic vapors of the atmosphere were the chief causes of epidemic disease dates back to the days when Hippocrates lighted fires in the streets of Athens to purify the air and put a stop to the plague. This view persisted as a dominant factor in medical teaching for 1 500 years, and

when the medical faculty of Paris reported on the epidemic of plague which ravaged that city in 1348, they attributed the disease as Hippocrates did, primarily to malign conjunctions of the planets and, secondarily and immediately, to a corrupted atmosphere. A free translation of one paragraph of this famous document would read somewhat as follows:

"Although there is danger of pestilential sickness from corrupted water and food as is observed in times of famine and drought, yet there is greater danger of sickness from the corruption of the atmosphere. For bad air is more injurious than food or drink on account of the velocity with which it penetrates to heart and lungs with its evil qualities. We believe that the present epidemic or plague arises immediately from air not only qualitatively altered but corrupted in its substance. By this I mean that normal air existing in its clear and pure state cannot decay or become corrupted unless from the admixture of evil vapors.

"Because therefore many corrupt vapors from the aforesaid conjunction from earth and water have been raised up and disseminated through the air by the blasts of the heavy and turbid southerly winds, on account of extraneous humid vapors which they bring or have brought with them, the air itself has become corrupted. Air thus corrupted necessarily penetrating to the heart corrupts the substance of the heart according to its nature, and decays the surrounding tissues in similar manner, finally according to nature extends and corrupts the principle of life. This is the immediate cause of the present epidemic."

Even fifty years ago it was still held that such diseases as typhoid fever and cholera were pythogenic in origin, that is, that their causative miasms were generated by the decomposition of organic filth. Murchison, one of the leading physicians of his time, taught in 1873 that such fevers "may be generated independently of a previous case by fermentation of fœcal, and perhaps other forms of organic matter" and that in these diseases "the poison is not like that of smallpox, given off from the body in a virulent form, but is developed by the decomposition of the excreta after their discharge." Dr. John H. Griscom, of New York, writing in 1854 displays an even more exact knowledge of the etiology of such diseases. He says*, on what authority the speaker knows not:

"Dr. Carmichael Smith discovered that the vapors of nitrous acid neutralize and render harmless the typhus miasm, which he ascertained to be sulphuret of ammonia, and on the utility of this discovery being duly tested and confirmed, a sum of 5 000 pounds was voted to him by the British Parliament, as a national donation."

The development of the science of bacteriology gradually dispelled these miasms which certainly existed as corrupting influences in the thoughts of men even if they did not influence their physical health; and to-day it is known that the communicable diseases are no more influenced by atmospheric vapors than by the conjunctions of the stars. The fear of noxious odors is, however, very deeply implanted in human psychology and even after the germ theory was well established the old terror persisted under a new guise—that of the alleged predisposing influence of organic emanations—morbific matter, anthropotoxin—in the atmosphere. The physiologist, Brown-Sèquard, was perhaps the most famous proponent of this theory but all efforts to demon-

* "The Uses and Abuses of Air," 1854.

strate the presence of subtle organic poisons in air vitiated by organic decomposition have for the present failed.

The ordinary effects of the air of a badly ventilated room, as has been pointed out, have been clearly shown to be due to physical rather than chemical factors. Fortunately, however, we are safeguarded from a too arbitrary and dogmatic dismissal of the whole problem of bad smells to the limbo of chimeras by the experiments of the New York State Commission on Ventilation; and it is always a relief to find at least some experimental justification for widely held beliefs, since empirical opinion is so likely to prove itself in the long run not without justification. These New York studies indicated that the stale air of an occupied room, even when artificially controlled from the standpoint of temperature and humidity, did have two slight but distinctly measurable effects upon human behavior. Subjects exposed to such stale air showed a definite disinclination to physical activity accomplishing 9% less work in stale than in fresh air; and they showed a marked diminution in appetite, the quantity of food eaten at luncheon served in stale air being from 4 to 14% less than in fresh air of the same temperature and humidity. Experiments in the laboratories at Yale University have similarly shown that the growth of young guinea pigs is quite definitely retarded by exposure to the odors from decomposing organic matter.

As to the discomfort and at least minor derangements of health produced by more highly concentrated odors there can of course be no question. The most exhaustive study of this problem with which the speaker is familiar is contained in a "Special Report of the Rhode Island State Board of Health on the Cause, Prevalence and Effect of Offensive Odors in Providence, Cranston, East Providence, and Warwick during 1921." The odors in question were produced by the burning of high sulfur fuel oils and by the distillation and refining of petroleum. Their influence extended for distances of 3 to 4 miles from the plants at which they originated and was felt by a population of 126 000 people. Record was obtained of definite symptoms affecting 96 persons, such as irritation of the throat, suffocation or choking sensation, nausea, headache, dizziness, and malaise, reasonably attributed to the odors in question. Mr. Stephen DeM. Gage, the Chemist and Engineer in charge of this investigation, discusses the situation as follows, in a passage which deserves much wider publicity than that attained by the usual State report:

"When our investigation was first started certain physicians and public health men with whom we discussed the matter expressed the opinion that the subject of offensive odors was not a health matter; that the effect, if any, was purely psychological. This is largely a question of the interpretation placed on the term health. With a very few exceptions the symptoms of the cases cited in the previous pages were temporary and not lasting. But, while the effect lasted, it was very real and in certain cases sufficiently severe to incapacitate the victim and of sufficient duration to interfere seriously with his regular avocation. In other words it reduced his mental and physical efficiency for a greater or less period of time at intervals which were dependent on the frequency of recurrence of the odors. It is well recognized by physiologists that intense and offensive odors may cause reflex actions affecting the nervous, digestive, and the secretory systems. Such reactions frequently

repeated cannot help but have an unfavorable effect on the general health. Even if the effect is psychological rather than physiological, if the effects produced upon the system are the same, the ultimate effect on the health must be the same.

"From the results of our own investigation we know that there are at least 96 persons who are or have been more or less seriously affected by these odors. From physicians have been received reports of 32 persons made ill and of 15 persons suffering from other diseases whose illness has been aggravated and perhaps their chance of recovery lessened from this cause. All or nearly all of these persons show a record of symptoms identical in character with those known to be caused by fumes or vapors of certain substances which from all available evidence are probable constituents of the odors which these persons have been forced to breathe at least a material portion of the time for a period of many months. The consensus of opinion among Rhode Island physicians, as expressed in their replies to our inquiry, is to the effect not only that the course of a number of diseases may be unfavorably affected by certain of the constituents of these odors, but also that the effect of these odors or fumes would be prejudicial to the continued good health of normal persons."

It may be concluded, therefore, that although polluted air does not bear the germs of disease and does not carry with it mysterious and specific organic poisons, an atmosphere laden with offensive odors is not only objectionable on æsthetic and economic grounds, but under certain circumstances may constitute a real menace to the public health.

It is of primary importance, therefore, that sewage and garbage disposal plants, packing houses, and oil refineries should be designed and operated so as to minimize the discharge into the atmosphere of products of organic decomposition or of offensive fumes from the combustion and handling of odoriferous materials. The task is exceedingly difficult when one is dealing with such substances as hydrogen sulfide which the nose can detect when present to the extent of 1 gramme in 1 000 000 000 000 cu. cm. of air, or as mercapton which is detected twenty-five times more readily than hydrogen sulfide. In the selection of processes for the disposal of sewage and garbage the possibility of aerial nuisance must always be a determining factor; and where the production of offensive odors is unavoidable, the possibility of deodorization by the use of chlorine or by some other method should be given the most careful consideration.

The Rôle of the Engineer in Promoting the Purity of the Atmosphere

In bringing this somewhat lengthy dissertation to a close, the speaker would point out that success in the effort to maintain a pure atmosphere under the difficult conditions of urban civilization depends on the co-operative effort of engineering experts of all the various types represented in the Society, and is not merely the affair of the Sanitary Engineering Division alone.

It is for the sanitary engineer to conduct intensive studies of the odor nuisance problem such as Mr. Gage has made at Providence, to determine the source and nature of the odors at work, their range of influence, and their effect upon health. It is primarily for him, also, to see that sewage and garbage plants, which are among the most prolific sources of complaint, are designed and operated with a minimum of resulting nuisance.

For the control of odors from industrial plants, however, the sanitary engineer will often be forced to call in the chemical engineer, either to re-arrange processes so as to diminish odor elimination, or to destroy the odors before they are discharged into the atmosphere by chemical means.

For the protection of the industrial worker against toxic fumes and dust and against the deadly influence of siliceous particles, the ventilating engineer must design special local exhaust systems, and it is to the heating and ventilating engineer—educated in the newer physiology and freed from the fetish of 30 cu. ft. per min.—that the public must look for the maintenance of comfortable air conditions in schools, offices, churches, theaters, and other places of assembly.

The mechanical engineer can help in the task of promoting atmospheric purity by the provision of automatic stokers and smoke consumers. Even the electrical engineer has his physiological functions. The speaker can see no evidence that the ozone he offers as an adjunct to ventilation is of the slightest value; but his ultra-violet rays are indispensable in every well-equipped hospital.

Finally, even the railroad engineer and the highway engineer must come into the picture, for the problem of securing fresh air and sunlight for the dweller in great cities is a problem which can only be solved by intelligent town planning and by the skilful use of transit facilities which shall distribute urban populations over a wide area.

In a dozen different ways a thing so common as "the air we breathe" must be safeguarded and controlled by the watchful hand of applied science. Thus only can truth, which as engineers and scientists we serve, free all mankind from the shackles of preventable disease and death.

DISCUSSION

GEORGE A. SOPER,* M. AM. SOC. C. E.—The speaker feels that he has been on a long journey. From the stellar spaces to the bowels of the earth, from the abrasives factories of Niagara Falls to the fogbound cities of Europe, from the Gobi Desert of Mongolia to the hermetically sealed testing chamber of a New York university, is certainly a great distance. He hopes he has escaped from all the dangers of breathing air which Professor Winslow has told about, but he is not sure.

Carbon monoxide is a subtle poison, and since city dwellers are compelled to breathe it day after day and year after year, there ought to be more certainty as to its effects. The danger, if it is one, is increasing. It is increasing with the size and complexity of urban life. With the growth of traffic and of industry the air is receiving more and more carbon monoxide every day and, with the greater height of buildings, the opportunities for its diffusion and removal are becoming less.

One of the principal sources of carbon monoxide may be mentioned to show how rapidly the increase in this gas is taking place in the city atmosphere. In 1923, the number of motor cars registered in New York, N. Y., was 363 590. In 1924, the number had grown to 440 218. The increase was 21% in this one year. The volume of gasoline consumed by these cars in 1924 has been estimated at 185 181 000 gal.

It may easily be that many persons are suffering from slow poisoning and that effects of which there is as yet no proof are being produced that are decidedly injurious. It is a curious fact that people so seldom attribute to the air the sensations of discomfort, if not of ill health, which they experience from it. They sit in crowded and ill-ventilated rooms, until they get tired, little reasoning that it is the state of the atmosphere which fatigues them.

There can be no disagreement as to the significance of carbon monoxide as a cause of sudden death, when present in sufficient concentration. Dr. Charles Norris, Chief Medical Examiner of the City of New York, in an address read March 25, 1924, before the New York Academy of Medicine, and again before the Annual Meeting of the Police and Fire Surgeons of the City of New York, said:

"Carbon monoxide poisoning is of immense interest to the community, for the reason that it claims yearly, in all modern cities, more deaths than occur from any other one cause (including accidental and suicidal gas asphyxiation), except perhaps, in New York City, the deaths from vehicular accidents."

The speaker has said that people are slow to ascribe to the air about them the discomfort which they feel from it. This is particularly true of carbon monoxide. A person may fall over, gassed to unconsciousness, without ever having realized that anything was the matter with him. He may sit and breathe air which is contaminated with this gas and not be made in the least uncomfortable by it until he starts to walk home in the outer air when the symptoms may attack him with such violence that he is unable to proceed. A motor

* Cons. Engr., Great Neck, N. Y.

idling in a closed garage for 20 min. will produce so much carbon monoxide that a person who is working on the car may suddenly fall, lose his senses and die without ever crying out. Many deaths occur from gas heaters and stoves, which give off carbon monoxide to the atmosphere of sleeping-rooms, the lives of the sleepers passing quietly away without the victims waking up.

Some time ago a family was asphyxiated on the eighth floor of a house from a leak in a pipe in the cellar. Recently, there were two deaths in an apartment house in New York ascribed to a small leak in a main in the street. The speaker has broken down a door and dragged out a man near the point of death from a bathroom in which a gas water heater had produced enough carbon monoxide to overcome him. In none of these instances has there been any reason to suppose that the victims have struggled or made any effort to get better air. The gas has robbed them of the power of doing so.

So much for the poisonings of a tragic character which are due to carbon monoxide. Engineers are not unfamiliar with them, for they occur occasionally in mines, tunnels, and some other works of engineering construction. When these accidents are spoken of, however, the tale is not all told. It is necessary to consider the large amount of poisoning from this gas which does not lead to fatal consequences but produces effects which nevertheless may be harmful. Just what these effects are and how harmful they may be is a question that should be considered.

Carbon monoxide is a practically odorless, tasteless, invisible gas, of almost the same specific gravity as pure atmospheric air. It is discharged from automobiles in large quantities, owing to the incomplete combustion of their gasoline.

Are the poisonous gases diluted sufficiently under the everyday conditions of city life? In answering this question two conditions must be taken account of: First, the state of the general atmosphere of the streets and other places where the gases are encountered, in which diffusion is fairly uniform and complete; and, second, the air close to an automobile before the exhaust gas has had time to become thoroughly mixed with the air about it. All know from experience how unpleasant it is to get a breath of the odorous gases direct from an exhaust. One instinctively stops breathing for an instant in order to avoid it. The concentration in this second case is many times that of the first and is hazardous in the extreme.

The standard adopted for the New York-New Jersey Vehicular Tunnel is 4 parts per 10 000, it being expected that this standard will apply only to passengers making the transit of the tunnel in less than 1 hour and usually in less than ½ hour. Employees who are exposed longer are to be provided with an extra amount of ventilation at their stations.

A word as to the way in which carbon monoxide does injury. As every one knows, oxygen is so necessary to life that to be deprived of it causes instant death. It is absorbed from the atmosphere by the hemoglobin of the blood as it passes through the lungs. Hemoglobin will absorb carbon monoxide also. It has a decided preference for it. It takes up the carbon monoxide three hundred times more readily than it absorbs oxygen. It accumulates it and keeps it for a long time. A person recovering from carbon monoxide poison-

ing gets well very slowly. The gas is not irritating or corrosive. It is obstructive. It keeps the blood from absorbing the amount of oxygen which the person needs.

The intensity of the effects depends not only upon the amount of carbon monoxide in the atmosphere but also upon the length of time that one is exposed to it and upon various other conditions. One person may be much more quickly affected than another. In mine accidents where a number of men are killed, others may extricate themselves from the group and walk away. Long exposure to low concentrations produces worse effects than short exposure to high concentrations, even though the total absorption is the same.

Increased breathing facilitates the absorption, so that walking and other exercise accelerates it. Anything that lowers the physical fitness of a person, as fatigue and overwork, contributes to the harmful effects. Children and persons in ill health are ordinarily the first to be affected.

For the blood to take 60 to 80% of the greatest amount of carbon monoxide which it is capable of absorbing is fatal. To absorb 30 to 40% is to produce severe headache, weakness, dizziness, dimness of vision, nausea, and collapse. To absorb 10 to 20% produces a sense of tightness across the forehead, perhaps a slight headache.

When the concentration of carbon monoxide in the air is from 2 to 3 parts per 10 000, the blood will absorb from 23 to 30% in from 5 to 6 hours. When the concentration is 4 to 6, the percentage becomes 36 to 44 in 4 to 5 hours. When the concentration is 7 to 10, the percentage rises to 47 to 53 in 3 to 4 hours.

In other words, if a person breathes air containing between 2 and 3 parts of carbon monoxide per 10 000 for 5 or 6 hours, he is very likely to have a headache with throbbing of the temples, and long before that he may feel a tightness across the forehead and a slight headache, and the blood vessels of his skin may be somewhat dilated.

Now as to the concentrations of carbon monoxide which are to be met with in city streets.

In 1923, Professors Henderson and Haggard of Yale were employed by the Committee on Public Health of the New York Academy of Medicine to make tests of the amount of carbon monoxide in the air of the streets of New York. This they did, between the middle of July and the following March. The samples were collected into suitable containers, which were carried about in a motor car along the streets.

The investigators showed an inclination, as many visitors do, to remain on Fifth Avenue. They went repeatedly up and down that thoroughfare, once as far north as 135th Street, although it is to be noted that, from 59th to 110th Streets, one side of the Avenue is entirely open to Central Park and the air is better for this reason. Following are some of the results of the analyses. One July afternoon, with light traffic, the carbon monoxide among fourteen samples ranged between 0.1 and 0.64 parts per 10 000. On a rainy day, with light traffic, sixteen samples were taken and had a range between 0.5 and 2.6. On a winter morning, with moderate traffic, twenty-seven samples ranged between 0.2 and 4.6. A series of nineteen samples, with considerable traffic, ranged

between 0.2 and 3.1. On still another day, with moderate traffic, fifteen samples varied between 0.2 and 2.9. The average of these ninety-one samples of air from Fifth Avenue was 1.1 parts of carbon monoxide per 10 000 parts by volume of air.

In various parts of the city, on a clear, cool day with light traffic, samples were collected gradually over a period of 20 min. and ranged between 0.6 and 2.1. Finally, on a very cold, gusty day, seven samples were collected which ranged between 0.65 and 1.5.

The tests were made on samples of air collected at times of moderate traffic and, for the most part, on a broad and open thoroughfare under conditions which may be said to have been, on the whole, favorable to low carbon monoxide concentrations. On every day except the first one, when the traffic was light, the samples contained more carbon monoxide than is safe and some of them so much as to be positively alarming.

The investigations should be repeated on a larger scale, now that the number of automobiles has increased so greatly, and they should include a greater variety of places, times, and concentrations of traffic. Certainly, they should cover times when the traffic is heavy, for this is a very common condition and one to which particular interest attaches. Not only is more carbon monoxide produced then but there are more people in cars and on foot to be affected by it.

Not only the maximum and minimum and average condition of the air but the worst conditions which frequently occur should be known. For example, Sixth Avenue under the elevated railroad in the lower Forties is often filled from curb to curb with cars, either moving slowly or idling. Overhead is the low-lying structure of the elevated railroad to interfere with the diffusion of the exhaust gas to the upper atmosphere.

How much carbon monoxide is present when the avenues are blocked with traffic? How much is present when the blue, oily smoke from the motor cars hangs, a thick haze in the streets and there is no wind to blow it away? What is the concentration when traffic is jammed on the narrow cross streets? What is at the taxi platforms of the railroad stations, when it seems, as it usually does, so stifling?

The information collected by a thorough investigation would apply not only to people temporarily on the streets, important as that would be, but it would have more far-reaching value. It is not to be forgotten that the subways get their air from the streets. In fact, they get it from the surface of the pavement, which is the level at which the automobiles exhaust their gases.

The information would have relation to those who work, many of them for long hours, in the shops on each side of the streets, to the shoppers, and to the large numbers of people on the floors above the street level in the buildings which line the streets. For the shops and the buildings above them get much of their air from the streets and at the level of the front doors and cellar openings. Except in summer and under unusual circumstances at other seasons, the air which gets into the buildings enters, not so much through the windows, as does the light, but at the ground floors and cellar levels. It is heated and rises upward. Studies of the ventilation of tall office buildings of

Manhattan have provided a surprising amount of interesting information upon this subject. As Professor Winslow has pointed out, the proper ventilation of schools and shops and apartment houses and, in fact, all buildings, is not to be accomplished by fans and ducts and dampers, so much as through a better utilization of the natural currents of air which exist or may be brought about under the conditions of actual occupancy. In buildings, more attention must be given to what, out of doors, is called meteorology, and out of doors more attention must be given to what, indoors, is called ventilation. If there is to be good air indoors, there must be good air outdoors. It is of paramount importance that cities should be well ventilated.

In connection with the investigation of the air of streets, a study should be made of the health of the people who spend their working hours there. Traffic officers and the motormen and conductors of busses and trolley cars, although obviously selected as much for their rugged health as for the possession of mental qualities, would afford a profitable field for a carefully made and thorough physiological investigation.

Such an inquiry as is proposed would not be without precedent. Studies of subway air and ventilation were made some years ago at the instance of the Rapid Transit Commission of New York, in which many thousand analyses were made and these were supplemented by thorough examinations of the respiratory apparatus of the motormen and conductors on the trains.

Is the speaker exaggerating the possibility of danger from carbon monoxide in city streets? It is possible that he is. It may be that most people can stand a certain amount of carbon monoxide without much harm, just as they can stand tobacco smoke and, in former times, some stood alcoholic intoxication. Perhaps the only bad effects of carbon monoxide in the city is to make people tire more easily, to be a little less correct in judgment, a trifle less certain in mental and bodily co-ordinations, somewhat more irritable, and a bit more susceptible to accidents and disease. In the aggregate this may or may not be important. It has been suggested that some of the very numerous accidents which occur in the streets may be ascribable to the subtle effects of unsuspected carbon monoxide, but nobody really knows.

A word as to the remedy, if a remedy is required. The speaker thinks the remedy should be left to the Engineering Profession. It is really an engineering question how to make better use of the gasoline and thus reduce the amount of carbon monoxide to a minimum. It is an engineering question how to discharge the spent gases so that they will not be driven into the faces and so into the lungs of drivers of other cars and pedestrians. It has been suggested that motor cars carry stacks and send their exhausts upward, as do locomotives. It has been proposed that absorbing chambers, filled with suitable chemicals, be carried by automobiles, for the purification of the gases which they produce.

It is to be hoped that something better than any of these solutions may be brought forward, for, aside from the question of life and health, an improvement over present conditions would certainly be beneficial from the standpoint of reasonable comfort.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1579

DETECTION AND ELIMINATION OF ODORS

A SYMPOSIUM*

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* Presented at the meeting of the Sanitary Engineering Division, New York, N. Y., January 22, 1925.

LAWS RELATING TO OBNOXIOUS ODORS

By Irving I. Goldsmith,* Esq.

The problem of safeguarding health and preserving the comfortable enjoyment of property is becoming increasingly complex. While remedies grow more efficient, standards mount higher and new dangers are added by commercial and industrial development and the steady influx of population from rural districts to urban centers. The topic under consideration concerns the contamination of the air. Possibly no single natural right is more closely related to the preservation of health, the enjoyment of life, and the comfortable occupation of property than the privilege to breathe clear, pure, and wholesome air. Although modern medicine and science justify and demand relief from conditions which tend to pollute the air, it was anciently recognized that malodorous fumes and stenches impaired health and disturbed the enjoyment of property. Early legal protection was confined largely to the effect of such contamination upon the enjoyment of property, but as far back as Coke's time, the common law had extended its protection to the public in a limited way.

The law relating to the suppression of dangerous and distasteful fumes and gases is found in that division of substantive law known as "nuisance". According to Blackstone, a nuisance is "anything done to the hurt or annoyance of the lands, tenements or hereditaments of another." The present conception of nuisance is broader, however, and "is anything that works an injury, harm or prejudice to an individual or the public."† Nuisances are generally known as private and public:

"They are private when the injury resulting from them violates only private rights and produces damage to a few persons only; or even to one person only. They are public when they violate public rights, and produce a common injury; when they injure or annoy that portion of the public which necessarily comes in contact with them."

There is a third class called mixed nuisances "which are both public and private in their effects—public, because they violate public rights, and injure many persons, or all the community; and private, in that they also produce special injury to private rights."

Again, there is a classification as to the nature of nuisances:

"First, those which in their nature are nuisances *per se* or are so denounced by the common law or by statute; second, those which in their nature are not nuisances, but may become so by reason of their locality, surroundings, or the manner in which they may be conducted, managed, etc.; third, those which in their nature may be nuisances, but as to which there may be honest differences of opinion in impartial minds. If an instrumentality or condition is a nuisance at all times and under all circumstances, irrespective of location or surroundings, it is said to be a 'nuisance *per se*'."

* Deputy Attorney General, State of New York, Albany, N. Y.

† Definitions taken from "Ruling Case Law," Vol. 20, pp. 380, 383.

These definitions cover the basic law governing nuisances. The rules are of ancient origin in jurisprudence, but they are as responsive to present conditions as they were at the time of their conception. The law is progressive and attunes itself with the onward march of civilization. The uninitiated, the impatient, and the radical frequently dub the law as reactionary; but their judgment is based upon either lack of knowledge or wilful blindness. Legal reasoning does follow established principles and honored precedents, but the decisions of the high Courts are undisputed proof of the flexibility of these dogmas in meeting current needs. The Chief Justice of the great Court of Appeals of New York State has said:*

"I shall follow the thought that administration of the law is a practical matter and that where possible, it must with reasonable flexibility adjust itself to new and changing conditions and, in accordance with recognized principles, must attempt to solve the problems of life in a well-ordered, fair and reasonable way which will secure the approval of well informed and intelligent opinion."

And, pursuing the same subject further, he said:

"In somewhat general terms, we may safely define progressiveness in accordance with generally accepted concepts as a disposition on the part of the Courts toward liberal interpretation of remedial statutes to the end of accomplishing the purposes for which they were enacted; a disposition toward that interpretation, extension, and, if necessary, modification of legal rules and precedents so far as permissible which will secure what the sensible and well informed student would regard as a greater measure of justice between private litigants; and a liberal, but justifiable and faithful, interpretation of Constitutional provisions in the direction of upholding the legislative function to adopt enactments for the greater protection and welfare of the public."

The legal principles governing the regulation and suppression of nuisances from the earliest days of the common law, have constantly yielded to the exigencies of progress, and the limited application to property rights has been developed to cover the broader rights of the individual as typified by the public. The subject assigned to the speaker relates to pollution of the air and this discussion will be confined to those offenses in the category of nuisances which render the air unwholesome and unsanitary and constitute a menace to health and the reasonable and peaceable enjoyment of property. It is evident that in a paper of this character only broad general principles can be considered and the distinctions and differences so common to the law must be passed over with the comment that they exist.

The early cases involving private rights pertained very largely to the noxious odors emanating from pig sties and other animal enclosures, vaults, and privies, and from such commercial enterprises as kilns, smelters, coke ovens, and apothecaries. These structures or trades have not generally been considered nuisances *per se* because they constituted an essential part of domestic life and business activities; but objectional and deleterious odors emanating from these sources have always been held to be actionable and, if sufficiently harmful, subject to injunctive relief in equity. The law in this respect has

* *Cornell Law Quarterly*, Vol. 9, No. 4, p. 371.

been established for centuries and has been particularly well summarized in a leading case of the United States Supreme Court:*

"There is no doubt of the general proposition that a man may do what he will with his own, but his right is subordinate to another, which finds expression in the familiar maxim: *sic utere tuo ut alienum non laedas.* His right to erect what he pleases upon his own land will not justify him in maintaining a nuisance, or in carrying on a business or trade that is offensive to his neighbors. Ever since Aldred's case, 9 Coke, 57, it has been the settled law, both of this country and of England, that a man has no right to maintain a structure upon his own land, which, by reason of disgusting smells, loud or unusual noises, thick smoke, noxious vapors, the jarring of machinery or the unwarrantable collection of flies, renders the occupancy of adjoining property dangerous, intolerable or even uncomfortable to its tenants. No person maintaining such a nuisance can shelter himself behind the sanctity of private property."

Although the growth of community life and the development of industrial activities have created new conditions which often menace the health and comfort of the occupants of adjoining premises, the Courts have continued to extend the rule cited to these situations for the protection of neighboring property and the occupants thereof.

Of course, it must be borne in mind that an owner of land is always entitled to the reasonable use of his premises and if damage results to his neighbor from such use it is not actionable. Probably the contamination of the air by soot and gases resulting from the burning of soft coal presents the most vivid example of the effects of modern industry when conducted in or near a village or city. The New York Courts were called upon to enjoin such a nuisance almost twenty years ago and held that in the case before it the burning of soft coal was not necessary for the practical running of the plant and constituted an unreasonable use of the manufacturer's property.† The facts are interesting in defining the scope of the case. It appears that the plant in question was located on the outskirts of a village of 13 000 inhabitants in a sparsely settled neighborhood, but one suitable for residential purposes. The nearest dwelling was owned and occupied by the plaintiff and was distant 840 ft. When the wind was in the right direction, the soot and gases from the defendant's plant settled upon the premises of the plaintiff, causing discomfort and slight discoloration of the residence property. The proof disclosed that hard coal could be used to operate the plant, although the expense would be greater. Under these circumstances, the Court awarded damages to the plaintiff and enjoined the defendant from further operating its plant with soft coal. This case created wide comment at the time of its determination as the damage proved was quite inconsequential, but it firmly established the rule in New York State that no one may make an unreasonable use of his property to the injury of his neighbor. The Court, in its opinion, discusses private nuisances as follows:

"The law relating to private nuisances is a law of degree and usually turns on the question of fact whether the use is reasonable or not under all

* Camfield *vs.* United States, 167 U. S., 518, 522.

† McCarty *vs.* Natural Carbonic Gas Co., 189 N. Y., 40.

the circumstances. No hard and fast rule controls the subject, for a use that is reasonable under one set of facts would be unreasonable under another. Whether the use of property to carry on a lawful business, which creates smoke or noxious gases in excessive quantities, amounts to a nuisance depends on the facts of each particular case. (21 Am. and Eng. Ency. of Law, 2d ed. 692.) Location, priority of occupation and the fact that the injury is only occasional are not conclusive, but are to be considered in connection with all the evidence and the inference drawn from all the facts proved whether the controlling fact exists that the use is unreasonable. If that fact is found, a nuisance is established and the plaintiff is entitled to relief in some form. Unless the fact is found, or it is an inference of law from other facts found, no nuisance is established, even if the plaintiff shows that he has suffered some damage, annoyance and injury. Those evils are at times incidental to civilized life and the sufferer finds compensation in the arts and agencies of civilized society. What is reasonable is sometimes a question of law and at others a question of fact. When it depends upon an inference from peculiar, numerous or complicated circumstances it is usually a question of fact. Whether the use of property by one person is reasonable, with reference to the comfortable enjoyment of his own property by another, generally depends upon many and varied facts; such as location, nature of the use, character of the neighborhood, extent and frequency of the injury, the effect on the enjoyment of life, health and property and the like."

And, further, referring particularly to the nuisance created by the smoke:

"The defendant's business is lawful and not a nuisance *per se,* although it has been found that as carried on it is a nuisance in fact. The extent more than the nature of the injury, the *quantum,* rather than the *damnum,* constitutes the nuisance. Some smoke is generally created by the natural and ordinary use of land near a village or city, and while this may sometimes be annoying to neighbors it is part of the price paid for living where there are neighbors. But when the smoke is so unusual and excessive as to materially interfere with the ordinary comfort of human existence, the trier of the facts, taking into account all the circumstances, such as public utility, locality, immediate surroundings and the like, may find the use unreasonable. This is not a case where the defendant cannot carry on its business without injury to neighboring property, for all damage can be avoided by the use of hard coal, as is done by one of its competitors in the same kind of business in the same locality, or possibly by the use of some modern appliance such as a smoke consumer, although either would involve an increase in expense. It is better, however, that profits should be somewhat reduced when he did not *come to the nuisance* but was there before. 'The safety of property generally is superior in right to a particular use of a single piece of property by its owner. It renders the enjoyment of all property more secure by preventing such use of one piece by one man as may injure all his neighbors' (Sullivan *vs.* Dunham, 161 N. Y. 290, 300)."

This case has been dealt with at length because it so clearly sets forth the law relating to private nuisance caused by carrying on a lawful business, and also because it deals particularly with the burning of soft coal which undoubtedly constitutes the most pronounced nuisance in populous communities.

Like the law relating to private nuisances, that governing public nuisances has its origin in the common law. Blackstone defines public nuisances as "offenses against the public order or economical regimen of the State, being either the doing of a thing to the annoyance of the King's subjects or the

neglecting to do a thing which the common good requires."* According to current decisions a public nuisance exists whenever a common or public right is invaded.† The annoyance must affect a substantial number of persons in order to constitute it a common or public nuisance.‡ Public nuisances for the most part are concerned with the broader and larger common rights, although frequently a public nuisance may also become a private one when special damage is inflicted upon the individual in the occupation of his premises.§ In illustration, a stable may be so negligently maintained in a well settled neighborhood that the entire community suffers discomfort from the disgusting odors emitted and, at the same time, the occupants of adjacent premises may suffer special damage by being denied the comfortable enjoyment of their property.

A public nuisance, as its character indicates, is indictable‖ and may even be enjoined or abated.¶

Although public nuisances have long been recognized by the common law, it is probably a fair statement that most of the invasions of common rights are characterized by statute or ordinance as public nuisances. This does not signify that statutes and ordinances comprehend all acts which constitute public nuisances, but they usually include those acts which are most likely to impair common rights. Neither does it mean that every act declared by the legislature or municipal government to be a public nuisance will receive that construction when reviewed by the Courts.** Even when legislative or local governing bodies have declared certain acts to be nuisances, such acts must be in fact nuisances known in the law.††

The effect of the legislative or municipal declaration is to eliminate any question of the resulting injury being a common one, but it does not make an unlawful act lawful.‡‡ When the determination rests upon particular facts, and they present a question fairly debatable, the judgment of the legislature will usually be adopted and confirmed.§§

The right to brand certain acts as public nuisances by statute or ordinance rests in the police power of the State. The law is well settled that legislation for the protection of the health, morals, and welfare of the people is a proper exercise of the police power of the State. The validity of such statutes intended to suppress noxious odors was challenged in the famous "Slaughter House Cases" and sustained by the United States Supreme Court.‖‖ The cases merit special comment. In 1899 the Legislature of the State of Louisiana by statute provided that the business of slaughtering all animals in the City

* 4 Com. 166.

† Jones *vs.* State, 38 Okla. 218; 132 Pac. 319.

‡ Flynn *vs.* Butler, 189 Mass. 377, 386; *in re* Debs, 158 U. S. 564.

§ Quinn *vs.* Lowell Electric Light Corp., 140 Mass. 106; Acme Fertilizer Co. *vs.* State, 34 Ind. App. 358; 72 N. E. 1037.

‖ Commonwealth *vs.* Packard, 185 Mass. 64, 66.

¶ Delaware Division Canal Co. *vs.* Com., 60 Pa. St. 367.

** Lawton *vs.* Steele, 152 U. S. 133, 140.

†† Jones *vs.* State (*supra*).

‡‡ Commonwealth *vs.* Boston, 97 Mass. 555; Richards *vs.* Washington Terminal Co., 233 U. S. 546.

§§ Hadacheck *vs.* Sebastion, 239 U. S. 394.

‖‖ Slaughter House Cases, 16 Wallace, 61.

of New Orleans should be confined to certain restricted areas and granted to a particular corporation the exclusive privilege to erect and maintain slaughter houses to carry on this trade and the right to exact fees for this service, prohibiting all others from erecting and maintaining such structures or engaging in such business. The Court affirmed the judgment of the Legislature that the slaughtering of cattle in a great city constituted a public nuisance and that it was a proper exercise of the police power of the State to locate slaughter houses where health and comfort required. A part of the opinion of this Court is illuminating in defining the scope of public nuisances and the inherent power of the sovereign to prohibit them:

"'Unwholesome trades, slaughter houses, operations offensive to the senses, the deposit of powder, the application of steam power to propel cars, the building with combustible materials and the burial of the dead, may all', says Chancellor Kent (2 Commentaries 340), 'be interdicted by law, in the midst of dense masses of population, on the general and rational principle that every person ought so to use his property as not to injure his neighbors; and that private interests must be made subservient to the general interests of the community.' This is called the police power, and it is declared by Chief Justice Shaw (Commonwealth *vs.* Alger, 7 Cushing, 84) that 'it is much easier to perceive and realize the existence and sources of it than to mark its boundaries, or prescribe limits to its exercise. This power is, and must be from its very nature, incapable of any very exact definition or limitation. Upon it depends the security of social order, the life and health of the citizen, the comfort of an existence in a thickly populated community, the enjoyment of private and social life and the beneficial use of property'. 'It extends', says another eminent judge (Thorpe *vs.* Rutland & Burlington R. R. Co., 27 Vermont 149), 'to the protection of the lives, limbs, health, comfort and quiet of all persons and the protection of all property within the State', * * * and persons and property are subjected to all kinds of restraints and burdens in order to secure the general comfort, health and prosperity of the State. Of the perfect right of the Legislature to do this, no question ever was, or, upon acknowledged general principles, ever can be made, so far as natural persons are concerned."

The police power is very broad. It was even held by an Illinois Court and affirmed in the United States Supreme Court* that where a fertilizer plant was established by Act of the General Assembly, and subsequently a community grew up about the plant, that under a village ordinance to suppress the operation of the plant, a bill in equity to prevent the enforcement of the ordinance would be denied. The pecuniary injury which directly resulted to the company from the stoppage of its works was held no bar to the exercise of the police power of the State.

Statutory regulations to prevent nuisances are less common, however, than local ordinances. The answer is obvious. Most cities and villages in their charters are delegated with power to adopt ordinances for the promotion of the health, safety, and welfare of the community. Inasmuch as conditions are so different in each locality the local governing body is better able to appraise the local situation and to make its rules accordingly. Of course, the

* Fertilizing Company *vs.* Hyde Park, 97 U. S. 659.

grant to the municipality is limited by the language of the charter and probably never includes matters affecting State policy.

Neither the State nor the municipality can make an unlawful act lawful by statute or ordinance. Such action may rob the act of its character as a public nuisance, but if injury results to person or property, the act still constitutes a private nuisance. In this connection reference is called to a case wherein a municipality erected a pumping station in a residential locality, burning soft coal to produce power, thereby damaging the plaintiff's residence. The defense asserted that, in establishing the pumping plant and thereafter operating it, the defendant was exercising a governmental function delegated to it by the State, and, therefore, it was not responsible for the misuser of its delegated power. The Appellate Court affirmed a judgment for the plaintiff on the ground that the delegation of authority must be exercised in conformity with private rights, and if their destruction be involved, in such a case, payment must be made.*

The law has also been clearly stated by the nation's highest tribunal:†

"There are many lawful and necessary occupations which, by the odors they engender, or the noise they create, are nuisances when carried on in the heart of a city, such as the slaughtering of cattle, the training of tallow, the burning of lime, and the like. Their presence near one's dwelling-house would often render it unfit for habitation. It is a wise police regulation, essential to the health and comfort of the inhabitants of a city, that they should be carried on outside of its limits. Slaughter-houses, lime-kilns, and tallow-furnaces, are, therefore, generally removed from the occupied parts of a city, or located beyond its limits. No permission given to conduct such an occupation within the limits of a city would exempt the parties from liability for damages occasioned to others, however carefully they might conduct their business."

It is encouraging that the Courts thoroughly recognize the importance of maintaining the air free from pollution. In a renown case the State of Georgia filed a bill in equity in the United States Supreme Court to restrain a corporation maintaining and operating its plant in the State of Tennessee from permitting noxious odors and gases from escaping therefrom and being carried by the winds over the forests, orchards, and crops of Georgia, causing destruction.‡ In granting the desired relief, the Court wrote:

"It is a fair and reasonable demand on the part of a sovereign that the air over its territory should not be polluted on a great scale by sulphurous acid gas, that the forests on its mountains, be they better or worse, and whatever domestic destruction they have suffered, should not be further destroyed or threatened by the act of persons beyond its control, that the crops and orchards on its hills should not be endangered from the same source."

This case is also noteworthy in that it not only recognizes the jurisdiction of the Federal Courts to entertain in equity the petition of a State in its quasi-sovereign capacity, but it establishes a more liberal system of law where the State is a suitor. As to these features of the case, the opinion reads:

* Gordon *vs.* Village of Silver Creek, 127 App. Div. (N. Y.), 888.

† Baltimore & Potomac R. R. Co. *vs.* Fifth Baptist Church, 108 U. S. 317, 334.

‡ Georgia *vs.* Tennessee Copper Co., 206 U. S. 230, 238.

"This is a suit by a State for an injury to it in its capacity of quasi-sovereign. In that capacity the State has an interest independent of and behind the titles of its citizens, in all the earth and air within its domain. It has the last word as to whether its mountains shall be stripped of their forests and its inhabitants shall breathe pure air. It might have to pay individuals before it could utter that word, but with it remains the final power. * * * Some peculiarities necessarily mark a suit of this kind. If the State has a case at all, it is somewhat more certainly entitled to specific relief than a private party might be. It is not lightly to be required to give up quasi-sovereign rights for pay; and apart from the difficulty of valuing such rights in money, if that be its choice it may insist that an infraction of them shall be stopped. The States by entering the Union did not sink to the position of private owners subject to one system of private law. This Court has not quite the same freedom to balance the harm that will be done by an injunction against that of which the plaintiff complains, that it would have in deciding between two subjects of a single political power."

Upon the foregoing authority, the State of New York through the Attorney General in 1916 instituted actions in equity in the Federal Courts to restrain certain corporations operating in Edgewater, N. J., opposite Riverside Drive, New York, N. Y., from discharging over said city, quantities of noisome and disagreeable smoke, fumes, effluvia, and noxious and poisonous vapors and gases, thereby endangering and injuring the comfort and health of its citizens. These actions were the culmination of investigations conducted by the Health Department of New York State, commencing by order of the Governor in 1908 and carried on almost continuously until the beginning of the litigation. The reports rendered upon these investigations disclosed that, because of the fumes and gases cast over from the New Jersey shore, the air was quite generally badly contaminated over that section of New York bounded by Central Park West and the Hudson River and from 72d to 135th Streets. The State of New York thereupon made demand on the State of New Jersey to abate the nuisance, and, no change in conditions being apparent, the bill referred to was filed. The defendants named were the Bulls Ferry Chemical Company and the Corn Products Company, but the Barrett Manufacturing Company, Valvoline Oil Company, and General Chemical Company were also contributing to the nuisance. Definite action on the part of the State of New York, strengthened by the decisions of the United States Supreme Court, produced immediate relief.* The defendants asked for delay in order to make arrangements to abate the nuisance and, within a year, the defendants and the other offenders had remodeled their plants and installed new methods of manufacture and mechanical appliances, so that the nuisance was practically abated. The cases were never tried, but it was the force of the law which actually caused the suppression of this notorious nuisance.

The decisions referred to in this discussion indicate that, except where constitutional or property rights outweigh public necessity, the Courts will prevent the pollution of the air either through the inherent power vested in them under the common law or by virtue of legislative mandate. The law seems

* A contributing cause was Chapter 292, Laws of New York 1917, authorizing the forfeiture of the charter of a domestic corporation and the revocation of the certificate of authority of a foreign corporation to conduct its business in New York State whenever it appears that such corporation is maintaining a nuisance.

to be adequate to protect the public welfare and enjoyment of property, but resort to the Courts for relief appears to be on the wane for the reason that the skill and ingenuity of the Engineering Profession are eliminating the causes which contributed so largely to the undesirable and unhealthy odors and vapors permeating the air in and about cities and villages. Modern sewage disposal, septic closets, garbage incinerators, smoke appliances, refuse reducers, and manufacturing methods designed to absorb or eliminate disgusting and obnoxious smoke, gases, and effluvia are making communities cleaner and healthier and removing any reasonable excuse for pollution of the air. To the engineering fraternity great credit is due, as their solution is the elimination of nuisances fouling the air without injury or harm to any one, whereas the suppression of nuisances through law must necessarily cause inconvenience and expense to the parties besides loss of full exercise of liberty and property rights.

METHODS FOR DETERMINING THE ORIGIN, PREVALENCE, AND EFFECT OF OBNOXIOUS ODORS AND THE EVALUATION OF AN ODOR NUISANCE

By Stephen DeM. Gage,* Esq.

During 1920-21 the writer was commissioned to make an investigation of an alleged nuisance caused by certain oil refining plants in the Metropolitan District of Providence, R. I. In planning this investigation it had to be borne in mind that although it was desirable to obtain as full and complete information as possible on all correlated points, only a very limited appropriation was available for the purpose.

Before starting the investigation, therefore, a search was made through health and sanitary journals and reports to ascertain what methods had been followed in other investigations of similar import. These researches proved of little assistance, however, as descriptions of methods used were omitted from the few reports on odor nuisance investigations that were found. It was necessary, therefore, to rely largely on initiative and to formulate methods of investigation. This paper outlines the writer's idea of how such an investigation should be conducted and points out some of the difficulties which may be met and the manner in which they may be overcome for the guidance of those who may be called upon to conduct a similar investigation.

A complete investigation and report on an odor nuisance should cover the following points:

(1) The origin of the odors, and the cause thereof.
(2) Degree or magnitude of the nuisance. Under this head are included the area and population affected, the frequency and duration of the odor manifestations, and the intensity of the odor at different times and different places.
(3) Effect upon health.
(4) Direct and indirect damage to property.
(5) Remedial measures.

Evaluation of an Odor Nuisance

For purposes of record and also for use in legal proceedings it is desirable to have some means of expressing quantitatively the degree or magnitude of the nuisance caused by odors arising from any specific source. No one will question the fact that an intense odor is more of a nuisance than a mild one. A faint disagreeable odor which persists over considerable periods of time or which is of frequent occurrence must be considered a greater nuisance than a strong odor which occurs only at long intervals, or which lasts only a few minutes at a time. Furthermore, an odor, no matter how offensive it may be, is a nuisance only when there are people to smell it. A strong disagreeable

* Chf. Chemist and San. Engr., Rhode Island State Board of Health, Providence, R. I.

odor which can be smelled by only a small number of people must, therefore, be considered less of a nuisance than an odor of less intensity which spreads out over a large and thickly populated territory. The quantitative expression of the magnitude of the nuisance should therefore include four variables, intensity of odor, frequency of odor, duration of time odor is offensive, and population affected.

If each of these variables be expressed quantitatively, their product should furnish a more representative idea of the magnitude of the nuisance than can be obtained by simply comparing the various figures individually.

The frequency may be represented in such a formula by the number of days the odor occurred in a year or some other stated period. As it is not always convenient to continue odor observations for any fixed period, it is probably better to express frequency as the ratio between the total days when the odor occurred and the total days included in the period of observations.

Duration may readily be represented by the average number of hours per day which the odor lasted.

Population affected might be expressed in direct figures, but in order not to have too many figures in the final result, it is better to express population in thousands.

For odor intensity, there is as yet no generally accepted unit of measurement. This might satisfactorily be expressed as parts per million of hydrogen sulfide or some other substance found to be a common component of a number of different odors, or by a factor representing two or more such substances. This combination of factors may be expressed by the formula:

$$M = \frac{D}{T} \times \frac{H}{T} \times \frac{P}{1\,000} \times I = \frac{D\,H\,P\,I}{1\,000\,T^2}$$

in which,

M = the magnitude of the nuisance;
T = the total days of observation period;
D = the number of days odor was noted;
H = the total hours of odor;
P = the population affected;
I = the average intensity of odor.

In compiling the results obtained in the Providence odor survey, no reliable data on "duration of odor" were available and the records of "intensity of odor" determined by the sense of smell were not considered sufficiently accurate to warrant their being used in a formula in combination with other dissimilar data. It was impossible, therefore, to compute the "magnitude of the nuisance" caused by each source. There were reasonably accurate data on the area and population affected, however, and by combining the daily reports of the special observers with the more or less erratic reports from a considerable number of volunteer observers, fairly representative figures were obtained on frequency of odor. As at least eight odor-producing sources which were causing more or less of a nuisance were under observation, it was desirable to have some method of expressing the relative degree of the nuisance originat-

ing from each source. For this purpose the following simplified formula was used to obtain a semi-quantitative expression of the relative degree of nuisance:

$$N = \frac{P\ D}{100\ T}$$

in which,

N = the degree of nuisance;
P = the population affected;
D = the number of different days on which odor was reported from that source;
T = the total number of days on which odor observations were made.

The value of combining even these incomplete data into a single expression is illustrated by Table 1.

TABLE 1.—Relative Degree of Nuisance Caused in Metropolitan Providence by Odors from Various Sources.

Source of odor.	Total days covered in observation, T.	Number of days odor was observed, D.	Population affected, P.	Relative degree of nuisance, $N = \frac{P\ D}{100\ T}$.
Oil Refinery No. 1	148	53	87 700	283.0
Oil Burning Plant No. 1	158	40	32 650	83.1
Oil Refinery No. 2	121	43	21 200	76.0
Tar Products Manufacturing Plant	158	16	6 800	6.8
Gas-works	148	6	11 100	5.0
Oil Burning Plant No. 2	158	16	2 200	2.2
Slaughter-house	158	6	4 700	1.8
Oil Burning Plant No. 3	158	11	1 850	1.3

Selection of Observers

In determining the source, prevalence, intensity, etc., of an odor nuisance chief reliance should be placed on the observations of trained persons. The reports of complainants are usually so biased that they are likely to be untrustworthy, whereas the reports of volunteer observers are likely to be so erratic or so irregular that no complete record of cause and effect can be deduced from them. Volunteer observers, however, may be used in order to supplement the data obtained by the paid men. In the Providence investigation, complainants and other interested persons were supplied with return envelopes and blank forms on which they were requested to report location, date, and time that odors were prevalent. These reports, although they were of little statistical value, served as a check on the regular observers and were also of considerable assistance in plotting the distribution of odors at certain times.

As the acuteness of the olfactory sense varies widely in different persons, special pains must be taken to insure that reports of different observers are comparable in an investigation of this kind. In selecting observers, care must be taken to obtain men whose sense of smell is reasonably acute, and then to

train all these men to record odors of the same intensity in the same manner. Engaged in this investigation were two men who, through long association, had learned to report odors on waters on the same base. All observers were trained by these men in methods of expressing odor intensity. As the investigation progressed, the work of each observer was checked at intervals by one of these men, and, in addition, two or more observers were occasionally sent out together in order that they might check each other.

Vagaries of Odor Characteristics

In attempting to trace odors back to their source, due regard must be given the fact that there may be a decided change in the characteristics of the odor with varying dilution. This variation in character proved for a time particularly troublesome in the Providence investigation. In one section a distinct odor resembling scorched sugar was at first attributed to a large bakery located near-by, but was eventually proved to be derived from the incomplete combustion of oil high in sulfur at a power plant some distance away. Near their source, these same odors when strong smelled like rotten eggs or hydrogen sulfide, whereas at a little distance they resembled burning rubber. The reason for these varying characteristics could not be determined. Probably these particular odors were a mixture of different substances some of which were more overpowering or had greater carrying properties than others. It is well known that some apparently simple odors change materially with dilution and that a blend may have entirely different properties from any of its components.

Peculiarities of Odor Travel

Knowledge of the conditions affecting odor travel is far from complete and peculiarities will frequently be observed which are difficult or impossible to explain. If there is little or no wind, odors usually travel very slowly from their point of origin. On the other hand a strong wind appears to dissipate them. In the experience of the writer an odor nuisance is most likely to occur under the condition of what are generally known as light airs, that is, at times when the velocity of the wind is between 5 and 15 miles per hour. In the Providence survey, an odor was observed in only one instance when the recorded wind velocity was more than 25 miles per hour, and in only nine instances when the wind velocity was greater than 20 miles per hour. In some instances odors travel high and at other times with apparently similar conditions they may travel low. In one instance an odor was traced from hilltop to hilltop for some distance, but could not be detected in the intervening valleys. In certain instances an odor settled in what was probably an air pocket and remained a source of offense for some time after it had ceased to be produced at its source, this condition generally occurring when there was some fog and little wind. Strong odors were also observed on the wind blowing up certain streets while parallel streets only a block distant were practically free from odor. Such apparent freaks of odor travel may have been caused by differences in air currents or humidity produced by relatively small variations in topographical or other surface conditions. Variations of this kind, especially when they are of regular occurrence, may lead to wide differ-

ences of opinion among residents of different parts of an odor-affected area as to whether a nuisance does or does not exist. For this reason all such peculiarities should be noted in an odor survey together with the concomitant facts by which they may be explained.

Use of Meteorological Records

It may not be out of place to record a word of caution about the use of official Weather Bureau meteorological records. The accuracy of these records cannot be questioned and it is not the purpose of the writer to cast any aspersions upon the men who make the meteorological observations. It must be borne in mind, however, that the stations at which they are obtained are at a specific point usually at an elevation and that the records apply only to conditions at that point.

During the Providence odor survey good use was made of official records of wind direction and velocity in checking up reports of odor at times when the paid observers were not on duty. In many cases, however, the reports of the observers in sections within four or five miles of the Providence Weather Bureau Station were at variance with the official record. The Providence station is on the top of a sixteen-story building in the heart of the city, whereas the observers were on the ground. Under the conditions which prevailed when the odor nuisances were most prevalent, that is, when the wind velocity was low, the ground currents, especially in urban districts, are likely to be very different from those which prevail at an elevation of 100 to 200 ft. The writer has frequently noticed wind blowing from one direction in one street when in another street only a few hundred feet distant the direction of the wind was entirely opposite. In many cases observers in districts three or four miles apart would report wind blowing from diametrically opposite points of the compass. There is no question that air currents on the ground may vary in direction from those which prevail at a slight elevation. It is these ground currents, however, which govern the movements of odors.

Humidity is another factor which undoubtedly affects odor travel and considerable variations in humidity may exist at a time of light airs in sections which are relatively close together. The presence of water surfaces in the vicinity undoubtedly affects not only humidity conditions, but also the direction of ground currents during the night. It is possible that tidal movements may also have more or less effect on these ground currents. In the Providence survey it was observed that the direction of the wind often changed at about the same time that the tide turned.

Determination of Odor Intensity

For the purpose of recording odor intensities in the Providence studies, the terms "very faint", "faint", "distinct", "strong", and "very strong", were used. These terms may be defined as follows: A "very faint" odor is one which would be observed by some persons, but would pass unnoticed by the majority. A "faint odor" would be detected by the majority and would be disagreeable to a few. A "distinct" odor is one which would be distinctly noticeable to

almost everybody and would be disagreeable to the majority. A "strong" odor is one which would be objectionable to practically everybody. A "very strong" odor would be decidedly objectionable to all except those with a defective sense of smell.

Records of odors described in this manner are very useful for comparative purposes, but cannot be readily used as a basis for mathematical expression of odor intensity.

As a matter of convenience odors of water are frequently expressed by the numerals, 1 to 5, inclusive, a "very faint" odor being recorded by the figure, 1, and a "very strong" odor by the figure, 5. For lack of a more accurate method these numbers are sometimes averaged to obtain some idea of the mean odor intensity during a stated period. It is apparent from the results obtained by Allinson and Katz in their studies of stench producers, that the concentration of odor judged by the olfactory senses is not proportionate to the amounts of odoriferous substances actually present in the air at the time. There is, therefore, no real basis for assuming that a "faint" or "No. 2" odor is twice as intense, or that a "distinct" odor is three times as strong as a "very faint" or "No. 1" odor. In fact, there is some basis for the assumption that when dealing with simple odoriferous substances the mathematical ratio between these five degrees of olfactory sensation is geometric rather than arithmetic. When dealing with compound odors caused by two or more substances, however, it is quite probable that these ratios may vary considerably, depending upon the nature of the stench producers present.

There is need for exact analytical methods for determining the amounts of stench-producing substances in air. Unfortunately, the amounts of such substances required to cause offensive odors are extremely small and their detection and accurate measurement require the examination of very large volumes of air. For this reason the collection of samples in bottles or gas bags as is customary for ordinary purposes of gas analysis is not practicable. The practice of drawing or forcing the air through the test solution, as used in some types of air-analysis apparatus, is also impracticable for field work owing to the fact that air movement through the absorbtive medium must proceed very slowly and a rather lengthy stop at each sampling point is thus required when large volumes of air must be examined.

Attempts have been made to adapt to purposes of chemical air analysis the apparatus devised by Palmer of the New York State Ventilation Commission for collecting the dust from considerable volumes of air. In this apparatus the liquid used as an absorbent is broken into a fine spray by the inflowing air and conditions are thus made favorable for absorption of the gaseous ingredients from the air. In operation, the air flow through the absorption bulb is maintained by a small motor-driven fan, measurement of volume being obtained by passing the air through a standard orifice under constant pressure.

The great advantage of this form of apparatus is that large volumes of air can be passed through it in a relatively short time, thereby permitting

the determination of extremely minute quantities of gaseous and other ingredients. Weston used this apparatus in his investigations at Fall River, Mass., and reports that small amounts of H_2S and other sulfides could be determined with reasonable accuracy. The chief objection to the apparatus in its present form is that the motor requires the use of 110-volt current which limits its use to points where connections can be made with the domestic lighting circuit. It has occurred to the writer that if the motor were wound to be operated by current from a storage battery, the apparatus could then be used at any point which could be reached in a motor car, and possibly the battery charge could be maintained from the generator of the starting and lighting system. Weston's assumption was that the foul smell of the odors which he was studying was largely due to hydrogen sulfide or other sulfur compounds. It is quite possible that this assumption may also be true of obnoxious odors from sewage treatment plants and perhaps other sources. There is no reason, however, why many other chemical ingredients of air should not be determined by the use of the Palmer apparatus. Knowledge of the chemistry of odors, particularly those that are obnoxious, is very meager and there is a broad field for research work to determine what chemical substances are present and to provide suitable methods for their estimation. As has already been stated, the measurement of odors by the olfactory sense is far from accurate, and methods for exact measurements of intensity are necessary if odor surveys are to be placed on a definite basis.

Determination of Prevalence of Odors

The determination of the prevalence or frequency of odors in order that the nuisance caused may be evaluated by the formula previously given necessitates that a sufficient number of observers be on duty to cover the affected district at all hours of the day and night. A man in an automobile with windshield and windows open can make odor observations at frequent intervals over a considerable territory. It may be objected that exhaust odors from the automobile may tend to cause errors in the observations, but if the car is driven at a fair rate of speed, the exhaust odors are left so far behind that they have no practical effect. In the Providence investigation, the observers traveled on bicycles in order to avoid this theoretical error, but in a supplementary investigation made by Dr. Chapin, Health Officer of Providence, the use of motor cars was found to be a distinct advantage.

Information as to the prevalence of odors in various parts of the odor-affected districts was derived both from observations made each night by regular employees and from reports sent in from time to time by volunteer observers living in those districts. Whenever possible, the regular employees traced odors which they noted to their source of origin, and from their reports it was possible to determine both the total frequency of odor during the middle night hours, and the relative responsibility of the odor-producing sources. The reports of the volunteer observers served as a check on the reports of the regular observers and also furnished information as to the occurrence of odors at other times than those when the regular observers were on duty. As might be expected, reports from volunteers were received at irregular

intervals when the visitation of odors was especially marked, and except in one or two instances no attempt was made by these observers to keep a complete record of the occurrence or non-occurrence of odor. In many of these reports it was stated that odors came from certain specified sources, although as a rule the volunteer observer did not trace the source of odor. However, the probable source of odor reported by volunteers could frequently be estimated by a study of the direction of the wind at the time and comparison with other contemporary reports. The volunteer reports, therefore, although of relatively small value individually, did furnish collective evidence of the frequency of odors which proved to be of considerable statistical value.

Determination of Affected Area and Population

There are often times when obnoxious odors are much more intense and affect a much wider area than usual and a special effort should be made to determine the spread of odor under the particular conditions of wind and weather which prevail at such times.

In the Providence investigation, a number of special surveys were made to determine the limits of the territory affected by odors at particular times. The method used was to follow the odor in one direction as far as it could be detected and to record this point. The car was then run some distance farther and turned back toward the odor area, and the point at which the odor could again be detected was recorded. After running well into the odor area and recording the odor intensity at various points, another turn was made and the odor limit at another point was determined. This procedure was repeated until a sufficient number of points had been determined to permit the odor-affected area to be plotted accurately on the map, together with data as to odor intensity at various places. By running outside the odor area at frequent intervals the sense of smell of the observers was revivified and the car freed from odors, thus facilitating the determination of the limits of the odor-affected area and the distance of odor travel.

Theoretically, the area which may be affected by odors from any source is a circle the radius of which is the maximum travel distance of the odors from that source. Owing to the effect of prevailing winds, topographical conditions, and other factors which are not clearly understood, odors usually travel farther in certain directions than in others. The map of the area actually affected, therefore, is likely to be irregular in shape, with a large proportion on one side of the odor source. By platting all the locations at which odors have been noted at various times on a map of the affected sections, diagrams can be made showing the total area subject to influence from any source, and this area can then be measured. In computing the extent of such areas, water surfaces should usually be eliminated, because in a study of this kind, it is the land surfaces with which one is principally concerned. If the same territory is subject to visitation of odors from two or more sources, separate plots should be made of the area affected by each odor source. Maps with the affected areas blocked out furnish a picture of the effect of any odor-producing source which can be obtained in no other way,

and are invaluable, both as a matter of record and as evidence in case action through the Courts is necessary.

From a nuisance viewpoint, it is the population of an odor-affected district, rather than the area itself, which is of major importance. There are a number of ways by which the population of such an area can be estimated with reasonable accuracy. When a house-to-house canvass is made for any purpose as part of the investigation, the number of inhabitants in each house should be ascertained and recorded as a basis for a population count. When it is not feasible to obtain a direct population count recourse must be had to estimates. When the area affected is residential, and not too large in size, a count of the number of houses can be made. Two persons, in an automobile, can cover a considerable territory in a few hours counting and recording the number of houses on each side of the various streets. In such a count, the object is to determine the number of families and a two-family house should be counted as two. Even in closely populated tenement or apartment districts, a close count of the number of families can quickly be obtained in this manner. In some cases, the original street by street data obtained in a special census are available, or the local tax assessors have data which can be used. In a good residential district or in a high-class apartment district it is usual to assume that the average family consists of four persons, but in closely built tenement districts this factor should be increased to five, and in densely populated sections occupied largely by people of foreign birth the factor may be six or even more persons per family. In many communities an enumeration of children of school age in each school district is made every year, and knowing the ratio of school children to total population, a fair estimation of the population of any section can be made from such data. In Providence, where the affected areas were large and thickly populated, and where the influence of various odor sources frequently overlapped, the population estimates were based on the counts obtained by enumerators in different districts in the census of 1920. In arriving at these estimates, the odor-affected and the census-enumeration districts were both plotted on the same map. After blocking out territory known to be uninhabitated, the proportionate part of the census district which lay within and without the odor-affected district was then determined with a planimeter and the census population distributed accordingly. The total population of the odor-affected area, obtained by adding the sectional figures as determined by the method described, was then corrected for the presumed increase in population since the census was taken. In this method of computation it was necessary to assume an even distribution of population within the inhabited portions of the border-line enumeration districts, and also that growth of population was the same in all such districts. In a few selected localities the population estimated by this method was checked by a house or family count and the figures obtained by the two methods were found to be in reasonably close agreement.

When the odor-affected area contains factories and large mercantile establishments the employees of which are subjected to the odors during working hours, the average number of such employees should be ascertained and a

correction made in the total population affected. This is contrary to census-enumeration practice in which only resident population is counted in order to avoid duplication. In an odor survey, however, it is probably fair to assume that approximately one-half the employed population lives outside the odor-affected area, but as these people are subjected to the odors during working hours they should be included in the estimate of population affected.

Effect on Health

Although it is generally admitted that chemical fumes emanating from various industrial processes may cause illness and even death, it is a disputed point as to whether offensive odors *per se* can actually be considered unhealthy. Nevertheless, the claim will usually be made by complainants that their health is being unfavorably affected by offensive odors and it may also be claimed that the symptoms of persons ill from other causes are being seriously aggravated as a result of the nuisance and that their recovery is thereby being delayed or prevented. For these reasons, therefore, an investigation of a major odor nuisance cannot be considered complete until all the facts bearing on its possible effect on health have been collected and correlated. This part of the investigation should be carried out by a physician or at least by one who has had sufficient training in physiology and public health to determine to what extent the symptoms of illness are psychological and to what extent they are actually physical.

Information relating to the health phase may be obtained by a house-to-house inquiry throughout the odor-affected district or by inquiry only among persons who are reported to have been made ill. Frequently an inquiry in one family where illness of this kind is reported will bring forth information about members of other families who have been similarly affected. When feasible the house-to-house canvass is preferable as it is thus possible to obtain definite statistics as to the proportion of the population in whom symptoms of illness are being produced. The inquiry among residents should be supplemented by an inquiry among physicians practising in the affected area to obtain all possible information as to the effect of the odors on patients under their care. If by chance there should be a hospital in the odor area, an effort should be made to enlist the aid of the hospital authorities in the investigation to the end that the effect of odor visitations on various patients may be noted and made a part of the case history. In studying the health phase of this question and particularly in making inquiries among both physicians and laymen, it must be constantly kept in mind that definite facts and not opinions are desired and also that to the layman nausea, sick headache, vertigo, malaise, etc., are real illnesses, irrespective of whether the cause is physical or psychological.

Injury to Property

It has long been recognized by the Courts that property damage may be caused by odors or fumes. Injury of this kind may be either direct or indirect. Direct injury to property may result from damage to buildings, household furnishings, etc., or to trees, shrubbery and other vegetation. Indirect injury

may result from the fact that owing to the existence of a nuisance and the feeling of the people that the locality is unhealthy the property becomes less desirable as a residence or work place and its sale value is therefore depreciated. Although direct or indirect property damage is often considered outside the jurisdiction of sanitary authorities, proof of its existence is valuable as an argument to induce the owner of the source from which the odors or fumes originate to take measures to abate the nuisance. In an odor survey, therefore, it is wise to collect and record all obtainable information concerning both direct and indirect injury to property.

When buildings which have been painted with lead or zinc paints are exposed to fumes containing any appreciable amounts of sulfides, discoloration is likely to ensue. This action is usually more rapid on freshly painted surfaces. Household silver, automobile fittings, etc., may also be tarnished by sulfides. Complaints of such troubles, which at first may seem impossible, on investigation will often bring out information of a surprising nature. Information as to damage to paint may also be obtained by inquiry among master painters and paint dealers. Exposure to acid fumes may affect vegetation more or less seriously and also cause rapid deterioration of sheet metal, textile fabrics, paper, etc. Sometimes inquiry among roofers and sheet metal workers will yield information regarding excessive corrosion of metal roofs, conductor pipes, cornices, etc. Damage to trees can readily be detected by inspection in summer, either by discoloration and withering of foliage or by premature falling of the leaves. As not all vegetation is affected alike, care must be used before a report is made that such damage does not exist.

Indirect injury to property by depreciation of value is somewhat more difficult to estimate. When a nuisance has been of long duration, its effect may sometimes be observed by comparison of changes in valuation in property in different sections on the books of the tax assessors. Information about changing property values may often be obtained from real estate brokers. The financial aspect of a major odor nuisance is usually not appreciated either by city and town officials or by the public. As practically the entire revenue of the community is derived from taxes assessed on property, any material reduction in taxable valuation in one section must be compensated either by increased valuation in other sections or by a general increase in the tax rate. Every taxpayer in the community is, therefore, affected irrespective of whether or not he owns property in the nuisance area, and the presentation of the financial evidence of this nature can be used to influence public sentiment outside the odor-affected area in cases when municipal authorities are slow to take necessary action.

Remedies

This paper has dealt only with methods which may be followed in determining the cause, scope, and effect of an odor nuisance with the object of presenting all the facts clearly and forcibly to the Court or other official body which may be called upon to abate the nuisance. In order that such a report may be constructive, rather than destructive, the investigation should also

include a study of the conditions which prevail in the industrial plant or other source where the offensive odors originate, particularly to ascertain whether the emission of odors cannot be eliminated by changes in the processes or in the plant or whether the offensive character of the odors or fumes cannot be overcome by some method of treatment.

The scope of this paper does not include a discussion of methods of odor elimination. Suffice to say in closing, however, that an industry or other source which is the cause of a major odor nuisance is usually also a financial asset to the community in or near which it is located, and although it is not justifiable to consider such a nuisance as a necessary evil, it certainly is not justifiable to close down such a plant when the nuisance may be abated by the application of remedial measures.

ELIMINATION OF ODORS FROM GARBAGE DISPOSAL WORKS

By Samuel A. Greeley,* M. Am. Soc. C. E.

The disposal of garbage is an operation in which an odor hazard must be fairly recognized. Unless garbage disposal works are designed, built, and operated with care, odors may result. In fact, with some methods of disposal, odors are likely to result unless the design and operation are good. In this regard, garbage disposal is somewhat akin to the odor-producing industries such as tanneries, packing houses, glue works, oil refineries, rendering plants, etc. Measures for odor elimination thus become pertinent.

A number of factors influence the general problem of the elimination of odors from garbage disposal works, one of which centers about the environment. In some locations, some odors apparently are not objectionable. In other locations, the same or less intensive odors might cause a nuisance. In recognition of this fact, the zoning map of Kansas City, Mo., indicates areas where odor-producing industries may be located.

Another factor is the cost of operation. Both construction and operation costs may be increased as odors are eliminated. If the disposal is by reduction, rates of drier operation may have to be curtailed and processes adopted for odor elimination which do not produce additional revenue and which add to the cost of disposal. If it is done by incineration, additional fuel, larger capacities, and sturdier construction may be required.

Other factors relate to the many phases of garbage disposal as a municipal enterprise, including group habits and viewpoints, the efficiency of the operating staff, the financial resources and budget allowances, competitive conditions in the field of garbage plant installations, patent limitations, etc. All these factors are closely interwoven. Local considerations are often justly important and cannot be entirely discarded. A helpful perspective may be secured by describing a few actual conditions.

In Rochester, N. Y., a garbage reduction plant of the most approved design as regards odor elimination is operated in a commercial and semi-industrial district near the center of the city. A similar plant was built on Staten Island, New York, in relatively open country, but not far from residential districts. The protests against this plant were sufficiently vigorous practically to force its abandonment shortly after it was built. The City of New York, however, operates an incinerator in a commercial district only a few blocks from residence and business sections. In two or three other cities, garbage reduction works are in such isolated locations that apparently reduction with less odor control and lower operating costs is permissible. Thus, from a practical standpoint, the attitude of the community, the relative economies of near-by or distant location, and the over-all costs of disposal by various processes must be considered and carefully balanced.

* (Pearse, Greeley & Hansen), Chicago. Ill.

Status of Odor Elimination

Relatively little progress in the development of methods for odor elimination can be recorded with tangible results expressed in definite terms. Inside feeding has been developed at hog farms, but capacities and designs are not generally understood and applied; and too little attention has been paid to the sanitary disposal of the unconsumed garbage and the hog manure. Fume-consuming grates have been proposed for incinerators and the danger of low temperatures has been commonly recognized, but many designs fail to consider needed capacities for grates, flues, and other parts of the incinerator under peak conditions. At reduction works, odorous gases have been washed with water and chlorinated, with a reduction of odors, but methods of applying the water and chlorine to secure the best efficiencies need further development.

It is apparent that the subject of odor elimination from garbage disposal needs technical study and development with a fuller realization on the part of those purchasing garbage plants that the odor problem is vastly important and is likely largely to control the capacity, arrangement, and cost of the works under design. In this paper the writer hopes to visualize the subject more clearly than has been done heretofore, to outline its fundamental aspects, to suggest means of odor elimination, and to stimulate further progress.

Fundamental Considerations

The problem of eliminating odors from garbage disposal works should not be entirely confined to methods of elimination after the odors have been produced. It is a fundamental consideration that odors which are difficult and costly to eliminate should be prevented or reduced as much as possible. The old adage as to the value of prevention before cure is true in this case. However, both the prevention and the cure should be studied and a well-balanced procedure developed with due regard to efficiency and cost.

There appears to be no common scale for determining permissible odor and, therefore, the location and environment of a garbage plant may be considered in the general scheme for odor elimination, although they affect the nuisance resulting from the odor rather than the odor itself. For instance, if it should be necessary in a town to burn garbage, night soil, screenings, or other such refuse materials with an additional fuel, it may be said that the elimination of odors bears a major relationship to the temperature of combustion and, therefore, to the quantity and cost of additional fuel. In this case, is it better to locate the plant far afield, involving an additional haulage cost, but a lower fuel cost, permissible through a somewhat lower temperature standard?

Thus, a fundamental consideration is the standard of odor elimination required or desirable in each locality. In a general way and speaking in practical terms, odors should be eliminated; but it is of the utmost importance that the economic and essential relationship be realized between the type of plant, its capacity and arrangement, and its location, on the one hand, and the degree of odor elimination and its cost on the other. A straight Cobwell reduction plant is likely to produce less odors requiring elimination than a

modified Cobwell or Cobwell-Merz plant in which the drier gases are added to those of the reducers, but the straight Cobwell plant is likely to cost more. The cost and efficiency of odor-eliminating devices thus become pertinent.

Terms Used

It is not necessary in this paper to define at length the terms used. The paper relates primarily to odors from garbage works. Other classes of municipal refuse such as ashes, rubbish, manure, street refuse, etc., are included only as they enter the garbage disposal problem. Garbage is here used to mean the animal and vegetable waste matter originating in houses, kitchens, restaurants, and hotels, and includes the natural content of moisture, and generally, also, the tin cans in which portions of the food were originally supplied.

Of the various methods of garbage disposal, special reference will be made to disposal by feeding to hogs, by incineration, and by reduction, as these terms are used in the broader sense to mean the process used rather than a particular patented device or arrangement. Hog-feeding and the reduction method are applicable to garbage only, but most refuse materials may be disposed of by incineration.

Comments on Odors

Garbage odors should be characterized or classified so as best to promote and facilitate a study of their elimination, as follows:

1.—Garbage odors as related to the food materials comprised, such as cinnamon, coffee, lemon, burnt meat, and many others. Such odors are characteristic of fresh garbage and might be termed the "fresh garbage" odors. Their predominance is relatively brief, depending on the condition of the garbage after its house treatment, the frequency of collection, and similar factors.

2.—The odors of decomposition. In warm weather, for instance, moist garbage decomposes quite readily with the production of odors which are more or less of a nuisance depending on the character of decomposition. Garbage which is wrapped and drained in the kitchen or mixed with ashes decomposes more slowly than otherwise. Odors from decomposing garbage may come from dirty collection wagons and disposal works.

3.—The process odors, an expression here used to include those odors which result from the various methods of garbage disposal and at times are more or less characteristic of them. Such odors are the "pig pen" odor, the odor of incomplete combustion, the odor of drier gases, etc.

Among these odors, some may be classed as pleasant and others as offensive. Some will be intermittent and others more or less continuous; and some may be seasonal. The odors of decomposition are, perhaps, likely to be more offensive, but less widely distributed than those from gases discharged from a chimney top.

Although this paper is not on the subject of odors, but rather on their elimination, it should be noted that scientific information about odors, *per se*, is likely to help materially in developing improved methods of odor elimination. Much of interest has been published on the subject of odors proper, such

as the interesting paper* by Louis L. Tribus, M. Am. Soc. C. E., and the several discussions of it. The following quotation† is of interest:

"The common belief that the olfactory stimulus consists of minute material particles suspended in the air current of the olfactory organ is supported by the observation that odors may be carried on the wind in a definite direction many miles. Odors do not emanate from a given center and disperse in all directions as sound and light do. Moreover, many substances, such as arsenic, that are odorless under ordinary circumstances, give out an odor after they have been heated sufficiently to volatilize. The fact, discovered in 1917 by Woodrow and Karpman, that the adaptation time for olfaction—the time needed for an olfactory sensation to wane completely—is directly proportional to the vapor tension of the odorous material shows that olfactory stimulation is due to the activity of gaseous particles. These and other like observations have led to the conclusion, now generally accepted, that the olfactory organs are normally stimulated by material particles, and not by disturbances of a non-material character.

"Some odorous bodies, such as musk, are well known to give out these material particles for a very considerable time without appreciably changing weight. From the standpoint of a receptor this indicates that olfaction is called forth by an infinitesimally small amount of substance, and measurements directed toward testing this question justify the conclusion. These measurements have been made in a variety of ways."

Facts about the chemical and physical characteristics of odors and odor-producing substances, although relatively meager in the garbage field, are, however, of considerable import in a study of odor elimination. Thus, Parker suggests that certain olfactory stimuli are more soluble in oil than in water and that, in fact, solubility in oil may be "of much more significance for olfactory stimulation than solubility in water". This further suggests that "scrubbing" gases from garbage plants with oil as well as water might prove advantageous.

Parker also indicates that olfaction is essentially a chemical process and that some progress has been made in discovering relations between chemical structure and olfactory sensation. Thus, "* * * in a general way types of chemical constitution can be indicated for the six groups of odors that he [Henning] was able to distinguish. * * * In the burnt odors, the ring is smooth as in pyridin, and in the foul odors the ring is fragmentary as in cacodyl". These comments and quotations are stimulating and show the possibilities before engineers in the understanding of the odor-elimination problem.

Sources of Odors in Garbage Works

The sources and causes of odors in garbage works may be related to the several parts of the works as follows:

(*a*).—*The House Treatment.*—Odors may come from accumulations of garbage in the can or house receptacle and from accumulated piles of garbage spilled from the can. In some apartment houses, the garbage is spouted to the ground and these spouts may become foul.

* "Odors and Their Travel Habits," *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 378.

† "Smell, Taste and Allied Senses in the Vertebrates," by George Howard Parker, 1922, p. 49.

(*b*).—*The Collection.*—If the collection wagons are not cleaned, or are difficult to clean, they may become quite odorous. Leaky and overloaded wagons may spread odor-producing material along the route. A proper covering of collection units is desirable and may be important.

(*c*).—*The Transportation.*—In cases of long haul, the garbage may be transported from a transfer station to the disposal works in scows, freight cars, or large trucks. Poorly designed and maintained transfer stations with inadequate facilities for washing are likely to produce odors. Such conditions may be aggravated if the garbage is not removed from the transfer station frequently.

(*d*).—*The Final Disposal.*—Garbage disposal works may cause odors through design and construction in the first place and neglectful operation in the second place. Such factors as the capacity and general arrangements of the plant are important.

To secure relative freedom from odors, these several sources should be appreciated and operations conducted to prevent odors as described more fully subsequently.

Effect of House Treatment

Local odors may result from poor house treatment and odors may be spread along collection routes if the interval between collections is too long, especially in warm weather. If the house receptacles are not cans with good covers, but are baskets, boxes, or barrels, they may leak and foul the ground on which they stand, or they may overflow, if they are too small. Local odors may result. A good method is to have the garbage drained and wrapped before it is placed in the can, but this method is not generally applicable to disposal by reduction or hog-feeding. It is practiced in Minneapolis, Minn., Milwaukee, Wis., Trenton, N. J., and elsewhere, and it facilitates incineration. In Madison, Wis., the garbage is wrapped by the housekeeper and then unwrapped at the hog farm before feeding, at a cost estimated at $2 500 per year. This, however, may result in a fresher garbage at the hog farm.

Effect of Collection

Unclean garbage wagons will carry odors along their route. Such odors are noticeable 100 ft. or more from the wagons. They can be eliminated by the selection of a type of wagon easily cleaned, with a good cover, and by washing the wagon at regular and sufficiently frequent intervals. A good type of cover is of canvas arched over the top of the wagon and arranged in sections which can be thrown back during filling.

The European practice is to wash the wagons every day and this practice is growing in the United States, particularly during the summer months. Washing the wagons on alternate days will materially reduce odors, but will not eliminate them. With only two washings a week, odors are likely to develop. At Milwaukee, the estimated cost at pre-war prices of having a man scrub wagons with water and a broom was about 4 or 5 cents per wagon.

Odors in the collection wagons and in disposal works will be reduced if the garbage is collected at sufficiently frequent intervals. A common practice

in this country is to collect once a week in winter and twice a week in summer, but this is hardly sufficient, and a collection every other day in summer is often desirable. In some cities, especially in England and Europe, garbage is collected daily at least from the business districts. This is a good practice if the garbage is to be fed to hogs. Additional collections add materially to the cost but they are often justified by the better service to housekeepers. If a collection unit can visit 250 houses in a working day, each unit will serve 750 houses with two collections per week, and with less garbage per collection probably somewhat more than 500 houses with three collections per week. Thus, more units will be required to make the more frequent collections, entailing an increased cost of less than 50% during the period of more frequent collections.

The collection of garbage, ashes, and rubbish mixed is practiced abroad and to some extent in this country. From the standpoint of odor elimination it is a good practice, but it limits disposal to mixed incineration except where sanitary fill may be possible. If dumps for ash disposal are relatively easy of access, mixed collection is likely to cost more than separate collection, because the former requires a more frequent collection of ashes than householders ordinarily require.

Effect of Disposal

Elimination of odors from garbage disposal works centers chiefly about the disposal plants proper. The sources of such odors may be stated as follows:

(*a*).—Odors from the raw garbage and its storage and handling about the plant. If the plant capacities are such that garbage must be stored at the plant for several days during peak seasons, odors will result. The age of the garbage as delivered to the plant also affects the odor situation. Lime, creosote, and other cleansing and deodorizing substances may be used to advantage. Provision should also be made for cleaning and washing the plant.

(*b*).—Odors may come from the processes. Thus, in hog farms, an incomplete disposal of hog manure and unconsumed garbage may cause serious odor troubles. At incinerating plants, incomplete combustion may permit odorous gases to escape from the chimney top. In the reduction process odors may come from the vapors escaping from open grease-skimming tanks, from hot materials when exposed, and from leaks about piping and tanks. Odors may also come from the digester vent gases and from the drier gases.

(*c*).—Some odors may come from the products of disposal, as, for instance, half burned garbage from incinerators delivered to ash dumps; or the accumulation of tankage, tailings, and sewage at a reduction plant.

The extent of odors from garbage disposal works is influenced more than is realized by details of design and construction. In Reduction Plant A, for instance, floors may be of concrete, piping may be easily kept tight, the capacity may be sufficient so that no garbage is held in storage, there may be sufficient driers so that scorching and burning of the garbage is negligible, and the plant as a whole may be kept clean and in good repair. Contrast such a situation with Reduction Plant B, where floors are rough and uneven, where in the summer heavy deliveries of garbage may be stored at the plant for several

days, where inconvenient arrangement renders pipe maintenance difficult, where driers are forced to operate at high temperatures, and where slovenliness prevails. The elimination of residual odors is likely to be much more difficult at Plant B than at Plant A.

Composition of Garbage and the Gases from Disposal

Little appears of record as to the characteristics and quantities of the gas and odor-producing components of garbage. Most of the analytical data relate to the composition of the garbage as regards incineration, reduction, or feeding to hogs. Thus, an average analysis of garbage in Washington, D. C., for 1915 is given in Table 2.

TABLE 2.

Item.	Percentage by weight.
Moisture	73.80
Combustible	22.50
Ash	3.63
Ether extract	5.12
Potash, as K_2O	0.28
Phosphoric acid, as P_2O_5	0.39
Nitrogen	0.71

An average analysis of garbage in New York, N. Y., from dry composite samples taken in 1905 and 1906 is given in Table 3.

TABLE 3.

Item.	Percentage by weight, dry basis.
Carbon	43.10
Hydrogen	6.24
Nitrogen	3.70
Oxygen	27.74
Silica	7.56
Iron oxide and alumina	0.41
Lime	4.26
Magnesia	0.28
Phosphoric acid	1.47
Carbonic acid	0.59
Lead	0.20
Tin	Trace sulfides
Alkalies and undetermined	4.45

Analyses of the products of combustion of mixed refuse incinerators have been made to determine the CO_2 in the chimney gases, but few have been made with garbage alone. Analyses have also been made of gaseous products from garbage reduction plants. The report on the elimination of odors from the Barren Island Disposal Plants (Osborn, 1915), furnishes data for the digester vent gases, as given in Table 4.

In addition, the following constituents were found to be present: Alcohols, acetic acid, fatty acids, fats, essential oils, and aldehydes.

Samples of the drier gases were found to contain 6.2 to 5.1% of carbon dioxide, 12.6 to 14.9% of oxygen, and 2.2 to 1.2% of carbon monoxide. No

sulfur dioxide was found in the drier gases, but acrolein and similar compounds were indicated.

TABLE 4.

Item.	Percentage before entering condenser.	Percentage After Passing Through Condenser.	
		A	B
Carbon dioxide.....	45.2	36.8	6.2
Oxygen.....	10.8	11.8	18.0
Sulfur compounds.....	Present.	Present	Present

It is likely that more analytical data would be helpful in approaching the odor problem, and very likely more are of record. However, so few are readily available that the foregoing are given in the hope that more will be published. It should be stated, however, that odors may be produced by very minute quantities which might not be shown by an ordinary analysis.

Incineration of Garbage

Odors from incinerating plants probably arise more often from the storage about the plant and from incomplete combustion. There are two general types of incinerators which, in a general way, embrace the various designs, for example, the "outside storage" and the "inside storage" types.

Outside storage incinerators are a development of the mixed refuse incinerators first studied in detail in this country by J. T. Fetherston, M. Am. Soc. C. E., in connection with the installation of two plants on Staten Island, New York, in 1908 and 1913. A considerable volume of technical information is available relating to the operation of this type of plant when burning a mixture of garbage, ashes, and rubbish, and also, but to a less extent, when burning garbage and rubbish only. Rates of burning per square foot of grate area, draft requirements, flue areas, and heat balances are quite well settled. Thus, odors from incomplete combustion can be generally avoided. The outside storage of the garbage makes careful design and operation necessary if odors from this source are to be avoided.

In plants of the inside storage type, a large part of the garbage is dumped into the furnace from the collection units as delivered. Outside storage at the plant is thus largely eliminated. However, steady uniform combustion is more difficult and lower rates of burning appear to be necessary. In this type of plant, safe maximum loadings have not been so definitely fixed for continuous complete combustion for the various designs.

Fig. 1 shows an estimated quantity of coal required per ton of garbage to maintain a temperature of 1 250° Fahr. In this estimate, the composition of the garbage, of the coal, and of the mixture for the yearly average, are given in Table 5.

Assuming 75% excess air and 20% loss of heat by radiation, this mixture results in a temperature of 1 240° Fahr.

All incinerators should have ample combustion and dust-settling chambers. Lack of such chambers increases the odor hazard and allows dust and sometimes paper to be carried up the stack. The combustion chambers at Milwaukee have capacities equivalent to 15.4 cu. ft. per ton of mixed refuse furnace capacity per 24 hours.

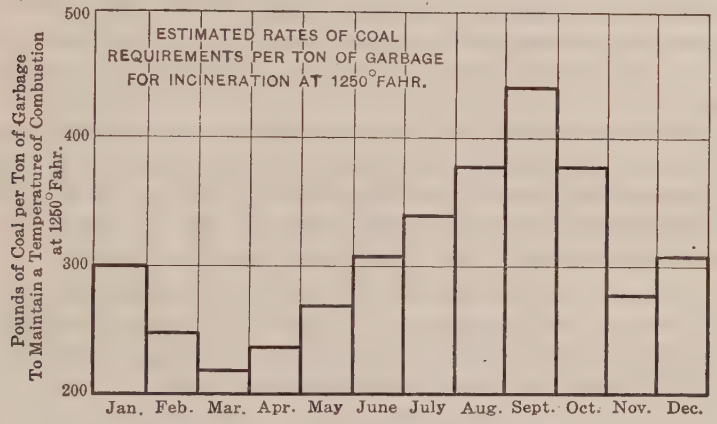


FIG. 1.

Satisfactory incineration over a number of years calls for durable construction with properly designed furnace structures to stand the relatively hard service. With leaky furnace fronts, cracked brick work, and inadequate draft control, cold air enters the furnaces and lowers the effective temperature of combustion.

TABLE 5.

Item.	Garbage.		Coal.		Mixture.	
	Pounds.	Percentage by weight.	Pounds.	Percentage by weight.	Pounds.	Percentage by weight.
Carbon	102	5.09	239	79.62	341	14.8
Hydrogen	61	3.03	14	4.65	75	3.3
Nitrogen	41	2.07	4	1.31	45	2.0
Oxygen	205	10.23	14	4.55	219	9.5
Sulfur	41	2.07	3	1.02	44	1.9
Ash	72	3.61	16	5.39	88	3.8
Water	1 478	73.90	10	3.46	1 488	64.7
Totals	2 000	100.00	300	100.00	2 300	100.0

Reduction of Garbage

The development of garbage reduction in this country has been influenced by the odor problem as well as by costs of operation and market conditions. Several groups of reduction processes have resulted, among which may be mentioned the following:

(*a*).—The cooking group, in which the garbage is first cooked by steam in digesters and then generally pressed and percolated for the quite complete recovery of grease, the waste liquor being evaporated and added to the tankage. In some plants, the process is limited by omitting gasoline extraction of grease or the evaporation of waste liquids, or both.

(*b*).—The drying group, in which the garbage is first dried and then percolated for grease recovery with a further processing of the residual solids for tankage. In this process, there is no cooking and the digester vent gases are eliminated.

(*c*).—The Cobwell group, in which the entire process is accomplished in closed tanks termed reducers. In the reducers, the garbage is treated with solvent, heated, dewatered, degreased, and dried. The process thus offers a minimum of odor hazard. Recently, the garbage has first been dried to increase the general plant capacities and to reduce net operating costs.

The so-called Morrison process has been adopted at Indianapolis, Ind., and Dayton, Ohio, for the preparation of a stock and poultry food and other products from garbage. It is a cooking process, using a steam-jacketed digester in which cooking, pressing, and drying are done.

At garbage reduction works several relatively well defined sources of odors have been noted. First are the garbage odors arising from the delivery and storage of the garbage. Second are the digester odors which are commonly described as having a caramel or burnt coffee odor. The odor of the digester vent gases is apparent at times and is different from the digester odors. Third are the drier odors resulting from the escaping drier gases. These generally have a sweet burnt vegetable odor with occasional whiffs which are acrid and irritating. By condensation, by washing with water, by heating to a high temperature, by treating with chlorine gas, and by removal of dust, these various odors have at times been largely eliminated.

A comparative analysis of these various reduction processes as they effect odors involves so many local factors that general typical operating schedules are not likely to be helpful. Obviously, a completely developed plant of the cooking type, requires more different, separate operations than one of the drying type. In the former method, the cooked garbage is usually pressed to a cake containing from 55 to 60% of moisture of fairly uniform character, and this cake is passed through driers. Under the latter or drying method, green garbage with an average moisture content about 75% is the material dried. There is, however, a wide range of moisture content in the different particles. Thus, although one process eliminates cooking and pressing, the other calls for heavier work by the driers. Therefore, the economies of operation and the likely success of measures for odor elimination are interrelated.

Factors influencing the costs of disposal are fixed charges which reflect the first cost, labor costs, maintenance costs, net sale prices of grease, and tankage after deducting freight, and the cost of odor elimination. This last factor includes the often considerable increase in first cost due to the type of driers, condensers, and other equipment permissible from an odor standpoint, as well as the cost of odor-eliminating devices.

Digester Operation

The cooking of garbage in digesters generates gases and vapors the treatment of which is important in the control of odors. There are a number of different digesters on the market. Some constitute a vertical cylindrical steel tank, generally lined with vitrified brick, with a false bottom over a drain. In these the garbage is cooked with live steam. The period of cooking varies from season to season with the condition of the garbage and, at times, with its age and state of decomposition. A frequent period is 8 to 10 hours. During cooking the excess steam is vented through small pipes to condensers. Some digesters contain arms or paddles which are rotated to stir the garbage during cooking. Water or waste liquors are added to promote the cooking. Other digesters are steam-jacketed and the heat from the jackets cooks the garbage and may be used to evaporate the liquid contents. In other processes, some of the liquids are pressed out by steam and, in some, the pressed material is partly dried by the heat of the steam in the jacket.

The gases and vapors from the digesters usually pass to condensers, where the steam in the vent gases is condensed and a large part of the gases is absorbed by the water. In some plants, the insoluble gases have been trapped and passed into the boiler furnaces, but it has been found that heating these gases to 2 000° Fahr. does not eliminate them, but makes them more irritating by changing the sulfur compounds to sulfur dioxide. When these gases are first condensed and then burned and discharged at the top of the boiler stack, it is probable that air dilution and dispersion is helpful in reducing the odor trouble. Based on studies by Osborn, Crowe, and Klein, it was concluded that by first heating the insoluble digester vent gases to about 1 200° Fahr. and then passing them through water all odors were eliminated. The extent and character of these odors have not been fully related to the condition of the green garbage and to the schedule of operating the digesters.

Drier Operation

As regards odors, the type and design of drier and the rate of firing and of feeding garbage or press cake are important. With direct heat driers in which the gases and flames of the fire pass directly into the drier, charring or burning of the garbage may result from contact between the intensely hot gases and the partly dried material. This is more likely to occur if the feeding of material into the drier is not uniform and if its distribution in traveling through the drier is uneven. Charring is also more likely to occur if there is a poor adjustment of the fire to the quantity of incoming material and to variations in its moisture content from time to time and throughout the mass.

A commercial rating for a direct heat drier, 5½ ft. in diameter by 40 ft. long, is the evaporation of 4 000 lb. of water per hour. In some garbage reduction plants, rates of direct heat drier operation have been noted as given in Table 6.

As odors such as acrolein (from burned grease) result from burning or charring the garbage, close control of the drier operation is needed. This requires relatively complete combustion in the furnace so that only hot gases reach the drier. The temperature can be controlled to some extent on the way to the drier by air inlets, brick regenerators, and like factors.

TABLE 6.

Plant.	Water evaporated, in pounds per hour.	Dimensions, in feet.	Approximate Moisture Content, Percentage by Weight.	
			Incoming material.	Product.
Pittsburgh, Pa.	3 400	6 by 40	60	10
Chicago, Ill.	6 500	5½ by 40	75	10
New Bedford, Mass.	3 000	5½ by 40	75	35

With indirect heat driers, the chance for burning the garbage by direct contact with the flames is reduced as the flames and hot gases from the furnace pass around the outside of the drier shell with the material on the inside. If, however, bits of grease and other substances stick to the hot shell, they will burn. In some driers, the hot gases pass first around the outside of the drier and then through the inside with the material to be dried. Such driers were recommended for the proposed plant at Cleveland, Ohio, and were rated at an evaporation equivalent to about 8.2 lb. of water per pound of coal, which compares favorably with a direct heat drier.

With steam driers, the moisture is driven off by contact of the material with the shell of the drier (or steam tubes through the drier) heated by live steam. Thus, the chances for burning are practically eliminated unless the dry material ignites. The rate of evaporation, however, is low, as steam driers require about twice as much fuel per pound of water evaporated. Steam driers are, therefore, less often used in garbage reduction plants. However, under some circumstances, it may be preferable to operate at a slower drying rate and a higher cost in order to reduce the odor hazard. In some plants, the garbage or press cake is first dried to about 30% moisture in direct heat driers and then finished in steam driers.

Treatment of Drier Gases

Even with the best of design and operation, it is probable that some odors will result from the driers, and these must be eliminated by special treatment. The most common method is by treatment with water, commonly termed scrubbing. The gases are passed through a masonry structure in which sprays or curtains of water are maintained. A large water surface should be exposed to the gases passing through the scrubber. The sprays of water should condense the steam and absorb the soluble gases and carry both to the sewer. If the velocity of the gases through the scrubber and into the stack is too great, the particles of moisture and some dust will escape at the chimney top, and if insufficient water is used, some of the steam and soluble substances will escape to the atmosphere. Thus, scrubber operation may not be entirely efficient. In scrubber design, the items of temperature, velocity, time, quantity, and exposed surface of water and the general arrangement are pertinent and should be related to the type and rate of operation of the drier.

Dust is probably best removed by sedimentation or centrifuging prior to scrubbing. In some cases, in addition to scrubbing, the drier gases are heated by passing them through the hot gases of a coal, oil, or gas furnace. As a result of the tests at Barren Island, it was stated that "the odor of drier gas can be completely eliminated by heating to a temperature of 1 850° Fahr." Here, again, the cost of heating is pertinent and should be related to the design and operation of the drier and scrubber.

Chlorine gas has been used for the elimination of odors from drier gases under the Henderson-Haggard patents. At New Bedford, Mass., it is estimated that about 1 lb. of chlorine is used per 1 000 000 cu. ft. of drier gases at 100° Fahr. The chlorine is introduced at the entrance to a small scrubber.

Experience with Plant Locations

The selection of sites for garbage disposal works has often been an arduous task, which is apparently more difficult in some cities than in others. Among the more important factors are the following:

(*a*).—The occupancy and use of adjacent areas. Unoccupied and unused areas do not suffer immediate damage from odor-producing industries. Residential districts where people spend their leisure hours and where children are growing up are probably the least desirable districts for locating garbage plants.

(*b*).—The type of disposal plant and the character of its design and construction. The details of the plant may properly be related to its location with due regard to future property developments and plant additions.

(*c*).—The distance to which odors are likely to travel. Odors from digesters and driers in garbage reduction works have been detected at distances of several miles.

A record of plant locations showing the distance from the city hall to the disposal works is given in Table 7. These distances range from a few hundred feet to 50 miles. Some engineers are of the opinion that plants should be located far afield where odor elimination is not required. Others feel that garbage should be disposed of in the community which produces it and in such a manner as to eliminate odors.

General Summary

In odor elimination, attention to detail is the secret of success. Thus, plant upkeep and cleanliness are of great importance. It seems almost axiomatic and certainly it is common sense, that a clean garbage plant tends to eliminate odors as compared with a dirty one; and yet the condition of garbage disposal plants covers a wide range from very dirty to clean. If plant operators would realize that clean means clean and that there is no compromise with dirt, much odor trouble would be eliminated. Having these points in mind, the following considerations will be helpful in odor elimination:

(*a*).—There should, without doubt, be a good general plan for handling garbage covering the house treatment, the collection, and the final disposal. This will result in a fresher garbage at the disposal works.

(*b*).—As part of the general plan, good house treatment should be developed and the collection frequency oriented to the method of disposal and the location of the disposal plant as well as to the convenience of the householders.

TABLE 7.—Data Relating to the Location of Incinerators, November, 1924.

City.	Population, 1920.	Refuse matter handled.	Number of plants.	Miles from business center.	Character of locality.
Atlanta, Ga.	200 616	G. R. A.	1	0.75	Railroad yards.
Charleston, W. Va.	39 608	G. R.	1	0.75	Semi-residential and commercial.
Dallas, Tex.	158 976	G. R.	4	1.5	Factory and residences.
Danville, Ill.	33 776	G.	1	....	Negro houses of good character within 500 ft.
Elgin, Ill.	27 454	G.	1	0.5	Residences and soap company.
Fort Worth, Tex.	106 482	G. R.	4	....	.....
Houston, Tex.	138 276	G. R.	3	1.5	.....
Miami, Fla.	29 571	G. R.	3	3.0	Residences but none within 500 ft.
Milwaukee, Wis.	457 147	.....	1	1.25	Entrance to harbor.
Minneapolis, Minn.	380 582	G.	1	4.0	In open country near city workhouse.
New Orleans, La.	387 219	.....	2	2.2	.....
Oak Park, Ill.	39 858	G.	1	0	Schools, city hall, residences.
Paterson, N. J.	135 875	.....	..	....	.....
Philadelphia, Pa.	1 823 779	.....	..	2.5	.....
St. Louis, Mo.	772 897	.....	..	1.2	Commercial.
San Francisco, Calif.	506 676	G. R.	1	....	.....
Savannah, Ga.*	83 252	.....	..	....	.....
Scranton, Pa.	137 783	.....	1	0.75	Rendering plant and factories.
Springfield, Ill.*	59 182	.....	..	....	Small residences.
Trenton, N. J.	119 289	G. R.	1	....	Center of city.

Note.—G = Garbage; R = Rubbish; A = Ashes.
* Discontinued.

(*c*).—Collection wagons should be regularly cleaned to eliminate odors and to reduce the chances of receiving decomposed garbage at the plant.

(*d*).—Garbage disposal works should be built on suitably chosen sites with reference to the character of disposal works and the over-all costs of garbage collection and disposal.

(*e*).—Of particular importance is the capacity of the garbage disposal plant. No garbage plant should be subject to serious overload when garbage collections are heavy, if odors are to be eliminated. This is a real factor too often not given serious and definite consideration.

(*f*).—Following a good general plan, clean operation and ample real capacity of disposal works suitably located, there should be added proper devices for the elimination of residual odors, including combustion and dust removal chambers at incinerators and scrubbers, condensers, burners, chlorinators, etc., at reduction works.

To secure these various measures for odor elimination properly calls for the best of technical engineering effort. To facilitate this work many more accurate data, carefully taken and recorded, should be made available on rates and results of operation for all the various devices. However, with proper

effort, a practical elimination of odors from garbage disposal works can and should be expected.

Tables 8 and 9 give data relating to garbage reduction plants and garbage hog farms, respectively.

TABLE 8.—Data Relating to the Location of Garbage Reduction Plants, November, 1924.

City.	Population, 1920.	Miles from business center.	Character of locality.
Akron, Ohio*	208 435	2.0	Near main highway.
Boston, Mass	748 060	4.0	Island, Boston Harbor.
Bridgeport, Conn	143 555	3.0	.....
Buffalo, N. Y.*	506 775	6.5	.....
Chicago, Ill	2 701 705	4.5	Stockyards.
Cincinnati, Ohio	401 247	6.0	Scattered dwellings.
Cleveland, Ohio	796 841	8.0	Sparsely settled.
Columbus, Ohio	237 031	4.0	Sewage disposal plant in undeveloped district.
Dayton, Ohio	152 559	8.0	Open country.
Detroit, Mich	993 678	20.0	Isolated near main highway.
Dubuque, Iowa	39 141	4.0	Isolated.
Indianapolis, Ind	314 194	3.0	Open country near main highway.
Los Angeles, Calif.*	576 673	2.5	Warehouses.
Michigan City, Ind	19 457	4.0	Near main highway.
New Bedford, Mass.*	121 217	2.5	Sparsely settled.
New York, N. Y.*	5 620 048	12.0	Isolated.
Philadelphia, Pa	1 823 779	2.5	Industrial.
Pittsburgh, Pa	588 343	22.0	Isolated.
Rochester, N. Y	295 750	0.75	Commercial and industrial.
Schenectady, N. Y	88 723	1.2	Mixed.
Syracuse, N. Y	171 717	1.6	Sewage disposal plant.
Toledo, Ohio	243 164	3.5	Stockyards.
Washington, D. C	437 571	35.0	Isolated.

* Discontinued.

TABLE 9.—Data Relating to the Location of Garbage Hog Farms, November, 1924.

City.	Population, 1920.	Miles from business center.	Character of locality.
Akron, Ohio	208 435	7	Farms.
Denver, Colo	256 491	7	Near city limits.
Grand Rapids, Mich	137 634	35	Farms.
Lansing, Mich	57 327	3.5	Farms.
Los Angeles, Calif	576 673	50	Open country.
Madison, Wis	38 378	3.4	Near sewage plant in undeveloped district.
Newark, N. J	414 524	2	Commercial and industrial.
Salt Lake City, Utah	118 110	....	.....
Utica, N. Y	94 156	5	Farms.
Worcester, Mass	179 754	3	Developing residential.

CONTROL OF ODORS FROM SEWAGE TREATMENT PLANTS

By John F. Skinner,* M. Am. Soc. C. E.

Synopsis

A study of this subject presupposes a knowledge of the general subject of odors, their nature, production, and means of transport. The writer has had no opportunity for experimental work and, therefore, can treat the subject only in the most general manner. A line of research, however, is suggested which may produce results.

The subject has been studied under the following heads:

Character of Odors,
Source of Odors,
Control of the Production of Odors,
Control of the Escape of Odors,
Conclusions.

General

Sewage, at best, is water-carried filth, the wastes from the population and culinary and industrial processes. Such material may give off odor, usually unpleasant *per se* and often reminiscent of its source.

A sewage treatment plant handling such wastes may be expected to make its presence known in the immediate neighborhood through the sense of smell, but the necessity of dealing promptly, effectively, and continuously with such matter demands considerable tolerance. Efforts should be directed toward minimizing discomfort, nuisance, and damage, by control of the operating process, by the design of the plant, and by its location. Wherever possible, it should be isolated at a distance from occupied territory and the frequency, direction, and intensity of prevailing winds should also be considered. It should be surrounded by a wide area under municipal ownership or control and preferably be masked or screened by vegetation.

Character of Odors

Just as the odor of fresh feces varies with the food and the condition of the individual's digestion, so when the putrefactive process is continued at a sewage treatment plant, odors will vary with the character and condition of the influent sewage and with the regimen of the continuing processes of bio-chemical change and digestion throughout the plant.

The principal gases given off by putrefying sewage are methane, carbon dioxide, and hydrogen sulfide. Only the latter possesses appreciable odor. All of them, however, may act as carriers of other odorous material. As an ex-

* Cons. Engr.; Deputy City Engr., Rochester, N. Y.

ample of this transporting power of gases, pure carbon disulfide has a rather pleasant ethereal odor, but the commercial article carries impurities that give it its well-known smell which is exceedingly disagreeable.

Water vapor may carry gases and other products of the decomposition of organic matter in solution or it may transport such products in colloid form, or in the form of finely divided suspended solids. If these malodorous particles reach human olfactories the fact of the existence of odors is at once established.

Another source of odor is the presence of moulds which reside and grow on walls, floors, hand railings, and all exposed surfaces in the immediate neighborhood of the works. These are readily transmitted by contact with the flesh or clothing. If one touches such surfaces, his hands or gloves will have a noticeable and disagreeable odor for some time thereafter.

Source of Odors

The odor of fresh sewage, when not confined and when flowing tranquilly in channels and tanks, or when remaining quiescent in tanks, is not particularly offensive, except to the hypersensitive. As such sewage becomes older, its odor is more pronounced and its condition passes on through the stale to the septic state, when gas can be seen to bubble from the surface and an odor of hydrogen sulfide may be perceptible and also an odor often described as a "sweet-sickish smell." If sewage is confined, as when it flows through an illy-ventilated sewer or chamber, the air may become very foul.

When the influent sewage flows rapidly over steps or down an incline where the smooth flow is broken by baffles—as is or was the case where lime is mixed with the sewage at Worcester, Mass.—in advance of the chemical precipitation tanks, the odors liberated are very strong. A decided cessation or reduction in intensity of odor is observed at this station as one proceeds along the walk between the tanks, but it again becomes intense on going back to the vicinity of the lime mixer.

Trash from coarse racks, if not complicated by the presence of offal or dead animals, can ordinarily be removed, and disinfected, burned, dried, or covered before foul odors are noticeable.

Detritus or grit from properly operating grit chambers does not emit unpleasant odors, as it contains little putrescible matter. If, however, sludge is allowed to collect in such chambers and remain and become septic, it may give rise to smells.

Fine racks and screens which intercept feces, or places where feces are present in large quantities floating in the screen chamber or accumulated in piles, may become offensive, as noticed in the screen house at Baltimore, Md. If such material arrives at the plant already comminuted, or if visible pieces are so few that it is practicable to break them with a hose stream and pass the fragments on with the effluent, the remaining screenings may vary from washed out garbage and offal to non-putrescible detritus and will only be seriously offensive if not promptly removed and disinfected, burned, dried, or covered.

The odor from sewage in a properly operated Imhoff sedimentation chamber is, as described, dependent on the freshness of the sewage. In a septic tank, however, where the sewage is generally older and where there is a mass of undigested sludge, both below the surface in the tank and floating on the liquid, there is usually a more disagreeable odor from the undigested sludge. This condition is best met by covering the tanks. When sludge is placed in tanks for separate digestion, the odor is more intense and becomes nauseating to many.

The effluent of an Imhoff tank when sprayed upon a sprinkling filter will be more or less odorous, depending on whether it is stale or fresh, concentrated or dilute. It may also be affected by the character of the raw sewage and by the thoroughness of the preliminary operations.

When the sprinklers are in display the odor is noticeable to the leeward, as might be expected. When the sprinklers are at rest and the bed is in good condition little odor is noticeable if sufficient sedimentation has been previously accomplished. If, however, proper sedimentation has not been performed, fragments of putrescible matter of considerable size may lodge on the surface of the bed and form accumulations which will putrefy and give off odors. With proper preliminary sedimentation, the smaller sized particles of suspended matter will pass through and below the surface of the bed and the air entrained by the falling drops will carry down any odors given off by such particles. Moreover, such particles themselves will be washed down into and through the bed.

In the activated sludge process, little odor is occasioned by the aeration tanks. In the direct oxidation process, as the electrolytic tanks are kept closed, little odor may be anticipated except at the screening plant, for the sewage before passing into the sedimentation tanks has been treated with an excess of lime, which also prevents odors from being given off by the sludge.

Thoroughly oxidized effluent from a sprinkling filter and in a final settling basin has no appreciable odor, but the secondary sludge when occasionally removed and allowed to stand until the larvæ and worms contained therein have died and their bodies have putrefied, is sometimes quite offensive for a few days. It has an odor similar to that of a bait box of fish worms which has been neglected until the worms have died from exposure to the sun and lack of nourishment.

Well digested sludge has a not unpleasant musty odor which does not travel far. Some describe it as a "choky odor," or as if it were occasioned by a fine dust.

Control of the Production of Odors

Screenings should be removed promptly and covered with earth. The value of composted screenings as a fertilizer has been suggested. A continuous belt-conveyor has been used at the Irondequoit Treatment Plant at Rochester, N. Y., since 1917 for removing the product of four 12-ft. Riensch-Wurl screens. The belt dumps the screenings into an industrial car which is hauled out when full. The cleanest method of removal is probably by pneumatic conveyor. By this method the screenings are least exposed to the air and to flies.

The writer has a plan for a plant to be constructed in which the only treatment of the sanitary sewage will be multiple screening. It is proposed, after passing coarse racks, to screen the fresh sewage twice through openings of diminishing size and then to convey the screenings, including feces, and grit if any, to the top of a closed silo tank for storage. They will later be discharged from the bottom of the tank into farmers' trucks when required.

Raw sewage and sewage in tanks and channels should be screened or skimmed so that no floating material will be present to putrefy on the surface. Bio-chemical changes in the sewage solids can best take place under water for gas rising through the water will be washed and partly dissolved so that less will be sent abroad into the atmosphere.

If the design of the channels is such that sewage will flow smoothly and not be broken up and vaporized, considerable odor will be avoided. If the design allows spray to be blown into the air, it may be condensed by providing a covering on which the spray may impinge.

Sprinkling filters have been rejected for some exposed locations near dwellings, and contact beds have been constructed rather than the sprinkling filters, because of the fear of odors to the lee of the filters. In such situations, advantage might be taken of the economy of the trickling filter over the contact bed by adopting a dripping distributor in place of the usual sprinklers which toss the spray into the air and thus furnish the greatest cause for complaint as to odors. Wherever possible a location should be sought which is not exposed to winds which may blow odors or flies to a distance. Sludge is more or less offensive as it is less or more thoroughly digested.

Separate sludge digestion tanks have been operated experimentally with a layer of oil to keep air from the surface and solids from exposure on the surfaces. The experiments have not yet been conclusive. As a general proposition, sludge digested under water is less productive of odors than when exposed above water.

Sludge cake from the pressing of undigested sludge is likely to be offensive, especially in warm moist weather. It may be covered with earth or ashes. Digested sludge is inoffensive and does not attract flies.

Lagooning undigested sludge is likely to produce odors, for either scum may form, or there may be insufficient water to cover the sludge which will then putrefy and give off odors.

In general, lagooning should only be adopted in winter or in remote locations where no nuisance will be occasioned to neighbors. Secondary sludge, following sprinkling filters, may be pumped into tanks for digestion, or removed when the weather is best for its disposal by lagooning.

Control of the Escape of Odors

Assuming that certain odors will be produced in a sewage disposal plant engineers are confronted with the problem of preventing their escape from the plant to the surrounding atmosphere at such intensity and with such frequency as to constitute a nuisance to persons residing in or passing through adjacent territory.

If it is assumed that odors are volatile emanations, gases, colloids, liquids, or finely divided solids which reach human olfactories, this part of the study must be directed to the means by which these attenuated gases or minute particles move, what gives direction to their motion, and what effects their intensity as observed odors, in various localities.

The elements which at once suggest themselves are:

(*a*) Diffusion of gases and vapors in still air.
(*b*) Relative temperatures of air, sewage, gas, and vapor (convection).
(*c*) Moisture and vapor tension of the atmosphere.
(*d*) Wind direction and intensity.
(*e*) Topography, elevation, and exposure.
(*f*) Screens, natural and artificial.
(*g*) Mingling of other more pleasant odors.
(*h*) Psychic considerations.

(*a*).—If the air is perfectly quiet and there are no disturbing temperature conditions, a gas or a vapor carrying odors may diffuse equally in all directions in the atmosphere, and its intensity will be inversely as the cube of the distance from its source; hence the odor will not travel far.

(*b*).—If, however, the sewage and the gas or vapor emitted by it is warmer than the surrounding air (a condition which may obtain in quiet winter weather), the warm gas or vapor will rise through the cooler and denser atmosphere until the odor-bearing gas or vapor has cooled to the temperature of the surrounding air. During this time diffusion has been taking place into the atmosphere so that one may assume an odoriferous column expanding upward in the shape of a plume, denser at the center than at the periphery and similar in form to the steam cloud above a volcano; not much odor would cause complaint under these conditions.

If, however, the air is warmer than the sewage, as on a summer day, the gas and vapor given off will diffuse into the atmosphere relatively near the ground and only rise and disperse when it has attained the temperature of the air.

The two opposite conditions of temperature just cited may account, in part, for the more intense odor near the disposal plant in summer than in winter, although doubtless there is the further explanation that more active decomposition is taking place in summer than in winter and greater quantities of gas are liberated during the summer.

There is also the case of the cool summer night when the sewage may be warmer than the denser air and warm enough for the maximum of bio-chemical activity. Being slightly warmer than the air, odoriferous gases will rise to some height, but possibly not far enough to be diffused in the upper air as described for quiet winter weather conditions. With a gentle breeze, these summer nights are often made unpleasant at some distance from the disposal plant, for windows are open and residents may be awakened by an unusual and nauseating odor and have their attention concentrated on this manifestation.

(*c*).—If the air is dry it will avidly take up vapor and as it is no respecter of odors, it will take the pleasant and the unpleasant alike. Winds may carry

these odors to considerable distance. With moist air, however, near the dew point or when rain is falling, sewage vapors will not be taken up and only gases will be transmitted which are being given off so voluminously that they are not entirely absorbed by the rain or atmospheric vapor.

(*d*).—The condition of absolute quiet is so rare as to be almost theoretical, for there is generally some wind movement, be it ever so slight.

A strong wind will usually dilute the odors to so great an extent that they become negligible at a short distance from the disposal plant.

A gentle breeze, however, wafts the odor cloud bodily to leeward before it has become sufficiently attenuated by diffusion to destroy its unpleasant character. This condition is more noticeable in summer when the disposal plant processes are most active and when, as already mentioned, windows are open. This indicates the advisability of not placing a plant to the windward of a summer population.

(*e*).—If the plant is in a protected valley, or one surrounded on the windward side by woods or high hills, the probability of blowing odors from it is considerably reduced.

(*f*).—Screens of natural forest trees not only prevent access of the winds which transport odors, but both forest trees and artificial plantings of trees and shrubs by means of their moist leaves furnish surfaces on which odoriferous gases and vapors may be condensed or absorbed.

(*g*).—If trees and shrubs which give off pleasant odors are planted about the premises in masses sufficient to affect the atmosphere, they will be noticed at times for their beauty and for their pleasant odors and at other times will serve to disguise unpleasant odors by their presence.

(*h*).—Out of sight is out of mind in sewage plants as in other things. Some citizens who see scum on a river at a distance will think they smell it, when in fact the scum and the stream may be odorless.

Neat and artistic planting about the station where it must be exposed, and screens of trees or shrubbery to hide it if possible, do much to prevent imaginary odors from causing annoyance. The nerve sensation called smell may be forgotten if other interests are paramount.

Conclusions

The sewage disposal plant should be designed liberally so that it will not be overloaded. It should be located in a protected and isolated spot. It should be operated to avoid uncovered, decomposing, solid matter and the walls and floors should be kept clean and washed down at least daily. Splashing and the needless production of vapors should be avoided if possible. The plant should be screened from view when practicable and visible parts should be made attractive by means of trees and flowering shrubs. Keep the sewage treatment plant away from the public and keep the public away from it, especially when it is not functioning properly. These measures will mitigate if not entirely prevent the escape of odors from a plant to the surrounding territory.

A suggested line of research is as follows: As exceedingly minute portions of a substance are carried in the air and constitute odors and as bacteria

exist in earth, in sewage, and in the air close above a sewage plant, may it not be conceived that the putrefactive odors which differ from pure gaseous odors are in part, at least, due to the presence of these bacteria in the air?

If this is true, will not the destruction of these bacteria result in the diminution of the odor? Chlorine, the best known disinfectant, would have to be distributed and released continuously in order to be effective, and moreover it would add an odor of its own.

It would seem that engineers might profitably, first, seek for some type of gaseous disinfectant which would be pleasant, or better still, odorless; and, second, devise means for its economical application at the sources of odors in the various parts of a plant.

THE DETECTION AND ELIMINATION OF ODORS FROM OIL REFINERIES

By Robert Spurr Weston,* M. Am. Soc. C. E.

In a commercial or practical sense, the word, petroleum, applies to natural, liquid hydro-carbons which, from a scientific standpoint, are not strictly liquid, but are mixtures of gases, liquids, and colloidally suspended solids.

Shortly after its discovery, the chief use of petroleum was in the preparation of illuminating oil. Following this occurred the development of petroleum lubricants which were found to be superior to the animal and vegetable fats and oils previously used. However, it is chiefly because of the development of the internal combustion engine and the increased use of oil as fuel that the petroleum industry has become so extensive. It is estimated that there are about 16 000 000 automobiles in use in the United States at present (January, 1925), and the number of Diesel, gasoline, and other motors of this general type is increasing for use on land and water, and the high prices of coal and labor have furthered the use of oil for fuel.

The world's production of petroleum from 1919 to 1923 was as follows:

Year.	Total Number of Barrels.
1919	558 892 000
1920	700 902 700
1921	762 241 000
1922	854 809 000
1923	1 010 995 000

Of the total petroleum produced, more than 60% came from the United States, and more than 90% from the United States and Mexico. The production of petroleum in Mexico is declining, but the loss is likely to be greatly compensated by increased production in Venezuela where extensive fields are being exploited.

With this extensive and increasing production of petroleum, it is not surprising that there has been a great increase in the number of refineries throughout the world. Most of the refineries in the United States are situated along an arc drawn from Texas to Massachusetts, in California, and in Wyoming. Until recently there were no refineries in New England, but in the past few years several large installations have been made in the vicinity of Narragansett Bay and Boston Harbor. The introduction of this new industry in thickly settled regions where the people are jealous of the purity of the atmosphere and the cleanliness of the shore waters, has created new sanitary and æsthetic problems.

The use of petroleum in the various grades is growing and has become general rather than local. Therefore, it must be expected that refineries, which have increased from 176 in 1914 to more than 500 in 1924, will be

* Cons. Engr. (Weston & Sampson), Boston, Mass.

located where economic conditions necessitate. Consequently, sanitary engineers must assist in making these plants inoffensive and thus contribute to the prosperity of these localities with the minimal impairment of a healthful and salubrious environment.

There are now three large oil refineries in Massachusetts and two in Rhode Island, and it is in these States that vigorous complaints regarding odors have arisen.

The crude petroleum oils differ greatly in composition, specific gravity, and viscosity, but the property that most concerns sanitary engineers is the sulfur content, which varies from less than 0.1% in Pennsylvania oils through less than 0.5% for Mid-Continent oils, to less than 3.0% for Wyoming oils, and to more than 5.0% for Mexican oils. Sulfur is important because it forms disagreeable compounds with the hydrogen, nitrogen, and hydro-carbons in the oils. These compounds, some of them gaseous, are given off during the process of distillation. The most difficult odors to control occur when a refinery is operating with Mexican or other oils high in sulfur. Certain refineries in New England began operations with Mexican oil, and with no devices to prevent the escape of odors. The reaction by the public was instantaneous and violent.

Typical Refinery Practice

Although this paper is not descriptive of oil refining, and although the practice of this art varies greatly, it seems well to outline the general practice in order that the methods of odor control may be understood more readily.

Refineries may be classified in two general groups, namely, skimming or topping plants which confine their operations to the separation of gasoline and the lighter oils from the crude oil, and complete refineries which split the crude product into gasoline, naphtha, kerosene, gas oil, light and heavy lubricating oil, fuel oil, paraffin wax, pitch, coke, and other products, as the markets demand. There are many modifications of these two kinds of plant. For example, either may be operated in conjunction with a so-called high temperature distillation or cracking plant which, through heat and pressure, splits off hydro-carbons of lower boiling point from heavier oils, that the yield of the more profitable gasoline and lighter products may be increased. Then, again, it is unprofitable for certain refineries to carry the distillation until only coke remains in the stills, and such may stop distillation when the residue in the still is suitable for fuel, or, carried further, is a semi-solid pitch or petroleum asphalt.

Figs. 2 and 3 show the flow sheet for a complete refinery and the main features of a crude oil distillation plant, respectively. Although the kinds of distillate from the still are numerous, this paper concerns itself chiefly with the lighter products, some of which may escape from the refinery in gaseous form, and the heavier oils, pitch or coke, which, when hot, are likely to give off strong odors. Coke cooled with water is apt to emit an odor when the steam which arises from it carries away the odor of the coke.

During ordinary distillation, carbon accumulates on the shell of the still and the scale thus formed has to be removed by hand at frequent intervals.

The cleaning of stills, or tubes, is particularly frequent with the cracking process, in which high temperatures and pressures favor the deposition of carbon and make delays in cleaning dangerous.

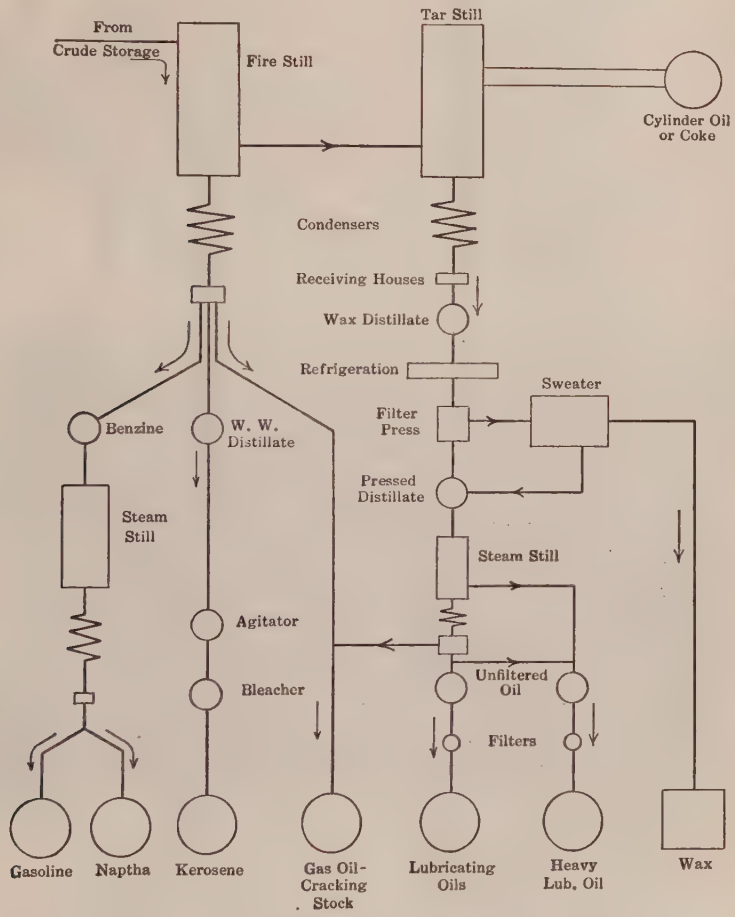
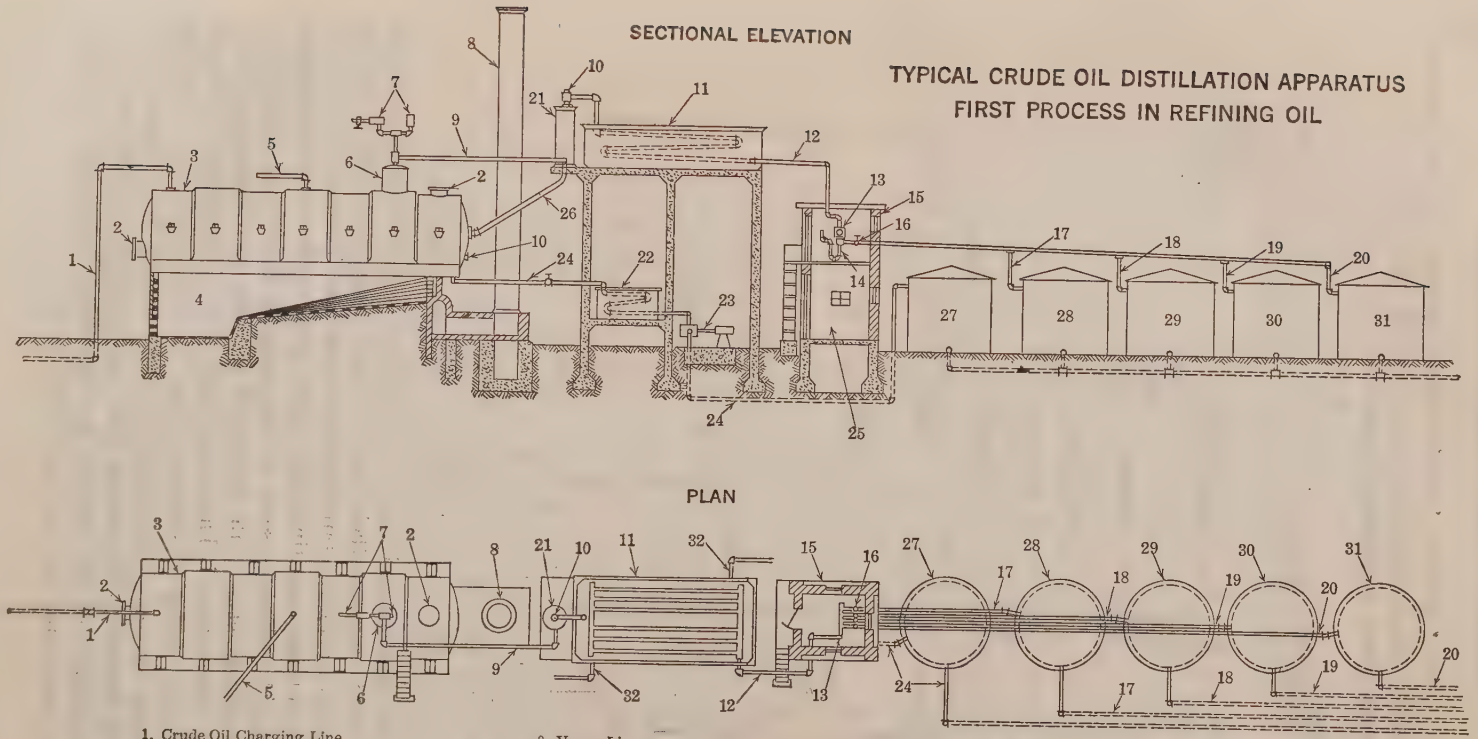


FIG. 2.—SCHEMATIC SEQUENCE OF OIL-REFINING PROCESSES.

Fuming sulfuric acid and lead oxide are used to treat gasoline and other lighter products to improve their odor. This is especially necessary if the crude oils contain sulfur and saturated hydro-carbons. This operation is usually conducted in tanks, operating either on the batch or the continuous system. The sulfuric acid carbonizes some of the asphaltic bodies and unsaturated hydro-carbons which settle to the bottom of the tank with the sulfuric acid, in which form the mixture is known as sludge acid. This mixture is washed with water and the sludge oil separated from the weak acid. Frequently, the acid is recovered by concentration and the process may be conducted so as to be extremely offensive. The sludge itself becomes acrid and offensive, especially in the vicinity of the refinery, but fortunately the odors are heavy, are soluble in water, and are not carried far away.



1. Crude Oil Charging Line
2. Manholes
3. Crude Still
4. Firebox
5. Steam Line
6. Dome
7. Pressure and Vacuum Valves
8. Stack
9. Vapor Line
10. Thermometers
11. Condenser Box
12. Condensate Line
13. Lookbox
14. Water Leg
15. Receiving House
16. Manifold
17. Lubricating Distillate Lines
18. Gas Oil Distillate Lines
19. Kerosene Distillate Lines
20. Gasoline Distillate Lines
21. Dephlegmator
22. Tar Cooler
23. Tar Pump
24. Tar Lines
25. Instrument Room
26. Runback
27. Tar Storage
28. Lubricating Oil Distillate
29. Gas Oil Distillate
30. Kerosene Distillate
31. Gasolene Distillate
32. Water Lines for Condenser

FIG. 3.

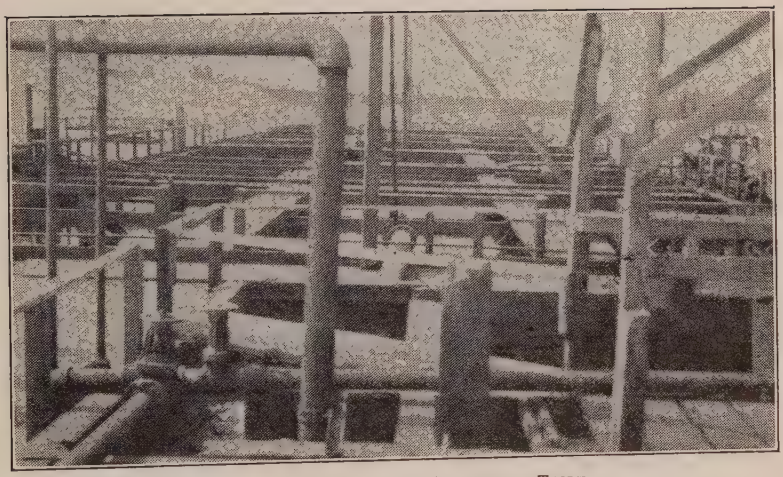


FIG. 4.—SKIMMING OR SEPARATOR TANK.

As will be seen from Figs. 2 and 3, nearly all the processes require water for cooling, and this, together with whatever oily waste may escape from the plant through leaky condenser coils, by accident or otherwise, is usually discharged from the refinery in a common drain, discharging in turn into a large subsiding basin (Fig. 4), in which ample time is given for the removal, by gravity, of the subsidable solids and the skimming off of waste oil.

The effluent from such a basin should not contain more than 20 parts of oil (ether soluble) per million. Frequently these basins, especially when the condenser waters are warm, give off objectionable odors. Especially is this true when there is accidental leakage of volatile products or when batches of distillate, improperly processed, are salvaged by discharge into the drain and by skimming from the subsiding basin.

Where lubricating oils are made, it is necessary to filter them through fuller's earth, diatomaceous earth, or similar material. It is sometimes necessary to filter gasoline, kerosene, and other products, also. After the filter material is exhausted or clogged, it is necessary to regenerate it by heating in a rotary kiln until the waxy accumulations are volatilized. If the dust and oily vapors from the kiln are discharged directly into the air, offensive odors may be carried for miles, apparently in the form of colloidal particles of oily dust.

The crude oil from ship, tank car, or pipe line is stored in large tanks, usually of steel, and covered. It is obvious that, during the process of filling, the gaseous products accumulated in the empty tank from a previous filling will be discharged into the atmosphere, the tanks inhaling air and exhaling petroleum vapors alternately. There is also evaporation, which is greater if uncovered tanks are used.

The boilers, stills, and cracking plants are oil or gas-fired and give off the products of combustion characteristic of oil-fired furnaces. These products possess the characteristic burned-fuel-oil odor so frequently noticeable in localities where fuel oil is burned, beneath the boilers of ships, locomotives, and stationary steam plants.

The main sources of odor may be listed as follows:

(1) The gases and vapors from crude oil.
(2) The volatile gases given off by petroleum when distilled.
(3) The vapor from heated oil, especially oil heated to a high temperature during the "cracking" process.
(4) The vapors and dust from kilns used to regenerate filter material.
(5) The heavy acid and acrid odors resulting from treating oil with sulfuric acid and other chemicals. This odor is especially strong when the oil and acid are agitated by compressed air in open tanks.
(6) The emanations from the surfaces of subsiding or separating basins and from the surface of water in gas-holders.
(7) The vapors given off from hot coke or pitch freshly drawn from the still.

(8) The vapors given off from stills when opened and steamed after a distillation has been completed.
(9) The stack gases from oil-fired furnaces; this is the most prevalent odor.

The substances included in these odors are petroleum and asphaltic hydrocarbons, hydrogen sulfide, carbon bisulfide, sulfur dioxide, sulfur trioxide, pyridin vapors, the oxides of carbon, and various volatile compounds of sulfur or sulfur and nitrogen, of which the offensively smelling mercaptan is an example.

That some of these odors may be present in the air in sufficient concentrations to affect the health was shown by the studies of Mr. S. DeM. Gage,* who reported that during 1921 at least ninety-six persons in the vicinity of Rhode Island refineries had been more or less seriously affected by them. The symptoms most frequently noted were nausea, headache, vomiting, sore throat, and insomnia, and it was the opinion of the majority of physicians consulted that asthma, respiratory diseases, and nervous diseases were unfavorably affected by petroleum and sulfur odors and also by the sulfur dioxide and trioxide gases.

Nature of the Odoriferous Matters

The matters which produce the odor may be divided into two classes, namely, true gases and vapors. The true gases diffuse rapidly and the intensity of the odor roughly varies inversely as the square of the distance. Not so the odor produced by vapors which, in the form of minute particles, "odorphores", impinge directly upon the olfactory organs and seem to lose their intensity very slowly with distance within the sector where their presence may be observed. The distances to which they may be observed seem to depend more upon the humidity and the character of the air currents which carry them than upon any other factors.

For example, the writer has observed the acrid odor from sludge acid when it was so strong that it was impossible to remain a whole minute in its presence. Yet this odor could not be observed ½ mile to the leeward of its point of origin. On the other hand, the sulfur compounds in the vapors given off from Mexican oil were at least one-fourth as intense 2 miles away as they were in the refinery yard where they were produced.

The odors from the kilns in which material used to filter the various distillates is ignited to regenerate it, also have the power of traveling great distances. This is probably due to the fact that the dust particles from the kiln become coated with a film of oil, and in this state become dispersed in the air, much as pollen and other dust particles are dispersed.

Character and Intensity of Odors

It is a fact of physiology that the nose detects two classes of odors: First, the true odors which are observed by organs located in the upper nasal passages behind the superior turbinated bones; and, second, the irritants which are observed by a separate set of nerve endings in the anterior part of the

* Report of Rhode Island State Board of Health on the Cause, Prevalence and Effect of Offensive Odors in Providence, Cranston, East Providence and Warwick during 1921.

nose. For example, perfumes are perceived by one set of nerves, whereas irritants like ammonia and pyridin, affect the other set. Certain substances like tobacco smoke, for example, are both odoriferous and irritating, and the odors given off by oil refineries are in either or both of the two classes.

Oily vapors, for example, volatilized vaseline, possess true odors which are not irritating, whereas sulfur trioxide is sometimes recognized by its irritating effect, before being detected by the olfactory organs. Certain volatile hydro-carbons are also highly odoriferous, but are not irritating unless present in high concentrations.

It is a peculiarity of the olfactory organ, by which the true odors are detected, that it tires rapidly and that ordinarily sniffing is required to carry odor to the portion of the lining membrane of the nose where in a small area (25 mm. square on each side) the special organs are located.

Names of Odors.—Various observers have given names to the various odors, and sometimes various names to the same odor observed about oil refineries. For convenience, the following adjectives have been used in the writer's practice:

Adjective.	Description of Odor.
Sweetish	The odor of gasoline and refined petroleum products.
Turnipy	The odor of well-diluted petroleum gases containing sulfur and nitrogen.
Onion or garlicy	The same odor in higher concentration.
Sulfide	The rotten-egg-like odor of hydrogen sulfide gas.
Sulfur trioxide	The characteristic odor of fuming sulfuric acid. In considerable dilutions this is detected by its effect upon the mucous membrane and without the accompaniment of any true odor.
Oily	The odor of volatilized petroleum distillate as from a fuel oil tank.
Burned oily	The characteristic odor from oil-fired furnaces.
Burned rubber	The burned oily odor in higher concentration.

Any of these odors may exist singly, or in combination, and the combinations may possess odors and irritants which are difficult to analyze.

Intensity of Odors.—The intensity of odor may be graded on a scale of 5, as follows:

Number.	Corresponding Adjective.
1	Very faint
2	Faint
3	Distinct
4	Decided
5	Strong

Detection of Odors.—Two methods of detection are usually practiced, namely, smelling and chemical analysis.

Smelling.—The nerve ends in the nose not only are delicate qualitative instruments, but when the results of their trained use are classified and the adjectives pertaining to quality are given numerical values, they do more

reliable work than any mechanical instrument; because, after all, the reaction of a complaining public depends upon nasal observations and not the qualitative results of chemical air analyses, which are of little practical meaning to them. Furthermore, an observer becomes expert with training and can detect and describe odors which baffle a lay observer. His observations are of especial value when a differentiation of sources of odor is required.

Gas Analyses.—Chemical gas analysis methods are more effective for the detection of the irritating sulfur gases, it being possible to measure concentrations which are about one-tenth of those which may be detected by the nose. For example, hydrogen sulfide may be smelled in concentrations of 100 parts in 1 000 000 000 parts of air, whereas the same gas may be detected in concentrations of 15 parts in 1 000 000 000 parts of air, by chemical analysis, using 100-liter portions for each determination.

The Nasal Method.—The first requisite for this method is an observer of keen perceptions, one who is quick to notice any change in atmospheric conditions and who can make judicial records of his observation.

The automobile is a great convenience for making observations, but care should be taken to select a machine which emits the minimum of oily odors, and when making observations, the observer should step out of the automobile so as to avoid any error due to observing the products in the exhaust of the motor in place of those present in the air at the point of observation.

While making record of nasal observations, together with those of wind and weather, a sheet, Table 10, has been used by the writer's firm. It will be noted that there are spaces for recording the velocities and the direction of the wind at various times of observation, the weather, and the time, place, kind, intensity, and probable source of any odors observed. Space is also provided for smoke observations for the reason that the burned oily odor from stacks connected with oil-fired furnaces, is more or less proportional to the density of the smoke emitted from them.

At Fall River, Mass., eight stacks were observed. Some of these were within the refinery yard, and others were connected with various mills within two miles of the refinery. Ringelmann's chart was used to record the percentage of black smoke in the emanations from the various stacks. The relative humidity was observed at one station only.

Having made the observations, the next step is to express them in diagrammatic form. For this purpose, reproductions of a map of the vicinity of the refinery about which observations are made, are useful, and Figs. 5 and 6 show the map-diagrams which were used at Fall River and at East Braintree, Mass., and vicinity, respectively.

The Fall River map-diagram (Fig. 5), which records the observations for 24 hours ending at 7 A. M. on July 25, 1922, shows a summary of the weather, wind, smoke observations, the intensities and kinds of odors, and the areas within which the odors were observed. It will also be noted that odors were observed as coming from three sources, one of which was the oil refinery and the others manufacturing plants where boilers were fired with oil. Compared with a large power house, an oil refinery is a small consumer of fuel oil, and

TABLE 10.—Report of Odor Observations.

(For 4 Hours Ending at 7 A. M., July 25, 1922, at Fall River, Mass., New England Oil Refining Company.)

Wind.				Weather.
Velocity.	Direction.	From.	To.	
Light.....	N—NNE	3	4	Cloudy
"	NNE—N—NE	4	5	"
"	NNE—NE	5	6	Partly cloudy
"	"	6	7	"

Odors Observed.

Time.	Place.	Kind.	Intensity.	Source.
3:45	Lincoln Ave. between Rock and Belmont Sts.....	Onion—sweetish.	Faint.....	N. E. O. R. Co.
4:00	President Ave. between N. Main and railroad.....	"	Very faint.....	"
4:15—4:30	Cedar to Pine on N. Main—N. Main to Dartmouth on Pine.	Burnt oily.....	Faint to distinct...	Electric Light Plant
6:00	Brightman between railroad and St. Mary St.....	Onion—sweetish.	Very faint.....	N. E. O. R. Co.
6:30	Bradford Ave., Dimon to Day Sts.....	Burnt oily.....	Faint traces only..	American Print Wks.

Smoke Observations—Densities according to Ringelmann's Chart.—Per Cent. Black.

Time.	S 1	%	S 1*a*	%	S 1*b*	%	S 2	%	S 1/10	%	S 1 1/20	%	S 2 6/30	%	C k S	%	S	%
5:45		20		40		10		10		10		10		0		10		

Remarks

Observer.

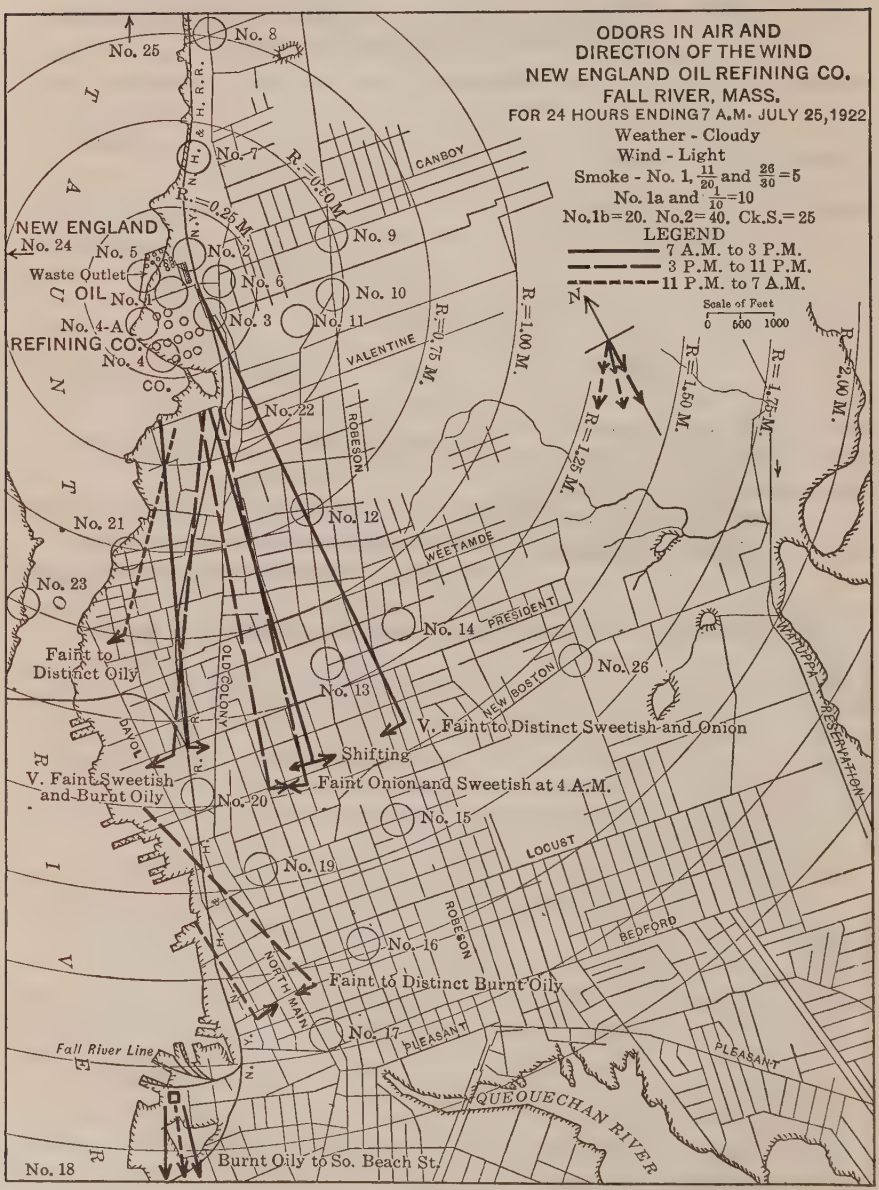

ODORS IN AIR AND
DIRECTION OF THE WIND
NEW ENGLAND OIL REFINING CO.
FALL RIVER, MASS.
FOR 24 HOURS ENDING 7 A.M. JULY 25, 1922
Weather - Cloudy
Wind - Light
Smoke - No. 1, 11/20 and 26/30 = 5
No. 1a and 1/10 = 10
No. 1b = 20. No. 2 = 40. Ck.S. = 25
LEGEND
7 A.M. to 3 P.M.
3 P.M. to 11 P.M.
11 P.M. to 7 A.M.
Scale of Feet
0 500 1000
NEW ENGLAND
OIL
REFINING CO.
Waste Outlet
No. 24
No. 25
No. 8
No. 7
No. 9
No. 10
No. 11
No. 6
No. 5
No. 4-A
No. 4
No. 3
No. 2
No. 1
No. 22
No. 12
No. 21
No. 23
No. 14
No. 13
No. 26
No. 20
No. 15
No. 19
No. 16
No. 17
No. 18
R. = 0.25 M.
R. = 0.50 M.
R. = 0.75 M.
R. = 1.00 M.
R. = 1.25 M.
R. = 1.50 M.
R. = 1.75 M.
R. = 2.00 M.
CANBOY
VALENTINE
ROBESON
WEETAMDE
PRESIDENT
NEW BOSTON
OLD COLONY
DAVOL
LOCUST
BEDFORD
PLEASANT
NORTH MAIN
WATUPPA RESERVATION
QUEOUECHAN RIVER
Fall River Line
Faint to Distinct Oily
V. Faint to Distinct Sweetish and Onion
Shifting
Faint Onion and Sweetish at 4 A.M.
V. Faint Sweetish and Burnt Oily
Faint to Distinct Burnt Oily
Burnt Oily to So. Beach St.
T A U N T O R I V E R

FIG. 5.

frequently odors detected in the vicinity of a refinery are properly chargeable to the furnaces of public service and manufacturing plants.

The diagram used at East Braintree, Mass. (Fig. 6) shows a little different method of recording results. The observations at this place benefited by the use of a recording weather-vane, which avoided many errors in observations of wind direction, especially those due to topographical features near the points of observation.

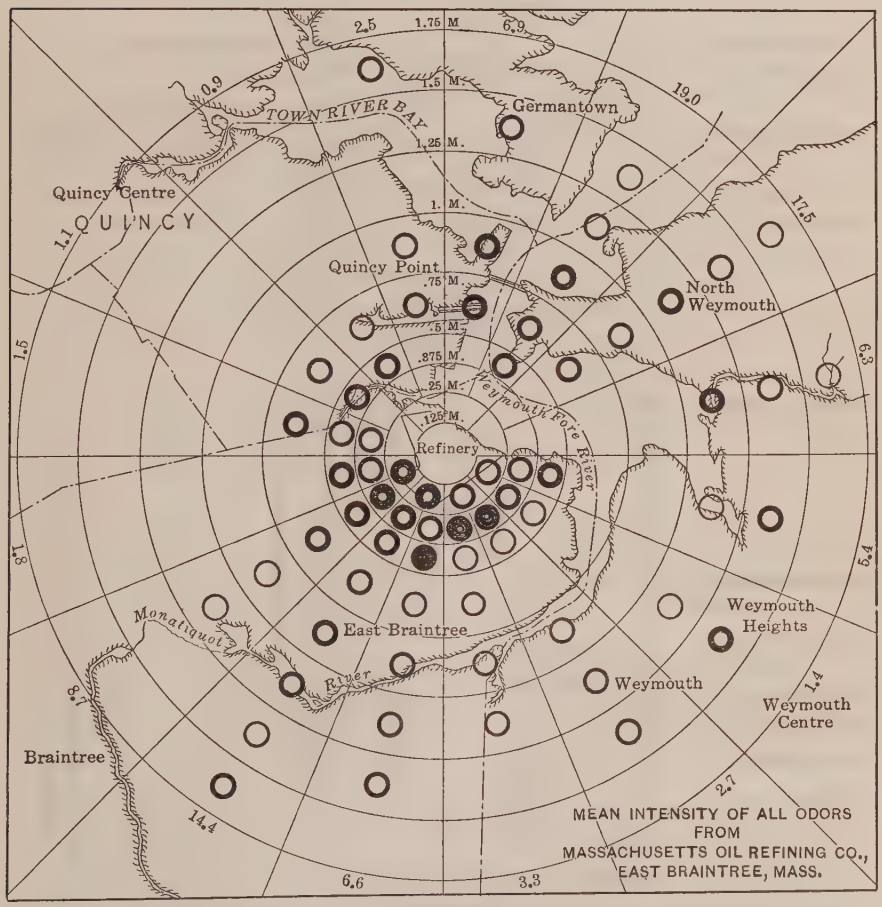


FIG. 6.

On the diagram referred to (Fig. 6), various data were recorded by the use of symbols, and the illustration is one showing the mean intensity of all odors in each sector of 22.5° and the relative mean intensities of these odors in the section of each annulus lying within the sector, the mean intensities in each sector being expressed by figures, and those in each section of an annulus by circles of varying heaviness.

Chemical Analysis of Air.—The modified Palmer machine (Fig. 7), which has been described previously by the writer* has been used quite successfully

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 420.

for the estimation of the amount of sulfur gases contained in the air. This machine is operated by a small electric motor, and unfortunately requires the installation of numerous lighting sockets, preferably on the poles of the electric lighting system, in order to operate it over any considerable area of country surrounding the refinery under observation. The stations at Fall River and East Braintree were located at the points indicated on the map-diagrams (Figs. 5 and 6) by numbered rings.

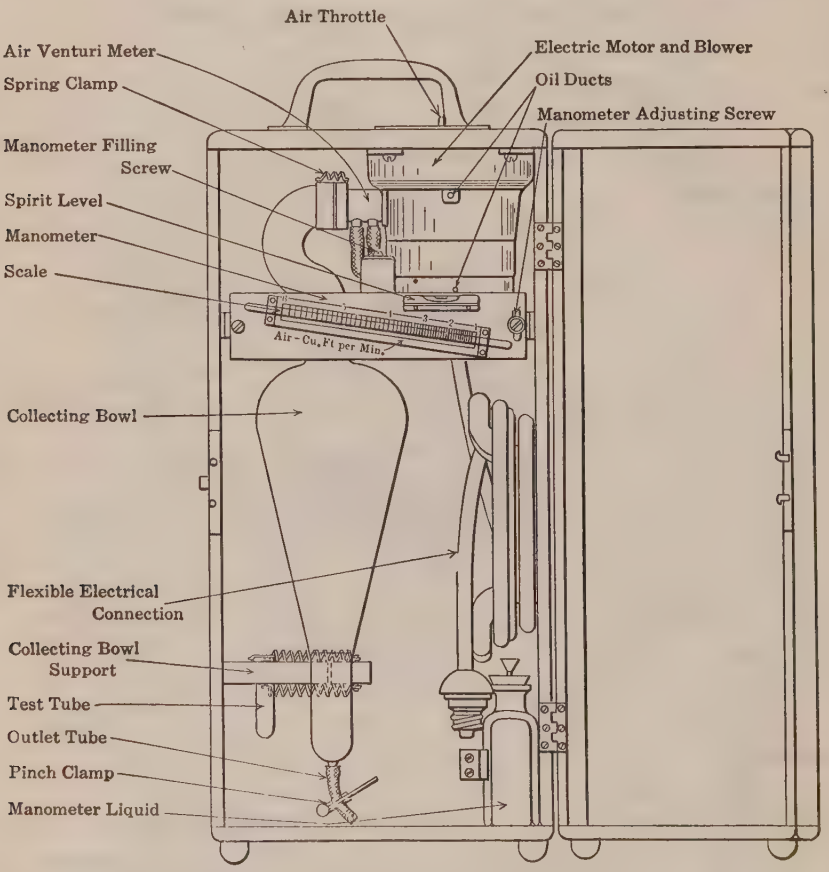


FIG. 7.—DUST SAMPLER, PALMER WATER SPRAY TYPE.

By means of the Palmer machine, 100 liters of air could be aspirated through nebulized caustic-soda solution and the sulfur gases absorbed determined by the usual chemical method. This method is similar to the one used by the copper companies in the West for the detection of sulfur dioxide in the gases emitted from smelters. Usually the concentrations of gas found in the air were very minute, although concentrations as high as 50 parts per 1 000 000 were observed in the vicinity of sludge acid tanks, but beyond a mile of the refinery conditions are unusual when 1.0 part of sulfur gas, computed as hydrogen sulfide, was found. Unfortunately, this method of absorption cannot distinguish between sulfur dioxide and hydrogen sulfide.

The presence of a faint burned-oily odor in the vicinity of a refinery, and extending some distance to the leeward, is to be expected, just as the same odor is observed to the leeward of any manufacturing plant using oil in the power house. It is desirable, however, that no other odor be present, and it is possible to conduct a refinery so that the presence of any other odor will be the result of accident only. How the control and elimination of these odors may be accomplished is described subsequently.

Control and Elimination of Odors

General Methods.—There are two general methods used to eliminate odors from petroleum gases and distillates. One method is to pass all the gases through a scrubber and absorb them in water which in turn is discharged into the nearest large body of water, usually through a skimming tank or separator. The objection to this method is that the water used to scrub the gases is as capable of liberating the gases under certain conditions as it is of absorbing them under others. Consequently, objectionable odors are frequently absorbed, only to be given off again later.

Petroleum distillates have been used to scrub the gases, in place of water, the aim being to re-distill the combination and recover a part of the volatile matter in the form of gasoline, thus increasing the gasoline production of the refinery. This method often fails because of the gaseous character of the compounds which refuse to remain absorbed in the finished product. Consequently the method only postpones the final disposition of the odoriferous compounds.

The best method yet devised for eliminating the gaseous petroleum compounds from the air, particularly those offensive ones containing sulfur and nitrogen, is to collect them by means of a vacuum system, and to burn them beneath a still or boiler. This method is not only practicable, but effects a considerable economy of fuel.

The danger in carrying out this process is one of forming explosive combinations of air and petroleum vapors. This can be guarded against, however, by placing Monel metal screens (Figs. 8, 9, and 10), at various convenient points in the collecting piping. These screens act upon the safety-lamp principle and prevent backfiring of the vapors.

Another requisite of the process is to provide for variations in the rate of gas emanation, for it is obvious that when a still is freshly charged, the rate of gas evolution from it is greater than later in the distillation process. On the other hand, the rate of burning gas beneath boilers is more nearly constant. Consequently, it is necessary to store the gas during the peaks in production for use during periods of low production. For this purpose, gas-holders similar to those used in illuminating gas-works are used. The gas-holders are connected with the gas mains, and maintain a practically constant pressure at the burners beneath the stills, furnaces, or boilers. A partial vacuum of a few inches of water is maintained throughout the system by exhausters similar to those used in illuminating gas practice. These exhausters must be of ample capacity to exhaust the gas regardless of the rate of production.

It is usual to provide two vacuum systems, one connected with the stills, from which gases rich in combustible material are collected, and another system connected with the tanks, the collections from which are comparatively weak in combustible gases. The collection system for the tanks is somewhat different from that for the stills in that some air is collected at all times.

Each tank is usually provided with a seal or trap (Figs. 11 and 12), having a water seal equivalent to a depth of water of about 1 in. Consequently, when the tanks are connected with the vacuum system, a slight partial vacuum is maintained at each tank where odors are likely to be produced. Exhausters on the gas main maintain the desired vacuum in the tanks. A higher negative pressure is maintained on the still gas lines.

With the whole refining process conducted under a partial vacuum, little or no petroleum gas can escape into the atmosphere unless its combustion beneath stills and boilers be incomplete. Figs. 13 and 14 show the types of burner used beneath the boilers of a refinery in Massachusetts.

Naturally different burners have to be provided for the combustible gases distilled from the petroleum than for the enriched air exhausted from the storage tanks. The first is utilized like any other gaseous fuel. The second is used as if it were a semi-inflammable air used to support and accelerate combustion. The difficulties of caring for either the petroleum gases or the enriched air are those connected with maintaining a proper temperature in the furnace. In the case of the enriched air, it is usually necessary to burn it in conjunction with fuel oil, and in both cases the construction of a suitable checker-work of brick to make sure that all the gases are raised to the point of ignition is necessary.

Occasionally stack gases will show evidences of having been imperfectly consumed in the furnace, and where oils high in sulfur are refined, may contain the objectionable nitrogen-sulfur compounds. The usual remedy in such cases is to raise the temperature of the furnace, although experiments made under the direction of the writer have shown that the introduction of a small amount of chlorine gas into the stack gases will so change their character that they will become unobjectionable. This process is the one so successfully applied by Professors Henderson and Haggard to garbage plants to overcome the disagreeable odors emanating therefrom.

Odors from Sludge Acid.—The disagreeable acrid odor given off by sludge acid is difficult to control. In some plants, the material is removed and covered with earth. This requires a large area, usually not available in connection with a refinery. The sludge is washed in water to recover the acid and the weak acid is concentrated by distillation in a closed system provided with devices for maintaining a partial vacuum by a method similar to that used in the petroleum stills. By the use of this apparatus, in which the weak acid may be agitated with superheated air, or heated in a retort, the emanation of disagreeable odors may be eliminated. Furthermore, the recovered acid may be re-used for refining oil.

Because the actual volumes of sludge acid discharged from a large plant are small, it is frequently advisable to ship acid from several refineries to a

FIG. 8.—TEE CONTAINING SAFETY SCREEN.

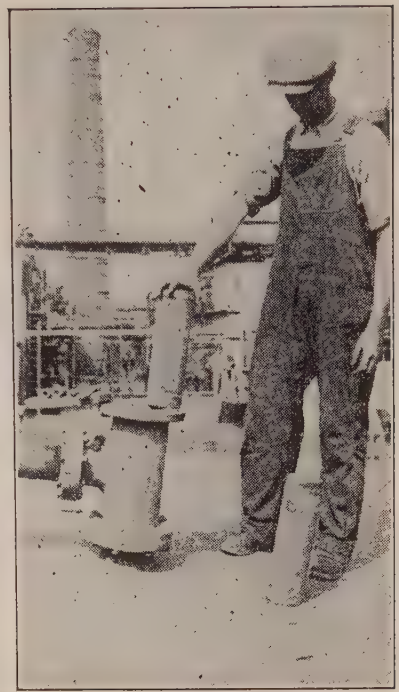


FIG. 9.—SAFETY SCREEN REMOVED FOR INSPECTION AND CLEANING.

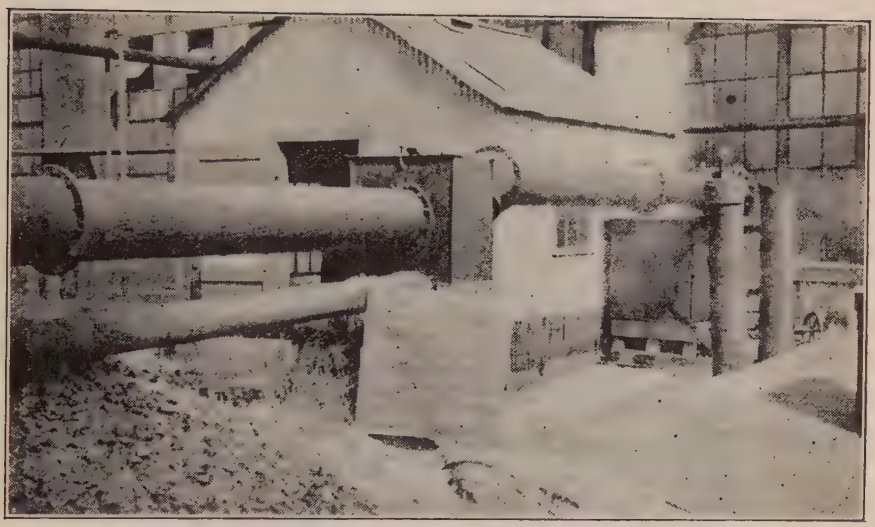


FIG. 10.—SAFETY SCREEN BOX AND SCREEN IN MAIN EXHAUST LINE FROM TANKS, ALSO CHAMBER FOR INTERCEPTING LIQUID.

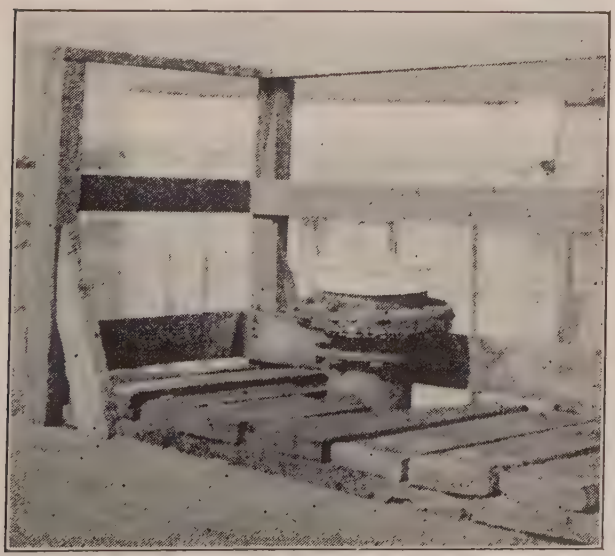


FIG. 11.—GAS TRAP, CLOSED AND IN SERVICE.

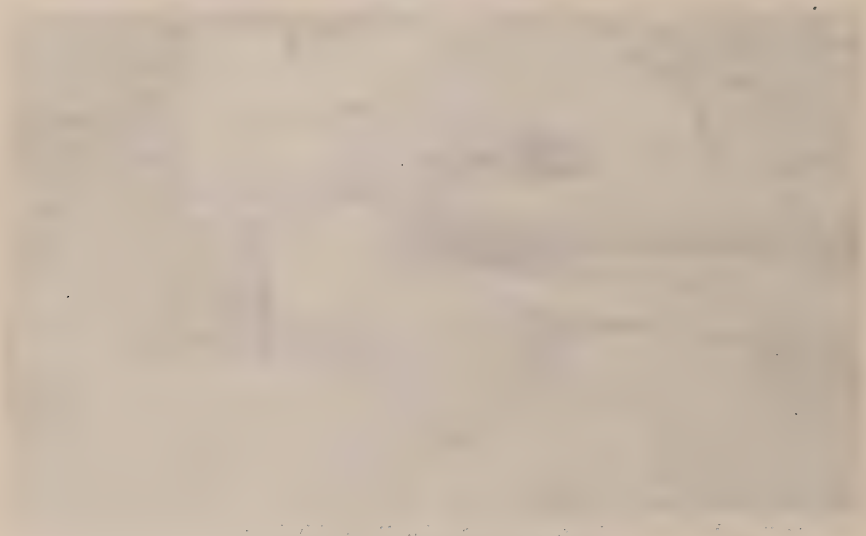


FIG. 12.—GAS TRAP ON CRUDE OIL TANK OPENED FOR INSPECTION AND FILLING WITH WATER.

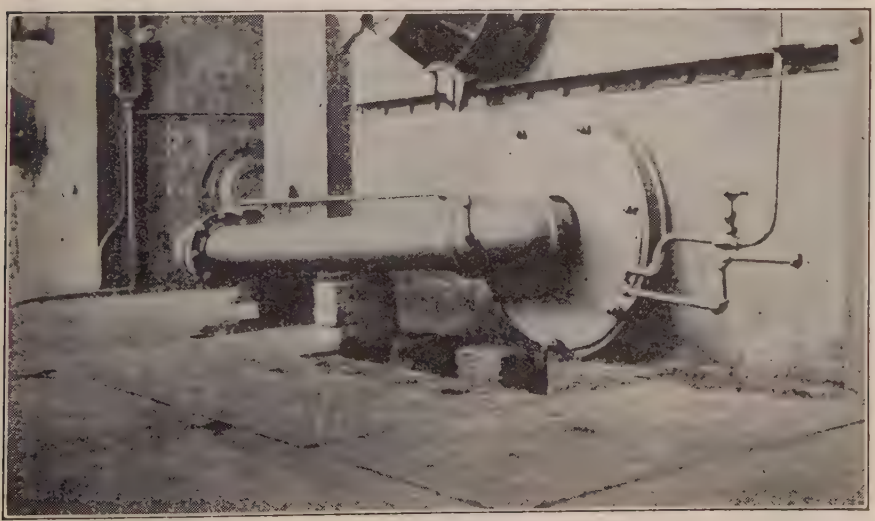


FIG. 13.—EXHAUST PIPE FROM TANKS DISCHARGING BENEATH BOILERS.

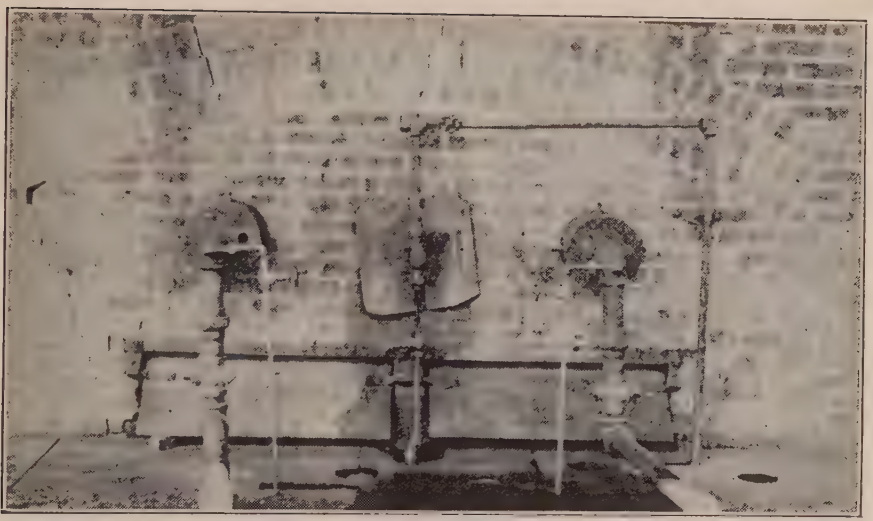


FIG. 14.—TWO GAS BURNERS AND ONE OIL BURNER (CENTER) FOR HEATING CRUDE OIL STILL.

FIG. 15.—VACUUM AND OTHER PIPING IN THE REAR OF A BATTERY OF CRUDE OIL STILLS.

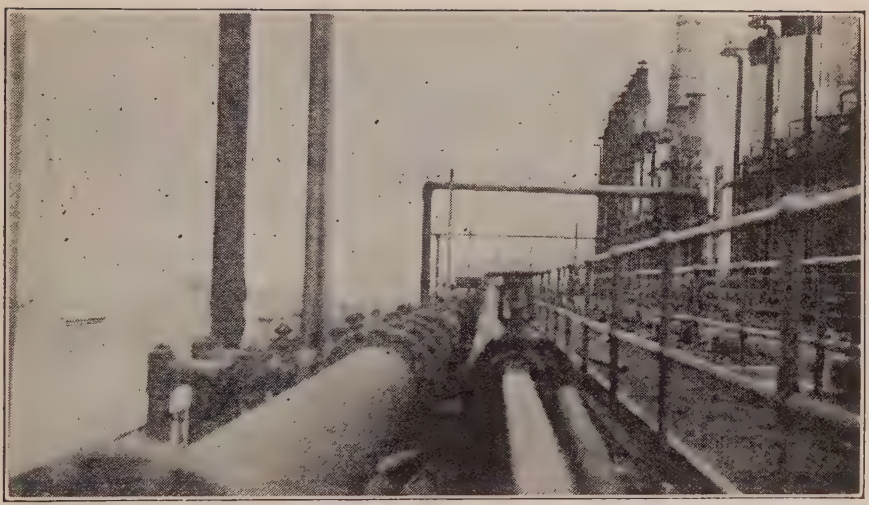


FIG. 16.—VACUUM AND WATER PIPING IN THE REAR OF DEPHLEGMATORS AND ABOVE CONDENSERS.

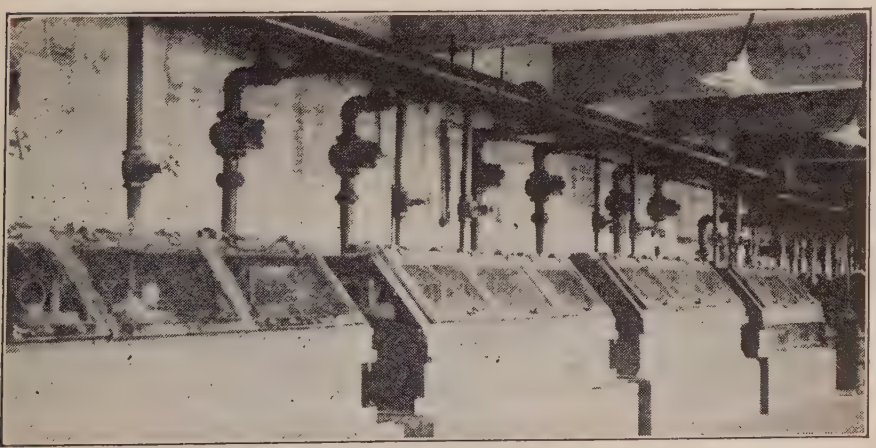


FIG. 17.—VACUUM PIPING AND "LOCK BOXES" IN RECEIVING HOME WHERE DISTILLATES ARE DISCHARGED.

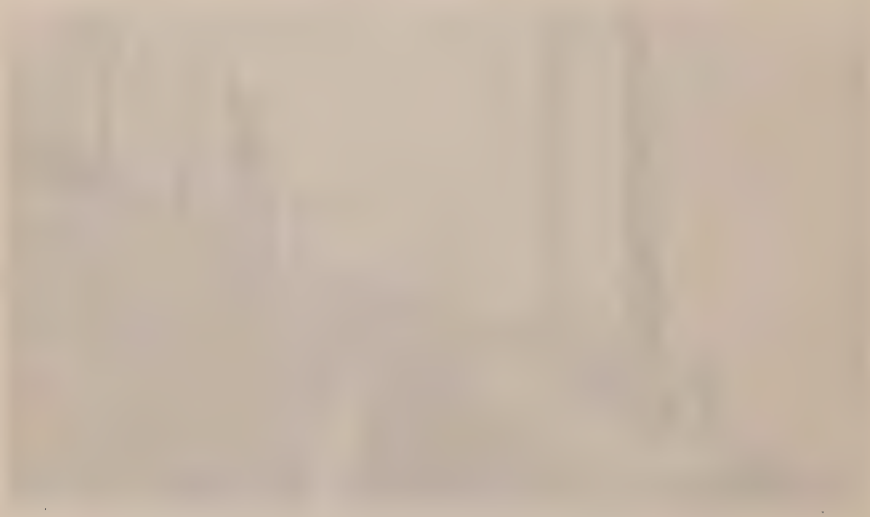


Fig. 18.—Trap in Vacuum Gas Line Between Stills and Exhauster.

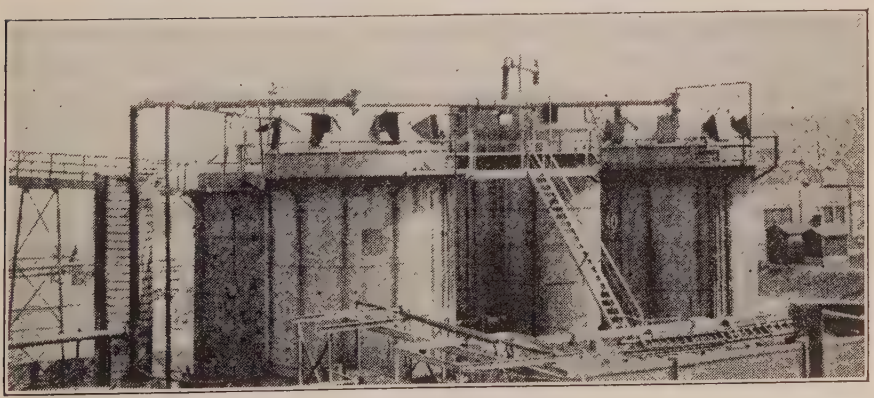


Fig. 19.—Agitators for Treating with Chemicals. Note Tank Covers and Exhaust Piping.

central plant for the reclamation of the acid. This plant may be located where there is the minimal danger of nuisance.

The Discharge of Oil into Watercourses.—It is probable that the befouling of shore waters and bathing beaches by discharges of oil has given rise to as serious complaints as the discharge of odor into the air. It is beyond the province of this paper to discuss this nuisance except to state that the discharges which come or might come from an oil refinery are readily controlled by skimming off separating tanks, the effluent from which should not contain more than 20 parts per 1 000 000 of ether soluble oil, the discharge of which in so great dilution into any considerable body of water may be made with impunity. Usually the befouling of shore waters and beaches is from shipping and sources other than oil refineries.

Although the principle of odor-control by operation *in vacuo* is simple, its carrying out is a matter requiring care and involving large expense for construction, maintenance, and inspection. It is not enough to have a single connection with each still or condenser, but each high point must be connected with the general system through a separate pipe, each pipe provided with a shut-off valve for use in case of accident. (Figs. 15, 16, and 17.) Furthermore, the workmanship must be of the best to avoid gas leakage. In addition, frequent traps (Fig. 18) for the interception of liquids are necessary, and also devices for separating the intercepted petroleum products without discharge into the air. Fig. 16 shows the extensive piping required to exhaust the gases from a battery of stills.

Not only must the stills be vented, but the use of open tanks for chemical treatment of distillates must be avoided. Fig. 19 shows the method of covering tanks and exhausting the gases arising from a group of agitators.

The control of the odors emanating from the skimming basin is seldom attempted. Fortunately these odors are not usually strong except when accidental discharges of the lighter distillate happen to occur. Usually the odors are imperceptible beyond the refinery yard. The method of guarding against their spreading is to construct a high fence about the basin to minimize the movement of air currents.

Naturally the water in a gas-holder absorbs and gives off disagreeable odors. These may be largely controlled by renewing the water in the gas-holder and by preventing the circulation of the exposed surface water as much as possible. In some cases the addition of chemical absorbents has been practiced, but not generally.

Summary

Briefly, it may be stated that unless controlled, the odors emanating from an oil refinery may create a nuisance. These odors are due to various vapors and gases emitted during the process of refining, and the best method of controlling and eliminating them is to collect them at all possible points of discharge by means of vacuum-collecting systems, to store the gases so collected, and to discharge them into properly designed furnaces in which, with or without the aid of other fuel, they will be completely consumed.

Travelers have reported that in the "Black City" near Baku, in Russia, where there are 200 refineries which treat the whole of the petroleum of the district, the odor of petroleum is perceptible at a great distance and the town is always covered and surrounded with dense, black smoke. It is obvious that this condition ought not to exist in the vicinity of refineries which are near large centers of population in the United States, and they should be controlled by proper methods, even if their installation may cost 10% as much as the total cost of the refinery construction. It is fortunate, on the other hand, that the investment in a gas-collecting system is compensated in great part at least by the large saving in fuel due to the utilization of combustible gases for heating stills and boilers.

Air cleaning, like river and harbor cleaning, is a problem connected with the concentration of population and has become an art which all communities that wish to maintain a high degree of civilization, with health and happiness for all, must practice.

In conclusion, the writer wishes to thank the officers of the New England Oil Refining Company, and particularly Mr. D. T. Schulz, General Superintendent, for permission to publish certain information, and assistance in preparing the illustrations in this paper.

DISCUSSION

JOSEPH A. WARREN,* ESQ.—Except in England and the United States the system of law prevailing among all the civilized Caucasian races is what is known as the Civil Law. That law takes general abstract principles and applies them to each state of facts as it comes up. It makes for rigidity, and it makes for certainty.

The American system of jurisprudence, however, is known as the Common Law, and that takes a given state of facts as it happens and applies to the interpretation of such facts the custom and the general understanding during past and present times of the community in general. As can readily be seen, under the latter system the law is continually in a state of flux, and it is very difficult to say in advance what the Courts are going to do in regard to any given state of facts. A very striking example of that is had in this very law of nuisance.

As stated by Mr. Goldsmith, in the early cases the law looked at the injury to property almost exclusively. It was the damage to an individual's property that laid the foundation of all action, and that was true not only with respect to the law of nuisance, but generally. The old common law looked at the damage to property—damage to an individual suitor. With the change of the customs of the people, with the growing civic consciousness, there has been a gradual getting away from the emphasis formerly placed on property damage, and the Courts look more to the interests of the community at large than to those of the individual property owner; and if the individual property owner's rights interfere or injure the community at large, the Courts are very active in finding some way of serving the general interest even at the expense of the individual property owner.

The most striking example of that attitude in recent years is the upholding of the rent laws of New York by the Court of Appeals of that State and by the Supreme Court of the United States. Formerly, the emphasis was on the injury to property, but now most emphasis is placed on the health and interest of the public or of any considerable part of the public.

In the same category is the rule mentioned by Mr. Goldsmith that one cannot move into a nuisance. At common law, if a business existed, that might be a nuisance, and any individual who, after the establishment of that nuisance, moved into the zone of influence knowing it was there, had no remedy. Where emphasis was on the property right, this was only fair; but that law is almost extinct.

Where a nuisance exists, because a community has grown up around it, the law, not to protect any individual who has moved there knowingly, but to protect the community, will make available the remedies that exist in the case of nuisance.

There was a rather striking example of the way this law works in the Edgewater case, referred to by Mr. Goldsmith. It is true that New York State in 1916 started suit in the United States Supreme Court against certain

* New York, N. Y.

companies in the State of New Jersey; but it is not quite accurate to state that the settlement that has resulted came from the litigation. Consider the peculiar problem there. One of the finest residential sections of the great State of New York was directly opposite those factories. Some of the factories were there before the upper part of Riverside Drive had developed into a residential section. All these factories were engaged in a kind of business that gave forth fumes, odors, and smoke. They were in another State. Jurisdiction of them could not be obtained in the State of New York. They were large commercial enterprises—the Corn Products Company, the General Chemical Company, the Barrett Manufacturing Company, and others of almost equal magnitude.

Officials started dealing with the problem in 1908. Nothing very definite was accomplished until 1917, when the Legislature of the State of New York passed an Act (Chapter 292 of the Laws of 1917) that gave to the State Commissioner of Health power to investigate those alleged nuisances and, if he found nuisances that affected the life, health, and safety of the people of the State of New York, to file in the office of the Secretary of State a certificate to that effect. If within thirty days those nuisances were not abolished, the charters of those corporations in the State of New York, if they were New York corporations, or their certificates to do business in the State of New York, if they were foreign corporations, would be automatically revoked, and receivers appointed for their property. In other words, a sentence of capital punishment was imposed on those corporations for maintaining nuisances. It would have meant the destruction of certain of those great corporations.

The remarkable thing about that legislation is that no one raised the question of its constitutionality, drastic as it was. Under the direction of the then Commissioner of Health, Dr. Biggs, elaborate investigations were conducted in the first instance to ascertain the source of the particular trouble in the City of New York, and then remedies were sought and suggested. Over a period of two years the hearings went on, and a solution was gradually worked out, so that, now, with a few occasional and rare exceptions, that nuisance in the City of New York is practically abolished.

In this proceeding the so-called psychological effect of nuisances was given much consideration. Probably no one went further in holding what might affect the health of the people than did Dr. Biggs in that particular case. The matter was adjusted so that the Courts have never had occasion to pass on Dr. Biggs' ruling, but without doubt they would have sustained him, particularly on the evidence in that case. It was held unnecessary to prove, for instance, that a considerable number got sore throats, or got any particular disease; but if the odors, the fumes, so disturbed residents that they lost sleep and that their general health and vitality was lowered because of this disturbance, this alone was sufficient ground to exercise the health authority of the State.

This whole case well illustrates the serious propositions that will come before sanitary engineers as the law develops in the future. In the first place, before this bill was passed various remedies had existed for years. There was the remedy by abatement in the Courts; the remedy by injunction in the

Courts; action for damages in the Courts; and the machinery of the Health Department for the direct suppression of nuisance without resort to the Courts. It is thought that in the future the remedies will be modeled on the last method, that is, abatement of nuisances through the administrative powers of the State rather than through the judiciary.

The Edgewater case further illustrated the fact that, when the present remedies are not adequate, either the legislatures or the Courts are going to devise some method that will be adequate to control the situation; and the tendency in all cases nowadays is to assign the remedy to the health authorities. Now, health authorities are rather uncertain; some of them are rather extreme. One can imagine that a very serious situation would have arisen in the Edgewater case if Dr. Biggs had been an arbitrary and radical Health Commissioner who at once, without proper hearing and consideration, would have endeavored to solve the difficulty by applying the very drastic measures of that law.

It is well to remember this—when the legislature refers to a health authority the determination of a question of fact as to whether a certain business does constitute a nuisance injurious to the health of the people, the determination by that health official will hardly ever be overturned by the Courts.

This experience suggests, therefore, that when any considerable commercial enterprise is being considered, not only the immediate vicinity of the location of the plant should be carefully investigated, but the whole probable zone which might be affected by any of the processes that are proposed to be conducted.

Further, in view of the fact that a future development may make a nuisance out of what previously was not a nuisance, it seems that, in so far as is reasonably possible, an investigation should be made as to the probable development of the locality, because, once a plant is established and developed, a situation may easily arise where the whole plant would have to be torn down and moved to some other location, or where the State might have to do it.

Charles A. Holmquist,* Esq.—In presenting this paper Mr. Gage has rendered a valuable service to the Sanitary Engineering Profession, more particularly to the Engineering Divisions of State Departments of Health which are perhaps called upon to deal with problems of this kind more often than any other organization. As indicated by Mr. Gage, nuisances created by odors are among the most difficult problems with which health departments have to deal, owing largely to the intangible and elusive nature of the subject and to the lack of scientific or standard means of measuring quantitatively the magnitude of such nuisances. The detailed description of the procedure and methods used in the investigation conducted by Mr. Gage of the alleged nuisance created by certain oil refineries in the Providence Metropolitan District should be very helpful to those who may be called upon to carry on similar investigations. The development of the formulas to determine the magnitude and degree of a nuisance is a long step in the right direction. The value of such formulas, however, would be considerably increased if

* Director, Div. of Sanitation, State Dept. of Health, Albany, N. Y.

numerical values could be given to intensities of odors similar to those given to tastes and odors of water supplies that are now in general use by water analysts. The speaker realizes, of course, the difficulty of doing this. In the first place there is no such thing as a standard nose. No two persons would probably re-act in exactly the same way to the same odor due not only to the physiological but also to the psychological difference of individuals. In fact, an odor that is objectionable to one person may be unobjectionable and even pleasing to another. Furthermore, all so-called odors are not true odors and do not stimulate the same sets of nerves. True odors, such as artificial musk and oil of roses, affect the cells of the olfactory nerves in the nasal cavity, whereas sulfurous acid and similar substances have no true odor, but are irritants which affect the ends of the trigeminal nerves. Other so-called odors affect both sets of nerves.

In view of the rapid increase in the number of industrial developments in centers of population, the ever-increasing number of sewage, garbage, and waste disposal plants in this country which are likely to give rise to nuisances due to the emanation of objectionable odors and the growing demand for higher standards of sanitation, it is evident that this problem will have to be given more attention in the future than in the past and that this subject might well be worthy of study by a committee. A great many tests have already been made to determine the concentration of different chemicals and other substances that are detectable in given quantities of air, and their relative intensities, but additional studies would be necessary to give definite values to such odors.

Mr. Gage points out that he has dealt with the methods of determining the cause, scope, and effect of odor nuisances and has left the discussion of methods of odor elimination to others. Considering the large financial investments and the great number of persons that are usually involved, remedial phases of the problem in most cases will be very important. The speaker will endeavor to discuss these phases of the problem from the point of view of State health departments and will cite a few instances that have come under his personal observation where odor nuisances have been successfully abated. In fact, it has been the experience of the New York State Department of Health that there are relatively few odor nuisances that cannot be abated by changes in operation of the plant, or the treatment, utilization, and recovery of waste products. Very often the abatement of such nuisances results in increased financial returns and sometimes in more sanitary and healthful conditions in the works themselves.

It has been the consistent policy of the Department in dealing with odor nuisances to co-operate as far as possible with the industrial plants in abating such nuisances and to issue orders and start Court proceedings only as a last resort. This spirit of co-operation is usually appreciated by the manufacturers and meets with their hearty response. Recommendations for improvements have often been carried out with relatively little delay. The owners of many of the plants investigated have probably felt, and some have admitted, that it would be cheaper in the long run to make voluntarily the improvements or

changes suggested by the Department rather than to take the question to the Courts with the possibility that the improvements would ultimately have to be made at their direction. The following cases are some of the odor nuisances that have been abated through the efforts of the State Health Department without the necessity of issuing orders or bringing action in the Courts.

Carbon Disulfide Works.—These works were situated in a narrow valley extending in a generally northerly and southerly direction. Complaints were received from residents of the valley that whenever the wind was from the direction of the chemical works very objectionable and stifling odors, fumes, or gases, were noted and that the fumes discolored paint on houses more than a mile from the works. From the investigation, it was found that the trouble was caused almost entirely by the liberation of hydrogen sulfide (H_2S) in the process of manufacture of carbon disulfide and that houses in exposed areas within 1½ miles of the works were discolored, especially those painted with lead paints. The carbon disulfide (CS_2) was made in the usual manner by passing sulfur vapors through charcoal heated to a cherry red in iron retorts. The uncombined sulfur vapors were conveyed to iron vessels in which these vapors were condensed and the sulfur was recovered. Some sulfur dioxide also escaped when the retorts were cleaned, but this gas did not appear to be an appreciable factor in the creation of the nuisance which was caused almost entirely by the liberation of hydrogen sulfide.

When the plant was first visited an attempt was made to eliminate this gas by passing it through a tank into which lime-water was pumped. Although some of the hydrogen sulfide was probably eliminated by absorption by the water and by reaction with the lime in the production of calcium hydro-sulfite ($Ca\ H_2\ O_2 + 2H_2\ S = Ca\ H_2\ S_2 + 2H_2O$), this method was not effective in abating the nuisance and was abandoned. Another attempt was made to eliminate the hydrogen sulfide by passing it through a wooden tower packed with lime and iron oxide, but the tower was found to be too small and the heat of reaction developed was sufficiently high to set the tower on fire. Finally, the nuisance was abated by passing the hydrogen sulfide through a Claus kiln consisting of two condensing chambers and an exit tower operated in series. The reaction chamber contained a layer of iron oxide through which the hydrogen sulfide mixed with a definite quantity of air, was passed after the oxide had first been heated to a dull red. This caused the reaction, $H_2S + O = H_2O + S$, to take place. The sulfur vapors were condensed in the second chamber of the kiln as liquid sulfur and in the third chamber as flower of sulfur. The water passed off through the exit of the third chamber in the form of steam. Some sulfur dioxide was carried over with it, but not in sufficient quantities to give rise to a nuisance. This installation resulted not only in the abatement of the nuisance, but also in the recovery of considerable sulfur which had formerly gone to waste.

Acetic Acid Plant.—Acetic acid was manufactured at this plant by distilling grey acetated lime with sulfuric acid. A pungent acetic acid odor sufficiently strong to cause a very uncomfortable irritating sensation in the nose and throat was found in the building in which the acid was made, and this odor was carried a considerable distance from the plant. A similar pungent

pyroligneous odor resembling acetic acid was also given off from the residue from the acetic acid stills and this odor seemed to carry very much farther than the acetic acid odor. When this plant was first inspected it was customary for the company to dump the residue from the stills on the floor of the still house, shovel the residue into dump carts, and haul it to a dump on the water-front where it would be left until a barge load or two had accumulated. This dump seemed to be the principal source of the odors.

The nuisance was abated by collecting and scrubbing the air in the building in which the acetic acid stills were located, by cleaning up the water-front, and by installing mechanical conveyors by which the residue from the stills was conveyed through covered chutes to covered gondola cars on a railroad siding outside the plant. The cars were removed as soon as they were filled. The scrubber through which the air from the building was passed consisted of a wooden tub about 10 ft. in diameter and 10 ft. high, filled with coke. Numerous sprays of water played continually on the surface of the coke, and the exit from the scrubber was connected to the stack of the power-house plant which was about 200 ft. high. Although no waste products were recovered, there was an appreciable saving in the cost of handling the residue by the mechanical conveyor.

Sulfuric Acid Plant.—In one plant of this kind against which complaint was made, the sulfuric acid was manufactured from pyrites by the contact process. Objectionable conditions were created by the escape of sulfur dioxide and a reddish dust from the burned pyrites which was stored on a dump outside the plant. Sulfur dioxide was found to be escaping from the exits of the absorbing towers and was so strong as to be almost unbearable within a few hundred feet of the plant when in line with the wind from the exits. The nuisance from this source was abated by utilizing the waste sulfur dioxide in the manufacture of sodium bi-sulfite. The exits through which the uncombined sulfur dioxide from the sulfuric acid plant formerly discharged into the atmosphere were closed, and the gas was conveyed to an adjoining building where it was passed through absorption towers through which a concentrated solution of sodium carbonate was circulated which resulted in the production of sodium bi-sulfite. About 80% of the sulfur dioxide was absorbed in these towers and a large part of the remainder was removed by scrubbing the exit gases from the bi-sulfite plant with water. The dust nuisance from the pyrites dump was eliminated by spraying the dump with water. Crude sulfur was later substituted for the pyrites in the process of making sulfuric acid.

At another plant where sulfuric acid was made by the chamber process, the acid was concentrated in platinum stills. The gases and acid escaping from the stills carried for a considerable distance and gave rise to a serious nuisance. These conditions were abated by changing the method of concentration and by installing a Cottrell machine for the condensation of the acid fumes.

Linseed Oil Works.—Characteristic odors given off from the process of making linseed oil although very pronounced near the works did not seem to carry any considerable distance from them. Very pungent odors, however, were given off from the process of boiling or aging the oil. This was done in covered steam-jacketed kettles in which the oil was heated to a temperature of

about 350° Fahr. for several hours while air was blown through the oil. When the oil works were first visited the oil vapors from the boiling kettles were conveyed to the breaching of the boiler plant. Owing, however, to the relatively low temperature at this point, and probably also to deficiency in oxygen, the oil vapors were not consumed but were simply scorched. Although the resulting odors could not be detected on the premises, they were very objectionable at a distance of a mile or more from the works. This nuisance was abated by connecting the vent pipe from the kettles with the firebox of one of the boilers. This seemed to produce complete combustion and eliminate the nuisance.

Coal-Tar Products Plant.—At a plant where coal-tar products were manufactured, large volumes of yellowish pungent oil vapors were given off when the stills were blown into the coolers. This nuisance was abated simply by closing the manholes on the coolers and installing vents by which the vapors were conveyed to coil condensers.

Starch and Corn Products Plant.—In this plant the corn was cleaned and then soaked or steeped with water containing sulfurous acid for the purpose of softening the kernels. After steeping, the corn was passed through cracker mills where it was partly crushed and the germ removed from the body of the kernel. The germ was separated from the remainder of the corn by gravity in shallow tanks filled with water and was dried in rotary driers, and ground, heated, and pressed to produce corn oil. The corn from which the germ had been removed was then ground in burhstone mills, and the mixture was washed and passed through fine mesh screens and silt reels a number of times in order to separate the starch from the gluten and other fibrous matters. The gluten and fibrous matters were passed through mechanical squeezers to remove the water, after which they were dried in rotary steam-heated driers to form gluten feed. The starch was settled out in long inclined troughs and then dried. Glucose was made by boiling starch paste with mineral acid, neutralizing the acid and evaporating the liquid.

A number of gases and odors were given off from the process of manufacture, including sulfur dioxide and sulfurous acid from the sulfur burners and the sulfurous acid plant and from the screen and reel rooms. A peculiar odor was also given off when the contents of the glucose retorts were blown into the coolers. The odor that seemed to cause the most trouble, however, was that discharged from the rotary driers where the gluten feed and the germ were dried. There were twenty-two of these driers which were vented into eleven large wooden stacks. Vapors approximating 40 000 cu. ft. per min. were discharged from these stacks. The odor of these vapors although difficult to describe resembled a combination of acid and organic matter not much unlike the smell of sour bread. These odors carried for a considerable distance and seemed to be nearly as intense a mile or two from the works as they were at the source. Considerable chaff was also discharged from the drier stacks with the vapors. A large part of this chaff was deposited on the adjoining buildings and premises, and some of it was carried long distances by the wind.

It was first thought that these odors could be eliminated by collecting them and passing them through a large water scrubber. Owing to the structural difficulties involved and the very large-sized scrubber necessary to care for the

vapors, tests were carried on with a new type of scrubber. This consisted of two concentric cylinders connected by a spiral plate arranged so that the vapors would take a spiral course in traveling from the inlet to the outlet while water was sprayed into the space between the two cylinders in such a way as to keep the surface against which the vapors came in contact wet. The vapors were forced through the scrubber by a fan at a velocity of about 5 000 ft. per min. While these tests were being made, an incident occurred which indicated very clearly that scrubbing the vapors by water alone would not eliminate the nuisance. One day while observations were being made at a point about a mile from the starch works, a very severe rain storm occurred. In order to determine the effect of the rain on the odors, the observer continued his observations in the rain for about 30 min., and it was found that, although the odors passed through a natural mammoth scrubber about a mile in length, they were not appreciably diminished in intensity.

An attempt was then made to deodorize the vapors by applying different volumes of chlorine gas to the base of the drier stack but, although sufficient chlorine was applied to be distinctly noticeable a considerable distance from the plant, the odor was not reduced to any appreciable extent. This was probably due in part to the inefficient mixing of the chlorine gas with the vapors and the short time of contact, but largely to the inability of the chlorine to deodorize or oxidize the particles of chaff carried out with the vapors. Tests were then made with the application of chlorine gas in connection with the experimental scrubber. Chlorine was applied first to the inlet and then at the outlet of the scrubber. Although the application of the chlorine gas to the inlet of the scrubber seemed to have no marked effect on the odors, its application near the outlet of the scrubber, but ahead of the fan, seemed largely to eliminate the odors, especially when sufficient water was used practically to condense the vapors and remove the chaff.

The company found, however, that the scrubbing and chlorination of the large volume of vapors given off from the exits of the driers would involve a considerable expenditure and would be costly to maintain and, therefore, abandoned that project. The vapors were finally collected and after passing them through scrubbers of the type experimented with, in order to remove the chaff which would otherwise have collected in the conduit, were conveyed to the forced-draft conduit of the power plant where they were mixed with about 200 000 cu. ft. of air per min., and burned. The burning of these vapors under the fireboxes of the boilers and the installation of a condenser on the exits of the glucose blow-tubes seemed to abate the nuisance effectively. Although no by-products were recovered as the result of the abatement of this nuisance, the sanitary conditions around the plant were improved appreciably, and it is no longer necessary to employ men to clean up the chaff which formerly settled on the roofs of adjacent buildings.

Many other cases could be mentioned but a sufficient number have been cited to indicate, in general, the nature of the problems involved in abating odor nuisances. Some of the problems are very simple; others are more complicated and difficult, but most of them can be solved by the intelligent application of chemistry, engineering, and common sense.

GEORGE H. SHAW,* M. AM. SOC. C. E.—As Chief of the Division of Housing and Sanitation in the Department of Public Health of Philadelphia, Pa., the speaker has encountered conditions similar to those described by Mr. Gage and Mr. Holmquist.

Philadelphia, with 2 000 000 inhabitants, is one of the largest manufacturing cities in the United States. In many places, manufacturing plants are located in close conjunction with residences; probably the conditions in this respect are as bad as those in any city in the country.

One of the serious nuisances is the manufacture of red oxide paint. In one large plant sulfurous acid is driven off and discharged from a tall stack spreading the fumes over the neighborhood. Everything possible was done to get the management to control this nuisance without success. It was finally necessary to take the case to Court and secure action requiring the manufacturers to abate this nuisance, which they did by discontinuing the "roasting" of the raw material in Philadelphia and transferring that operation to another city.

Another troublesome type of plant is that used for rendering. In one case it was necessary to institute criminal proceedings. Realizing that the law was sufficient to compel it to correct this condition, the company came to terms and spent a considerable sum of money installing control apparatus to abate the nuisance. Since then there has been no trouble.

Packing-house supply companies now sell rendering apparatus so constructed that the odors do not escape into the atmosphere. The apparatus is air-tight, and has an outlet to the sewer; so there is no further trouble with the cooking odors from open jacketed kettles. Many improvements in the handling of material have also been developed by packing-house engineers.

Another nuisance-producing industry that has given a great deal of trouble in Philadelphia is the oil refinery. In one large refining plant, sulfuric acid is used, which causes a nuisance when it is being recovered by distillation. To correct this condition a Cottrell electric precipitator was installed and has given satisfactory results.

In Philadelphia, there is a nuisance that Mr. Holmquist did not mention, that is, one due to the manufacture of electric storage batteries. Even workmen living close to such plants, who liked to sit on their porches in the evening and enjoy the fresh air, could not do so because of the presence of irritating acid fumes given off from the "forming room", where battery plates are manufactured or "formed" by electrolytic action. The co-operation of the company was secured and satisfactory control apparatus was developed. The air from the "forming rooms" is forced by fans through a condensing chamber filled with staggered fluted glass plates. The air, as it is drawn from the "forming room", goes through a tortuous passage, depositing the acid on the glass plates whence it trickles down the fluted surface and is collected at the bottom. A considerable quantity of acid is thus recovered.

It is believed that nuisances of the types mentioned have an indirect if not a direct effect on health and that their regulation properly comes within the

* Philadelphia, Pa.

"police power of a community." Experience has shown that they can be corrected and that rarely is it necessary to take drastic Court action. Helpful co-operation with the offending plant usually brings the desired result.

L. L. Tribus,* M. Am. Soc. C. E.—Mr. Gage has brought out one of the important features in considering odors, namely, the psychological effect—the effect that may be positively injurious and constitute actual legal nuisance. This was discussed somewhat in the speaker's paper entitled "Odors and Their Travel Habits."†

There is a peculiar difference in the effect of odors on different individuals, their personal equation. An odor might well be profoundly disturbing to the whole nervous system of some strong man, and, at the same time, be fully tolerated by some otherwise weak woman.

Another condition might be considered. How long does an odor affect an individual? Does the effect continue after the actual atoms have become dissipated, and memory reproduces the condition? The speaker thinks that it does, that even nausea can be reproduced long after the original smelling of the odor, if the first experience had caused such a condition.

John V. Lewis,‡ Esq.—Mr. Greeley brings out two points of prime importance which must be apparent to every sanitary engineer who has been concerned with the problem of refuse disposal at some time or other in his career.

The first point is the need for a collective technical effort and thorough engineering study of this ever-perplexing and acute problem which almost every growing municipality now faces. With due respect to those who have "blazed the trail", it is to be regretted that the same combined effort has not been accorded the disposal of rubbish, garbage, or mixed refuse, as has been given to sewage disposal. Had that been done, it is safe to say that the status of refuse disposal in its entirety would be quite another story.

The second point—and indirectly it is really a part of the first one—is that it should be understood that the problem, particularly as regards the control of odors, is one of engineering in its broader sense, namely, the education of the public and the injection of economics, just as much as the pure and applied sciences involved.

As mention has been made by Mr. Greeley of the Rochester, N. Y., plant as one of the most approved design and free from complaint as to obnoxious odors, some additional information may be of interest. From 1907 to 1921, the garbage of the city was reduced in a so-called modified Arnold plant. It was first cooked in large vertical steel digesters, and the resulting mass was then passed through hydraulic presses for the recovery of the grease. Drying and subsequent percolation were not used. The pressed tankage was sold for fertilizer.

The speaker is informed that the odors emitted from the plant, situated as it was in a commercial and industrial district about ¾ mile from the business center of the city, were the cause of numerous complaints during the period

* Cons. Engr. (Tribus & Massa), New York, N. Y.

† *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 378.

‡ San. Engr., in Chg. of Refuse Disposal, Dept. of Public Works, Rochester, N. Y.

of operation. Most sanitary engineers are quite familiar with the difficulties which beset any one who advocates a new location for a garbage or mixed refuse disposal works, once it has been previously established. Thus, in constructing the new Rochester Reduction Plant, which began operation in 1921, the so-called straight Cobwell system was adopted as that which would best eliminate the cause of previous complaints and enable the existing site to be utilized. This site, which is at the geographical center of the city, greatly influences the haulage and several other important questions. The speaker believes that the success of this new plant and the failure of a similar one on Staten Island, New York, was essentially a question of geography and economics and not a lack of understanding of the technical side of the problem itself. The City of Rochester has the most up-to-date and sanitary garbage plant of its size in the country to-day, free from objectionable odors which may be the source of formal complaint. There is a constant endeavor to exercise technical control of the process used; to reduce still further the slight odors which do exist; and to place the collection system and the plant on an economic business basis. Such an attitude and effort were intended by those responsible for the inception of the new system.

As Mr. Greeley has pointed out, the control and possible elimination of odors from garbage disposal works must begin at the garbage pail itself, continue through the collection and haulage system, and end finally in the disposal works proper. If either of the modern sanitary methods of disposal called incineration and reduction are used, every effort exerted to collect the raw material in its fresh state and with the least water content means better control of odors and a reduction in the operating costs, both as regards haulage and the method of final disposal.

When through proper education of the public and diligent co-operative engineering effort, the collection and disposal of mixed refuse or garbage can be said to approach present-day methods for the collection and treatment of sewage, the problem will be nearer to a satisfactory solution.

I. S. Osborn,* M. Am. Soc. C. E.—In his paper on the elimination of odors from garbage disposal plants, Mr. Greeley brings out the fact that each stage in the treatment has a bearing on the subject as a whole. The house treatment, methods of collection, types of equipment, and methods of disposal are so interrelated as to require consideration of each.

If the most desirable solution from a sanitary standpoint is expected all phases of the problem must be considered. Mr. Greeley has indicated that the location of disposal plants in some cases should be determined by the type selected, and that the choice of a plant emitting odors requires an isolated site, whereas one free from odors may be nearer the center of production. Too little attention is given to the selection of the type of plant which can be erected in a central location when other factors enter and appear to dominate or control, such as length of haul, or cost of delivery of the refuse, and possibly ownership of the site where the right to operate has been established.

* Vice-Pres. The C. O. Bartlett & Snow Co., Cleveland, Ohio.

Two cases of this kind have come to the speaker's attention. In one case, the accepted location increased the cost of haul by $30 000 per year over that of another available site, although the site selected had no advantages in its surroundings over the one rejected. In the other case, a site outside the city limits was selected and by so doing the hauling costs were increased approximately $50 000 annually over what would have obtained if another available site had been chosen. In each case the type of plant adopted was such that it could be operated in many locations without producing odors and without giving rise to complaint.

In many cities, the type of disposal plant is selected before a site has been chosen on which the plant can be located. However, the plant at Rochester, N. Y., mentioned by Mr. Greeley, was selected by a method directly opposite. The city owned the site and selected the type of plant to meet its surroundings. No other reduction plant is so well situated from the standpoint of cost of delivery of the garbage to the plant.

Mr. Greeley's statement with reference to the location of a reduction plant in isolated places in order to lessen the necessity for odor control is hardly supported by the facts. There is not a reduction plant disposing of garbage in an isolated location, and causing odors, to which those in the community where it is located do not object. There are, no doubt, fewer complaints due to the smaller number of people affected. In some cases the few tolerate the nuisance rather than assume the burden incident to a protest.

The establishment of districts under a zoning law, where odor-producing industries may be located, is done to protect property and industry. If plants, such as packing houses, rendering plants, and garbage disposal plants, are permitted, it is done to protect the industry as well as the public and not to legalize a nuisance from plants producing odors which may be eliminated.

With the Court rulings which have been made there is a question whether, even in an isolated location, the operation of a plant giving off odors will be permitted unless it is conducted in a manner to create as little nuisance as possible.

In discussing incinerators, Mr. Greeley states that "ample combustion and dust-settling chambers" should be provided, and that "satisfactory incineration over a number of years calls for durable construction with properly designed furnace structures to stand the relatively hard service."

Some of the fundamental features that should be incorporated in furnace design to give satisfactory service are, as follows.

Temperature.—The furnace should be designed so that a uniform temperature can be maintained when burning material with low heat value and high percentage of moisture, with assurance of temperature at all times sufficient to obtain complete combustion or oxidation without part of the gases passing off by distillation.

Fuel.—The design should be such that material containing a small quantity of combustible matter and a high percentage of moisture will burn without using additional fuel. This can be accomplished only by utilizing at maximum efficiency the heat developed in its operation.

Maintenance.—The cost of the maintenance of furnaces will depend on their design and the incorporation of features calculated to give proper protection under severe service. The design should permit of performing maintenance work without disturbing other parts of the furnace not in need of such work.

Thermal Efficiency.—The design should be such that the thermal efficiency will be high, using materials and construction that will insure only a small loss from radiation, with temperature developed to a maximum.

Draft.—The design should insure that all parts of the furnace can be balanced as to draft, including the furnace chamber, combustion chamber, flues, and chimney.

Simplicity.—A furnace for burning refuse adapted to the service for which it is intended, as well as the class of labor to be employed, and the material to be burned, should be simple in design.

Combustion.—Furnaces to insure proper combustion, should have:

(*a*) Ample space for combustion and the burning of material carrying a large quantity of hydro-carbon.
(*b*) Maintenance of steady high temperature in the fuel bed and in different parts of the furnace chamber.
(*c*) An adequate air supply, thoroughly distributed at the point where combustion takes place.
(*d*) Charging of the furnace and the manipulation of the fire without opening the fire doors or charging doors.
(*e*) The maintenance of a fuel bed on the grate whereby a variable quality of material will not influence the combustion or cause fluctuation in temperature.
(*f*) A high rate of combustion for maintenance of temperature and satisfactory operation.

Pre-Heating Forced Draft.—The pre-heating of forced draft air aids in combustion by supplying heat, thus permitting the burning of a lower grade of material; and, due to the expansion of the air in pre-heating, it permits of better distribution and, at the same time, reduces the excess air that would be required if it were not heated.

Structure.—To withstand the continued heating and cooling of the furnace, as well as to take care of the stresses developed under high temperature, the furnace should be designed as a structural unit with all the stresses that are developed in it taken up by the structural parts, the brick housing being installed for refractory and insulating purposes.

Mr. Greeley presents a table (Table 5) of analysis of organic garbage with the estimated quantity of coal that would be required to supply fuel and maintain a temperature of 1 240° Fahr. At this temperature, all gases or combustible materials will oxidize and be destroyed without producing odor.

A furnace, however, operating under variable conditions, such as charging and cleaning, will not hold a uniform temperature. The usual practice in high temperature furnaces is to maintain a temperature higher than is ordinarily required in the combustion chamber so as to insure that the minimum temperature will be sufficiently high to guarantee complete combustion.

The burning of mixed refuse in incinerators will insure better results than the burning of wet organic waste, which requires a large quantity of additional

fuel. It should never be necessary for any city to burn only organic wastes (kitchen garbage).

With a mixed collection of rubbish and garbage disposed of by incineration, in furnaces which consume the mixture at high temperature, the odors, with proper treatment in collection, storage, or combustion, will be prevented.

Investigation of plants burning organic waste (kitchen garbage) and using large quantities of additional fuel will show that there is a tendency to economize on fuel at the expense of operating with odors due to incomplete combustion.

Reverting to the question of odors, there are certain fundamental considerations which should govern the design and operation of disposal plants, varying, of course, with the system selected.

The adoption of preventive methods will prove more satisfactory than the elimination of odors after they are produced. Such preventive methods are applicable to all kinds of garbage disposal, including feeding, dumping, burial, reduction, and incineration.

Mr. Greeley's statement that relatively little progress has been made in the development of methods for odor elimination is hardly borne out by the facts, if a study is made of the beneficial results which have been obtained. The results obtained and the development made should not be confused with the failures resulting from neglect to apply existing knowledge.

The greatest criticism of present practice is that engineers are not using the experience gained, but continue to follow obsolete methods. When the best known practice in design and operation of disposal plants irrespective of the type or method is applied, more satisfactory results will be secured and continued improvement will follow. Study will show that to dispose of garbage satisfactorily and in a sanitary manner, without offensive odors, the cost will be higher either in capital investment or operation, or both. This will apply to practically all methods of disposal.

Probably no other branch of municipal service has received so little real study to determine the results that can be obtained or the improvements that can be made. Many officials and engineers look on the problem as one to be side-stepped. In many instances the question is considered from what is assumed to be an economic standpoint, the selection of type of plant or method of disposal being based on the least possible capital outlay.

The sanitary engineering problems of municipalities have received study and improvement and gradual development have been made. When municipal officials and the Engineering Profession realize that the disposal of garbage in a sanitary manner is a function requiring study, and look on it with the same interest as they do on other municipal problems, improvement will be made. When the problem is considered from this viewpoint plants will be constructed and operated which are free from odor, and there will be improvements in design and operation from every standpoint.

MORRIS M. COHN,* ESQ.—The subject of the control of odors from sewage plants has always been one of vital interest to those associated with the disposal of the city sewage at Schenectady, N. Y. The treatment plant was

* Supt., Bureau of Sewage Disposal, Schenectady, N. Y.

turned over to the City and operation commenced in January, 1915. The odors arising from the untuned Imhoff tanks, trickling filters, and sludge beds caused several property owners in the vicinity of the works to bring suit against the City to restrain it from operating the plant. The complaint stated an action for an injunction with fee damages, in view of the "exceedingly disagreeable, obnoxious and unhealthy odors" wafted to the plaintiffs' lands, about 1 500 ft. from the plant site. The action was begun in July and tried in November, 1915. Many lurid descriptions of the horrible odors in question were presented by witnesses. One day during the trial the Court and attorneys visited the plant. The cold weather had caused a cessation of intense bacterial activities in the treatment units and no odors of any kind were detected at a distance of less than 100 ft. It was the opinion of the Court that it had not been shown that the City could not run the plant without nuisance in the future and an injunction was denied. Fee damages were not allowed as no injury to health or no expense had resulted from the period of nuisance. The Court, however, advised the plaintiff that another action would be entertained if a repetition of such nuisance occurred.

Thus, at the beginning of 1916, a grave problem confronted the City. On the ability of the operators to control the odors from the plant depended the life of an installation that had cost approximately $500 000. The success that has resulted from the intelligent and careful manipulation of the treatment units is best shown by the fact that no formal complaint has been lodged against the plant in the past nine years, although most of those who signed affidavits of avowed nuisance in 1915 are still residents of the district in which the plant is located. It will be interesting to note the conditions existing at the treatment works at the present time.

Crude Sewage.—The crude sewage that comes to the plant is representative of the average American weak domestic flow. It seldom contains more than 200 parts per million of suspended solids and the daily variation is slight. The sewage is fresh and always contains an appreciable quantity of dissolved oxygen. About 65% of the total flow treated is pumped at a station about 2 miles from the works and little, if any, unbroken feces reach the plant. The odor of the liquid is soapy and has no tendency to become disagreeable unless the flow in the trunk sewer is greatly diminished and the sewage takes on stale characteristics on its way through the conduit. Whenever the pumping station is not in full operation, the flow in the trunk sewer is maintained by cutting in a 24-in. line that ordinarily is by-passed to the river because of the inability of the plant to handle the entire sewage flow of the city.

Bar Rack.—The crude sewage is passed through a stationary coarse bar rack that removes on an average ½ cu. yd. of wet trash per day. These screenings consist of rags, waste, paper, and all forms of garbage, which are disposed of on a dump. It has been found that this deposit gives off a distinct odor of garbage in the spring when it begins to dry out, but when dried it smells only like humus. In order to overcome this putrefactive tendency, the deposit is spread, covered with dry hypochlorite of lime, and then composted with sand. The odor is thus controlled with great certainty.

Present plans call for spreading the trash in the field and periodically plowing the accumulation under.

Imhoff Tanks.—After passing the bar rack, the sewage flows through the influent piping and submerged shear gates into the sedimentation compartments of the Imhoff tanks. This method of influx now replaces the old open influent channels that exposed the sewage to the air and retarded the velocity of flow to such an extent as to produce a deposition of solids in the invert. Overcoming this possible source of odor has done much to improve the sanitary conditions about the works.

The surface of the tanks are kept clean at all times by skimming, and the solids thus removed are thrown into the gas vents and hosed. The water supply of Schenectady contains about 160 parts per million of hardness and produces great quantities of insoluble soap curd that float on the tank surfaces. This grease has a tendency to give off a rancid fat odor under summer heat and great care is taken to skim off this material and to hose the sedimentation compartments frequently. All concrete surfaces and channels are kept clean by hosing.

Fine screens of ¼-in. mesh placed at the effluent weirs of the tanks and auxiliary screens of ½-in. mesh placed in the effluent channels leading to the dosing tanks prevent much solid matter from passing over to the filter stone and there producing odors.

The old influent channels now act as effluent carriers, and, therefore, one set of such channels is always out of service and filled with stagnant sewage. Metal flash-boards are placed at the dead weirs to keep floating solids from being blown into this quiescent liquor and there putrefying. In summer, there is a tendency for this stagnant sewage to become stale, and, therefore, flow is reversed once each week to avoid any chance for septization and the consequent escape of sulfide gases.

As is generally known all Schenectady sludge rises into the gas vents to produce a scum blanket that is more than 6 ft. thick in the spring. This scum is driven down with a 1¼-in. stream of water during the entire sludge-drawing period. The first hosing of the year produces a sour odor that may be caused by the fermentation of the great quantities of mash that clog the gas vents, but subsequent hosings result in only a slight tarry odor about the tanks. A phenol-derivative disinfectant was used with some success to offset the slight odors produced, but the cost proved prohibitive. The gas vents have been raised by means of wooden chimneys, 1 ft. high, closed with hinged covers. This retards the foaming over of the scum into the sedimentation compartments and prevents the dissemination of scum odors into the air.

Dosing Tanks.—On passing into the dosing tanks, the tank effluent spills over three 30-ft. weirs and cascades down steps that break up the liquid into a thin film and spray. Contrary to expectation, no distinct odor, except that of fresh sewage, has ever been detected at this point. The agitation has a distinct advantage. Tests show that there has been an increase in dissolved oxygen saturation of more than 40%, produced by the action of the weirs and steps. It would appear that, given a fresh, weak sewage, the greater the

agitation, the less the tendency for septization and the production of odors. Slime and grease foam coat the walls and appurtenances of the siphon chamber and are hosed as soon as the characteristic musty odor is detected.

Trickling Filters.—The oxidation effected by the dosing tank overcomes the tendency of the liquid distributed on the filter stone by the Taylor nozzles to produce odors, and only slight evidence of such odors can be found to the lee of the filter area. Tests have further shown that there is an increase in dissolved oxygen in the tank effluent affected by the filter distribution piping before the liquid leaves the orifices of the nozzles which increase often amounts to more than 1 part per million.

The black gelatinous growth on the stone surface gives off no odor except when the beds are cut out of service, but this has not been done in two years. Hypochlorite of lime was used successfully during the summer of 1924 to control the development of *Psychoda.* This chemical removed much grease from the nozzle spindles and some of the gelatinous growth on the stone surface. From experience at Schenectady, the use of bleach on troublesome filters is to be recommended.

Sludge Beds.—Great care has been taken to keep all unripe sludge from the drying beds. Small beds, designed to drain the sludge force line, are used for draining test quantities of sludge, as any tendency for the material to give off odors can easily be controlled on the few square feet of these areas.

Conclusion.—All sewage plants have slight odors that are noticeable at the site to the trained observer, and the Schenectady installation is no exception. Odors at the plant are of inconvenience only to those on duty there, but the same odors when carried off plant property are of interest to the public. The prevailing wind at Schenectady blows over the plant in the direction of habitation only 1 000 ft. distant. The topography does not deter the transportation of odors; little obstruction is set up by trees. Thus, the absence of complaint is sufficient proof that the plant is functioning without the production of such odors as would be carried beyond city property.

Most odors reported from sewage plants have little to do with the olfactory senses. They originate in the brain and are a result of over-active imagination. It is indicative of the prejudiced opinion the public has of sewage plants that so many express sincere surprise at the beauty and freshness of the sewage plant grounds and of the clean and inodorous condition of the treatment features. If any particular installation is functioning properly, the speaker would recommend that an invitation be given all taxpayers to visit the works. In no other way is it possible to combat the disgust that most laymen have for things related to the disposal of the liquid wastes of the community.

C. E. KEEFER,* Assoc. M. AM. SOC. C. E. (by letter).—The sanitary engineer is beginning to realize more and more the importance of controlling the odors from sewage treatment works. It is necessary not only to reduce the contamination and the pollution of natural watercourses, but also to render the air which one breathes pure and wholesome. This awakened consciousness

* Engr. of Sewage Disposal, Sewer Div., Baltimore, Md.

of the engineer is brought about by at least two factors: First, the public is demanding a more hygienic and satisfactory environment; and, secondly, the engineer himself is continually striving to advance the science of sanitation. For a number of years he has had at his disposal any one of a number of types of sewage treatment that would produce effluents to meet different chemical and bacteriological standards. However, it has been only within the past few years that the problem of the control of odors from sewage work has been treated and discussed before engineering societies. In addition to this, little practical work has been done. As regards the control of odors, sewage works are being designed almost the same to-day as they were ten or fifteen years ago.

The reason for this condition is not difficult to understand. The inherent nature of sewage is such that it will produce an odor and, under many conditions, an unpleasant one. Furthermore, during the course of treatment, the sewage is exposed to the atmosphere, and an excellent opportunity is afforded for the escape of unpleasant gases. One reason odors of any kind are so difficult to control is that little is known about them. As yet, it has not been determined definitely whether they are present in the gaseous or the liquid phase. However, the former assumption seems to be the correct one. Moreover, there is no satisfactory way to measure the degree of an odor. Not infrequently such small quantities of a substance have to be present to produce an odor that it is difficult to make any scientific study of the subject. Until some of these difficulties are overcome—especially determining the phase of the odor and establishing some standard by which odors can be measured—the engineer or scientist will be handicapped in their control and elimination.

If the available data are limited, however, efforts can still be made to reach a solution—even if it is a partial one. In the realm of sewage treatment much can be done to relieve the situation, as Mr. Skinner has ably pointed out, although such relief will not necessarily be complete. It is often possible to locate the sewage works in a sparsely inhabited district so that few people will be affected, to select that type of treatment which will produce a minimum of odors, and to pay particular attention to the details of design so that there will be a reduction in aerial nuisances. However, with the greatest care paid to design and operation, a sewage works consisting of sedimentation and oxidation units of any appreciable size will produce odors that will vary with a number of factors.

Not infrequently persons living in the vicinity of sewage plants will seek redress through the Courts for damages. Such has been the experience of a large sewage works of which the writer has knowledge. Since this plant was first put in operation, approximately $100 000 has been paid for damages caused in part by obnoxious odors. Thus far there seems to be no satisfactory and economical solution for this problem. The actual area of the various treatment units covers several acres, and even if the odors coming from the sewage could be collected and treated successfully, the cost of the undertaking would be prohibitive.

Experiments at Baltimore, Md.—At the Baltimore plant a careful study made by those in close contact with the work has led to the belief that most of the odors come from the preliminary settling tanks. After these tanks have been in service for a short time, a heavy scum forms, which gives off a strong and penetrating odor. If this odor could be eliminated successfully, the situation would be relieved.

With this in mind a series of experiments was begun in the summer of 1924, which consisted in treating the sewage gases with chlorine. One of the preliminary settling tanks was used for this experiment. This tank, which treats the sewage after it passes through bar screens, is approximately 420 ft. long and 103 ft. wide, with a working depth of from 9 to 14 ft. It has a capacity of about 3 800 000 gal. with a detention period of approximately 3.5 hours. The tank is divided into two compartments. The sewage, usually in a stale condition, first enters the smaller compartment, which is 105 ft. long, 103 ft. wide, and from 9 to 11 ft. deep. This compartment was covered with 2-in. tongue-and-groove planking and was thus practically air-tight. At opposite ends of the tank, two small wooden buildings were erected, one of which contained a blower and chlorine control apparatus. There was direct connection from this building to the air space between the top of the sewage and the under side of the wooden covering of the tank. The sewage gases passed into the building, and thence through the blower. The chlorine was applied to the discharge side of the blower which was connected direct to one end of a wooden conduit, or flue, 100 ft. long and 2 ft. square. This flue entered the second building, where an observer could study the treated odors.

The quantity of chlorine used was varied from 5 to 15 lb. per 24 hours, and the volume of gas treated from 630 to 1 480 cu. ft. per min. A number of test runs were made both with the gases from sewage and from the sewage sludge. The tank was operated for a number of weeks so as to produce a thoroughly septic condition and a more intense odor. In order to ascertain the difference between the untreated and the treated gases, the observer first went into the building in which the blower was stationed; after observing the odor, he went into the second building into which the treated gases were passing. Here, he was able to isolate himself from the odors coming from the various parts of the sewage works, and thus breathe only the treated gases, which were being blown from the interior to the exterior of the building.

Observations Made.—One of the most noticeable characteristics of the untreated gas was that it no longer had the characteristic smell of sewage. The odor was considerably less objectionable. Perhaps covering the tank and excluding the sunlight played a part in this new condition. Another interesting change was that little scum formed on the surface of the sewage. This latter condition has prevailed during the warmest summer months up to the present time.

Observations of the chlorinated gases were made by a number of persons, and various opinions were expressed as to the efficacy of the chlorine. Some held that it improved the odor little, if any; others thought that it practically eliminated any unpleasant odor; and still others took an intermediate ground

and contended that the application of the chlorine was a material benefit, but that it had not fully corrected the condition. No definite conclusions were reached.

One of the unfortunate things in connection with any experiment on odors is that there is no satisfactory standard or unit by which they can be measured. There will often be a wide diversity of opinion with reference to any particular odor. For this reason, any conclusions that are reached, will lack a certain definiteness and finality. However, the experiments conducted at Baltimore are felt to be in the right direction, and will probably increase interest in the elimination of odors from other sewage plants. If tests of this character could be conducted at other places, additional data would be available from which more definite conclusions could be formulated.

C. G. Gillespie,* M. Am. Soc. C. E. (by letter).—It is significant of an important exaction in the higher standard which the public is setting up in regard to clean surroundings and clean air, that all the papers presented before the Sanitary Engineering Division at the meeting of January 22, 1925, relate to the odor problem associated with industry and the disposal of putrefying wastes. A relatively untrodden field of endeavor, not at all simple, and fraught with the danger of the imposition of huge expense ultimately falling on the whole public, is opened in the engineering control of odors. In admirable fashion Mr. Skinner presents his own observations and directs engineers to seek out the principles affecting the behavior of odors so that efforts at control may proceed intelligently.

The principles of course are best ascertained where odors from particular sources exist unmixed with those from other sources. This calls for considerable isolation, a condition not common in or near metropolitan areas. Then, too, local temperatures, air movement, variations in the sewage, kind and condition of works, and exactions of the neighboring public, will color experiences throughout the country.

In the absence of any guide to isolation requirements for prospective sewage plants in California, studies were begun in 1923, to ascertain the odor zones around a number of existing and typical plants. This was done by procuring a map of the particular locality showing the homes, roadways, and landmarks, and establishing on the map a number of points surrounding the disposal plant or area, at which points odors in noticeable degree had been experienced by residents. Corroboration of experiences is nearly always possible along public roadways. The extent of odor travel found from these tests is given in Table 11 and a typical nuisance map showing a survey for odors, in Fig. 20.

In California, at least, the observations indicate that odors are most annoying in the summer and in the evening. Daytime inspections are invariably negative as to odors, except at most within a few hundred feet, whereas odors at evening travel many times as far. There is a noticeable tendency for odors to confine themselves to narrow, invisible ribbons of rather uniform intensity stretching out for as much as ½ mile, or more. These ribbons shift in direction and thus establish quite a remote periphery to the odor zone. High

* Director, Bureau of San. Eng., State Board of Health, Oakland, Calif.

or strong winds, of course, dissipate odors rapidly. Low, gentle breezes, especially in the morning and evening, tend to shift the whole body of the nuisance zone only slightly leeward, as indicated by the relation of maximum to minimum nuisance radius. There is a tendency for odorous gases to seek low swales or pockets.

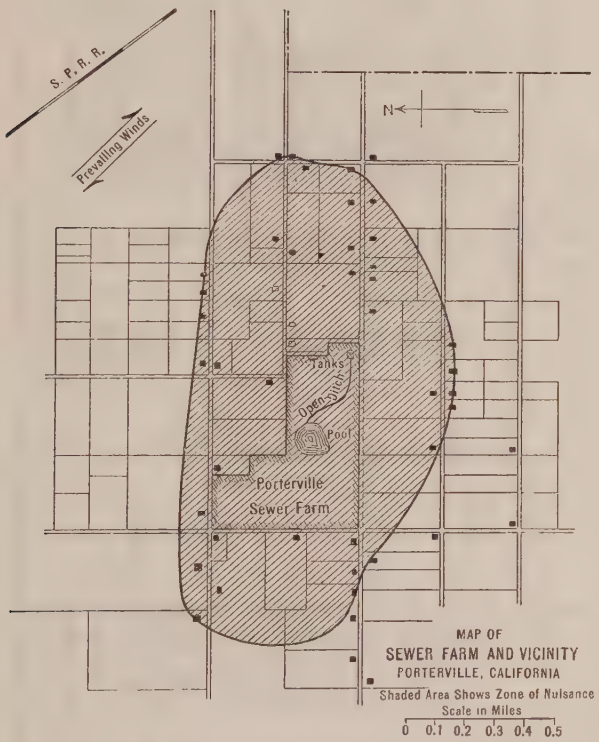


FIG. 20.

The quantity of organic matter, its state of decomposition, and the time or area exposed to the air before it disappears in the ground or is stabilized, markedly affect odor production. Cannery and creamery wastes, perhaps the most prominent industrial wastes in California, produce exceedingly strong odors. The waste from a modest plant is equivalent in its organic matter to the sewage of a large city. Odors are correspondingly intense in all the cases observed. Gas-house wastes impart a heavy but disagreeable odor in sewage, reminiscent of moth balls.

The sulfates in sewage appear to have a marked effect on the production of odor through their conversion to hydrogen sulfide. Along the outfall sewer in El Centro, Calif., intense sulfide odors emanate from the manholes. Sulfates in the 3-mile outfall drop gradually and uniformly from 330 parts per million ½ mile below El Centro to 160 parts per million at the Imhoff tank, and 155 parts per million in the tank effluent. It is curious to note that the sulfate-

TABLE 11.—Results of Odor Surveys Around Certain Sewage Treatment Plants and Disposal Areas in California.*

Place.	Population.	Treatment.	Disposal.	Nuisance Radius, in Miles.			Character of sewage.
				Average.	Maximum.	Minimum.	
Hollister	3 000	Raw sewage	Sewage farm	0.75	1.25	0.25	Contains gas-house waste and cannery waste
Vacaville (South Farm).	1 200	Raw sewage	Sewage farm	0.50	0.75	0.25	Domestic sewage
Vacaville (North Farm).	1 200	Septic tank	Sewage farm	0.75	1.00	0.25	Domestic sewage
Santa Rosa	13 000	Septic tanks	Sewage farm	1.00	1.25	0.25	Contains cannery, tannery and gas-house wastes
Fresno	55 000	Septic tanks	Sewage farm	2.00	2.50	1.50	Principally domestic sewage.
Selma	3 500	Imhoff tank	Sewage farm	1.75	3.00	1.00	Contains excessive cannery waste
Clovis	1 000	Septic tank	Sewage farm	0.37	0.50	0.25	Domestic sewage
Exeter	2 000	Imhoff tank	Sewage farm	0.62	0.75	0.50	Domestic sewage
Lemoore	1 500	Septic tank	Sewage farm	1.00	1.25	0.25	Domestic sewage
Porterville	4 500	Septic tank	Sewage farm (sandy)	0.75	1.00	0.50	Domestic sewage
Lindsay	2 800	Septic tank	Sewage farm	0.50	0.75	0.25	Domestic sewage
Corcoran	1 000	Imhoff tank	Sewage farm	0.50	0.75	0.25	Domestic sewage
Hanford	6 500	Imhoff tank	Sewage farm	1.00	1.50	1.25	Contains much cannery waste
Visalia	6 500	Raw sewage	Sewage farm	1.50	1.75	0.75	Contains much gas-house and cannery waste
Reedley	3 500	Sprinkling filter	Kings River	0.37	0.75	0.25	Fresh domestic sewage
Kingsburg	1 200	Sprinkling filter	Kings River	0.75	1.25	0.50	Septic domestic sewage
Dinuba	3 500	Septic tank	Sewage farm	1.25	1.50	1.00	Domestic sewage
Dinuba	3 500	Sprinkling filter	Drain ditch	1.00	1.50	0.75	Domestic sewage
Paso Robles	2 000	Imhoff tank	Salinas River (bottoms)	0.37	0.50	0.25	Domestic sewage and hydrogen-sulfide swimming pool water
Tulare	3 500	Septic tank	Sewage farm	1.5	2.00	1.00	Domestic sewage and much milk factory waste
Sonoma	1 200	Septic tank	Sewage farm	0.75	1.50	0.50	Domestic sewage

* By California State Board of Health, Bureau of Sanitary Engineering, C. G. Gillespie, Director, dated January 2, 1925.

consuming organisms are reported to accumulate in this outfall until it becomes nearly choked. Application of copper sulfate at the head of the outfall destroyed and loosened the growth, allowed it to float down the outfall, and temporarily corrected the odor.

At Coalinga, Calif., where the sulfates in the water supply have approximated 1 000 parts per million, a crust of precipitated yellow sulfur has formed below the water line in the outfall sewer beyond the septic tank and the concrete of the tank itself is practically destroyed, probably due to the attack of associated acids. At the outlet of the tank intense odors of pure hydrogen sulfide are given off.

Long force mains between pumping plants and sewage treatment works of small towns in California contribute noticeably to the production of odor for the reason that over-size pumps must be used to prevent clogging and pipe lines are correspondingly large. The time of passage through this section of the sewer system may be as much as five times that through a gravity sewer of the same length, and the sewage not only ages as much as 3 or 4 hours in them, but becomes charged with putrefactive gases given off in the sewer.

For the same reasons, flat sewers, and sewers in which the manholes have a depressed bottom ostensibly for intercepting sediment, have intensified the production of odor. The writer's experiences with high groves of sheltering trees, reinforced by a 30-ft. levee, is that these barriers failed to confine the odors. One experience with the housing of sewage treatment works was not satisfactory. This was over a small Imhoff tank and sprinkling filter at Los Gatos, Calif. The housing so intensified the temperatures in the sprinkling filter as to interfere seriously with its results. The housing was opened and the plant subsequently abandoned because of the odor annoyance to neighbors.

Experience at Dinuba, Calif., where a sprinkling filter and Imhoff tank superseded a septic tank and sewage farm, tends to show that the zone of odor was practically the same in both cases.

As to the activated sludge process the writer has made observations on only two plants, at Lodi and at Pasadena, Calif. The Lodi plant treats the sewage of a town of 7 500 people, including gas-house waste, and in summer also the waste from a 50-ton peach and tomato cannery. The sewers in Lodi are laid on extremely flat grades. Sludge is spread on sedimentary soil overlying hardpan and when dry is plowed under as fertilizer. Odors are not noticed at a residence 500 ft. away at any time, nor have they even been noticed beyond a few feet from the plant or from the sludge area. Returning the sludge from the clarifiers to the sewer ahead of the fine screen, lessened the odors in the screen room, but caused more watery screenings.

The Pasadena plant treats a strong domestic sewage for a population of about 100 000. The worst fault of this plant is that it is undersized for a rapidly increasing load. Odors, however, were not a factor beyond 1 000 ft. or more, until deep lagooning of sludge was tried.

There seems to be a singling out of sewage odors for complaint by the neighboring public, probably through the old fear that sewage gases are dangerous to health. It seems to the writer that the public needs to know that

sewage odors are merely a nuisance, more or less annoying and discomforting, not essentially different from the odor from corrals, pig pens, factories, fertilized fields, and the like. In equity, although one may be more intense or more widely annoying than the others, they are all nuisances differing only in degree. Unless there is considerable tolerance and this noticeable movement for cleaner air proceeds prudently, a financial burden altogether too heavy may easily result. In the final analysis the expense shifts to the whole public. Agitators may easily become inimical to the best public interest. A real odor nuisance generally depresses property values. If the total cost of abating all the nuisances responsible for that particular damage even approximates the estimated damage, it seems only logical that correction of the odor is in order and this principle may afford some means of judging the merit of a proposition to control odors.

T. Chalkley Hatton,* M. Am. Soc. C. E.—The speaker would like to challenge one statement of Mr. Skinner, in which he says:

"Lagooning undigested sludge is likely to produce odors, for either scum may form or there may be insufficient water to cover the sludge, which will then putrefy and give off odors.

"In general, lagooning should only be adopted in winter or in remote locations where no nuisance will be occasioned to neighbors."

At Houston, Tex., a nuisance has not resulted from the lagooning of activated sludge either in winter or in summer and little or no scum has formed on the surface of the liquor after several years of operation.

At several sewage treatment plants in England, undigested sludge has been lagooned without creating a material nuisance which might be considered detrimental to the surrounding neighborhood. The large and numerous digestion tanks at Saltly in the Manchaster plant, which contain highly concentrated undigested sludge, could not in July be termed offensive judged from 200 ft. away especially after the formation of the heavy top scum.

It is doubtless true that while the undigested sludge is being transported from one tank to another, the sulfureted hydrogen escapes and creates an objectionable odor but when the sludge becomes quiescent the odors arising from its decomposition, when in the presence of water, are scarcely perceptible a few hundred yards away at most.

N. E. Loomis,† Esq. (by letter).—In carrying out most of the suggestions made by Mr. Weston, the writer feels certain that sanitary engineers will meet with hearty co-operation from the oil industry. In fact, in the largest and most up-to-date refineries they have already been carried out. This is for the reason that the escape of odors is immediately connected with other losses which the efficient refinery wishes to avoid.

For example, Mr. Weston has emphasized the importance of collecting the gases from the various distillation processes and burning them under stills, in order to obviate odor troubles and at the same time to effect fuel economies. As an alternative the scrubbing of gases with a solvent oil was

* Chf. Engr., Sewerage Comm., Milwaukee, Wis.

† Director, Experimental Div., Standard Oil Co., Elizabeth, N. J.

mentioned with the objection that the gasoline recovered was so "wild" as to escape subsequently and thereby create a nuisance. As a matter of fact, in most of the first-class refineries all gas is collected from the distillation operations, scrubbed to recover the stable gasoline that it always contains, and then the dry gas is burned under boilers or stills. There are numerous methods for insuring that the recovered gasoline be free from dissolved gases which later might cause odor trouble. It frequently happens that the recovery of gasoline alone in such a manner pays a substantial profit in addition to having the fuel value of the dry gas, which value may amount to more than 20% of the total fuel consumed in the entire refinery. The combined operation of recovering the gasoline from the gas and using the dry gas as fuel represents an economy over wasting the gas that a large refinery cannot afford to neglect.

Mr. Weston recommends the use of a vacuum system on tankage. There are two kinds of tankage used in refinery operations—storage tanks for crude and refined products (which may be as large as 80 000 bbl. in capacity), and working tankage. The working tankage is smaller in size and, as a rule, comprises a much smaller number of tanks than the storage tankage. It is often feasible to tie the working tankage into a vacuum system as recommended, but the present construction of most storage tankage would make this impracticable. Further, the escape of gases from storage tanks can be minimized in other ways so as to make it unnecessary.

Odors from the burning of oil under boilers and stills should not be objectionable provided complete combustion is obtained; here again there is a community of interest between the general public and the oil refiners. In many refineries it is the custom to keep a crew of men constantly studying the combustion conditions of the various pieces of equipment. Appearance of smoke at the stacks is an indication of improper operation.

Mention has also been made of the odors resulting from treating oils with acid and from the concentration of the recovered weak acid. In treating, the use of tight agitators and the mechanical mixing of oil and acid prevents the escape not only of odors but also of valuable fractions of the oils. The odor difficulties encountered in acid concentration are being minimized both by the use of proper types of concentrators and also by the proper conditioning of the weak acid to eliminate organic material.

In trying to eliminate odors from oil refineries sanitary engineers are fortunately facing a different situation from that in some other branches of industry. With the refineries they are dealing not with the prevention of a nuisance which can be corrected only at a monetary loss, but rather with a condition in which the economics effected will actually pay a profit in almost every case cited. This higher operating efficiency is certainly the strongest argument which can be used by sanitary engineers and public health officials in forwarding the work in which they are primarily interested. By proper co-operation based on the community of interest, sanitary engineers and refinery managers should be able easily to correct whatever troubles actually exist. In no case, however, do American conditions resemble in the faintest

manner the picture that has been drawn of Russian conditions. The efficiency which has always maintained the American refining industry in a position of unchallenged world leadership would make such conditions in the United States absolutely impossible.

Edward Wright,* M. Am. Soc. C. E.—The importance of the detection and elimination of odors from oil refineries is perhaps more fully realized by a comparatively small number of sanitary engineers in Eastern Massachusetts, in the vicinity of Providence, R. I., and certain districts in New York State and New Jersey, than by the profession in general. The number of complaints on account of odors from the refining of high sulfur oils since the World War has been unparalleled in the recent records of the Massachusetts Department of Public Health.

To indicate the size of the oil refining industry, it may be said that one of the refineries in Eastern Massachusetts has been the heaviest shipper on the New York, New Haven and Hartford Railroad System. Although the number of persons engaged in the industry is not great, an immense amount of capital is invested in the 500 to 600 refineries in the United States, and the industry is an increasingly essential one.

Soon after the World War, the New England refineries used great quantities of Mexican crude oil which contained 1.5 to 6% sulfur. More recently, and simultaneously with greater knowledge and improvement of odor control apparatus, considerable quantities of crude oil rarely exceeding 0.5 of 1% of sulfur have been shipped from California and Texas and from other fields. Extensive prospecting is now under way in Venezuela where crude oil having a sulfur content from 1 to 3½% is being obtained in increasing quantities. If large quantities of this crude oil are used, the objectionable conditions which arose with the use of Mexican crude may occur again unless adequate control devices are operated. Not all the sulfur is removed during the refining process; in fact, fuel oil may contain 3 to 4% of sulfur, and the speaker has noted hydrogen sulfide odors from the combustion of Mexican fuel oil which were more marked than the odors from a poorly managed sewage disposal works situated adjacent to the power plant. This indicates that the time may come when it will be necessary to remove some of the sulfur, possibly by chemical means, from certain fuel oils.

It may be of interest to describe briefly some of the methods and difficulties of operating odor-control devices that have come to the speaker's attention in examinations for the Massachusetts Department of Public Health. The installation of odor-control devices, like that of sewage disposal works, is only the beginning. Painstaking operation of the devices is essential. The operating force under which these various gas systems are controlled in one of the larger and more complete refineries in Massachusetts consists of ten men: a supervisor and a night supervisor; an experienced man and two helpers employed in the inspecting, cleaning, and changing of screens in the various gas lines; an experienced man and a helper employed in the inspection and maintenance of the water seals and recording instruments; and three gas-

* Asst. Engr., Massachusetts State Dept. of Health, Boston, Mass.

system operators working in three shifts, who make hourly inspections of the gas-collecting systems and the plant in general. Detailed reports of the inspections are submitted in writing or in tabular form to the supervisor at the end of each 8-hour shift, and the essential data are subsequently submitted to the management.

In the complete refinery, there are eight to ten recording instruments on the pipe systems for collecting non-condensable still-gas together with a number of additional instruments for determining temperatures and other data that are of value to the refinery, but of no particular significance in the odor elimination problem. Parts of the instruments must be renewed from time to time because of the corrosive effect of the gases. The records not only show the quantity of gas, but also immediate evidence of any defect in the gas collection or gas-pressure system, such as leaks or the formation of deposits. The instruments are placed so as to be visible, particularly to the operators of the gas exhausters.

In a refinery handling 15 000 to 20 000 bbl. of crude oil daily the non-condensable still-gas may amount to 1 500 000 cu. ft. per day, but the quantity varies greatly from hour to hour even when the stills are operated continuously.

The quantity of air in the non-condensable still-gas is estimated three times a week at one of the refineries in Massachusetts, and determinations are made less frequently for hydrogen sulfide and other constituents. The quantity of air varies from 10 to 35% by volume in the low-pressure still-gas, and when a weekly run of about one-third Mexican and two-thirds Californian crude is being used, the hydrogen sulfide varies from 3 to 15% by volume.

The gas pressure lines leading to the boilers are provided with one or more water seals which must be kept under constant supervision, with a seal of 2 in. of water. Running water is usually provided in these seals to compensate for evaporation, and care must be exercised to prevent freezing.

In disposing of the gases of evaporation from the crude oil storage tanks an equilibrium in pressure is provided on the tanks while they are being filled, whereas on the storage tanks for hot oil a constant vacuum of ¼ in. to 3 in. of water is maintained. U-tubes are provided at various points throughout this system, and in the winter these tubes are filled with a solution of gas-oil to prevent freezing. The liquid in the U-tubes is usually colored with red ink to facilitate reading, and readings are made hourly and properly recorded.

Thirty-mesh Monel metal screens are installed on the gas-line outlets of all the oil tanks and containers, mainly for fire protection. The screens are given several inspections weekly, which indicate that the surfaces sometimes quickly become coated with deposits, largely of sulfur. The life of these screens averages not more than three weeks and proper management requires that they be kept in good condition in order to prolong the periods between cleaning and renewal of the gas lines. At one of the refineries in Massachusetts, 3-in. gas lines on the hot-oil, storage-tank systems have recently been replaced with 6 and 8-in. lines because of deposits. The gases of evaporation are passed through suitable seals in which running water is maintained. The water seals on top of the tanks require inspection twice daily.

The quantity of air in the gases of evaporation, which is estimated three times weekly, varies from 40 to 80 per cent. Additional determinations are made less frequently for hydrogen sulfide and other constituents.

The maintenance of the odor-control devices in the plant for chemically treating raw gasoline is also troublesome, particularly because of the acids contained in the gases. All parts of this apparatus are inspected frequently for leaks.

Odors often result from deterioration of the metal parts of refining apparatus allowing the escape of the contents. Corrosion and deposits made up largely of sulfur present great difficulties in operating the gas-disposal systems at oil refineries, particularly in the non-condensable gas lines from the low-pressure stills where the gas contains as high as 35% of air. Corrosion is less marked in the high-pressure still lines where air is not present. The mixture of the air and gas with sulfur compounds readily corrodes wrought-iron pipe so that its average life is not more than three to four years. The mixture of the gases of evaporation from various grades of petroleum products and air also corrodes the tops of the metal tanks so that it is necessary to replace them every three to five years. In gas-holders for storing the non-condensable gas, the corrosion is also marked and careful inspections and renewals of parts of the holders are necessary in order to prevent leaks.

Regular inspections are made of the still settings to determine the presence of breaks in the walls due to the contraction and expansion of the stills, and when the stills are used on the batch process, as is necessary when the heavy crude oils are refined down to coke, very careful attention is necessary to prevent leaks and to provide good combustion of the fuel beneath the stills.

The management of one of the refineries in Massachusetts requests the salesmen and others to make such observations in the locality about the refinery as are reasonable and to call up the management if offensive odors are noted. A representative of the Odor Control Department is on hand at all times to make observations in the field concerning complaints and to determine and correct the cause.

Samuel A. Greeley,* M. Am. Soc. C. E. (by letter).—Mr. Lewis properly emphasizes the need for beginning studies at the garbage pail itself and including both collection and disposal for the control and proper elimination of odors.

Mr. Osborn comments on the relation between the location of a garbage disposal plant and the cost of haul. This is undoubtedly an important item. As indicated in the paper,† zoning boards have recognized certain odor-producing industries by outlining so-called nuisance and semi-nuisance districts. Presumably the amount of odor control for such industrial establishments is related somewhat to their location. Just how far such practical considerations should influence the design and cost of a garbage disposal plant must be determined for each special case. The writer has investigated such a situation in a city of 140 000 population. An old established site, well isolated, was available for the garbage disposal plant. The use of another site would have

* (Pearse, Greeley & Hansen), Chicago, Ill.

† See p. 361.

permitted the collection work with one less wagon, effecting a reduced annual cost amounting to about $5 000. However, it was decided that the security of the existing site against nuisance hazards was worth the extra cost.

Mr. Osborn's comments on the fundamental features of design and his statement that engineers are not using available experience, are sound. However, records of operating experience with garbage disposal plants are not as well advanced as other branches of sanitary engineering work. Undoubtedly, a great deal of such experience has been had, but relatively little of it has been published in a definite way, as yet, with reliable yardsticks. In comparison with the large sums that have been expended by municipalities in testing sewage disposal processes, there is much yet to be done in the field of garbage disposal. Mr. Osborn's statement, therefore, that the disposal of garbage in a sanitary manner is a function requiring study on which municipal officials and engineers should look with the same interest as they do on other municipal problems, is a good summary of the situation.

JOHN F. SKINNER,* M. AM. SOC. C. E. (by letter).—The writer appreciates the value of discussion elicited from engineers in charge of different types of plants and illustrated by conditions in widely separated localities.

Mr. Cohn refers to the sour odor "caused by the fermentation of the great quantities of mash that clog the gas vents." At the Irondequoit plant at Rochester, N. Y., "considerable quantities of grain are found in the grit, the probable source being the wastes from home-brewing processes." This material is deposited with the grit and if promptly removed causes no offense. This is one of the reasons why a small grit chamber may be desirable in a plant limited to the treatment of sanitary sewage. Mr. Cohn's observations indicate that care and cleanliness about the plant remove most causes of complaint.

Mr. Keefer's work at Baltimore, Md., including the confining of gases and their treatment with chlorine, is most interesting and points the way to further experimentation. It is in line with the writer's suggestion.

Mr. Gillespie's odor surveys are excellent in furnishing comparative results which may be studied in the light of location, exposure, prevailing winds, type of sewage, and care in operation. His statement, "daytime inspections are invariably negative as to odors, except at most within a few hundred feet, whereas odors at evening travel many times as far", corresponds with the writer's observation and his explanation of the phenomena due to convection.

The writer has assumed that any considerable quantities of putrefying industrial wastes should be treated before admission to the public sewers. In general, the expense of such treatment should be borne by the interests involved rather than by the general public.

Mr. Gillespie's observations at El Centro, Calif., indicate how a trained and observant operator may relieve an acute condition by a specific remedy. The plowing under of activated sludge at Lodi, Calif., and its local use as fertilizer, is doubtless the best solution for a small plant suitably situated.

Mr. Hatton's criticism anent lagooning of activated sludge is interesting. He states that at Houston, Tex., little or no scum has formed, while at Saltley

* Cons. Engr.; Deputy City Engr., Rochester, N. Y.

in the Manchester, England, plant he refers to the formation of heavy top scum.

J. C. McVea, M. Am. Soc. C. E., City Engineer of Houston, writes that his sludge lagoons are 18 in. to 10 ft. deep, and of various sizes up to nearly 1 acre in area:

"No appreciable amount of scum forms on the surface * * * and it will be necessary for them [lagoons] to stand several months before they are entirely dry. Odors are not noticeable during the normal operation of the lagoons, but upon allowing them to stand for several months and to dry out a very offensive odor is created upon again filling with sludge unless the old material is entirely removed."

This is with a range of temperature from 50 to 105° Fahr.

Mr. Gillespie writes that at Pasadena, Calif., the activated sludge was lagooned 2 or 3 ft. deep in a sandy river bottom:

"The sludge soured badly and made a terrific nuisance, reaching about 6 000 ft. away where it made a nuisance to some 7 000 people. Examination of the beds seemed to indicate that they remained sour for at least a month * * *. No scum formed on the surface of the lagoon. It was rather a pasty sludge. There was a thin, dry crust which was evidently not sufficient to confine the odors."

The lagooning has now been discontinued and the present practice is to coagulate the sludge with alum, dry on an Oliver filter, and bury in deep trenches. The latitude of Pasadena is 34° N.

At Houston, the lagoons are operated continuously at considerable depth practically as open septic tanks. In Pasadena the odor from sour sludge in shallow lagoons became unbearable. Odors from lagooning apparently depend on the condition of the sludge, the temperature, the depth of supernatant water, the formation of scum, and the method of operation. The writer, therefore, feels justified in his statement that, "in general, lagooning should only be adopted in winter or in remote locations where no nuisance will be occasioned to neighbors."

Robert Spurr Weston,* M. Am. Soc. C. E. (by letter).—Mr. Loomis has done well to call attention to the necessity for co-operation between oil refiners and the public and to the economies resulting from proper odor-preventing devices. Regarding the use of a vacuum system on storage tanks, the writer still believes it to be an excellent method for the control of odors from these structures and one which is best adapted to a critical environment where the minimizing of the escape of odors rather than the prevention thereof does not suffice.

The use of oil for scrubbing is of course sound scientific practice and ought to produce no odor. The writer cannot presume to approve or disapprove; he has seen cases where its operation was faulty. This again emphasizes the equal importance of good construction, good inspection, and careful operation of all odor-preventing devices.

Naturally, the word picture of Baku, Russia, quoted by the writer, will never be exhibited in the United States. It stands in contrast to the condition toward which American refinery practice is growing.

* Cons. Engr. (Weston & Sampson), Boston, Mass.

Mr. Wright's emphasis upon the necessity for vigilance in inspecting instruments, screens, traps, and other devices so subject to corrosion, is a point well made.

After all the oil industry is as necessary to the community as any other purveyor to civilization, and it is particularly incumbent upon sanitary engineers to assist in those adjustments of plant, of regulations, of methods of control, and of public attitude, which are necessary to maintain the rights of all with due regard to the rights of every one—refiner, petroleum user, navigator, bather, pleasure seeker, and the ordinary air-breathing householder.

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Paper No. 1580

THE ST. LAWRENCE WATERWAY TO THE SEA*

By Francis C. Shenehon,† M. Am. Soc. C. E.

With Discussion by Messrs. H. de B. Parsons, H. C. Sadler, F. P. Williams, Thomas H. Hogg, L. C. Sabin, Gardner S. Williams, E. P. Goodrich, David B. Rushmore, Maurice W. Williams, L. H. Hart, A. Lindblad, Walter M. Smith, W. M. Black, G. B. Pillsbury, L. F. Harza, Elliott J. Dent, Kenneth L. De Blois, C. S. Riché, E. G. Walker, and Francis C. Shenehon.

Synopsis

The writer assumes the only substantial justification for a deep waterway connecting the Great Lakes with the salt seas is the deep penetration of the continent to Duluth, Port Arthur, and Chicago. The navigable right of way of the whole Great Lakes System, therefore, is involved in the discussion.

The 1924 project-drafts in Lakes Erie, Michigan-Huron, and Superior with the connecting rivers, are 20 ft. The waterway to the sea contemplates initially 25-ft. and ultimately 30-ft. drafts.

The St. Lawrence River is the geological avenue to the sea through which outflow the surplus waters of the Great Lakes. This river is, therefore, the natural avenue for a navigable waterway to the sea. The water itself, with a volume of 220 000 sec-ft. and a descent of over 220 ft. from the level of Lake Ontario to Montreal, is one of the great power resources of the world. The writer assumes that the development of water power co-ordinately develops and subsidizes a navigable waterway.

This paper describes the Port of Duluth and the various navigable ways through Lake Superior, St. Marys River, Lake Huron, the St. Clair and De-

* Presented at the Fall Meeting, Detroit, Mich., October 23, 1924.
† Cons. Engr., Minneapolis, Minn.

troit Rivers, Lake Erie, through the Welland Canal descending over the Niagara Escarpment 326 ft. to the level of Lake Ontario, and then through Ontario and the Upper St. Lawrence to the Rapids Section of this river. This section, beginning at the Galops Rapids and terminating at the foot of the Lachine Rapids at Montreal, is the locus of the great constructions contemplated at an initial cost exceeding $250 000 000. Little detailed description of the development of the Rapids Section contemplated in the report of the International Joint Commission of 1921 is embodied; but some general principles involved in the best co-ordination of navigation and water power are suggested.

A new International Board of Engineers is studying the report of the Government engineers of 1921, having in mind a review of the earlier conclusions. It has seemed appropriate, therefore, to devote the major portion of the paper to those elements of the problem not within the purview of the report.

The water supply tributary to the St. Lawrence is fully discussed and works are suggested to increase the dependable volume of flow.

Two routes from the Great Lakes to the sea investigated by the Board of Engineers on Deep Waterways in 1900, are touched upon. The route by the way of Oswego and the Mohawk to the Hudson, thence to the Port of New York, was preferred in 1900. Since that time, the near completion of the Welland Canal and the desirability of building at least the International Section of the St. Lawrence River Waterway, make preferable a route to the sea through the St. Lawrence River to the foot of Lake St. Francis, thence descending to Lake Champlain and through the Hudson River to the Atlantic.

It is shown that the deeper drafts in the Great Lakes necessary for a waterway to the sea, are also economically desirable for the immense existing internal navigation. It is shown that regulating works, in addition to those already built at the foot of Lake Superior, may be constructed at the head of the St. Clair and Niagara Rivers. These works will raise the lake levels and diminish the excavation needed for 25-ft. drafts.

The regulating works recommended will convert Lakes Superior and Michigan-Huron into vast storage reservoirs as aids to water power in the Niagara and St. Lawrence Rivers, stabilizing the flow and adding 20 000 sec-ft. to the volume of prime flow. The fact is emphasized that in a state of Nature it takes several years for available storage water in Lake Superior to be transmitted for power use in the Niagara and St. Lawrence Rivers; whereas with a complete series of manually controlled regulating works the equivalent of Lake Superior water may be instantaneously made available in these great power streams. And conversely the equivalent of excess water supply in one of the lower lakes, Ontario, for example, may be instantaneously transmitted to the Superior Reservoir or to the Michigan-Huron Reservoir.

Water power at Niagara Falls is touched upon as competitive in a limited market with St. Lawrence power.

The international aspects of the various constructions are discussed, and certain suggestions made for desirable elements to be incorporated in a new Boundary Waters Treaty between Great Britain and the United States.

The St. Lawrence Waterway to the Sea may be narrowly visualized as the St. Lawrence River improved for 25 or 30-ft. navigation between the smooth, stately river at Montreal, and the smooth, stately river above the Galops Rapids. In the intervening 115 miles, the river descends roundly 224 ft. and has a normal flow of 230 000 cu. ft. per sec. Under average river-stage conditions, ocean liners of 30-ft. draft penetrate the continent as far as Montreal—1 000 miles inland from the open Atlantic—making Montreal one of the great ports of the world. Advancing this inland travel of ocean liners to Toronto, near the west end of Lake Ontario, serves no adequate navigational purpose. The Great Lakes still reach nearly 1 000 miles inland after climbing, by locks and canals, 326 ft. up and over the Niagara Escarpment and entering the foot of Lake Erie. The east end of Lake Erie, with Buffalo as its principal port, is really the water origin for present west-bound commerce and the water terminal for east-bound cargoes.

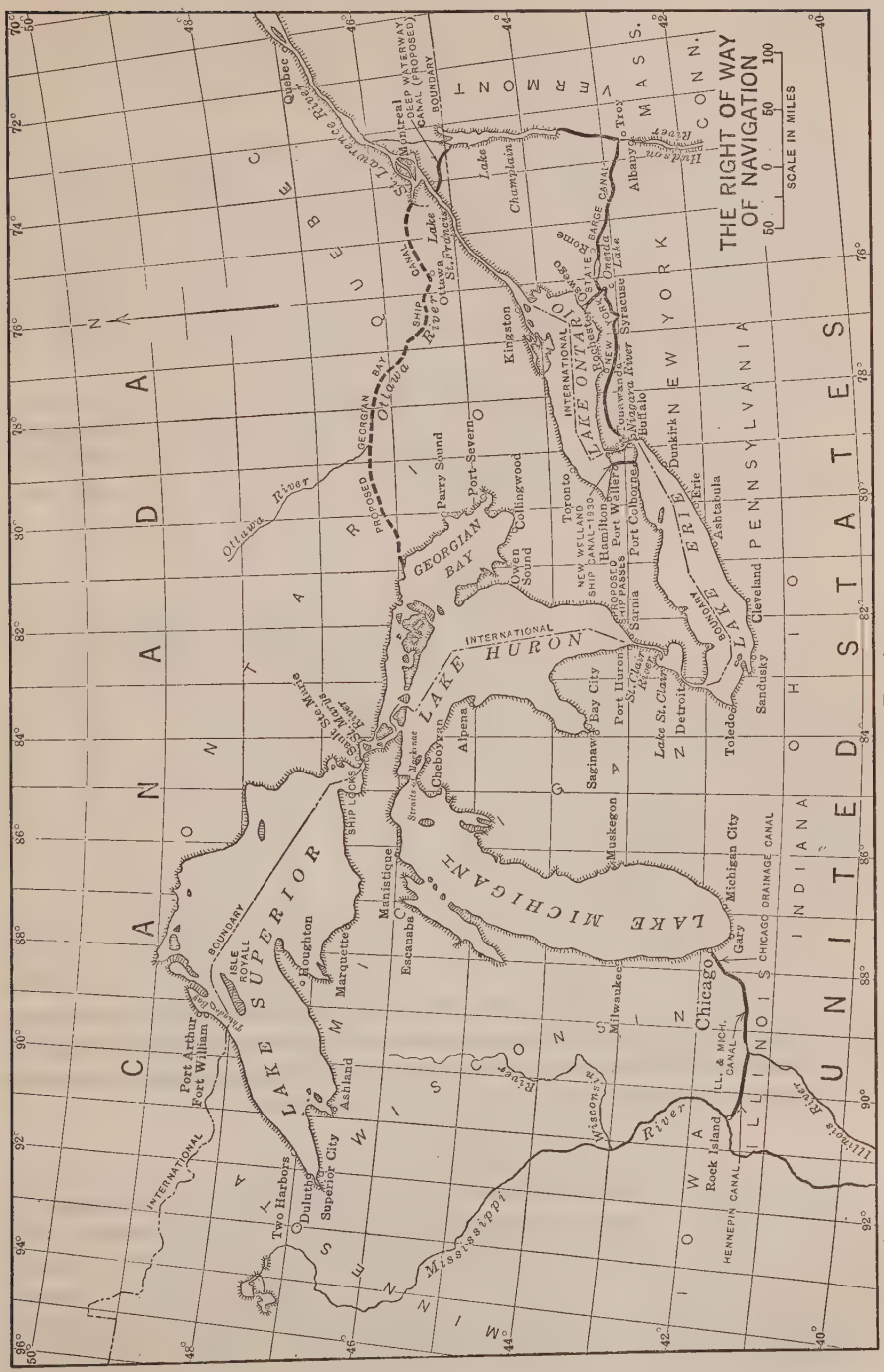
In the largest sense the St. Lawrence Waterway to the Sea must be visualized as the right of way leading to the Gulf of St. Lawrence from Duluth and Port Arthur, near the head of Lake Superior, and from Chicago, at the extreme south end of Lake Michigan. Deep penetration of the continent is the only adequate justification for great expenditures in the name of navigation. (See Plate III and Fig. 1.)

The St. Lawrence Waterway to the Sea contemplates deep-draft navigation—perhaps 25-ft. depths first, with 30-ft. drafts some decades later. The present navigation scheme for Lake Erie, and the rivers and lakes westward to Chicago, Duluth, and Port Arthur, is for 20 and 21-ft. depths—20 ft. in sheltered channels with soft bottom, 21 ft. where exposed to the surge of wave action and in rock channels. The Deep Waterway to the Sea, therefore, must include, as an essential part of its project, the securing of better depths in all artificial channels, harbors, canals, and locks which deep-draft vessels may need to traverse in the whole Great Lakes System. These depth betterments will be secured by two methods: First, by the comprehensive method of the maintenance of higher lake and river levels; and, second, by the piecemeal, localized method of dredging in trunk-line channels and canals and in important harbors. In many cases both methods will be necessary—the volume of dredging being diminished by the prism gained by the maintenance of higher water-surface levels.

Near Available Mileage

One of the notable things in this waterway project is the relatively large mileage already available for 25-ft. drafts; the mileage which will become available by the maintenance of higher water-surface levels; and the major projects, small in mileage but large in cost, already advancing toward com-

SHENEHON ON
THE ST. LAWRENCE WATERWAY
TO THE SEA.


CANADA
UNITED STATES
LAKE SUPERIOR
LAKE MICHIGAN
LAKE HURON
LAKE ERIE
LAKE ONTARIO
GEORGIAN BAY
Quebec
Montreal
Ottawa
Kingston
Toronto
Hamilton
Port Colborne
Niagara River
Buffalo
Rochester
Syracuse
Albany
Troy
Lake Champlain
Lake Ontario
Lake Erie
Lake St. Clair
Lake Michigan
Lake Superior
Port Arthur
Fort William
Isle Royal
Houghton
Marquette
Manistique
Escanaba
Duluth
Superior City
Two Harbors
Ashland
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Sarnia
Detroit
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Cleveland
Ashtabula
Erie
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Muskegon
Milwaukee
Chicago
Gary
Michigan City
Rock Island
Sault Ste Marie
Welland Canal
Ship Canal
Barge Canal
Chicago Drainage Canal
Ill. & Mich. Canal
Hennepin Canal
Mississippi River
Illinois River
Wisconsin River
Ohio River
Genesee River
Oswego
Lake Ontario
Lake Erie
Lake St. Clair
Lake Michigan
Lake Superior
Lake Champlain

pletion. The project as a whole awaits only the breaking through of a few impediments and the construction of works at the heads of the outflow rivers. The penetration of the principal impediment in the Rapids Section of the St. Lawrence will be eventually subsidized in whole or in part by the water power simultaneously developed.

The mileage of the vessel tracks of the Great Lakes is infinite and the mileage of the principal avenues or trunk lines is very considerable. Confining attention to the trunk line with Duluth as the initial westernmost port of the waterway and a point 1 000 miles below Montreal as the eastern gateway to the ocean, the total trunk-line length is 2 340 miles—with Liverpool still 2 200 miles away across the Atlantic. The mileage available after lake-level maintenance is effective and the Welland Ship Canal completed is given in Table 1.

The mileage requiring improvement, dredging, or cleaning up of obstructions, is about 9½% of the 1 338-mile vessel track from Duluth to Montreal; and about 5½% of the 2 341 miles of trunk-line waterway between Duluth and Belle Isle Straits. Outside this trunk line is the right of way of the salt seas encircling the globe.

TABLE 1.

Section.	Near available, miles.	Requiring improvement, miles.
Duluth to ship locks, St. Marys River	391	3
Ship locks, St. Marys River, to Detour	30	20
Detour to head of St. Clair River	213	3
St. Clair River	37	3
Lake St. Clair	10	8
Detroit River	16	11
Lake Erie to entrance of Welland Canal	227	3
Welland Ship Canal completed about 1930	26	0
Lake Ontario to Tibbetts Point	156	0
St. Lawrence River to Galops Rapids	66	3
St. Lawrence, Galops Rapids to Montreal	40	72
St. Lawrence, Montreal to Atlantic	1 003	0
Totals	2 215	126

For practical use of the waterway, the mileage of tracks in the principal harbors utilizing deep drafts must be added. Mostly the mileage given in Table 1 and mostly the work in harbors and harbor entrances are in sheltered areas where dredges may work in all weather conditions.

Cost of Improvements

The aggregate yardage to be removed is comparatively small to change 20 and 21-ft. channels, based on extreme unregulated low-water levels of the Great Lakes and interlake rivers, to 25 and 26-ft. channels, based on higher regulated lake and river surface levels. The channel depths which will come with maintained surface levels will be for the most part 22 and 23 ft. between Duluth, Port Arthur, Chicago, Detroit, Buffalo, Toronto, Prescott and Ogdensburg. The remaining 3 ft. of depth for 25 and 26-ft. channels will be secured

by cleaning up some of the channels and by dredging to an additional depth of 3 ft. in others. The volume in a mile of prism, 3 ft. thick and 300 ft. wide, is less than 200 000 cu. yd. One-half, at least, of the 50 miles of the Great Lakes is in channels with soft bottoms, as in Lake St. Clair; a large fraction is in glacial drift formations with boulders, as in St. Marys River above the locks; while a comparatively small mileage is in sand rock or limestone formations, as in the Neebish Channels in St. Marys River, in St. Marys Falls Canal and in the Lower Detroit River. Harbor channels are for the most part in soft materials. Only the major harbors of the Great Lakes need improvement for 25 and 26-ft. drafts.

The cost of deepening St. Marys River represents the largest item of dredge work. Louis C. Sabin, M. Am. Soc. C. E., U. S. Civilian Engineer in charge of the construction, maintenance and operation of the locks, canals and channels of St. Marys River (1924), has provided the following unit prices for submarine excavation:

	Pre-war.	1924.
Earth per cubic yard.....	$0.11 to $0.15	$0.18 to $0.30
Medium rock per cubic yard....	2.00	3.00
Hard rock per cubic yard.....	3.00	4.00 to 4.50

In the West Neebish Channel, excavated in the dry in a coffer-dam, the prices, 1904 to 1907, were $1.36 per cu. yd. for limestone rock and $0.56 for earth. Pre-war prices in the Detroit River compiled by Charles Y. Dixon, M. Am. Soc. C. E., for shallow cuts in submarine rock and glacial drift, indicate about $2.15 per cu. yd. At present this might cost $3.50 per cu. yd., and is comparable with the work still necessary in the Detroit River and in the rock and glacial drift sections of the channels of the Great Lakes and rivers above the Galops Rapids of the St. Lawrence River. Taking into account the sections of soft bottom in Hay Lake and Mud Lake of St. Marys River and the fine sand in Lake St. Clair, the cost of improving all trunk-line channels above the Galops Rapids to Chicago, Port Arthur and Duluth, may be between $30 000 000 and $40 000 000. Adding to this sum, the cost of breakwaters, regulating works, and navigable passes at the head of the St. Clair River, $10 000 000; and regulating works and excess-capacity channels in the Niagara River, $10 000 000; it appears that the total cost of 25 and 26-ft. navigation will be somewhat less than $70 000 000.

Benefits

These betterments will serve not alone the commerce seeking an outlet to the salt seas and seeking to pierce deep into the continent from the salt seas, but will serve also the inland commerce now utilizing, on impaired drafts, the right of way of the Great Lakes—with a volume of 100 000 000 tons in a maximum year. It will appear as this discussion develops that the regulating works already built and in operation at the outlet of Lake Superior and those proposed by the writer for the head of the St. Clair River, will make available for water-power volume and continuity in the Niagara and St. Lawrence Rivers

the vast storage reservoirs of Lake Superior and Michigan-Huron; and that the regulating works proposed for the head of the Niagara River will nearly double the volume of water permissible for power use at Niagara Falls, will permit the peak-load use of inefficient plants, and will solve some difficulties coming with the season of ice. As regulating works are part of the proposed St. Lawrence River development, only this mention of the works in that river, as the lowermost link in the series, is at this point desirable.

Scope

This discussion has three phases:

First.—The internal commerce of the Great Lakes as it now exists as an economic asset of two nations, with the physical limitations which handicap it, and the proposed engineering remedies.

Second.—The outlets to the salt seas existing and contemplated, including the St. Lawrence avenue.

Third.—Water power at Niagara Falls and on the St. Lawrence River as an economic asset of two nations, with the physical and international elements which handicap it—and the engineering and international remedies.

It appears obvious that if an expenditure of $70 000 000 in the Great Lakes System above the Galops Rapids, in addition to the new Welland Ship Canal now nearing completion (1924), did not serve as a paying investment for the movement of the 100 000 000 tons of fresh-water cargoes, it might be a venture hinging on a problematical salt-water commerce; and if the expenditure of $200 000 000 or $300 000 000 to make possible a problematical volume of commerce between fresh water and salt water, did not co-ordinately bring into use upward of 4 000 000 e.h.p. to enrich two nations, the St. Lawrence avenue to the sea might be forgotten for a generation or more. If, however, the expenditure of $70 000 000 in the Great Lakes System above the Galops Rapids serves economically the fresh-water commerce of the United States and Canada and, at the same time, vitalizes and multiplies the water power at Niagara Falls and on the St. Lawrence River, it ceases to be a doubtful venture; and if the water-power development of the St. Lawrence subsidizes the cost of the navigable avenue of this river above Montreal, the St. Lawrence Waterway to the Sea will not need to be forgotten for a generation.

These three things—the right of way of the Great Lakes, the various avenues to the sea, and the water power generated by the excess outflowing waters of this vast drainage basin—are the elements to be discussed.

Standards

The right of way of the Great Lakes, between Duluth, Port Arthur, Chicago and Lake Erie ports, with Buffalo as the present easternmost terminal, will be touched upon first. Before doing so, however, it may be well to state that the subject matter of this paper deals with projects so vast that $1 000 000 is the lowest unit to be given consideration. If this is not clearly apprehended at

the outset, the large expenditures suggested may be set aside as extravagant and visionary. It is well, also, to state that where estimates of the cost of improvements recommended by the writer are given, they are not true detailed estimates, but are more in the nature of illuminated guesses. It is well, also, to state that the ton of freight wherever referred to in this paper means a ton of 2 000 lb. Miles are always statute miles. Wherever levels are mentioned, the reference is to height in feet above mean tide at New York in terms of the adjusted levels of 1903. This adjustment was made by the U. S. Coast and Geodetic Survey working in collaboration with the U. S. Lake Survey. Wherever the Lake Survey is referred to in this paper it means the Survey of the Northern and Northwestern Lakes, an organization under the Corps of Engineers, U. S. Army, with headquarters at Detroit, and with jurisdiction for surveys, chart-making and hydraulic and other investigations covering the whole Great Lakes System, extending westward and including the Lake of the Woods and eastward and southward to include Lake Champlain and connecting waterways.

The writer has mentioned casually the expenditure of something approaching $70 000 000 within the Great Lakes System to secure 25 and 26-ft. navigation. The United States Government has already expended in developing the right of way of the Great Lakes, including channels, canals, ship locks, breakwaters, harbors, light-houses, and aids to navigation, upward of $150 000 000. The new Welland Ship Canal, now under construction, to be completed perhaps by 1930, will cost fully $100 000 000. The engineers' report for the St. Lawrence development between a point a little above the Galops Rapids and Montreal is over $250 000 000. It is probable that any of these figures may vary in ultimate estimates by 10% or more, with a possibility always that the variation may be upward.

Navigation Values

To make understandable and justifiable proposed expenditures comparable with the cost of the Panama Canal, a quotation is made from the Presidential Address of the late Alfred Noble, Past-President, Am. Soc. C. E., for many years Dean of American engineers. This address,* entitled "The Development of the Commerce of the Great Lakes," was delivered before the Society in 1903.

"This wonderful development of a few leading articles of freight has resulted in cheaper transportation than known elsewhere, except on long ocean routes. The most reliable and complete data existing are obtained at the St. Mary's Falls Canals by the co-operation of the Governments of Canada and the United States. The average freight rate on all classes of freight during 1902 was 0.89 mill per ton-mile, the average haul being 827.4 miles. If the freight had been carried by rail the average rate could hardly have been less than 3 mills per ton-mile, and the average haul would have been about the same as by water. On this basis the saving in transportation by the water route during one year was nearly $63 000 000. This amount of saving on Lake Superior commerce alone, during a single year, is within $5 000 000 of the entire amount appropriated by the United States for all harbors and waterways on the Lakes above Niagara Falls from the formation of the Govern-

* *Transactions*, Am. Soc. C. E., Vol. L (1903), p. 327.

ment; if the commerce between Lake Michigan and Lake Erie be included, the annual saving greatly exceeds the amount thus appropriated. Indeed, the saving during one year on a single article, iron ore, would repay all the money expended up to this time on the waterways and harbors which it traverses on its way from mine to furnace. Viewed in this way, no great investment was ever made by a corporation or government with such a magnificent showing of profit. But a far broader view may be taken justly; this water route has not merely afforded a saving in expense of transportation, but has been creative of a large part of the wealth of the nation. Without it the development of our manufactures of iron and steel and other industries dependent on them or related to them, all constituting a great share of the national wealth, would have been retarded or impracticable; without it the prairies of the Northwest would have remained unproductive and unpeopled for a much longer time. With these broad considerations in view we may place to the credit of Lake transportation a sum perhaps hardly definable, but compared to which the largest above mentioned is a mere trifle."

In the year 1902, referred to by Mr. Noble, the commerce through the Soo Locks amounted to about 36 000 000 tons with a valuation of a little over $358 000 000. During the same year, the estimated freight passing through the Detroit River was over 44 000 000 tons with an estimated value of nearly $441 000 000. Fourteen years later, in 1916—a year when commerce was stimulated by the World War—the freight passing the Soo Locks was nearly 92 000 000 tons and that through the Detroit River exceeded 100 000 000 tons, with a valuation of nearly $1 070 000 000.

Applying the same rule for estimating the economic value of the right of way of the Great Lakes as used by Mr. Noble, the saving for the single year, 1916, was $175 000 000 for commerce passing the Soo Locks.

Water Values

It will appear later that the outflowing waters of the Great Lakes have incredibly large values. It has been estimated that the diversion of 10 000 cu. ft. per sec. in the Drainage Canal is worth $8 000 000 a year to Chicago. The total ultimate usable power in the Niagara River is nearly 3 000 000 e.h.p. Should the installations for the use of this power cost $100 per e.h.p.—a low figure—the investment in that river alone will be $300 000 000. The present capitalization of the Niagara Falls Power Company is more than $75 000 000. On the St. Lawrence River upward of 4 000 000 e.h.p. will ultimately be developed. This will warrant an expenditure of more than $400 000 000.

Justification

After these vast sums are fully comprehended and digested, the justification of the expenditure of considerable sums in securing the highest carrying capacity of the vessels of the Great Lakes, through better and deeper channels —and, hence, cheaper freight rates—becomes apparent; and the advisability of utilizing for water-power betterment the vast storage reservoirs of Lake Superior and Lakes Michigan-Huron becomes obvious.

A great temptation exists for the writer to go into the historical phase of the development of the commerce of the Great Lakes. Instead, reference is again made to the Presidential Address of the late Mr. Noble previously

noted. The beginnings of the navigation of the Great Lakes reach back to a period of exploration, adventure and romance, perhaps more interesting than the cold recital of statistical facts and the cold development of engineering projects to add to the economic value of the right of way of the Great Lakes. This paper deals with the present and with the future and utilizes the past only to make the future understandable.

During the 1923 season of navigation—which may be regarded normally as having a duration of a little less than eight months, from April 15 to December 15—the freight passing the locks at Sault Ste. Marie amounted to over 91 000 000 tons, with a value exceeding $1 000 000 000. The total freight paid was nearly $81 000 000, which included loading and unloading, except in the case of coal. The average distance carried was roundly 800 miles. The average freight charge per ton for this distance was 88 cents, which indicates an average cost of 1.1 mills per ton-mile. It is a fair assumption that this freight if carried by rail would have cost over four times as much.

Deferred Maintenance

This land-locked waterway—the right of way of the Great Lakes—should be maintained in the same excellent condition as the right of way of a great railway, such as the Pennsylvania System or the New York Central Lines; but the fact is that the levels of the Great Lakes, on which the drafts of vessels depend for their loading capacity, have been permitted to lower, following the caprice of natural rainfall conditions; and have been artificially lowered by diversions for sanitation at Chicago, by diversions for power and navigation purposes in the Welland Canal and in the Niagara River, and have been lowered by the additional outflow capacity of the rivers themselves, created by the deepening of river reaches for ship channels. The depth betterment, for 20 and 21-ft. channels between Buffalo, Chicago, and Sault Ste. Marie, has been secured by excavating the lake and river bottoms below the level of extreme low water rather than by the elimination of extreme low water through the maintenance of the surface levels of the lakes.

The first step toward the St. Lawrence Waterway to the Sea is to bring up the levels of the lakes and the inter-lake rivers between the foot of Lake Erie, the southernmost extremity of Lake Michigan, and the westernmost extremity of Lake Superior. The maintenance of the surface levels of Lake Ontario is an integral part of the St. Lawrence River project and will be discussed in the general review of that project as part of the St. Lawrence Waterway to the Sea.

During the present season of navigation (1924), the St. Marys River, below the foot of the Soo Locks, and Lakes Michigan-Huron have been at their lowest recorded stage in a period of 65 years. The reason for this low stage involves the various diversions already mentioned, but is caused predominantly by the temporary absence of abundant rainfall in the Great Lakes System and by the absence of regulating works at the head of the St. Clair

River, where the lake waters outflow toward Lake St. Clair, the Detroit River and Lake Erie. (See Plate IV.)

It must be understood that the dearth of precipitation shown by the low levels of Lakes Michigan-Huron and Erie is not confined to this drainage basin but is widespread over the United States. California is having a water famine without precedent. The Mississippi River at Minneapolis in January, 1924, showed its lowest recorded flow. Most frequently, precipitation meagerness is somewhat localized. The drainage basin of Lake Superior may have an abundant water supply at a time when the Lake Erie Basin is deficient. It will appear later that the diversity factor applies to the basins of the Great Lakes and that advantage of this factor may be secured only by two operations: First, by conserving the flood waters in the upper reservoirs; and, second, by arranging for the transfer of these stored waters without time loss to the lake where navigation or water power requires additional supply. This obviously is the function of regulating works which will be discussed in detail in subsequent paragraphs.

Value of Draft

It is desirable to apprehend clearly what draft means in the carrying capacity of the freighters of the Great Lakes. The largest of these freighters exceeds 600 ft. in length. The tonnage of freight for a vessel drawing 20 ft. of water is carried by a displacement of perhaps 10 ft., the weight of the vessel and its non-productive cargo requiring the displacement of the other 10 ft. This means for this type of vessel that a loss of 1 ft. in draft involves a loss of upward of 1 000 tons of freight.

The elevation of Lakes Michigan-Huron for the season of navigation of 1924 is 579.3. The mean stage of Lakes Michigan-Huron during the seasons of navigation over a period of 65 years was 581.5. Based on the mean elevation for a period of 65 years, this indicates that in 1924 Lakes Michigan-Huron are deficient in navigable depth 2.2 ft., which means a loss of 2 200 tons in carrying capacity for each large lake freighter on each trip, with little change in the cost of operation. It is obvious that this must mean increased freight rates.

Expressing the loss of draft in inches as a matter of convenience, the season of navigation shows a deficiency in draft of 26 in. Chicago is chargeable with 5 in., through its diversion to the present time of about 9 000 cu. ft. per sec. Diversions in Lake Erie for purposes of navigation and water power are chargeable with perhaps a back-water reflection of 1½ in. Enlargements in the channels of the St. Clair and Detroit Rivers are responsible for perhaps 3½ in. The remaining loss of draft of 16 in. is chargeable to lack of normal precipitation and the absence of regulating works at the head of the St. Clair River.

The economic value of 1 in. in draft has been estimated as $500 000 per year. While this figure may be a little on the large side, it will be used in this paper. It must be understood that the large types of freighters on the Great Lakes are capable of loading to 24 ft., that they load to the last

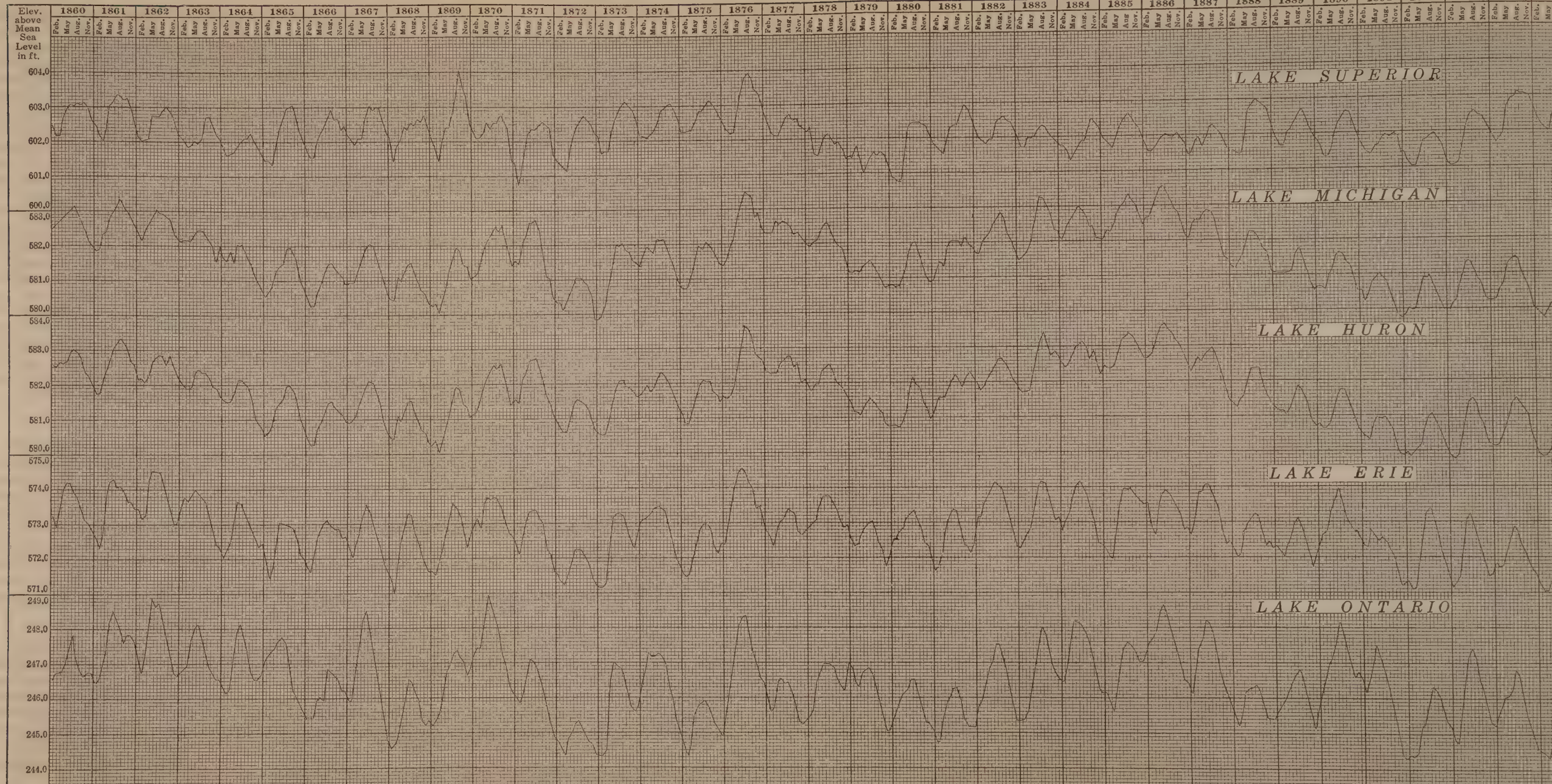
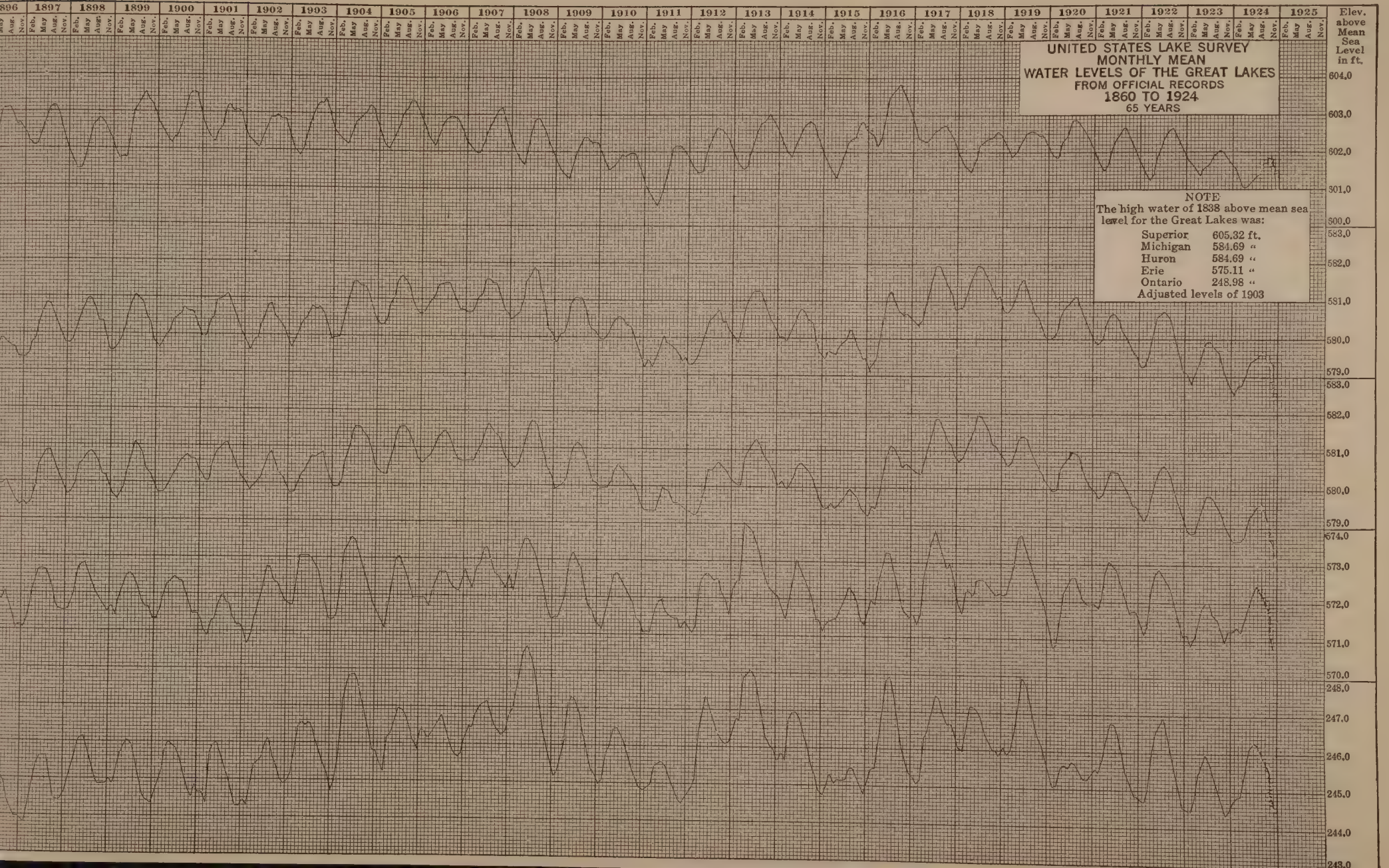

Elev. above Mean Sea Level in ft.
1860
1861
1862
1863
1864
1865
1866
1867
1868
1869
1870
1871
1872
1873
1874
1875
1876
1877
1878
1879
1880
1881
1882
1883
1884
1885
1886
1887
1888
1889
1890
1891
1892
1893
1894
Feb.
May
Aug.
Nov.
604.0
603.0
602.0
601.0
600.0
583.0
582.0
581.0
580.0
584.0
583.0
582.0
581.0
580.0
575.0
574.0
573.0
572.0
571.0
249.0
248.0
247.0
246.0
245.0
244.0
LAKE SUPERIOR
LAKE MICHIGAN
LAKE HURON
LAKE ERIE
LAKE ONTARIO

PLATE IV.
TRANS., AM. SOC. CIV. ENGRS.
VOL. 89, PAPER No. 1580.
SHENEHON ON
THE ST. LAWRENCE WATERWAY
TO THE SEA.



inch available in the channels they are to traverse, and that the loading is directed by telegraphic advice.

Assuming the correctness of this figure of $500 000 for a 1-in. draft, the loss for the 1924 season of navigation, based on 26 in. of deficiency, is $13 000 000. With reasonable reductions for a conservative estimate, it is certain that this loss under ordinary trade conditions would mean $10 000 000 for the navigable season of 1924. It happens, however, that depressed conditions have resulted in smaller shipments than usual and that vessels are available to carry the tonnage to be moved on lesser drafts. For this reason, the real economic loss on the Great Lakes for 1924 is somewhat less than the figure given.

During the season of 1923, the elevation of Lakes Michigan-Huron was 579.7. Based on a mean water-surface elevation for these lakes of 581.5, the deficient draft for this season of heavy shipments was 22 in., with a loss of upward of $10 000 000 for the year.

Assuming that these estimates of economic loss coming from deficient drafts in the channels of the St. Marys River, Lakes Michigan-Huron, and in Lake Erie, are anywhere nearly correct, it appears that the maintenance of lake levels by regulating works at the head of the St. Clair River, installed at the cost of perhaps $10 000 000, and regulating works at the head of the Niagara River at a cost of perhaps $10 000 000, is a most obvious economic necessity; and if it appears that some part of the cost of these works may be properly charged to water-power betterment at Niagara Falls and in the St. Lawrence River, the economic desirability is still further emphasized.

Low Levels Anticipated

The degraded levels of Lakes Michigan-Huron in the years 1923 and 1924, which have been cited, were a reasonable expectancy based on the water-supply conditions of the Great Lakes over a period of 65 years beginning in 1860. In the year 1895, the level of Lakes Michigan-Huron during the season of navigation was only 1 in. higher than in 1923; and in the year 1896, the level of these lakes was the same as in the season of 1923. These low levels of more than a quarter of a century ago preceded the deeper excavations in the St. Clair and Detroit Rivers, preceded the opening of the Chicago Drainage Canal and preceded any large diversions for water-power purposes in the Welland Canal and in the Niagara River.

Desirable Elevation

During the seasons of navigation for a 5-year period, 1883 to 1887, inclusive, the elevation of Lakes Michigan-Huron was 582.8. The writer suggests the regulation of these lakes during the season of navigation at the same elevation, 582.8, with a possible pull-down of 1 ft. of storage for water-power betterment in the Niagara and St. Lawrence Rivers in periods of meager precipitation.

Assuming the normal regulated stage of Lakes Michigan-Huron as 582.8 during the season of navigation, the deficient drafts during the season of

1923 amounted to 37 in. and during the season of navigation of 1924 to 42 in. These figures may be lessened to 25 and 30 in., based on the lower regulated level of these lakes after an occasional pull-down of 1 ft. for the use of the accumulated storage.

Conditions at Soo Locks

The effect of these low levels, and particularly that of 1924, in the lower approach to the ship locks at Sault Ste. Marie, should be noted. The water-surface elevation in the river just below the locks in a period of small river flow, such as that at the present time, is about 9 in. above the water-surface elevation of Lakes Michigan-Huron. In times of ample flow, the water level at the lower entrance to the locks may be 1 ft. or 1½ ft., under present conditions, above the level of these lakes. With the present degraded mean levels for the season of navigation of 1924, the permissible draft of vessels passing through the Poe Lock is about 17.5 ft. and the draft of vessels passing the Canadian Lock is limited to 17.7 ft. These locks were built in connection with the project in which the 20 and 21-ft. drafts provided for, were based on mean lake levels rather than on extreme low lake levels. It is for this reason, and because the level of the St. Marys River at the lower approach to the locks has been further degraded by excavating deeper channels throughout the lower river, that these two ship locks are of no value this year (1924) for other than light-draft vessels. Vessels loading to 18 or 19 ft. must pass through the two new American locks—the Davis Lock and the Sabin Lock. It is obvious that bringing Lakes Michigan-Huron levels up 3 ft. or more by regulation, will make available again these two ship locks for 20-ft. navigation.

During the present season, the conditions in St. Marys River and in Lakes Michigan-Huron control drafts. During other seasons, the drafts of vessels have been limited by Lake Erie channels, which will be discussed later in connection with regulation works at the head of the Niagara River.

The Port of Duluth

The zero point for the St. Lawrence Waterway to the Sea is at Duluth, at the westernmost extremity of Lake Superior, 950 miles westward and 410 miles northward from the Port of New York and 1 420 miles eastward from Seattle. (See Plate III.) By a somewhat remarkable natural coincidence the St. Louis River enters Lake Superior at this westernmost point and a delta formation of alluvial material forms a natural breakwater 8 miles in length, protecting the estuary of the river from storms. This spit or breakwater is pierced by harbor entrances in two places, one toward its northwestern extremity leading to the inner basin of the Harbor of Duluth. Southeasterly from this entrance about 6¼ miles is an entry first, through artificial breakwaters to an outer harbor, and, then, through a passage 400 ft. wide, to the inner basin of Superior Harbor. As the St. Louis River forms the boundary line between Minnesota and Wisconsin, the twin harbors connected by a vessel channel behind the breakwater, are interstate. Aside from the 4-mile channel connecting the two basins, there are more than 13 miles of channels leading to the various docks of the two harbors.

The harbor entrances are already deep enough for 25-ft. navigation. For the most part, the various channels and the basins of the harbor will require dredging to a depth of perhaps 3 ft. to provide for 25-ft. navigation, with higher regulated levels of Lake Superior. The material to be excavated in deepening the harbor channels is largely silt and sand, underlaid by boulder clay. All the material may be handled by suction dredges, using cutters in the clay, at a probable cost not exceeding 20 cents per cu. yd.

The Government has already spent—or is completing the expenditure of—a little over $8 500 000, including maintenance. The cost of maintenance for 1925 is estimated at $52 500. Private companies and the City of Duluth have expended about $200 000 in breakwaters and trunk-line channels. An expenditure of about $1 000 000 will provide 25 and 26-ft. depths in the twin harbors.

The following statements are briefed from the 1923 reports of E. H. Marks, Major, Corps of Engineers, U. S. A., Assoc. M. Am. Soc. C. E., in charge of the work:

The entire protected harbor area aggregates 19 sq. miles, with about 49 lin. miles of frontage, only 6 miles of which are occupied by wharves. Nine iron ore docks, aggregating 8 720 ft. in length, have a total combined storage capacity of 1 080 000 tons; 24 coal docks have an aggregate storage capacity of 11 305 000 tons; 26 elevators have a total storage capacity of 36 325 000 bushels; a modern cement elevator has a storage capacity for 114 000 bbl. of cement, with an unloading capacity of 1 000 bbl. per hour. Most of these docks and elevators are up to date in facilities for the quick handling of ore, coal, and grain, and are probably not surpassed by those of any other port. Practically all the wharves, docks, and elevators are privately owned. The facilities for handling package freight, lumber and general merchandise were primitive up to 1924, when a new terminal plant, privately owned, but open to the public, was made available. Three shipbuilding yards and forty-three wharves handle freight other than iron ore, coal, and grain. Ten railroads, either direct or through affiliated lines, connect with the water terminals.

Major Marks sums up the value of improvements in the Duluth and Superior Harbor as follows:

"The effect of the work done under the existing and previous projects has been to facilitate navigation by means of deeper and wider channels, with commodious anchorages and turning basins and safe entrances to the harbor, as well as to give refuge to vessels seeking shelter from storms. Through these improvements, which permit the use of larger vessels, freight rates have been materially reduced."

Inter-lake navigation, based on records for the past 20 years, begins normally on April 20, and terminates on December 14, showing a navigation season of 239 days for Duluth.

Duluth Commerce

In 1923, the commerce of the Duluth-Superior Port shows over 59 000 000 tons, with a valuation of nearly $450 000 000. Of this commerce, the outbound tonnage amounted to a little over 45 000 000, while the inbound com-

merce amounted to a little over 14 000 000 tons. This shows an outbound balance of roundly 31 000 000 tons, indicating that many upbound vessels travel light or with water ballast. Iron ore, with a value of $4.32 per ton, constitutes 58% of the outbound valuation and 94% of the outbound tonnage. Wheat, high in value, but low in weight, constitutes 13% in value and 7½% in tonnage. Shipments of animals and animal products show a valuation of $14 500 000; vegetable and food products, including wheat, $17 500 000; wood and wood products, $800 000; unclassified freight, $3 900 000; and copper nearly $8 000 000. The dominant inbound cargo is coal, 12 700 000 tons in 1923, with a value of $72 245 000. The total inbound commerce has a valuation of nearly $132 000 000.

Canadian Ports

About 200 miles northeast of Duluth are the harbors of Port Arthur and Fort William in Thunder Bay. These are the terminals for the railways that tap the fields of Canada's great western granary and bring wood products as well. This harbor is landlocked against Lake Superior seas by Isle Royal lying across the entrance to Thunder Bay, by Pie Island lying midway of the entrance, and by Thunder Cape. This westernmost Canadian port, with its capacious roadstead, has unlimited capacity and ample drafts for ocean vessels.

Type of Vessel

The largest portion of the commerce of these ports is carried in a typical vessel of a length of 600 ft., with a beam of 60 ft., and built for drafts of 24 ft. The engine is carried aft, and the freight hold extends nearly the full length of the vessel. The deck is largely made up of hatches for rapid loading and unloading. The economic speed of travel in open water is about 11 miles per hour. The engine has about 3 000 i.h.p.

Duluth to the Soo

One of these freighters, carrying 10 000 tons of cargo on a 20-ft. draft, leaving Duluth, traverses the deep waterway of Lake Superior—1 000 ft. deep in places—and reaches the ship locks at Sault Ste. Marie, 394 miles away, in about 36 hours. It requires an hour for the freighter to pass the ship locks, descending 20 ft. to the level of St. Marys River below the rapids.

The Sault de Sainte Marie

Fig. 2 is an airplane view of the layout of the various works at Sault Ste. Marie, and Fig. 4 explains the photograph and gives much desirable information.

It should be noted that regulating works, consisting of a battery of Stoney gates, control the outflow and levels of Lake Superior. These regulating works and St. Marys Falls Canal are about 14 miles down stream from the foot of Lake Superior. Upper St. Marys River is for the most part 1 mile or more in width and runs deep and still. The descent from Lake Superior

FIG. 2.—AIRPLANE VIEW OF WORKS OF NAVIGATION AND WATER POWER AT SAULT STE. MARIE.

FIG. 3.—OPENING OF THE SABIN LOCK, SEPTEMBER 18, 1919.

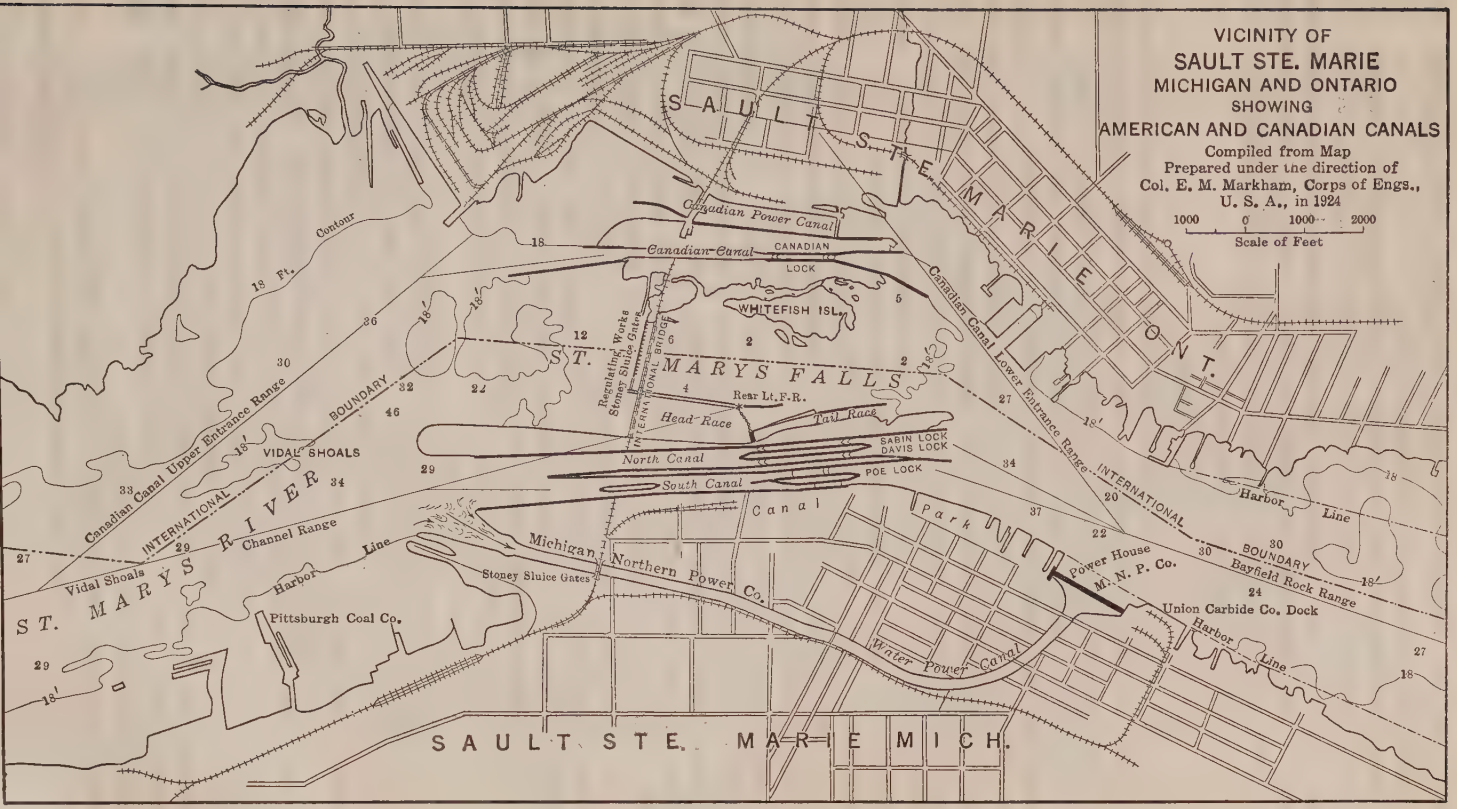

VICINITY OF
SAULT STE. MARIE
MICHIGAN AND ONTARIO
SHOWING
AMERICAN AND CANADIAN CANALS
Compiled from Map
Prepared under the direction of
Col. E. M. Markham, Corps of Engs.,
U. S. A., in 1924
1000 0 1000 2000
Scale of Feet
SAULT STE MARIE ONT.
Canadian Power Canal
Canadian Canal
CANADIAN LOCK
WHITEFISH ISL.
Canadian Canal Lower Entrance Range
Regulating Works Stoney Sluice Gates
INTERNATIONAL BRIDGE
ST. MARYS FALLS
Rear Lt. F.R.
Head Race
Tail Race
SABIN LOCK
DAVIS LOCK
POE LOCK
North Canal
South Canal
Canal
Park
Canadian Canal Upper Entrance Range
VIDAL SHOALS
INTERNATIONAL BOUNDARY
ST. MARYS RIVER
Channel Range
Harbor Line
Vidal Shoals
Stoney Sluice Gates
Pittsburgh Coal Co.
Michigan Northern Power Co.
Power House M. N. P. Co.
Bayfield Rock Range
Union Carbide Co. Dock
Water Power Canal
SAULT STE. MARIE MICH.
Contour
18 Ft.

FIG. 4.

level to the level of the river at the regulating works varies somewhat with the volume of flow, but may be taken as 0.3 ft.

The writer suggests the regulation of Lake Superior at Elevation 603 as a mean during the season of navigation, with a pull-down of 1 ft. for storage water in case of navigation need in the lower lakes or in the water powers of the Niagara and St. Lawrence Rivers. Based on these regulated lake surfaces, about 3 miles of dredging—deepening 22-ft. channels to 25-ft. channels—would be required in the upper river.

Ship Locks

Four ship locks on the American side of the river and one ship lock on the Canadian side are in operation. Down-bound vessels approaching these locks pass through about a mile of canal with ample mooring space. The Canadian Lock, completed in 1895, is 900 ft. long and 60 ft. wide, and is intended for 22-ft. drafts, but during the mid-season of 1924, due to depressed river pool conditions, low lake levels, and small volume of river outflow, the draft over the limiting barrier at the lower entrance of the locks was only about 17.7 ft. On the American side, the Weitzel Lock, 515 ft. long and 80 ft. wide, narrowing at the gates to 60 ft., was completed in 1881 and was intended for 14-ft. navigation, but has only about 12 ft. over the lower breast-wall. The Poe Lock is 800 ft. long and 100 ft. wide, built for 22-ft. drafts, but in 1924 had about 17.5 ft. in the lower entrance. The writer had a small part in the building of this ship lock. It was intended as a 4-vessel lock, each vessel having a length of 400 ft. and a beam of 50 ft. It was completed in 1896 and in the interval of less than 30 years, it has become a single-vessel lock for a typical lake freighter when traveling light. The Davis Lock was completed in 1914, has a 1 350-ft. length and an 80-ft. width, with 24 ft. of water on the lower breast-wall despite lower-pool depression, small outflow, and low lake levels. The Sabin Lock (Fig. 3), of the same dimensions as the Davis Lock, was opened to traffic in September, 1919. It is in gracious appreciation that this lock bears the name of Mr. Louis C. Sabin, in local charge of construction and operation of the works in St. Marys River since 1906. The other American locks bear the names of officers of the Corps of Engineers, U. S. Army. (See Fig. 2.)

The Poe Lock had a pre-war cost of $3 000 000 and the Sabin Lock, built in 1913 to 1919, cost $2 500 000. The total cost of improvements in St. Marys River to July, 1923, aside from the practically obsolete Weitzel Lock, amounts to $30 000 000. The cost of operation of the locks and canals on the American side was $141 200 in 1923 and the maintenance cost was, roundly, $105 000. The total cost of operation and maintenance during the past 10 years has amounted to 2¾ mills for each ton of cargo passing the ship locks.

It should be noted that the necessary time spent by vessels in traversing 1.6 miles of canal and passing the ship locks, averages roundly 1½ hours. This means that practically 1 hour is necessarily spent in the lockage. This time is in part utilized in taking on supplies.

With regulated Lake Superior and Lakes Michigan-Huron levels, the Davis and the Sabin Locks will become available for 25 and 26-ft. navigation.

although this will probably involve a lowering by perhaps 1 ft. of the breast-wall and the miter-sills of the upper approach. It should be accentuated that increased drafts and larger cargoes mean fewer vessels and, therefore, fewer necessary locks. The Canadian Lock and the Poe Lock will still normally remain available for 20-ft. navigation, while the two new American locks will care for deep-draft vessels. This link in the St. Lawrence Waterway from Duluth to the Soo is, therefore, substantially existent.

St. Marys Falls Canal Statistics

An average of 92 vessels per day passed the five locks during the season of 1923, of which 55 passed through the Davis and Sabin Locks and 17 through the Canadian Lock.

In 1923, the total freight carried was 91 380 000 tons. This freight had a valuation of $1 026 045 064, and the freight charges amounted to $80 843 000. The east-bound freight passing through the Soo Locks was 71 236 000 tons, and the west-bound, 20 144 000 tons. The valuation of 774 registered vessels using the locks was $254 630 000. The average distance which freight was carried was 801.3 miles and the average cost per ton for freight transportation was 88 cents, showing a cost per ton-mile of 1.1 mills. In 1923, American vessels carried 92.7% of the total freight, and Canadian vessels, 7.3 per cent. The total valuation of registered Canadian vessels is given as $29 128 350, which is 11.4% of the total valuation of all registered vessels passing the Soo Locks.

The American Canal was operated 231 days during the somewhat shorter than usual season of 1923. The normal navigation season during the past 20 years has been from April 18 to December 19, or 246 days. In 1923, over 98% of the freight was carried during May to November, inclusive, the month of July showing the largest tonnage.

The statistical report of lake commerce passing through the canals at Sault Ste. Marie, Mich., and Ontario, during the season of 1923, with a supplementary report of commerce passing through the Detroit River, prepared under the direction of E. M. Markham, Colonel, Corps of Engineers, U. S. A., with headquarters at Detroit, is a most interesting informative document. Students of water transportation need this pamphlet.

In 1922, of the total of 600 vessels, 158 carried cargoes of 10 000 to 14 000 tons. In 1923, with lower lake levels, only 110 vessels carried cargoes of these sizes.

The dominant east-bound tonnage for American vessels is iron ore and for Canadian vessels flour, wheat, and other grain. The dominant west-bound cargo for American vessels is coal, 10 tons of soft coal to 1 ton of hard coal. The dominant west-bound cargo of Canadian vessels is general merchandise.

The following tabulation shows the tonnage and percentage of tonnage of the total east-bound traffic for 1923 through the Soo Canals:

Iron ore	59 200 000 tons	83.2%
Grain	10 000 000 "	14.0%
Flour	1 500 000 "	2.1%
Miscellaneous	0 500 000 "	0.7%

For west-bound cargoes, coal, 18 400 000 tons, constitutes 87% of the tonnage.

Perhaps the valuation is a better criterion of the use of a waterway than the number of tons carried. Of the total valuation of $1 026 045 064, the valuations of various classes of cargo are, as follows:

Lumber	0.69%
Flour, wheat, and other grain	41.93%
Copper	1.65%
Iron ore, pig iron, and manufactured iron and steel	30.51%
Coal, soft and hard	9.98%
Miscellaneous merchandise	15.24%

It should be noted that the valuation of all freight carried, outside of iron ore, is $720 356 808.

The freight rates for various commodities in 1923 were as follows, reduced to cost per ton, including loading and unloading except in the case of coal:

Lumber	$2.75
Flour	3.50
Wheat	1.27
Other grains	1.52
Copper	5.00
Iron ore	0.83
Pig iron	2.85
Manufactured iron and steel	2.50
Coal	0.45*
Salt	1.25
Oil	1.25
Stone	0.50
General merchandise	5.00

These freight charges bear some relation to the value of the commodity itself. Lumber has a value of $18.65 per ton; flour, $62.50; wheat, $35.30; grain other than wheat, $29.60; copper, $281.00; iron ore, $5.15; pig iron, $30.85; manufactured iron and steel, $85.00; coal, soft, $5.00; coal, hard, $11.20; salt, $8.90; oil, $41.00; stone, $1.10; and general merchandise, $225.00.

It is obvious also that the space occupied by freight of a given class, as well as the difficulties of loading and unloading, enter into the freight rates charged. Taking 100 cu. ft. as the measure of the registered ton of vessel capacity, 1 ton of wheat occupies 37⅓ cu. ft. of space; 1 ton of lumber occupies 40 cu. ft.; and 1 ton of general merchandise is understood to occupy 100 cu. ft., which figure, however, has wide variations. The freight passing the Soo Canal in 1923 shows 1.43 tons of freight carried for each registered ton of vessel capacity. This means that the average ton occupied 70 cu. ft. of space.

These statistical figures are tedious and need to be read only by those who wish to analyze the water-transportation problem in its bearing on the land-locked commerce of the United States and the possibilities of emergence of this commerce into the salt seas. The big problem aside from carrying capacity, appears to be that of increasing the load factor of vessels of the Great Lakes

* Does not include loading and unloading.

by the stimulation of up-bound cargoes and by 12-month operation. Coal shows a marked upward volume of tonnage, the coal carried in 1900 being less than 4 500 000 tons, while that carried in 1923 was more than 18 400 000 tons. Grain cargoes are also increasing, showing for a period about 1900, 73 000 000 bushels and for 1923, 370 000 000 bushels. Lumber, on the other hand, shows a declining tonnage. The growth in the transportation of general merchandise is very small.

Forecast

For the coming decades it is probable that the shipments of high-grade, commercial, hematite iron ore will decline. This decline will be replaced, in part at least, by the shipment of ores of lower grade which will be concentrated before shipment by the removal of some non-metallic elements. The shipment of coal will increase largely with each decade, tending more nearly to equalize up-bound and down-bound cargoes. The greater carrying capacity of vessels which will come with deeper drafts, the near-balance of up-bound and down-bound cargoes, and the final utilization of some vessels now limited to an 8-month year for ocean service during the four ice months, will all tend to the further reduction of freight rates.

It is interesting to note the economic value of the present commerce of the Great Lakes, excluding iron ore and coal. Aside from these commodities, the 1923 shipments on the Great Lakes aggregated, for the Soo Locks, 13 627 404 tons, and the freight paid was $23 398 840. These shipments, assuming an average travel of 800 miles, represent a ton-mile rate of 2.14 mills. A normal rail rate per ton-mile is 11 mills. The saving by this part of the transportation on the Great Lakes over rail on these commodities alone represents an excess of more than $90 000 000 for 1923.

Assuming the elimination of iron ore alone, and considering coal and all other commodities, and taking a rate of $1.00 per ton on up-bound coal, the aggregate carried through the Soo Locks in 1923 is 32 022 715 tons and the freight charged would be $41 694 150. The average rate per ton-mile then becomes 1.63 mills. This is less than one-sixth the normal rail rate. It should be understood that if it were not for the cheap freight rates on the Great Lakes, much of the iron ore of Minnesota might lie unused in the ground, the ores coming from other sources; and coal might come by rail from Illinois and Kentucky instead of from the Pennsylvania mines. About 80% of the iron used in the United States comes from the mines tributary to Lake Superior. It is probable that ultimately iron ore will be exported from Minnesota.

The Soo to Port Huron

The 600-ft. freighter coming down Lake Superior from Duluth and reaching the Soo Locks 36 hours later has spent 1½ hours in passing through the locks and canals, including the examination of statistics. The descent in the locks is about 20 ft. under normal conditions. Leaving Sault Ste. Marie, down-bound, the vessel course follows for 2 miles the partly improved river with a

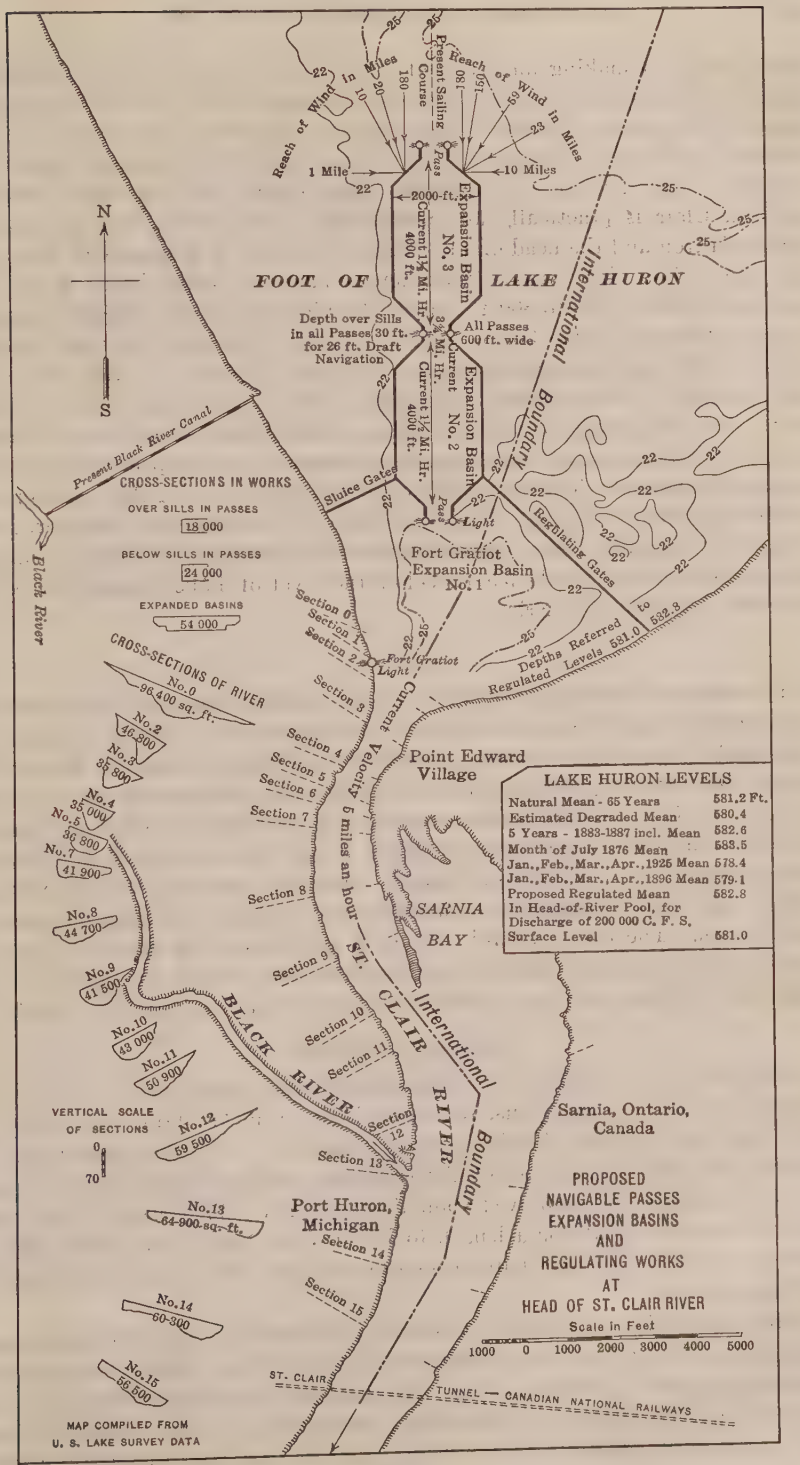
width of ¾ mile, then for a distance of 5 miles in a dredge cut through Little Rapids into Hay Lake. This lake is about 1 mile wide and, for 4 miles, has depths requiring little or no dredging to secure 25-ft. drafts. The channel splits about midway of Hay Lake, down-bound vessels taking the West Neebish Channel and up-bound the Middle Neebish Channel for a distance of 14 miles, when the course leads into Mud Lake and the estuary of St. Marys River—upward of 2 miles wide. From here to Point Detour is 25 miles of clear sailing in a channel the greater part of which is 1 mile or more in width. At Point Detour, vessel courses radiate toward Lake Michigan and Lake Huron ports. Vessels bound for Chicago, work to the westerly in Lake Huron and through the Straits of Mackinaw to the foot of Lake Michigan, 423 miles away. The down-bound 600-ft. freighter passes down Lake Huron with depths as great as 400 ft., reaching Port Huron, Mich., at the head of the St. Clair River, in 216 miles of travel and 20 hours after leaving Point Detour. This is 62 hours from Duluth.

Suggested Works

At the foot of Lake Huron—at the head of the St. Clair River—the writer suggests regulating works to maintain lake levels and create storage. Such regulation means that the elevation of Lake Huron will be normally 1 to 2 ft. or more higher than the water in the head-of-the-river pool opposite Fort Gratiot Lighthouse. The elevation of the water at Fort Gratiot primarily determines the outflow through the St. Clair River; and conversely the surface elevation or stage of Lakes Michigan-Huron is determined by the abundance or meagerness of the water supplied to these lakes. It is purposed to curtail the natural prodigal outflow from the Michigan-Huron Reservoir in times of opulent water supply and during the winter so as to store the water against periods of meager water supply; and it is proposed to release reservoired water, when Lakes Erie and Ontario need navigational uplift and when hydro-electric plants in the Niagara and St. Lawrence Rivers require volume maintenance. Regulating works above the head of the St. Clair River will co-ordinately require navigational passes to permit up-bound vessels to ascend from the head-of-the-river pool level to the lake level. (See Fig. 5.) The 600-footer, down bound, will enter the 600-ft. wide pass and traverse without delay the three expansion basins proposed. Up-bound vessels will encounter no inconvenience or delay. No ship locks will be required.

Port Huron to Detroit

At present, the down-bound 600-ft. freighter passes directly into the gorge of the St. Clair River at Port Huron, 800 ft. wide, 50 ft. deep and flowing with a velocity of 5 miles per hour. Just below this rapid descent of the upper river is 1 mile of duplicate channels. For the greater part of 27 miles the river is more than ¼ mile wide, with water varying from 25 to 50 ft. deep. The current, except at Port Huron, is leisurely. Just below Algonac, Mich., 28 miles below Port Huron, the river enters a delta formation radiating into a series of channels somewhat similar to those at the mouth of the



LAKE HURON LEVELS

Natural Mean - 65 Years	581.2 Ft.
Estimated Degraded Mean	580.4
5 Years - 1883-1887 incl. Mean	582.6
Month of July 1876 Mean	583.5
Jan., Feb., Mar., Apr., 1925 Mean	578.4
Jan., Feb., Mar., Apr., 1896 Mean	579.1
Proposed Regulated Mean In Head-of-River Pool, for Discharge of 200 000 C. F. S.	582.8
Surface Level	581.0

FIG. 5.

Mississippi. In making an artificial channel the material to be excavated is alluvial. For a distance of 11 miles, the channel is 500 to 1 000 ft. in width, with some curvature. Ultimate deep channels will involve rectification as well as deepening of some portions of this reach. The 600-ft. freighter passes into Lake St. Clair through the St. Clair Flats Canal with duplicate channels. Lake St. Clair is practically an enlargement of the rivers between the foot of Lake Huron and the head of Lake Erie. The bottom is largely of fine sand, with some clay. The vessel track across Lake St. Clair is 15 miles long and has a width of 600 to 800 ft., much of which has been artificially deepened by the stirring up of the bottom by the propeller wheels of the many passing vessels. Although the natural depth of at least 10 miles of Lake St. Clair was about 21 ft., the channel dredged by vessel travel has a depth of about 24 ft. This deepening of channels by propellers means simply the roiling of the water, with a portion of the material thrown into suspension depositing outside the channel. This condition is possibly pertinent in the consideration of the maintenance of Lake St. Louis, Lake St. Francis, and Lake St. Peter in the St. Lawrence River where the alluvial material is somewhat similar. Lake St. Clair is normally 5.5 ft. below the level of Lake Huron and 3.1 ft. above the level of Lake Erie.

Detroit

The center of the Port of Detroit is 6.5 miles below the head of the Detroit River and the foot of Lake St. Clair. The river at this point is ½ mile wide, with depths from 30 to 45 ft. and a current of perhaps 2 miles per hour. The harbor front extends practically from the head of the river down for a distance of 13 miles. On the north bank of the river is the Canadian City of Windsor. Additional harbor frontage is given by the improvement of the Rouge River—5 miles down from the center of Detroit—where Mr. Henry Ford has extensive works. The Port of Detroit is 724 miles from Duluth, involving 66 hours of vessel travel. The name, Detroit, means a strait, and this river, 27 miles in length, connecting Lake St. Clair with Lake Erie, varies in elevation with Lake Erie heights which are reflected by back-water into Lake St. Clair.

Detroit to Buffalo

Southward, for a distance of 14 miles below the center of the Port of Detroit, the river is capacious, requiring little or no dredging to develop it. Then the river shallows, showing estuary conditions. A little lower down, the channel splits, down-bound vessels following the Livingstone Cut, and up-bound vessels taking the old improved Canadian Channel. This reach of the river requiring improvement has a length to Bar Point, Ont., of 6.5 miles; and for 3 miles more through the delta formation of Lake Erie additional dredging will be required.

From Bar Point to the Port of Buffalo is 240 miles of clear sailing in open Lake Erie. This lake is the shallowest of all the Great Lakes, having depths for the most part of 100 ft. or less. The west end of the lake shows depths of 25 to 35 ft. for a distance of 30 miles. On account of the tendency of Lake Erie to tilt up in times of severe storms, channels have been dredged to greater

depths than in the other more stable lakes. It will be shown subsequently that the betterment of Erie levels is involved in this storm-tilting of the lake.

The Port of Buffalo is 985 miles and 90 hours travel from Duluth. The entrance to the Welland Canal at Port Colborne, Ont., is 10 miles less. These ports represent the present eastern limit of deep-draft navigation.

The waters of Lake Erie discharge through the Niagara River with its head at Buffalo. The Board of Engineers for Rivers and Harbors of the Corps of Engineers, U. S. Army, recommended in 1920, immediate construction of regulating works at the head of the Niagara River to maintain Lake Erie levels. The writer will discuss works at this point in connection with the control of all the outflow rivers of the Great Lakes for the dual purposes of the maintenance of the best navigable depths and at the same time the creation of storage for water power in the Niagara and St. Lawrence Rivers.

Outlets to the Sea

The navigable waterway of the Great Lakes with its deep penetration of the continent (see Plate III), and its large commerce cannot remain always a land-locked interior basin without adequate connection with the salt seas. For a century the desirability of this connection has been obvious and considerable steps toward its achievement have already been made. The Erie Canal was begun in George Washington's day and about the same time the Canadians were building waterways entering the Great Lakes System by the way of the St. Lawrence River and Lake Ontario and then over the Niagara Escarpment through the Welland Canal to Lake Erie. In the summer of 1924, the centenary of the beginnings of the Welland Canal was celebrated. The present Welland Canal with 14-ft. drafts and lock chambers, 270 ft. long by 45 ft. wide, descends from Lake Erie to Lake Ontario with a definite, although small, volume of commerce, perhaps showing a maximum of 3 500 000 tons per year; and 2 000-ton vessels passing through the Welland Canal go on down to Montreal through the Laurentian Canals which have drafts and locks much the same as those of the Welland. The New York State Barge Canal leaving the Niagara River at Tonawanda, N. Y., a few miles below Buffalo, is an escape to the Hudson River, and thence to salt water in the Port of New York. The Illinois Waterway at present under construction, will pass through the Drainage Canal at Chicago to the Des Plaines and Illinois Rivers with 9-ft. drafts and thence by the Mississippi River to the Gulf of Mexico. The use of any one of these three canals involves either the utilization of light-draft vessels on the Great Lakes or transshipment from the deep-draft lake carriers. Neither of these alternatives makes for the most economic through transportation.

The Welland Ship Canal

Before discussing the various deep waterways to the sea, proposed a quarter of a century ago, it may be desirable to note the near completion of the new Welland Ship Canal, assume that this is the year 1930, and continue toward Montreal on the 600-ft. freighter down-bound from Duluth.

The Welland Canal entrance is at Port Colborne. It is protected from storms, and mooring space is provided by an extension of breakwaters and piers for a distance of more than 1 mile out into Lake Erie. The entrance to this harbor and the harbor itself are dredged to a depth of 30 ft. below ordinary Lake Erie level. The object of this dredging is to provide for the tilting of Lake Erie in storm-time, so that when the water level drops 5 ft., vessels drawing 25 ft. of water may still enter the harbor. This deep dredging is continued to the Guard Lock, 2 miles from the harbor entrance, where vessels will descend normally about 5 ft. The inner summit canal has a depth of 25 ft., a bottom width of 200 ft., and slopes of various inclinations—the normal water surface width being 305 ft. The canal has a cross-sectional area of 6 000 sq. ft. and is capable of passing, without obstructive currents, about 10 000 cu. ft. per sec. The water-power capacity of this canal will be touched upon later in discussing Niagara River power.

Lift Lock No. 7 is 20 miles from the Erie Harbor entrance. The descent in this lock is, as in the remaining six locks, 46.5 ft.; and the chamber in all the locks is 800 ft. long by 80 ft. wide, with 30 ft. depth capacity. Another ½ mile brings the 600-ft. freighter to tandem Locks Nos. 6, 5, and 4, which are twin locks, as well. The twinning of these locks is necessitated by water supply conditions, all other locks being single-chamber locks. After 5 miles of further travel, including the passing of Locks Nos. 3, 2, and 1, the freighter has descended 326 ft. to Lake Ontario level, which is normally about 246 ft. above mean tide at New York; then 9 600 ft. through a broad canal and an inner harbor—over 1 mile long and 800 ft. wide, with adequate mooring space—to the 400-ft. entrance into Lake Ontario.

The work done on the Welland Canal in its various elements represents the highest engineering achievement. As already stated, it will be completed about 1930 and will cost upward of $100 000 000. The original undertaking was pre-war and the ultimate expenditure of so much money as the final cost was not anticipated. Whether or not the St. Lawrence River itself is canalized for sea-going vessels, this adequate link over the Niagara Escarpment brings Lake Ontario into the deep-water basin of the Great Lakes System leading to Buffalo, Detroit, Chicago, Port Arthur, and Duluth. The estimated time consumed in passing through this 27 miles of waterway, including harbors at each end, is about 12 hours.

Port Weller to St. Lawrence

From Port Weller, the Lake Ontario exit from the Welland Canal, the course is straightaway toward Tibbetts Point at the head of the St. Lawrence River, 156 miles distant. Northerly from Port Weller is the Canadian City of Toronto, 29 miles away; and the City of Hamilton is about the same distance to the westward. Charlotte, the lake port of the City of Rochester, N. Y., is 92 miles to the eastward, while Oswego, N. Y., the Lake Ontario entrance to the New York State Barge Canal, is 141 miles to the eastward of Port Weller. Lake Ontario has depths exceeding 600 ft.

Upper St. Lawrence

The 600-ft. freighter passes the Tibbetts Point Lighthouse and enters the St. Lawrence River, 14 hours after leaving Port Weller—1 157 miles and 115 hours from Duluth. At its head the river is 10 miles across from the New York shore to the Ontario mainland, with the waterway split by islands occupying 6 miles of the width. The formation here is igneous with characteristic pinnacle rocks forming submerged obstructions and creating the Thousand Islands. This condition extends down stream 51 miles to Morristown, N. Y. It should be understood that the description of channels relates to the usual vessel track, crossing back and forth in some places over the International Boundary. In this 51-mile reach natural channel depths range from 30 to 180 ft., and the deep vessel right of way is 1 000 ft. or more in width—except that a 7-mile stretch from Thousand Islands Park to Alexandria Bay and a 3-mile stretch, beginning 4 miles above Brockville, Ont. (which is 1 mile above Morristown), have widths ranging from 300 to 600 ft.—but with very deep water. (See Fig. 6.) Some cleaning up is indicated in this 10 miles of right of way.

Below Morristown, the river flows 50 ft. deep and ¾ mile or more wide for a distance of 11 miles to Prescott, Ont., and Ogdensburg, N. Y.—at the mouth of the Oswegatchie River. These cities are 62 miles down the St. Lawrence—1 219 miles and 120 hours, or 5 days—from Duluth.

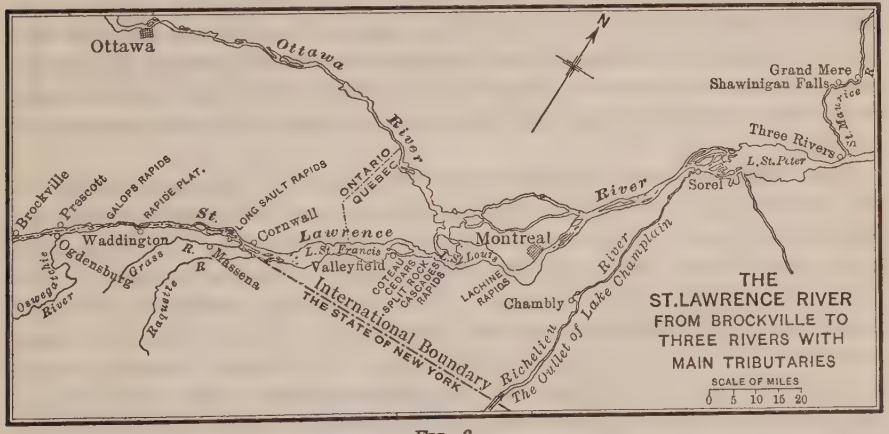


Fig. 6.

The deep wide river extends 3.5 miles down stream from Prescott to the entrance of the Canadian Cut known as the North Channel—with the head of the Galops Rapids 3 miles farther down stream. On the New York side, a deep channel extends 6 miles to the head of the American Galops Rapids. A limestone ledge, overlaid in part with glacial drift, forms the natural weir which creates Lake Ontario. (See Plate V.) The upper 69 miles of the St. Lawrence River is hydraulically an arm of the lake—with an open-season descent of only 1 ft. from lake level to river level at Ogdensburg.

As the St. Lawrence River takes a general northeasterly course, it is well to note the fact that the latitude of Ogdensburg is 44° 42′, and the latitude

of Liverpool is 53° 24′—so that Liverpool is still 600 miles to the north of Ogdensburg; and when an ocean liner finally passes from the Gulf of St. Lawrence into the open Atlantic, it is still 100 miles to the south of Liverpool, and about 200 miles to the north of Havre.

International Rapids Section

The zero point for the Rapids Section of the St. Lawrence Waterway begins about 4 miles below Ogdensburg. It is desirable to note the characteristics of the St. Lawrence River and the flanking canals used by lake-going vessels with a draft of 14 ft. and carrying perhaps 2 000 tons of freight.

Down-bound vessels pass through the North Channel, an artificial rock cut, for a distance of 1½ miles, then another 1½ miles to the entrance of the Galops Canal at the head of the Galops Rapids. Some down-bound vessels chute the rapids following an improved channel through the rock barrier. Up-bound vessels take the Galops Canal. The descent in the Galops Rapids is 8 ft. in a distance of 3 miles. The canal extends for a distance of 7 miles flanking the swift, curving river, descending 14 ft. in the terminal lock.

At the Town of Iroquois, Ont., vessels enter the river again, following the deep channel in a swift current for a distance of 4 miles to the entrance of the Morrisburg Canal at the head of Rapide Plat. This canal has a length of 4 miles, making a descent of 12 ft. in the locks at Morrisburg, Ont. Down-bound freighters sometimes chute these rapids. The visible bottom in the upper portion of these rapids is of red sandstone and it is in this vicinity that the engineers of the International Joint Commission have recommended regulating works, for the maintenance of Upper St. Lawrence River and Lake Ontario levels, the creation of storage, and the manual control of the water supplied to the river.

After passing through 10 miles of swift-flowing river with depths of 20 to 40 ft. or more, the narrow channel and swift current at Croil's Island is reached. Down-bound vessels follow the river through this swift water, descending 3 ft. in less than 1 mile, while up-bound vessels take the flanking Farran's Point Canal. At the head of Croil's Island, the channel splits, leading to the right toward the South Passage of the Long Sault Rapids and the entrance 4 miles down from the splitting point to the Massena Power Canal. Vessels passing out of the Farran's Point Lock take the capacious, deep, north channel for a distance of 4.5 miles to the entrance of the Cornwall Canal and the head of the Long Sault Rapids. Only a few passenger steamers chute these and the other magnificent rapids between this point and Montreal. The Cornwall Canal makes a descent of 50 ft. in 11.5 miles. The Long Sault Rapids make the major descent in a main-channel distance of 3 miles. The descent to Lake St. Francis, with a volume flow of 230 000 cu. ft. per sec., creates the dominant water power of the International Section of the St. Lawrence River.

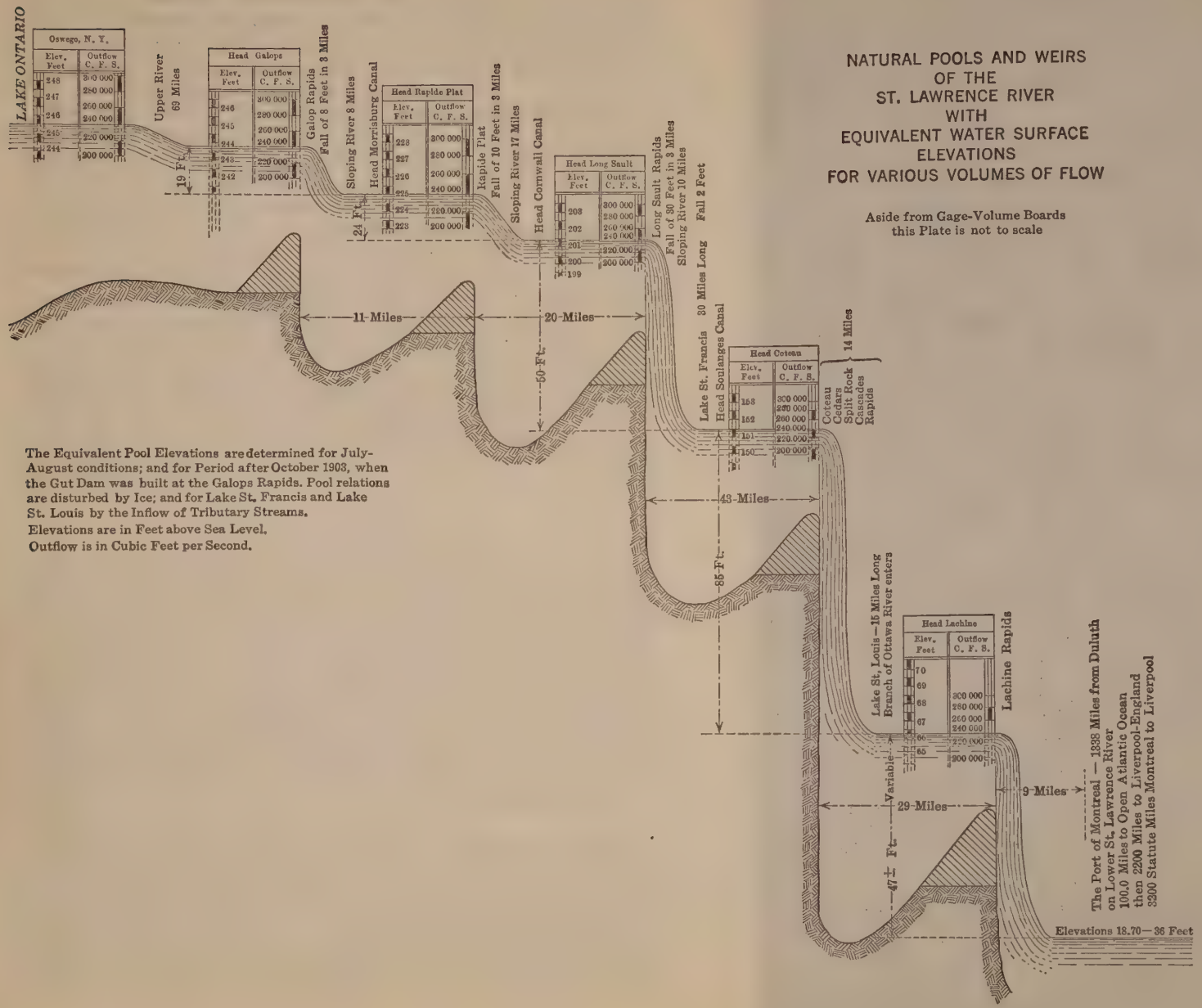
The Canadian Rapids Section

About 1.5 miles below the lowermost lock at Cornwall, Ont., the International Boundary Line leaves the St. Lawrence River and proceeds overland, following eastward on the 45° parallel of latitude.

PLATE V.
TRANS., AM. SOC. CIV. ENGRS.
VOL. 89, PAPER No. 1580.
SHENEHON ON
THE ST. LAWRENCE WATERWAY
TO THE SEA.

NATURAL POOLS AND WEIRS OF THE ST. LAWRENCE RIVER WITH EQUIVALENT WATER SURFACE ELEVATIONS FOR VARIOUS VOLUMES OF FLOW

Aside from Gage-Volume Boards this Plate is not to scale



The Equivalent Pool Elevations are determined for July-August conditions; and for Period after October 1903, when the Gut Dam was built at the Galops Rapids. Pool relations are disturbed by Ice; and for Lake St. Francis and Lake St. Louis by the Inflow of Tributary Streams.
Elevations are in Feet above Sea Level.
Outflow is in Cubic Feet per Second.

From this point, 46.5 miles below the zero point of the International Rapids Section, the Canadian National Section begins. This is at the head of Lake St. Francis and is 50.5 miles down stream from Ogdensburg and Prescott and 1 270 miles from Duluth. From the level of the St. Lawrence River at the head of the Galops Rapids, the descent to the head of Lake St. Francis is 91 ft.; and Lake St. Francis is still 152 ft. above sea level.

The conditions in the Canadian National Section are exceedingly simple: First, Lake St. Francis—30 miles long, with a width of 1 mile to 5 miles, and water abundantly deep for 25-ft. navigation—with the exception of about 3 miles which need dredging or cleaning up. The total area of the lake is about 90 sq. miles which will give excellent forebay capacity for peak-load operation of hydro-electric plants in the tandem rapids at its foot. These outflow rapids in series, beginning at the entrance of the Soulange Canal at Coteau Landing, are the Coteau, the Cedars, the Split-Rock, and the Cascades—the full series making a descent in 14 miles of 84 ft. to Lake St. Louis. Lake St. Louis is another capacious, deep lake, 15 miles long, with widths ranging from 1 mile to 6 miles. It has a forebay value of over 50 sq. miles to stabilize varying volumes of flow in the hydro-electric plants of the future in the outflowing Lachine Rapids. The Lachine Rapids, beginning at the head of the Lachine Canal, make a normal descent of 45 ft. in 9 miles to the Lower St. Lawrence at the ocean port of Montreal—with the Atlantic 1 000 miles away down a magnificent, still-flowing river—with 30-ft. drafts on the trans-Atlantic liners an accomplished fact.

The Port of Montreal

The Port of Montreal has been created by the expenditure of nearly $40 000 000. It has deep-draft berths to accommodate more than 100 large modern ocean steamships simultaneously, with more than 8 miles of wharves and piers capable of docking vessels drawing over 25 ft., and more than 5 miles of these will receive vessels drawing 30 ft. and over; 60 miles of railway tracks to serve these wharves; 35 of these steamship berths are modern concrete wharves built in the past few years; 4 large modern fire-proof elevators with a capacity of 12 000 000 bushels, having conveyor systems to 26 steamship berths at which 19 vessels can be loaded with grain at one time; 24 permanent fire-proof transit sheds; modern cold storage warehouse; complete and valuable construction and repair plants; and near-by shipbuilding plants, including a dry dock. These facts are taken from the 1923 report of the Montreal Harbor Commission.

Having in mind the large financial investment in creating the modern facilities of a great terminal port, where vessels discharge their import cargoes for transshipment to smaller lake vessels or to rail carriers, and where large quantities of freight for export are received from the Great Lakes either by the small canal type of vessel or by rail, it may be readily appreciated why Montreal may view apprehensively the creation of a deep-draft waterway interfering with this great and profitable business of transshipment. The

situation here is somewhat the same as the situation in Buffalo, the present down-bound Lake Erie terminal of the Great Lakes, where cargoes are transshipped to barges of the New York State Canal or to railway carriers.

Since the waterway to the sea between Duluth and Montreal contemplates through shipment without stopping at Montreal for other than supplies, no further detailed statement of the harbor at Montreal is desirable in this paper.

However, as Montreal is the eastern limit of the contemplated improvement, the conditions of navigation and its strategic position with respect to the seaports of the world will be reviewed, before touching on other avenues to the sea, and before discussing the proposed constructions for navigation and water power in the Rapids Sections of the St. Lawrence River for a distance of 115 miles up stream from this port.

St. Lawrence Ship Channel

Below Montreal, dredged channels, 450 ft. wide, with greater widths on curves, and with depths ranging from 30 to 35 ft., are encountered for a distance of 150 miles to Quebec and for a further distance of 175 miles to Father Point, Que. Below this are few dredged areas. At Three Rivers, Que., 80 miles below Montreal, tidal movements are discernible. To April, 1921, the Dominion Government had expended about $24 000 000 on the St. Lawrence Ship Channel below Montreal. The season of navigation begins in late April and ends in early December, having a normal length of 224 days, or 61% of the year. The hazards of navigation are not large and insurance rates will not be prohibitive.

Montreal's Strategic Position

The Port of Montreal is 370 miles nearer Liverpool than the Port of New York. The distance to Gibraltar, the gateway to Mediterranean ports and to the Suez Canal, is approximately the same for the two ports. From the foot of Lake Erie to Liverpool by way of New York is 443 miles by rail and 3 576 miles by sea, a total of 4 019 miles; while the all-water route through the St. Lawrence is 350 miles shorter. The loss of time in passing through the Welland Canal—about 9 hours more than in open channel running—and the loss of time in the St. Lawrence River Section—perhaps 9 hours more—are probably less than the time lost in transshipping cargoes. The handicap of any water route in northern latitudes is the embargo of the ice season of 140 days.

Montreal Commerce

The 1923 commerce of the Port of Montreal amounted to 6 372 400 tons, the exports being a little less than twice the imports. The imports, however, were mostly from the United States and consisted largely of coal and petroleum. The dominant export cargo is wheat and other grains, showing a total volume of 3 278 100 tons out of the total export volume of 4 152 900 tons. Other large exports are automobiles and parts, dairy products, flour—346 100 tons—fruit, and meats. Of the exports of wheat and other grains amounting to 120 108 000

bushels in 1923, nearly 75 000 000 bushels reached Montreal by boat. During the same year, the Port of Buffalo received by lake 182 000 000 bushels, of which 114 000 000 bushels were Canadian grain. It is interesting to note the fact that 294 000 000 bushels of grain were shipped from the Canadian Lake Superior Ports of Fort William and Port Arthur between September 1, 1922, and September 1, 1923. A part of this grain was moved by rail. It is also interesting to note that the Duluth-Superior Port shipped in 1923 about 70 000 000 bushels of grain and that the volume of grain passing through the locks at Sault Ste. Marie in 1923 was about 370 000 000. During 1923 Buffalo shipped by the Erie Canal about 20 000 000 bushels of wheat. The total commerce through the Erie Canal was 1 626 100 tons in 1923.

If the value of exports in the Port of Montreal is taken as $50 per ton, the total valuation appears to be $200 000 000.

Lake-and-Ocean Vessels

It will be assumed that engineers will solve the problem of composite vessels to traverse the fresh-water seas and the salt seas; that the evolution of this vessel will come in the ten years between the definite decision to construct the waterway and its completion; and if not in this decade then in some of the early decades of the century. Such a vessel was designed nearly a quarter of a century ago by an eminent naval architect and shipbuilder of Detroit, Frank E. Kirby, and his conclusions—published in 1900 in the report of the Board of Engineers on Deep Waterways—indicated the cost to be practically 10% more than the cost of a vessel built for exclusively Great Lakes navigation. Any economies in transportation by the opening of this waterway might be cancelled by the time losses, by the cargo damage and losses, and by the labor and overhead costs of transshipment. The scheme of water transportation connecting Liverpool, Continental Atlantic, and Mediterranean ports, must contemplate fundamentally the continuous movement of an unbroken cargo. It should be understood that special types of boats will continue in special service, such as ore and coal-carrying, between Lake Superior and Lake Erie ports. It is of little concern whether the Rapids Section of the St. Lawrence Waterway to the Sea has, in its first decade, a commerce of 5 000 000 tons or 20 000 000 tons. The analysis of possible traffic and the savings to be made by a through route from Duluth or Chicago to Montreal and overseas is interminable and speculative. It will appear that the construction of this waterway is inevitable, that the cost will be largely carried by the co-ordinately developed water power, and that a demonstration of its commercial feasibility is unnecessary.

Other Waterways to the Sea

Turning briefly to deep waterways other than through Montreal to pass from the Great Lakes to the Atlantic, it should be recalled that under an Act of Congress of February, 1895, a Deep Waterways Commission was appointed to make a preliminary inquiry concerning waterways between the ocean and the Great Lakes. It was provided that the commissioners appointed under this

Act should serve without compensation. The Commission appointed by the President of the United States consisted of James B. Angell, then President of the University of Michigan, John E. Russell, and Lyman E. Cooley. This Commission made an investigation from existing sources of information and reported voluminously in 1897.*

This Commission recommended the appointment of a Board of Engineers on Deep Waterways. Congress acted on this recommendation and made appropriations ultimately aggregating nearly $500 000. The Board of Engineers appointed consisted of the late Charles W. Raymond, Major, Corps of Engineers, U. S. Army, M. Am. Soc. C. E., Alfred Noble, Past-President, Am. Soc. C. E., and George Y. Wisner, M. Am. Soc. C. E. This Board, after two years of surveys, examinations, investigations, and studies, reported in June, 1900.† The report is probably the most elaborate existing discussion of the elements entering into deep waterways.

Routes

The routes contemplated by the Board of Engineers take into consideration the Great Lakes System from Duluth and Chicago to the foot of Lake Erie, with Lake Erie regulated. The descent over the Niagara Escarpment followed the Niagara River to Lasalle, N. Y., for a distance of 16 miles below Buffalo and re-entered the Lower Niagara River at Lewiston, N. Y., substantially at Lake Ontario level, 25 miles from the regulating works at Buffalo; then proceeded through the deep, spacious Lower Niagara River and Lake Ontario to Oswego 139 miles away. Here the canalization climbed up 133 ft. by lockages to the level of Oneida Lake and followed the Mohawk Valley descending to the Hudson at Troy, N. Y. From Troy to the Battery in New York it follows the established tidal Hudson River route for a distance of 140 miles. This route beginning at Oswego and ending in the Port of New York will be referred to as the Oswego-Mohawk Route.

Another route followed Lake Ontario past Oswego, entered the St. Lawrence River at Tibbetts Point, and proceeded to the Galops Rapids. Here, the route followed through a series of river channels and canals, including four ship locks, to Lake St. Francis at the International Boundary. The route then crossed the Boundary Line and followed for a distance of nearly 30 miles down Lake St. Francis to a point on the south bank of the lake opposite Coteau Landing and near the head of the Coteau Rapids. The route then proceeded through the divide for a distance of 40 miles in Canadian territory and for a further distance of 8 miles in American territory to Kings Bay, Lake Champlain—15 miles north of Plattsburg, N. Y. Lake Champlain is about 50 ft. lower in elevation than Lake St. Francis and the descent is made in five ship locks. The route then proceeded 124 miles through deep water in Lake Champlain and through canalized waterways and entered the upper courses of the Hudson River below Fort Edward, N. Y. It met the Oswego-Mohawk route at Troy and proceeded to New York through the tidal Hudson.

* H. R. Doc. No. 192, 54th Cong., 2d Session.
† H. R. Doc. No. 149, 56th Cong., 2d Session.

Preferred Routes

The Board of Engineers on Deep Waterways recommended the route by the way of Oswego; but since that time the Dominion Government has undertaken and is completing the construction of the new Welland Ship Canal over the Niagara Escarpment, and the development of the St. Lawrence itself to the foot of Lake St. Francis and beyond to Montreal is recommended. The International Section of the St. Lawrence Waterway to the Sea will be subsidized by water power, and may be eliminated in any estimate of cost chargeable to navigation.

Duluth, Chicago, Montreal, and New York

The preferred route from the Great Lake System to the Port of New York for deep-draft ocean-going vessels now appears to be the St. Lawrence-Champlain Route; and the aspirations of the citizens of New York for a deep waterway, entering salt water through the Hudson River portal, appears to be in parallel with the aspirations of those seeking a deep waterway following the St. Lawrence River to the Atlantic. The trunk line is the same for both routes from Duluth for a distance of nearly 1 300 miles, the dividing point being 40 miles above Montreal.

Comparison of Routes

Having in mind this most remarkable community of interest between the State of New York and other States tributary to the Great Lakes, it is well to compare the Oswego-Mohawk and the St. Lawrence-Champlain routes to the sea. From Oswego to the Battery in New York through Oneida Lake and the Mohawk Valley is 313 miles, involving the passage of 31 ship locks, and requires about 46 hours. Reduced roughly to 1924 prices—assumed to be about 50% more than those of 1900—a waterway with 25-ft. depths in canals, canalized rivers, and lakes, and with ship locks, 740 ft. long, 80 ft. wide, and 30 ft. deep, would cost approximately $287 000 000.

By the St. Lawrence-Champlain Route, from the division point at the foot of Lake St. Francis, the distance to New York is 363 miles, involving a descent through 12 ship locks, and requires 42 hours. The cost, reduced to 1924 prices, would be about $221 000 000.

From Buffalo, the distance to New York by the Oswego-Mohawk route is 477 miles, requiring 64 hours' travel. From Buffalo by the St. Lawrence-Champlain route to New York is 685 miles, requiring 76 hours of travel. The St. Lawrence-Champlain route from Buffalo to New York is 208 miles longer than by the way of the Oswego-Mohawk route; and it takes 12 hours longer to traverse it. On the other hand, it costs $66 000 000 less than the shorter route. It would have the advantage of traversing and serving additional territory in New York and New England; and it would not conflict physically with the permanent use of the Barge Canal. If 1924 prices are taken as 75% more than those of 1900, the Oswego-Mohawk route would now cost $335 000 000 and the St. Lawrence-Champlain route $77 000 000 less.

Other Proposed Routes

No other outlet proposed for deep-draft navigation to the sea needs more than casual mention. The Georgian Bay Canal, planned to depart from Lake Huron through the French River, and to pass over the divide to the Ottawa River and thence to Montreal, appears to be disadvantageous in cost, short season of navigation, and in the fact that it would short-circuit the important traffic cities of the Detroit River, Lake Erie, and Lake Ontario. A railway to Hudson Bay has long been advocated and considerable money spent in surveys and investigations. The short season of navigation—about 150 days—and the transshipping elements, are probably fatal to this route in competition with all-water routes.

The St. Lawrence-Montreal Project

The project contemplated in the report of the International Commission* will be lightly touched upon. The investigations of the engineers were limited to a single year, a period much too short to reach other than tentative conclusions.† A new International Board of Engineers has been appointed to review the project, to report in October, 1925. The personnel of the United States and Canadian Government Engineers reporting to the International Joint Commission in June, 1921, was, as follows: For the United States: W. P. Wooten, Colonel, Corps of Engineers, U. S. Army, M. Am. Soc. C. E., H. G. Roby, M. Am. Soc. C. E., Assistant; for Canada: The late W. A. Bowden, M. C. I. E., D. W. McLachlan, Assoc. M. Am. Soc. C. E., Assistant.

Canadian Section

It appears to the writer that the treatment of the St. Lawrence River for navigation and water power from the foot of Lake St. Francis, with a descent of 83 ft. to Lake St. Louis and from Lake St. Louis with a descent of 45 ft. or more to the river at Montreal, is a matter for the Dominion Government to decide. The recommended project contemplates by-passing in lateral canals the rapids from Lake St. Francis to Lake St. Louis and from Lake St. Louis to the lower river. The two proposed 25-ft. depth canals have lengths of 26½ miles and require three lift locks and two guard locks. These locks would have a usable length of 800 ft., be 80 ft. wide, and 30 ft. deep over the breast-walls. The total estimated cost for these two portions of the Canadian National Section is $92 373 000.

The proposed project contributes nothing toward the ultimate water-power development—stated to be 2 420 000 e. h. p.—in these two great descents. The solution of the problem has been based on the absence of a market capable of absorbing the great volume of visible water power in the St. Lawrence River. An alternate navigation route shown in the plans, follows the river channels through the rapids. This would involve the construction of at least three dams, each 40 ft. or more in height, creating deep slack-water pools, with ship locks passing through each dam to the pool below. From the point of view of navigation this appears sounder than the use of lateral canals, but the build-

* Senate Doc. No. 114, 67th Cong., 2d Session.

† The report of the engineers is printed in Senate Doc. No. 179, 67th Cong., 2d Session.

ing of these dams, in the view of the engineers, should await a receptive market for the electric current produced.

It is the writer's conviction that water power should subsidize navigation. The undertaking is, therefore, a commercial project and the United States does not need to contribute toward the cost. The water power belongs to Canada and ultimately should do its share toward financing the necessary ship locks. The solution of the ice problem of the St. Lawrence River probably means the eventual elimination of all rapids.

It appears from the report of the engineers of the Joint Board, that the alternate navigation route following the river channel, with one or two dams in the Lachine Rapids, would cost $7 000 000 more than the navigation route by 13 miles of lateral canals; and the alternate navigation route through the rapids from Lake St. Louis to Lake St. Francis, with two dams in the rapids, would cost $13 119 000 more than through 13½ miles of lateral canals. The addition of $20 119 000 to a navigation project estimated to cost $92 400 000 should be given careful consideration, provided the slack-water pools created give ampler and faster navigable ways, and provided the total cost of the improvement and its maintenance may be paid eventually by water-power rentals. It is difficult to understand how a project of lateral canals, in no wise aiding water-power development, is logically chargeable to anything else than transportation. If, on the other hand, the best and largest water-power development incidentally creates navigable ways—only the operation and maintenance expenditures and interest on the cost of four ship locks—perhaps $20 000 000—are chargeable to navigation. The operation, maintenance, and depreciation estimated by the Commission for 26½ miles of 25-ft. lateral canals—with all permanent structures built for ultimate 30-ft. drafts—is $750 000 per year. This is 0.8% of the capital cost. For the alternate project of river channel slack-water pools this sum should be less. These comments are based on the proposition that all constructions for the navigable waterway should be steps toward and be chargeable to water-power development. The expenditure of $1 158 000 on dredging in the Canadian National Section of Lake St. Francis needs mention only.

International Section

Division No. 4 of the Commission report embraces the International Rapids Section, from the boundary line at the head of Lake St. Francis to the deep river about 5 miles up stream from the Galops Rapids, covering a distance of 48 miles with a difference of 91 ft. in elevation. Beginning at the head of the Galops Rapids the descent to the head of Rapide Plat—11 miles down stream—is 19 ft.; from this point to the head of the Long Sault Rapids the descent—in 20 miles—is 24 ft.; and from the head of the Long Sault to the boundary in Lake St. Francis, the descent is 48 ft. in 13 miles. (See Plate V.) The International Joint Commission project for navigation provides a lateral canal, 6 miles long, with two locks, on the Canadian side of the river, following in part the present alignment of the Cornwall Canal around the

Long Sault Rapids. A dam at the head of Barnhart Island, raises by 30 ft. the present level of the pool at the head of the Long Sault Rapids to Elevation 231. This gives a total lift from the head of Lake St. Francis of 78 ft. at normal flow stage; and a total power fall of 76 ft. This slack-waters the river for a distance of 17 miles to the foot of a regulating dam in Rapide Plat. The summer slope in the river in this 17 miles would be about 1 ft. Here, vessels pass through a ship lock, with a lift of 15 ft. to Elevation 247, which is within 1 ft. of the regulated level of Lake Ontario. The regulating dam slack-waters the river to the zero point at the entrance to the North Channel—drowning out the Galops Rapids. The reformed International Section then consists of 41 miles of open-river running, 7 miles of canals, and 3 ship locks.

Duluth to Liverpool

For the whole Rapids Section from the zero point at the North Channel, 3 miles above the Galops Rapids, to Montreal—a distance of 115 miles—the reformed waterway shows 81 miles of open river running, 33 miles of canals, 7 lift locks, and 2 guard locks. An ocean-going vessel with a normal speed of 11 miles per hour will require 20 hours to traverse the Rapids Section to Montreal. This then shows from Duluth to Montreal, 1 338 miles and 140 hours, or 5 days and 20 hours, in transit. The average speed is about 9½ miles per hour. The Board of Engineers on Deep Waterways in discussing the various waterways to the sea based the time on a vessel traveling in open running 12½ miles per hour—and the Board used a quicker locking schedule than the writer. Using the Board's values the time from Duluth to Montreal would be 124 hours, or 5 days and 4 hours. Adding for Montreal to Liverpool 10 days and 20 hours, shows 16 days in transit; and with a day at each end for lay-over—loading at Duluth and unloading at Liverpool—shows a total time of 18 days. With a cargo of 10 000 tons, the work done is 45 400 000 ton-miles. It is probable that bulk freight may be carried between Duluth and Liverpool at a rate not exceeding 0.7 mills per ton-mile, or $3.18 per ton. This means for wheat a rate of about 10 cents per bushel. The rate in 1923 through St. Marys Falls Canal was 3.8 cents per bushel from Superior to Erie ports, including loading and unloading.

Exporting Raw Materials

An economic principle enters in considering a project which facilitates the export of raw materials. This principle was enunciated by the millers of both the United States and Canada at the time of the hearings before the International Joint Commission: A country's prosperity lies in converting, in its own domain, raw materials into finished products. Wheat should not be exported until it has been converted into flour. The flour mill employs labor, it employs the makers of machinery, sacks, and barrels. It sends the feed-stuffs to the farm for poultry and cattle. Export flour, dairy products, poultry, beef—not wheat; export—not ore—but steel!

It is not improbable that on the completion of a waterway to the sea a special type of vessel will carry flour direct from the Great Lakes ports to overseas ports. The present discrimination of rates on flour, as compared with wheat, seriously handicaps export flour trade. In 1923 the total exports of wheat flour from the United States as reported by the Department of Commerce, amounted to 16 310 000 bbl. Of this, 5 218 000 bbl. passed through the Port of New York. During the same year the export of wheat amounted to 98 523 000 bushels. In 1921 the export of wheat was 280 058 000 and, in 1922, 164 692 000 bushels. The largest portions of the 1921 exportation went to Belgium, France, Germany, Italy, The Netherlands, and England, a considerable portion going to Canada, doubtless for export through the Port of Montreal. The largest portion of flour exports goes to the same countries, with additional exports to China and Japan.

Waterway Desirable

The immediate construction of the final link of the navigable waterway from the foot of Lake St. Francis to Montreal at a cost of about $100 000 000, the United States sharing in this expenditure, will probably be in the interest of economic transportation for overseas trade. The most careful investigation, however, should be made, weighing the desirability of spending all the money in a way helpful to ultimate water power. If a somewhat larger expenditure will count toward the water-power development of a decade later, it may be advisable to make this larger expenditure now. The part of the United States would then be to pay the interest on a portion of the bonds until water-power rentals do so.

Water Power and Navigation

Water-power projects are sometimes suspected of wearing the guise of navigation projects. A navigation project appeals more strongly to the minds of the people in the trade districts tributary to the Great Lakes far from the water power. The radius of economic benefits from water power may be limited to 200 miles, while the economic benefits of navigation, reaching the west through Chicago, Duluth, and Port Arthur, may have a radius of 2 000 miles. Many of the people looking for benefits in transportation find it difficult to understand that a project may be dual—that the development of the navigable waterway and the development of the water power go hand in hand, each assisting the other. If the water-power element of the St. Lawrence Waterway to the Sea is too greatly accentuated, it arouses suspicion in the minds of the people of the agricultural regions of the Middle West. It is best, however, to state the case as it exists.

Water Power

The Government engineers find the total potential energy available in the Rapids Section of the St. Lawrence River to be 4 064 000 e. h. p. This is as follows:

Canadian:	Lachine Rapids	860 000 e. h. p.
	Coteau, Cedars, Split-Rock, Cascades	1 560 000 " " "
	Total	2 420 000 e. h. p.
International:	Long Sault Rapids, initial	1 464 000 e. h. p.
	" " " , later	180 000 " " "
	Total	1 644 000 e. h. p.
	Rapids Section, Grand total	4 064 000 e. h. p.

These estimates of potential power are based on a volume of continuous flow of 210 000 sec-ft.; and the use of an over-all efficiency of 88%, with the performance of new units at Niagara Falls as a precedent for this high efficiency. Only the development of 1 464 000 e. h. p. in the Long Sault Rapids, amounting to 36% of the whole, is contemplated in the initial project. Of this amount, 58 000 e. h. p. is to care for existing needs of the Massena Plant of the Aluminum Company of America.

The volume of continuous or prime flow, taken as 210 000 sec-ft., assumes the diversion in the Drainage Canal at Chicago as discontinued. The writer believes it is safer to assume that the diversion of 10 000 sec-ft. will continue at Chicago for a number of years. The available continuous or low-water flow, as determined by the Government engineers, may be taken as 200 000 cu. ft. per sec. Perhaps the over-all efficiency of 88% is somewhat optimistic. The writer prefers 84 per cent. With these values the available prime power in the St. Lawrence River becomes 2 190 000 e. h. p. in the Canadian Section and 1 410 000 e. h. p. in the International Section. This latter value assumes a head of 74 ft. for all water used at the Long Sault Rapids.

Heads

The heads estimated by the Government engineers in deriving the values first stated are for the Lachine Rapids, 41 ft.; for the Coteau, Cedars, Split-Rock, and Cascade Rapids, 74 ft.; and, initially, for the Long Sault Rapids, 74 ft. The development at the Long Sault Rapids contemplates the ultimate raising of the head about 8 ft. The limitation on available head comes with the increased slope due to greater frictional resistance of the river running under an ice cover. The following may be regarded as reasonably conservative values for the ultimate winter head:

Canadian:	Lachine Rapids	30 ft.
	Coteau, Cedars, Split-Rock and Cascade Rapids	82 "
	Total	112 ft.
International		88 "
	Total	200 ft.

The reduced slope giving the highest usable head means slower current and less ice trouble during the season of zero weather. The reduction of slopes

with deeper channels and slower velocities is also desirable from the point of view of navigation.

Prime Water

The dependable volume of river flow is a most important element in the determination of the power content of the St. Lawrence River. In a state of Nature, monthly mean volumes range from as low as 190 000 sec-ft. to 320 000 sec-ft. The outflow follows the caprice of natural automatic control of the waters of the Great Lakes. Colonel Wooten found, by a careful investigation, that the use of regulating works on rock foundations at Rapide Plat, would increase the dependable flow to 210 000 sec-ft. In a report on the regulation of the St. Lawrence River, in 1919, the writer reached conclusions which do not differ greatly from those of Colonel Wooten, although his analysis had in mind navigation as prior to water power in benefits.

These studies of the possibilities of making uniform and dependable the flow of the St. Lawrence River are illuminating as pointing out the possibilities of converting secondary power into primary power by conserving the water and stabilizing the flow. It appears that the regulation of Lake Ontario, with a pull-down of 3 ft., adds 20 000 cu. ft. per sec. to the prime outflow of the St. Lawrence River. It is obvious that the full utilization of all the great natural reservoirs of Lakes Superior, Michigan-Huron, and Erie, added to the Lake Ontario reservoir, will yield the largest volume of prime flow. Although Lake Ontario is a considerable reservoir in itself, with an area of 7 240 sq. miles, the combined area of the three other great reservoirs mentioned is more than twelve times as large. The storage reservoirs and forebays of the Great Lakes will be discussed later in some considerable detail. The available dependable prime flow of the St. Lawrence River with full Great Lakes regulation is given in Table 2.

The years included in this hydrological analysis (Table 2) are more precise than earlier years, in the necessary lake elevations. The first year is 1891 and the last, 1924—34 years in all. The gauge readings shown in Column (2) of Table 2 are of January 1 of the year concerned, determined by taking a mean of the mean for the month of December of the year before and of January of the year concerned. They mark the fluctuations of the lakes from the first day of each year to the first day of the following year. They may be regarded as readings on gauges having the following zeros for the several lakes: Lake Superior, 600.5; Lakes Michigan-Huron, 580.0; Lake Erie, 571.0; and Lake Ontario, 245.0. These may not represent the best final values for these gauge zeros, as subsequent analyses made more in detail may indicate slight changes. The values used will give an additional depth of approximately 2 ft. over the improvement planes for navigation in the lakes, except for Superior, where the benefit may be little over 1 ft.

The approximate mean elevations of the various lakes during the important months of navigation, May to November, inclusive, may be 0.7 ft., the cresting elevation, 1 ft., and the annual mean, 0.5 ft., above the mean elevation for January 1 of each year as shown in Column (2) of Table 2.

TABLE 2.—St. Lawrence River Outflow with Great Lakes Regulation.

Year.	Lake surface, gauge readings, in feet.	Water supply, in second-feet.	Outflow, in second-feet.		Storage changes, in second-feet.	
			Prime.	Excess.	Saved.	Spent.
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1891	2.00	187 000	230 000	.....	.....	43 000
1892	1.49	238 000	" "	.....	8 000	.....
1893	1.59	284 000	" "	10 000	44 000	.....
1894	2.11	241 000	" "	10 000	1 000	.....
1895	2.12	150 000	" "	.....	.....	80 000
Means of 5 years.....		220 000	230 000	4 000	10 600	24 600
1896	1.17	234 000	" "	.....	4 000	.....
1897	1.22	230 000	" "	.....	0 000	0 000
1898	1.22	233 000	" "	.....	3 000	.....
1899	1.26	247 000	" "	.....	17 000	.....
1900	1.46	255 000	" "	.....	25 000	.....
Means of 5 years.....		239 800	230 000	0 000	9 800	0 000
1901	1.76	202 000	" "	.....	.....	28 000
1902	1.43	236 000	" "	.....	6 000	.....
1903	1.50	247 000	" "	.....	17 000	.....
1904	1.70	280 000	" "	20 000	30 000	.....
1905	2.06	257 000	" "	20 000	7 000	.....
Means of 5 years.....		244 400	230 000	8 000	12 000	5 600
1906	2.14	235 000	" "	10 000	.....	5 000
1907	2.07	251 000	" "	10 000	11 000	.....
1908	2.20	212 000	" "	.....	.....	18 000
1909	1.99	252 000	" "	.....	22 000	.....
1910	2.25	177 000	" "	.....	.....	53 000
Means of 5 years.....		225 400	230 000	4 000	6 600	15 200
1911	1.62	238 000	" "	.....	8 000	.....
1912	1.72	292 000	" "	30 000	32 000	.....
1913	2.10	275 000	" "	30 000	15 000	.....
1914	2.28	184 000	" "	.....	.....	46 000
1915	1.73	235 000	" "	.....	5 000	.....
Means of 5 years.....		244 800	230 000	12 000	12 000	9 200
1916	1.79	310 000	" "	40 000	40 000	.....
1917	2.27	249 000	" "	30 000	.....	11 000
1918	2.15	268 000	" "	30 000	8 000	.....
1919	2.24	201 000	" "	.....	.....	29 000
1920	1.89	228 000	" "	.....	.....	2 000
Means of 5 years.....		251 200	230 000	20 000	9 600	8 400
1921	1.86	198 000	" "	.....	.....	32 000
1922	1.48	215 000	" "	.....	.....	15 000
1923	1.30	204 000	" "	.....	.....	26 000
1924	1.00	197 000	" "	.....	.....	33 000
1925	0.61	.....	.....	.....	.....	.....
Means of 4 years.....		203 500	230 000	0 000	0 000	26 500
Mean of 34 years.....		233 700	230 000	7 100	8 900	12 300

The water supply shown in Column (3) of Table 2 is the yield or outflow in the St. Lawrence River after converting the storage changes in the Great Lakes System into second-feet. It is the yield which would have occurred in the absence of any rise or fall in the lake surfaces. It should be understood

in interpreting the water supply that the volume of diversion at Chicago, since the opening of the Drainage Canal in January, 1900, has been credited as outflowing in the St. Lawrence River instead of at Chicago. Account was taken of the lessened outflow of the St. Lawrence River due to natural ice conditions.

The dependable flow with limited rise and fall in the various lakes is 230 000 cu. ft. per sec. It was found that the excess flow, with a mean value of 7 100 sec-ft., is undependable. For periods of from 1 to 9 years no excess water may be available. Storage changes are shown in Columns (6) and (7) of Table 2.

From the volume of prime flow should be deducted 10 000 cu. ft. for Chicago. This leaves 220 000 sec-ft. for continuous water-power uses in the St. Lawrence River. This volume will be available every day of every year and in every desirable hour of every day. The Reservoir System of the Great Lakes will guarantee the mass volume, and the forebay of Lake Ontario will absorb the minor irregularities of this water. The regulation of all the Great Lakes contributes 20 000 cu. ft. per sec. more of prime water than was determined by the Government engineers from the regulation of Lake Ontario alone.

The mean volume of water supply is precise to 2 per cent. It is an even chance whether it is 2% larger or 2% smaller. The mean value of the water supply for each 5-year period has a precision a little less than the mean flow of 34 years. For a single year, the possible error may be as much as 5%, sometimes too large, sometimes too small. The effect of these possible divergences is not cumulative.

The computations of level changes in the various reservoirs and forebays, as shown in Column (2), Table 2, have assumed in all the same depth of storage saving or loss. In practice, the storage in Lakes Erie and Ontario will be used only in low-supply years such as 1891, 1895, 1901, 1910, 1914, 1919, and 1924. In four years out of five, Lakes Erie and Ontario will be maintained at a high level with little surface variation. This will mean a gain of perhaps 1 ft. in head for water-power uses and increased depths in the upper river channels, with beneficial effect on velocities and ice formation.

The mean regulated outflow values of the water supply of the Great Lakes and the yield, in cubic feet per second, for each square mile of tributary drainage basin are given in Table 3.

The supply diverted at Chicago after 1900 is counted as present in Table 3.

Gain by Regulation

It should be accentuated that the gain in electric energy from the increased outflow of 20 000 cu. ft. per sec. is prime power. The installation already estimated by the Government engineers is sufficient to use all the prime water all the time, with some margin for excess water and peak-load use. The increments of power from increased flow, using the same heads as the Govern-

ment engineers, but using the coefficient 84% for over-all efficiency, are as follows:

Canadian: Lachine Rapids	78 000 e. h. p.
Coteau, Cedars, Split-Rock, Cascade Rapids	141 000 " " "
Total, Canadian	219 000 e. h. p.
International Section: Long Sault Rapids	141 000 " " "
Total, St. Lawrence River	360 000 e. h. p.

The installation at the Long Sault contemplates 52 units, each rated at 32 250 e. h. p. on the station busses. At full gate, the turbines are estimated to use 4 580 sec-ft. The total output may be 1 680 000 e. h. p., and the maximum water use is 233 000 sec-ft.

TABLE 3.—ST. LAWRENCE WATER SUPPLY AND YIELD OF GREAT LAKES.

Periods.	Total supply, in second-feet.	Yield per square mile of drainage basin, in second-feet.
1860–1864	241 000	0.838
1865–1869	252 000	0.876
1870–1874	232 000	0.807
1875–1879	249 000	0.866
1880–1884	260 000	0.904
1885–1889	235 000	0.817
Mean of 30 years	245 000	0.852
1891–1895	220 000	0.765
1896–1900	239 800	0.833
1901–1905	244 400	0.850
1906–1910	225 400	0.784
1911–1915	244 800	0.851
1916–1920	251 200	0.871
1921–1924	203 500	0.708
Mean of 34 years	233 700	0.813
Mean of 64 years	239 000	0.831

It will be observed that the installation contemplated is sufficient for the continuous flow of 220 000 cu. ft. per sec., all used in the main power house at the Long Sault Rapids, without any diversion going to Massena. Under this assumption, the prime power to be developed in the Long Sault Rapids under a head of 74 ft. is 1 550 000 e. h. p.

The ultimate capitalized value of the additional 360 000 e. h. p., without additional installation, will not be less than $50 000 000.

COST OF ENERGY

The estimated cost of the initial power development, including all structures, canals, ship locks, regulating works at Rapide Plat, and channel dredging,

is $159 197 200. The estimated operation and maintenance cost, including depreciation, is $1 782 000 per year. The unit cost is $103 per e. h. p., based on the prime volume of river flow. Let it be assumed that the cost may be $17 per e. h. p. more than estimated. This will care for some over-run, adjust matters with Massena, and take care of additional overheads.

With money at 5% and all other annual expenditures 3%, the possible rental price for the sale of electric power available every year for 365 days and every hour of every day is $9.60 per e. h. p.-year. In case the costs should prove to be $150 per prime e. h. p., the rental price would be $12 per e. h. p.-year. For the lesser cost, the sale price per kilowatt-hour, with 90% utilization, will be 1.63 mills and with 50% utilization, 2.94 mills. For the larger cost, the price per kilowatt-hour, with 90% utilization, is 2.04 mills; and with 50% utilization, 3.67 mills. These rentals will be competitive with those for Niagara Falls power, which the writer is informed, average about $15 per e. h. p.-year, and for new contracts about $20 per e. h. p.-year, on the United States side.

With energy per electric horse power-year selling for $12, it is well to note the value of 1 ft. of additional head. It means 21 000 e. h. p. greater output for installations already paid for. This means not less than $200 000 per year added income, which, capitalized at 5%, shows a value of $4 000 000 per ft. of added head. Where additional head may be had for the expenditure of a less sum than this, it should be secured.

St. Lawrence Ice

The St. Lawrence has a bad name as a stream with ice complications. Frazil accumulates and drowns out lesser and sometimes major rapids. The back-water rise at the foot of the Long Sault Rapids is at times 15 ft. or more. It is believed this frazil formation is due to exposure in the rapids and in the shallow swift river and will terminate with the conversion of the river into a series of deep, still, slack-water pools. The writer has observed the vanishing of the ice in the deep, still St. Lawrence River above Ogdensburg, and in the deep, still St. Marys River above Sault Ste. Marie, Mich. The brilliant sunshine characteristic of late March and early April in these regions, and the temperature of the water mass itself, tend to honeycomb the ice. It breaks into needles and melts. It is spoken of colloquially as "rotting." The unreformed Niagara River is a more serious ice stream than the reformed St. Lawrence. In the Niagara many millions of dollars have been spent in hydro-electric construction, but up to the present time nothing has been done toward fundamental ice protection at Buffalo. Ice engineering is a phase of practice which will have to be given more attention in the future than in the past.

Market for Power

In order to visualize the meaning of the 1 550 000 e. h. p. to be initially developed in the International Section, it is well to compare other installations and other outputs. The allotment of each country will be about 880 000 e. h. p. It is stated* by F. A. Gaby, M. Am. Soc. C. E., Chief Engineer of the Ontario

* *Journal*, Eng. Inst. of Canada, March, 1924, p. 14.

Hydro-Electric Power Commission, that, in 1924, the market for electric current in Ontario is 720 000 e. h. p.; that in 1926 there will be a market for 1 250 000 e. h. p., with a visible shortage of 150 000 e. h. p. in hydro-electric installation; and that the 1934 market will be about 2 650 000 e. h. p. This indicates a forecast of growth of 1 500 000 e. h. p. in 8 years, which is more than doubling the electric current consumed in this period. The market is tributary to both Niagara and St. Lawrence power. The 1924 Year Book of the Commonwealth Edison Company serving the Greater Chicago District shows 1 224 000 e. h. p. operating under a load factor of about 50% to be installed by 1925. The Year Book also shows that the Greater New York District had a maximum load in 1922 of about 1 418 000 e. h. p. The Niagara Falls Power Company of New York has a total installed capacity of 698 400 e. h. p., which is used mostly for electro-chemical and electro-metallurgical industries.

The growth in the absorbing capacity of various communities is very clearly indicated by the citations from Mr. Gaby. In the Greater Chicago District, the increase in consumption in 1923 over 1922 was over 15 per cent. The State of Illinois has an absorptive capacity of 1 e. h. p. per year for each 4 people of its population, Chicago and the surrounding territory within a radius of 150 miles for each 4.43 of population, and New York within a similar radius, for each 4.5 of population.

It should be understood that the region of Canada within 150 miles of the Long Sault Rapids is rich in water power and that comparatively few of the available sources have been developed. The Ottawa River entering the St. Lawrence River at Montreal has many available water-power sites; the St. Maurice entering the St. Lawrence at Three Rivers, 95 miles below Montreal, has much unused power and developments at Grand Mere, Shawinigan Falls, and La Gabelle. The extensive developments at Shawinigan Falls generate and transmit current for aluminum and public utilities, and grind wood pulp, with a down-hill rail haul of 21 miles to reach water navigation in the St. Lawrence. La Loutre Dam, recently constructed, creates a reservoir with a surface area of 300 sq. miles near the headwaters of the St. Maurice River and to some extent increases the low-water flow. It must be understood that the developments mentioned have the disadvantages of flashy streams, instead of the eternally even-running flow of the St. Lawrence River. The development of St. Lawrence power will flood the market and possibly degrade the values of many of these undeveloped sites.

On the United States side, it has already been demonstrated that a considerable block of power may be advantageously transmitted to New York State and to Southwestern New England. New York City is about equidistant from the Long Sault and from Niagara Falls—300 miles away. Schenectady is 160 miles from the Long Sault and 260 miles from Niagara Falls. Albany is 175 miles from the Long Sault, and Boston is 270 miles.

Prime Current

The very nature of the continuous even flow of the St. Lawrence River indicates the use of prime power for 24-hour industries operating 365 days

per year and requiring electric current at a cost not exceeding 3 mills per kw-hr. The electric furnace is the dominant consumer. Perhaps the growth in the use of electric-furnace current has not been as rapid as that of public utilities in the past; but for the future it will serve an infinite variety of purposes.

Prime current permitting 100% utilization of great manufacturing plants is worth considerably more than intermittent current. In public utilities, prime current is worth about three times as much as excess current. For public utility uses, the cost of long-distance transmission lines indicates the wholesaling of base load current to various corporations operating in the region tributary to the St. Lawrence, while peak loads of short duration may be carried by local inexpensive steam installations. In the Greater Chicago District the electric current generated by steam averages 2.23 lb. of coal per kw-hr. of output. This means a fuel cost, at $5 per ton of coal, of 11.4 mills per kw-hr. This is a public utility, with a load factor of about 50%, retailing about 60% of the energy for residential and commercial lighting and to small power users.

Canadian Policy

The energy to be generated in the Long Sault Rapids plant will be absorbed shortly after the development is completed. The remaining power of the St. Lawrence is Canadian. It will be an economic question for Canada to decide whether she will patiently await the inevitable coming of industries to her own territory with the inevitable building up of her commercial importance, or will transmit this power, with losses, to available markets largely in the United States. Questions of customs duties will enter, but the chances are that, in the end, the Mohamet of industry will be compelled to go to the Canadian mountain of power.

Niagara Water Power

The real immediate competitor of the St. Lawrence River power is Niagara River power, with a probable cost of production of less than 3 mills per kw-hr. The question is whether the great electro-chemical and electro-metallurgical industries will prefer first Niagara Falls or the vicinity of the Long Sault Rapids. Ultimately, no choice will exist; it must be Montreal.

Electric current from the Long Sault Rapids and from Niagara Falls will meet at Oswego, practically 135 miles from each development. The probable place of the interconnection of the two great hydro-electric systems will be at Syracuse, already reached by high-tension transmission lines from Niagara Falls, and substantially 150 miles distant from each of the developments.

Present Developments

The water power installations on the Niagara River aggregate about 570 000 e. h. p. for the American side and 700 000 e. h. p. for the Canadian side of the river. Only one of the present installations utilizes the water of the Niagara River under a head of over 300 ft. The total fall between Lake Erie and

Lake Ontario is 326 ft. Between Lake Erie elevation at the Buffalo Lighthouse and the head of the Rapids approaching the cataracts at Niagara Falls, the difference in elevation is about 10 ft. This leaves a gross head of 316 ft., utilizable, with some slope losses, for hydro-electric development. (See Plate VI.) The new Chippawa-Queenston plant of the Hydro-Electric Commission of Ontario operates under a head of 304 ft. The head utilized by the Niagara Falls Power Company in its major installations on the New York side of the river is 212 ft. The effective head utilized by the Ontario plant of the Hydro-Electric Commission, taking water from near the head of the Rapids and discharging, with penstock losses, into the Maid-of-Mist Pool in the Gorge, is about 170 ft. The three other installations, one on the New York side and two on the Canadian side of the river, have effective heads of from 130 to 150 ft. These three plants are doubtless marked for relegation to peak-load and standby purposes, when new plants are built, to use efficiently the limited volume of flow.

Treaty

The volume of flow for hydro-electric uses at Niagara Falls is limited, by the Boundary Waters Treaty of 1910, to 20 000 sec-ft. for the United States and 36 000 sec-ft. for Canada. The limitations imposed contemplate the preservation of the scenic grandeur of Niagara Falls. The combined water use of all present installations would exceed the limitations imposed by this Treaty. The use of water, therefore, in the more efficient plants must compel the stopping of the wheels in the less efficient plants.

Assuming for all developments at Niagara Falls an output of 29 e. h. p. for each cubic foot of water permissible under the Treaty of 1910 (56 000 sec-ft.), the total power output would be over 1 600 000 e. h. p. The actual output from the use of this volume of water is about 1 000 000 e. h. p., indicating that without reforming the treaty in quantity of water, but extending it to include diversions from the Maid-of-Mist Pool, an additional 600 000 e. h. p. will become available. This is 75% of the allotment of the United States at the Long Sault Rapids.

Future Power

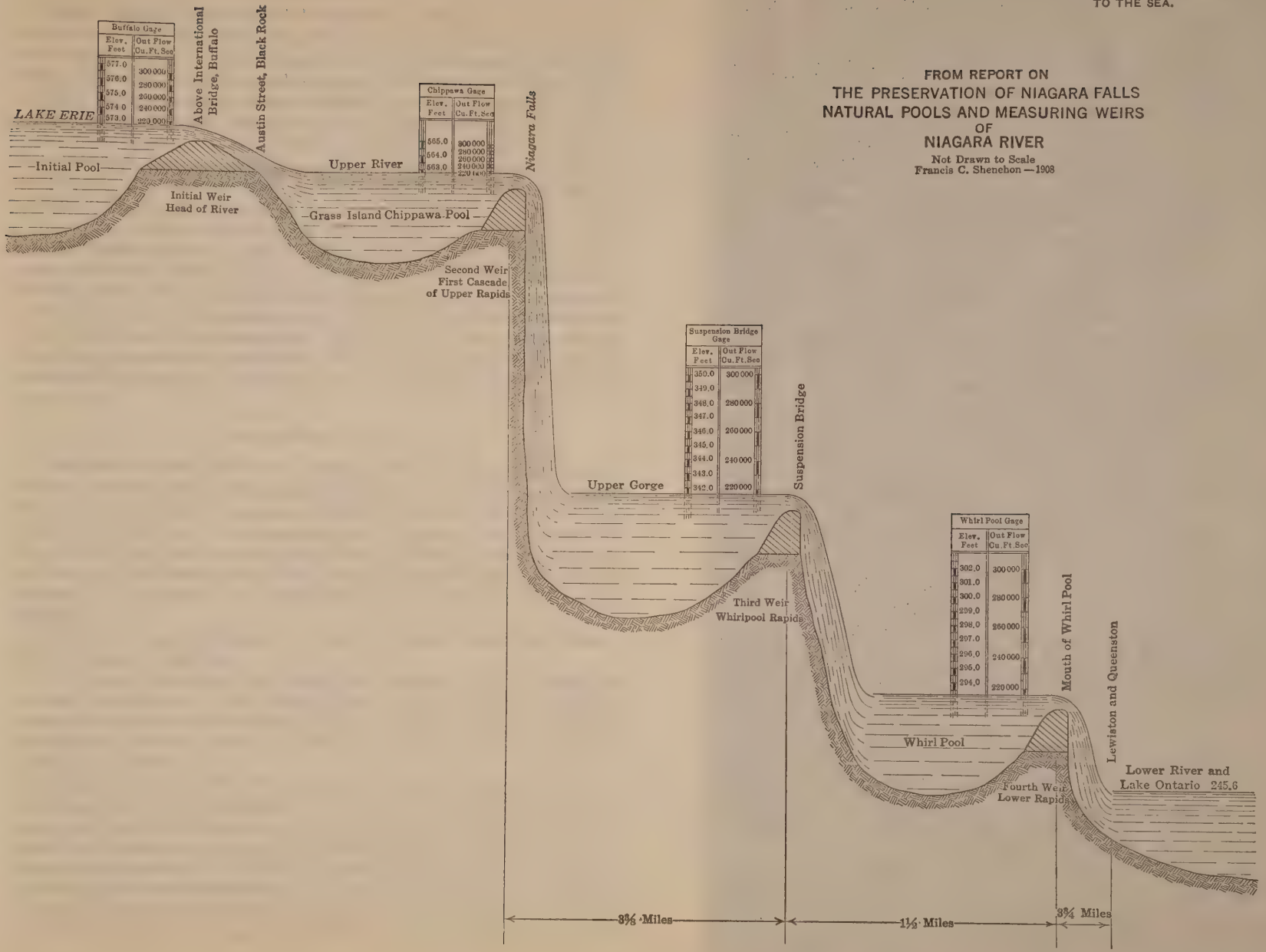
It will be practicable to utilize, at Niagara Falls and in the Niagara Peninsula, after regulating works have been constructed, about 40 000 cu. ft. per sec. more water than was used in 1924 for all purposes of water power and navigation. Should this additional diversion become permissible under the terms of a new treaty with Great Britain, 1 160 000 e. h. p. more will become available. The efficient use of the 56 000 sec-ft. permissible, added to the proposed diversion, will make the ultimate volume about 2 800 000 e. h. p., which is 1 800 000 more than is now (1924) in use. As before stated, Niagara power, under a reformed treaty, is the greatest competitor of St. Lawrence power. The writer in a report on the preservation of Niagara Falls in 1908,* stated, "* * * the possibilities of continued and extended use of power at the Falls are conditioned upon the construction of regulating works in the Niagara River to avoid the wasteful outflow of the water of Lake Erie."

* Senate Doc. No. 105, 62d Cong., 1st Session, p. 75.

PLATE VI.
TRANS., AM. SOC. CIV. ENGRS.
VOL. 89, PAPER No. 1580.
SHENEHON ON
THE ST. LAWRENCE WATERWAY
TO THE SEA.

FROM REPORT ON
THE PRESERVATION OF NIAGARA FALLS
NATURAL POOLS AND MEASURING WEIRS
OF
NIAGARA RIVER

Not Drawn to Scale
Francis C. Shenehon — 1908



Power Uses

Before passing to the matter of the conservation and control of the waters of the Great Lakes for water-power betterment on the Niagara and St. Lawrence Rivers, it will be illuminating to note some uses of power at Niagara Falls: Abrasives, including carborundum; refractories, including fused alumina, high temperature electric furnaces, crucibles and pyrometer tubes; silicon carbide; firesand, electrically sintered magnesia; aluminum and its alloys in an infinite variety of forms; ferro-alloys, including ferro-silicon, ferro-chromium, ferro-tungsten, ferro-molybdenum and ferro-titanium; sodium bichromate and potassium bichromate; sodium; chrome alum, chromium sulfate, chromic acid, all three used in tanning leather; hydrochloric acid; carbon tetrachloride, sulfur chloride, caustic potash, caustic soda, liquid chlorine, bleaching powder, graphite, carbon electrodes, sodium chlorate, potassium perchlorate, barium chlorate, hydrogenated oils, titanium aluminum bronze, copper calcium alloy, potassium chlorate, formaldehyde; cyanamid, used as a fertilizer or for ammonia and nitric acid. It will be observed that these are all electric-furnace products. In addition, are the grinding of pulp wood and the making of paper with boilers electrically steamed for heating purposes, the grinding of wheat and grain, and an infinite number of small power uses.

Niagara Water Supply

The Niagara River, like the St. Lawrence River, has suffered seasons and years of low flow, because Nature appears to have little interest in the maintenance of the lakes for purposes of navigation, or the maintenance of an even flow in its rivers for water-power uses. The natural unrestrained outflow of the Great Lakes has served the elemental purpose of transferring from the basin of the Great Lakes to the ocean the eternally recurring rainfall surplus. For navigation and water-power uses, guidance by engineers is essential.

The water supply outflowing in a state of Nature through the Niagara River is shown in Table 4.

In Table 4, the outflow since 1900 in the Drainage Canal at Chicago is counted as outflowing in the Niagara River. The volumes for navigation passing through the Welland and Erie Canals, and the volume passing through the Welland Canal for water-power uses, are likewise counted as part of the water supply shown.

Comparing Table 4 with Table 3, showing the water supply of the St. Lawrence River, it will be observed that the earlier 30 years and the later 34 years show very little change in supply, demonstrating that the water supply of the Great Lakes is not failing. The indication is that changes at the Galops Rapids have modified the St. Lawrence outflow. Based on this comparison of the supply of the two streams, the water supply for the St. Lawrence River has been adjusted by subtracting an obviously excessive quantity. It will be observed that the local supply of Lake Ontario for the mean of 64 years

amounts to about 32 000 cu. ft. per sec. more than the Niagara supply. The yield for Lake Ontario is somewhat in excess of its percentage drainage basin area.

TABLE 4.—Niagara Water Supply and Yield of Great Lakes, Excepting Ontario.

Periods.	Total supply, in second-feet.	Yield per square mile of drainage basin, in second-feet.
1860–1864	202 000	0.793
1865–1869	213 000	0.836
1870–1874	202 000	0.793
1875–1879	206 000	0.809
1880–1884	234 000	0.919
1885–1889	210 000	0.786
Mean of 30 years	211 200	0.830
1891–1895	186 000	0.730
1896–1900	211 000	0.828
1901–1905	210 000	0.824
1906–1910	198 000	0.777
1911–1915	218 000	0.856
1916–1920	215 000	0.844
1921–1924	175 000	0.687
Mean of 34 years	202 600	0.796
Mean of 64 years	206 700	0.811

Gain by Regulation

The result of utilizing all the Great Lakes as reservoirs shows a gain of 20 000 sec-ft. of dependable flow for the Niagara River. After deducting 10 000 cu. ft. per sec. for the Chicago diversion, the mean supply for a period of 64 years is 196 700 cu. ft. per sec. With the proper limitations in the pull-down of Lakes Superior, Michigan-Huron, and Erie, the volume of prime outflow may be taken as 190 000 cu. ft. per sec. If 100 000 cu. ft. per sec. be assigned to water power and navigation uses in the Niagara River and in the Niagara Peninsula, 90 000 cu. ft. per sec. remains available to perpetuate the scenic grandeur of Niagara Falls.

Preservation of Niagara Falls

Hydro-electric engineers do not always accept graciously the thought that a great volume of water which might serve industry, be devoted to the maintenance of a great spectacle. Perhaps the fate of the cataracts may be forecast by saying that when the poverty of the nation compels the conversion of the National Parks, including vast timbered tracts, to commercial uses, and when money is no longer spent for city parks and playgrounds, and architectural refinements in buildings are omitted, and the art institutes are closed—then all the water of the Niagara, as far as practicable, may be de-

voted to hydro-electric uses. For the present, not less than 90 000 cu. ft. per sec. of water must be reserved for the preservation of the scenic grandeur of Niagara Falls.

Value of Added Water

The value of 20 000 sec-ft. of dependable water added to the Niagara River as a result of Great Lakes regulation may be arbitrarily assigned at $58 000 000. It should be stated here that the value of water power at Niagara Falls or on the St. Lawrence is not the difference in cost between electric energy generated by steam or water for the reason that industries requiring cheap power would seek other water powers or use steam installations near the pit mouth. The market for the Niagara and St. Lawrence is created by the cheapness of the power. It should be observed also that there is no vital concern in regard to the saving of bituminous coal, because at the present rate of consumption the visible supply in the United States will last for more than 5 000 years.

Great Lakes Regulation

The writer has repeatedly referred to Great Lakes regulation. It is desirable to visualize clearly the Great Lakes System. It is made up of two immense reservoirs, Superior and Michigan-Huron, with little or no water-power development in the outflowing rivers, St. Marys and St. Clair. The Niagara River, with its great descent of 326 ft., has immediately above it the lesser reservoir of Erie as a forebay; and likewise the St. Lawrence River with its power drop of 200 ft. has above it the lesser reservoir of Ontario as a forebay. (See Fig. 7.)

The writer in 1923 had a part in the development of hydro-electric power on the western slope of the Sierras in the Eldorado Region of California. Terms used there are applied to the Great Lakes for clearer visualization. Near the crest of the Sierras, storage reservoirs conserve the rainfall and the melted snow of the wet season for use during the dry season. Reservoir water passes through ditches, canals, and flumes, to a smaller reservoir, called a forebay, having a supply good for 12 to 120 hours, immediately above the 1 800-ft. power drop. The forebay is used for peak-load operation and for emergency supply in case of interruption in ditch service. The water of the reservoirs and of the forebay is expended on a well planned budget system. On the Great Lakes, Superior and Michigan-Huron are the reservoirs in the High Sierras and Erie and Ontario are the forebays just above the power drops. Erie will serve magnificently as a forebay for Niagara power and will permit variations of flow in the Niagara River during the different hours of the day. Any interference with navigation in the Niagara River will be trivial compared to the vast economic gain from intermittent river flow. In the same manner, Ontario will serve magnificently as a forebay for St. Lawrence power, provided the developments in the St. Lawrence are made so as to constitute an uninterrupted series of slack-water pools.

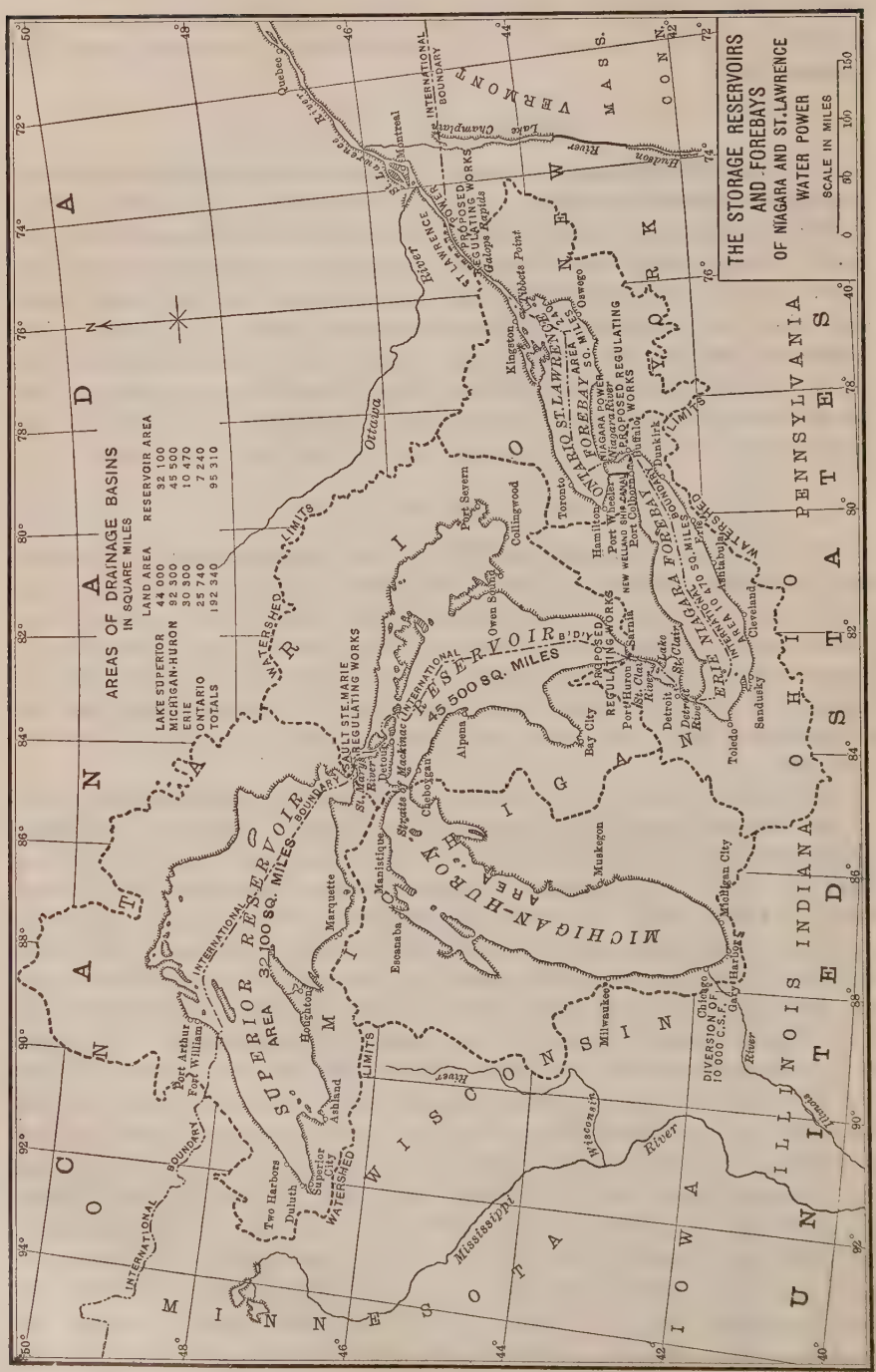

THE STORAGE RESERVOIRS AND FOREBAYS OF NIAGARA AND ST. LAWRENCE WATER POWER
SCALE IN MILES
0 50 100 150
AREAS OF DRAINAGE BASINS IN SQUARE MILES
LAND AREA
RESERVOIR AREA
LAKE SUPERIOR 44 000 32 100
MICHIGAN-HURON 92 300 45 500
ERIE 30 300 10 470
ONTARIO 25 740 7 240
TOTALS 192 340 95 310
SUPERIOR RESERVOIR AREA 32 100 SQ. MILES
MICHIGAN-HURON RESERVOIR AREA 45 500 SQ. MILES
ONTARIO-ST. LAWRENCE FOREBAY AREA 24 SQ. MILES
NIAGARA POWER
PROPOSED REGULATING WORKS
NIAGARA FOREBAY AREA 10 470 SQ. MILES
ERIE-NIAGARA FOREBAY
PROPOSED REGULATING WORKS
SAULT STE. MARIE REGULATING WORKS
INTERNATIONAL BOUNDARY
WATERSHED LIMITS
DIVERSION OF 10 000 C.F.S.
CANADA
MINNESOTA
WISCONSIN
MICHIGAN
IOWA
ILLINOIS
INDIANA
OHIO
PENNSYLVANIA
NEW YORK
VERMONT
MASS.
CONN.
Quebec
Montreal
Ottawa
Kingston
Toronto
Hamilton
Port Sheldon
Port Colborne
Port Huron
Sarnia
Detroit
Toledo
Sandusky
Cleveland
Ashtabula
Erie
Dunkirk
Oswego
Tibbets Point
Port Severn
Collingwood
Owen Sound
Alpena
Bay City
Muskegon
Michigan City
Chicago
Gary Harbor
Milwaukee
Escanaba
Manistique
Marquette
Houghton
Ashland
Superior City
Two Harbors
Duluth
Port Arthur
Fort William
Stratford
Detroit
Mackinac
Ceboygan
St. Lawrence River
Lake Champlain
Hudson River
Wisconsin River
Mississippi River
Illinois River
Lake Erie
Lake St. Clair
Detroit River
Niagara River
Galops Rapids
Long Sault Rapids
Cascades Rapids
Coteau Rapids
Lachine Rapids
Lake Superior
Lake Michigan
Lake Huron
Lake Ontario

FIG. 7.

Ontario Regulation

Beginning with Lake Ontario, it has already been stated that regulating works are a part of the project recommended by the International Joint Commission. The detail of these works is immaterial in the present discussion. The principles of budget outflow alone need consideration.

The writer suggests that Lake Ontario be regulated with a pull-down not much more than 1 ft., aiming at Elevation 247.0 at the beginning of each year. This will be good for navigation. No danger of high water will exist in Lake Ontario, because any large local supply such as occurred in the spring months of 1912 and 1913, will be offset by checked flow in the Niagara and St. Clair Rivers, or by the discharge of an excess volume to the sea through the St. Lawrence River.

It is desirable for purposes of control to increase the channel capacity of the St. Lawrence River to the extent of 50 000 cu. ft. per sec. This will mean no more dredging than will be required for the utilization of the highest head without too great slope losses, and for the efficient solution of the frazil ice problem. The volume of flow for which all artificial works should be constructed may be about 300 000 cu. ft. per sec.

It should be accentuated that the St. Lawrence will have no floods—because the volume of flow will be controlled by regulating works—and that the water is crystalline, carrying no sediment and little trash. An exception to this last statement is the Ottawa River water entering Lake St. Louis above the Lachine Rapids.

There will never be any deficient flow in the St. Lawrence River because increased channel capacities and maintained Lake Ontario levels will care for the tilting up of Lake Ontario and the level drop at the head of the Galops Rapids. With an increased flow capacity of 50 000 cu. ft. per sec. over the flow in a state of Nature, a volume of 220 000 sec-ft. may be passed down the St. Lawrence when the gauge at the head of the Galops Rapids reads 242.2, which is 2 ft. below the low-water winter level. It is not anticipated that large peak-load demands will be made on the various hydro-electric plants on the St. Lawrence River, but provision for some peak-load use is desirable.

In a report on the regulation of the St. Lawrence River, in 1919, the writer suggested works at the head of the American channel of the Galops Rapids with budgeted outflow for storage and navigational betterment.

Erie Regulation

In the report of the Board of Engineers on Deep Waterways, 1900, regulating works were recommended for the head of the Niagara River (see Plate VII), and the works themselves were designed under the direction of the late Mr. Alfred Noble. The scheme as a whole is not of great interest now, as the works were designed to serve navigation alone, the growth of hydro-electric development not being anticipated. The plans show sluices with clear openings of 80 ft. controlled by counterweighted Stoney gates of massive design. For present conditions, it is probable that the submerged weir section should be superseded by sluiceway sections so as to secure the highest flow capacity

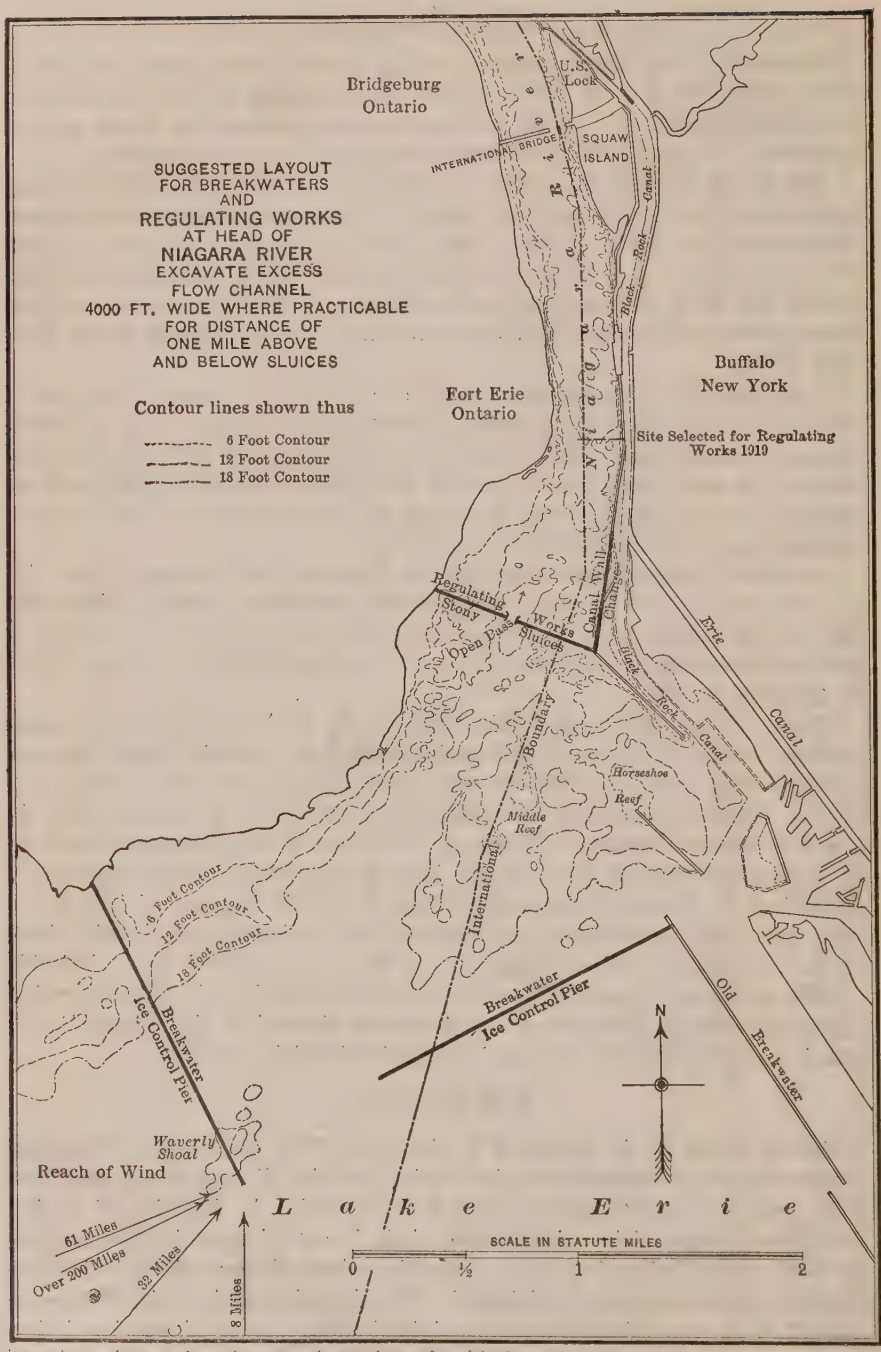
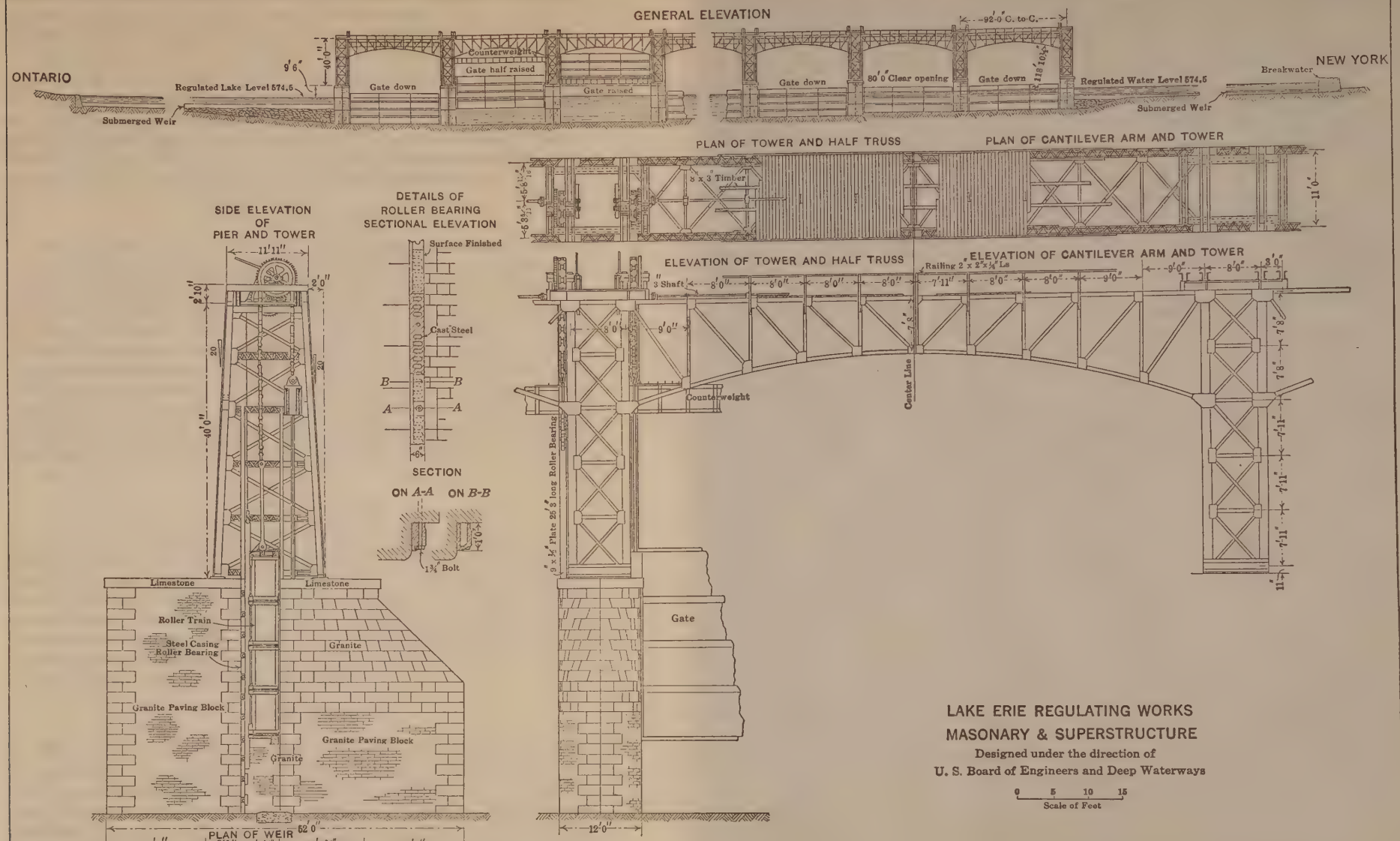

SUGGESTED LAYOUT
FOR BREAKWATERS
AND
REGULATING WORKS
AT HEAD OF
NIAGARA RIVER
EXCAVATE EXCESS
FLOW CHANNEL
4000 FT. WIDE WHERE PRACTICABLE
FOR DISTANCE OF
ONE MILE ABOVE
AND BELOW SLUICES
Contour lines shown thus
6 Foot Contour
12 Foot Contour
18 Foot Contour
Bridgeburg
Ontario
U.S. Lock
INTERNATIONAL BRIDGE
SQUAW ISLAND
Niagara River
Black Rock Canal
Buffalo
New York
Fort Erie
Ontario
Site Selected for Regulating Works 1919
Regulating Works
Stony Point
Open Pass
Sluices
Canal Wall
Channel
Black Rock Canal
Erie Canal
International Boundary
Horseshoe Reef
Middle Reef
6 Foot Contour
12 Foot Contour
18 Foot Contour
Ice Control Pier
Breakwater
Old Breakwater
Waverly Shoal
Reach of Wind
61 Miles
Over 200 Miles
32 Miles
8 Miles
N
Lake Erie
SCALE IN STATUTE MILES
0
½
1
2

FIG. 8.

TRANS., AM. SOC. CIV. ENGRS.
VOL. 89, PAPER No. 1580.
SHENEHON ON
THE ST. LAWRENCE WATERWAY
TO THE SEA.



and to avoid possible blocking by ice. The writer's suggestion is shown in Fig. 8. The minimum continuous flow is shown through an open pass, under normal conditions navigable down stream by small boats.

The aim of the regulation of Lake Erie would be Elevation 573.0 at the beginning of each year, with a possible cresting elevation of 573.8 ft., and a mean elevation through the season of navigation of 573.5 ft. These values in 1 year out of 5 years might be lessened about 1 ft. by emergency storage pull-down. Lake Erie like Lake Ontario will be safeguarded against unduly high water by the checking of the St. Clair River flow, or by sending excess quantities to Lake Ontario and through the St. Lawrence to the sea.

Ice in the Niagara River

The ice problem at the head of the Niagara River and violent storm conditions, with waves battering against the regulating works, need careful consideration. The writer suggests that ice should preferably rot in Lake Erie, as it rots at the head of the St. Marys and in the St. Lawrence, with no heavy ice flow in the rivers. The climatic influence on Buffalo of holding the ice above the head of the river, while it honeycombs, breaks up, and melts, would, in the writer's judgment, be negligible. The temperature of the surface water, with melting ice, will be little different from the surface temperature in April, without ice. It is certain that breakwaters enclosing the head of the river, as indicated in Fig. 8, will solve the question of the storm battering of the regulation works and, at the same time, will eliminate largely the congestion of ice.

Recommended Works

The Board of Engineers for Rivers and Harbors, U. S. Army, in a report dated August 24, 1920, and signed by Brig.-Gen. H. Taylor, U. S. A., M. Am. Soc. C. E., now (1924) Major General, Chief of Engineers, U. S. Army, has recommended the immediate construction of regulating works at the head of the Niagara River, estimated to cost $8 000 000, utilizing substantially the site chosen by Alfred Noble in 1900. The recommendations of the Board include channel enlargements to provide for a flow of from 300 000 to 400 000 sec-ft. to flush ice through the Rapids Section.*

Other Works

The writer's report of 1919 suggested a "daring" type of removable gates to be placed in the narrowest river section, 1½ miles down stream from the site now contemplated. These works had in mind the definite betterment of Lake Erie levels, and, by back-water reflection, the levels of the intervening rivers and Lakes St. Clair and Michigan-Huron. The writer is not now concerned with the mechanism which accomplishes flexible control of the outflow of the various rivers in the Great Lakes System. A clear comprehension of the function of regulating works is first essential and that already exists in

* Report on Diversion of Water from the Great Lakes and Niagara River, Washington, D. C., 1921.

the minds of the members of the Corps of Engineers, including Colonel Wooten and General Taylor. It is probable that the narrow section of the river—1 800 ft. wide—selected by the writer in 1919, will prove to be the best location and disappearing works a better type than Stoney sluices.

It must be understood that other recommendations have been made for the betterment of Lake Erie levels, mostly for the purpose of offsetting the lowering of 5½ in. due to a diversion of 10 000 ft. per sec. at Chicago. It is not desirable to examine the merits of the various suggestions. A comprehensive grasp of the vital principle of river control eliminates the use of throttling works which lie dead and inert in the river bottom or immovably constrict the river width. The vital principle of river control, for the purposes of navigation and water power, means enlarged outflow channels for the rivers of the Great Lakes to the extent of 50 000 cu. ft. per sec. over the natural outflow capacity; and it means manual control of the gates permitting augmented outflow or throttled outflow. These principles were stated definitely in the writer's report of 1919 on the regulation of the Niagara and St. Lawrence Rivers, and are re-affirmed here. Regulating works refer to works manually operated for elastic control; compensating works refer to inert submerged weirs or wing dams.

Intermittent Flow

In addition to the control of Niagara River flow for navigation and water power, is the problem of reasonable fullness in the cataracts and rapids during a portion at least of each day from May to October. The writer proposes for the cataracts a daylight volume of 150 000 sec-ft. during the months mentioned. While this large natural flow goes over the cataracts, water-power uses will be cared for by mill-pond water accumulated in the Erie forebay during 16 hours of diminished cataract and rapids flow. This may mean a river flow of 250 000 sec-ft. for 8 hours, and of 160 000 sec-ft. for 16 hours per day.

It is probable that it will be economical to build control gates at the head of the rapids above the cataracts to maintain a higher level in the Chippewa-Grass Island Pool. This will give greater depths in the water-power canals, with a gain in effective head, and also deeper navigable channels during the periods of lessened flow. These gates will serve also to re-distribute the flow over the crest of the Horseshoe Falls, lessening destructive action at the apex, and increasing depths at the extremities of the crest line. Re-distribution of the flow in the Horseshoe Falls was recommended in the writer's report of 1908, already referred to.

It should be observed lest it appear that this discussion is far afield from the St. Lawrence Waterway to the Sea that the regulating works recommended in 1900 were a part of all deep waterways to the sea, including that of the St. Lawrence to the foot of Lake St. Francis and thence through Lake Champlain to the Hudson River and New York. The maintenance of navigable depths in the lakes and rivers and the best utilization of the water supply of the Great Lakes are dominant elements in the St. Lawrence Waterway to the Sea.

Michigan-Huron Regulating Works

The general layout of regulating works at the head of the St. Clair River, to improve navigable depths in Lake Michigan-Huron and in St. Marys River to the foot of the rapids, including better depths over the lower breast walls of the ship locks at Sault Ste. Marie, is shown in Fig. 5; and these works, with the appurtenant navigable passes, have already been briefly described. The economic desirability and the physical practicability of the regulation of these immense lakes are certain. The same principles apply as in the case of the St. Lawrence and Niagara Rivers, that additional outflow capacity is desirable and that the regulating works shall give elastic manual control. (See Fig. 5 and page 506.)

Over-Emphasizing Minor Losses

For the past 25 years, during the season of navigation, Lakes Michigan-Huron have been from 18 to 40 in. lower than desirable. Since the opening of the canal of the Sanitary District of Chicago in January, 1900, however, more concern has been felt for the loss of level, increasing during the period from 2 in. to 5 in., than for the large degradation of levels coming from other causes, and engineers have given their attention to the reclamation of the 5 in. rather than to the larger needs. It is obvious that the lesser loss could be compensated by contraction works in the form of submerged weirs or wing dams. A Board of United States Engineers reported in 1914 on a series of submerged weirs in the St. Clair River to restore 6 in. The writer proposed in 1919 to raise Lakes Michigan-Huron, as did the Board of Engineers on Deep Waterways in 1900, by back-water coming from a super-elevation of Lake Erie. The International Waterways Commission, 1910, gave consideration to the regulation of Lakes Michigan-Huron, but reached a conclusion that the natural regulation of the lakes was so perfect that it could not be improved on by the works of man.*

Mr. Grunsky's Forecast

"The Low Stage of Lakes Huron and Michigan" has been discussed in a paper† by C. E. Grunsky, Past-President, Am. Soc. C. E. Speaking of the increased elevation of Lakes Michigan-Huron through back-water from the super-elevation of Lake Erie, Mr. Grunsky states:‡

"It is possible, moreover, to go a step farther and to provide works for the throttling of the waterway, or for the complete control of flow, within fixed limits, of the St. Clair River. The stage of Lakes Huron and Michigan could then be brought under adequate control, and the lake stage could be maintained at all times as high as required by navigation interests.

"That regulating works will sooner or later be constructed, in the rivers draining the several lakes, of such a character that the elevation of the lakes under proper management cannot fall below predetermined minima, is reasonably certain. It follows, therefore, in view of the benefit that will be derived from the use of Lake Superior as a storage basin, and of the control

* Report of the International Waterways Comm. on the Regulation of Lake Erie, 1910.

† *Transactions*, Am. Soc. C. E., Vol. LXIII (June, 1909), p. 31.

‡ *Loc. cit.*, pp. 46–47.

of water surface that will result from works below Lake Huron, that navigation interests will be but temporarily, if at all, affected by a diversion of water at Chicago, or elsewhere within reasonable limits as to amount, and that any deleterious effect will not continue beyond the time when the water elevation of this lake and Lake Michigan will be controlled by works below Lake Huron.

"The stages of Lake Ontario and of the St. Lawrence River are questions apart from the one that has here been discussed. Enough has been said, however, to show that here, too, the less than normal stages of recent years are to be ascribed to climatic conditions, supplemented in a very slight degree only by the diversion at Chicago.* The ultimate effect of the diversion of water from any of the lakes, offset by the equalizing effect of lake regulation, upon the stage of water in the St. Lawrence River is a study that will have to be made when the regulating works come under consideration."

Superior Regulation

The writer's suggestion is to regard the regulation of the lakes, not as piecemeal and individual, but as a continuous, interrelated series in which each lake serves all the others. As Lakes Superior and Michigan-Huron are treated as a unit of storage, it is well to pass immediately to the existing regulating works at the head of St. Marys River, giving manual control of the outflow of Lake Superior. The original design of these controlling works was made under the direction of the late Mr. Alfred Noble, very shortly after the design of the works at the head of the Niagara River. Sections of the works were built to compensate for water-power diversions in lateral canals, and these sections existed for a number of years as dead, inert, unoperated, compensating works, permitting diversions without unduly disturbing the level of Lake Superior. Later, water-power development on both sides of the river became the adequate reason for completing the works and operating them manually. These works now consist of a battery of sixteen sluices, each with a clear opening of 52.2 ft., with massive steel Stoney gates. In addition, there are three 33-ft. sluices in the dam of the United States power plant. These works are shown in Figs. 4 and 10. The original construction is detailed in a paper by G. F. Stickney, M. Am. Soc. C. E., entitled "The Compensating Works of the Lake Superior Power Company."* The recent works and their operation have also been described† in much detail by Mr. Louis C. Sabin.

It will be assumed that the regulating works, with the added flow in the water-power canals on each side of the river, have an excess outflow capacity of 50 000 sec-ft., or more, over that of the natural rapids. The volume of flow through the wheels, either running or blocked open, may be regarded as effective outflow capacity for the water-power plants on the St. Marys River. In considering the capacity of flood flow on flashy streams where abnormal conditions of ice or trash might prevent penstock flow, it is regarded as desirable to have other spillways of adequate capacity. For the streams of the Great Lakes, however, no comparable flood conditions exist. A brief shutdown of the power plants at Sault Ste. Marie at the time of maximum supply,

* *Transactions*, Am. Soc. C. E., Vol. LIV (June, 1905), p. 346.

† In an article in the *Military Engineer*, reprinted in the *Canadian Engineer*, March 21, 1920.

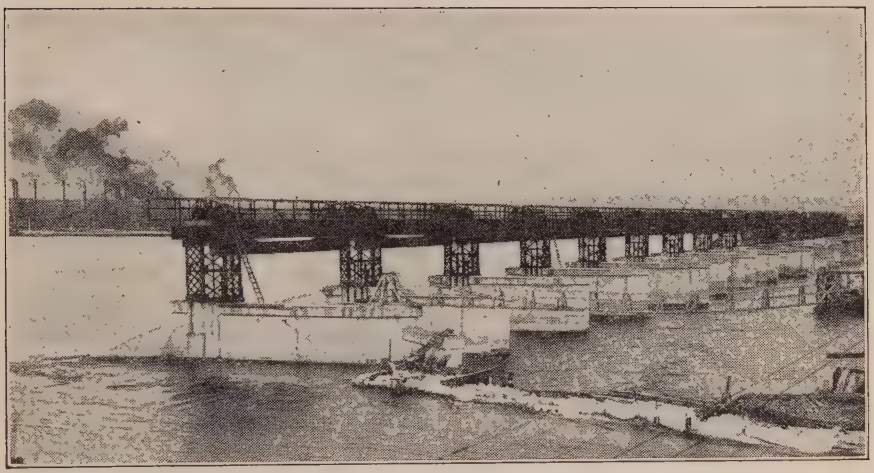


FIG. 9.—VIEW OF REGULATING WORKS IN ST. MARYS RIVER.

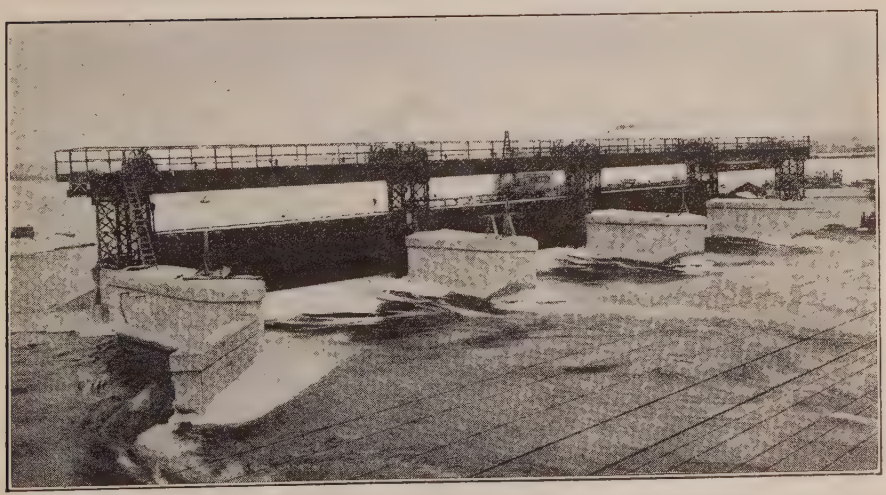


FIG. 10.—VIEW OF PART OF REGULATING WORKS AT SAULT STE. MARIE.

will have a negligible effect on the level of Lake Superior. On the Niagara and St. Lawrence, however, the well-being of the tandem series of plants should involve adequate spillway capacity so that lower plants may not be shut down in case of blockade in an upper plant.

It is noteworthy that no scheme of regulation to better navigable conditions by the maintenance of higher surface levels has originated with the shipping interests of the Great Lakes. With the exception of the Gut Dam at the Galops Rapids of the St. Lawrence, the compensation has always been suggested by diversions for sanitary or water-power purposes, rather than for the maintenance of levels when the supply conditions of the Great Lakes needed the throttling of the outflow toward the ocean.

The water power at Sault Ste. Marie contemplates a prime diversion of 50 000 sec-ft., with the use of 10 000 sec-ft., or more, of excess water when the Lake Superior supply warrants additional outflow. The use of this water for power purposes at Sault Ste. Marie, under a head of 20 ft., less slope losses in the canals, should amount to about 90 000 e. h. p. The actual development is somewhat less than this. The water power here, with perhaps an effective head of 18 ft., is not comparable with the great powers at Niagara with a head of over 300 ft., or with the ultimate great powers on the St. Lawrence with heads aggregating 200 ft. The release of Lake Superior water should contemplate the well-being of navigation and the lower water powers, rather than the over-accentuation of the local water powers of St. Marys River. The water power at Sault Ste. Marie, is largely used for the making of carbide, ferro-alloys, electric smelting, the grinding of wood pulp, and the making of paper.

Assuming a cost of $750 per ft. for works installed at Sault Ste. Marie, under post-war conditions, the total cost for the main river section is about $750 000. The cost on the American side was borne initially by the water power companies, but the United States is assuming the cost by the remission of water power rentals. It should be understood that at Sault Ste. Marie, under the decision in the famous Chandler-Dunbar case, all submerged lands were taken over by the Federal Government for purposes of navigation. This made the United States a water power proprietor, receiving each year as rentals for the use of the water, $2 per sec-ft. for prime water and $1 for secondary water. Here, then, exists a little subsidization of navigation by water power.

The operation of the regulating works in St. Marys River is under an International Board, the program contemplating the maintenance of Lake Superior levels between the limits of 601.5 and 604.1 ft. The program is based on the needs of Superior alone without necessary benefit to Michigan-Huron.

The final closing of the gap completing the work and placing Superior under full manual control is so recent that the value of regulation is not yet fully demonstrated. In the paper already referred to, Mr. Sabin writes:

"When the works are entirely completed the flexibility of the system will be increased, since the entire flow will be under control; the degree of regulation will not be materially improved, however, but a greater use of water for

power will be permissible. What will be accomplished, then, is the continuous use of the natural minimum flow, equal to about 75% of the mean discharge of the river, and a reduction of the fluctuation in monthly mean levels from a range of 3.8 ft. to a range of about 2.6 ft. The economic advantage of power development to the extent of some 80 000 h. p. is thus combined with improvements of navigation conditions."

Regulated Levels

Lake Superior has not been entirely immune from the low-water régime existing on the other lakes. The reason for this is the tremendous inertia of the lake, which will require a number of years of ample supply or restricted outflow to bring it up. Assuming that it is desirable to raise the surface level of Lake Superior 1 ft. over the present level, it will be necessary to throttle the outflow to the extent of over 28 000 sec-ft. for 1 year, or 14 000 sec-ft. for 2 years. At a time when the lower lakes need the water, it is not practicable to accomplish this restoration of Lake Superior level. This must await regulating works in all the Great Lakes, so that the supply may be simultaneously throttled in all.

In the scheme of regulation suggested by the writer, Lake Superior will preferably stand at about Elevation 602.5 at the beginning of each year. As the year progresses, the lake will normally rise to a cresting height of 603.3 ft. in September, and gradually drop to Elevation 602.5 by the end of the year. This is the normal cycle in a state of Nature. To what extent it will be modified under manual regulation is not yet definitely clear. The limiting cresting stage is about 604.1 ft. Between cresting at 603.3 ft. and 604.1 ft. is a depth of 0.8 ft. To fill the lake to this depth in 4 months will require an excess of roundly 85 000 sec-ft. more than the normal generous supply of these productive months. As the excess capacity of the outflow works permits a release of 50 000 sec-ft. over the release in a state of Nature, the actual rise might be held to Elevation 603.6.

Effect on Michigan-Huron

The dumping of 50 000 sec-ft. for 4 months will raise the level of Lakes Michigan-Huron, with manual control at the head of the St. Clair River, about 0.4 ft. The hazard of high water in Superior, with regulating works in operation and with 50 000 sec-ft. excess channel capacity, appears to be slight.

In case Lakes Michigan-Huron, under these circumstances, are at a stage where the addition of 0.4 ft. is undesirable, the excess volume of 50 000 sec-ft. may be simultaneously discharged through the St. Clair River to Erie, through the Niagara to Ontario, and through the St. Lawrence to the sea, without any level change in the intermediate lakes. The Superior Reservoir should be regarded as a unit with the Michigan-Huron Reservoir. When Michigan-Huron has a greater supply than it needs for its level maintenance within safe limits, the throttling of the flow of the St. Marys River will retain any desirable quantity in the Superior Reservoir. Conversely, when Michigan-Huron needs additional water, this may be secured by release from Superior.

The immediate passing of water to the sea by opening simultaneously all works to the maximum extent of 50 000 sec-ft. will dispose of the excess supply.

Storage Reservoirs

The areas of the land part of the Superior Basin and of the reservoir are shown on Fig. 7. A depth of storage of 1 ft. on Superior has a value of 28 000 sec-ft. over a period of 1 year, and the combined storage capacity of the Superior and Michigan-Huron Reservoirs, for a pull-down of 1 ft., amounts to 68 000 sec-ft. over a period of 1 year, or 20 000 sec-ft. for a period of 3.4 years. It will be noted in Table 2 that the maximum net storage release from all the Great Lakes, including the forebays of Erie and Ontario, during the years 1921 to 1923, inclusive, amounts to about 24 000 sec-ft.; and that the maximum net release required from all reservoirs and forebays in the exceptional low-supply, 4-year period, 1921-25, is 25 000 sec-ft. The storage value of the Superior and Michigan-Huron Reservoirs alone over a period of 5 years amounts to 13 600 sec-ft., while the capacity of all reservoirs and forebays for a depth of 1 ft. is nearly 17 000 sec-ft.

Michigan-Huron Supply

The outflow conditions in the St. Marys River are simple, and it is known that the average Superior supply amounts to about 80 000 sec-ft. It is known also (see Table 3) that the average supply outflowing in the Niagara River is 205 000 sec-ft. The average supply coming from Lakes Michigan-Huron and Erie is, therefore, 125 000 sec-ft. If it should be assumed arbitrarily that the Erie supply is 20 000 sec-ft., then Michigan-Huron shows an average supply of 105 000 sec-ft. The average outflow of the St. Clair River is then about 185 000 sec-ft. This outflow value is somewhat startling, because the outflow of the St. Clair, when Lake Huron is at its average stage of 581.1, is 205 000 sec-ft. There is, therefore, a discrepancy of 20 000 sec-ft. This is attributable to the fact that during the months of January to April, inclusive, the outflow of the St. Clair River under ice conditions is restricted to perhaps 150 000 sec-ft. In considering manual regulation of Michigan-Huron, the condition during the ice months mentioned must be given the fullest consideration.

St. Clair River Winter Flow

The small winter discharge capacity of the St. Clair River, suggested as perhaps 150 000 sec-ft., may doubtless be increased, if desirable, by channel improvements, more particularly in the delta section of the river. The hydraulic explanation of this small winter outflow is simple. In the open season the river utilizes 5.5 ft. of head as potential to meet the various energy expenditures in its 40 miles of channel. When winter comes, it finds itself flowing in part under an ice cover with diminished cross-sectional area. The various resistances must then be increased requiring a greater fall than 5.5 ft. The result, therefore, with a fixed head of 5.5 ft., must inevi-

tably be lessened volume of flow. Notwithstanding this lessened winter flow of the St. Clair River—somewhat augmented now by channel improvements—the Niagara River continues its outflow in ample volume, depending on Erie storage and local supply to supplement the lessened St. Clair River inflow; and the St. Lawrence River with its local supply, Ontario storage, and ice retardation, continues in ample flow.

Michigan-Huron Regulation

The suggested regulating works at the head of the St. Clair River for manual control, and the necessary vessel passes are shown in Fig. 5. The foundation for all works is boulder clay. The regulating works indicated, contemplate 3 000 ft. of 80-ft. sluices with massive Stoney gates, similar to those proposed by the Board of Engineers on Deep Waterways in 1900 for the head of the Niagara River. (See Plate VII.) Based on the estimate made by the Board of Engineers for the Niagara River, the cost of the regulating works will be about $1 000 per lin. ft., or about $3 000 000. In addition will be the cost of lateral pier work for the east section, $500 000; and 1 700 ft. of sluice dam west of the expansion basin walls, costing perhaps $500 000. The total cost of regulating works is about $4 000 000. The three navigable passes and the two concrete expansion basins, will cost about $5 000 000. Breakwater protection—not shown in Fig. 5—above the uppermost pass will cost about $1 000 000. The total cost of all works at the head of the St. Clair River will be about $10 000 000. No provision is made for excess outflow capacity other than that due to the superelevation of the water surface of Lake Huron, and that due to enlarged navigable channels throughout the St. Clair River. No specific item of cost is assigned for dredge work within the expansion basin, and forming part of the large project of channel excavation required for 25 and 26-ft. navigation. As the vessel passes are no-stop works, no costs of operation similar to the costs at the ship locks at Sault Ste. Marie are involved. The ultimate capitalized value of these works to navigation and water power will be not less than $70 000 000.

The regulation of Michigan-Huron contemplates an elevation at the first of the year of 582.0 ft., with a cresting elevation of 583.0 ft. These elevations are subject to an emergency pull-down of 1 ft. for the use of storage for lake-level preservation in Lakes Erie and Ontario and the maintenance of uniform flow in the Niagara and St. Lawrence Rivers for water-power purposes.

The inertia of this vast reservoir safeguards riparians against high water. With the Michigan-Huron Reservoir at Elevation 582.2, at the beginning of April of a given year, it will require an excess supply from its own water-shed of 120 000 sec-ft., continuing for 4 months, to raise the lake to Elevation 583.2, and this is still 0.3 ft. below the cresting elevation in July, 1876, and June, 1886, and 1.5 ft. below the high water of 1838. Should an excess flow capacity in the St. Clair River of 50 000 sec-ft. be invoked to check this rise, the elevation in the July cresting in the illustrative case will be 582.8 ft. instead of 583.2 ft. The normal rise, with St. Marys River inflow without control, is 0.7 ft. between the first of April and the last of July. After July, the level of the Michigan-Huron Reservoir almost invariably drops.

The geographical regions embraced in the water-sheds of the Superior and Michigan-Huron Reservoirs are so extended that the diversity element of supply enters to a considerable extent. When the Superior Reservoir is receiving an exceptional supply, it is by no means certain that the Michigan-Huron Reservoir in its own basin is receiving a comparably large supply. The diversity element is more conspicuous still, in the regional differences between Superior and Erie and Ontario.

Restoration

On January 1, 1925, Lakes Michigan-Huron were 3½ ft. below the desirable regulated stage of 582.0 ft. To restore these lakes to this elevation will require a throttling of the outflow to the extent of 28 000 sec-ft. for a period of 5 years. The probability is small of level restoration without artificially throttling the flow of the St. Clair River. Reference to Table 4 indicates that the probability of a net excess volume of 20 000 sec-ft. for a 5-year period is 1 in 12. In the period, 1880-84, the net excess was at the rate of 19 000 sec-ft. The restoration of the Superior and Michigan-Huron Reservoirs, therefore, must be accomplished by lessened flow in the St. Clair, Niagara, and St. Lawrence Rivers over a period of from 5 to 10 years. Regulation in the Niagara River by works built simultaneously with regulating works above the head of the St. Clair River should precede the throttling of the outflow of Lake Huron for level restoration. This throttling should be accomplished in as large a measure as practicable during the non-navigation period between mid-December and mid-April.

Instantaneous Water Delivery

The infirmity of the natural operation of the Superior and Michigan-Huron Reservoirs is in the fact that it is impossible to deliver water to Lakes Erie and Ontario without the lapse of a long time interval. Should the St. Lawrence River need additional water at a time when Lake Superior is filled to the limit of its storage capacity, it would take 5 years to transmit this water in some such volume as 20 000 sec-ft. to the head of the St. Lawrence River. The Superior water released by the regulating works at Sault Ste. Marie must first raise the level of Lakes Michigan-Huron practically 1 ft. before the St. Clair River is able to discharge the 20 000 sec-ft. inflowing from St. Marys River, and Lake Erie likewise must rise nearly 1 ft. before it, in turn, will be able to transmit the water to Lake Ontario.

The outstanding value of co-ordinated regulation from the point of view of lake-level maintenance in Lakes Erie and Ontario and water-power use in the Niagara and St. Lawrence Rivers, arises from the fact that storage in the Superior Reservoir may be made immediately available, without a day's loss of time, for use in the St. Lawrence River. The procedure is as follows:

The St. Lawrence River needs 20 000 sec-ft. additional water. Instantaneously release 20 000 sec-ft. through the St. Marys River, the St. Clair River, the Niagara River, and the St. Lawrence River. The St. Lawrence then uses immediately the needed 20 000 sec-ft., without any change in the surface elevations of Lakes Michigan-Huron, Erie, or Ontario. This pro-

cedure is like cashing at Ogdensburg a draft on funds at Duluth. The ideal development at the St. Lawrence River in a series of four slack-water pools corresponds very closely with the geological layout of the four slack-water pools of the Great Lakes, and the transmission of water for peak-load use through the St. Lawrence from pool to pool will follow the instantaneous procedure described. Having in mind the fact that at present it takes a number of years to transmit available storage water from the Superior Reservoir to the head of the Niagara River, the tremendous economic gain in water use in the instantaneous transmission proposed by the writer is obvious.

At the Annual Convention of the Society in July, 1898, a paper by the late Brig. Gen. Hiram M. Chittenden, U. S. A. (*Retired*), M. Am. Soc. C. E., then Captain, Corps of Engineers, U. S. A., entitled "Reservoir System of the Great Lakes of the St. Lawrence Basin",* with a "Mathematical Analysis of the Influence of Reservoirs upon Stream-Flow", by the late James A. Seddon, M. Am. Soc. C. E., was presented by Mr. Seddon. In this paper the lag in time between the release of water from Lake Superior and its outflow in the St. Lawrence River was fully discussed.

Erie Regulating Works

The regulating works at the head of the Niagara River, with breakwater protection, as shown in Fig. 8, might cost approximately as follows: Sluices with Stoney gates, 4 000 lin. ft., $4 000 000; breakwaters, 3 miles in length, $3 000 000; channel enlargements for excess capacity flow, $3 000 000; total, $10 000 000. It will be understood that these estimates are roughly approximate. The Black Rock ship lock recommended by the Board of Engineers on Deep Waterways in 1900, to by-pass works at the head of the Niagara River, has already been built.

Expedients

Doubtless preliminary inexpensive measures may be used to restore, in part, the levels of Lakes Michigan-Huron. Submerged weirs in the St. Clair River and regulating works with gates in the Niagara River, to raise Lake Erie level and by back-water reflection the level of Lakes Michigan-Huron, will restore water-level losses chargeable to the diversion of 10 000 sec-ft. at Chicago. If no further betterment of lake levels or of water supply is needed, these simple expedients will serve. The future, however, will require the adequate regulation suggested by the writer in this paper. The permanent use of submerged weirs or wing dams violates the fundamental principle of regulation, which requires excess channel capacity in the St. Clair, Niagara, and St. Lawrence Rivers.

Canals as Excess Outlets

It should be understood that all continuously flowing canals—including the Drainage Canal at Chicago, the Black River Canal at Port Huron, the Welland Canal, and the Erie and Black Rock Canals at Buffalo—count as

* *Transactions*, Am. Soc. C. E., Vol. XL (1898), p. 355.

excess capacity channels required by the ultimate effective regulation of the Great Lakes System. To the extent of the volume of flow permissible in these various canals, the enlargement cost of the natural channels of the St. Clair and Niagara Rivers will be lessened. This is clearly the case in the St. Marys River, where the entire excess capacity flow needed, has already been secured in lateral water-power canals. The use of 10 000 sec-ft. of continuous flow in the Welland Canal for water power, if that be practicable, would also lessen the cost of rock excavation in the Niagara River. To this extent, in the ultimate analysis, it is obvious that excess-flow canals serve beneficially.

Data

In order that those discussing this paper may have available full information, the following diagrams are incorporated: Plate IV is the hydrograph of the U. S. Lake Survey, showing the water surface elevations of the Great Lakes for 65 years; Fig. 11 shows curves of discharge of the St. Clair River in terms of the elevation of Lake Huron and Lake St. Clair; Fig. 12 shows the storage capacity of Lake Ontario; Fig. 13 is the curve of discharge of the Niagara River in terms of the elevations of Lake Erie height at Buffalo; Fig. 14 gives the discharge curves on the St. Lawrence River before and after the building of Gut Dam at the Galops Rapids in 1903; Fig. 15 gives a series of curves expressing relationship between discharge in the St. Lawrence River and storage in Lake Ontario; Fig. 16 shows the theoretical division of discharge between the American and Canadian channels at the Galops Rapids of the St. Lawrence River; and Plate VIII shows the detail of discharge measurements in the Niagara River, giving some indication of the precision of measurements.

Financing the Work

When the time comes to adjust between the United States and Canada the cost of the various constructions of the St. Lawrence Waterway to the Sea, the full length from Duluth, Port Arthur, and Chicago, to the foot of the canalized sections of the St. Lawrence River below Montreal, must be taken into consideration.

Should a flow of 10 000 sec-ft. be utilized for water-power purposes in the Welland Canal, securing perhaps 250 000 e. h. p., rentals might be charged for the delivered water, and the Welland Canal would then be subsidized, in part, by water power. It is permissible under the Federal Water Power Act to assess money costs and charges for operation, maintenance, and depreciation of head-waters improvements against the water powers benefited, which might include those at Niagara Falls. The Act expressly states that a charge may be made by the United States for head-waters improvements to be paid by licensees. An administrative fee of 25 cents per h. p. is now paid by the Niagara Falls Power Company. This must amount to nearly $100 000 per year. On the Canadian side, rentals are charged for the use of the water, ranging from 50 cents to $1.00 per e. h. p.-year. The writer is not informed as to whether or not this includes all water power on the Canadian side at

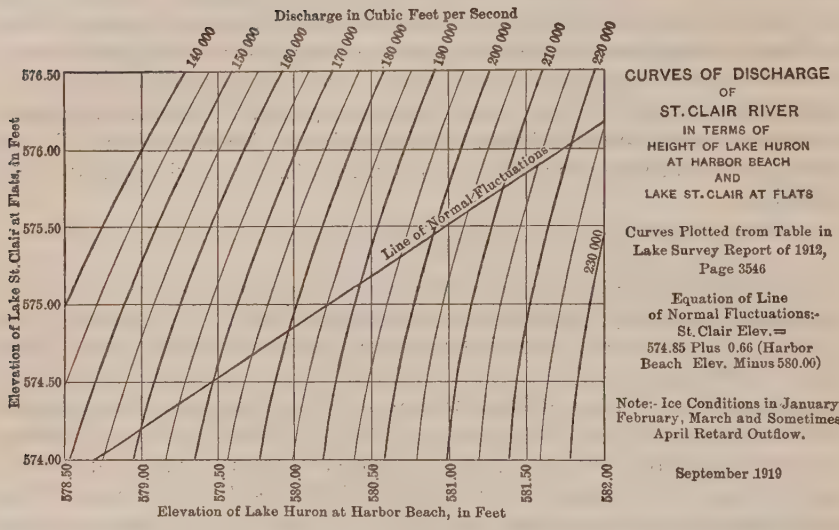

Discharge in Cubic Feet per Second
140 000
150 000
160 000
170 000
180 000
190 000
200 000
210 000
220 000
230 000
576.50
576.00
575.50
575.00
574.50
574.00
Elevation of Lake St. Clair at Flats, in Feet
Line of Normal Fluctuations
578.50
579.00
579.50
580.00
580.50
581.00
581.50
582.00
Elevation of Lake Huron at Harbor Beach, in Feet
CURVES OF DISCHARGE
OF
ST. CLAIR RIVER
IN TERMS OF
HEIGHT OF LAKE HURON
AT HARBOR BEACH
AND
LAKE ST. CLAIR AT FLATS
Curves Plotted from Table in
Lake Survey Report of 1912,
Page 3546
Equation of Line
of Normal Fluctuations:-
St. Clair Elev.=
574.85 Plus 0.66 (Harbor
Beach Elev. Minus 580.00)
Note:- Ice Conditions in January,
February, March and Sometimes
April Retard Outflow.
September 1919

FIG. 11.

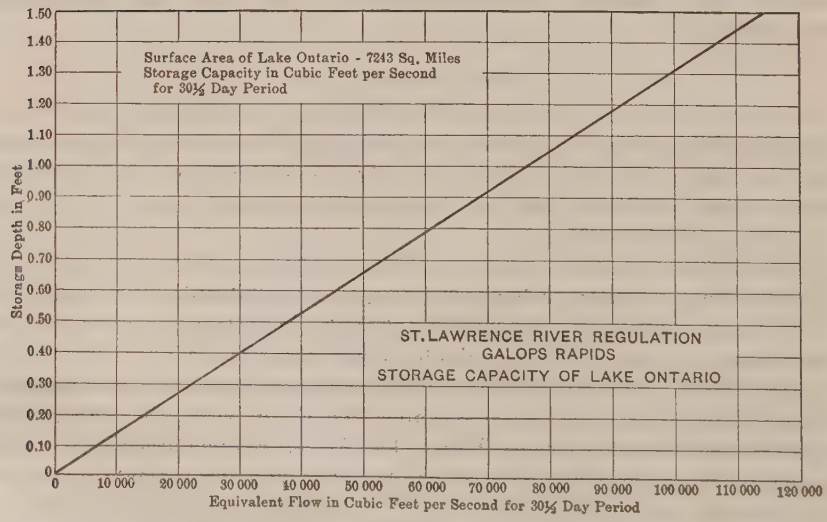

Surface Area of Lake Ontario - 7243 Sq. Miles
Storage Capacity in Cubic Feet per Second
for 30½ Day Period
Storage Depth in Feet
1.50
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0
ST. LAWRENCE RIVER REGULATION
GALOPS RAPIDS
STORAGE CAPACITY OF LAKE ONTARIO
0
10 000
20 000
30 000
40 000
50 000
60 000
70 000
80 000
90 000
100 000
110 000
120 000
Equivalent Flow in Cubic Feet per Second for 30½ Day Period

FIG. 12.

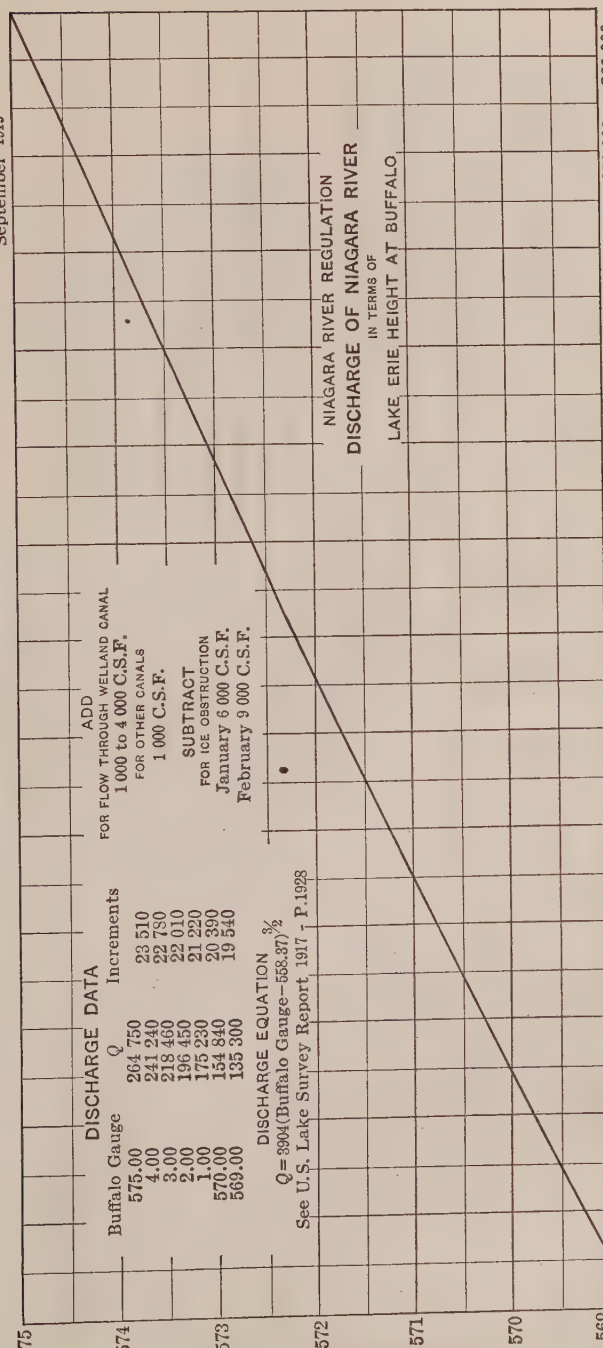

September 1919
NIAGARA RIVER REGULATION
DISCHARGE OF NIAGARA RIVER
IN TERMS OF
LAKE ERIE HEIGHT AT BUFFALO
DISCHARGE DATA
Buffalo Gauge
Q
Increments
575.00 264 750
23 510
4.00 241 240
22 780
3.00 218 460
22 010
2.00 196 450
21 220
1.00 175 230
20 390
570.00 154 840
19 540
569.00 135 300
DISCHARGE EQUATION
Q = 3904(Buffalo Gauge − 568.37)^3/2
See U.S. Lake Survey Report, 1917 - P. 1928
ADD
FOR FLOW THROUGH WELLAND CANAL
1 000 to 4 000 C.S.F.
FOR OTHER CANALS
1 000 C.S.F.
SUBTRACT
FOR ICE OBSTRUCTION
January 6 000 C.S.F.
February 9 000 C.S.F.
Elevations of Lake Erie at Buffalo
575
574
573
572
571
570
569
130 000
140 000
150 000
160 000
170 000
180 000
190 000
200 000
210 000
220 000
230 000
240 000
250 000
260 000
Discharge in Cubic Feet per Second

FIG. 13.

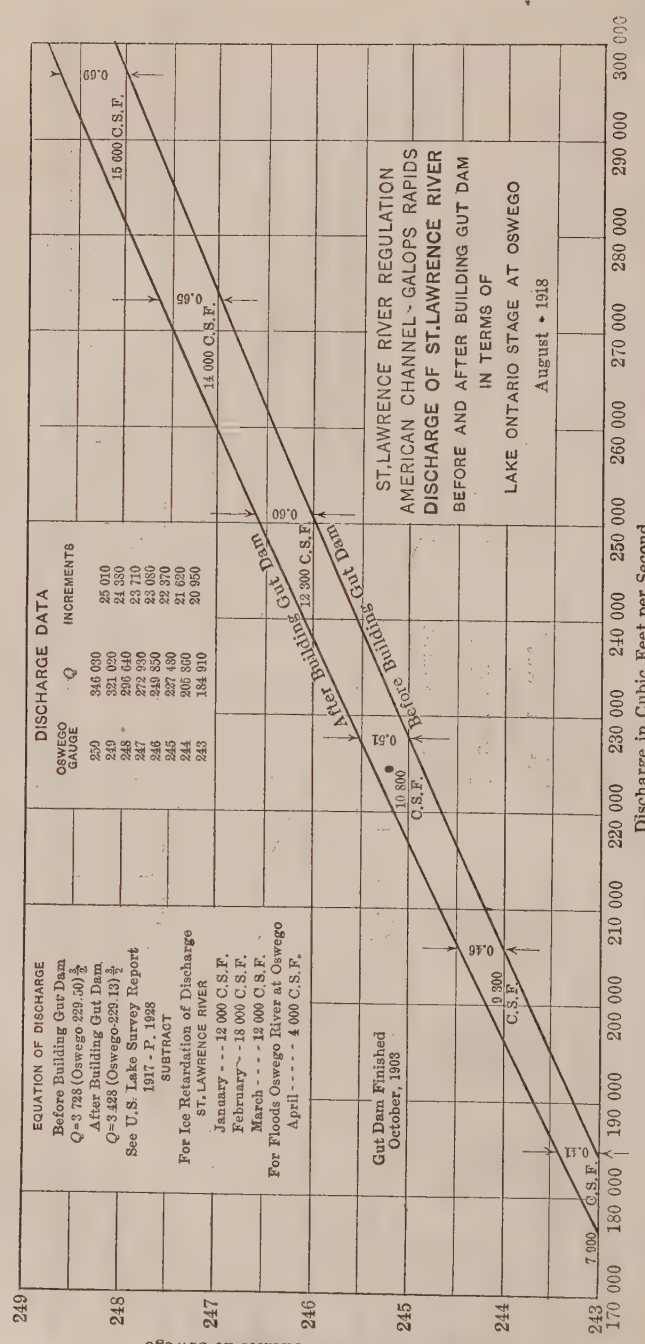
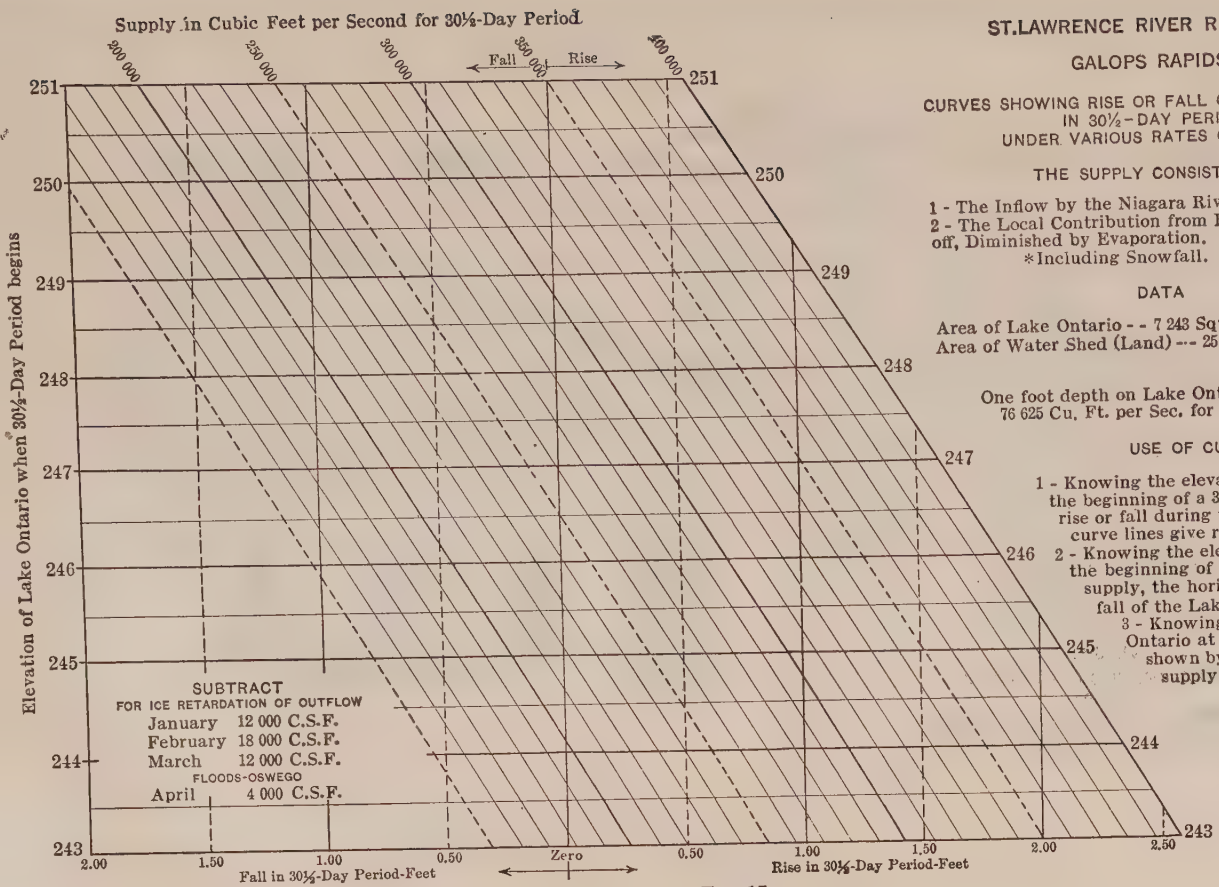

ST.LAWRENCE RIVER REGULATION
AMERICAN CHANNEL - GALOPS RAPIDS
DISCHARGE OF ST.LAWRENCE RIVER
BEFORE AND AFTER BUILDING GUT DAM
IN TERMS OF
LAKE ONTARIO STAGE AT OSWEGO
August • 1918
DISCHARGE DATA
OSWEGO GAUGE
Q
INCREMENTS
250 346 030
249 321 020 25 010
248 296 640 24 380
247 272 930 23 710
246 249 850 23 080
245 227 480 22 370
244 205 860 21 620
243 184 910 20 950
EQUATION OF DISCHARGE
Before Building Gut Dam
Q=3 728 (Oswego 229.50) 5/2
After Building Gut Dam
Q=3 428 (Oswego-229.13) 5/2
See U.S. Lake Survey Report
1917 - P. 1928
SUBTRACT
For Ice Retardation of Discharge
ST. LAWRENCE RIVER
January - - - 12 000 C.S.F.
February - - 18 000 C.S.F.
March - - - - 12 000 C.S.F.
For Floods Oswego River at Oswego
April - - - - 4 000 C.S.F.
Gut Dam Finished
October, 1903
After Building Gut Dam
12 300 C.S.F.
Before Building Gut Dam
10 800 C.S.F.
9 300 C.S.F.
7 000 C.S.F.
15 600 C.S.F.
14 000 C.S.F.
0.69
0.65
0.60
0.51
0.46
0.41
Elevations of Lake Ontario at Oswego
Discharge in Cubic Feet per Second
249
248
247
246
245
244
243
170 000
180 000
190 000
200 000
210 000
220 000
230 000
240 000
250 000
260 000
270 000
280 000
290 000
300 000

Fig. 14.



ST.LAWRENCE RIVER REGULATION

GALOPS RAPIDS

CURVES SHOWING RISE OR FALL OF LAKE ONTARIO
IN 30½-DAY PERIOD
UNDER VARIOUS RATES OF SUPPLY

THE SUPPLY CONSISTS OF:-

1 - The Inflow by the Niagara River.
2 - The Local Contribution from Rainfall* and Run-off, Diminished by Evaporation.
*Including Snowfall.

DATA

Area of Lake Ontario - - 7 243 Square Miles.
Area of Water Shed (Land) --- 25 737 Square Miles.

One foot depth on Lake Ontario is equal to 76 625 Cu. Ft. per Sec. for a 30½-Day Period.

USE OF CURVES

1 - Knowing the elevation of Lake Ontario at the beginning of a 30½-day period, and the rise or fall during the period, the slanting curve lines give rate of supply.
2 - Knowing the elevation of Lake Ontario at the beginning of a 30½-day period, and the supply, the horizontal scales give rise or fall of the Lake in that period
3 - Knowing the elevation of Lake Ontario at any time, the outflow is shown by the intercepts of the supply lines on the zero line.

September-1919

For conditions after building Gut Dam. (Completed in 1903)

FIG. 15.

ST. LAWRENCE RIVER REGULATION

THEORETICAL DIVISION OF DISCHARGE
between
American and Canadian Channels
GALOPS RAPIDS
ST. LAWRENCE RIVER
in Terms of Depth on Sill
Galops Canal - after 1903
Elevation of Sill - 234.14
August, 1918

DISCHARGE VALUES			Approximate Oswego Gauge Equivalents
Depth on Sill	Amer. Channel	Can. Channel	
12.00	137 600	157 800	248.64
11.00	123 600	147 600	247.56
10.00	110 250	137 500	246.48
9.00	97 200	127 780	245.40
8.00	84 600	118 340	244.32

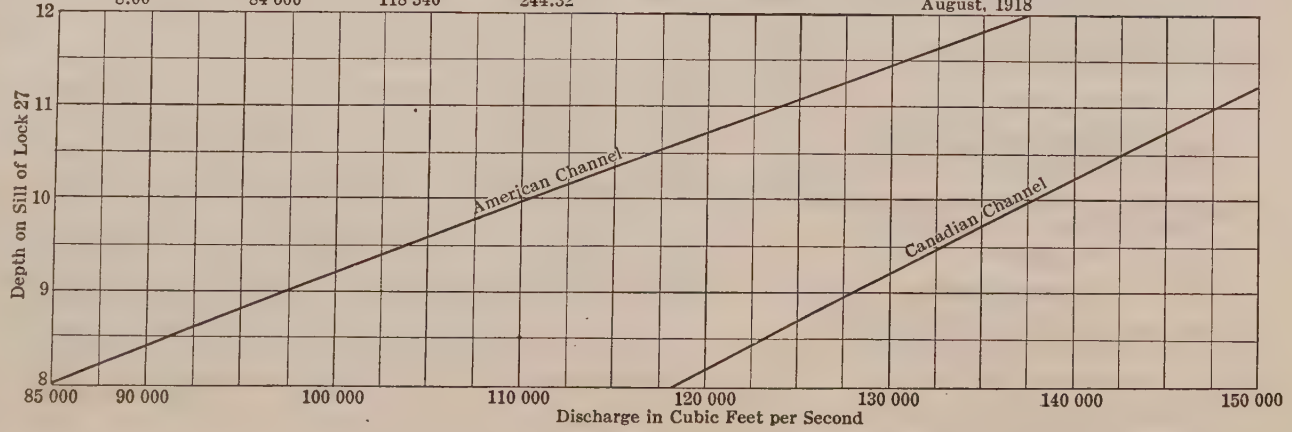
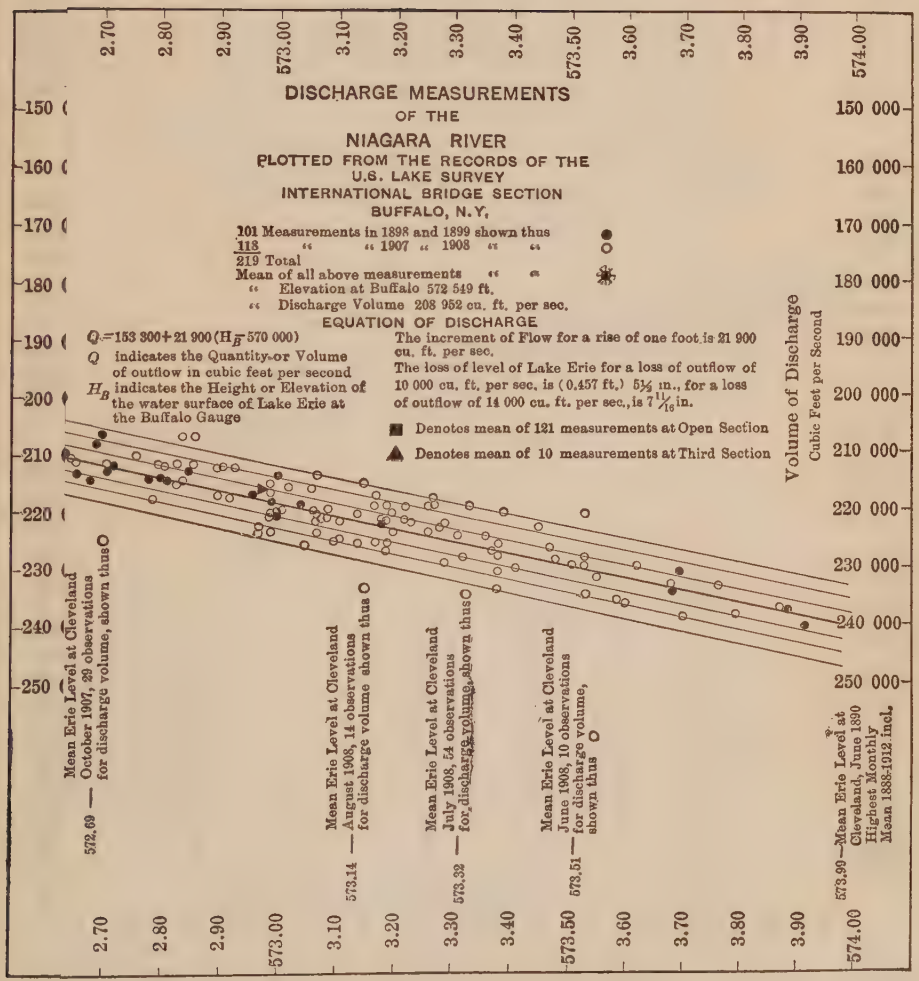


FIG. 16.

PLATE VIII.
TRANS., AM. SOC. CIV. ENGRS.
VOL. 89, PAPER No. 1580.
SHENEHON ON
THE ST. LAWRENCE WATERWAY
TO THE SEA.



Niagara Falls. After the full development of 2 800 000 e. h. p. at Niagara Falls, an assessment of 10 cents per e. h. p.-year would bring in an annual return of $280 000, which capitalized at 5% is $5 600 000. Should St. Lawrence River water power ultimately pay a similar fee for head-waters improvements, the annual return there would be over $400 000 per year, with a capitalized value of $8 000 000. As navigation interests are large beneficiaries of the head-waters improvements suggested, the charge of 10 cents per e. h. p.-year against water power appears to be sufficient. The Sanitary District of Chicago has already offered to pay over $2 000 000 toward works for lake-level restoration.

International

The international aspect of the St. Lawrence Waterway to the Sea, with the various engineering works involved, needs careful consideration. Regulating works in St. Marys River, at the head of the St. Clair and Niagara and in the St. Lawrence, bestride the boundary line and modify the natural outflow of the rivers on both sides of the boundary and the natural levels of the lakes above them. The real beginnings of regulating works in St. Marys River were on the Canadian side, and the one real compensating work which has benefited lake levels, is the Gut Dam at the Galops Rapids of the St. Lawrence River, built by the Canadians with the acquiescence of the United States.

The planning and building of works to restore and improve the levels of Lakes Superior, Michigan-Huron, Erie, and Ontario and the connecting rivers, should be carried forward under a comprehensive plan, satisfactory to all parties at interest.

It should be understood that under the treaty between Great Britain and the United States of 1871, all Canadian channels, including the Welland and Laurentian Canals and the St. Lawrence River to the Atlantic Ocean, were opened forever to vessels of the United States on the same terms as to Canadian vessels; and all American channels and canals, including the locks at Sault Ste. Marie, were opened on equal terms to Canadian vessels. American vessels have long utilized Canadian waterways in the Welland and on the St. Lawrence and Canadian vessels have long used the improved waterways of Lakes Superior, Michigan-Huron, and Erie, and the connecting rivers, St. Marys, St. Clair, and Detroit. As a matter of fact, vessel tracks are no respecters of the International Boundary line, following for the most part deep water and least distance.

Delay

Probably the reason for a delay of a quarter of a century in building works at the head of the Niagara River, is the terror in the minds of laymen lest any interference with these tremendous reservoirs may lead to disaster to property on the shores of the lakes. Assurance as to the exact limit of high lake levels, and, therefore, the absence of riparian damage, must come from the intelligent analysis of the hydrological and hydraulic data accumulated over a period of 65 years. Few hydraulic engineers have been blessed with such abundant and precise data for the determinate solution of

their problems as are contained in the records of the levels of the Great Lakes from 1860 to 1924, and in the extensive river measurements leading to the precise establishment of the laws of discharge of the several outflowing rivers concerned. Perhaps the vastness of the reservoirs and the superficial complexity of the problems have blinded even the eyes of engineers to the simplicity, certainty, and desirability of the changes suggested in this paper.

New Boundary Waters Treaty

The present Boundary Waters Treaty of 1910 between Great Britain and the United States needs to be fundamentally rewritten; and when a new treaty is made, the counsel of hydraulic engineers will be needed. It should provide for the construction of the various works suggested in this paper, and it should insure the most effective use of all regulating works, including those in the St. Marys River, for the dual purpose of navigation and water power. Diversions should contemplate such elasticity as will permit the best economic use of the water, with intermittent flow for peak-load conditions and augmented flow, if desirable, for power purposes during the winter season.

Such new treaty should, in general, determine and adjust all the various and interrelated matters which have to do with the entire question of the passing of ships from the western end of Lake Superior along all possible routes eastward to the Atlantic Ocean. The policy to be determined is one in which two nations are vitally interested and a new treaty must be broad and just. No real progress can be made and no substantial advantage can be secured except through negotiations, based squarely on equality of opportunity. Considerations of statistics, of expediency, of benefits, or of strategy of position must not be permitted to delay the solution of this great problem which looks to the economic welfare of the people of two countries.

Conclusion

This paper is of necessity long. It attempts to deal comprehensively with great projects for navigation and water power. It is intended to be constructive, provoking discussion to the end that a clearer comprehension of the engineering problems of the Great Lakes System may exist in the minds of engineers and of laymen, and that the best solutions may be devised. It is not intended to prescribe finalities, but rather to suggest the vital principles which should obtain in the engineering ways of securing ultimate benefits in this tremendous international resource of the St. Lawrence Waterway to the Sea.

DISCUSSION

H. DE B. PARSONS,* M. AM. SOC. C. E.—The biggest engineering project now (1924), under discussion concerns the development of the St. Lawrence River. The project contemplates the canalization of the river so that ocean-going vessels may enter the Great Lakes, and the development of hydraulic power between Montreal and Lake Ontario.

Much has been written and spoken about the St. Lawrence project, but a large proportion of it has been propaganda designed to awaken popular fancy. The canalization of the St. Lawrence and of the rivers joining the Great Lakes, coupled with hydraulic power development, is such a gigantic undertaking that the problem requires intensive study based on facts before any sound opinion can be reached. The engineering difficulty of damming an immense river, situated in a cold climate and lacking a low-water period to facilitate coffer-dam construction, is not the only problem. A greater difficulty is that of correctly conceiving future conditions of finance and trade, in order that the completed project may be of greatest National benefit without excessive cost.

The northern part of the Middle West is a "big" country, it has had a "big" growth—even phenomenal—and it is easy to comprehend why its people should be interested in a "big" undertaking. Projects of great magnitude always have to go through a long period of discussion and duplication of investigation. Probably it is right that they should be subjected to such scrutiny, as to remedy an oversight, error of judgment, or mistake in application would be most costly once the works were started. Mistakes and errors usually are attributable to a lack of clear understanding and appreciation of underlying principles. It is easy to sink capital in an enterprise which will not pay a return on the investment, whereas it is difficult to forecast future requirements in an age of progress, such as the present.

This paper covers the chain of the Great Lakes as well as the St. Lawrence River. The speaker concurs with the author regarding benefits that would accrue from a proper regulation of the lake levels. He is not able to follow the author's assumption that ocean-lake transportation cost will approximate the present cost of lake freight charges, as this conclusion has not been fully accepted by disinterested parties. He is not in accord with the author on costs of construction, as mentioned in the paper, because he believes they will be greater than stated, and also because the costs of deepening lake harbors and of building harbor protection works and dock facilities have not been given sufficient consideration. Nor can he subscribe to the conclusion that hydraulic power development along the waterway will subsidize canalization, because it is by no means certain that the Governments of Canada and of the United States, after constructing such power developments, can operate or lease them for the purpose of subsidizing navigation. In brief, he believes that the paper is an excellent introduction to his argument, namely, that

* Cons. Engr.; Prof. Emeritus, Rensselaer Polytechnic Inst., New York, N. Y.

the whole subject should be fully considered by an International Commission composed of disinterested and unbiased engineers.

The speaker always has believed that any project which benefits a large section of a country and a large proportion of its inhabitants is of advantage to all, even to those who cannot make use of the project nor receive any direct service from it. Prosperity or depression in one section is reflected in other sections, although it may not be evident at the time. While people in the great divisions of the United States have different aims, different interests, and different viewpoints, they are linked together by economic consideration in such a manner that their common welfare is of greater import than sectional benefit.

The entire project is in reality two projects linked together, namely, first, a ship canal through the St. Lawrence to Lake Ontario, utilizing such power as can be generated at the dams, remodeling the Welland Canal, and deepening interlake rivers and lake harbors by dredging and by lake-level regulation works; and, second, an hydro-electric power development between Montreal and Lake Ontario, using the ponds above the dams for navigation purposes, while the remodeling of the Welland Canal and the deepening of the interlake rivers and lake harbors would follow the use made by ship navigation.

Briefly, the questions are: Should navigation or power generation be the primary factor in the development of the river between Montreal and the outlet of Lake Ontario, and, which primary consideration would be the better for the States and the Dominion? These two primary factors, although closely related, should be kept separate. Dams best suited for canalization necessarily would not be the same in height or location, as those at which power generation was the dominating consideration.

The St. Lawrence is the natural drainage outlet of the Great Lakes. These lakes form immense storage reservoirs, so that the yearly flow of the river is remarkably even. The annual variation of the surface of Lake Ontario is only a few inches, with a maximum at long intervals of about 5½ ft. The uniform flow of the river is a condition favorable for power development, as it will permit the use of units of large capacity with a steady output. In this respect the St. Lawrence is unique, its normal flow being greater than that of any other river on which hydraulic power can be developed advantageously.

The normal flow of the St. Lawrence bears a relationship to the maximum flood and to the minimum discharges of other large rivers approximately as shown in Table 5.

This comparison is more favorable to the St. Lawrence than is apparent from the data given, because the maximum flood discharges last only a few hours whereas the flows approach the minimum for an appreciable part of each year.

This remarkably uniform natural flow can be improved by regulation works near Lake Ontario. It has been estimated that the flow can be controlled between 210 000 and 280 000 sec-ft. Such regulation would be of great benefit to both navigation and power development.

TABLE 5.—Relation of Normal Flow of the St. Lawrence River to the Maximum Flood and the Minimum Discharges of Other Large Rivers.

	Maximum flood discharge.	Minimum discharge.
Susquehanna, at McCall Ferry, Pa.	One-fourth	100 times greater
Mississippi, at St. Paul, Minn.	One-third	10 " "
Nile, at Assuan Dam, Egypt	Equal	10 " "
Connecticut, at Hartford, Conn.	Equal	160 " "
Delaware, at Lambertsville, N. J.	One and one-half	190 " "
Hudson, at Mechanicville, N. Y.	Twice	200 " "

The best available figures on flow of the St. Lawrence River, as measured near Ogdensburg, N. Y., are:

Maximum discharge	320 000	sec-ft., or	152%
Mean "	210 000	" " "	100%
Minimum "	191 000	" " "	91%
When ice-blocked	185 000	" " "	88%
High water in Lake Ontario (1870), in feet			248.95
Mean level of Lake Ontario (1860-1923), in feet			246.15
Extreme low water in Lake Ontario (1895), in feet			243.41
Discharge at mean level, in second-feet			240 000
Increase per foot rise of lake, in second-feet			21 500
Entire area of drainage basin, in square miles			298 080
Of which the water surface is, in square miles			95 160

The area of the drainage basin is divided between the United States and Canada, as follows:

In the United States:

Water	60 950 sq. miles
Entire	176 050 sq. miles

In Canada:

Water	34 210 sq. miles
Entire	122 030 sq. miles

The only tributary of any size is the Ottawa River, part of which enters Lake St. Louis, just above Montreal. Although this river has a wide range of flow, its effect on the general project is of minor importance, and would only affect hydraulic works near Montreal.

Some international discussion has taken place in regard to the 10 000 sec-ft. required by the City of Chicago, Ill., for its Drainage Canal. However, Lake Michigan is entirely within the United States and its basin has an area of 69 040 sq. miles. If it is assumed that the mean run-off is the same per square mile as for the entire basin of the St. Lawrence, namely, 0.8 sec-ft., then the Lake Michigan Basin contributes to the general flow 55 232 sec-ft., or 5½ times the quantity that Chicago is asking permission to take. The lowering by this taking of the general water surfaces of the lakes below Michigan, however, is a condition that affects Canada as well as other places in the

United States besides Chicago. The surface levels of the lakes (except Superior) appear to be lowering. This may be due to deforestation and to deepening of the channels connecting the lakes. This subject should be officially studied. If lake-level regulation works are required, they should be built at once, as the gradual shallowing of channels and harbors is a serious menace to navigation.

The Canadian Provinces of Quebec and Ontario have an area of about 563 700 sq. miles and a population of about 5 500 000. The States bordering on the Great Lakes (excluding New York and Pennsylvania, because they would benefit but little by the canalization) are Minnesota, Wisconsin, Michigan, Illinois, Indiana, and Ohio. These six States have an aggregate area of about 332 400 sq. miles, or about 11% of the total area of the United States, and a population of about 22 500 000 or about 22% of the total population. If the grain-growing States of Kansas, Missouri, Nebraska, Iowa, South Dakota, and North Dakota, are included as being benefited by the canalization, then the area mentioned would be increased to 765 860 sq. miles and the population to 32 900 000, which would be 25% and 31% of the corresponding figures for the entire United States.

As years pass, this nation's demand for foodstuffs will become greater, and less surplus will be available for export. Possibly, the Dominion will have surplus grain for export when that of the United States is required for home consumption.

Even under most favorable conditions only a part of the grain and other export business would seek an outlet *via* the St. Lawrence, and the question is "What part?". The business is largely bulk freight. Return cargoes would be largely package freight, requiring different conditions for handling.

Going from lake ports *via* the St. Lawrence rather than *via* New York the saving in distance to Liverpool is about 450 miles and to Southampton about 225 miles. There is no saving in distance to many foreign ports. Physical disadvantages—ice and fogs, dangers due to narrow channels and locks, and increased insurance—would offset the gain in distance. There would be, however, a saving in transshipment, and possibly a cleaning of ship bottoms, as barnacles and other sea-water growths would have a tendency to drop off in the fresh water of the lakes. The question then arises, "Would the traffic make it worth while for ocean-going vessels to navigate the St. Lawrence?". The distance from the Atlantic Ocean to Montreal, *via* Cabot Strait, is approximately 797 miles; and from Montreal to Lake Ontario, 182 miles. The approximate distance from Chicago to the ocean is 2 040 miles, *via* Cabot Strait on the south of Newfoundland, and from Duluth, 2 139 miles. These distances would be increased about 200 miles *via* Belle Isle Strait on the north of Newfoundland.

The speed of vessels using the canalized river probably would not exceed 6 miles per hour, while delays at the locks, and loss of time through fog would diminish the average speed below this limit for transit from the ocean to Lake Ontario. Assuming that loss of time by reduced speed and delays at locks would be compensated by increased speed while traversing the open water of

the Great Lakes, a vessel would require about 11 to 12 days for passage from the Atlantic to Chicago or Duluth.

The St. Lawrence River is closed to navigation by ice about four months each year. Practically the foreign shipping season would be about seven months, as ocean ships would leave port before there was danger of closing and would not enter until after the river had been opened.

The lake trade has developed a wonderfully efficient type of vessel. These vessels can carry about three times the tonnage on the same draft as a sea-going vessel, and their large hatchways permit rapid loading and discharging of bulk cargo like grain, ores, or coal. It is not uncommon for a lake freighter to discharge 10 000 tons of iron ore in 3 hours with four 15-ton electric buckets, each operating about once a minute. Lake vessels can load in even shorter time. This speed makes a quick "turn around", which is an economical feature for marine transportation. Ocean navigation is more expensive than lake navigation—a larger crew is required (about 60% larger), inspection regulations are more stringent, surface condensers are required in place of jet condensers, large hatchways are unsuitable for ocean service, and sea-going vessels cost per ton capacity from two to three times that for lake steamers. To-day's lake freight charges per ton-mile, therefore, are not applicable to vessels that are suitable for a trans-oceanic service. It might be possible to develop a new type of vessel—a combination, so to speak, between sea-going and lake types—but could such specially designed vessels afford to be laid up for more than one-third of each year while the waterway was ice-bound?

The St. Lawrence below Montreal has a minimum depth of about 30 ft. Between Montreal and the outlet of Lake Ontario, the river is navigable in part, and Canada has a series of canals about 14 ft. deep around natural obstructions. The Welland Canal has a depth of about 25 ft., with 30 ft. on the lock sills. The Detroit River has been excavated in rock to a normal depth of about 23 ft. The lake harbors have different depths, in general, from 16 to about 22 ft.

Before undertaking the canalization of the St. Lawrence, primarily for the purpose of navigation, it will be well to make sure that ocean-going ships will patronize it. As the power possibility is great, it would seem a pity to minimize it for a canalization project that was not founded on sound judgment. There is little doubt that the full power of the river is needed, or soon will be needed, especially in the United States. The question to be settled is: "Should not the river be developed to best advantage for power generation, and then make use of the pondages for navigation?".

The St. Lawrence water powers are not equalled by any river in North America. Its nearness to a power market is undisputed. There is little danger that private interests alone will benefit. The Government will safeguard its rights, and public service commissions will have a voice in regulating prices and in determining the manner in which current is sold. There can be no question that the cost of development by private enterprise will be less than by the Government. As interest on the cost forms one of the largest items of annual charges, it is clear that the lower cost will make cheaper rates for power.

Rated power of prime movers in the United States is growing more rapidly than the population; in other words, there is more horse-power per capita now than formerly, and there will be still more in the near future. The Federal Power Commission has compiled data as shown in Table 6.*

TABLE 6.—Data Relative to Population and Power Consumption in the United States, 1905 to 1930.

Year.	1905.	1910.	1920.	1925.	1930.
Population	85 000 000	92 000 000	109 000 000	113 000 000	119 000 000
Power consumption, total rated horse-power	20 000 000	26 000 000	42 000 000	50 000 000	59 000 000
Rated horse-power per capita	0.235	0.282	0.385	0.442	0.495
Increase in percentage	.....	20	64	89	110

The figures given in Table 6 are significant. The rated power of prime movers per capita has increased in 20 years (1905-25) about 89%, and will increase in 25 years (1905-30) about 110 per cent.

The figures in detail indicate a decreasing tendency in horse-power per capita generated by manufacturers, and a rapid increase in rated capacity for electric railways and central stations. The figures show the importance of large central stations for the generation of power, with their inherent economies, and the demand that will be made on the St. Lawrence for the utilization of its power.

The chief link of the project is the 182 miles between Montreal and Lake Ontario. As the lake is about 247 ft. above mean sea level, and as the river at Montreal varies in depth from about 16 to 23 ft., the gross available fall is about 224 ft. Of this reach of the river, 113 miles, with a fall of 93 ft., forms the International Boundary, while 69 miles, with a fall of 131 ft., is entirely within the Canadian border. In brief, 62% of the distance and 41½% of the power is divided equally between the United States and the Dominion; and 38% of the distance and 58½% of the power is owned entirely by the Dominion. Therefore, the United States, or more correctly speaking the State of New York, owns 20¾% of power, while the Dominion owns 79¼ per cent. The State has riparian rights covering 62% of the total distance; while the Dominion has rights covering 100 per cent.

On the basis of a mean flow of 210 000 sec-ft., a total fall of 224 ft. between Lake Ontario and Montreal, and an efficiency of 70% (to allow for mechanical losses and losses of head that cannot be recovered), there would be 3 741 000 potential h.p. This estimate is subject to refinement by load factor, diversity factor, transmission-line losses, conditions governing a super-power system, and loss of water due to lockages by passing ships. Regulation works would increase the available horse-power, as the lakes would be storage reservoirs.

Of the total stated, the Dominion, with a fall of 131 ft., has about 2 188 000 potential h.p. The power that lies on the International Boundary has a fall

* "Super-Power and Its Public Relations," by Herbert Hoover, Hon. M. Am. Soc. C. E., *Military Engineer*, July–August, 1924.

of 93 ft., which would give 1 553 000 potential h.p., of which the State of New York (or the United States) would own one-half.

There would have to be an agreement between the United States, the State of New York, and the Dominion of Canada, before any construction work were undertaken, either for canalization or power development. If the two National Governments equally divided the cost of construction and cost for flooded lands, the United States would have to pay for deepening its own lake ports and harbor protection works, as well as its share of blasting and dredging the interlake rivers and canals. If the canalization were made for 30-ft. draft and the full hydraulic power developed, the total cost to both Governments might exceed $1 000 000 000, without any allowance for transmission lines.

There are many vital questions to be settled, such as, what shall be the minimum depth of canal? Shall it be for vessels of 21-ft., 25-ft., or 30-ft. loaded draft? Where should the dams be located? What is the geologic structure at the proposed sites? What is to be size for locks? Should lake-level regulation works be constructed? What should be the rated capacity for generators? What load factor can be expected? What would be the cost?

The speaker believes that both Governments should unite in the appointment of an International Commission, composed of the highest grade of engineers, which should outline its recommendations in interim reports. These interim reports should be published so that constructive criticism might be received. When its final report is accepted, and an agreement between the Governments is reached, the works could be started. If a "plan and policy" for a comprehensive development of the St. Lawrence were adopted, there would be no good reason why part could not be done by the Governments and part by private interests, should such a division be deemed the most economical and the quickest procedure.

Let the investigation be full, because ordinary propaganda will not finance a great undertaking. Facts alone must guide those seeking the solution of a complicated problem.

H. C. Sadler,* Esq.—In all large undertakings, especially those of a public character, numerous and conflicting interests are bound to be involved. When, in addition, projects that affect international as well as sectional interests are introduced, the field for discussion and criticism is expanded still further. Therefore, in the case of the Great Lakes-St. Lawrence Waterway, it is only natural to expect not only criticism, but opposition, and from its very nature, opposition of a somewhat violent character.

Although it will be generally conceded that certain points are debatable, on account of the mass of facts which can be summoned to support each side of the controversy, it is necessary that, in order to receive any consideration in the argument, the facts as stated should be true. Otherwise, the whole

* Prof. of Marine Eng. and Naval Architecture, Univ. of Michigan, Ann Arbor, Mich.

fabric, although for a time appearing to be substantial, must eventually collapse, as its foundation rests on a quicksand of imaginations or mistakes.

A sage of old time once said, "Would that mine enemy would write a book", and in connection with the project under discussion, many books, pamphlets, reports, etc., have appeared which contain arguments against the proposition. In this mass of literature one finds numerous statements which, to say the least, are misleading and others that are absolutely untrue.

In this discussion the speaker proposes to deal simply with matters affecting ships and their operation, and leave the other points of controversy, which involve power developments, international and sectional questions, to those whose training and experience qualify them to pass an opinion; if this method had been followed in the past, the amount of chaff in the wheat of the literature would have been reduced considerably.

Although some of the speaker's statements may have appeared before, they will bear repetition, first, because they have been somewhat lost in the mass of the literature; and, second, because such repetition may help to brand as utterly false certain other statements that may have passed muster as the truth.

Perhaps one of the most important points at which those who are interested in the questions relating to the engineering features and those of transportation meet, is in regard to the question of depth of water in the channels and canals. This ultimately affects the draft available for ships. Rather than go into a mass of figures and technicalities as to sizes of vessels, the speaker will present the actual facts of overseas transportation, to show what the world has done and is still doing.

Every maritime country in the world has several ports that may be classified as "major" and "minor". An examination of "Lloyd's Register of British and Foreign Shipping" will show the depth of water available in every part of the world. With comparatively few exceptions this depth varies from about 30 ft. to 24 ft. for the major ports and from 24 ft. to 14 ft. for the others. All these ports are doing business, so it is somewhat hard to reconcile with the actual facts such statements as the following: "24 ft. is the minimum draft that can be operated profitably", or "there are no vessels * * * profitably operated in ocean service drawing as little as 18 ft.", or "all sizable ships engaged in foreign commerce draw in excess of 30 ft.", or, again, "the average draft of ocean-going vessels is from 30 to 35 ft."

The great mass of the world's commerce is not carried by the *Leviathans, Majestics,* etc., but by the cargo or tramp steamer, and as a matter of fact more than 70% is carried in vessels of less than 5 000 tons or, say, 7 500 tons dead weight, and on a draft of less than 25 ft. If a channel 30 ft. deep is provided, it would be available to nearly 90% of the vessels of the world. The figures can be verified by reference to "Lloyd's Register".

There has been a great deal of comment on and objection to the fact that even with a 25-ft. channel, the speed of a vessel would be considerably reduced. Others have made it appear that at least 5 to 6 ft. of water would be required below the keel of a vessel to maintain a speed of 12 miles per hour, and from

this they argue that a draft of only 19 to 20 ft. could be used on a 25-ft. channel. Although it is true that, in restricted channels and shallow water, the speed would be reduced, even to, say, 5 miles per hour, the percentage of time so lost as compared with the whole voyage is insignificant. What again is the experience of vessels operating in the open seas? It is common knowledge that through stress of weather, fogs, etc., vessels are often delayed one or two days or even longer on a given voyage, and yet vessel owners do not give up their business because of this fact. It is absurd therefore to state that a vessel requires 5 or 6 ft. below her keel to maintain full speed through channels, even if this were possible from other consideration as to the canal banks, etc.; and, moreover, actual experience with ships shows that owners will load their vessels to the maximum safe limit of draft, and take the small delay due to reduced speed. The profit due to the extra cargo thus carried would seem to be evident, even to a layman of ordinary intelligence. On the Great Lakes the vessels operate through the shallower channels, not with some feet of clearance, but more often with inches, and do so successfully.

It is absolute nonsense therefore to make a mountain out of the molehill of reduced speed over an insignificant part of a voyage, particularly when one considers that the average speed of a freight train is about 4 miles per hour, or less, when terminal and other delays are taken into account; and yet the speaker ventures to suggest that, in proposing a new railroad, these delays would not deter its construction.

Another objection has been raised to the proposed waterway, in that it would be open for only about 7½ months in the year, and, therefore, that the business would be seasonal. With the exception of the trade by regular liners to the larger ports of the world, all the general ocean trade is seasonal. A prominent ship-owner once told the speaker that operating a freight vessel itself did not require much effort, but the really successful ship-owner was the man who knew the geography of the world and where cargoes were to be had at different times of the year. One need only call attention also to the Baltic trade where somewhat similar conditions pertain.

Cargo vessels to-day are navigating the whole globe, and often go long distances, even in ballast, to pick up a cargo. To argue that a vessel would not come up into the Great Lakes because it is so far, is laughable, and to state that: "Vessels will not find it profitable to gather up partial cargoes", and "such attempted service would be entirely unprofitable and uneconomic", shows a pathetic ignorance of facts. When one considers the long ocean voyages that vessels make, without any port of call, it is inevitable that the great area of the Middle West economically adjacent to the Great Lakes, with its numerous ports, and with a tremendous producing and absorbing power, would prove an attractive and profitable field for the cheap form of transportation offered by ships. Again, the speaker would call attention to the successful operation of vessels from the eastern coast of the United States to the Pacific ports, in competition with the much shorter rail haul, to emphasize the fact that actual distance does not act as a deterrent.

Finally, also, even with the restrictions of the present Welland Canal, with its limits of about 250 ft. in length and 14 ft. in draft, Norwegian, Swedish,

Danish, Canadian, and other vessels have been operated in the past and are to-day operating through the Great Lakes. This last consideration brings up another argument, of a somewhat specious character, that has been used against the proposed waterway, namely, that ocean-going vessels could not compete with those of the Great Lakes, or that the latter could not operate on the open seas.

All the world over there are special types of vessels designed to do specific work. If changes were made in design so that they would be a compromise between two different sets of conditions, they would probably be failures in both. The lake freighters are the last word in a type of vessel especially developed to do the business of carrying bulk commodities, such as iron ore, coal, etc., in the most economical way, between the different lake ports. Their business is on the lakes, not on the ocean, and experience shows that, even with the bogie of a restricted season, they can be run profitably. To alter the design to allow them to go to sea would be suicidal, and almost as absurd as to suggest putting a passenger vessel of the Hudson River Day Line into the North Atlantic trade, when the summer excursion season was over. No, the function of the ocean-going vessel would be to carry commodities to and from the Great Lakes and the remainder of the ports of the world, including even those on the east and west coasts of the United States. She would not, and, if a foreign vessel, she could not, according to the coastwise shipping law, engage in inter-lake commerce; but lest the opponents' arguments should be taken too seriously, the speaker will quote two which are destined to appeal to the sense of humor of any one connected with ships. The first is as follows:

"Lake vessels with their flat bottoms have a capacity of 14 000 tons, but ocean-going vessels with different keels would be limited to a capacity of not more than 4 000 tons, * * * ."

The shape of the mid-ship section and the bottom is identical in both lake and ocean cargo vessels, and although some of the former may be of a little fuller form at the ends, there is absolutely no difference for the middle body. Again, in criticizing a statement in McElwee and Ritter as to the cost of a large lake vessel, an enthusiast makes bold to say* that "the present cost of such a vessel is about $400 000, or a little over one-third of the amount stated [$1 100 000]". At one time, before the World War, a vessel of 10 000 tons was built for about the sum quoted, but not since then, as any one can find out by inquiring of any lake ship-builder. Consequently, the critic's statement that the figures quoted in the book are "utterly unworthy of trust", should be politely returned to him with the suggestion that it is always the part of wisdom to make sure of one's own facts before contradicting others that are true.

A final argument which is somewhat related to the foregoing, is that ships of a peculiar type would have to be specially designed for this service. This is nonsense because any vessel within the limit of draft available could use the Welland Canal and the great open waterways of the Great Lakes, in precisely the same way as it does at present all over the world. One does not

* Twelfth Annual Rept., New York State Waterways Assoc., 1921, p. 44.

hear of specially designed vessels to navigate the Delaware River, or the River Clyde, or the Suez or Manchester Ship Canal waterways where ships take the time and risk of restricted navigation in order to reach the final port. It is nevertheless true that vessels for some particular class of work might require special designs, in order to obtain the maximum efficiency for that special service. If so, the problem presents no difficulty to the naval architect or ship-builder, and is nothing more than he is continually being called on to solve. Most of the coasting types and lake package freighters could be used for coastwise service on this route without any modification. Perhaps one might interject the thought here that, after all, large numbers of the trans-Atlantic liners are very specialized types for that particular trade and, as such, do not represent the trading vessels of the world.

The following general conclusions may be drawn, not necessarily from masses of minute computations, or timid predictions as to the future, but from a common-sense consideration of what vessels are actually doing to-day all over the world, and from past experience of developments due to increased water-transportation facilities:

1.—There is nothing to hinder existing sea-going vessels up to about 7 500 tons dead weight, or even a little higher, from utilizing the proposed St. Lawrence route to reach the great productive Middle West.

2.—Delays from locks, fogs, ice, and reduced speed in shallow waters, representing only a small percentage of the time of a voyage of an overseas vessel, are difficulties that have fallen to the lot of the ship-owner from time immemorial and form no serious obstacle. Fog particularly is rapidly losing its menace with recent developments in submarine signaling.

3.—Vessels will always go where trade is to be had, irrespective of the fact that it may take a little longer to reach a certain destination; and the Middle West, with its numerous lake ports, has already shown its productive and absorptive power, as reference to trade statistics will show.

4.—If special types of vessels for the intercoastal trade should be desired for any specific purpose, their design would present no difficulties. In this connection, it may be remarked that the actual coast line of the United States and Canada available for coastwise shipping will be increased thousands of miles by opening up the Great Lakes to the Atlantic Ocean. This alone has enormous possibilities for increased trade.

5.—It is absurd to talk about competition between special lake types and ocean-going or overseas vessels, as each performs an entirely different kind of service.

In conclusion, and referring again to past experience, it is the man with the broad vision, undimmed by the immediate existing order of things but looking to the future, who has been responsible for the progress of the world.

The United States and Canada are new countries still possessing tremendous possibilities for future development both in material and population. Cheap transportation is the essence of trade, and the speaker ventures to suggest that, in the years to come, the proposed Great Lakes-St. Lawrence Waterway, the New York State Barge and Erie Canals, and the Mississippi River,

will not only prove inadequate for the tasks imposed on them, but that even other waterways already suggested will be necessary for the economic life of the two countries; and, then as now, the two great banking centers of Montreal and New York will still continue to feel an enhanced benefit from a prosperous and busy Middle West, even if they may lose a few dollars in tolls on shipping.

F. P. WILLIAMS,* M. AM. SOC. C. E.—In this paper there is presented an extended description of the St. Lawrence navigation project, the cost of which it is proposed to pay from power developed along the Niagara and St. Lawrence Rivers.

Much convenient data regarding present conditions are furnished. The description of present conditions along the route from Duluth to the sea is of special interest. Many phases of the subject are discussed, with references extending back twenty-five or thirty years, showing the author's familiarity with the subject.

Although the author (page 469), refers to the control works on the Niagara River for the dual purpose of maintaining the best available depths, and, at the same time, of creating storage for water power in the Niagara and St. Lawrence Rivers, he does not make it clear in this or in the varied references to navigation and water power intermixed throughout the paper, that the principle of regulation shall conform to the requirements of navigation at all times, and that the development of water power shall be subordinate to navigation requirements. The accepted principle is that water power shall be subordinate to navigation.

In the following discussion the speaker desires to consider various points under their respective headings.

The Subsidizing of Navigation by Water Power.—In the paper it is stated, in numerous places, that water power should subsidize navigation. For instance (page 475):

"* * * that the construction of this waterway is inevitable, * * *, and that a demonstration of its commercial feasibility is unnecessary."

Again (page 475):

"It is of little concern whether the Rapid Sections of the St. Lawrence Waterway to the Sea has in the first decade a commerce of 5 000 000 or 20 000 000 tons."

Again (page 479):

"* * *. The undertaking is, therefore, a commercial project and the United States does not need to contribute toward its cost."

The advantages accruing to navigation by using power to subsidize it are gained at a corresponding sacrifice of the rightful advantages belonging to power. Both navigation and power are important necessities. There has been a great deal of looking forward to advantages to be obtained from large quantities of power commercially developed on the St. Lawrence River.

* Secy. Engr., New York Water Power Comm., Albany, N. Y.

No general advantage is obtained by helping navigation at the expense of power.

With a liberal supply of power ranking high as an essential of economic welfare and modern comfort, and with navigation a prime necessity, it is well to consider the benefits derived from each, and also to consider financing that will not sacrifice the benefits to be derived from power to the cause of navigation, important as navigation is. The subsidizing of navigation by water power, as advocated by the author, is opposed to this and is open to serious criticism on that account.

Comparison of Waterway Improvement with Cost of Power Development.—It is important to note the trend in plans for the improvement as shown by a comparison of the Government Deep Waterway plan for navigation, made in 1900 and revised in 1918, with the Wooten-Bowden plans submitted to the International Joint Commission in 1922. In the first plans, navigation alone was considered. The scheme was to improve the International Section of the river by a series of dams and locks, and to construct a channel so as to reduce the velocities to conform with the requirements of navigation. It was not planned to extend the dams across the river, forming power dams, and thus change conditions by flooding. The total cost of such plans, revised to 25-ft. draft, in 1918, was estimated at $33 962 772. This estimate is doubtless low on the basis of current prices.

The plans of the engineers reporting to the International Joint Commission in 1922 assume the construction of extensive dams that will raise and change the river elevation extensively, and create power possibilities of almost 2 000 000 h.p. Such plans are natural in view of the trend in the power industry. The tremendous growth in the demand for electric power and the improvements in transmission make it feasible to develop the St. Lawrence power at this time, whereas, at the time of the early studies, such development was out of the question. The cost of the International Section, according to the engineers' estimate, is $201 850 583, of which $135 047 264 is chargeable to power. Thus, it is seen that a navigation project, with navigation alone in view, may be had for about one-fifth of the present-day estimate for the combined project. Incidentally, in the latter project, two-thirds of the cost is chargeable to power.

No one desires for this river waterway improvements that cannot be adapted to the full utilization of power. Similarly, no one desires water-power improvements that do not fully conform to navigation requirements. The point worth considering is the relative amounts of money required to secure power and to secure navigation. This unique proposition as offered urges undertaking a navigation project and combining with it a power project costing from four to five times the amount of the navigation project alone and, incidentally, making power pay for the entire improvement.

New York State's Interest in Waterway.—In recent years, New York State has paid more than $160 000 000, without outside aid, to connect the St. Lawrence River and the Atlantic Ocean, for the purpose of lowering rates and for carrying American shipments through American ports. The State owns

one-half the bed of the St. Lawrence River along its border, and is the natural agency to develop and control the power on the St. Lawrence.

New York State will be required to pay a large percentage of the cost of the proposed project, in the ordinary procedure of paying her share of Government expenditures. The plan proposed by the author amounts to making New York State alone pay the entire cost of the navigation project in so far as the Federal Government's share of the enterprise is concerned. This to be brought about through increased charges for power due to the increased cost placed on power developments to cover navigation improvements. It is one thing to contemplate a great enterprise such as this, when the advantages of the main objective—the waterway—stand out as warranting the investment; it is another thing to contemplate this enterprise in so far as New York State's interests are concerned, as appraising the Barge Canal as an "escape" from the Niagara to the Hudson River, and as giving New York interests in the water power along the Niagara and St. Lawrence Rivers over to the benefiting of navigation interests, without regard to whether or not the improvements in the waterway are warranted.

Chicago Diversion.—Under the heading, "Water Power" (page 482), the author assumes that the diversion of 10 000 cu. ft. per sec. at Chicago will continue for a number of years. No reference is made to the fact that a case is pending in the United States Supreme Court between the Government and the Sanitary District of Chicago to enjoin the Sanitary District from diverting more than 4 167 cu. ft. per sec. from Lake Michigan.

Subsequent to the presentation of Mr. Shenehon's paper in October, 1924, the Supreme Court granted to the United States an injunction restraining the Sanitary District of Chicago from withdrawing more than 4 167 cu. ft. per sec. from Lake Michigan after March 5, 1925. A revocable permit was issued by the Secretary of War dated March 3, 1925, based on the recommendations of the Chief of Engineers. The significant part of the permit relating to the author's assumption of a continued diversion of 10 000 cu. ft. per sec. from Lake Michigan at Chicago, is as follows:

"* * * to divert from Lake Michigan * * * an amount not to exceed an annual average of 8 500 cubic feet per second * * *."

* * * *

"That the Sanitary District of Chicago shall carry out a program of sewage treatment by artificial processes which will provide the equivalent of the complete (100%) treatment of the sewage of a human population of at least 1 200 000 before the expiration of the permit.

"That the Sanitary District shall submit for the approval of the Chief of Engineers and the Secretary of War plans for controlling works to prevent the discharge of the Chicago River into Lake Michigan in times of heavy storms. These works shall be constructed, in accordance with the approved plans, and shall be completed and ready for operation by July 1, 1929.

"That if, within six months after the issuance of this permit, the City of Chicago does not adopt a program for metering at least 90% of its water service and provide for the execution of said program at the average rate of 10% per annum, thereafter, this permit may be revoked without notice.

"That if, in the judgment of the Chief of Engineers and the Secretary of War, sufficient progress has not been made by the end of each calendar year in the program of sewage treatment prescribed herein so as to insure full compliance with the provisions of Condition 4, this permit may be revoked without notice.

"That this permit, if not previously revoked or specifically extended, shall cease and be null and void on December 31, 1929."

The significant part of the recommendations of the Chief of Engineers relative to the assumption by the author of the continued diversion of 10 000 cu. ft. per sec. from Lake Michigan at Chicago, is as follows:

"7. * * *. (The execution of sewage treatment program which will relieve the load on the Drainage Canal by the equivalent of a population of 1 200 000 * * * will make possible a reduction in the amount of diversion to 7 250 cu. ft. per sec., or lower, by the end of 1929). This condition looks to a reduction to 4 167 cu. ft. per sec. by 1925."

* * * *

"12. * * * Condition (3) (The construction of controlling works to prevent the Chicago River from discharging into Lake Michigan in times of heavy storms * * * should be ready for operation July 1, 1929), is considered necessary to permit an ultimate reduction of the diversion to 4 167 cu. ft. per sec."

Engineers are left to their own conclusions regarding the appropriateness of such an assumption by the author, that the diversion of 10 000 cu. ft. per sec. at Chicago will continue for a number of years.

Conservation of Coal Supplies.—Under the heading, "Value of Added Water" (page 493), the author states that there is no vital concern in regard to the saving of bituminous coal, because at the present rate of consumption the visible supply in the United States will last for more than 5 000 years.

The following is quoted from a *Bulletin* dated August 1, 1922, issued by G. H. Ashley, State Geologist of Pennsylvania:

"Bituminous Coal Reserves in Pennsylvania, in Short Tons, 1922.

"Original deposit	75 259 055 000
Mined out	5 519 665 000
Recoverable	43 830 860 000"

If the mining of bituminous coal at the rate of 170 000 000 tons per year is continued, the bituminous coal in Pennsylvania should last about 250 years. For years the State of Pennsylvania has supplied about one-half the coal used in the United States. It is true that the coal originally estimated to be underground in Pennsylvania constitutes only a very small percentage of that in the whole United States, but the point is that (1) the State of Pennsylvania is depended on to-day to furnish a large quantity of coal because it exists there in thick veins; and that (2) the cost of coal is mounting. There is only one conclusion, that while estimates may show a supply of coal sufficient to last a long term of years, there should be vital concern about how long the supply of cheap coal will continue. All water power should be developed as early as it is economical to do so, for the purpose of conserving the coal supplies.

Radius of Economic Benefits from Water Power and from Navigation.—Under the heading, "Water Power and Navigation" (see page 481), the

author states that while the economic benefits of water power may be limited to 200 miles, navigation may benefit an area in a radius of 2 000 miles. It is difficult to assign specific distances, as the author has, to either range of benefits when the indirect and intricate benefits involved in manufacturing aided by navigation or by power, or by both, are considered.

The manufacture of some of the electro-chemicals, paper, etc., is only feasible when large quantities of cheap hydro-electric power are available. In such instances, instead of limiting the benefits from hydro-electric power used in such manufacture to 200 miles, it is more nearly correct to estimate that the benefits are country-wide. It is possible that the author has in mind simply the feasible distance that power may be transmitted. Plans advanced for disposing of St. Lawrence power contemplate transmitting it 300 miles to New York and elsewhere.

Summary.—Only a part of the paper is here discussed, the speaker's remarks being made to give students of the project information from which they may see what the situation is, namely, that the author is advancing a plan to nationalize the water-power interests of the State of New York along the Niagara and St. Lawrence Rivers, making the power subsidize navigation improvements, without even taking the trouble to justify such action from either the legal or economic standpoint. The speaker heartily disapproves of this plan.

The author has spoken of opposition as stimulating. It is hoped that as a result of this suggestion, a discussion of all phases of the points touched upon regarding this important subject will be had.

Thomas H. Hogg,* M. Am. Soc. C. E.—The author is to be congratulated on the excellent manner in which he has presented his description of the projected waterway to connect the Great Lakes and the sea.

The speaker will endeavor to point out certain fallacious reasoning that has led to serious erroneous conclusions on the part of the author. He assumes that water power should be charged with the cost of navigation development on the St. Lawrence River. There are two possible conditions under which a justification of this assumption might be considered:

1.—That the development of water power causes existing navigation structures and channels to become inoperative.

2.—That the benefits derived from the power developed are shared by districts and interests identical with those benefited by the navigation works.

Present conditions on the St. Lawrence River provide for navigation with a draft of 14 ft., whereas the author's proposal is for deep-draft navigation. It would be reasonable, under certain circumstances perhaps, for power developments on the river, if they destroyed existing works, to be charged with facilities equivalent to those existing, but it is quite unreasonable to expect a power development to go beyond the existing facilities and pay the whole of the enormous increased cost occasioned by the construction of greatly improved river channels, canals, and locks suitable for vessels six or eight times as large as those now accommodated by the existing canals.

* Chf. Hydr. Engr., Hydro-Elec. Power Comm. of Ontario, Toronto, Ont., Canada.

At present, power is transmitted to points about 250 miles distant from generating stations, so that potential developments are of possible value throughout a circle described by this radius. This transmission distance may, perhaps, be somewhat extended in years to come, but at present, and to a greater or less extent in the future, it gives a measure of the district benefited by a water-power development, and also a measure of the district which must pay through power rates for the development and for all the works provided for in the construction cost of such development.

On the other hand, navigation benefits in this particular instance extend far beyond this radius of 250 miles, and, in fact, reach their utmost value in a district greatly removed from the locality of the power development. The districts that would be especially benefited by the construction of deep-draft navigation works on the St. Lawrence River are the eastern section of the Prairie Provinces in Canada and the Central United States.

The conclusion to be reached, therefore, is that, according to the author's assumption, industry and other power users in the district bounded by Toronto, North Bay, Quebec, Boston, and Rochester, should alone pay all the cost of these navigation works, which would benefit primarily the district west and southwest of Detroit.

Engineers and financiers are quite conversant with the economic aspects of the construction of water-power plants and of the somewhat temporary nature of the market and employment involved therein. The relation of the water-power development to the permanent market for raw materials, production of manufactured goods, and development of permanent employment in other industries, is not so well appreciated and, in fact, by many, is not appreciated at all.

An analysis of these general aspects of the problem under consideration, aided by a special census in the United States, shows that of the whole investment in a water power and the industries built up by it, only 13.4% is found in the water-power development, and 86.6% in the applications of the power. In the matter of employees, less than 3% of these are engaged in the production of power, and more than 97% in industries utilizing that power. The wages paid are 2.3% of the total for water power, and 97.7% in application of this power.

Very great and widespread benefits are received by all people and interests in the district affected by the production of power through the application of that power to industry; but the benefit derived from the power *per se* is usually small, accounting as it does in its tolls for less than 3% of the business of the district. Thus, power development is in the same category as navigation works, and instead of being forced to pay tribute, water power might more fairly be subsidized.

The author indicates that by regulation works constructed at the outlets to Lake Huron, Lake Erie, and Lake Ontario, in addition to those existing on the St. Marys River, a dependable flow of 230 000 cu. ft. per sec. can be obtained for the St. Lawrence River on the assumption that 10 000 cu. ft. per

sec. now diverted at Chicago passes to the sea by its former and natural course. The argument appears to be—although not so stated—that, since the power capacity on the Niagara River and on the St. Lawrence River is increased by this regulation of the Upper Lakes, the people in districts to be served by these power developments should pay the whole cost of the necessary regulating works which are so vital for navigation.

A comparison is made by the author of the results to be obtained by regulation of the St. Lawrence River, according to the Government engineers' report on the St. Lawrence Waterway, and the results, he asserts, can be obtained by complete regulation of the Upper Lakes. It must be kept in mind that, in the various studies of regulation already made, different objectives are set. In some cases this objective has been the maintenance of a large flow in the Lower St. Lawrence River during September, October, and November; in others, the maintenance of proper winter operating conditions; or, again, the maintenance of a high uniform flow. In passing, it may be noted that it is quite possible to obtain a higher minimum flow by the regulation of Lake Ontario alone than the author quotes from the Commission Engineers' report, for purposes of comparison with his figure of 230 000 cu. ft. per sec. It must always be kept in mind that, although artificial regulation, even after a subtraction for the Chicago diversion, may give better results than those obtained under natural conditions, yet the results would be just so much better were there no diversion at all at Chicago. In other words, the diversion at Chicago and the regulation of the lakes are two perfectly independent matters.

There are certain other points on which exception might also be taken, for example, the author's assumption that there would be no break in cargo at Montreal after the construction of the deep waterways. It is conceivable that economic operation will involve the use of large lake freighters from the Upper Lakes to Montreal and transshipment there to sea-going freighters.

One must, of course, at this time speak with caution respecting the diversion of water at Chicago, for although this is a most important international matter, yet at the present time its domestic aspect, as far as it concerns the United States, is under consideration by the Supreme Court. It may be that the author's conclusion that for the next fifty years, that diversion will be at least 10 000 cu. ft. per sec., may not be borne out by that decision.

Although several relatively minor causes of lowered lake levels are noted by the author, nevertheless proper emphasis has not been placed on the level-lowering effect of the Chicago Drainage Canal, which removes water not only from the navigation channels, but entirely from the water-shed, thereby lowering all lake levels except that of Lake Superior and all river levels from that lake to the sea. In this connection it is not practicable to compensate the St. Lawrence River for its loss.

The author's apparent concern in regard to the failure of power development companies to expend money for ice protection at the outlet of Lake Erie is probably due to the fact that he has failed to recognize, in the present

instance, that power development engineers are unable to spend millions of dollars for accessory works that cannot be made to return a justifiable income on the money invested.

L. C. SABIN,* M. AM. SOC. C. E.—It is refreshing to have such a comprehensive paper devoted to the Great Lakes Basin and the water power possibilities of its final outlet to the sea presented by one who has given much time and thought to the collection of data concerning it, and to the subsequent study and application of these data.

The author is correct in his assumption that the justification of a deep waterway outlet from the Great Lakes to the sea must be found, from the point of view of navigation, in the length of ship channels already available and intensively used in the Great Lakes themselves which would be tapped by such a deep-water outlet. Duluth, at the head of Lake Superior, 1 000 miles by steamer from Buffalo, is only 200 miles nearer, by airplane, to the east coast of Maine than to Seattle. As the crow flies, Chicago is equi-distant from Denver, New Orleans, and Quebec.

The advisability of building the St. Lawrence waterway at this time has been the subject of much discussion. The promoters of the project doubtless have grossly overestimated, at least by inference, the amount of traffic which it would serve in the immediate future, and the resulting saving in transportation costs. The speaker has seen no well considered study of this economic question, and from the standpoint of navigation it is a study that should precede, or at least coincide with, the study of the physical facts.

If the St. Lawrence improvement were for the sole purpose of navigation, it is doubtful whether its cost could be justified, even with the advantage of its connection with lanes of commerce already established over which traffic is hauled at a cost hardly approached by any other system of transportation except on the ocean. Fortunately, however, it is not merely, and perhaps not mainly, a navigation matter. The value of the water power is such that whenever the large blocks of power developable can be sold advantageously, the returns should pay interest on the entire cost, and, in a term of years, amortize the investment as well.

While those in favor of the project may have painted too glowing a picture of the probable volume of commerce and the consequent benefits to be derived from cheap carriage, the opponents, on the other hand, have attacked the estimated cost on the ground that the improvement of the Great Lakes channels must be a necessary part of the plan, and that the cost of such improvements should be included in the estimate.

It is well known that a large part of salt-water commerce is carried on drafts of 25 ft. or less. The tentative plan for the St. Lawrence provides for this depth at present, with a possible increase to 30 ft. later should this become desirable. The same idea is applied in the new Welland Canal. It may be assumed that a navigable depth in excess of 25 ft. in the improved St. Lawrence will not be undertaken until ocean-going tonnage has so increased as

* Vice-Pres., Lake Carriers' Assoc., Cleveland, Ohio.

to make it an economic measure. Accordingly, present consideration may be confined to the justification for a navigable depth of 25 ft. on the present commerce lanes of the Great Lakes as well as on the St. Lawrence outlet.

The importance of the commerce on the Great Lakes is not generally appreciated. In volume it equals the foreign commerce of all ports on the Atlantic, Gulf, and Pacific coasts. It exceeds the freight carried on 297 rivers, canals, and navigable waterways of the United States, including 37 well-known streams, such as the Ohio and Mississippi Rivers, and their tributaries above New Orleans, the Hudson River above New York, the Delaware River above Philadelphia, etc. These figures exclude that part of the river traffic attributable to ocean ports, such as New York, New Orleans, Philadelphia, etc. More tonnage passes the Sault canals annually than the combined traffic of the Erie, Manchester, Kiel, Suez, and Panama Canals. The average eastbound cargo at the Sault canals in 1923 was nearly 7 000 tons, while the average cargo on the Panama Canal was about 5 600 tons.

Not only in volume, but particularly in length of haul, do the Great Lakes channels exceed other waterways, exclusive of ocean tracks. The ton-mileage of the Great Lakes is about sixteen times that of the thirty-seven waterways mentioned. For the year 1923, it was equal to about one-sixth the ton-mileage of all the railroads in the United States.

The author quotes from the Presidential Address of the late Alfred Noble, Past-President, Am. Soc. C. E., of 1903, in which, on the basis of the actual freight rate on the lakes of 0.89 mill and the estimated rail rate of 3 mills per ton-mile, the saving in freight charges for the single year, 1902, was estimated at $63 000 000. The question may be raised as to how this saving is distributed, and it may even be thoughtlessly suggested that the lake transportation interests are the principal beneficiaries. A moment's consideration, however, will show that it is the same kind of a benefit as that resulting from a time-saving improvement in the making of steel and the grinding of flour, and that the beneficiaries are those who buy the ultimate products, whether wash-boilers or steel bridges, brown bread or Parker House rolls.

In the twenty-one years that have elapsed since 1902, the year of Mr. Noble's comparison, that part of the Great Lakes commerce that passed to and from Lake Superior has amounted to 1 337 000 000 tons, and the freight charges to $1 000 000 000, with an average rate of 0.9 mill per ton-mile. Using the figure of 3 mills per ton-mile as comparative rail cost, the saving in freight charges amounts to $2 300 000 000. Subtracting from this the cost of all channel and harbor improvements on the entire Great Lakes system, there remains a net saving of $2 000 000 000, the interest on which at 3% is $60 000 000 per year. With such a surplus to the credit of Great Lakes channels no hesitation need be felt in advocating further expenditures whenever the needs of the growing commerce require more adequate facilities.

The author mentions the value per inch of draft as $500 000 for one year. This figure is not a guess but an estimate based on the actual number of passages of loaded vessels of the larger classes during 1923, which showed an increase in carrying capacity of more than 3 000 000 tons for an increase in

draft of 6 in. At the average freight rate of 88 cents per ton this would amount to very nearly $3 000 000 extra earning capacity, reduced only by a small additional cost for loading and unloading and a trifling longer delay in port.

About 30% of the freight traffic is carried by vessels capable of drawing from 23 to 24 ft., and a draft of 21 ft. could be used by vessels carrying 95% of the freight. With the present fleet it is estimated that a total gain of $15 000 000 per year would result were a 25-ft. channel made available.

To sum up the case for Great Lakes navigation, the savings by this system have piled up a surplus of $2 000 000 000 in the last two decades alone. The savings amount now to $100 000 000 per year. A further saving of $15 000 000 per year awaits the development of deeper channels.

Two fundamental methods present themselves by which deeper channels may be obtained. It is a physical possibility to raise all the lakes by 4 or 5 ft. With comprehensive regulation, meaning thereby control of river slopes as well as of lake levels, the deeper channels would result. If this were attempted and the St. Lawrence outflow entirely cut off, it would take about 20 months to accomplish the initial rise. If the St. Lawrence flow were reduced 20%, it would take 8 years. This plan, however, is manifestly impracticable as resulting land damages would be too great, and the value of existing port facilities would be almost entirely sacrificed.

The second method is the excavation of channels to the required depth wherever this depth does not naturally exist. The author suggests a combination of these two methods, which may be referred to as a third plan. The advisability of such a combination will hardly be disputed, although the question as to the extent to which one or the other fundamental method should be applied is one on which opinions will differ, and which merits discussion.

It will be well first to get a clear understanding of some physical facts concerning the Great Lakes—facts which are not always correctly stated and used in discussions of the hydraulics and hydrology of this great reservoir system. The fact that the water levels of the Great Lakes are now comparatively low, especially on Michigan-Huron with its thousands of miles of shore line, has attracted wide attention, and has led to much speculation regarding the cause, particularly among laymen. Having available definite gauge readings on each lake extending back to 1860, the extent of this lowering may be determined, and it appears that the average level for 30 years (1890-1919), is below the level of the preceding 30 years (1860-1889), by the following amounts for the several lakes: Superior, 1 in.; Michigan-Huron, 17 in.; Erie, 9½ in.; and Ontario, 8 in. For the last four years the supply has been unusually deficient; in fact, on some of the lakes deficient to an unprecedented degree for so long a period. Compared with the 60-year mean (1860-1919), the average level for 4 years (1920-1923), has been low by 2½ in. on Lake Superior, 15 in. on Lake Michigan-Huron, 7½ in. on Lake Erie, and nearly 9 in. on Lake Ontario.

Based on the discharge values of the St. Lawrence River given in the 1921 Report of the United States and Canadian Government engineers on the St.

Lawrence improvement for the years, 1860-1887, and the discharge values as determined by the U. S. Lake Survey for subsequent years, it appears that the supply to the water surface of all the lakes for the 30 years, 1890-1919, was 5.5 ft. less than for the 30 years, 1860-1889; or, in other words, that the average shortage in supply for the later 30 years was about 2¼ in. per year. This represents a decrease in supply of about 6 per cent. To indicate the proportion of this deficiency in supply for which Lake Superior is accountable, by comparing the two 30-year periods, the net supply for the later 30 years is found to be lower by 10.9 ft., or the equivalent of 3.6 ft. on the surface of all the lakes. Thus, two-thirds of the total shortage of 5.5 ft. is found on Lake Superior, although the area of its surface is only one-third of the total area of all the lakes. Accordingly, the intensity of shortage on Lake Superior was double that on the other lakes. With this proportion of the lack of supply in recent years attributable to Lake Superior, it is interesting to consider why the average level of Lake Superior was lowered only 1 in. and that of Lake Erie only 9½ in., while the average level of Lake Michigan-Huron was lowered 17 in.

As to Lake Superior, its level was interfered with by various changes in the outlet. The first obstruction took place about 1887 when bridge piers were built near the head of the Rapids and certain small channels were shut off. This occurred near the close of the first 30-year period of record. Other encroachments followed, modifying the equation of flow, and about 1900 the use of water in side outlets for water power began, but was partly or fully compensated by obstructions. Later, the compensating or regulating works were built, and since 1921 complete control of the outflow has been possible.

By the use of these regulating works, the level of Lake Superior has been held up, the present level being about 1 ft. higher than it would have been had the outlet remained in its original condition with no water used, for power or otherwise, in side canals. It should be noted that this result was accomplished in spite of the fact that the intensity of depleted supply was highest on Lake Superior. Had the restrained water making up this 1 ft. of storage been allowed to discharge gradually in accordance with the original conditions of the outlet, it would have raised the lower lakes a small amount, not, of course, in proportion to the areas, because the discharge of each outlet would have been increased.

A rather definite part of the depression in level of Lake Michigan-Huron in the later 30 years is due to the diversion through the Chicago Drainage Canal; but this diversion affects also Lakes Erie and Ontario. The discrepancy of about 8 in. between the average lowering of Lake Michigan-Huron and that of Lakes Erie and Ontario thus remains to be explained. This apparent discrepancy has sometimes been attributed to scour or to the excavation of navigation channels in the outlet of Lake Michigan-Huron, particularly in the Detroit River. The Board of Engineers on Deep Waterways concluded that scour had taken place at the head of St. Clair River, but comparison of surveys made by the speaker with an old map of 1857 gave no evidence of such a change.

Theoretical calculations indicate that the necessary fall in this connecting channel has not been reduced by channel improvements more than 2 or 3 in. In this connection it should not be overlooked that the improvement of a given reach of channel cannot reduce the fall in the reaches above and below. If in a channel 80 miles long, with a uniform slope and a fall of 8 ft., a reach of 5 miles is so improved as to make it practically a lake, the total fall cannot be reduced more than 6 in. It should also be considered that in dredging operations the material usually is put back in the original river channel, and that it easily may be so placed as to maintain the original slope.

Passing to practical considerations, there exists in the channel connecting Lakes Huron and Erie an enlargement called Lake St. Clair, the level of which, at least in the summer months, is controlled by the levels of the two larger lakes. The level of Lake St. Clair during ice-free months can be expressed by a formula making it a function of the levels of the upper and lower lakes. This formula holds good with residuals of 0.1 ft. or less for 40 years, except for short periods when known obstructions were temporarily in place. There is, therefore, practically no evidence of such a change in the channel as to indicate a reduction of slope between Lake Huron and Lake Erie.

The explanation of this apparent discrepancy in relative water levels of Lake Huron and Lake Erie was investigated by the speaker years ago and the results are shown in his report on the discharge of the St. Clair River made in 1902.* During the winter months the outflow from Lake Michigan-Huron is reduced by the action of ice in the St. Clair and Detroit Rivers. An ice cover usually forms over a considerable portion of this connecting channel, which reduces the discharge according to the laws of flow, but a much greater effect results from ice jams that occur at times, especially in the St. Clair River.

This ice effect was evaluated by discharge measurements taken from January to April, 1901, which showed individual determinations as low as 75 000 cu. ft. per sec., or considerably less than one-half the normal discharge. Connecting these discharge measurements with the readings of gauges established in 1899 at the head of the river resulted in an equation for winter discharge from which it appeared that for the three winters, 1901-03, there was an average reduction in flow of 50 000 cu. ft. per sec. for 4 months of each winter, equivalent to about 0.41 ft. per year on the surface of Lake Michigan-Huron. It is apparent that variations in this ice effect are sufficient to account for temporary changes in the relative levels of Lake Michigan-Huron and Lake Erie.

The gauges at the head of the St. Clair River have been maintained since 1900, but there is no reliable method of determining the winter discharge prior to that date. The effect of ice varies from year to year according to the severity of the winter, not following closely the mean temperature, but probably dependent on the number of short periods of extreme cold followed by temporary break-ups.

* Report, Chf. of Engrs., U. S. Army, 1902, p. 2825.

The speaker's explanation of this discrepancy in the levels of Lake Huron and Lake Erie is that the average ice effect in the earlier 30 years of record, which cannot now be evaluated, was greater than that of the later 30 years. This cannot be definitely proved, but some light can be thrown on it by a consideration of the indicated local supply to Lake Michigan-Huron for 60 years. The local supply to any lake is determined by adding the discharge to the storage and subtracting the inflow. The inflow to Lake Michigan-Huron from Lake Superior is known in winter nearly as well as in summer; but in determining the outflow from Lake Michigan-Huron prior to 1900 the figures for discharge for the ice period are generally too high, and accordingly give too high a local supply for the winter months.

A comparison of the net local supply to Lake Michigan-Huron for the 40 years, 1860-1899, with that for the 20 years, 1900-1919, shows the indicated net supply for the former period to average 3.036 ft. per year and for the later 20-year period, 2.816 ft., indicating a shortage in net supply of 0.22 ft. per year.

This retardation by ice, however, occurs entirely in the half year from December to May, inclusive. Dividing the yearly supply into two parts, the period from December to May being called the first half year and that from June to November the second half year, it appears that for the earlier period of 40 years the net local supply to Lake Michigan-Huron averaged 2.071 ft. in the first half and 0.965 ft. for the second half. Of the later 20-year period, 1900 to 1919, inclusive, there is a fair knowledge of the winter discharge. This shows the net local supply to average 1.712 ft. per year in the first half and 1.104 ft. per year in the second half. Therefore, while the average supply for the second half of each year during the later 20-year period actually exceeds that of the corresponding half of the year for the earlier period by 0.14 ft., the average supply for the first half of the year for the later 20 years is 0.36 ft. less than the indicated supply for the corresponding part of the year in the earlier period.

It is submitted that such a condition is not likely to have occurred and that the figures for the winter discharge of the St. Clair River prior to 1900 are probably too high by from 0.36 ft. to 0.50 ft. The result for 3 years (1900-1902) indicated a reduction of flow equivalent to 0.41 ft., as already mentioned. These results are mutually confirmative. Correcting the years prior to 1900 by 0.41 ft. each, shows the average net supply to Lake Michigan-Huron for the 30 years, 1860-1889, to be 2.74 ft. on the lake surface and for the 30 years, 1890-1919, 2.64 ft., a difference of but 0.1 ft. per year, or less than 4 per cent. Reduced to the area of all the lakes this represents a shortage of only 0.05 ft. per year, or about 1½ per cent.

Briefly then by comparing the record of 30 years (1860-1889) with that of the following 30 years up to 1919 there appears a total shortage in supply to all the lakes of 5.5 ft. in 30 years, an average reduction of 0.18 ft. per year, or 6 per cent. Of this shortage of 5.5 ft. Lake Superior's supply deficiency accounts for 3.6 ft., or 65%, Lake Michigan-Huron's deficiency for 1.5 ft., or 27%, leaving only 0.4 ft., or 8%, to be allocated to Lake Erie and Lake Ontario.

The author, using perhaps different data as to the outflow from Lake Ontario, finds a smaller deficiency in supply, only about 3%; but taking the larger figure of 6%, or 5.5 ft., in 30 years, an average of 2¼ in. per year, is this a sufficient change to warrant the conclusion that the trend is definitely downward, and to launch a discussion of changes in climate, permanently reduced precipitation, and the effect of clearing the forests? The speaker thinks not. On the other hand, he believes that the experience of the past, and particularly of the last three or four years, should not be allowed to pass without profiting by the lesson that natural water levels recede and that to whatever extent they may be artificially sustained, datum planes for improvement should be set low enough to assure continuous maintenance of the navigation depths found to be economically justified. This depth at the present time is considered to be that permitting a draft of not less than 24 ft., which the largest lake carriers are now able to utilize and for which many more would be built within the ten or fifteen years required to make the improvement.

In considering to what extent lake-level regulation may replace the improvement of channels by dredging, it is desirable to state some principles that appear axiomatic regarding the operation of reservoirs. Two of these were well stated by the late General Chittenden, in his paper* on the "Reservoir System of the Great Lakes of the St. Lawrence Basin", as follows:

"I.—The effect of a natural reservoir upon a stream passing through it is to reduce variations and irregularities of flow and to give the stream a more uniform discharge upon leaving the reservoir than it had before entering."

* * * * *

"II.—Any reduction of the normal oscillation of level in a natural reservoir will increase the variation of flow in the outlet; and any reduction of the normal variation of flow in the outlet will increase the oscillation of level in the reservoir."

The effect of a uniform lake level is primarily to retain harbor depths. An incidental effect is the stabilizing of the water level for a certain distance up stream from the mouth of the inlet from the reservoir above. The extent of this latter effect depends on the character of the outlet.

Lake Superior, already under regulation, is admirably adapted to this treatment. The rock rim restraining the lake furnishes excellent foundations, and a ¾-mile reach of rock apron below the works insures against scour from concentrated currents. The natural drop over this rock apron was 17 to 21 ft. The level of Lake Superior either under natural or control conditions is free from any effect of the level of Lake Michigan-Huron, the next reservoir below. The section at the regulating works will always remain in control as the lower river will carry any reasonable discharge without affecting the flow through the works which cannot be drowned out to any extent. On the other hand, the slope in the lower river, to accord with present datum planes, will be maintained by the use of the minimum discharge required for power.

With the proposed improvement of the St. Lawrence similar advantages for regulation will be found. The water levels in the river are seriously affected by ice, but under the suggested arrangements the control works cannot

* *Transactions*, Am. Soc. C. E., Vol. XL (1898), pp. 359 and 365.

be drowned out. When the St. Lawrence is improved for deep navigation these control works will form a part of the plan and the level of Lake Ontario can be regulated. Until the river is so improved, deeper channels below Buffalo are not important.

Turning to Lake Erie, the control of this lake has been the subject of a number of studies, the conclusion usually being favorable with certain limitations. One objection is that if control means a higher average level the occasional storms on Lake Erie would be somewhat more destructive, especially to Buffalo. The principal objection heretofore is that regulation of Lake Erie, without coincident regulation of Lake Ontario, would increase the fluctuations in level of the latter lake, and as concerns the two border countries it happens that the one more interested in the maintenance of levels on one lake is less interested in the maintenance of levels on the other. With the improvement of the St. Lawrence this diversity of interest and its effect would cease.

In the Niagara River the conditions for control are only less favorable than in Lakes Superior and Ontario. Here, again, the ice problem is important, but there need be little fear of the effect of the works being drowned out by the action of the outlet channel below, and if a temporary low controlled discharge depresses the water in the lower river, the effect will not be serious as the navigation there is not important. Accordingly, even if the improvement of the St. Lawrence should be indefinitely postponed, it is thought that the level of Lake Erie should be controlled, but without a radical increase in its mean level, say, within limits of 1½ ft., or at most, 2½ ft.

As regards the outlet of the largest single reservoir in the system, the St. Clair and Detroit Rivers draining Lake Michigan-Huron, this connecting channel is about 80 miles in length with an average fall of only a little more than 0.1 ft. per mile. The bottom in general is gravel, sand, and clay, except for a rock ledge overlaid with boulders near the lower end. In general, the river is about ½ mile wide and, except at and below the rock ledge mentioned, good navigation depths exist naturally; but where the width is increased to ¾ mile, shoals appear. The regimen, now established by the existing discharge with its relatively small variations, is remarkably stable. With a considerably increased flow capacity the new conditions might be less permanent.

In this lake outlet there is a natural controlling works, or perhaps, more properly, compensating works, set up at times by Nature in the form of retardation of flow by ice. As regards levels, the effect of this natural regulation has in general been good. The trouble is that Nature sometimes neglects to apply the remedy. This natural compensation, if made certain, would be ideal in some respects. It does not seriously affect water levels in the outlet. Whatever effect it has is exercised when the only navigation is in light-draft ferryboats. Thus far, it has not seriously affected any power development. It has to some extent depleted the flow over Niagara Falls and thus affected the grandeur of that spectacle, but Lake Erie fills so rapidly in the spring that by June the flow is restored, and no complaints are known to have resulted from this kind of regulation.

In this paper the mass curve is ingeniously applied to the Great Lakes System to show the maximum value of the continuous yield. By taking advan-

tage of whatever diversification in water-sheds may exist and by the use of the mass curve for mean annual yields, the author is able to show a possible increase of 10 000 to 20 000 cu. ft. per sec. of prime water for power.

It is quite proper that a paper on the St. Lawrence development should be written from a water-power point of view; but to secure this additional volume in actual operation is it not necessary to have the omniscience which is assumed in the method adopted in the paper? Knowing what is to take place in the next year, or the next five years, a decision as to the flow to assume for the earlier period is fairly simple. If any rule has been deduced whereby the several works can be operated without this omniscient mind, it does not appear, and the speaker's experience with similar but simpler problems leads him to doubt the equally smooth working of the theory under conditions of stated limits but of uncertain sequence.

Each one of these lake reservoirs has a considerable fluctuation in local supply. There frequently occur monthly maximum supplies of 1 ft. on Lake Ontario, 0.8 ft. on Lake Erie, 0.6 ft. on Lake Michigan-Huron, and 0.5 ft. on Lake Superior. On the other hand, negative supplies, meaning greater evaporation than local supply from the basin, occur almost every year, the values of these negative supplies being not infrequently 0.2 ft. or 0.3 ft. per month on Lakes Ontario and Erie, and 0.1 ft. on Lakes Michigan-Huron and Superior. With variations in local supply amounting to 1 ft. per month, the difficulty of maintaining levels within a total range of 1½ ft. may be seen.

No attempt is made in the paper to follow through the details of regulation, the author contenting himself with a broad presentation of the subject. It is of interest, however, to test the possibilities of regulation under the limits laid down. For the condition obtaining in 1910 and 1911, an attempt to restrain the lakes within these limits by the methods suggested by the author would appear to have required changes from the natural flow volumes of the outlets of from 50 000 to 75 000 cu. ft. per sec. in certain months, and still serious oscillation in the level of Lake Michigan-Huron could not have been prevented, but this level would have had a range of 1.75 ft., while in these two years the level would have fallen nearly 1 ft. below the proposed lower limit. This could not have been corrected by increased discharge from Lake Superior, because for these two years the natural rate of discharge from Lake Superior was reduced by regulation 15 000 cu. ft. per sec. below normal, and yet the lake fell 0.3 ft.

In 1924, the level of Lake Michigan-Huron fell 6 in. with an average discharge of only 152 400 cu. ft. per sec. During the four years, 1920-24, the levels of Lakes Superior and Michigan-Huron were depressed about 1.5 ft., with an average discharge of the St. Clair River of approximately 167 000 cu. ft. per sec. Since no excess supply could have been drawn from Lake Superior, this average discharge of the St. Clair River would seem to be as great as would be available for long periods under similar conditions with complete regulation in effect.

It is fully appreciated that the author is merely sketching works and methods, not specifying details, but the speaker is only attempting to point

out the possibility of disappointment in the working out of details when this becomes necessary.

With Lake Michigan-Huron under complete regulation, throttling the discharge of the St. Clair River at or near its head will at times result in quite serious depression of level below the works. An indication of the extent of this depression appeared in the plan originally proposed by the author which included locks with lifts of from 1 to 5 ft. As finally submitted, however, (Fig. 5), the locks have been replaced by navigable passes combined with expansion basins. With Lake Erie at a regulated stage of 573.0 ft., a fall of only about 6.5 ft. would be required to carry a reduced discharge of 160 000 cu. ft. per sec. through the Lake Michigan-Huron outlet, giving a level of 579.5 ft. at the head of the St. Clair River, or about the present low-water datum. Since it is mainly the connecting channels that are shallow the maintenance of levels in the rivers is more effective in reducing dredging costs than the preservation of the levels of the lakes. Under natural conditions high lake levels result in corresponding channel depths, but, as indicated, this condition easily may be disturbed by the character of regulation suggested for such a channel as the St. Clair River.

The proposed plan contemplates the enlargement of channels to take care of excess flow, the latter being required in order to assure the large prime volume of 230 000 cu. ft. per sec. It should be pointed out that in the case of the Lake Michigan-Huron outlet, 80 miles long, it may prove as much of a task to provide this additional flow capacity by dredging, as to provide by the same method the desired depths for navigation throughout the shorter reaches requiring improvement for this prime purpose.

In this connection, the suggestion that the Chicago Drainage Canal contributes an effective share of this needed additional capacity is not tenable unless it be assumed that the flow through that Canal is subject to regulation. To make this clear it is only necessary to assume the flow in the Chicago Drainage Canal to be, say, 175 000 cu. ft. per sec. There would then be, at times, no flow remaining in the St. Clair River to be regulated. Uncontrolled diversions are serviceable in regulation only when the flow is returned to the main outlet or to a lower reservoir of the system, a condition not applying to the Chicago Drainage Canal.

The value for power of the additional prime water which it is proposed to make available by an elaborate system of regulation, would be more impressive if every means were being taken to permit the use of more power at Niagara while preserving the scenic beauty of the Falls. It would also be more impressive if it was certain that all the water power developable on the St. Lawrence could find a ready market, and if, in the plans proposed, consideration were not being given to the entire omission of power works in an important part of that river.

In computations of power benefits the author seems to use a figure of about $7 per h.p. as the annual value. Assuming a final development of 7 000 000 h.p. on the Niagara and St. Lawrence Rivers combined, would give a total annual value of $50 000 000, or less than one-half the benefits now

resulting from the Great Lakes navigation. The time may come when the power value of this additional water can be fully realized, but this time has not yet arrived. When it does come, the value will still be below that resulting from navigational uses. Certainly to-day and in the immediate future the needs of navigation should not be sacrificed in the name of power. To require vessels to pass through locks at the foot of Lake Huron or to subject them to excessive currents would unquestionably be detrimental to the interests of navigation.

Partial regulation of Lakes Michigan-Huron through works located below the head of the river would seem to fit the present need of control at this point. These works should be located in a wide section of the river, perhaps in the vicinity of Stag Island. Their character should be such that a part of the flow could be restrained during the winter months when this restraint should be needed, while not ordinarily reducing the summer flow nor forming any hindrance to navigation. Works of this character would be operated to restore gradually the mean level of Lake Michigan-Huron, and could form a part of more extended works later should it develop that greater control were desirable and that it could be obtained without introducing other evils.

The author expresses the hope that the subject will provoke discussion. As the paper is general, the discussion must be of a similar character. Having in view particularly the needs of navigation, the speaker feels that its requirements have been subordinated to those of power. That an increase in prime water is given more weight than better continuous navigation depths. The speaker has attempted to bring out the enormous benefits derived now and in the past from navigation on the Great Lakes, indicating a surplus of $2 000 000 000 now standing to its credit, which is being increased at the rate of $100 000 000 per year, and to show that this navigation should be supplied with the 25-ft. depths it now requires for economical operation.

It has been shown that, whereas the present supply to the lakes is low, there is little to indicate that this condition is permanent; nevertheless, that steps should be taken to prevent the recurrence of extreme low water, but that these steps should be such as not to result in destructive levels during periods of high supply. It has been suggested that the control system proposed might be found disappointing in its final analysis and more expensive than present benefit to power would justify.

Finally, the opinion is expressed that the regulation of Lake Erie should be exhaustively studied, and, that if no insurmountable difficulties appear, it should be put in operation; that minor works should be placed at once in the Lake Michigan-Huron outlet, and that a program for depth betterment to the extent of 25-ft. channels on the Great Lakes should be immediately adopted and entered upon. Such a program would require fifteen years for its consummation. By that time control would have had its preliminary trial, power needs would have increased and presumably would have been supplied either at Niagara or on the St. Lawrence, or both, and during this process, by proper allocation of work on the navigation project, navigation benefits would have been progressively obtained.

GARDNER S. WILLIAMS,* M. AM. SOC. C. E.—The speaker is in harmony with the general expressions which have been made in the discussion of the Great Lakes-St. Lawrence Waterway. Referring to the remarks of Mr. Sabin, however, he would call attention to certain historical facts. The early explorers coming up the channels connecting the Great Lakes reported that those channels varied in depth from 8 to 12 ft. Mr. Sabin has stated that there is now a 21-ft. channel through the same reaches. Students of hydraulics can readily decide whether the deepening of channels that were between 8 and 12 ft. to 21 ft. in depth has or has not enabled more water to flow out of the upper into the lower lakes.

Attention is directed to the program which was recommended in the report on the St. Lawrence Waterway submitted to the International Joint Commission regarding the development of that section of the river below the International Boundary. As stated in the paper, that program contemplates bypassing the traffic through 28 miles of canal and 5 locks, counting the twin locks as one. It contemplates no power development within that reach, although there are about 2 000 000 potential h.p., and is estimated to cost $92 000 000.

It has been intimated that the reason the Commission adopted its method of treatment of this section of the river was the belief that there was not a market available for that amount of power in connection with what would come from the upper reach. In that same report, however, the Commission estimated the horse-power demanded within 300 miles of the Long Sault to be 13 350 000 e. h. p. in 1925, for the United States and Canada, and, in 1930, 15 375 000 e. h. p. is estimated, or an increase in 5 years of 3 025 000 e. h. p.†

The contemplated works will not be built in five years. If the estimated rate of increase of power demand continues, according to the Commission's own figures, that power would all be absorbed as soon as it could be put upon the wires. Referring now to the super-power report, which was an investigation of the demands for power in the region of the United States from Washington, D. C., on the south to Boston, Mass., on the north, the central part of this zone is within reach of the St. Lawrence power.‡ That report states§ that the energy economically to be taken from the super-power system in New England and New York and Upper New Jersey—all of which territory is within 300 miles of the generating points of the possible power developments of the Great Lakes-St. Lawrence Waterway—in 1930 from new sources will be 2 232 000 e. h. p.; that is, it is estimated that in 1930 there will be needed from a super-power system in this district new power to the extent of 2 232 000 e. h. p. It was further estimated by the Super-Power Commission, that the increase in steam-generated power in the area noted, from 1919 to 1930, would be 2 059 000 e. h. p.‖ That is about all the St. Lawrence River would yield if distributed over the wire, into that territory. It appears, therefore, that

* Cons. Engr., Ann Arbor, Mich.

† Senate Doc. No. 114, 67th Cong., 2d Sess., p. 175.

‡ "A Superpower System for the Region between Boston and Washington," by W. H. Murray and others, *Professional Paper No. 123*, U. S. Geological Survey, 1921.

§ *Loc. cit.*, p. 181.

‖ *Loc. cit.*, pp. 160–161.

the idea that there is no use for the power—that it is not needed, and could not be disposed of—is a mistake.

The development of this water power will be a source of revenue to Canada, and a source of revenue, through sub-station service, to the people of the United States.

There is, however, another element involved. The people in the Central States are not very much interested primarily in the development of water power to be used in New England and Northern New York; but a certain amount of power is to be absorbed in that territory, and whether it comes from the St. Lawrence and Niagara Rivers, or from the burning of coal, will make no difference. The power will be utilized. Now, if that power does not come from the St. Lawrence River, or from the Niagara River, and some minor water-power sources that are hardly worthy of consideration in connection with them, it will have to come from coal. The amount of fuel consumed by public service corporations in New England, New York, and the northeastern part of Pennsylvania, for 1919 was more than 11 000 000 tons of coal; or, more accurately, it was equivalent to more than 11 000 000 tons of coal. Some of this fuel was oil and some of it was gas; but the bulk of it was coal. The coal used by public service corporations in the States of Illinois, Indiana, Ohio, Michigan, Minnesota, and Wisconsin for the same period was about 10 700 000 tons.* In other words, if sufficient St. Lawrence power went over into New England, New York, and Northeastern Pennsylvania to replace the coal that is now burned by the public utilities in that district, there would be sufficient coal released to supply the public utilities in the States of Illinois, Indiana, Ohio, Minnesota, Wisconsin, and Michigan, and that is why the people of the Central States are interested in the utilization of St. Lawrence power in New England and New York.

Simply as a matter of comparison, the total output of fuel for the United States is the equivalent of about 600 000 000 tons of coal per year; and the railroads use 180 000 000 tons. The Super-Power Commission previously referred to, in investigating the cost of the operation of railroads by means of central stations and electrical locomotives, showed that nearly 75% of the fuel burned by the railroads could be saved if they were operated from central generating steam stations.† If the railroads in this district are supplied with electricity generated by water power, there will result a further saving, which becomes of interest.

Turning now to another phase of the subject, it has been stated that the estimates for this project were probably inadequate. The speaker most fully concurs in that opinion. He has taken some pains to compare the costs of similar works where such have been built under the direction of the Corps of Engineers of the United States Army as far as similar work can be found, and from the records, his estimate, or his guess, is that the cost of this development, including the water power features, will be more nearly $600 000 000 than $500 000 000, which latter sum is about twice as much as the last report indicated would be involved.

* U. S. Geological Survey, Monthly Reports of Fuel Consumption.

† Super-Power Report, *Professional Paper No. 123*, U. S. Geological Survey, p. 62.

Opinions have been expressed as to whether water power should build this waterway, or whether water power should be subsidized, or whether water power should be considered at all, or whether navigation should be considered at all, and perhaps the speaker may also be permitted to express an opinion, that is, the water power of the St. Lawrence River is needed now, and can be absorbed as soon as it can be put upon the wires. The water power development of the St. Lawrence River will provide almost everything that navigation needs except the locks and the deepening of the channels at the upper ends of the pools. Therefore, let the water power interests build the dams and develop the water power as fast as they will, under the most liberal terms that the Governments can grant them, and let the Governments provide the cost of constructing that part of the work which is necessary for navigation. If the problem is developed in that way, the burden will not be so great but that Canada, even with the enormous load that she is carrying, can take a reasonable share in this project. The people of the United States can hardly realize what conditions must be across the border, but if they examine the expenditures of Canada during the World War, and the vast projects in public works which Canada has been carrying on, they cannot blame her for hesitating to bind herself at this time to the expenditure involved in the development of the St. Lawrence Waterway. If that cost can be put where those who will profit the most by it can bear it, however, the cost will be reduced, as far as the Governments are concerned, to a figure that seems to be entirely within their reach.

Next to the Great Lakes-St. Lawrence Waterway, there is no subject before the people of the United States to-day about which a greater amount of misinformation has been disseminated than the so-called Chicago Drainage Canal, and although the Canadian Government—and this is said not in any spirit of antagonism or of criticism—is demanding, or at least suggesting, that the Chicago diversion be prohibited or restricted, attention should be called to the fact that in a treaty which now exists between the United States and Great Britain, covering the use of the boundary waters, there is a provision that the Dominion of Canada may have 16 000 cu. ft. per sec. more water to use at Niagara than the United States. If one will go back to the records, and those records are in Canadian publications, it will be found that such allowance was made to compensate Canada for a diversion supposed to be 10 000 cu. ft. per sec. at Chicago.

Attention may also be called to the fact that, whatever the effect of a diversion of 10 000 cu. ft. per sec. on the navigable depths of the Great Lakes may be, the proposed diversion of approximately 15 000 cu. ft. per sec. through the Chippawa Canal, around Niagara Falls, if uncompensated, will lower Lake Erie by approximately 50% more than the diversion at Chicago; and that the lowering of Lake Erie will be accompanied by a lowering of Lake St. Clair and the Detroit River, of approximately the same amount as that due to the Chicago diversion, and by a lowering of Lakes Michigan and Huron by approximately one-half the latter amount. Attention is further called to the fact that Canada is diverting about twice as much water through the Welland

Canal at present as was so diverted prior to the World War. When Canada contends that the diversion at Chicago is lowering the Great Lakes, and wants it stopped, it may be proper to suggest that consideration be given to the effect of its own diversions on the levels of Lake Erie and the Upper Lakes.

The speaker does not oppose diversions of water through the Canadian developments for water power below Niagara. He considers it more economical to take that water through the Queenston-Chippawa Canal to utilize the large head that is possible by obtaining it near Lake Ontario, than to take it at the Falls and derive only one-half the power. He is in favor of all that, but let there be fairness to all sides.

E. P. Goodrich,* M. Am. Soc. C. E.—It is obvious that Government expenses for deeper waterways and for harbor improvements are subsidies of one kind or another. The speaker would suggest another kind of subsidy for consideration at least by the present International Joint Commission, and possibly by the board or commission which Mr. Parsons has suggested might be appointed as an auxiliary investigating body. This alternative is a railroad subsidy to cover what might be called full train operation through from origin on one of the Great Lakes ports to some seacoast port, on an export tariff basis. It might be handled, for example, by a Federal commission, like the Wheat Commission, which would provide funds for by-passing the congested railroad throats on the railroad systems, as they exist to-day, so that the operation of a through train could be expedited and thereby decrease the time required of freight cars, the extra return accruing to the railroads, which, in their turn, could distribute this saving, at least in part, to the shippers through a lowered export rate.

Certain investigations made in connection with the project for deepening the Upper Hudson to provide a Troy-Albany Port gave statistics which may be of interest in this connection. The present probable total export Gulf and Pacific commerce, to and from the district that was tributary to the St. Lawrence, as well as to the Upper Hudson port, was carefully estimated on several bases. A figure of 15 000 000 tons per annum, as a total of all types, was finally reached. Obviously, a ship will not come inland, making a special trip, unless it can have a large proportion of its cargo assured. In other words, there must be facilities at any lake port for collecting tonnage of a considerable amount destined for one or perhaps a group of foreign ports, so that the vessel can have sufficient inducement to make the trip. Studies were made to determine as to what commodities and in what quantities there was sufficient inbound and outbound tonnage to induce a vessel to make such a trip, at least as far as the Upper Hudson port. It was calculated that a tonnage not to exceed 5 000 000 would be thus available, and that there might be diverted additional tonnage not to exceed 1 500 000, with a possible increase to 5 000 000 in the course of the next decade, estimating on the rate of growth which has taken place in the Panama Canal, and other ship canals, and harbors in various parts of the world.

* Cons. Engr., New York, N. Y.

It may be of interest to discuss the eventual tonnage which may be derived in that way. Working on the basis of population estimates of the United States as a whole, and the proportion which it is expected will find itself in the tributary area of the hinterland of the St. Lawrence, say, 100 years hence, and basing these estimates on studies by Professors R. Pearl* and E. W. East,† and by the speaker,‡ it is probable that three times as much tonnage will be found available in the next 100 years. Assuming 6 000 000 tons per year, and 40 tons per car, this gives 150 000 cars per year; and assuming a 1 200-mile haul, at 4 miles per hour, 300 hours will be required. If by means of detours around traffic throats and the same methods of operation used to expedite freight as are common in expediting through traffic across the continent, for example, with reference to passenger trains, this time were to be cut in half so that an average speed of 8 miles per hour were secured, a saving of 150 hours, or 22 500 000 car-days would be made. At $1 per day, which is the usual rate, there is obviously an annual saving of $22 500 000, simply in the use of cars. Capitalizing this, and assuming that $5 000 000 is required for each throat to provide facilities to expedite travel, there still remains a saving of $162 000 000, capitalized, or $8 100 000 per year. On the basis of the 6 000 000 tons assumed, there is an immediate saving indicated of $1.35 per ton, which might be returned directly to the farmers in their shipment of grain, or to other shippers and receivers of freight, provided the plan suggested could be put into effect.

The speaker does not wish to be misunderstood as making this suggestion with reference to railroad operation. He has long been interested in the development of water terminals and waterways and believes that, eventually, the St. Lawrence project may be economically feasible, but it is obviously unwise for the two countries as a whole to expend moneys for waterways if the capital already invested in railroads might be used possibly to better advantage.

The paper is of considerable interest because it points out some of the things which have not heretofore been discussed, and the author is to be complimented for providing the opportunity to discuss these various details.

David B. Rushmore,§ M. Am. Soc. C. E.—Although forecasting economic and social conditions is only an approximation, it is certain that by the time hydro-electric development in connection with the St. Lawrence Ship Canal can possibly be completed, all the electric power produced can easily be consumed, if less than one-half of it in Canada, surely much more than one-half on the American side. There is no engineering difficulty of great magnitude in connection with the design of the power-house, of transmission systems, or of the distribution and use of this power. What is lacking is an understanding and an appreciation on the part of the average citizen of New England, New York, New Jersey, and Pennsylvania—the situation dis-

* "On the Rate of Growth of the Population of the United States Since 1790 and Its Mathematical Representation", by R. Pearl and L. J. Reed, *Proceedings*, National Academy of Sciences, Vol. VI, No. 6, June, 1920, p. 275; also, "A Further Note on the Mathematical Theory of Population Growth", by R. Pearl and L. J. Reed, *Proceedings*, National Academy of Sciences, Vol. VIII, pp. 365–368.

† "Mankind at the Crossroads", by E. W. East.

‡ In a paper presented before the American Statistical Assoc., October 30, 1925.

§ New York, N. Y.

cussed so well by Mr. Gardner S. Williams—of the benefits to be received by this power.

There is no man who, if he saw a coal mine on fire, would not endeavor to put it out, or to stimulate the efforts of those responsible for the fire to extinguish it. Wherever water power remains undeveloped, within reach of markets that are burning fuel, as at Niagara Falls and on the St. Lawrence, there is a coal mine on fire. At the World Power Conference, held in London, England, in July, 1924, one of the most striking features brought out was the many ways in which the people, through the Governments, are co-operating with the people as organized through private corporations. The great expenses necessary for the St. Lawrence investigation were contributed by people interested, who felt that it would be desirable to consider all its possibilities and that some satisfactory way should be evolved whereby the Government and private parties could co-operate to a satisfactory outcome.

Consider the demand for greater publicity; incidentally, that is one thing that can be done by the members of the Society. The great benefit from presenting such an interesting paper is that the public can be educated regarding the engineering facts and economic benefits to be received. Then there is a demand for a new kind of engineering—a presentation of engineering facts with as accurate a forecast of the economic and social conditions as possible, and a design of the project that is going to bring together and harmonize all the elements involved and that will permit in a reasonably short time of carrying this great project through to completion.

Maurice W. Williams,* M. Am. Soc. C. E.—Reference has been made to the diversion of water from Lake Erie through the Welland Canal as being greater in quantity than certain other amounts which are diverted into avenues lying wholly within the United States, such as the Chicago Drainage Canal. The Welland Canal simply happens to lie in Canadian territory, but is open to the free use of the vessels of both countries. That being the case, certainly Canada cannot properly be charged with the total quantity of water used in the Welland Canal. It may very properly object to the diversion of water for the use of any municipality in the United States, however desirable such diversion may appear to that municipality, the more so as all water passing through the Welland Canal is retained within the St. Lawrence drainage area while all water diverted through the Chicago Drainage Canal is permanently lost to the St. Lawrence.

L. H. Hart,† Esq.—The speaker heartily agrees with the great majority of those who are discussing the St. Lawrence Waterway project, that it is inevitable. He wishes, however, to summarize briefly the reasons why the money should not be spent during the lifetime of this generation. All these reasons fall under the general head of putting first things first.

On the navigation side of the question, the control works mentioned by Mr. Shenehon needed to raise and stabilize the lake levels, and the deepening of harbors and channels, are essential to the general plan. Let them be con-

* Cons. Engr., New York, N. Y.
† Buffalo, N. Y.

structed first, as they are within the United States and will be worth many times their cost, whether or not the St. Lawrence River in Canada is developed. Let full use also be made of the existing 14-ft. canals in the St. Lawrence, the Ohio, and the Mississippi Rivers, and of the 12-ft. canal from Buffalo to New York. New York State has contributed this 12-ft. channel to the National welfare free, at a cost of $200 000 000. Many residents of the State feel that before they are asked to pay a heavy tax for new plans, the existing one should be given a trial. Some feel that the cities of Toronto, Quebec, and Montreal, would be benefited by the St. Lawrence plan, at the expense of American taxpayers, and that it would withdraw commerce from cities in New York State. The speaker does not hold this view.

At present, the bulk of traffic leaving the Great Lakes goes, not to Europe, but to the New York District. The St. Lawrence Channel and the New York State Barge Canal are now being used to only a very small fraction of their capacity. The tonnage which goes from the Great Lakes Districts to Europe could easily be carried on the New York State Barge Canal, even if it is only 12 ft. deep. In short, the time would not seem to be ripe for spending American money for improvements in any foreign country. It should be noted that six of the nine locks which are proposed on the St. Lawrence are entirely within Canadian territory.

Relative to the power side of the question: Before developing power in Canada, let full use be made of Niagara Falls. Only one-fourth of the possible power output is being developed there. The obstacle is that the scenic effect might be marred. The proposed control works, however, could easily regulate and distribute the flow, so that no beauty will be lost. At present, $100 worth of water is wasted for every visitor who enjoys Niagara. The Lower Niagara River in the Gorge is merely awaiting capital for its development. The possibilities there are very great, in comparison with those on the St. Lawrence River. There are also great possibilities in the super-power project and in the proposition to burn coal at the mouth of the mine. These are strictly American, and of known value.

The wisdom of developing power before the market is in plain sight is very questionable, even for a wealthy Government Treasury. The Muscle Shoals Project, almost completed at a cost of $100 000 000, goes begging for lack of a bid of more than $5 000 000. If, as Mr. F. P. Williams claims, the power industries can subsidize navigation, American and Canadian power interests will be willing to do it on their own account.

A. Lindblad,* Esq.—The speaker is not a civil engineer and thus is not handicapped by any knowledge of the engineering difficulties in developing this inland waterway for navigation purposes. He will discuss this project, therefore, only from the standpoint of its shipping possibilities.

Several statements have been made regarding the kind of ships that can be used. In his discussion, Professor Sadler made this point clear, when he stated that he would never expect large ocean-going vessels of the passenger type to come to the Great Lakes. Such vessels are not wanted there;

* Prof., Dept. of Naval Architecture, Univ. of Michigan, Ann Arbor, Mich.

but the Great Lakes should be connected with salt water, so that all those vessels which do not have too large a draft can come to the lakes. There is no need of constructing new vessels. If a depth of 25 ft. is obtained in the channels, that is sufficient to permit approximately 70% of the world's tonnage to enter; and if that depth is increased to 28 ft., practically all ordinary cargo vessels will have access to the Great Lakes.

It has been stated that the cost of ocean-going tonnage is at present two or three times the cost of tonnage on the Great Lakes. That is really not correct. That it is about one and one-half times that cost would probably be a safer statement. Mr. Parsons also stated that the cost of running ocean-going vessels on the Great Lakes would be too great, partly because the vessels were more expensive in first cost, owing to the need of a surface condenser, etc., and partly because they need a larger crew than the lake freighters. It is true that they need surface condensers, but fitting a surface condenser instead of a jet condenser is really a rather small item compared with the cost of other parts of the machinery, and does not add much to the first cost. It is also true that there is a small increase in the numbers of the crew on ocean-going vessels as compared with lake boats. In other words, the cost of running ocean-going vessels is somewhat greater than the cost of running the present lake freighters, but that does not justify the statement that it would be uneconomical to use ocean-going vessels in combined lake-and-ocean traffic.

To obtain the full benefit of a deeper channel it is, of course, necessary also to have increased depth in the Great Lakes ports; but the cost of deepening these ports is not so enormously large as some people would imply. It must be remembered that the shipping on the Great Lakes is not evenly divided among the lakes, but is concentrated and handled at only a few ports. To deepen these ports to about, say, 25 ft., will not, in the speaker's opinion, be such an enormously costly undertaking.

In opposition to the St. Lawrence Waterway project, it has also been stated that it was not practical because it could provide for navigation during only for seven to eight months of the year. Everybody connected with shipping knows that a very large part of the world's commerce is seasonal, and the speaker believes that certainly the largest part of manufactured products could be handled easily in a navigation season of seven months. Consider a parallel from Europe. The whole region around the Baltic, including Sweden, Finland, Russia, Germany, and the new countries like Poland and Esthonia, have to be satisfied with approximately eight or nine months of navigation. Nevertheless, all these countries have been able to develop quite a foreign trade. The speaker thinks that it is an absolute mistake to say that shippers necessarily must depend on twelve months of navigation.

It has also been said, that ocean-going vessels coming to the Great Lakes would not be able to use the present terminal facilities. The grain elevators in particular are said to be suited only for lake ships and would need to be changed. That is another incorrect statement. It is true that ocean-going vessels are not constructed, like the lake freighters, with a very large number of short hatches spaced 12 ft., center to center, which hatches fit the 12-ft.

spacing of the loading spouts of the grain elevators. It is not necessary to be able to load through all the loading spouts. Only those spouts which fit the hatch arrangement of an ocean-going ship need be lowered and used. It is hard for the speaker to believe that any greater changes are needed than to increase the draft at the elevators. The other changes are unimportant.

In discussions of the St. Lawrence Waterway, the speaker has heard a great many statements to the effect that salt-water vessels would not be able to navigate on the Great Lakes, but he has not yet been able to find a naval architect or a marine engineer who said so.

WALTER M. SMITH,* M. AM. SOC. C. E.—The diversion of water at Chicago has little bearing on the St. Lawrence Waterway.

Three days during the past year (1925), the water flowing through the Chicago River and the Drainage Canal has been turned back into Lake Michigan because of heavy rainfalls. Generally, an increase in the number of typhoid fever cases occurred in the neighborhoods supplied by the water. The first case, which occurred in the spring, was at the mouth of the Calumet River. Before the Health Commissioner could issue orders to boil all the water drawn from the crib located nearest this point, the damage had been done. The boiling had to be continued for about two weeks, just because of only a few hours' change in the direction of the flow of the river. Later, there was a very heavy rainstorm in the center of the city, and the flow of the Chicago River again was reversed, this time for about six hours. Immediately, similar orders went out as to boiling the contaminated water. As a rule, people observe these directions rigorously because they know the danger. If Chicago should be deprived of the use of this lake water, the city could not exist for very long. It would have to be abandoned, temporarily at least.

The City of Chicago is now building sewage purification plants as rapidly as they can be constructed. Possibly, engineers are not familiar with the fact that many of these plants are already in existence. It is hoped to build them with sufficient speed so that it will never be necessary to divert more than 10 000 cu. ft. per sec. from the lake. Several years ago when the question of diversion was under discussion, the International Waterways Commission wanted to give the City of Chicago the right to divert 10 000 cu. ft. per sec., and it so recommended. Some of the representatives of the United States and of Chicago objected at that time, because they thought it might be necessary to use more than this in the future, and they did not want such limitation written into the treaty.

The discussion of this paper, with one exception, has been entirely on the question of the profit of building the St. Lawrence Canal from the standpoints of navigation and of water power. Engineers, however, should take a broader view of this subject, and realize that they are dealing with the lives and the property of 3 000 000 people in the question of the diversion of this water from Lake Michigan. It is more than a financial matter. Lake Michigan is an American lake. There is not one drop of water entering it that comes

* Chf. Designing Engr., Div. of Waterways, Dept. of Purchases and Constr., State of Illinois, Chicago, Ill.

from Canadian soil. All the other lakes are along the boundary line, midway between the two countries.

It seems to the speaker that if the question were submitted to an International Court of Law, the decision would be that the United States Government could divert any quantity of water that it cared to from Lake Michigan, without reference to Canada.

There is still another problem for the City of Chicago in the diversion of lake water. When this question first was agitated, the State of Illinois thought that it was the only party concerned, and that neither Canada nor the United States at large had anything to do with it. For a long time this view seemed to be accepted. The State of Illinois, before it would allow Chicago to divert water and reverse the flow of the Chicago River, passed a law that it must divert 3.33 cu. ft. per sec. per 1 000 of population. Now, if Chicago complies with the State law, it must take about 10 000 cu. ft. per sec. from the lake; if it is limited to the quantity allocated by the United States, it may take only 4 167 cu. ft. per sec. The city must break one of the laws.

W. M. Black,* M. Am. Soc. C. E. (by letter).—This paper and its discussions give many interesting and more or less well-founded figures of costs and of expected benefits. They also show the marked diversity of conclusions reached.

The entire subject is being re-investigated by an able board of American and Canadian engineers. Doubtless its report, when submitted, will contain data which will be as full and exact as are now obtainable. The writer believes that it would be wise to await the publication of that report before attempting to form definite judgments on the many controversial questions involved.

As stated by Mr. Sabin, the commerce of the Great Lakes causes for the people of the United States a saving of $100 000 000 annually in transportation costs. If the contention that the proposed deeper St. Lawrence Waterway would result in similar transportation economies is well founded, the desirability of the improvement is established—provided the net cost of making the improvement is not too great.

In general terms it is proposed to regulate the lake flow by suitable works and to concentrate the general slope of the St. Lawrence River into a series of falls, by means of dams, causing an increase of navigable depth in the pools above the dams and utilizing the head formed at the dams for the production of power. In certain localities, where the pool depths are insufficient, side channels are proposed for navigation. It is manifest that, within certain natural limits, the higher the dams, the greater their cost as balanced against the deeper navigable channels of the pools and the greater possible power production.

It is admitted that there is an immediate market for additional power. The question of how quickly an increase of power to the full potential of the fall of the St. Lawrence could be sold is as yet unsettled, and is dependent in

* Major-General, U. S. A. (*Retired*); Cons. Engr. (Black, McKenney & Stewart), Washington, D. C.

a measure on the cost. This, in turn, is a function of the depth of the navigable channel desired.

In so far as is known to the writer all the projects submitted have had for an object the provision of a navigable channel of a definite capacity, with a corresponding power development. Opinions differ as to the channel capacity required. The project depths considered have been 25 and 30 ft. It has been claimed that 25 ft. is an uneconomical depth for ocean carriers and *per contra* the claim is made that there would not be sufficient commerce to justify the cost of a 30-ft. improvement.

A study of this shipping question will disclose the following facts:

1.—For long voyages, when cargoes are available, the greater the cargo capacity of the ship, the less will be the cost per ton of transportation. It is due to this that the average tonnage of both ocean and lake vessels under construction has increased from year to year, and that the relative number of ships of small tonnage is growing less.

2.—For a given tonnage, and within proper limits, an increase of draft provides for a stronger and more seaworthy ship than an increase of beam. Structurally, the ship resembles a truss in this respect.

3.—The strains to which a ship is subject are much greater on the ocean than on the Great Lakes. Lake vessels can be built more lightly, and, consequently, more cheaply, than those designed for ocean navigation.

4.—The ship's complement of men as required by law is greater for the ocean than for the lake carriers.

5.—The draft of the ship is somewhat greater in fresh water than in salt water. For example, a 10 000 dead weight ton cargo steamer which draws, loaded, 27 ft. 2.75 in. in salt water, will have a draft of 27 ft. 6.2 in. in fresh water, or, in other words, with the same draft she can carry 161.5 long tons of cargo less in fresh than in salt water.

For the foregoing reasons transportation costs for a lake carrier in the lakes and in the Upper St. Lawrence River will always be less than for an ocean carrier in the same waters. *Per contra,* ocean carriers of larger tonnage, drawing from 26 ft. to 30 ft. are, to-day, the most economical means of transportation in the ocean highways.

It is claimed that a ship can be designed and built which will be equally good in the lakes and on the ocean. To the writer this seems doubtful. The conditions of navigation differ too widely. Certainly, a channel depth of more than 25 ft. would be necessary for a long-voyage ocean carrier. Such a depth would require expensive and extensive changes in the lake harbors and channels, as well as expensive works in the St. Lawrence River.

There exists to-day an established and efficient freight service between the interior of the United States and its Atlantic seaboard and, *via* the seaboard ports, with all parts of the world. Traders are conservative and a change of trade routes can be established only when the new routes offer positive and marked economies. Witness the slow growth of the Panama Canal traffic where the canal route provides enormous economies in time and costs! To make a deeper St. Lawrence Waterway a great trade highway, the saving resulting from its use must be manifest and marked.

In considering the project for a deeper St. Lawrence Waterway, the liabilities are (1) the first cost, and (2) the cost of operation and maintenance; the assets are (1) the power produced, and (2) the savings resulting from a lessened transportation cost for traffic between the interior, the Atlantic seaboard, and the world at large. The costs to the people of the United States and Canada must be weighed against the benefits to be received. The cost items can be estimated closely; the items of benefit do not admit of exact calculation.

Under these conditions would it not be well to advance slowly? Cannot a project be formed which will permit a progressive development, limiting the works to be constructed at first to those necessary to constitute a fair test of the economic advisability of the entire plan?

The experiment of connecting the lake cities with the Port of Montreal by a line of boats through the St. Lawrence has been tried. The limited channel capacity, however, requires boats that are too small to provide the economic transportation necessary to enable the fixing of the combined lake, river, and ocean freight rates, including transfer charges at Montreal, at a figure enough below the existing rates (by rail to the seaboard and transfer there to the ocean carrier) to be attractive. The experiment was a failure.

Carefully made estimates show, however, that by using the large type of lake carrier to Montreal and transferring there to an ocean carrier the costs of transportation to Atlantic Coast cities or to Europe, plus those of the transfer at Montreal, are considerably less than by the present routes.

The building up of new trade routes requires time. It is improbable to-day that any one of the lake cities is prepared to supply or to absorb the cargo of an economical European carrier in package freight, with the short interval between successive trips necessary in such a trade. The cargo volume is too great. Later, should such a trade route prove to be advantageous, the traffic might grow to truly great proportions. The combined traffic of several of the lake cities might reach a sufficient volume at once. The 4 000 to 6 000-ton lake carriers could serve one or more cities economically and the combined cargoes could supply the ocean carriers. In earlier days it was quite customary for outgoing cargoes to be collected in the warehouses of a port throughout a week or more, until enough tonnage was assembled to load a ship for the port of destination. This is no longer the custom. Ships sail from the great seaports at frequent intervals for all parts of the world. An interior manufacturer to-day ships his products just in time to reach the port before sailing day and avoids storage delays and charges. This fact is one of the causes of the decay of small ports and of the growth of great ports. The port warehouses of to-day are used mainly for imports awaiting sale.

With the lakes and their hinterland to provide and absorb cargoes, there is no reason why the well-equipped and well-managed Port of Montreal should not become an ideal transfer point.

It will be noted that the writer is not considering the transportation of bulk freight. Much grain moves now by water to Montreal, but the volume available for export from the United States grows less as the population increases. Part cargoes of grain would doubtless be carried from time to time to make up for

a deficiency of package freight. The traffic that would justify the deeper St. Lawrence Waterway must be of a character more similar to that of the Atlantic ports than to the bulk of the present lake traffic.

The cost of fitting the St. Lawrence River for lake boat navigation would not be excessive. It would be but little in excess of the value of the power produced. With such an improvement a thorough trial could be given to the project of direct water carriage between the lake ports and Europe and a new trade line built up. Later, if the increased expenditure for increased channel depths were shown to be justifiable, the deeper channel project could be carried out.

G. B. Pillsbury,* M. Am. Soc. C. E. (by letter).—This discussion relates to those parts of the author's paper dealing with the regulation of the Great Lakes; its purpose is to point out the engineering problems that must be solved before the economic feasibility of such regulation can be established, and before the plans for the necessary works can be drawn up, should regulation be found feasible.

Many members of the Society have regarded it as almost axiomatic that the regulation of the Great Lakes is inherently economically feasible; therefore, the writer may seem either bold or reactionary in asserting that the feasibility is yet to be proved. A consideration of the limitations that must be imposed on the levels of the lakes will, the writer believes, show that their regulation is not as simple and obvious a question as is sometimes assumed.

The interests concerned in the regulation of the Great Lakes are lake navigation, navigation of the St. Lawrence, power, and the owners of lands and improvements along the shores.

Obviously, lake navigation is interested in an increase in lake levels, that it may carry greater drafts through the channels connecting the lakes and into the harbors. Its greatest concern is the elimination of the extreme low levels, such as are now occurring. Within limits, present lake navigation is benefited by an increase in level at any stage of the lakes, as the short voyages and thorough organization of the traffic permit the utilization of the full draft available at the time. Lake navigation, however, would be adversely affected by any such increase in level as would hamper the operation of the loading or unloading wharves; for delays in loading and unloading are exactly as costly as delays and reduced cargoes en route. Navigation interests look also with extreme suspicion on any works in the connecting channels that would introduce delays to shipping, or that would tend to increase the hazards of navigation.

Navigation on the St. Lawrence is concerned with the minimum discharge from the Great Lakes System. The great Port of Montreal is not a tidal port, but a river port. There is about 20 ft. of fall in the St. Lawrence below Montreal, and the water level in Montreal Harbor depends on the volume of the discharge from the St. Lawrence and the Ottawa Rivers. The channel depths to and in Montreal Harbor have been won by the expenditure of millions of dollars in dredging, and are not lightly valued.

* Lt.-Col., Corps of Engrs., U. S. A.; U. S. Lake Survey, Detroit, Mich.

Power is obviously concerned, eventually, with increased minimum discharges. As long, however, as existing power installations use only a fraction of the minimum discharge of the river there can be no object in increasing this minimum discharge; nor will an increase benefit power until the development at some point on the river has reached a capacity corresponding to the minimum existing discharge. Considered from the economic viewpoint of the world at large, it will not be economical to expend large sums for regulation to increase the minimum flow for power purposes until all the developments on the river are beginning to reach the limit possible with the present minimum discharges.

The last interest concerned in the regulation of the lakes is the riparian owner, and this interest has sometimes been overlooked. At first glance it might seem that the increased lake levels proposed in any scheme of regulation that has been advanced are so small in amount, reaching a few feet only, that they could have no effect on the lands and the populous cities that border the lake. The investigation thus far conducted indicates, however, that the contrary is the case. Where land is valuable, improvements have been pushed beyond the absolute safe limit, as indicated by prior high water, to the reasonably safe limit, which would be surpassed by high water only at long intervals. There are many localities where a recurrence of past high waters would back up water in sewers, inundate the machinery pits of unloading wharves, and flood improved property in general. There is also the question of increased erosion of shore property at high lake levels.

The improvements effected have been made, in many cases, with full knowledge that they are subject to occasional damage by high water. It may be expected that the owners will insist that the frequency of such high waters will not be increased.

When the regulation of Lake Superior was under consideration by the International Joint Board, hearings were conducted to determine the fluctuations of the lake permissible under regulation. The so-called "high water of 1838" on Lake Superior is recorded at Elevation 605.32. From 1860 to 1914 the mean monthly level of the lake had been above 603.6 on four occasions, reaching in one month above 604. Nevertheless, riparian owners concerned, including municipalities and corporations owning wharf facilities, established at the hearings that any level above 603.6 would seriously injure their improvements, and the scheme of regulation prescribed by the Joint Board provides that the maximum regulated level of Lake Superior shall not exceed 603.6, except under such conditions of supply as would normally produce higher levels, in which case the levels should not exceed the level that occurred in the unregulated lake.

It may be confidently asserted that the riparian owners on any lake will impose a similar restriction on the regulated levels of the lake; that is, these will be limited to some level for normal operation which is lower than the actual highest levels of the past, with the provision that above this limit the lake must never rise oftener or higher than it would rise with the same supply, in Nature.

The writer has seen the view advanced that the riparian owners would have no legal claim for damages provided that the maximum levels heretofore reached were not surpassed as a result of regulation. He is not sufficiently versed in law to express an opinion whether damages would lie if the frequency of high waters were increased; but he is convinced that the Governments of the two countries concerned would never sanction wilful injury to a considerable number of their respective citizens.

As a consequence it is necessary to increase the low levels of the lakes for the benefit of navigation, without increasing the high levels, and to manipulate the reduced reservoir capacity remaining so as to improve the minimum outflow.

A brief consideration will show that this can be made possible only by increasing the discharge capacity of the outlets of the lakes to be regulated. With increased discharge capacity it will be possible to hold the lakes at higher minimum and mean levels, and, at the same time, to discharge an excess supply, whenever it may arrive, without raising the lake above the level to which it would have risen, in the state of Nature, with such supply. It is because the power canals at the St. Marys River provided an ample excess discharge capacity that regulation of Lake Superior, made necessary by the power works themselves, became feasible. A large increase in the discharge capacity of the St. Lawrence at the control section at the Galops is an inherent part of any future improvement of the International Section of the river for power and navigation, and, consequently, the regulation of Lake Ontario will naturally be combined with such an improvement.

It may be well to correct any misconception as to the operation of the regulating works at the outlet of Lake Superior. These works were, in their conception, for the purpose of diverting the water from the St. Marys River into the power canals. Since no such diversion works can automatically restore the natural regimen of discharge, they are of necessity operated under Governmental control. The operation is so directed as to decrease, in ordinary years, the fluctuations in the level of Lake Superior, while affording power users practically the entire river flow during dry years. These beneficial results are obtained, however, at the expense of increased fluctuations in the discharge from Lake Superior, made permissible by the existence of the enormous equalizing reservoir afforded by Lakes Huron and Michigan. The scheme of regulation, in the preparation of which the late Alfred Noble, Past-President, Am. Soc. C. E., had a prominent part, does not contemplate any hold-over storage from year to year. On the contrary, it provides for drawing down the lake each year in advance of the wet months in such manner that the inflow during those months can be safely handled.

The economic feasibility of the regulation of the lakes in general hinges, therefore, on the following questions:

(1) To what levels may the respective lakes be normally held without substantial damage to riparian interests?

(2) With an economically feasible outlet enlargement, what degree of improvement of lake levels and of discharge can be realized by regulation?

(3) Will the improvements realized be worth the cost of the outlet enlargement and of the works necessary to control the flow?

(4) What type of works, if any, acceptable to navigation interests, can be placed in the St. Clair River to control the outflow from Lake Huron?

The first question can be answered only by a thorough canvass of the situation at the principal harbors and towns along the several lakes.

Question (2) involves a computation of the cost of varying amounts of channel enlargement, with consequent increases of flow. It does not require much investigation to show that for an outlet such as the St. Clair River the cost mounts quickly with the increased discharge and soon becomes clearly prohibitive. The discharge through the St. Clair River during the winter months will certainly be less than the summer discharge, whatever the enlargement proposed; and it appears certain that the discharge capacity of the St. Lawrence during the winter months will also be limited, whatever the form and extent of the improvement. In any event no scheme of regulation can be considered that would increase the flow down the St. Lawrence at the time of the spring break-up, when, at Montreal and below, the river is packed with gorged and shoving ice.

With the limitations established, it then becomes possible to formulate rules for regulation, to test them by applying the supply factors of the several lakes established by past records, and to ascertain what improvement in present condition, both as to levels and as to discharge, can be realized.

In preparing and testing a suggested method for regulation the temptation is strong to assume that it will be possible to predict, in advance, the probable supply. Any scheme of regulation is necessarily based on prediction, which limits itself, however, to the assumption that the future will be no worse than the past, so far as maximum and minimum supply is concerned. To predict the future supply to a lake from the present rainfall records is a different, and the writer believes, an impossible task. The net water supply to the lake is the inflow into the lake, plus rainfall on the lake, less evaporation from the lake surface. Whatever hopes may be had that the inflow into a lake from its drainage basin may be predicted in advance from the rainfall records on its water-shed, no advance prediction can be hoped for regarding the variations in the rainfall on the lake and the evaporation from its surface. The latter two add to or diminish the supply as soon as they occur. Thus, the rains of August will have some effect on the inflow during September, but the temperature and humidity records of August will have no effect on evaporation during September. If it is realized that the average loss to Lakes Huron and Michigan from evaporation exceeds the inflow of the St. Marys River, it will be easy to see how uncertain any prediction of supply must be.

In estimating the benefits to navigation resulting from a proposed regulation (Question (3)), the cost of regulating works must be compared with the cost of securing a like measure of improvement by dredging. The author has shown the very large benefits that can be secured to navigation by deeper channels. It is quite immaterial, however, so far as navigation is concerned, whether depth is secured by dredging or by raising the stage. On the other

hand, a free, open and unobstructed channel through the St. Clair River is an unquestionable asset to navigation, the value of which must be debited against any regulation scheme in proportion to the obstruction or constriction imposed by the works in the St. Clair proposed in that scheme.

In summary, the writer hopes he has shown that the economic feasibility of regulating the Great Lakes is not axiomatic, but that it can be established only after due and full investigation. The instructions to the present Joint Board of Engineers on the St. Lawrence Waterway require such an investigation, which is in course, and which should form a basis for a determination of this vital question.

About fifteen years ago a board of eminent engineers answered the question in the negative. Whether or not present conditions and new facts warrant a reversal of this prior finding is a matter that can well be given the best engineering thought of the Society.

L. F. Harza,* M. Am. Soc. C. E.—It has been said by Guy E. Tripp, Chairman of the Board of Directors of the Westinghouse Company, that the general prosperity of any country or section of any country is directly proportional to the amount of mechanical power in use by that country. It is mechanical power only that makes the difference between the present and the time when a man worked 12 or 15 hours a day to make a living for his family, and when his wife spun the yarn and made the clothes, and the children worked from early childhood; it is only mechanical power that makes the difference between the standard of living and the average wage then and now. If that is the case—and it has been very well demonstrated—and if the eastern section of the United States is to receive all the benefit of this power development, why not let the Middle West have the benefit of the transportation development if there is any benefit to be derived.

Elliott J. Dent,† M. Am. Soc. C. E. (by letter).—Attention has been called by Mr. Gardner S. Williams to the fact that the navigation channels through certain reaches have been deepened, and he states that:

"Students of hydraulics can readily decide whether the deepening of channels that were between 8 and 12 ft. to 21 ft. in depth, has or has not enabled more water to flow out of the upper into the lower lakes."

The paragraph in which this statement occurs applies more accurately to the channels through the Lower Detroit River than to any others. The writer is now in charge of this improvement and desires to lay a few additional facts before the members of the Society.

The aggregate width of all the natural channels through a section selected as more or less typical of the reach in question was approximately 11 000 ft. The improved channels are two in number, one being 600 ft. wide and the other 450 ft. wide, making the combined width of the deepened channels 1 050 ft., or about one-tenth the natural width. Part of the rock excavated from the improved channels has been carefully distributed in such a manner as to obstruct portions of the natural channels with a view to neutralizing

* Cons. Hydro-Elec. and Hydr. Engr., Chicago, Ill.

† Lt.-Col., Corps of Engrs., U. S. A., Detroit, Mich.

the effect of the deepening. The question whether complete compensation has been effected is one that is still under consideration and measurements are made annually to supply the data for studying the subject.

The data thus far accumulated fail to demonstrate conclusively any change in level at the Fort Wayne gauge about 10.5 miles above the head of the deepened channels. Any effect that may eventually be found at Fort Wayne will necessarily be considerably reduced before the lower end of Lake Huron is reached, as the intervening length of river channels is about 38.75 miles.

KENNETH L. DE BLOIS,* JUN. AM. SOC. C. E. (by letter).—There are several fundamental principles that should be considered before embarking on the improvement of any inland waterway.

Experience has proven that ocean steamships will go as far inland toward the center of production and industry as physical conditions will permit. As examples of ports developed at the head of ocean navigation may be cited: Philadelphia, Pa., 101 miles up the Delaware River; Baltimore, Md., 190 miles up the Chesapeake Bay; New Orleans, La., 100 miles up the Mississippi; Baton Rouge, La., 190 miles up the Mississippi; Portland, Ore., 113 miles up the Willamette River; and Montreal, Que., Canada, 1 000 miles up the St. Lawrence River.

Recognition of this fact in the past would have prevented uneconomical expenditures on such ports as Halifax, Nova Scotia, and Montauk Point, Long Island. The St. Lawrence Waterway fulfills this condition in that it will permit ocean steamships to go into a territory rich in industry and agriculture. The fact that such steamships already come to Montreal, navigating a tortuous channel for many miles in preference to making Halifax or St. John, N. B., Canada, their western terminal, supports this claim.

Transfer charges are a very important consideration and a waterway to be economically feasible must cut the number of transfers to a minimum. The New York State Barge Canal was built to insure the transfer of grain at Buffalo, and New York, N. Y., and not to eliminate them. Because of this the grain trade has practically disappeared from the canal. The St. Lawrence Waterway will eliminate most of the existing transfer charges by permitting boats to be loaded at the head of the lakes and proceeding to their destination without further transfer.

As a substitute for the St. Lawrence Waterway a deep canal across New York State has been suggested. Even if the immense sum of money were at hand to build a canal it is extremely doubtful whether ship-owners would risk their vessels navigating a narrow channel for several hundred miles across the State. This project should not be seriously considered as a substitute. The proposed St. Lawrence Waterway must not be confused with a narrow ship canal. The author clearly outlines the use of dams and pools to give a wide channel for a greater part of the distance.

It has always puzzled the writer to understand why a certain group who are so adverse to the Great Lakes-St. Lawrence Waterway, where a large freight

* Cincinnati, Ohio.

traffic already exists, at the same time advocate the expenditures of huge sums of money on barge canals where little or no traffic exists and where there is no prospect of any. On the editorial page of nearly every Middle Western newspaper will be found a platform advocating a barge canal as the one means of creating a permanent prosperity for their community. It is not the writer's intention to discuss these various projects but merely to emphasize that money should be spent on waterways which will be used when completed. The Great Lakes-St. Lawrence Waterway with ocean traffic at Montreal and the large existing traffic on the lakes can leave no doubt but that the proposed waterway will be used.

C. S. Riché,* M. Am. Soc. C. E. (by letter).—Although the opening of the Great Lakes to ocean traffic seems inevitable, it has nevertheless been under discussion for a long time, and there seems to be enough opposition in certain quarters to keep it "under discussion" for many years longer.

The situation is somewhat similar to that which existed 35 or 40 years ago, with respect to the Gulf of Mexico. For many years there was opposition to the creation of deep-water ports in the western part of the Gulf. The Southwest had then only recently been linked to other parts of the country by rail. Long rail hauls to Eastern ports were desired by many interests. Nevertheless the improvement of Galveston Harbor was actively undertaken in the early Nineties and was followed by the improvement of adjacent harbors at Sabine Pass, Port Arthur, Beaumont, Orange, Bolivar, Texas City, and Houston, Tex.

All these places are relatively near together. Their access to the ocean is either by Galveston Entrance or by Sabine Pass. They form the nearest ocean outlets for approximately one-fifth the area of Continental United States. Their commerce in and out, foreign and domestic, was very small before these harbors were deepened, and was subjected to lightering and other transfer charges which none but the most valuable freight could bear.

A record of this commerce has been kept by the U. S. Engineer Department, and appears in the Annual Report of the Chief of Engineers for 1924, the record for 1923 being the last one given.

Beginning in 1893, a few years before the first harbor was deepened, and listing the annual totals at intervals of five years discloses that the freight passing in and out of Galveston Entrance and Sabine Pass was as follows:

Year.	Short tons
1893	415 403
1898	3 168 464
1903	4 867 965
1908	(Record incomplete)
1909	5 825 608
1913	9 915 829
1918	11 090 405
1923	24 622 522

* Col., U. S. A. (*Retired*), St. Louis, Mo. Colonel Riché died on March 20, 1926.

The general development of the Southwest has had much to do with this great increase, but *vice versa* this development has chiefly resulted from the improvement of these harbors. There is no indication that the commerce of the Port of New York and other Eastern ports has suffered from this development but rather has profited and increased by reason of it.

The Great Lakes penetrate into the continent nearly as far to the westward as the Gulf of Mexico, into a section of greater natural resources than the Southwest. Although the North and Middle West have already been extensively developed, a much greater progress is possible and their accessibility to ocean traffic would stimulate this further development.

New commerce, increasing as has that of Galveston and Sabine Pass, could be expected at the Western Lake ports of Port Arthur, Ont., Canada; Duluth, Minn., and Chicago, Ill. Adjacent territory would develop more intensively, and Montreal, New York, and other Eastern ports, together with Canada and the United States as a whole would be benefited and not injured as a result.

Opposition to this improvement because of the fear of injuring existing business is ill-advised. Any development that would directly benefit so large a territory could not fail to benefit the continent as a whole. One such benefit not often mentioned would be the ability to build sea-going ships at lake shipyards. This can be done to-day but, with the exception of vessels of the small St. Lawrence Canal size, these lake-built boats have to be towed to the seacoast in sections and there assembled. Consequently, little such work is now done on the Great Lakes.

Steel is manufactured at a number of points on the lakes and there is an advantage in having shipyards close to the steel mills. With easy egress from the lakes for the product of the shipyards, it would be surprising if such an industry did not soon become extensive.

There would also be a benefit, as Mr. Shenehon points out, if lake vessels could be available for ocean service during the winter instead of lying idle for five months each year. The additional revenue thus obtainable might easily pay for such strengthening as would adapt these vessels to ocean service. At times, there are demands for additional carriers on the ocean, which would make such service quite profitable.

It is unnecessary to go back further than the dark days of the World War, when the submarine menace was at its height, to recall the demand for vessels for ocean service. Old boats, many far less seaworthy than the average lake vessel, were pressed into service and helped to save the day. Meanwhile, for five months each year, the great fleet of lake vessels lay idle, incapable of getting to the ocean. Had it been possible to transfer this fleet or any large part of it to the sea at that time it seems probable that the resulting saving would have paid the cost of improving the St. Lawrence River and the Great Lakes several times over.

It is to be hoped that these conditions will not recur, but they may, and it would seem advisable next time to be ready for them. Where that readiness will also result in a large commercial benefit, it would seem wise not to

keep the subject "under discussion" indefinitely, but to take early steps to bring about results.

E. G. WALKER,* M. AM. SOC. C. E. (by letter).—The writer has read this paper with interest, for there are few individual engineering proposals in the limelight at present which have so general an interest, or which, if carried out, would have such far-reaching consequences. To appraise fully the magnitude and nature of the proposals for regulating the Great Lakes and the St. Lawrence River requires complete inspection and first-hand acquaintance with all the sites referred to by the author. The writer is at a disadvantage in this discussion in not possessing such acquaintance; hence, he prefers to confine his statements to more general matters.

Critics of plans for the greater utilization of the St. Lawrence question not so much the possibility of carrying out the works as the likelihood of their full use when completed. Criticism of details is seldom or never heard. No doubt this is due in part to the fact that none of the works proposed is of a startling nature or exceeds in general form and scope the very fine results which have already been achieved both in regard to power development and as to the provision of navigation facilities. At the same time the construction problems involved are not simple. The condition which makes the St. Lawrence unique as a power-producing river is the enormous magnitude of the reservoirs out of which it flows. While this gives it a minimum fluctuation of flow, it also means that there are no slack-water periods, such as occur on most rivers, of which advantage can be taken to carry out under-water works, and the whole flood volume has to be provided for in any plan for controlling the water during construction. It is not suggested that these and similar difficulties cannot be overcome. The writer merely raises the point because it is one which is apt to be overlooked. On the other hand he considers that a very great reason why the whole proposition should be given the fullest and most rational consideration is because it is one that does not involve any new or startling departure, but only requires the construction of works similar in character to those which have been carried out already in various parts of the world. The total expenditure proposed is vast and obviously cannot be undertaken without a very full study by men of high qualifications; but should the work be decided on, each unit into which it is divided is a solvable engineering problem. Although, at present, the total cost is merely a guess, it is certain that the proposals can be carried out in detail, even if some of them may prove to be difficult.

The main object of the paper being to advocate the construction of a deep-water navigation from the head of Lake Superior to the sea, it is disappointing to find inadequate consideration paid to the primary features of such a waterway. The paper leaves an impression that the premises on which any plan for a waterway must be built have been considered far too lightly. The idea of being able to take a large ocean-going ship into the heart of a continent is one which inevitably must make a strong appeal even to the dullest imagination, and the glamor of the idea is apt to spread in an unwarranted manner

* London, England.

and to mask opposing features which otherwise would stand out in a clearer light. Enthusiasm for the greatness of the development and for its possible widespread influence seems in the present instance to have drawn attention away from material facts which are of much too great importance to be passed over lightly.

As evidence of this the writer notes how little idea is given by the author of the type of ship which it is proposed to use on the waterway. He states:

"It will be assumed that engineers will solve the problem of composite vessels to traverse the fresh-water seas and the salt seas; that the evolution of this vessel will come in the ten years between the definite decision to construct the waterway and its completion; and if not in this decade then in some of the early decades of the century."

Instead of making the assumption that it would be possible to find a type which can be used on the waterway when built, why not give some consideration to the matter now? If the search for a type of ship suitable to the special conditions of the case and for methods of handling such ships and their cargoes is successful, then it is at least known that if the waterway is provided it can be used.

From the discussions on this point the writer concludes that the familiarity of the people near the Great Lakes with the craft they use has distorted the first of these problems into one of needless complexity. The Great Lakes freighter is a unique type of vessel which has been evolved to suit the special conditions obtaining there. It differs from the ocean-going steamer as much as a street car differs from a railroad coach and for a similar reason. The lakes freighter is free from many of the conditions imposed by ocean service. The result is that it has been possible to evolve a vessel of comparatively light dead weight and of high efficiency both in regard to stowage of cargo and to handling it. With such ships the local traffic of the Great Lakes is transported in the highly economical manner that has justified the installation of expensive special machinery for loading and discharging. It is agreed, and rightly, that these ships are not suitable for general ocean travel, but by some inexplicable inversion of reasoning, it is then assumed that the opposite proposition is also true, namely, that ocean steamers would not be suitable vessels for going up the St. Lawrence Waterway to ports of the Great Lakes.

To any engineer who has been concerned with ships or with the provision of harbor facilities for sea-borne commerce, this idea must appear untenable. Admittedly the ocean-going ship could not compare with the lakes freighter for interlake services. It would not be used for such work; it would deal with cargoes which the lakes ships could not handle; but both types of vessel use similar prime movers, propel themselves by screws of similar kind, and are steered and maneuvered similarly. The ship which can cross the high seas can equally well traverse the Great Lakes and negotiate any connecting waterways, provided the dimensions of these are suitable. One is at a loss, therefore, to understand whence arises this fetish of the hybrid ship. If cargo is to be consigned overseas it must be sent in an ocean-going ship; if inland produce is not to be sent directly overseas there is no reason for constructing a deep-water navigation to the farthest possible point inland.

The only excuse for attempting to bring the hybrid into being is that it would aim at combining the sea-going qualities of the one type with the efficiency of stowage and ease of loading of the other. Such an aim is largely a myth. The basis of design of all cargo ships is a complex compromise between a number of factors, often mutually antagonistic, such as seaworthiness, speed, ease of handling, cargo capacity, kinds of cargo to be transported, radius of action, etc. and in arranging the details account has to be taken of extreme conditions. In the case of ocean-going ships designed for the projected St. Lawrence navigation such conditions are provided by the high seas in practically every detail, except over-all dimensions, even to the relatively minor condenser problem referred to in the discussion. The principal possible complicating condition which a waterway would cause would be tonnage measurement rules. These, of course, might materially influence the design of the ships, but can hardly be discussed at the present stage. There is no reason to anticipate that if the proposed waterway should be constructed it would be found necessary to introduce freak tonnage regulations. The inevitable conclusion, therefore, is that if the St. Lawrence Waterway should be opened as a work of the magnitude contemplated by the author, it will be used, as have all waterways of a similar kind thus far constructed, by the normal type of ocean steamers which would voyage between overseas ports and the various ports along the waterway.

Whether the present terminal facilities in the Great Lakes ports are suitable for the contemplated traffic is a matter which has not been mentioned in the paper. The ocean steamer differs considerably from the lakes vessel in the arrangement and dimensions of hatchways and the facilities available for stowing and discharging cargo, and must continue so to differ because of the provisions which have to be made to meet open sea conditions. Furthermore, the opening of the Great Lakes ports to overseas commerce would probably have the effect of causing a large increase in the amount of general cargoes passing through these ports in comparison with the special cargoes which are now the principal ones handled. Advocates of the proposed waterway do not seem to have considered adequately that one of the main effects of a deep-water direct connection to the sea might be expected to be the opening of numerous new inland ports to the exportation and importation of sea-borne goods, and that if this movement were successful, it would alter very greatly the average character of the shipping using the Great Lakes ports. This would mean that terminal and port facilities suited to overseas traffic would have to be provided at lake-side ports, and in many cases might entail considerable expenditure for re-arrangement of wharves, warehouses, transit sheds, cranes, sidings, etc., and for the provision of new works, in order to handle products which at present are not carried by water at all as far as the territory of the Great Lakes is concerned.

Assuming the proposed waterway to be constructed and provision made for accommodating any type of ship, it is unlikely that vessels would enter and incur the expenses of lockage and other dues (which, in view of the great total lift of 500 to 600 ft., must be heavy) unless they were on definite charters.

Statements that have been made as to the possibility of tramp steamers in search of cargo visiting Great Lakes ports in the same way as at ocean ports are far-fetched and unlikely. The costs of operating such a navigation as that outlined in the paper must be fairly high. They must be sufficient to increase very materially the cost of transport over the 1 340 miles above Montreal.

The author has taken for the purposes of this paper a depth of 25 ft. with provision for a future increase to 30 ft. and in so doing has used correspondingly moderate dimensions for the lengths and breadths of locks and other permanent works. Apparently, his basis is a lakes freighter 600 ft. long, with a 60-ft. beam, 24-ft. depth, and a cargo capacity of 10 000 tons, although as already pointed out it is unlikely that such a vessel would be used for overseas commerce. It is probable that provision for 10 000-ton ships would be more than sufficient to meet the requirements for a long time. Of the world's ships 50% do not exceed 1 000 tons and although there is a tendency to increase the size of vessels of between 5 000 and 10 000 tons it is unlikely there will be, at any rate for as long a period as the present generation need consider, more than a small minority which could not use a waterway large enough to accommodate a 10 000-ton, ocean-going ship. To make certain of this, however, it might be well to give increase of breadth to all permanent works, since the vessel selected by the author for his standard is narrow in proportion to its length. A 10 000-ton intermediate ship would normally have a beam of about 60 to 70 ft. and be appreciably shorter than the lakes steamer. The dimensions proposed for locks, namely, 800 by 80 by 30 ft., should be ample.

In discussions on the power development aspect of the St. Lawrence Waterway, objection to the plans has been taken on the score that so large an output could not be used. Accepting the estimate given in the paper, the author appears to conclude that an ultimate total of nearly 7 000 000 h.p. is available, made up as follows:

Present development:

	Horse-power.
Canadian	700 000
American	570 000
Total	1 270 000
Possible additional power (with regulation)	4 424 000
Possible improvements at Niagara	1 160 000
Total	6 854 000

The writer has no figures at hand relating to American power developments, but taking the current figures for the Canadian Provinces of Ontario and Quebec only, the following water power is already in use:*

	Horse-power.
Ontario	1 445 000
Quebec	1 116 000
Total	2 561 000

* Annual Report, Dominion Water Power and Reclamation Service, 1923–24.

This power serves a population of about 5 500 000 and includes the 700 000 h.p. given in the author's figures. It is equivalent to about 0.47 h.p. per person. On this basis there would remain 6 154 000 h.p. to be utilized, requiring a population of more than 13 000 000. The power requirement in the Provinces of Ontario and Quebec is greater than the average for the United States and Canada, which works out as 0.44 and 0.35 h.p. per head, respectively. It is probably greater than the average for the whole territory tributary to the Great Lakes and the St. Lawrence. Hence the development of the two countries would have to be greater even than is implied by the indication of this simple statistical calculation. Bearing in mind that this population figure is of the magnitude of the total increase of the population of the United States in the last ten years, one gets some impression of the size of the problem involved in harnessing the whole of the St. Lawrence power.

There are two potential competitors with the present plan that have to be reckoned with, namely, (1) other hydro-electric power; and (2) power from coal. Roughly speaking, these resolve themselves into Canadian and American competition, respectively. The large undeveloped sources of power in Canada and particularly in the Province of Quebec are well known and the 5 000 years' supply of bituminous coal in the United States to which the author refers may well be formidable competitors. Remarkable advances have been made in recent years in the economical use of coal as a source of power and it is noteworthy that some of the most economical power stations in the world are situated in the area treated in the paper. The writer does not agree, therefore, with the reasoning that because supplies of coal are ample, no concern need be felt about saving coal. No hydro-electric power plan can oust steam power unless it can make an appreciable reduction in the cost of supply of current. The fact that such large deposits of coal are available seems to the writer to make the use of that material as a source of power a big factor in considering the development of the huge hydro-electric propositions within a relatively short distance of the largest coal districts of the country. One would like to have much closer estimates of costs of power than are given in the paper, and also to have for comparison figures as to costs of generating from coal, before venturing on definite opinions.

In the Province of Quebec the power supply situation is different, but none the less subject to conditions which must be very carefully considered in connection with this St. Lawrence project. The Province of Quebec is only at the beginning of its development, and it is hardly possible to forecast exactly along what lines such development will take place; but the northern parts, which are yet almost unexplored, contain much mineral wealth that must inevitably be developed in the course of time and there are large timber and agricultural resources likewise awaiting development. Further, the Province contains sources of hydro-electric energy that are certainly much more than sufficient for its present needs, and that may well afford all the power required for mining and other developments for a long time to come. In the majority of cases the potential industrial developments are placed nearer the undeveloped power resources of Northern Quebec than to the sites referred to in the paper. It is certainly more likely that the power required for opening up what must

develop some day into one of the most important metalliferous areas on the whole Continent will be supplied from the sources near at hand before having recourse to the more remote potential supply from the upper reaches of the St. Lawrence.

Most engineers will regret that the utilization of this wonderful water system cannot be treated as a purely economic and technical problem. It is unfortunate that politics, international as well as national, are so dominantly involved, for the considerations thus brought into the subject are not those best calculated to advance the interest of humanity. Be that as it may, however, the situation remains. The writer is not concerned here with political matters except so far as their influence on the large engineering proposition is concerned. From his examination he has reached a few simple conclusions:

1.—As the greater part of the channel (including most of the proposed power development) of the St. Lawrence is entirely within the Province of Quebec, the attitude of that Province must be inevitably a dominating factor in the scheme.

2.—Quebec opinion generally is that the Quebec Port of Montreal would lose much of its trade if shipping were free to pass on up to the head of the Great Lakes.

3.—The people of Quebec realize the potentialities of their Province and also the overwhelming importance, living as they do in a cold country with no coal deposits, of controlling fully all the sources of power they have, and they are not likely to agree to any proposal for generating power for export.

4.—With the passing of time the industrial activities of Quebec will extend more and more toward the north and northeast, that is, in general, down the St. Lawrence away from the part of the river available for power generation.

That the opening of a waterway for large ships up to Lake Superior would injure the Port of Montreal, the writer does not believe. To extend navigation above Montreal would doubtless divert a certain amount of traffic from that port to points above it; but, on the other hand, the waterway may quite well be expected to compensate by creating traffic not now existing and to bring into being new lines of commerce, from which Montreal will benefit in common with all the other ports along the waterway.

On the question of power development, however, the writer does not believe that at present progress will be served best by the St. Lawrence plan. Doubtless it will come, for ultimately the world will not be able to afford to let any possible source of energy run to waste; but before this time, much more development will have occurred in the northern parts of the St. Lawrence Basin, and the sources of power in these areas may well be utilized before economic necessity brings about the harnessing of the St. Lawrence. Development is gradually pushing seaward along the north bank of that river, and power plants exist on a number of its northern tributaries. The writer believes that there is greater probability that this development will continue than that the St. Lawrence power plan as outlined in the paper will be undertaken in the near future.

He does not lose sight of the fact that on the south side of the St. Lawrence, in a direction east from Montreal, there is an area of constantly increasing industrial development. Here, however, the proximity of Montreal on the one hand and of industrial areas over the American border on the other has been the cause of growth rather than the exploitation of natural resources that must ultimately direct the trend of progress in Northern Quebec. If, as seems likely, export of power is prohibited by the Government of Quebec, it is open to question whether the territory south of the St. Lawrence will be able, for many years to come, to utilize more than a small fraction of the power developed under the author's proposals.

Regulation of the Great Lakes in the interests of lake navigation is obviously a matter that stands on quite a different footing. There is everything to be said in favor of proposals that seek to maintain the water levels of the various lakes to relatively constant heights; but this is a matter that can be considered apart from the waterway proposal or the development of power on the St. Lawrence.

Francis C. Shenehon,* M. Am. Soc. C. E. (by letter).—The writer is grateful for the discussions, which supplement and illuminate his paper. These are mostly fair-minded and helpful in evaluating the several elements entering into this great project.

Let it be set down that the writer is not promoting this Waterway to the Sea. Instead, he states its ultimate economic desirability and therefore its inevitability. And he does not fix an early date of completion, because too many political and regional elements arrest the free play of economic forces; but he does anticipate immediate action on the restoration, deepening, and maintenance of the right of way of the Great Lakes. And he does advocate immediate action on the International portion of the Rapids Section of the St. Lawrence River, extending from above the Galops Rapids to the head of Lake St. Francis, primarily as a water-power project, but so developed that the ultimate deep-draft waterway will be provided for adequately.

When 25-ft. drafts become available in the Great Lakes System, in the Welland Canal, and through the International St. Lawrence River, the remaining developments, between Lake St. Francis and Lake St. Louis and through the Lachine Rapids to Montreal, will follow in due course, possibly depending on the time when additional electric energy is needed in Canada.

It appears unnecessary to answer any discussions bearing on the commercial feasibility of the navigable waterway. The paper itself is clear and explicit as to the writer's views, which are still unchanged. The blending of navigation and water power on the St. Lawrence River is obvious. To what extent the opulence of the energy element will subsidize the transportation element is a matter of opinion. The utilization of all the water power as a commercial enterprise is certain; the utilization of the navigable waterway to the limit of its capacity is still uncertain. The water which will create untold millions of kilowatt-hours and horse-power-years, comes in largest part from Lakes Superior and Michigan-Huron and represents the contributions

* Cons. Engr., Minneapolis, Minn.

of the States of Minnesota, Wisconsin, and Michigan, and the middle and western portions of the Province of Ontario. The writer proposes to utilize, in the interest of larger continuous flow in the Niagara and St. Lawrence Rivers, the storage capacity of these great reservoirs. Navigation interests do not view favorably any draw-down in these reservoirs for the benefit of water power; and water-power interests do not view favorably any subsidization of navigation.

It is significant that navigation is the vital economic element for the United States rather than water power; and that for Canada, water power is the vital economic element rather than navigation. More than 90% of the commerce of the Great Lakes is American. With equal division of water at Niagara Falls and on the International Section of the St. Lawrence, approximately 67% of the ultimate water power will be Canadian.

Notwithstanding lesser commerce and lesser population, the Canadians are showing high optimism and enterprise in extending deep drafts down to Lake Ontario and into the Upper St. Lawrence by the expenditure of more than $100 000 000 in the New Welland Canal. On the other hand the United States—with a lake commerce eight times greater, with a population in the States bordering the Great Lakes more than four times greater and in the country as a whole more than ten times greater—has had neither the optimism nor the enterprise for a quarter of a century to secure and maintain even reasonably good drafts in this vital internal waterway; and has added no link—aside from the Davis and Sabin Locks at the "Soo"—to the ultimate waterway to the sea. And the Canadians have outstripped the United States in the volume of developed water power in the Niagara and St. Lawrence Rivers.

In reply to the suggestions of Mr. Hogg, reference is invited to the judicial Engineering Reports (1925) of the Board of Review of the Sanitary District of Chicago. This Board was the most eminent group of engineers ever called together to investigate a great sanitary and hydraulic problem. It is well to point out the significant fact that the diversion of 50 000 cu. ft. per sec. in St. Marys River has yielded some beneficial horse-power years; and after the obvious engineering device of regulating works became effective, has made Lake Superior better for navigation. The local conservation use of the water is therefore twofold; and, in addition, the reservoir value of Lake Superior will eventually help to stabilize the energy output of the St. Lawrence River and the navigational use of the St. Lawrence Waterway. In any use of the water of the Great Lakes true economic analysis must balance the value of working water against wasting water; or must compare the present worth of water serving several purposes with the present worth of water to serve dual purposes twenty or thirty years hence. Engineers may leave certain legal questions to lawyers and statesmen and evaluate the service of the water, after regulating works have restored losses of depth and created storage to conserve in the future the wastes characterizing the past and the present.

Perhaps the writer's proposal to stabilize the flow of the St. Lawrence River by utilizing the reservoir capacity of Lakes Superior, Michigan and Huron, with some sacrifice of navigable depths in these lakes, represents

more cosmopolitanism than patriotism. In analyzing a physical problem it is not easy for the engineering mind to give due weight to such intangibles as political boundaries and regional psychology. It may be well, however, to retain a detached world citizenship, leaving to statesmen and diplomats the allocation of the economic benefits possible with the best physical solution. Perhaps optimism and enterprise should be accorded the little sacrifice suggested.

The writer's thesis might be entitled "Conserving the Water Wealth of the Great Lakes System"; because in the paper itself and in the discussions evoked the value of the water and the ways of storing and utilizing it are developed. The water is valuable for navigation within the lakes—to the Gulf of St. Lawrence and to the Gulf of Mexico—and for water power in the Niagara, St. Lawrence and Illinois Rivers.

Hydrological events, the views of discussing engineers, and months of further study have modified in some respects the tentative conclusions of 1924. The chief hydrological event is the unprecedented prolongation and accentuation of the period of meager supply in the Great Lakes System, beginning in 1921 and still (March, 1926) prevailing. The mean supply for 35 years—1891 to 1925, inclusive—is 231 500 cu. ft. per sec. The supply and the St. Lawrence River outflow for the past 5 years are:

Year.	Mean supply, in cubic feet per second.	Mean outflow, in cubic feet per second.
1921	198 000	229 000
1922	215 000	229 000
1923	204 000	214 000
1924	197 000	221 000
1925	168 000	209 000
Mean	195 200	220 400

Assuming that the levels of all the lakes were maintained without any accumulated lowering and that 9 000 cu. ft. per sec. were diverted into the Illinois River, the mean outflow in the St. Lawrence River would have been 186 200 cu. ft. per sec. Conversely, assuming that the St Lawrence flow had been maintained uniform with a volume of 220 000 cu. ft. per sec., the levels of all the lakes would have been lowered 2.0 ft. to supplement by storage the deficient supply; and in order to recover the 2.0 ft. of surface-level loss in these 5 meager years, it will be necessary—with fixed St. Lawrence River outflow—to have 5 years of opulent supply as much in excess of normal as the meager years are below normal. Such a compensation in hydrological fortunes is vastly improbable.

It is interesting to note the actual maximum and open-season minimum monthly outflow volumes of the St. Lawrence River in this period of comparative water famine as given in Table 7. The largest unregulated outflow comes in months when the Harbor of Montreal is deep because of the large Ottawa River discharge; and the smallest outflow comes in the fall months when navigation needs ample river volume.

In these fairly generous outflow volumes for the St. Lawrence River is an excellent illustration of the antithesis of uniformity of reservoir levels and uniformity of outflow volumes. In 1925 the supply is only 73% of normal, but the outflow is more than 90% of normal. The reservoirs of the lakes automatically waste the waters of the lakes to the ocean. The utilization of regulated Lake Ontario alone for storage, with the natural automatic drop in the big upper reservoirs, would have sufficed to maintain a reasonably uniform outflow in the St. Lawrence River. In the years 1926 and 1927, however, the St. Lawrence outflow must be meager, unless extraordinary water abundance prevails; because the St. Clair River with the present low Michigan-Huron elevation cannot outflow its normal supply. This must cause a deficit equal to the volumes to be stored in Lakes Michigan-Huron. This deficit in the St. Clair River contribution to the St. Lawrence flow will lessen each year until Lakes Michigan-Huron reach some elevation such as 580.2 ft. in the beginning of the year 1936. The elevations of the various lakes are as given in Table 8.

TABLE 7.—Maximum and Minimum Open-Season Discharges of the St. Lawrence River.

Year.	Month.	Outflow, in cubic feet per second.	Month.	Outflow, in cubic feet per second.
1921	May	253 000	November	213 000
1922	June	258 000	December	210 000
1923	July	237 000	November	203 000
1924	June	244 000	December	208 000
1925	May	230 000	October	202 000

The problem of the co-ordinated regulation of the Great Lakes is the adjustment between the maintenance of the best surface elevations and the largest practicable minimum outflow in the St. Lawrence River. The degree of uniformity in the Niagara River is of relatively lesser importance, because water power may use a fixed volume of, perhaps, 100 000 cu. ft. per sec., while the volume applied to the preservation of the scenic grandeur of the Falls may be diminished in times of restricted flow.

TABLE 8.—Elevations of the Great Lakes.

Lakes.	Elevations, in Feet.		Deficiency, in feet.
	January 1, 1926.	Desirable.	
Superior	600.2	602.5	2.3
Michigan-Huron	577.5	582.0	4.5
Erie	570.2	573.0	2.8
Ontario	244.4	247.0	2.6

If the volume of prime flow in the St. Lawrence had been limited to 220 000 cu. ft. per sec. instead of the 230 000 cu. ft. per sec. computed in Table 2, the elevations of the Great Lakes on January 1, 1926, would have been as follows:

Superior, 601.0 ft.; Michigan-Huron, 580.5 ft.; Erie, 571.5 ft.; and Ontario, 245.5 ft. The present unregulated deficiency, spread uniformly over all the lakes, would be 3.4 ft. Under co-ordinated regulation with a minimum flow in the St. Lawrence of 220 000 cu. ft. per sec., the deficiency would have been only 1.5 ft. The difference of 1.9 ft. represents the gain in navigational depth possible with co-ordinated regulation. Its value in the year 1926 would be not less than $10 000 000.

It is illuminating, if hazardous, to forecast the regain of surface elevations in Lakes Michigan-Huron based on average supply conditions and unregulated outflow in the St. Clair River. In making this forecast (Table 9) it is assumed that the restoration of regulated Lake Superior will be secured at the rate of about 2 in. per year—representing storage at the rate of 5 000 cu. ft. per sec.

TABLE 9.—Assumed Return of Lake Elevations.

Year.	Elevations, in Feet, January 1.		St. Clair River.	
	Superior.	Michigan-Huron.	Outflow, in cubic feet per second.	Quantity below normal, in cubic feet per second
1926	600.2	577.5	140 000	45 000
1927	600.4	578.5	156 000	29 000
1928	600.6	579.1	166 000	19 000
1929	600.7	579.5	172 000	13 000
1930	600.9	579.7	176 000	9 000
1931	601.1	579.8	.....	.....

Forecasting the unregulated outflow in the St. Lawrence River on the basis of normal or average supply, the mean volume of discharge for 1926 should be about 185 000 cu. ft. per sec., and for 1927 about the same quantity. It is an even chance that it will be somewhat greater and an even chance that it will be somewhat less. A portion of this outflow must come from the further draining and lowering of Lakes Erie and Ontario.

The serious deficiency in water supply of the past five years (1921-25) with resultant low lake levels and crippled transportation capacity demonstrates clearly the need of engineering control to conserve in years of abundant yield the waters of the Great Lakes and to expend by budgeted release in lean years the accumulated savings. The present concern over the depleted condition of the lakes had its parallel in the low-water period of 1895-96. The engineering remedy proposed then—more than a quarter of a century ago—was regulating works in the Niagara River.

Will regulation work? Colonel Pillsbury expresses doubts and Mr. Sabin after more than thirty-five years spent in Great Lakes problems—much the same as the writer's years of experience with these problems—is not sure. It is certain that the natural automatic operation of the lakes has failed to maintain the best navigable waterway.

Mr. Sabin refers to the years 1910 and 1911 as illustrating the complexity of regulation. As a matter of fact a detailed analysis of the hydrology of

these years—and of 1912 as well—demonstrates the practicability and necessity of regulation. In 1910 the supply naturally tributary to the St. Clair River was 68 000 cu. ft. per sec. less than the normal. The reserve storage of Lake Superior contributed 25 000, and Michigan-Huron, 28 000 cu. ft. per sec. The supply from the Erie and Ontario Basins was a little greater than normal. The outflow to salt water was at the rate of 234 000 cu. ft. per sec. Lake Superior by January 1, 1911, had fallen to Elevation 601.24 and Michigan-Huron to Elevation 579.33. Had super-regulation been in effect, these elevations would have been 601.8 and 581.3 ft., respectively. The gain of 2 ft. for vessel drafts in the St. Marys River and Lakes Michigan-Huron is significant. During 1911 the lakes would have risen slightly; and complete level restoration would have been accomplished in the generous supply year of 1912.

Mr. Sabin clings to the thought of single-river regulation illustrated by the works in St. Marys River—necessitated by water diversions—and suggests a tryout of regulating works in the Niagara River. Such works will help, but the principle to be followed if works are to be built in sequence is to build the lowermost works first; because the operation of works in the outlet of Lake Ontario cannot affect adversely the upper lakes. The operation of the Lake Superior outflow works has at times damaged the lakes below; and works in the Niagara River alone may damage Lake Ontario and the St. Lawrence River. Super-regulation, with works in the St. Clair, Niagara, and St. Lawrence Rivers making effective the existing works in the St. Marys River, is the sound engineering solution for the Great Lakes System. The cost of such works and the cost of operating them is negligible, because the vast financial importance of water transportation warrants adequate maintenance of this internal waterway, even aside from its ultimate destiny in overseas commerce.

Does the operation of the scheme of super-regulation require an intelligence transcending that required in the operation of a super-power system or other engineering work? Will "omniscience" be required, as suggested by Mr. Sabin? Four elements enter the problem: First, the release of a reasonably constant minimum volume of outflow through the regulating gates in the St. Lawrence River; second, the utilization of the forebays of Lakes Ontario and Erie as equalizing basins; third, the release of water through the St. Clair River to supplement the Erie and Ontario supplies in maintaining the fixed minimum volume of outflow through the St. Lawrence River; and, fourth, the operation of the reservoirs of Lakes Superior and Michigan-Huron as a unit of storage, with restrictions or enlargements of flow in the St. Marys River to keep the two basins in step with each other.

A detailed demonstration of the operation of the super-regulation of the Great Lakes System is beyond the limitations of this paper. The works to be utilized for outflow control do not need to be precisely detailed now. The writer would add to the works at the head of the St. Clair River a fourth expansion basin and navigable pass (see Fig. 5) to lessen current velocities and secure larger control volume.

No menace of unduly high water will exist in Lakes Erie and Ontario, because the relatively small areas of these lakes permit rapid disposal of surplus water by excess capacity channels; and these channels may be easily secured. Nor will any menace of unduly high water exist in Lakes Superior and Michigan-Huron, provided the base plane of control is conservative. Lake Superior has ample excess outflow capacity in lateral water-power canals. If a volume of 40 000 cu. ft. per sec. flows continuously through these canals, by-passing the rapids in St. Marys River (and this volume is never diminished), the value of the excess outflow capacity is unquestionable. And this same principle of continuous supplemental outflow creating excess capacity applies to the Chicago Drainage Canal and to the Welland Canal. Mr. Sabin questions the specific application of this principle.

Colonel Pillsbury raises the question of legality in stabilizing the levels of the lakes, suggesting that a riparian owner may insist on only the frequency of high-water periods shown by the hydrograph since 1860. The rights of riparians to natural sequences in lake levels and in volumes of river flow affect many engineering undertakings. In cases where the interests of shore owners in submerged lands clash with the necessities of navigation, the right of the riparian to compensation disappears. (See the case of United States *vs.* The Chandler-Dunbar Company.) The governmental right to modify the natural outflow of the St. Lawrence River in the interest of navigation and water power must be supreme.

The possibilities of the St. Lawrence Waterway to the Sea will continue to kindle the imagination. Some of the vital engineering principles are recorded in this paper.

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Paper No. 1581

MUNICIPAL WATER SUPPLY PROBLEMS OF ATLANTA, GEORGIA*

By Paul H. Norcross,† M. Am. Soc. C. E.

With Discussion by Messrs. J. N. Chester and Wingfield Nisbet.

Synopsis

The water supply of the City of Atlanta, Ga., is of interest from several angles, as it is the largest development of a surface water supply in the Southeastern States, furnishes a water unusually free of dissolved solids and, therefore, peculiarly well suited for industrial uses, and contains, in addition, some unique features of design and operation.

The supply is derived from the Chattahoochee River, a Cherokee Indian name meaning the "Chatter", or "Hooting of the Owl". Its source is on the eastern slopes of the Blue Ridge Mountains, at elevations ranging from 3 000 to 4 000 ft., and it flows in a southwesterly direction, through a basin singularly free of pollution.

The supply works consist of a 48-in. intake conduit, a steam-operated river pumping station, having a capacity of 106 000 000 gal. per day, pumping against a maximum head of 275 ft., including friction, through three parallel conduits, 30, 36, and 48 in. in diameter, respectively, each approximately 18 500 ft. long, into two raw water or settling basins, having a combined capacity of 393 000 000 gal.

From the raw water basins, the water flows by gravity through a battery of five coagulating basins, having a combined capacity of 13 000 000 gal., and then to two gravity filtration plants, operated in parallel. One plant, the original installation, is of the pressure type, containing 48 units, 12 vertical and 36

* Presented at the meeting of the Sanitary Engineering Division, Atlanta, Ga., April 10, 1924.

† Cons. Engr., Atlanta, Ga. Mr. Norcross was drowned at the time of the sinking of the *Norman* in the Mississippi River on May 8, 1925.

horizontal, having a total rated capacity of 21 000 000 gal. daily, and the other, completed in 1923, a plant of equal rating of the reinforced concrete gravity type, having 7 units, making a total combined rated capacity of 42 000 000 gal. daily.

From the filtration plants, the water passes through a 60-in. conduit, to a new reinforced concrete covered reservoir, of 10 000 000 gal. capacity, which is cross-connected to a steam-operated high-service pumping station, having a rated capacity of 105 000 000 gal. per day, delivering water direct to the distribution system against a maximum head of 100 to 120 lb.

Some of the features of design and operation include both inverted and overhead siphons of the raw water conduit, either in tunnel or aqueduct; flexible operation of the raw water conduits, raw water basins, etc.; a new, open-gallery type filtration plant and appurtenances, and a new reservoir and turbo-centrifugal pumping units that have developed unusually high efficiencies for loads varying from 50 to 110 per cent.

The successful and economical operation in parallel of two widely different types of filter plants is worthy of comment.

The problem of keeping pace with increasing demands, due to the rapidly growing population and industrial, commercial, and municipal consumers, while maintaining service, is being accomplished without interruption thereof.

Introduction

The purpose of this paper is to describe briefly the history and development of Atlanta's municipally owned water utility which, because of its importance and certain unusual features of design and operation, seems worthy of attention, and also to stimulate renewed interest in the broad field of water treatment. This subject is of increasing importance, particularly in the southeastern section of the United States which, due to its equable climate, wealth of raw materials, wonderful agricultural possibilities, and growing economic advantages, is gradually assuming supremacy in many fields, including the textile industry, and where numerous other classes of manufacturers and commercial concerns are prospecting for locations.

The extent of domestic and industrial use of public water supplies in the United States has been the subject of an investigation by the United States Department of the Interior, and the result of this study is set forth in a paper entitled "Utility of Public Water Supplies in the United States",* which publication is a most valuable addition to water supply data. This paper serves to emphasize the value of water free of minerals especially in industrial lines, including boiler plants, and "the manufacture of soft drinks, bakery products, and other foods." As Atlanta is the "birthplace" of probably the most widely known "soft drink" manufactured to-day, the significance of this statement is obvious.

It is, of course, needless to emphasize the importance of a high sanitary quality for public water supplies, or to review the decrease in "typhoid toll", due to advance in the science of water treatment. It is worthy of note, how-

* *Water Supply Paper No. 496*, U. S. Geological Survey.

ever, that the growing and more exacting demands of the consumer of to-day is for water not only free of suspended matter and polluting organisms, but also one more acceptable from the standpoint of mineral content.

The distribution of "hard" and "alkali" waters of various degrees of hardness throughout the country is general, occurring in twenty-seven States, comprising probably 75 to 80% of the total area of the United States. The localities providing so-called "soft water" are confined to nineteen States located in three groups, situated in the Northwest, Northeast, and Southeast, respectively, with the last predominating in area. Georgia occupies the geographical center of this latter area. That part of those States in the Southeast above the coastal plain obtain their supplies principally from the eastern and southeastern slopes of the Blue Ridge and Appalachian Mountain Ranges, and Atlanta's supply ranks foremost in size and character of development in this group.

It is not intended to review in detail or to describe, except briefly, those parts of the utility which are commonplace, but it is the desire of the writer to present a comprehensive view of the problem and to describe such parts of the development as seem interesting and unusual.

Historical

The real history of Atlanta's water utility originated in 1872, when the first Water Commission of the city contracted for the construction of a water supply on the head-waters of a stream having its source within the city limits, flowing in a southeasterly direction, and being a tributary of the Altamaha Drainage Basin, with its outlet in the Atlantic Ocean.

This supply consisted of an impounding reservoir on the head-waters of South River, about 3 miles south of the center of the city, of 250 000 000 gal. capacity, covering an area of 51 acres, with a drainage area of approximately 5 sq. miles. The water from this supply was originally untreated but, about 1886-87, Hyatt pressure type filters were installed (probably one of the first pressure type installations in the United States), from which the water was collected in a clear-water basin having a capacity of 800 000 gal. and then pumped direct to the distribution system by steam-driven Holly quadruplex type pumps, one of 2 000 000 and the other of 4 000 000 gal. daily capacity.

This supply, the first water from which was pumped into the mains on September 8, 1875, was supplemented in 1884 by an Artesian well, located in the center of the city, which was drilled through about 2 000 ft. of granite, or gneiss—the underlying bed-rocks of Atlanta. This well produced a limited supply of water from four veins, 102, 225, 450, and 1 160 ft. below the surface, respectively.

Records of water used daily and annually prior to 1888 are very meager, but the total quantity pumped from South River during that year was 629 000 000 gal., supplemented by 10 500 000 gal. from the Artesian well supply, which latter quantity represented the total available from this source. It is interesting to note that all domestic consumers were metered in 1888, and that a uniform charge of 17 cents per 1 000 gal. was made.

As a part of the drainage area of South River was within the city limits, and as unusual growth of the city began between 1880 and 1890, it was soon obvious that South River water, although originally of unusually good quality, was becoming grossly polluted. In addition, the supply available from this stream, with its limited drainage area, was not adequate to meet the existing and increasing demands made upon it. It, therefore, became necessary to obtain another and more ample supply.

In 1889, the city officials engaged the late Rudolph Hering, M. Am. Soc. C. E., to make surveys, investigations, and report on a new supply. Mr. Hering's report submitted under date of October 15, 1889, recommended the development of a supply from the Chattahoochee River. This stream has a drainage area of about 1 500 sq. miles with its head-waters in the Blue Ridge Mountains of North Georgia. It is a tributary of the Apalachicola Drainage Basin and flows in a southwesterly direction into the Gulf of Mexico, which course brings it, at its nearest point, within about 6½ miles northwest of the center of the city. This stream, although carrying large percentages of turbidity during certain periods and nearly always highly colored with clay and silt, was of splendid quality, with only nominal pollution and with a chemical analysis indicating a water almost free of mineral content.

The analysis of Chattahoochee River water, as made by the U. S. Geological Survey, is shown in Table 1.

TABLE 1.—Analysis of Chattahoochee River Water, in Parts per Million.

Total dissolved solids.	Loss on ignition.	Silica, SiO_2.	Iron, Fe.	Calcium, Ca.	Magnesium, Mg.	Sodium and potassium, Na = K.	Bi-carbonate radical, HCO_3.	Sulphate radical, SO_4.	Chloride radical, Cl.	Nitrate radical, NO_3.	Total hardness as Ca Co_3.
32	1.5	10	0.08	2.4	1.1	Na, 2.3 K, 0.6	6.6	7.8	1.8	0.33	11

The topography of Atlanta is peculiar and interesting. It consists of a series of ridges averaging 950 to 1 000 ft. above sea level, that divide the drainage of streams flowing either into the Atlantic Ocean or the Gulf of Mexico. This is an advantage from a sanitary standpoint, but the deep valleys and high ridges naturally affect the pumping problem and add to the cost.

Mr. Hering recommended the construction of a pumping station on Chattahoochee River, with an intake at the point at which it is at present located, just above the confluence of Peachtree Creek, as this latter stream drained a large part of the area within the city limits, and the construction of raw water storage reservoirs, treatment plant, and a high-service pumping station about half way between the river and the center of the city. The point on the river selected for development was about 300 ft. lower in elevation than the highest ridges in the city, thus making necessary two pumping stations to provide adequate pressures. Table 2 gives the elevation of points in the city and on the river.

The wisdom of selecting the Chattahoochee River as the source of the ultimate and final supply for Atlanta has been proved. The character of the region at its source, consisting of unpopulated mountain ranges between 3 000 and 4 000 ft. above sea level, with unusually high annual rainfall, and the fact that it flows through a sparsely settled area and drains practically no towns or cities of size, combined with the excellent chemical quality of its water, are factors in its favor.

TABLE 2.

	Feet above sea level.
Low water, Chattahoochee River, at mouth of Peachtree Creek	758
Raw water reservoir site	985
High levels in city	1 050–1 065
Ordinary level of domestic pressure	1 100–1 150
Fire-pressure level	1 200–1 250

Fig. 1 shows the drainage area of the Chattahoochee River at the intake. It will be noted that no large towns are in its basin. The diagram also shows the location of the South River development—the original public water supply of Atlanta—and the present outlines of the city limits.

Original Plant

The City began the development of Chattahoochee River in 1891 and placed the new supply in service in September, 1893. The original development consisted of a gravity-operated, 48-in. cast-iron pipe intake about 700 ft. long, discharging into an open intake well, and a brick pumping station surrounded by an earth dike to prevent flood overflows. Steam-driven Holly horizontal compound pumps, operated against a suction lift of about 16 ft., delivered the water through a 30-in. cast-iron main, about 18 500 ft. long, to an unlined raw water storage reservoir of 177 000 000 gal. capacity constructed in cut and fill and by damming several ravines.

The water was then passed by gravity through concrete coagulating basins to a battery of horizontal pressure type filters, operated as closed gravity filters with an average operating head of 30 ft., and thence to an uncovered clear water well. The total loss of head between high-water level in the raw water reservoirs and the high-water line in the clear water well was 40 ft. From there, the water was conducted to the high-pressure pumping station known as the Hemphill Avenue Station. The reservoirs, coagulating basins, filter plant, and pumping station were located on the main line of the Southern Railway about three miles northwest of the center of the city. This station was similar in type, design, and equipment, to the South River Station.

It is interesting to note that the old Hyatt filters, twelve in number, which were of the vertical pressure type, were removed from the South River Station and installed, and operated in parallel, with the horizontal filters at the new plant.

Plant in 1920

Between 1893 and 1920, this plant was expanded along the lines of the original development and, in 1920, the pumping equipment at the Chattahoochee River Station consisted of two 10 000 000-gal. Holly horizontal compound pumps, one 18 000 000-gal. Holly horizontal compound pump, one 10 000 000-gal. Worthington motor-driven centrifugal pump, and one 28 000 000-gal. DeLaval turbo-centrifugal pump, the last having been installed in 1918-19, to meet emergency war demands due to the construction of Camp Gordon, near Atlanta.

The raw water conduits consisted of the original 30-in. and an additional 36-in. cast-iron main, delivering water against a head of 260 to 290 ft., including friction, to two raw water reservoirs with capacities of 177 000 000 gal. and 216 000 000 gal., respectively.

The treatment plant consisted of five coagulating basins having a combined volumetric capacity of about 13 000 000 gal., and the filter plant contained thirty-six horizontal type pressure units of 500 000 gal. daily capacity each, and twelve Hyatt vertical type pressure units with rated daily capacities of 250 000 gal. each, making a total combined rated capacity of 21 000 000 gal. daily, discharging into two circular, open-type, clear water wells having a combined available capacity of 2 000 000 gal.

The Hemphill Avenue, or High-Service, Station, contained two 10 000 000 and one 15 000 000-gal. horizontal compound pumps, one 20 000 000-gal. vertical triple expansion pump, and one 30 000 000-gal. DeLaval turbo-centrifugal pump installed in 1920.

A brief résumé of the demands of service and the capacities of several of the elements composing the plant in 1920 should aid in a clear understanding of the major problems that confronted the city at that time. These are shown in Table 3.

TABLE 3.

	Gallons daily.
Average rate of consumption (1920)	24 000 000
Maximum rate of consumption (1920)	40 000 000
Maximum capacity of supply mains from river (1920)	33 000 000
Rated filter capacity (1920)	21 000 000
Clear-water storage (1920)	2 000 000

In addition to the fact that parts of the plant were wofully inadequate in size and arrangement to meet existing demands, much of the equipment had reached the end of its useful life, or was inefficient and obsolete.

Investigations and Studies

In 1920, the City of Atlanta authorized complete surveys and investigations to determine the necessary improvements and extensions to the water supply of the city. This investigation included study of the river intake, the pumping station at Chattahoochee River, supply mains between the river and raw water reservoirs, coagulating basins, filter plant, and high-service pumping

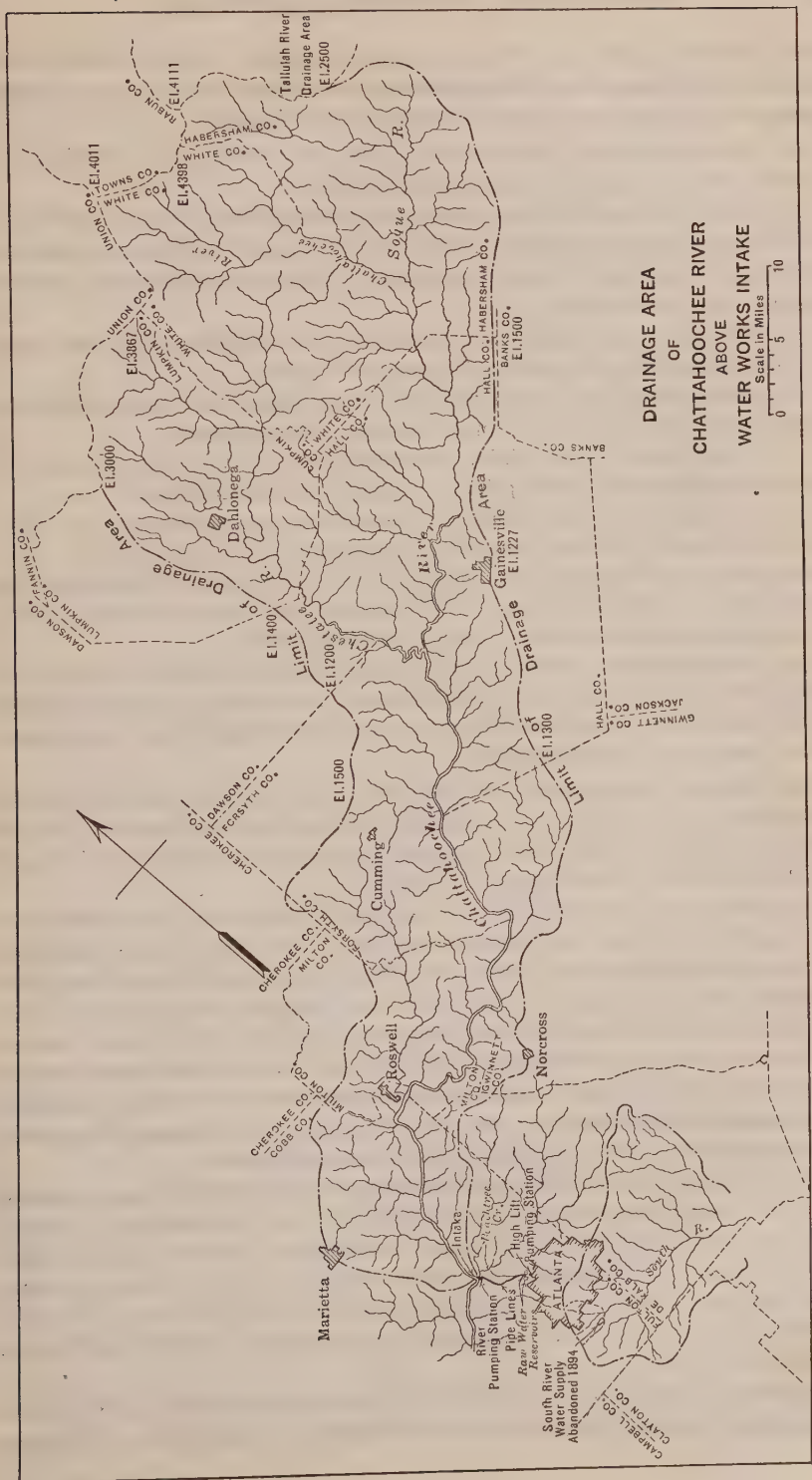

DRAINAGE AREA
OF
CHATTAHOOCHEE RIVER
ABOVE
WATER WORKS INTAKE
Scale in Miles
0 5 10
Tallulah River Drainage Area El.2500
El.4111
El.4011
El.4398
El.3867
El.3000
El.1400
El.1200
El.1500
El.1300
El.1227
El.1500
Dahlonega
Gainesville
Cumming
Roswell
Norcross
Marietta
Soque R.
Chattahoochee River
Limit of Drainage Area
Chestatee R.
Intake
High Lift Pumping Station
Pipe Lines
Raw Water Reservoirs
River Pumping Station
South River Water Supply Abandoned 1894
ATLANTA
South R.
RABUN CO.
HABERSHAM CO.
WHITE CO.
TOWNS CO.
UNION CO.
LUMPKIN CO.
HALL CO.
BANKS CO.
FANNIN CO.
DAWSON CO.
FORSYTH CO.
CHEROKEE CO.
MILTON CO.
GWINNETT CO.
JACKSON CO.
COBB CO.
CLAYTON CO.
CAMPBELL CO.
FULTON CO.
DE KALB CO.

FIG. 1.

station, including all connections and appurtenances. These studies were started in the latter part of 1920 and extended through 1921 and, as a result, recommendations were made, which briefly were as follows:

(*a*).—Construction of an impounding dam, new intake, and pumping station to supplement the existing intake and station.

(*b*).—Remodeling of the Chattahoochee River Station, including additions and changes in the boiler plant, remodeling of steam headers, new chimney, installation of new and efficient pumps, and the re-arrangement of suction, discharge lines, and headers.

(*c*).—New raw water supply main from the River Station to the raw water reservoirs, paralleling the existing 30 and 36-in. mains.

(*d*).—Installation of gate-house and cross-connection, controlling and making more flexible the operation of the raw water reservoirs.

(*e*).—Construction of a mixing chamber and a new chemical house for the proper treatment of water prior to coagulation and filtration.

(*f*).—Addition of a filter plant of the gravity type to the existing filter plant and the construction of a covered reinforced concrete clear water reservoir of ample size to meet peak demands, constructed at a higher elevation than the old reservoirs to reduce cost of pumpage, and the abandonment of the old clear water reservoirs, except for use in emergencies.

(*g*).—Remodeling of the Hemphill Avenue, or High-Service, Station, including changes in boiler plant, new chimney, remodeling of steam headers, and the installation of new and efficient pumping equipment, and re-arrangement of suction and discharge lines.

The foregoing recommendations were the result of an extensive study and investigation of the physical plant and operating costs and records, supplemented by studies of population, pumpage, consumption rates, and forecasts of future demands. (See Figs. 2, 3, and 4).

It is worthy of mention that although the demands on the water-works had outgrown the plant, the sanitary quality of the treated water furnished the consumers had remained, with few exceptions, at a high average. Red or colored water, however, was becoming more frequent, and it was obvious that the intake facilities, raw water pumping plant, raw water supply mains, pre-filtration and coagulating facilities, filter plant, and clear water storage had reached their ultimate capacities, and that the high-pressure pumping station needed early attention. A break-down at any one or more points would have precipitated a water shortage with disastrous results.

The preliminary cost estimates of the proposed improvements approximated $4 600 000, which included an item of $150 000 for the extension of mains within the city. Due to the customary impecunious condition of municipal treasuries, and the crying needs of other city departments, the amount provided for these water-works improvements was only $3 100 000, of which $250 000 was to be provided by a special tax assessment and $2 850 000 by a bond issue.

All these funds, except about $100 000, were made available in November, 1921, and in January, 1922, the actual construction work began. With the

exception of certain items of the improvements that are not yet under construction, due to delays and the failure to provide funds, several of the major improvements are either completed and in operation, or will be within short time.

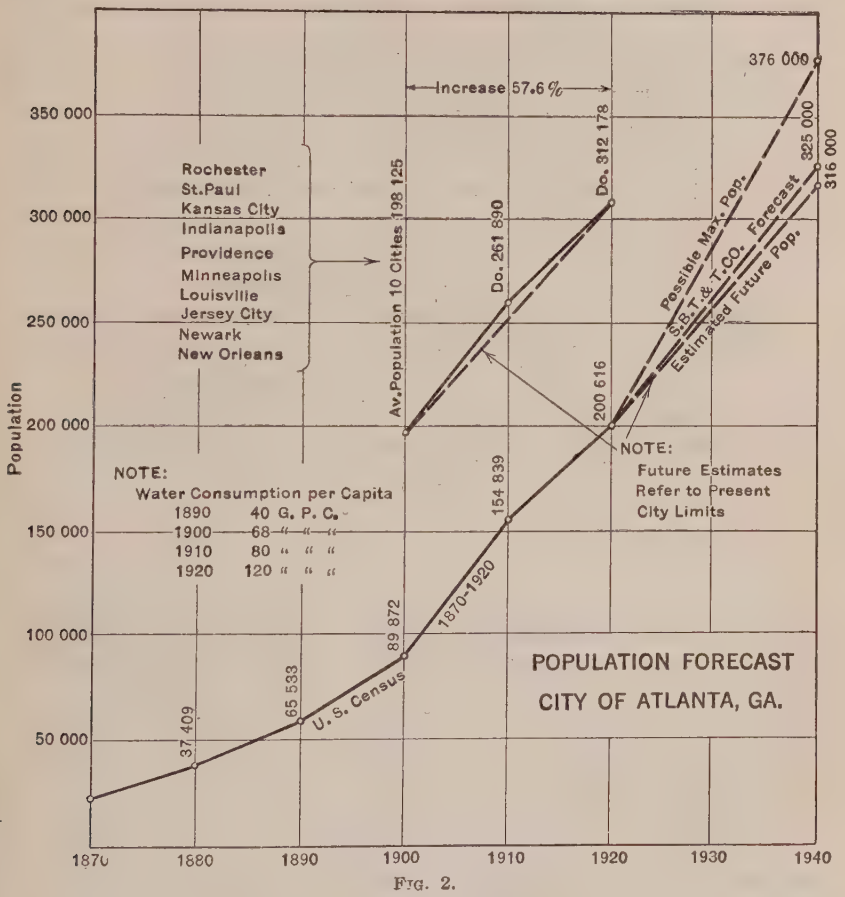


FIG. 2.

DESCRIPTION OF IMPROVEMENTS

New Intake and Impounding Dam.—The present intake which consists of one line of 48-in. cast-iron pipe terminating in a brick pier, with iron grating inlets, located in the bed of the river, is not submerged completely at all periods, as low-water conditions often partly uncover it. The normal water level of the stream has been raised by constructing a rip-rap and a pile dam, but due to increasing demands, it is extremely important that new intake works be provided at the earliest possible date. The present intake line is poorly located and crosses under Peachtree Creek, a stream that flows into the river just below the intake, and is otherwise not susceptible of much improvement. However, within 2 miles up stream, there are several ideal locations for the new intake works. When the new intake is constructed, the

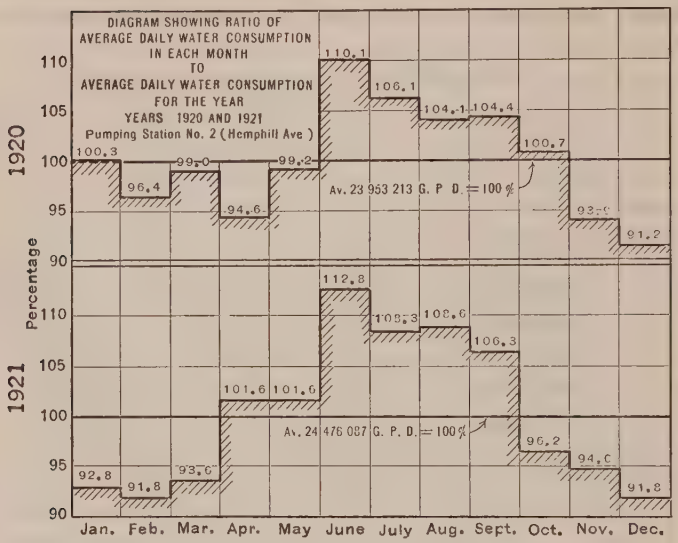
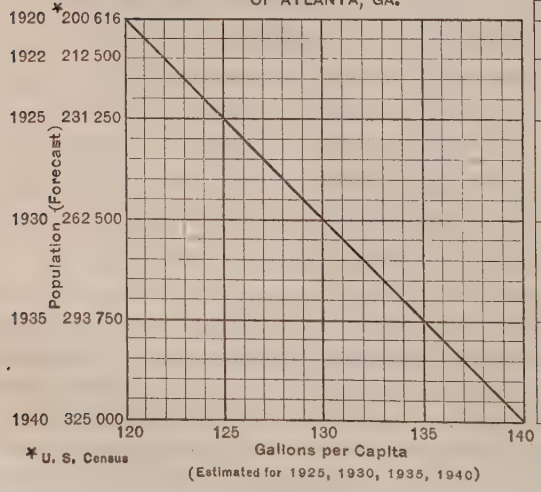


FIG. 3.



HEMPHILL STATION			RIVER STATION
Av.	Av. Max.	Av. Min.	
M.G.D.	M.G.D.	M.G.D.	M.G.D.
23.95	33.53	14.37	
25.92	36.28	15.55	28.8
28.90	40.46	17.34	32.11
34.13	47.78	20.48	37.92
39.65	55.51	23.79	44.05
45.50	63.70	27.30	50.55

NOTE:
Estimates under "River Station" include Allowance for Leakage, Evaporation, Etc.
NOTE:—"Av. Max." = 140%
"Av. Min." = 60%

FIG. 4.

City will also build an impounding dam to provide necessary storage to meet future requirements during low-water and heavy draft periods. The Chattahoochee River, due to its mountain source and the characteristics of its basin and run-off, has wide fluctuations of flow. This variation is further complicated by a hydro-electric plant about 11 miles up stream, which impounds water at night and discharges through its water-wheels during daylight hours. These factors emphasize the necessity for an impounding dam. The head created by this dam will reduce the pumping lift and provide hydraulic power which will be utilized by the City for pumping, thus reducing operating costs. The construction of the new dam and intake, with connections to the Chattahoochee River Station, has been delayed by negotiations between the City and riparian owners and the usual lethargy of municipalities in providing the necessary funds.

It is proper to state here that the selection of the original intake and pumping station sites was probably influenced by considerations of economy and expediency. The fact remains, however, that the present arrangement and design preclude additional expansion along good engineering lines. The location of the pumping station about 25 ft. below the flood-plain of the river, thus requiring the maintenance of dikes and emergency attention to prevent flooding through the intake well at high-water periods, supplemented by the fact that the sole dependency for supply is on one single inlet separated from the pumping station by an intervening stream, is a condition requiring prompt correction.

Remodeling Chattahoochee Station and Appurtenances.—To provide capacities, economies, and flexibility, and, at the same time, maintain service, it was found necessary to make radical alterations and extensions and additions to the Chattahoochee Station. These improvements consist of a new radial brick chimney, changes and additions to the boiler plant, re-arrangement of steam headers, abandonment and removal of the two 10 000 000-gal. pumps and the one 10 000 000-gal. motor-driven unit, installation of efficient pumping units, remodeling of suction and discharge lines and headers, and the reconstruction of the interior of the pumping station.

Practically all this work has been accomplished without interruption of service, and the present equipment of the Chattahoochee Station consists of the following:

One boiler plant, having a rated capacity of 1 850 h.p.
One 18 000 000-gal. compound condensing Holly unit (installed in 1904).
One 28 000 000-gal. DeLaval turbo-centrifugal unit (installed in 1919).
Two 30 000 000-gal. DeLaval turbo-centrifugal units (installed in 1918-19).

All the pumps lift water from the open suction well and discharge either into a new 48-in. header, or directly into the raw water supply mains.

The two 30 000 000-gal. turbo-centrifugal units were purchased in competition with 15 000 000-gal. turbo-centrifugals and reciprocating units. The load on the Chattahoochee Station is constant and susceptible of little varia-

tion, consequently, the use of smaller units represented no economy. The guaranties made by the successful bidder were based on the use of steam at 175 to 190 lb. pressure, with 100° superheat, and are as follows:

110% load	163 400 000	ft-lb. per 1 000 lb. of steam
100% load	166 000 000	" " " " " "
75% load	151 000 000	" " " " " "
50% load	126 900 000	" " " " " "

The units were completed in the fall of 1923. Duty tests were made on them during the latter part of December, 1923, and in January, 1924; in all these tests they have met or exceeded their guaranties and are notable examples of recent development in this type of equipment.

The re-arrangement of the suction and discharge lines provides ample facilities for flexible operation and increased demands. These lines are arranged with the idea of preventing interruptions of service from accidents and other emergencies. In addition, the station is provided with indicating and recording instruments to facilitate efficient operation, and the keeping of proper records.

The present intake facilities have not been increased, principally because the arrangement and physical location are poorly suited for further development. The capacity of the 48-in. cast-iron intake is limited by its size and the head of water over its inlet, which head, as previously stated, varies not only during the wet and dry seasons, but fluctuates daily and hourly due to the existence of the hydro-electric plant of the Georgia Railway and Power Company at Morgan Falls, several miles up stream, which stores the water at night and discharges during the day.

The problem of increasing the water level over the intake is complicated by the existence of a shifting sand-bar, or island, in mid-stream, known as DeFoor's Island, just opposite the water-works plant. In years past, the elevation of the water over the inlet of the intake had been increased by the construction of a rip-rap dam between the east bank of the stream and the Island, and a pile dam with rock-fill, between the north point of the Island and the west bank of the stream. During 1924, additional rip-rap will be added to the existing dams to increase the water level, but this will provide only temporary relief, as it will be necessary to supplement this intake by the construction of another one which will be located up stream at a more suitable site.

New 48-In. Cast-Iron Raw Water Main.—The necessity for new raw water supply conduits was emphasized by the fact that the peak demand which was increasing, exceeded the combined daily capacities of the existing 30 and 36-in. cast-iron mains. Detail surveys of existing and available routes and study of the future demands showed the advisability of installing an additional raw water main not less than 48-in. in diameter, along the same route as the existing mains from the Chattahoochee Station to the raw-water reservoirs.

After receiving competitive bids on steel, reinforced concrete, and cast-iron conduits, the last was selected. The conduit was constructed in sections

and placed in operation as each section was completed. It is now finished and in operation. Construction of this line was somewhat complicated and delayed by the necessity of crossing the tracks of three railroads and two interurban electric railways. Inverted siphons were used under two of the railroads, one siphon crossing eleven tracks and the other nine. The problem of maintaining water-tight conduits was perplexing and constituted a perpetual hazard with an economic loss due to constant leakage and occasional breaks.

The delay in the completion of these railroad crossings was due to a long series of conferences with the railroad officials as to the merits and costs of underground conduits in open cut or tunnels, and overhead crossings, and the existence of previous contracts between the City and the Railroad Company, which required the Railroad Company to pay for maintenance. Eventually, new and equitable contracts were executed, and the remaining portion of the new 48-in. main, between the Chattahoochee River and the raw water reservoirs, is now complete. Fig. 5 is a view of the pipe gantry used in laying the 48-in. cast-iron main.

Some of the more interesting details of this main include the construction of a differential surge tank at the high point on the line, with individual connections to the 30, 36, and 48-in. mains; and the elimination of one of the inverted siphons of both the existing and new mains by the construction of an overhead aqueduct, consisting of a steel span with steel girder approach on one end and a reinforced concrete span on the other end, carrying two lines of 48-in. cast-iron pipe, over the tracks of the Seaboard Air Line Railroad. Fig. 6 shows the differential surge tank and a view of the raw water mains.

The new main is also cross-connected with the 30 and 36-in. mains at four intermediate points with numerous fittings and gate-valves, which permit economical repairs with a minimum loss of time and also provide flexibility of operation.

Gate-House and Cross-Connections, Raw Water Reservoirs.—Previous to 1921, the physical connection between the raw water conduits and the two raw water reservoirs was very limited. To provide flexible operation of the two basins and also permit the lowering of the level of one or both without undue delay, new cross-connections with a centrally controlled gate-house were planned. This was all the more necessary in view of the increased capacities that were being provided by the installation of the new 48-in. raw water main.

As designed and constructed, the gate-house located between the two reservoirs, receives the water from the three raw water conduits through two 48-in. inlet pipes discharging into a common well of reinforced concrete construction. This well is cross-connected by a cast-iron pipe conduit 60 in. in diameter to the raw water reservoirs, the invert of which is 7 ft. below the normal high-water surface of the basins.

The raw-water inlets and the outlets to the two reservoirs are controlled by two 48-in. and two 60-in. sluice-gates, electrically operated, with hand-wheel

connections for emergency, all of which is housed in a substantial brick building.

Mixing Chamber and New Chemical House.—The pre-filtration processes of water treatment have grown in importance and magnitude in recent years. More extensive and detail operating records, supplemented by technical data obtained from numerous tests and investigations stimulated by various agencies, have emphasized the necessity for more scientific and comprehensive methods of preparing the water for filtration.

The Chattahoochee River carries variable and, sometimes, large quantities of suspended matter so finely divided that it will not settle even after a long period. It is necessary, therefore, to treat the water with a coagulant, which facilitates the removal of this finely divided material in coagulating basins. The process generally consists of injecting a solution of aluminum sulfate which due to the normal alkalinity of the water produces the usual flocculent precipitate and deposition of the suspended matter. Good coagulation cannot always be obtained by the use of this chemical alone, as chemical and physical changes in the water, supplemented by the large range of turbidity content due to heavy rainfalls, etc., prevent the adequate formation of the flocculent precipitate and, consequently, the delivery of unclarified water to the filters. In many instances these changes take place rapidly. The effect of these changes are of more importance particularly in water similar to that of the Chattahoochee River, containing a small quantity of dissolved solids.

These varying factors affecting the Chattahoochee River water, including reduction in alkalinity content and intensity of acidity—expressed as the hydrogen-ion concentration—supplemented by high turbidity, have often caused the delivery of discolored and unsatisfactory water to the distribution system, as the failure to clarify the water properly in the coagulating basins and the delivery of partly treated water with a large quantity of suspended matter to the filters, interferes with their normal operation.

The reaction between aluminum sulfate and the carbonate or bi-carbonate of natural waters often sets free a quantity of carbon dioxide, thus making the intensity of acidity great enough to cause corrosion in iron pipes and consequently red water. This is unfit for use by both the domestic and industrial consumer and will eventually damage the pipes.

The facilities for chemical treatment in 1920 were housed in a small brick building, at one end of the coagulating basins. In addition to the chemical and bacteriological laboratory, this building housed two wooden tanks for chemical solutions and provided storage space for alum, lime, etc. Normally, the alum solution was delivered by gravity through two parallel lines of lead pipe about 500 ft. long, to the 60-in. raw water conduit conducting the water from the raw water to the coagulating basins. Facilities for providing artificial alkalinity consisted of emergency preparations of solutions of soda ash, or lime, in one of the chemical tanks. This "hit-or-miss" method, without properly designed structures and equipment, has been the source of much trouble in the treatment of water at Atlanta. In 1921-22, the City installed a dry feed machine for applying lime, at a point adjacent to the 60-in conduit. When the new filter plant was placed in operation, it was necessary to re-

FIG. 5.—VIEW OF PIPE GANTRY FOR LAYING 48-INCH MAIN, LEAD GANTRY IN BACKGROUND, ATLANTA, GA.

FIG. 6.—VIEW OF DIFFERENTIAL SURGE TANK; RAW WATER MAINS, ATLANTA, GA.

arrange the operation of the coagulating basins and advisable to abandon the method of injecting the alum solutions, as inadequate mixing and other factors were preventing the proper removal of suspended matter.

In 1923, a small temporary wooden building for housing two alum tanks and the dry feed lime machine was constructed at the outlet tower of Raw Water Basin No. 2, and experiments were made in the injection of chemical solutions into the entrance of the 60-in. raw water conduit to the coagulating basins. The results obtained justified the change; but the necessity for a structure to facilitate proper mixing under varying conditions prior to coagulation still exists, as occasionally "red" and "muddy" water is produced by the filtration plant.

After careful investigation and study, it was decided to install a gravity-operated mixing chamber, between the raw water and coagulating basins, with a chemical house superimposed over the inlet to the mixing chamber. The design contemplates a structure built in three sections, which can be operated at rates of 30 000 000, 45 000 000 and 60 000 000 gal. per day, respectively, with the detention time element ranging from 7 to 45 min. The length of travel for the several rates of application varies from 1 200 to 3 000 ft., with velocities ranging from 1.8 to 3.45 ft. per sec.

The design gives a most flexible and satisfactory arrangement and will eliminate the delivery of unsatisfactory water to the filters. Bids have been received and work on this structure which is to be reinforced concrete with cast-iron pipe influent and effluent conduits, is to be started as soon as the steps necessary to legal condemnation of the necessary land area are completed.

New Filter Plant and Clear Water Reservoir.—The pressure type filters of the existing filter plant having a combined rated capacity of 21 000 000 gal. per day, discharged into two open circular clear water wells of a combined capacity of approximately 2 000 000 gal. The elevation of water in the coagulating basins is 985.0 and that of the water surface in the clear wells, 945; therefore, 40 ft. of head was lost in operation.

Because of the daily demands in excess of the normal rating of the filters and the limited capacity of the clear wells, it was necessary to vary the rate of filtration to meet the fluctuations in rate of draft or pumpage, and this constant change of the filtration rate, together with the overload on the filters which was approximately 100% during periods of peak demand, was often reflected in the delivery of a discolored and partly purified water. The maximum rate of draft had reached 40 000 000 gal. per day in 1920, and the maximum rate of filtration through the pressure filters had reached approximately 250 000 000 gal. per acre per day. It became imperative, therefore, to construct additional filters and a clear well of sufficient capacity to permit a more uniform filtration rate.

It was decided to construct a covered clear water basin of 10 000 000 gal. capacity, and a gravity filter plant of 21 000 000 gal. daily capacity designed for future expansion to 60 000 000 gal. per day. Construction was started March, 1922, and the new plant was placed in operation in July, 1923.

The water surface of the new filters was at Elevation 977, or 8 ft. below the high-water elevation of the coagulating basins, in order to provide for

loss of head in operating and a draw-down capacity of raw water and coagulating basins for future pre-filtration processes and emergencies.

The high-water elevation of the new clear water basin was at 967, or 10 ft. below the water surface in the filters. This elevation was 22 ft. higher than that of the water surface in the old clear water wells and necessitated the abandonment of these wells. The total loss in head through the filter plant now became 18 ft., and a saving of 22 ft. of head was obtained.

This higher elevation did not, however, render the pressure type filters inoperative, because a differential of 18 ft. with full clear water basin still existed, and this head is more than enough to operate them at their normal rate.

The filter plant (Figs. 7 to 11) consists of seven 3 000 000-gal. daily units, arranged in a straight line with an open pipe gallery. They are of reinforced concrete, 28 by 42 ft. in plan, have a net sand area of 1 040 sq. ft., or 0.0239 acre each, and are designed to operate at the rate of 125 000 000 gal. per acre per day. Each filter is equipped with twelve concrete wash-water troughs, 7 ft. center to center, which discharge into a central drain dividing the filter into two parts. The walls of this drain are 6 ft. high and carry a concrete walkway which extends from the operating platform to the rear of the filter. (See Figs. 8 and 10.)

The under-drain system consists of a cast-iron manifold, 10 in. in diameter, divided into eight sections and having eight 10-in. outlets to the effluent main. The laterals are of 2½-in. cast-iron pipe spaced 6 in. center to center. Laterals are drilled in the bottom quadrant with $\frac{5}{16}$-in. holes at an angle of 45°, center to center, and staggered. (See Fig. 8.)

The sand bed is 30 in. deep, and the sand has an effective size of 0.36 mm. and a uniformity coefficient of 1.5. The gravel supporting this sand is placed in five layers and is 20 in. deep.

The filters are entirely covered with a brick superstructure, 76 by 202 ft. At the west end of the building (Fig. 7) is located a head-house, laboratory, and office, 60 by 76 ft., which eventually will occupy a central position when the plant reaches its ultimate capacity.

The coagulated water enters at the west end of the plant through a 48-in. cast-iron main and Venturi meter. The level of water on the filter beds is controlled by a 42-in. hydraulic valve set in this line, which is operated by a float and pilot-valve mechanism set on the operating platform in an open chamber, 3 by 4 ft., connected with the coagulated water conduit. This chamber is also equipped with an emergency overflow weir and is connected with the main drain conduit to prevent the overflow of the filters if the control valve fails to operate.

The four main conduits in the filter building below the filter gallery are of reinforced concrete, side by side, and were poured monolithic in sections in one operation. These conduits were designed for an ultimate capacity of 60 000 000 gal. per day. The tops of the conduits are level with and form a part of the lower floor of the building. The bottom of the filters is about 5 ft. above this floor and is supported on concrete columns.

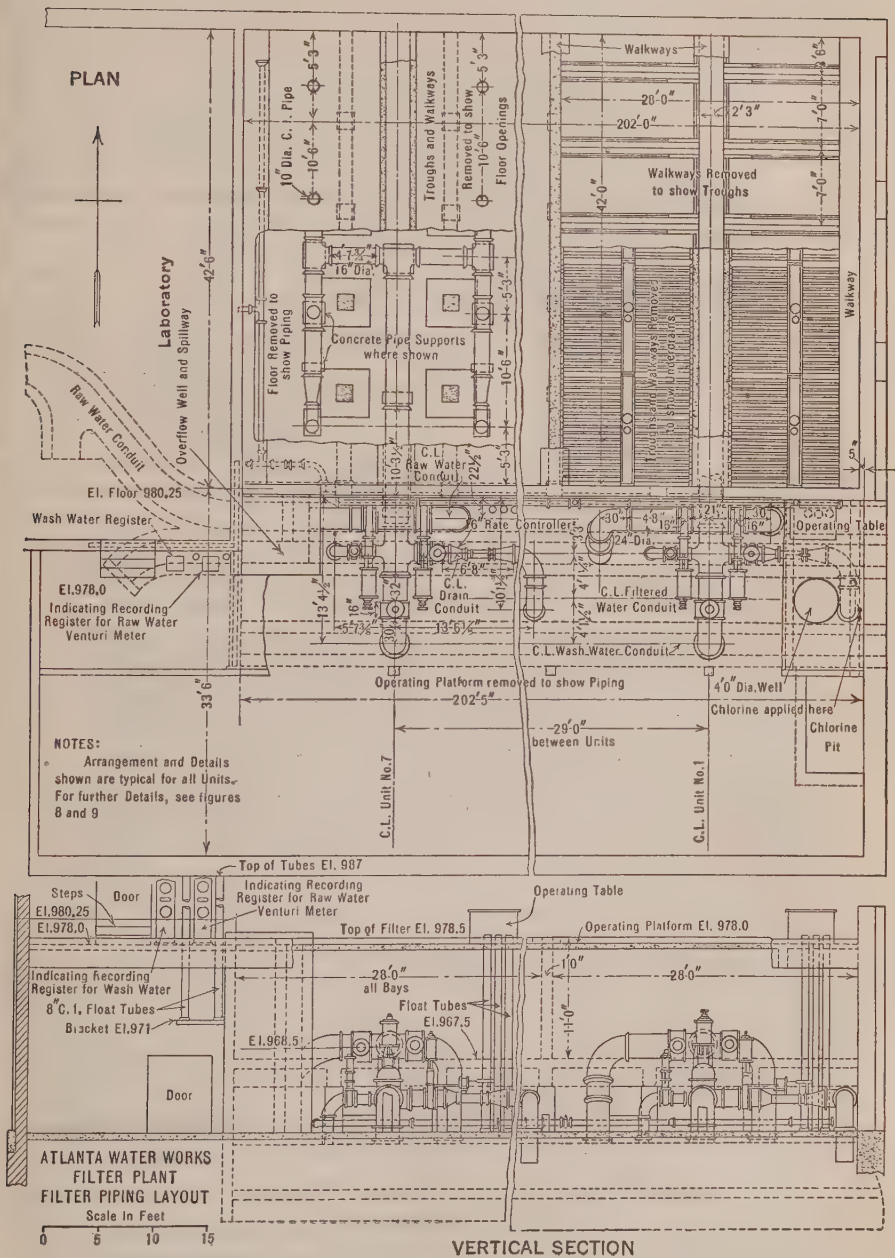

PLAN
Laboratory
Overflow Well and Spillway
Raw Water Conduit
El. Floor 980.25
Wash Water Register
El.978.0
Indicating Recording Register for Raw Water Venturi Meter
Floor Removed to show Piping
Concrete Pipe Supports where shown
Troughs and Walkways Removed to show Floor Openings
Walkways
Walkway Removed to show Troughs
Walkway
C.L. Raw Water Conduit
Rate Controllers
C.L. Drain Conduit
C.L. Filtered Water Conduit
C.L. Wash Water Conduit
Operating Table
Operating Platform removed to show Piping
4'0" Dia. Well
Chlorine applied here
Chlorine Pit
29'0" between Units
C.L. Unit No.7
C.L. Unit No.1
NOTES:
Arrangement and Details shown are typical for all Units. For further Details, see figures 8 and 9
Top of Tubes El. 987
Steps
El.980.25
El.978.0
Door
Indicating Recording Register for Raw Water Venturi Meter
Top of Filter El. 978.5
Operating Table
Operating Platform El. 978.0
Indicating Recording Register for Wash Water
8" C. I. Float Tubes
Bracket El.971
28'0" all Bays
Float Tubes El.967.5
El.968.5
Door
ATLANTA WATER WORKS
FILTER PLANT
FILTER PIPING LAYOUT
Scale in Feet
0 5 10 15
VERTICAL SECTION

FIG. 7.

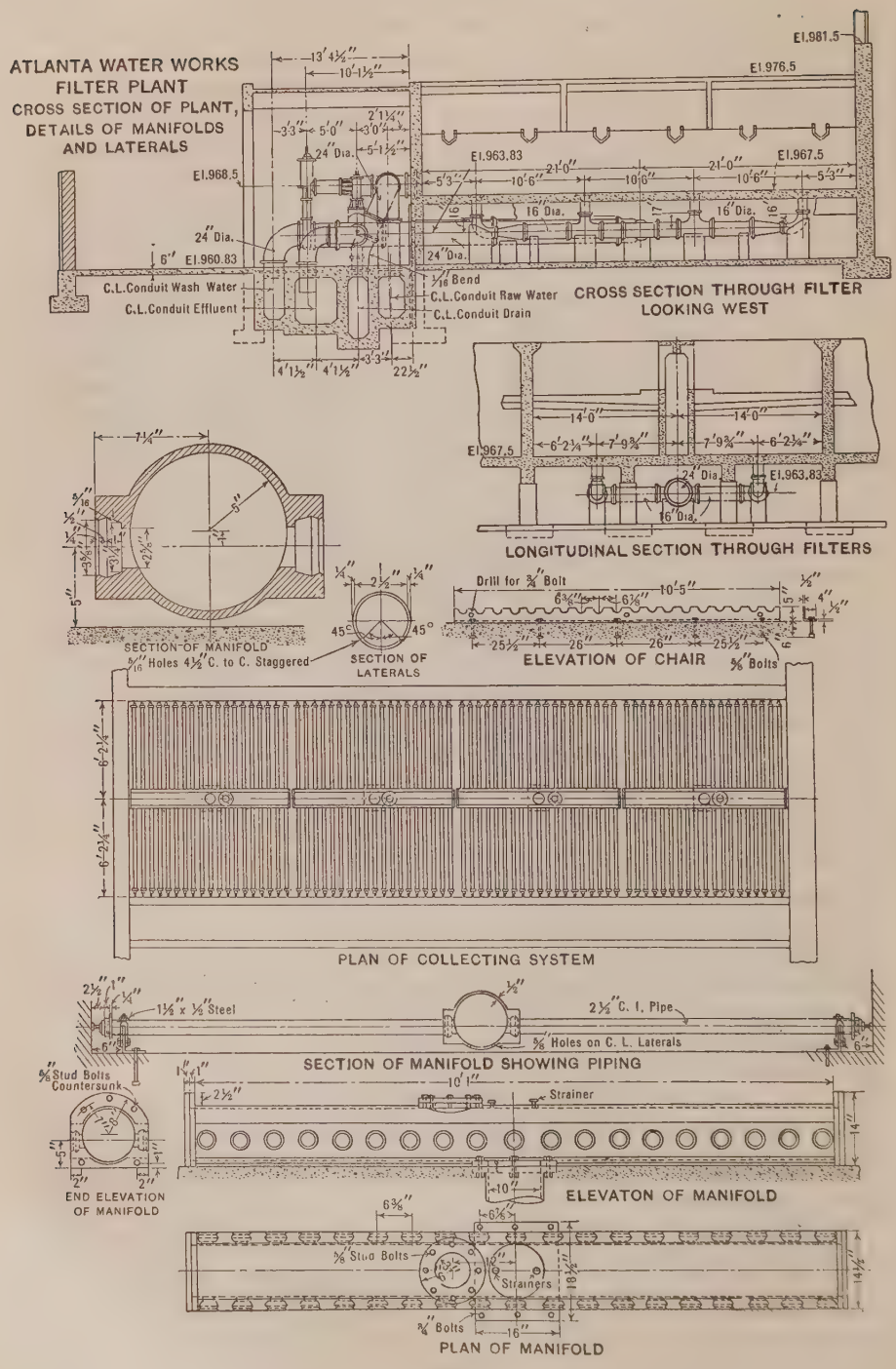
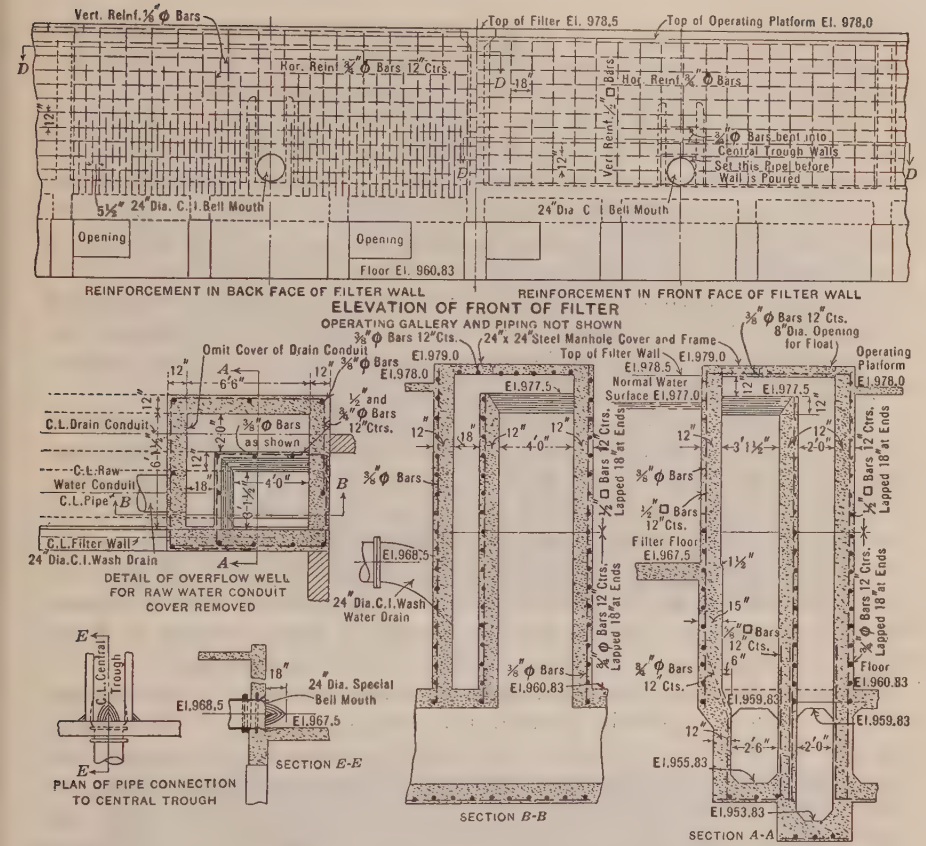

ATLANTA WATER WORKS
FILTER PLANT
CROSS SECTION OF PLANT,
DETAILS OF MANIFOLDS
AND LATERALS
El.981.5
El.976.5
El.968.5
El.963.83
El.967.5
El.960.83
24" Dia.
16" Dia.
C.L.Conduit Wash Water
C.L.Conduit Effluent
C.L.Conduit Raw Water
C.L.Conduit Drain
CROSS SECTION THROUGH FILTER
LOOKING WEST
LONGITUDINAL SECTION THROUGH FILTERS
SECTION OF MANIFOLD
Holes 4½" C. to C. Staggered
SECTION OF
LATERALS
Drill for ¾" Bolt
ELEVATION OF CHAIR
⅝" Bolts
PLAN OF COLLECTING SYSTEM
1½" x ½" Steel
2½" C. I. Pipe
⅝" Holes on C. L. Laterals
SECTION OF MANIFOLD SHOWING PIPING
⅝" Stud Bolts
Countersunk
Strainer
END ELEVATION
OF MANIFOLD
ELEVATON OF MANIFOLD
⅛" Stud Bolts
Strainers
¾" Bolts
PLAN OF MANIFOLD

FIG. 8.



ATLANTA WATER WORKS
FILTER PLANT
STRUCTURAL DETAILS

FIG. 9.

All collector piping beneath the filters is exposed and easily accessible. The under-side of the filters is shut off from the main part of the building by a 6-in. concrete curtain-wall, but a door is provided for each filter.

All operating piping, hydraulic valves, controllers, etc., are exposed to view from the main floor and are arranged so that all are readily accessible for repair or adjustment. This arrangement is perhaps somewhat greater in first cost than the usual pipe gallery with filters on both sides, but it certainly results in more efficient operation and permits inspection and ease of maintenance because of its accessibility. The operating platform is 15 ft. wide, and is supported above the conduits by concrete columns. It is reached from the main floor by a concrete stairway in the center, and by circular steel stairways at both ends, of the building. Fig. 11 is a view of the influent, effluent, wash, and drainage connections.

Wash water is supplied to the filters by a reinforced concrete wash-water tank of 425 000 gal. capacity, located about 50 ft. north of the filter building, with the normal water surface at Elevation 1 000, or 23 ft. above the water surface in the filters. Filtered water is supplied to this tank by two 4 000-gal. per min. motor-driven centrifugal pumps arranged in duplicate. The pumps take suction from a well connected to the main effluent conduit and are automatically controlled by float switches in the tank and suction well. The pump intake is arranged so that the water in the effluent conduit cannot be drawn down below the outlets of the filters, which would break their seals.

The tank discharges into the wash-water conduit through a 30-in. cast-iron main in which is set a 30-in. Builders Iron Foundry controller. The register for this controller is mounted on the operating platform and indicates the rise of wash water in the filters, in inches per minute and in gallons per minute, and records the time, duration, and rate of wash. A change in the rate of wash can be quickly and easily made by adjusting the instrument. It also prevents a sudden rush of water due to quick opening of the hydraulic valve on the wash-water main, because the normal position of the controller is closed, and it requires at least 1 min. to open it.

The effluent from the filters enters the filtered water conduit through a 16-in. cast-iron line in which is set a 16-in. Builders Iron Foundry controller. The effluent conduit is sealed by a weir to prevent the lowering of the water below the filter outlets when the clear-water reservoir is drawn down below the effluent conduit.

At the east end of the filter building, chlorine is injected into the filtered water, and the water then passes through a 60-in. cast-iron main through a Venturi meter and a controlling gate-house to the clear water basin, or the pumps, the clear water reservoir floating on the line.

In the gate-house are located the various valves for controlling the flow from the clear water well and the filters. The 60-in. line is here reduced to 48 in. in order to decrease the valve cost and make operation easier. All valves have been arranged for electrical operation in the event this becomes necessary or advisable in the future.

The pressure filters have been connected to the clear water basin, and the effluent line from the new filter plant, and the supply line between the gate-

FIG. 10.—WASH-WATER TROUGHS, NEW FILTERS, ATLANTA, GA.

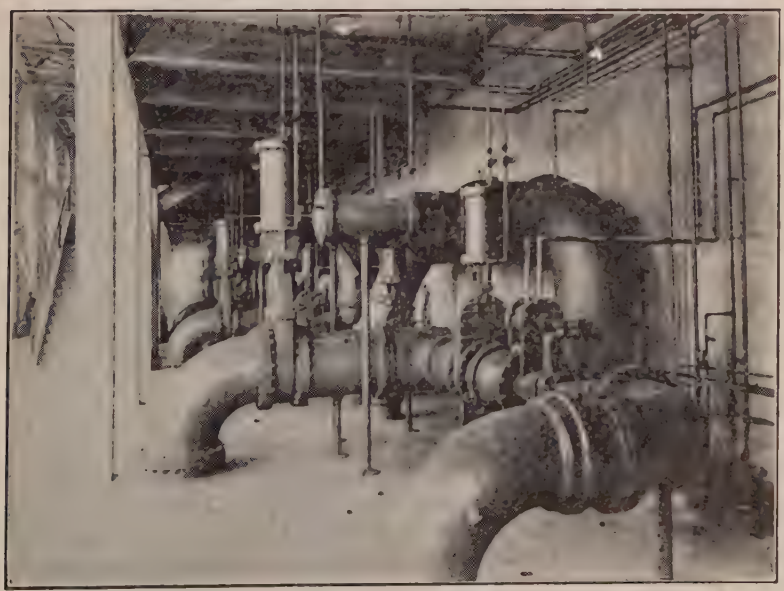


FIG. 11.—INFLUENT, EFFLUENT, WASH AND DRAINAGE CONNECTIONS, NEW FILTRATION PLANT, ATLANTA, GA.

house and the high-service pumps. Provision has also been made at the valve-house for an additional suction main to be laid in the future. When this is constructed, there will be two 48-in. and one 36-in. suction lines to the high-service pumping station.

The operation of the pressure filter plant in parallel with the gravity plant is accomplished by connecting the effluent from the pressure plant to the conduit conducting the water from the new clear water basin and gravity plant to the high-pressure pumping station, by a 36-in. cast-iron main. The pressure filters are located at Elevation 945, and the water surface of the clear water reservoir is at Elevation 967. To prevent back pressure on the pressure filters when the clear water basin is full, a check-valve has been inserted in the 36-in. line. The pressure filters must build up sufficient head to open this check-valve against the pressure from the new clear water basin, or the pressure from the gravity plant, before the filtered water from the pressure plant can pass into the system. With a full clear water basin a maximum differential of 18 ft. exists between the influent and the effluent, and this head has been found sufficient to overcome the losses due to the filters and pipe friction and to permit the operation of the pressure filters at their normal capacity in parallel with the gravity plant.

The old filter plant has no loss-of-head gauges, or rate-of-flow controllers, and the filters are washed by coagulated water. Detail plans for these improvements, including connection to the new wash-water tank of the new filter plant, have been prepared, and bids received, but contracts have not yet been awarded. This improvement is most necessary, and will probably be carried out during 1925.

Clear Water Reservoir.—The clear water reservoir has a capacity of 10 000 000 gal. It is trapezoidal in plan, 215 by 388 ft. at the top with 2 to 1 slopes, 22 ft. deep, constructed entirely in cut, and has a reinforced concrete roof of slab-and-beam construction which is covered with 18 in. of earth.

Small entry houses are provided at both ends of the structure, and concrete stairs extend along the slopes from the entry houses to the bottom of the reservoir. Fig. 12 is a view of the interior of the new clear water reservoir.

The floors and slopes are of concrete 6 in. thick and reinforced with electrically welded steel fabric of No. 4 wire, 4 in., center to center. In order to insure water-tightness, concrete beams or sills, 12 by 12 in., were first poured on the side slopes, 18 ft., center to center, with the tops of the beams level with the under-side of the slab. Into these beams, at intervals of 6 ft., were inserted bolts to hold forms. Bulkheads were erected on the center line of beams, and the slabs were then poured with a moderately wet mix, the forms being carried up 18 in. in advance of the concrete. Three slabs were usually poured simultaneously, and in this way the pressure on the forms was kept within reasonable limits. At the joints in the slabs over the beams, No. 18 gauge, galvanized-iron strips were inserted, extending 4 in. into the adjoining sections. Before the adjoining section was poured, the joint was covered with $\frac{1}{8}$ in. of hot pitch.

After completion, tests extending over a period of 20 days indicated a drop in water level with full reservoir of $\frac{1}{2}$ in. per 24 hours. Some of this leakage was traced to the 48-in. valve in the discharge line from the reservoir.

Water enters and leaves the basin at the south end through a 60-in. cast-iron pipe laid under the floor, which is later reduced to 48 and 36 in. From this line are taken three sets of 24-in. cast-iron laterals which extend at right angles to the main conduit and discharge at the foot of the side slopes. These laterals are fitted with 90° bends and extend about 2 ft. above the floor of the basin. The bends on each pair of laterals are turned in opposite directions to facilitate circulation.

The clear water basin acts as a balancing unit for the filters, and the filter plant is now being operated at a uniform rate of approximately 26 000 000 to 30 000 000 gal. daily, the new plant supplying 21 000 000 gal. and the pressure plant the difference.

Operating Results

The new filter plant (Fig. 13) has been in operation since June, 1923, and the results produced from both the bacterial and the operating standpoint have been excellent, except when the water delivered to the filter plant was overloaded with suspended matter. The operation of the filters at a uniform rate has reduced some of these troubles, and, although the pre-filtration processes are seriously in need of improvement, the entire plant is now functioning much more satisfactorily than it did under a constantly changing rate.

There has been no evidence as yet of the formation of mud balls, or cracking or shrinkage of filter beds, and the distribution of wash water is quite uniform, which indicates that the design of the under-drain system is satisfactory.

Table 4 is a daily operating log of the plant for the six-month period ending January 31, 1924. These data indicate unusually good results, but inasmuch as they are the combined record of both the old pressure type and new gravity filter plant, they do not show some of the more interesting records of the operation of the latter plant.

The average length of run between washings of the gravity units varies from 60 to 70 hours, with maximums of more than 120 hours. The quantity of wash water required for these units has been less than 1 per cent.

The open type filter gallery with its ease of inspection, operation, and maintenance, justifies this type of design.

The new plant is provided with every facility and is heated from the boiler plant at the high-service station. Ample radiation produces satisfactory temperature during zero weather, which occurs very rarely.

Improvements at Hemphill Avenue Pumping Station (High Pressure)

The improvements at the Hemphill Avenue Station consisted of a new radial brick chimney with a steel flue for future extensions; remodeling the steam headers; equipping the boilers with superheaters and mechanical stokers; installing a de-superheater to provide saturated steam for certain units; remodeling of suction and discharge headers; removing one of the old 10 000 000-gal. pumps; and installing one 30 000 000-gal. turbo-centrifugal DeLaval

Fig. 12.—View of Interior of New 10 000 000-Gallon Clear Water Reservoir, Atlanta, Ga.

Fig. 13.—View of New Gravity Filtration Plant, Atlanta, Ga.

TABLE 4.—SUMMARY OF OPERATING RESULTS, ATLANTA WATER-WORKS PURIFICATION PLANT, JUNE TO DECEMBER, 1923.

	June.	July.	August.	September.	October.	November.	December.
Million gallons coagulated, daily	27.89	28.94	28.76	27.03	26.05	25.59	25.36
" " filtered daily	27.45	28.71	28.25	26.62	25.50	25.21	25.00
Wash water, gravity filters, in gallons per day	.....	571 416	400 846	356 296	378 965	335 833†	261 429‡
Wash water, pressure filters, in gallons per day	408 600	230 022	181 900	178 867	149 179	136 915	123 442
Percentage of wash water to total	.....	2.8	1.8	1.8	1.9	1.6	1.4
Maximum run, gravity filters, in hours	68½	76½	93¾	122½	98	99½	93
Minimum run, gravity filters, in hours	10	8	10½	66¾	40¾	56¼	51½
Average run, gravity filters, in hours	39¼	39	45¼	94¾	68⅛	70	73
Maximum run, pressure filters, in hours	36	44	37	30	26	24	24
Minimum run, pressure filters, in hours	24	13	14	24	24	17	24
Average run, pressure filters, in hours	28¼	.....	27½	27	24	23¾	24
Alum used, pounds per day	3 132	3 954	5 952	2 858	1 428	1 296	1 359
Chlorine used, pounds per day	48.6	42½	44.9	40.6	42.7	41.0	40.0
Lime used, pounds per day	1 865	1 802	2 863	1 543*	.....	.....	.....
Alum used, in grains per gallon	0.89	0.96	1.04	0.80	0.38	0.35	0.37
Chlorine used, in parts per million	0.22	0.18	0.20	0.18	0.20	0.20	0.20
Lime used, in grains per gallon	0.57	0.44	0.60	0.39*	.....	.....	.....
River water turbidity	309	412	763	252	92.5	720	161
Raw water turbidity	242	274	395	234	63.6	37	59.9
Coagulated water turbidity	34.5	37.3	42	29.5	24	24	25
Alkalinity, raw water	13.9	15.3	15	15.2	21.3	23.6	19
Alkalinity, coagulated water	11.6	15.5	15.5	12.9	17.5	21.5	17
Bacteria per cubic centimeter on agar at 37½° cent.:							
Raw water	772	1 142	514	565	271.1	163	186
Coagulated water	79	128	85	29	31.4	17	26
Filtered water	6.1	49	28	5.1	9.1	5.1	3.7
Tap water	5.1	7	0.9	1.5	3.1	1.4	1.7
Bacillus coli:							
Raw, in 1 cu. cm., percentage positive	60	48½	25.8	16.7	3.2	3⅛	25.8
Raw, in 0.5 cu. cm. percentage positive§	94.9	80.7	58.1	41.7	11.3	28.3	72.6
Filtered, in 10 cu. cm., percentage positive‖	0	13.0	0	0	0	3.3	11.3
Tap, in 10 cu. cm., percentage positive¶	3.3	0	0	0	0	0	0
Tap, in 100 cu. cm., percentage positive	0	0	0	0	0	0	0

* Lime applied for 14 days only.
† 24 days only.
‡ 28 days only.
§ Daily average of two samples.
‖ Daily average of two samples.
¶ Daily average of three samples.

Filter washing	Gravity.	Pressure.	
Average time, in minutes	6.26	17.9	Maximum for any washing, gravity, 26.5 in.
Average rise, in inches per minute	24.7	11.2	Minimum " " " " 19.7 in.

pumping unit, having practically the same guaranty as those at the Chattahoochee Station.

As this station pumps directly into the distribution system with no equalizing reservoir, the operating conditions are variable, the domestic demand ranging from a rate of 17 000 000 to 18 000 000 gal. per day at night to a rate of 45 000 000 to 50 000 000 gal. per day peak rate during daylight hours, and varying with the season of the year.

The construction of the new clear water reservoir (high-water elevation, 967) has placed all pump suctions under a positive head, which reduces the differential between water surface and maximum head, thus materially reducing unit pumping costs.

The average domestic pressure maintained at this station ranges from 80 to 100 lb. It became necessary to build this pressure up from 120 to 125 lb. during fire-drafts, due to inadequate feeders and supply mains to the distribution system. The unprecedented growth of the City of Atlanta and its suburban area, with rapidly increasing demands, emphasizes the necessity of a prompt survey of the distribution system and an analysis of flows, pressures, and drafts to determine the reinforcement necessary.

Cost of the Work

The following figures indicating the cost of the major items of the work completed, authorized, and contemplated, may be of interest:

Estimated cost, impounding dam, intake, etc.	$2 500 000
Mixing chamber and chemical house	176 000
48-in. raw water main and connections, including gate-house at reservoirs, aqueduct, tunnel, etc.	682 000
Filter plant, reservoirs and connections	699 000
Three 30 000 000-gal. turbo-centrifugal pumps erected (not including foundations)	192 000
Pumping station improvements, including steam improvements, and pump foundations, connections, cranes, etc.	300 000
Equipping thirty-six 500 000 gal. per day horizontal type filters, with loss-of-head gauges, rate controllers, and filtered water wash line with controller	42 000

General

The entire work has been under the general direction of W. Zode Smith, General Manager of the Atlanta Water-Works.

The 48-in. raw water main and connections, all suction and discharge headers, pump foundations, connections and exterior piping at the filter plant, were constructed and installed by the forces of the Water-Works Department with William M. Rapp, Superintendent of Construction, in charge. This work has been carried on efficiently and expeditiously, and well within estimates, and at a considerable saving over competitive bids.

The writer, acting as Consulting and Designing Engineer for the City of Atlanta, desires to give full credit and honor to his able corps of designers, resident engineers, and assistants, who have served so conscientiously and efficiently in the preparation of plans and the supervision of construction, including Frank J. Keis, M. Am. Soc. C. E., M. T. Singleton, M. Am. Soc. C. E., and Herman F. Wiedeman, Assoc. M. Am. Soc. C. E.; and Messrs. T. M. Sullivan, McDonald Lawrence, Lowell Cady, S. E. Arnold, and others. Mr. Wiedeman was Resident Engineer on the filter plant and clear water reservoir and during the period of "tuning up". Mr. J. N. Eley has co-operated with the writer in all mechanical and electrical matters, and supervised all duty and efficiency tests on the pumps and steam equipment.

DISCUSSION

J. N. CHESTER,* M. AM. SOC. C. E.—The definition of engineering has been stated as "the ability to adapt the forces of Nature to the uses of man", and to that might be added the provisions as well as the forces of Nature. As indicated in the paper, Atlanta has been especially favored with a remarkable source of water supply. The water is slightly contaminated, red with generous proportions of aluminum alkalinity, and the minimum run-off to date has been sufficient for the city, but, at times, the shortage has been acute. This, however, is not alarming considering that there are bountiful storage possibilities above and near the intake, and that in these storage possibilities there also appear to be power possibilities which may some day be developed to avoid the burning of coal for pumping.

With regard to early filtration plants, the Hyatt plant represents more nearly the early stage of the art than any the speaker has seen. There are a great many of these older plants still in operation, but most of them have been shorn of some of the elements of the early development and are operating much as the horizontal filters of the Atlanta plant; but the Hyatt plant shows how filtration was first conducted. At least one unit of that plant should be preserved so that engineers may be able to hearken back to where they started and demonstrate, in a measure at least, that the original filtration plants can produce as good an effluent as the modern ones with the actual process that was used in the original filter.

Every engineer who designs a new filter plant incorporates something that is not common to others. The outstanding features of the filter plant at Atlanta appear to be the single gallery and the open gallery as found to-day in few filter plants. The over-all filter house would come next, although that has been used in Cincinnati, Ohio, St. Louis, Mo., and Minneapolis, Minn., the last being the oldest of any material capacity. The extreme climate at Minneapolis was probably a strong argument for this type. In a southern climate, there are many features that can be adapted and incorporated that would not always be successful in the more severe northern climate. One of these is the concrete wash tank.

The speaker's early experience convinced him that the inherent principle of filtration was the balancing of the water between the coagulating basins and the filter tubs to produce an unrestricted slow flow from basins to filters and thus preserve the floc already formed. This question has been discussed with the officials of the City of Norfolk, Va., for whom the speaker has recently done some work. There the water is pumped from the coagulating basin by centrifugal pump to the filters and the officials are insistent that that does not prevent good filtration. When a new filtration plant was built, however, that feature was not incorporated in it.

The speaker has never before seen any plant in which the indicator on the wash line was calibrated to indicate the inches of rise of wash water in the

* (The J. N. Chester Engrs.), Pittsburgh, Pa.

tub. This feature is to be commended highly as it shows the extent of the rise, and, consequently, the rate of washing.

One feature of the pumping station that impressed the speaker was the use of centrifugal pumps on direct pressure where there must be at least a variation of 50% in demand. A basic principle of that type of pump is that they are most economical when discharging against a fixed head. If the demand is not equal to the capacity of the pump, it must increase the head and, therefore, affect the economy. For this reason it will be more interesting to learn what the operating results of this plant prove to be, as compared with others, than what test duty these pumps produce, because when the machine is being tested it must be operated under the conditions on which the guaranty is made, regardless of the subsequent operating conditions. No manufacturer could compete on anything except plainly stipulated conditions. That, therefore, is a feature of test conditions to which engineers have to defer and await the results after a number of years of service.

Nisbet Wingfield,* M. Am. Soc. C. E.—The water for the supply at Atlanta is taken from the Chattahoochee River (about 7 miles from the center of the city) and pumped to two raw water reservoirs in the city; from these it flows by gravity through the coagulating basin and filters to a clear water basin, and thence to the pumps supplying the city. This basin, called the Hemphill Station, is 100 ft. below the high points in the distribution system. The pressure on the mains is maintained by the pumps at all times, there being no service reservoir. The lift from river to raw water reservoirs is 227 ft. From the Hemphill Station it is 100 ft. plus the pressure on mains, making a total lift of 185 ft. for domestic pressure and 285 ft. for fire pressure.

Many of the problems that have confronted the municipal authorities in maintaining an adequate water supply in Atlanta are similar to those arising in every growing community, but the growth of Atlanta, both numerically and industrially, has been excessively rapid in the last fifteen years, so that the situation at times has been acute. This applies to schools, transportation, water-works, and every other public service, and means that an extra burden has been put on those entrusted with these public utilities. How well they have stood the test has been indicated by the fact that no serious interruptions of the service have occurred, although at times the situation must have been very trying.

For years Atlanta has been outgrowing its income, that is, the needs of the public service institutions have been greater than the funds available. An effort to catch up at least partly in the Water Supply Department has been made by a recent bond issue of approximately $3 000 000. As explained in the paper, this was not sufficient to carry out the plans in their entirety, but has been sufficient to put the pumping, filtration, and storage capacities on a good basis, but the distribution system is not adequate by any means. New centers of distribution are being formed so frequently, on account of the building up of certain sections, that an adjustment in the pipe system is

* Cons., Municipal and Hydr. Engr., Augusta, Ga.

needed to meet these changed conditions, and additional funds should be provided for this purpose with as little delay as practical.

The City of Atlanta occupies the highest ground for miles, in either direction, hence there is no opportunity for a service reservoir sufficiently elevated to supply the city by gravity. Additional fire pressure in the mains (100 ft.) is not intended to furnish hydrant fire streams, its object being to force sufficient water through the small mains for use of fire pumping engines—this in lieu of larger street mains, which otherwise would be necessary.

The direct pumping and the varying pressure render it necessary to equip and maintain larger pumping units than would be necessary under ordinary conditions of service. The average consumption per day at this time ranges from 26 000 000 to 30 000 000 gal., but during the 24-hour period the rate of pumping varies from 17 000 000 to 50 000 000 gal. per day, and the pressure against which the pumps must work varies from 80 to 125 lb.

This varying condition of service does not apply, of course, from the river station to the raw water reservoir, where a constant rate of pumping may be maintained against a uniform pressure, the difference in elevation between the surface water in the suction well at the river station and the raw water reservoir being 227 ft. The difficulties which have been encountered heretofore in the operation of this station have been due to the fact that water is taken directly from the Chattahoochee River through a single conduit to a suction well of small capacity, and to the further fact that neither the 30-in. nor the 36-in. raw water discharge pipes had sufficient capacity to take care of this service alone, and an interruption of the service in either necessitated raising the pressure on the pumps above that for which they were designed, in order to overcome friction.

The trouble with the intake pipe is due to the tremendous variation in the flow of the river—the low flow being less than 400 cu. ft. per sec., whereas the flood flow is more than 100 000 cu. ft. per sec. During low flow the intake is not entirely submerged and the quantity of inflowing water is uncertain, while during periods of high water constant attention is necessary to prevent flooding through the intake well while still maintaining the proper flow to the pumps. When the impounding reservoir is constructed, as recommended in the paper, this trouble will be eliminated.

The present pumping capacity of the Chattahoochee River Station is 106 000 000 gal., and the three discharge mains have been cross-connected in several places, so that as soon as the intake from the river is reconstructed there should be no difficulty in delivering the required quantity of water into the raw water reservoirs.

At the Hemphill Station the pumping capacity is 95 000 000 gal. per day, with a clear water basin of 10 000 000 gal. capacity from which to pump, and filters of 42 000 000 gal. capacity constantly feeding it, so that to handle this plant properly all that is now necessary is to increase the sizes of the street mains, thus reducing the pressure to overcome friction.

Probably the most interesting feature of the Atlanta Water-Works is the filtration plant, which, to say the least, is unique in that the gravity filters

and pressure filters are operated in conjunction, this arrangement being made in order not to scrap the old pressure filter plant of 21 000 000 gal. capacity, which had proved through long years of excellent service that it had a right to live. Any one who has been actually in charge of the operation of a filter plant with a highly turbid raw water will appreciate the difficulty of suitably treating 30 000 000 gal. per day, having a peak draft of 50 000 000 gal. per day, with a filter capacity of 21 000 000 gal., and a clear water basin capacity of 2 000 000 gal. This was the condition of Atlanta before the construction of the filtration plant just completed.

The waters of the Chattahoochee River are, in all important particulars, similar to other rivers in the South Atlantic States, that is, the water is of good chemical quality, with few harmful bacteria, but carrying in suspension at all times large quantities of clay and silt of reddish-yellow color, varying in quantity with the different seasons. The principal work for the filtration plant, therefore, is to remove this color.

Under normal conditions the total suspended solids are approximately 43 grains per gal. From tests made by the speaker it was found that in 72 hours, with the water quiet, there was a precipitation of 32 grains, leaving 11 grains which would not precipitate. This residue of 11 grains is the exceedingly fine particles which give color to the water, so that if a settling basin of sufficient capacity is provided, the output of the filters will be largely increased. With water constantly passing through the basin the body is more or less disturbed, and precipitation will be slower. For this reason an additional length of time beyond 72 hours should be given.

The capacity of the raw water reservoirs in Atlanta is 393 000 000 gal. These reservoirs, acting as settling basins, gave sufficient time for settling, thus furnishing to the filters a water comparatively easy to treat, and making the 21 000 000 gal. capacity filter plant just about ample with a rate of filtration of 3 gal. per min. per sq. ft. At times, however, the filtered water had too much color. Many of the suspended particles giving the color are so minute that they will not be caught on the filter bed, unless the rate is extremely slow. Discolored filter water, therefore, means that the rate of filtration has been too rapid. This has now been cured, and there should be no further trouble until the rate of consumption is materially increased.

The old filter plant is interesting also, from the fact that it was the first mechanical filtration plant constructed to treat the whole supply of a municipality. The original plant consisted of twelve Hyatt filters, with combined capacities of 3 000 000 gal. in 24 hours. Previous to this time smaller units had been installed in several places in the East, but Atlanta has the distinction of being the first city supplied through a mechanical filtration plant. These filters had two chambers, an upper and a lower. The filtering was done in the lower chamber, and when it was necessary to wash the filter bed, the sand was lifted by hydraulic apparatus from the lower chamber to the upper chamber and washed, and then returned to the lower chamber. The operation of this plant was successful, due to the fact that the water was taken from a large impounding reservoir, where previous settlement had taken place.

Several years later (1887-90), in order to lessen the original cost of filtration works, attempts were made by other manufacturers to filter public supplies without previous sedimentation, the water being drawn direct from a turbid stream and forced under pressure through filters located on the discharge pipe. Notable examples of this were: New Orleans, La., taking water from the Mississippi River, Little Rock, Ark., with water from the Arkansas River, and Chattanooga, Tenn., taking water from the Tennessee River. In each case the attempt was unsuccessful, due to the fact that silt and other suspended matter quickly formed a mat on top of the filter bed, with the result that the more pressure applied from the pumps, the denser the mat became, almost entirely stopping the flow. In several instances the filter cases were torn open by pump pressure.

These plants and many other smaller ones of which the speaker has personal knowledge, where the filter installations were similar, were later used as gravity plants, by the construction of settling and clear water basins, so placed that the pressure filter cases and pipe systems could be used without alteration, the water passing through the beds by gravity instead of under pressure. In all cases the change from pressure to gravity filters, with previous sedimentation, had the desired effect and potable water was obtained without trouble.

When the source of water supply for Atlanta was changed from South River to the Chattahoochee River, and the existing raw water reservoirs and pumping plants constructed, the twelve vertical Hyatt filters were moved from South River and set up at their present location, between the raw water reservoir and the Hemphill Pumping Station, and a small clear water basin constructed, the water passing by gravity from the raw water basin through the filters to the clear water basin, with a fall from the raw water reservoir to the clear water basin of 40 ft.

The capacity of the filtration plant was increased by the addition of twelve New York horizontal filters with a rated capacity of 500 000 gal. each, giving a total capacity of 9 000 000 gal. During the past twenty years this capacity has been increased to a total of 21 000 000 gal. This plant worked satisfactorily until the consumption exceeded the plant capacity. In connection with the new plant just completed, it will no doubt continue to give excellent service, provided the raw water is given not less than 72 hours of sedimentation, and the rate of filtration does not exceed 2 gal. per min. per sq. ft. of filter bed.

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Paper No. 1582

HIGH SPECIFIC SPEED HYDRAULIC TURBINES IN THEIR BEARING ON THE PROPORTIONING OF THE NUMBER OF UNITS IN LOW-HEAD HYDRO-ELECTRIC PLANTS

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* Presented at the meeting of the Power Division, New York, N. Y., January 22, 1925.

HIGH SPECIFIC SPEED TURBINES

By George A. Orrok,* M. Am. Soc. C. E.

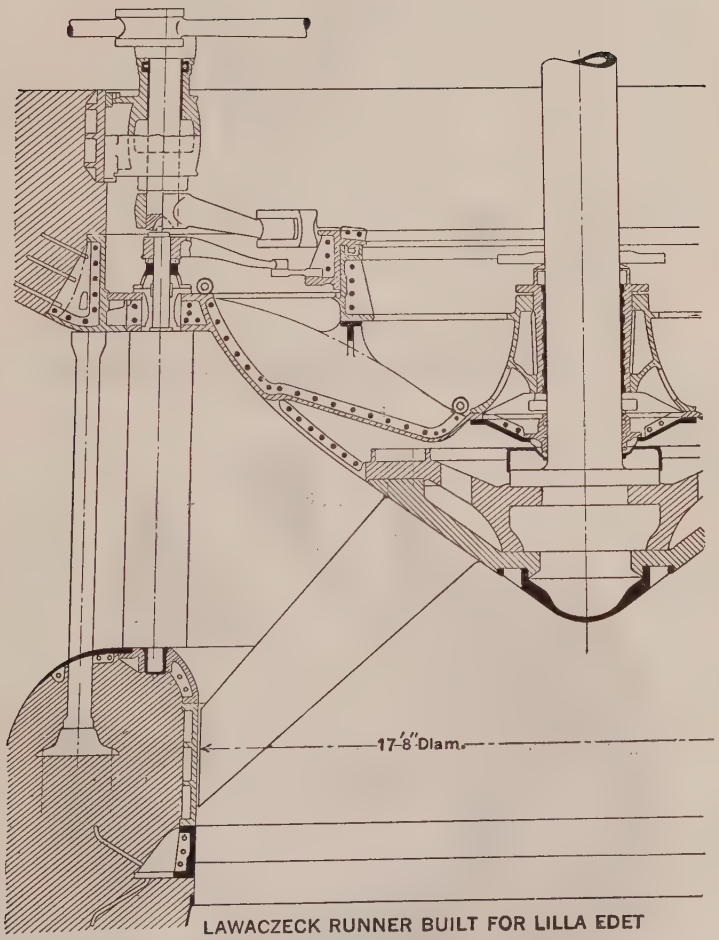
Among the many things of interest seen and heard at the World Power Conference (London, 1924), none perhaps was more striking than the variety of methods proposed and tried for the solution of the important problem of high specific speed turbines. From talks with experimenters and visits to European testing laboratories, from the papers on these laboratories and their work presented to the Conference, and from current literature in many languages, the speaker believes that although the subject is not new and is well known in America, yet the many excellent examples of good design and construction at present under way in Europe should be better known to members.

Among the larger plants, the Chancy-Pougny installation of five wheels (7 600 h.p.), on the Rhone, below Geneva, Switzerland, is most interesting. The turbines are 18 ft. 0 in. in diameter, under 26.7-ft. head, with a characteristic speed of about 120, just a trifle larger than those at Keokuk, Iowa. The wheels are a modified Francis type, retaining the outer rim, and were built by the Des Charmilles Company, of Geneva; they are guaranteed for an efficiency of more than 90% at the best gate-opening.

In comparison, the Lawaczeck wheels for Lilla-Edet, Sweden (Fig. 1), are 19 ft. 8 in. in diameter without the outer rim, and have a characteristic speed of 136; the head is 21.3 ft. instead of 26.7 ft., the power is 10 000 h.p., and the water used is 5 200 cu. ft. per sec., instead of 3 250 cu. ft. per sec. A similar wheel, 9 ft. 6 in. in diameter, at the Viereth plant in Germany gives 2 000 h.p., with a 17.4-ft. head and a speed of 93 (Figs. 2 and 3). This latter wheel was made by the Neumeyer Company, of Munich, Germany, and showed test results considerably better than runners of the Francis type for the same head and speed. About sixty runners of this type have been tested during the last year (1924).

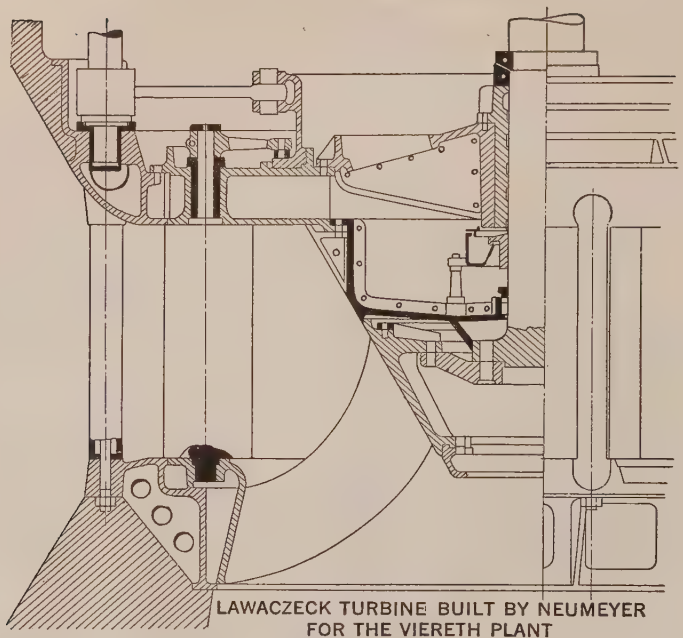
The Lilla-Edet plant has two of the 19 ft. 8-in. Lawaczeck runners installed, and the third unit is a propeller runner with adjustable blades of the Kaplan type. This third runner has about the same diameter and water capacity as the others, but the blades are adjustable, giving much better efficiency at low gate-openings. The Swedish Government has installed a testing laboratory at Lilla Edet and models of both wheels, 1 m. in size, were given an exhaustive test before the contracts were signed. Dr. D. Thoma, Professor of Hydraulic Engineering, Technische Hochschule, Munich, Germany, has stated that the form of the Lawaczeck runners is a mathematical surface with three parameters and that the test efficiencies may be related to the equations of the surface. He further states that this method of analysis will prove fruitful without restriction to any special theory of turbine design.

* Cons. Engr., New York, N. Y.



LAWACZECK RUNNER BUILT FOR LILLA EDET

Fig. 1.



LAWACZECK TURBINE BUILT BY NEUMEYER FOR THE VIERETH PLANT

FIG. 2.

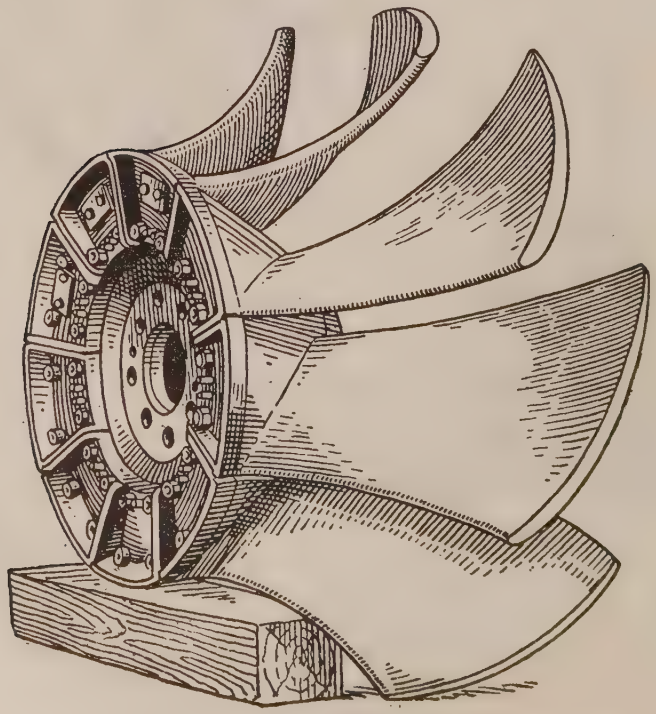


FIG. 3.—LAWACZECK RUNNER (9 FT. 6 IN. IN DIAMETER) FOR THE VIERETH PLANT.

The Kaplan runner at Lilla Edet is 19 ft. 0 in. in diameter and gives 11 200 h.p. It has four blades, adjustable while running. The design included a special draft-tube for each type of wheel. Actually, however, the second Lawaczeck runner was installed in a draft-tube designed for the Kaplan runner. The performance of the wheels at Lilla Edet is shown in Fig. 4.

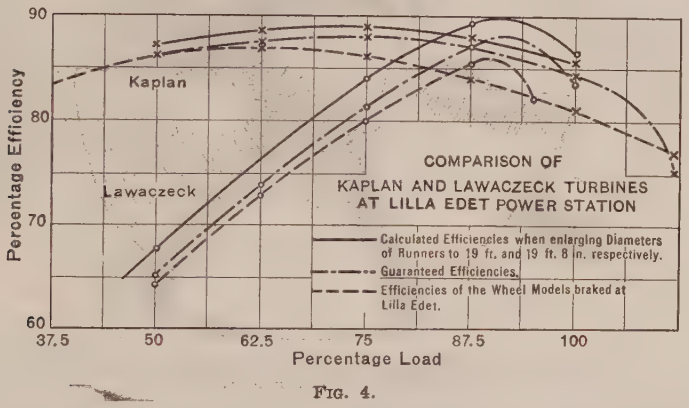


Fig. 4.

J. M. Voith and Company, of Heidenheim, Germany, has also manufactured a number of Kaplan runners, the first having been installed at Siebenbrunn, Austria (Fig. 6). This wheel is 6 ft. 3 in. in diameter, with 20.3-ft. head, and develops 400 h.p. at 250 rev. per min. The specific speed is about 212. There is a similar wheel, 5 ft. 1 in. in diameter, with a speed of 250 rev. per min., under a 13.75-ft. head, at Gävle, Sweden; apparently, a considerable number of these wheels have been made by other manufacturers.

The Voith Company has also been manufacturing wheels of the Kaplan type with fixed vanes. The installation at Kachlet, Germany, on the Danube, is quite interesting. Here, the wheels are 15 ft. 1 in. in diameter (Fig. 5), under a 25.1-ft. head, for 7 450 h.p., giving a specific speed of about 115. The runner has six blades covering nearly the entire area, whereas the four-bladed Kaplan designs do not cover much more than one-quarter of the projected area. Escher, Wyss and Company (Zurich, Switzerland), is also making a propeller turbine for which tests show good efficiency. In Fig. 7 is shown a six-bladed model; Fig. 9 shows performance curves of a four-bladed model. The tests of an interesting two-bladed helical runner (Figs. 8 and 10), made by Theo. Bell and Company (Kriens, Switzerland), have been published by Dr. F. Prazil, in Zurich. A turbine of this type for which the performance curve is shown in Fig. 10, is situated at the Matte Central Station in Berne, Switzerland. Dr. Hahn has given a table of test efficiencies of runners of this type collected from a number of European sources. Statistics of some of these high-speed European plants are given in Table 1 and comparative performances in Fig. 11.

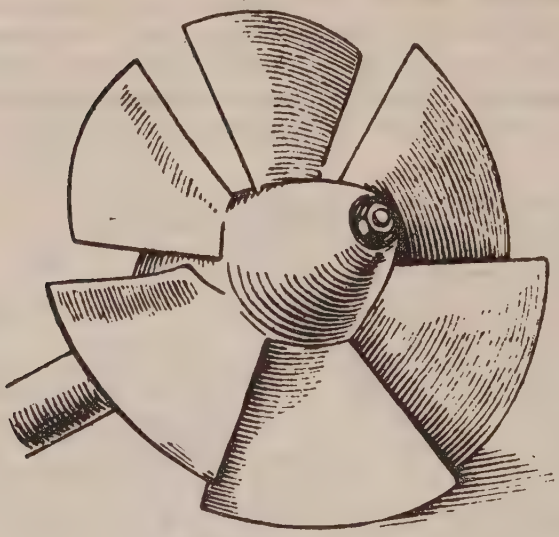


FIG. 5.—VOITH RUNNER (15 FT. 1 IN. IN DIAMETER) FOR THE KACHLET PLANT.

In his paper* before the World Power Conference, Dr. Thoma emphasizes the fact that the tests on small-sized, geometrically similar models give reliable figures for large-sized operations and that in the few cases of divergence noted a lack of geometrical similarity has been proven. He also states that the Camerer correction formula for efficiency has been found to be fairly correct. He gives a list of thirteen hydraulic testing laboratories actually running in Germany alone.

TABLE 1.—EUROPEAN LOW-HEAD TURBINES.

Manufacturer.		Location.	Diameter.	Discharge, in cubic feet per second.	Head, in feet.	Revolutions per minute.	Horse-power.	Specific speed.
Charmilles, S. A...	F	Chancy-Pougny, Switzerland	18 ft. 0 in.	3 250	26.7	83.3	7 600	120
A. B. Finshyttan*.	L	Lilla Edet, Sweden.....	19 ft. 8 in.	5 200	21.3	62.5	10 000	136
A. B. Finshyttan†.	K	Lilla Edet, Sweden.....	19 ft. 0 in.	5 300	21.3	62.5	11 200	140
Voith.....	K	Siebenbrunn, Austria.....	6 ft. 3 in.	.....	20.3	250	400	212
A. B. Karlstads....	K	Gavle, Sweden	5 ft. 1 in.	.....	13.75	250	400	219
Voith.....	H	Kachlet, Germany	15 ft. 1 in.	.....	25.1	.....	7 450	115
Neumeyer.....	L	Viereth, Germany.....	9 ft. 6 in.	.....	17.4	.....	2 000	93
Vevey.....	H	Wynau, Germany.....	10 ft. 10 in.	1 700	17.0	107	2 700	165

* Lawaczeck wheel.

† Kaplan wheel.

It is fair to state that Dr. Thoma emphasized the cavitation difficulty and is testing models under cavitation conditions. Half his paper was devoted to

* "Experimental Research in the Field of Water Power," *Transactions*, First Water Power Conference, 1924, Vol. II, p. 536.

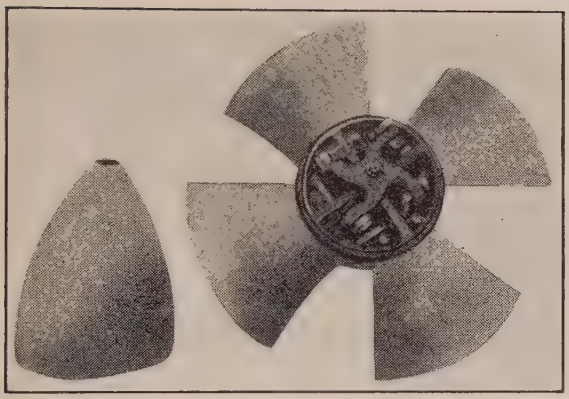


Fig. 6.—Kaplan Runner at Siebenbrunn Plant, Showing Mechanism for Moving Pivoted Blades.

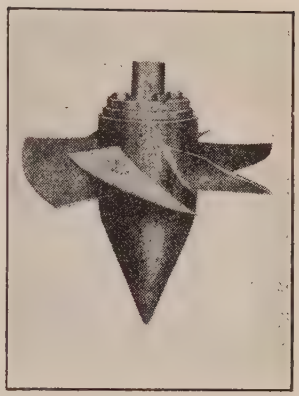


Fig. 7.—Dubs Propeller Runner Made by Escher Wyss and Company.

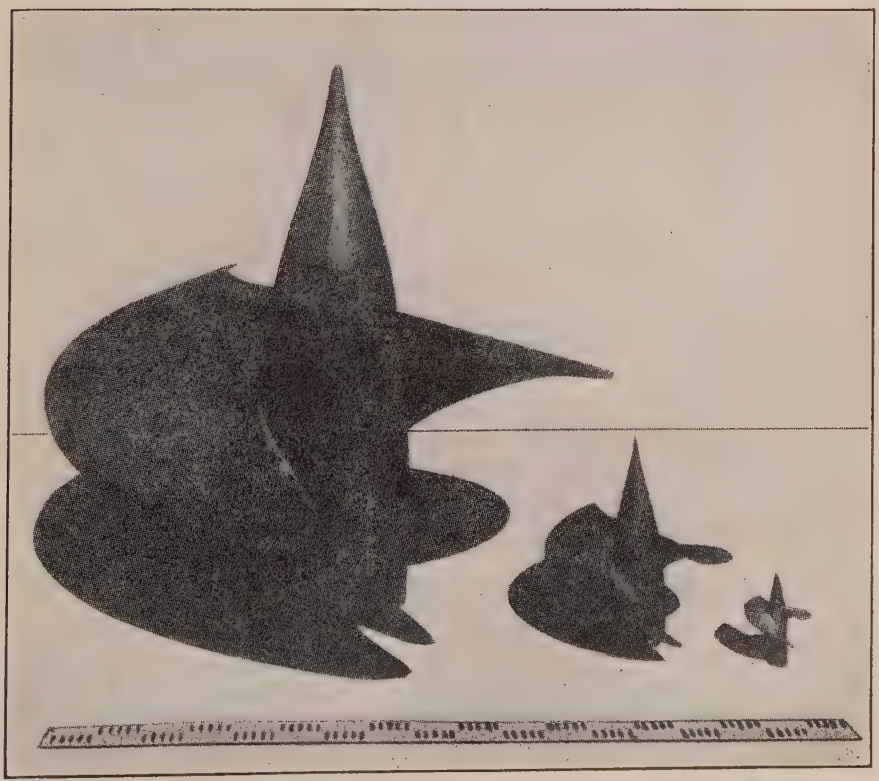


Fig. 8.—Bell Runner for the Matte Plant, with the Two Small Models Used in the Tests.

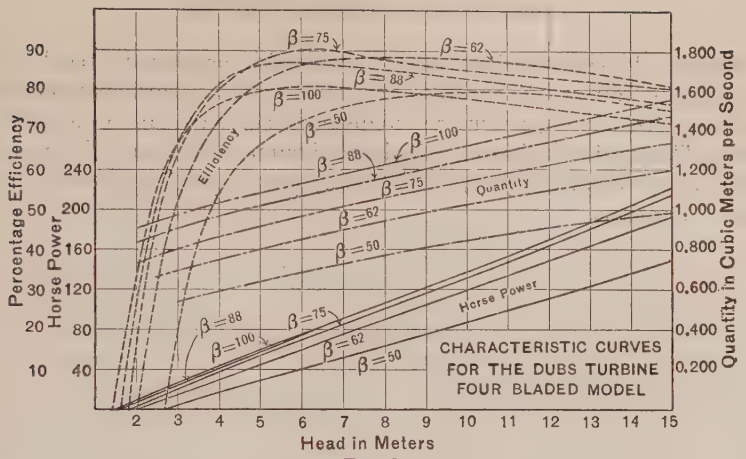


FIG. 9.

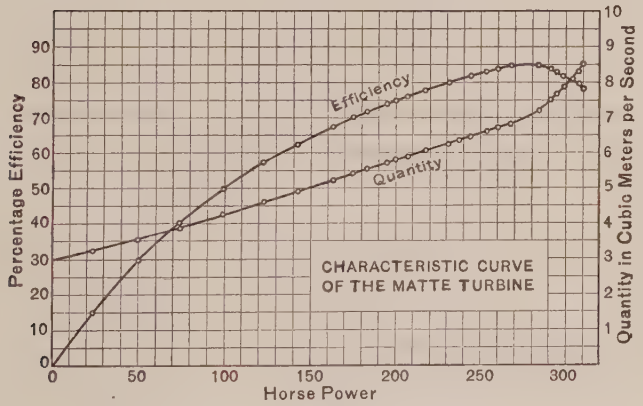


FIG. 10.

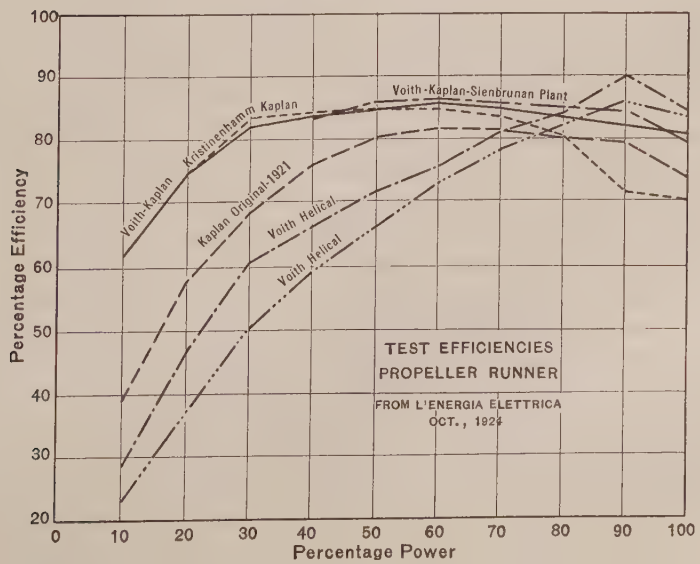


FIG. 11.

this subject. In conversation, he stated that this problem in the high-speed runner can be solved. Meanwhile, reports as to the performance and life of these large sized and particularly high-speed runners will be awaited with avidity. The speaker believes the high-speed runner has come to stay and that present difficulties with this type will be overcome, as with the older and better understood designs.

THE PROPELLER TYPE TURBINE

By Lewis F. Moody,* Esq.

To any one who has followed hydraulic power engineering it must seem surprising that a machine so highly developed as was the hydraulic turbine ten or fifteen years ago, should have suddenly entered a period of radical changes and encountered transformations in many of its essentials. The fact remains, however, that although practice in turbine design had become almost standardized in the period just prior to the World War, the development of this art has taken on new life and has already been revolutionized in many aspects.

The most striking changes have been those in runner and draft-tube design, resulting in a remarkable increase in the specific speeds. This paper is directed particularly to one form of high-speed turbine and to the description of a number of typical installations, representing some of the most recent developments in this field. Before describing particular installations, however, a few general relations will be discussed leading to some of the reasons which account for the adoption of the propeller type of runner and which explain at least in part the logic of its development.

The first point discussed is not directly pertinent, but is of some interest in itself, namely, the relation between size of turbine and efficiency; the second point then follows—the relation between type or form of turbine and its efficiency and specific speed. The results of several actual tests of models of the propeller type and of large-sized installations will then be presented.

Variation of Efficiency with Size of Turbine

In order to compare results secured on different sizes of models with full-sized turbines, the writer has believed it advisable to take up the question of the effect of size on turbine performance in a series of homologous turbines. In recent years calculations for this purpose have usually been based on the Camerer formula, of the form:

$$e' = 1 - (1 - e)\,\frac{0.12 + \dfrac{0.015}{\sqrt{\left(\dfrac{f_2}{U_2 D}\right) D'}}}{0.12 + \dfrac{0.015}{\sqrt{\left(\dfrac{f_2}{U_2 D}\right) D}}}$$

in which, e equals the efficiency of one turbine, e' that of another turbine; D and D' are their diameters; and $\dfrac{f_2}{U_2 D}$ is the ratio of the outflow area, f_2, of the

* Cons. Engr., I. P. Morris Dept., The William Cramp & Sons Ship & Engine Building Co., Philadelphia, Pa.

runner buckets to the product of the wetted perimeter and the runner diameter; that is, the ratio of the hydraulic radius of the runner bucket exit area to the runner diameter.

Although the Camerer formula has been found to be in fairly good agreement with actual results of tests, it is not altogether satisfactory for the following reasons:

1.—It involves the calculation of the hydraulic radius of the water passages through the turbine and cannot be accurately applied unless the design of a particular turbine is available. It is complicated in form and involves a number of constants or coefficients which can only be accurately determined by applying it to a number of installations of similar general character to any case under consideration. In determining the hydraulic radius to be applied in the formula, it would be necessary to arrive at some average value which would fairly represent the entire course of the water through the turbine, a matter of considerable complication even when the design is fully known. Camerer avoids this complication by using the minimum section which usually occurs at the outlet orifice of the runner buckets. Although the formula has been generally used with this assumption, it no longer remains entirely rational.

2.—The formula is also derived from the assumption that all the losses through a turbine are due to surface resistance or so-called "pipe friction", and discards the consideration, which was recognized by Dr. Camerer, that a part of the losses in a turbine is due to eddies, impact, and velocity head rejected at final discharge. The complicated form of the Camerer expression is due to the fact that it is based on Dr. Biel's formula for pipe friction which is an expression involving the sum of three terms.

Considering the fact that the losses in a turbine are dependent on many factors and involve many complications and that in comparing efficiencies only approximate results at best can be expected, the writer believes that a simple form of expression involving only the leading variables and giving a simple dimensional relation without including the details of design, can be used with a gain both in convenience of application and in accuracy. For this purpose the writer proposes in place of the Biel formula for pipe friction or other complex form of expression, such as the Kutter formula for channels, the use of an exponential formula, and for this purpose suggests either the recent Forcheimer formula:

$$V = k\, R^{0.7}\, J^{0.5}$$

in which,

V = velocity;
R = hydraulic radius; and
J = slope of hydraulic gradient;

which is equivalent to:

$$J = \frac{V^2}{k^2\, R^{1.4}}$$

or the still more recent Strickler formula:*

$$J = \frac{V^2}{K^2 R^{\frac{4}{3}}}.$$

The Strickler formula is based on rational grounds and has been shown to be in close agreement with a large range of actual measurements. If λ represents the proportional loss of head in a given turbine expressed as a fraction of the total head on the turbine, then $\lambda = 1 - e$, in which e is the turbine efficiency.

If $\lambda = 1 - e$ is the proportional loss of head in one runner, say, a small model, assume that a certain fraction, a, of this loss, that is, $a\,(1 - e)$, remains unchanged as the runner is increased (homologously) in size; while the remaining portion, $(1 - a)\,(1 - e)$, is due to surface friction, so that the proportional loss in another, say, a larger, runner, will be:

$$\lambda' = 1 - e' = a\,(1 - e) + \lambda'_f$$

Now the surface friction in any channel may be calculated by the exponential formula:

$$h_f = f \frac{L}{R^x} V^m$$

in which, except for very smooth pipes of small diameter, m may be taken as closely equal to 2, a value corresponding to cast-iron pipes, and x may be taken between 1.4 (Forcheimer) and 1.167 (Hazen and Williams), or as $1\frac{1}{3}$ (Strickler) (also proposed by Manning, 1890).

Calling $x = 1 + p$, in which $p = \frac{1}{3}$, if the Strickler value is used,

$$h_f = f \frac{L}{R} \frac{V^2}{R^p}$$

Expressed as a fraction of the total head on the turbine,

$$\lambda_f = \frac{h_f}{H} = f \frac{L}{R} \frac{V^2}{R^p H}$$

In homologous runners of all sizes, under any head, when operating at equal gate and corresponding speeds, V^2 varies directly as the head, H, so that $\frac{V^2}{H}$ = a constant; and in all homologous runners, $\frac{L}{R}$ = a constant; and as R is proportional to D, the runner diameter,

$$\lambda_f = \frac{k}{D^p} = (1 - a)\,(1 - e);$$

and

$$\lambda'_f = \frac{k}{D'^p}$$

* "Beitrage zur Frage der Geschwindigkeitsformel und der Rauhigkeitszahlen für Strome, Kanale und geschlossene Leitungen," by A. Strickler, *Schweizerische Bauzeitung*, June 7, 1924, p. 265.

Hence, putting:

$$k = (1 - a)\ (1 - e)\ D^P,$$

we have:

$$\lambda' = (1 - e') = a\ (1 - e) + (1 - a)\ (1 - e)\ \frac{D^P}{D'^P}$$

or,

$$\frac{1 - e'}{1 - e} = a + (1 - a)\left(\frac{D}{D'}\right)^P$$

From the application of this formula to a number of actual comparative tests, it was found that if P is taken as $\frac{1}{3}$, then a, the unvarying portion of the loss, is a comparatively small fraction, approximately $\frac{1}{4}$; and it is also found that the whole loss can be closely approximated for usual values by substituting for $a + (1 - a)\left(\frac{D}{D'}\right)^P$, the simpler expression, $\left(\frac{D}{D'}\right)^n$, in which a slight change in P to n closely compensates for the dropping of the constant term. Moreover, there is basis for believing that a varies with size of runner; in a small runner the vanes are relatively thicker, less sharply pointed, and less accurately formed, and have a greater eddy loss at their discharge edges and elsewhere. Hence, an expression of the form:

$$\frac{1 - e'}{1 - e} = \left(\frac{D}{D'}\right)^n$$

should be sufficient for all practical purposes in computing the effect of size on efficiency; and an application to a number of actual tests shows this to be true, with n varying but little from the value of $\frac{1}{4}$.

Therefore, the following formula may be used:

$$e' = 1 - (1 - e)\left(\frac{D}{D'}\right)^n$$

or,

$$e' = 1 - (1 - e)\left(\frac{D}{D'}\right)^{\frac{1}{4}}$$

Effect of Change of Form of Turbine on Efficiency and Speed

The preceding discussion applies to geometrically similar turbines. Consider, next, the effect of change of form or type of turbine. The particular change which it is desired to consider is that from a so-called Francis turbine to a turbine of the propeller type. This change in type involves the following principal alterations: (*a*) The elimination of the runner band or shroud ring, allowing the blades to rotate within a stationary wall; (*b*) the introduction of a large transition space between guide-vanes and runner, in which the water follows a free path and turns from a radial to a partly or completely axial direction; (*c*) the reduction of blade surface so that the blades either overlap each other only slightly or fail to overlap at all and leave clear openings between them; (*d*) a reduction in the number of blades; and (*e*) the use of draft-tubes symmetrical about the turbine axis and capable

of efficiently converting into useful head the velocity head of both the meridian and tangential components of the runner discharge velocity, thus permitting both components to be made relatively large.

The effect of each of these changes is to allow the runner to be operated at a higher speed under a given head, by permitting higher relative velocities to be used and a more inclined direction of the absolute velocity of runner discharge.

The reason for eliminating the band or shroud is simple. With the runner speeds and diameters used, the relative velocity of the water in contact with a moving band would be considerably greater than its absolute velocity along the surface of the stationary wall, so that there is less surface resistance on the inner surface of the band when it is stationary than when it rotates; and by replacing the rotating band by a stationary wall, all the frictional resistance of its outer surface is eliminated.

The effect of the transition space is to reduce the relative velocity between the water and the blades at all sections except at the blade tips. If this space is too much enlarged, however, or if the entrance edges of the blades are carried too near the axis, the relative velocity near the hub begins to increase again, the water at that point then tending to rotate faster than the runner, and, in addition, the surface friction due to the long absolute path along the stationary head cover partly discounts the decrease in blade friction. This consideration is one which has led to the writer's diagonal type of runner, intermediate between a radial and a purely axial flow runner. In a similar way, an undue reduction in blade surface while decreasing the surface resistance, if carried too far results in imperfect guidance of the flow and permits other losses which more than offset the gain. It has been found both from theory and experience that unduly small blade area greatly impairs the stability of operation, giving rise to surging, unstable flow, and loss of efficiency, and materially reduces the field of application of turbines of the propeller type in which it is found.

The effect of reducing the number of blades can be readily investigated along the lines of the analysis given previously for homologous runners. Assume a series of runners in which the same ratio of total blade area to disk area is used, but in some of which there are many blades with short and narrow passages between them and in others of which there are a few blades with long and wide passages, the blade sections being geometrically similar in all and having the same ratio of length to width of passages. The runners being unshrouded, the wetted perimeter of a runner channel may be taken as approximately equal to twice the blade length from hub to band plus the hub surface; and if the hub surface is neglected and offset approximately by taking the blade length as if it extended to the axis, the wetted perimeter can be called $2\,r$, in which, r is the tip radius of the blades, measured in a conical surface. The area of one runner passage is:

$$\frac{r \times 2\,\pi \frac{r}{2} \sin \beta}{z}$$

in which, β is the blade inclination and z is the number of blades. Hence, the hydraulic radius of a runner passage is:

$$\frac{\pi r^2 \sin \beta}{2 z r} = \frac{\pi r \sin \beta}{2 z}$$

With the same line of reasoning, the proportion of the total head on a turbine lost in blade friction will vary inversely as the hydraulic radius raised to the nth power, or using the Strickler value of $n = \frac{1}{3}$, the ratio of blade friction losses in two runners can be expressed as:

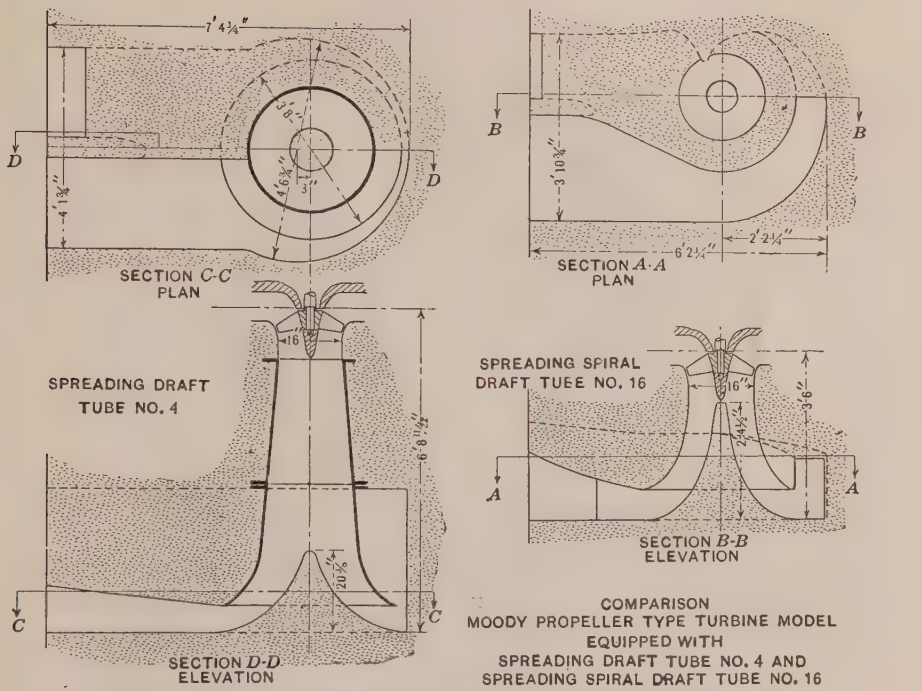
$$\frac{\lambda'_b}{\lambda_b} = \left(\frac{\frac{\pi r \sin \beta}{2 z}}{\frac{\pi r' \sin \beta}{2 z'}} \right)^n = \left(\frac{r}{r'} \frac{z'}{z} \right)^{\frac{1}{3}}$$

This relation shows that in such a series of runners as described the blade friction will vary directly as the cube root of the number of blades, and inversely as the cube root of the radius. For example, comparing a 16-vane runner with a 4-vane runner, the latter will have $\left(\frac{4}{16}\right)^{\frac{1}{3}} = 0.63$, or 63% of the blade friction loss of the first, or a reduction of 37 per cent. It must be kept in mind that this does not mean keeping the blades the same in size as their number is reduced, but enlarging them to keep the total blade area the same. Although a very small number of blades will be advantageous in reducing this particular loss, there are, of course, certain disadvantages in carrying this feature too far, such as the increase in axial length of the runner hub and blades, with resulting increase in weight of runner and the distance of its center of gravity from the bearing, increase in length of the surrounding ring, and greater loss in friction against the ring.

In considering the last factor, the change in form of draft-tube, no attempt at a general analysis of the effect of form of draft-tube on the performance of turbines of the propeller type will be given in this paper, as the subject has been rather exhaustively treated in other papers; it may be pertinent to point out, however, that one of the objects of recent studies on the draft-tube feature of these turbines has been the reduction in over-all dimensions without impairing the performance. To give an idea of what has been accomplished in this direction there are shown in Fig. 12 a model of the propeller type tested with a spreading draft-tube of large dimensions, in comparison with a recent design of a spreading spiral tube tested with the same model; and, in Fig. 13, the performance curves are compared.

Typical Test Results on Propeller Type Models

In Figs. 14 and 15 curves of efficiency plotted against specific speed are shown for two turbines of the propeller type of the form developed by the writer from Holyoke tests. Each of the individual curves on these sheets represents a particular gate-opening; the envelope shows the efficiency obtainable at various specific speeds.



COMPARISON
MOODY PROPELLER TYPE TURBINE MODEL
EQUIPPED WITH
SPREADING DRAFT TUBE NO. 4 AND
SPREADING SPIRAL DRAFT TUBE NO. 16

FIG. 12.

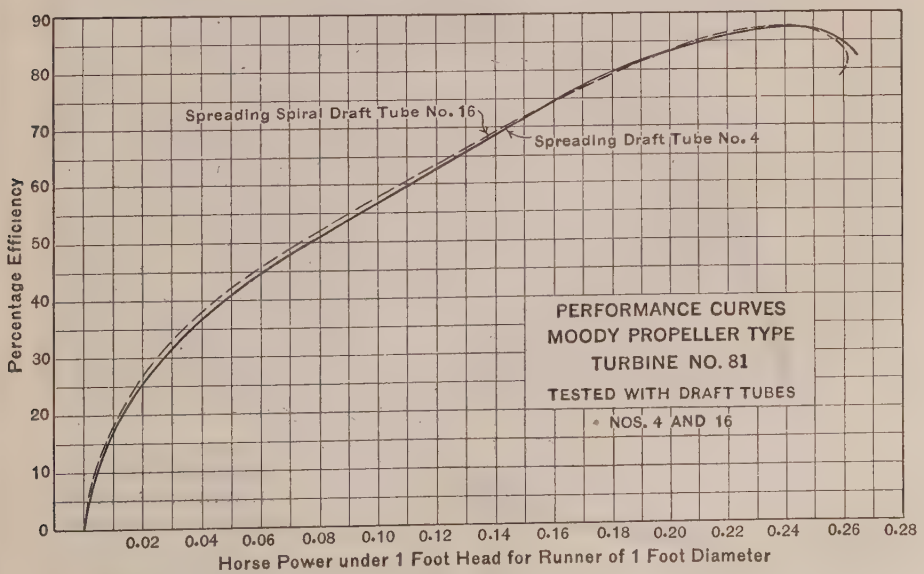


FIG. 13.

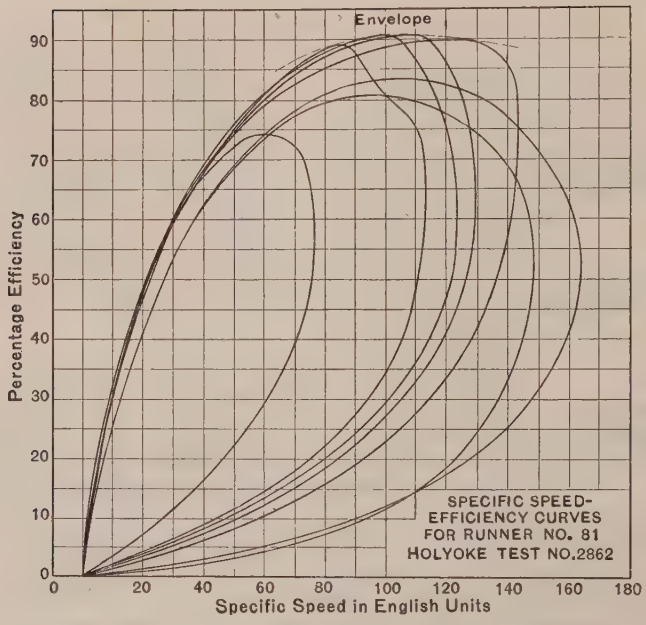
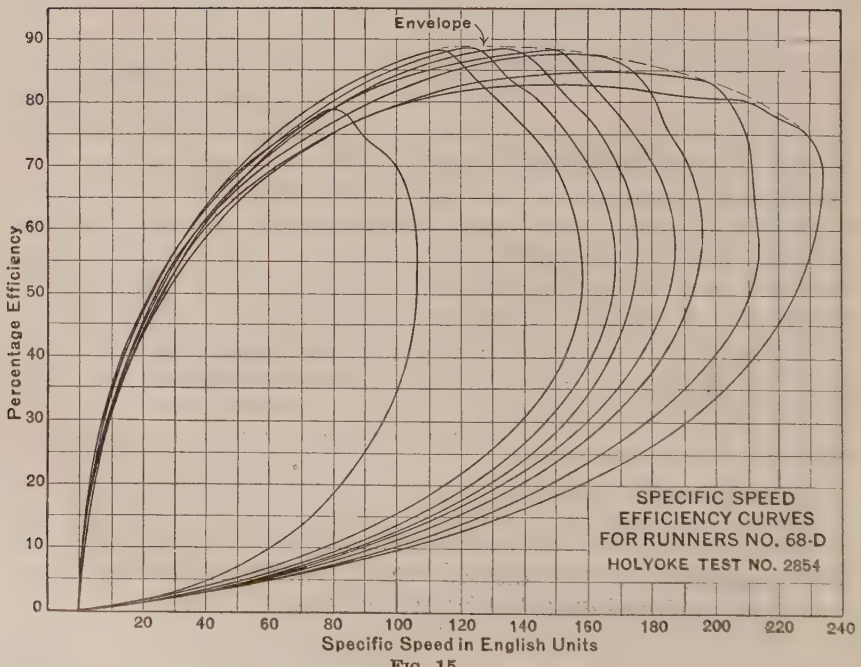

Envelope
Percentage Efficiency
0
10
20
30
40
50
60
70
80
90
0
20
40
60
80
100
120
140
160
180
Specific Speed in English Units
SPECIFIC SPEED-
EFFICIENCY CURVES
FOR RUNNER NO. 81
HOLYOKE TEST NO.2862

FIG. 14.


Envelope
Percentage Efficiency
0
10
20
30
40
50
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60
80
100
120
140
160
180
200
220
240
Specific Speed in English Units
SPECIFIC SPEED
EFFICIENCY CURVES
FOR RUNNERS NO. 68-D
HOLYOKE TEST NO. 2854

FIG. 15.

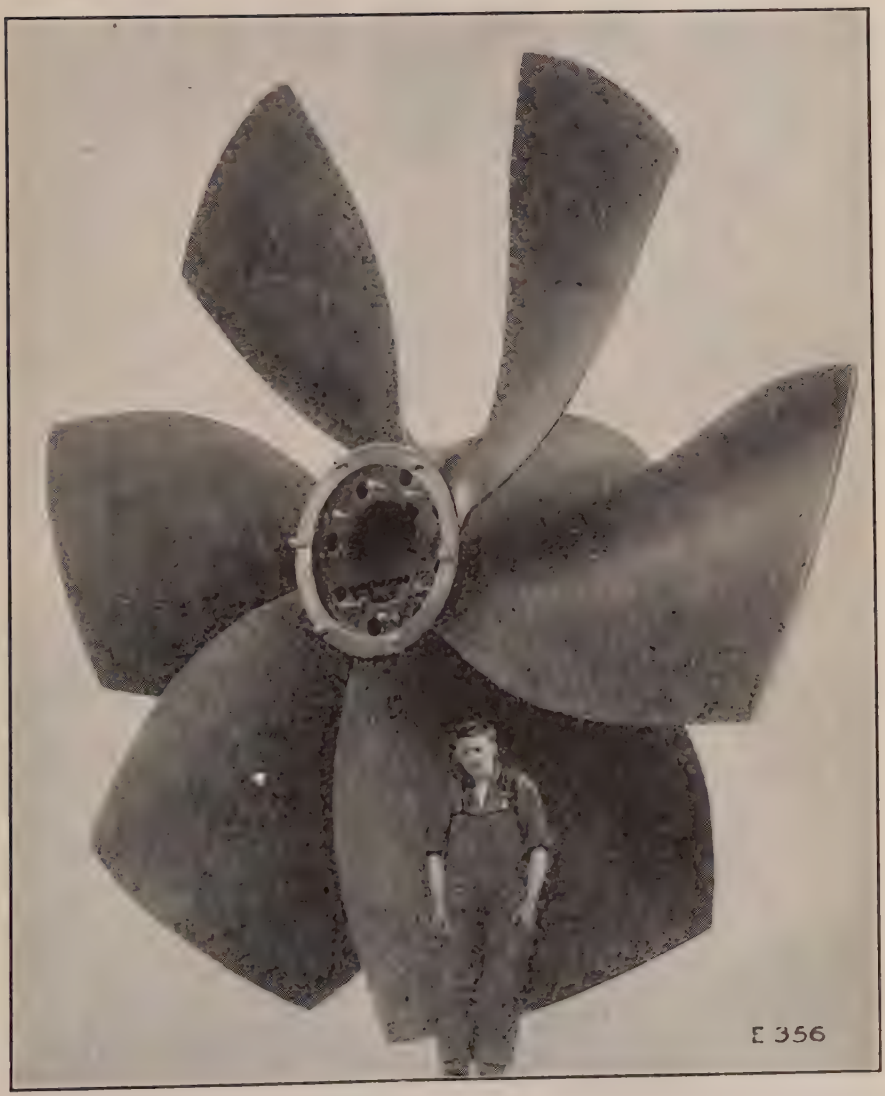


Fig. 16.—Propeller Type Runner (30 000 h.p.), for the La Gabelle Plant.

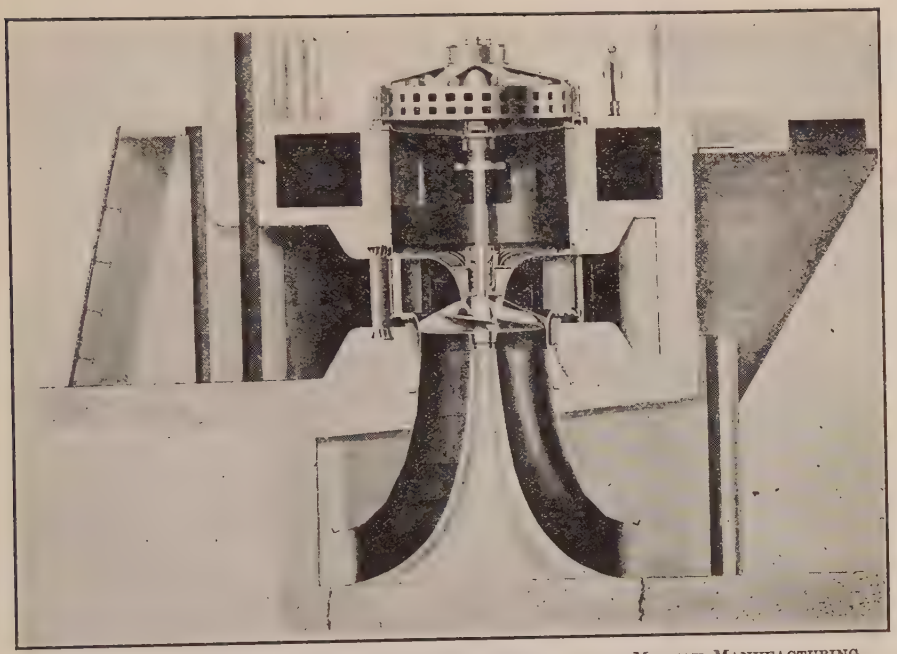


Fig. 17.—Sectional Elevation of 1 500-h.p. Turbines for Moreau Manufacturing Company.

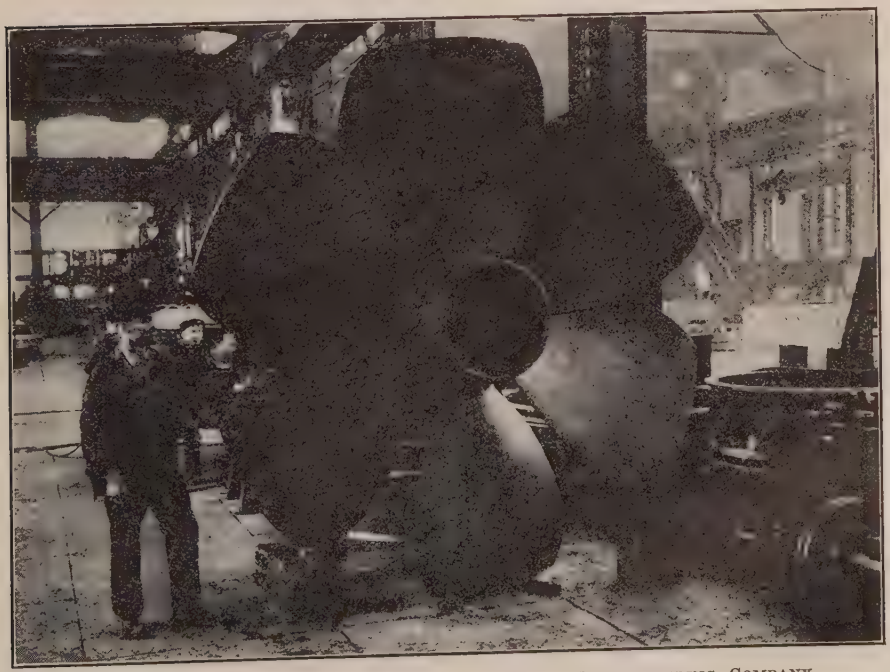
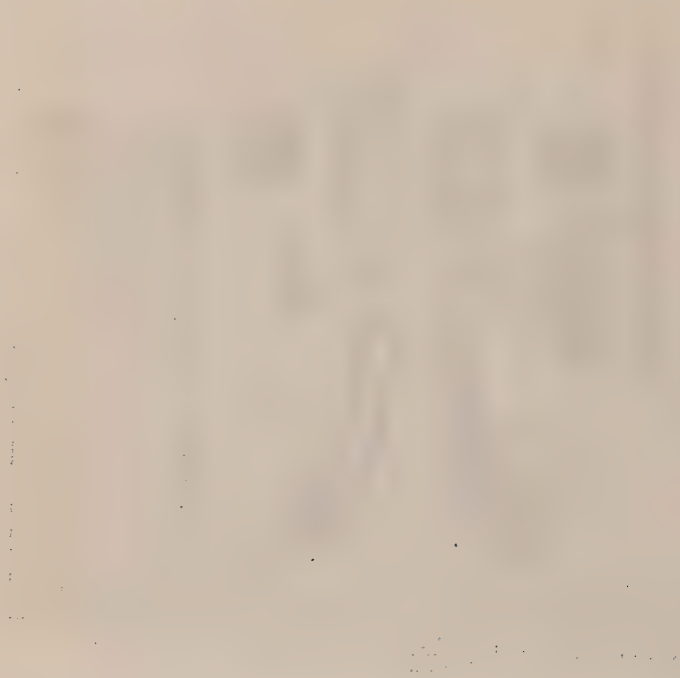


Fig. 18.—Propeller Type Runner for Moreau Manufacturing Company.



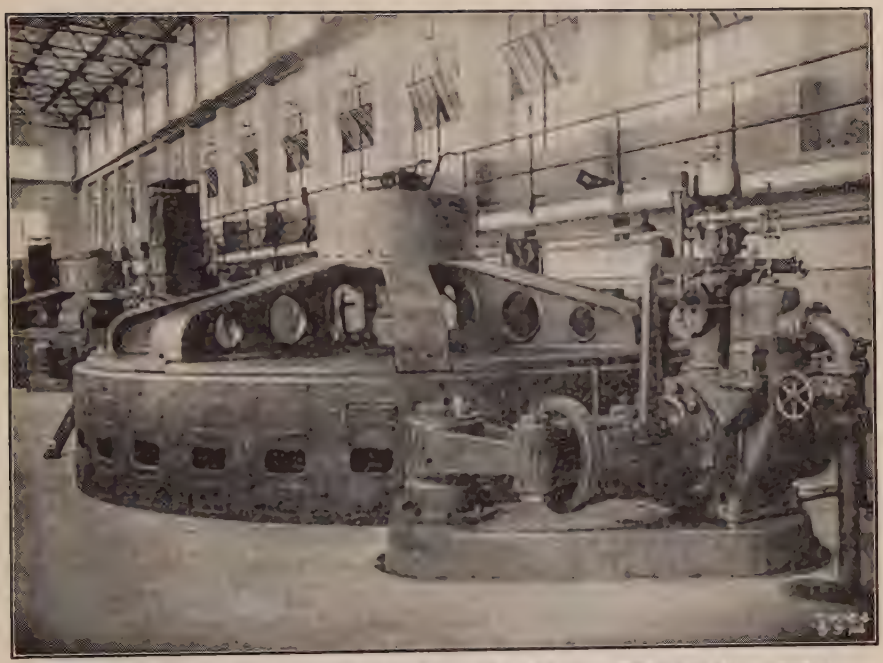


Fig. 19.—Interior View of Power-House, Feeder Dam Development, Moreau Manufacturing Company.

Fig. 20.—Interior View of Power-House, Anson Plant.

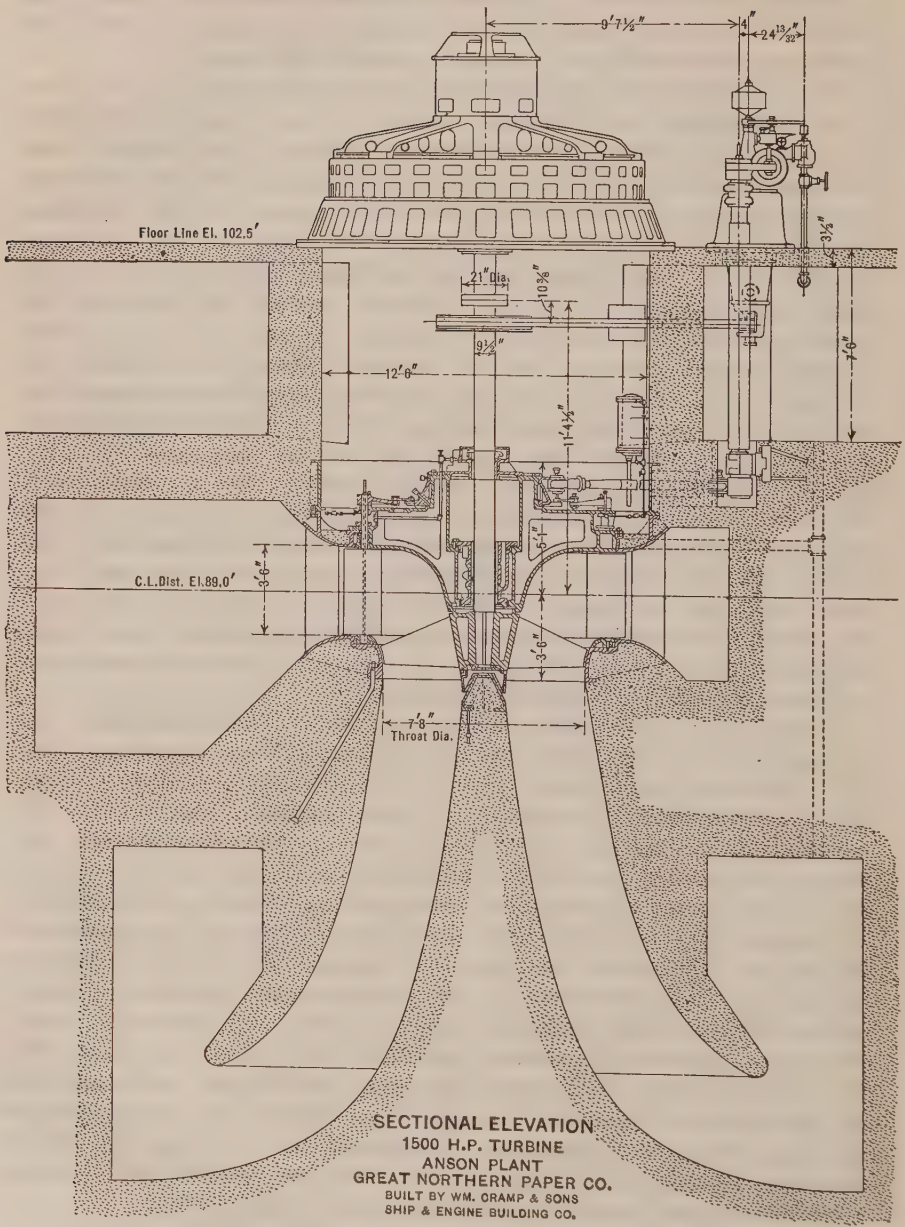
Some Typical Installations

Having discussed from a somewhat theoretical standpoint, a few of the factors affecting the design of these turbines, brief descriptions of a few recent installations will now be given to conclude this paper.

Fig. 16 shows one of the four 30 000-h.p. runners at the La Gabelle plant of the St. Maurice Power Company. These units have already been described in other papers, as have the similar 28 000-h.p. turbines of the Manitoba Power Company, Limited. It may be pointed out that the La Gabelle draft-tubes are closely similar to the spreading-spiral form shown in Fig. 12. The Manitoba units have been in operation under a wide range of heads—to more than 50 ft.—for about 2 years, and the La Gabelle units for nearly a year, under heads to more than 60 ft. It may be reported as a result of the experience with these units that the Manitoba runners have shown a slight tendency toward corrosion on the discharge edges of the blades, due perhaps to the abnormal operating heads, but the effect is local and not serious. The La Gabelle runners are in perfect condition. In both plants the concrete just below the runner has been eroded, and this has been corrected by the addition of plate-steel liners at the top of the draft-tube, a practice which will be followed in future installations of the propeller type under the higher heads. In all the remaining installations to be described subsequently, there has been an entire absence of pitting or corrosion at any point; and in all these installations, including the Manitoba and La Gabelle units, the operation has been characterized by quietness, steadiness, and freedom from surging or vibration or other difficulties, and in every case the calculated power output has been exceeded by a reasonable margin.

Figs. 17, 18, and 19 show the 1 500-h.p. units at the Feeder Dam plant of the Moreau Manufacturing Corporation on the Hudson River, near Glens Falls, N. Y. There are five of these units with a rated capacity of 1 500 h.p. each, operating under a normal head of 15.5 ft. and a maximum head of 19 ft. The speed is 120 rev. per min., corresponding to a specific speed of 151.5 (672 metric). The design has been simplified as far as possible and the object has been to keep the cost of turbines as low as possible without sacrificing mechanical dependability or the hydraulic conditions of operation. On account of the low head available for this station, a minimum cost of the development is naturally of great importance. In this plant an outside operating mechanism has been dispensed with, the gates being operated from links connected directly to the vanes, by means of a vertical gate-shaft. The runners are of cast iron, the casing stay-vanes are separate castings set individually in the concrete, and no continuous "speed ring" is used. The draft-tube is a relatively short spreading tube with the barrel or spreading portion of plate steel. These units have been in actual operation for several months and tests of power output have shown excellent agreement with the predicted values.

Fig. 21 shows a sectional elevation of one of the 1 500-h.p. turbines for the Anson plant of the Great Northern Paper Company, near Madison, Me. Fig. 20 shows an interior view of the power-house. This plant has been in operation for about fourteen months. Fig. 22 shows the efficiency curve secured



SECTIONAL ELEVATION
1500 H.P. TURBINE
ANSON PLANT
GREAT NORTHERN PAPER CO.
BUILT BY WM. CRAMP & SONS
SHIP & ENGINE BUILDING CO.

FIG. 21.

in a test of one of the full-sized units under actual operating conditions. This test was made by C. M. Allen, M. Am. Soc. C. E., using his salt-velocity method of water measurement, the water being measured in an open flume constructed in the tail-race. There are five of these units of 1 500-h.p. capacity each, operating under a normal head of 20 ft., at a speed of 150 rev. per. min. This speed corresponds to a specific speed of 137 (610 metric). These turbines have integral stay-vane rings or speed rings, outside operating mechanism actuated by a vertical gate-shaft, and the runners are of cast iron. The draft-tubes are of the spreading type.

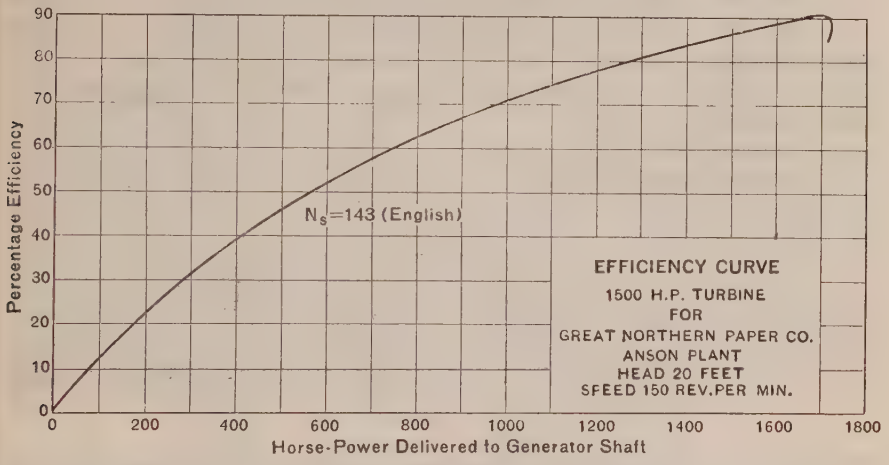


FIG. 22.

An installation very similar in design to the Feeder Dam and Anson plants is that of the Spruce Falls Company, Limited, at Kapuskasing, Ont., Canada. This plant was built by the Dominion Engineering Works, Limited, and involves a propeller type unit of 2 500 h.p., under a 30-ft. head, operating at a speed of 180 rev. per min. This corresponds to a specific speed of 128 (571 metric). Preliminary reports of a test made by Professor Allen indicate that the efficiency will probably be in excess of 90 per cent.

Other recent installations of the same type of turbine carried out in Canada are the following:

Two units for the Howard Smith Paper Company, of 350-h.p. capacity each, under a head of 8 ft. These have a speed of 99 rev. per min., corresponding to a specific speed of 134 (597.5 metric).

One unit for the Dryden Paper Company of 1 400-h.p. capacity, under a head of 29 ft., at a speed of 225 rev. per min., corresponding to a specific speed of 125 (556 metric).

These installations were built by the Dominion Engineering Works, Limited, of Montreal, Que., Canada, which also has under construction two turbines of 6 000-h.p. unit capacity for a head of 30 ft. normal, and 34 ft. maximum, for the Southern Canada Power Company. The draft-tubes are of the spreading type formed in concrete and will be provided with separate cast stay-vanes at the end of the spreading portion to carry the weight of the

superposed structure. Similar stay-vanes were used in the Manitoba and La Gabelle installations and in a large number of Francis turbine installations. These vanes have been found to be a valuable structural feature reducing the reinforcement in the power-house substructure which would otherwise have to carry the loads by beam action, and also proving useful during construction as they permit the earlier removal of forms. These turbines will have removable liner rings surrounding the runners; the runners will be of cast steel. The operating mechanism will be of the outside type. These units will operate at 138.5 rev. per min., or a specific speed of 153 (680 metric).

Among the recent installations in the United States may be mentioned a unit of 3 200-h.p. capacity for the Roanoke Rapids Power Company. This turbine operates under a head of 30 ft. at a speed of 163.6 rev. per min., or a specific speed of 131.5 (585 metric). This unit will also be equipped with cast-steel runner.

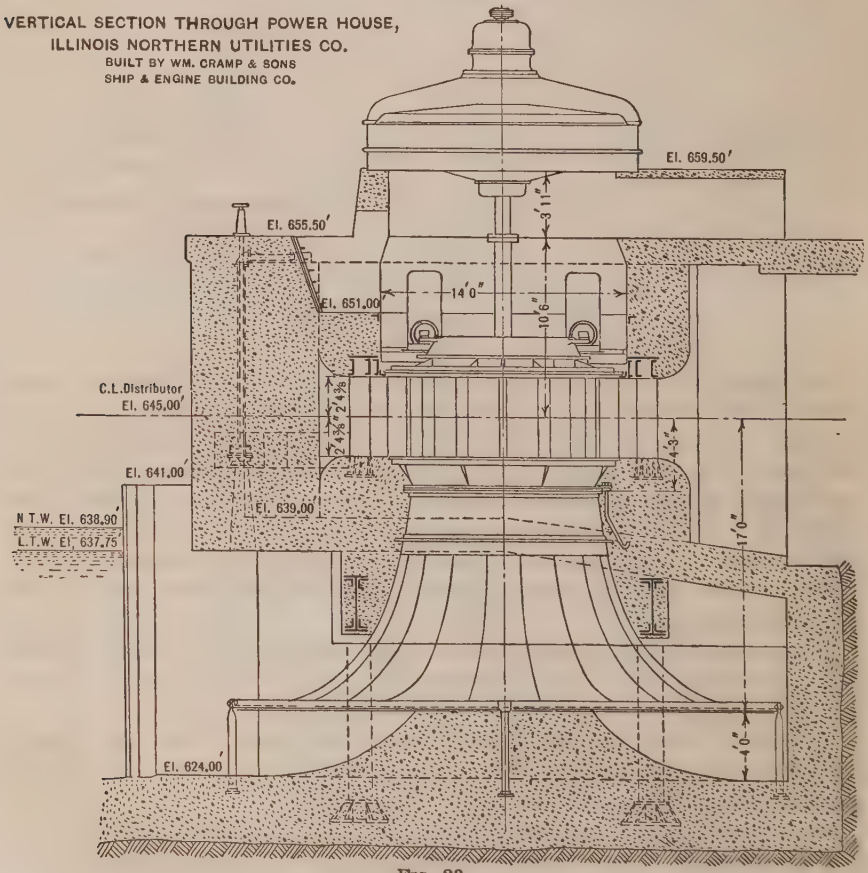


FIG. 23.

An installation of particular interest, the construction of which has recently been completed, is that of five units for the Illinois Northern Utilities Company, shown in section in Fig. 23. These turbines will have a unit capacity

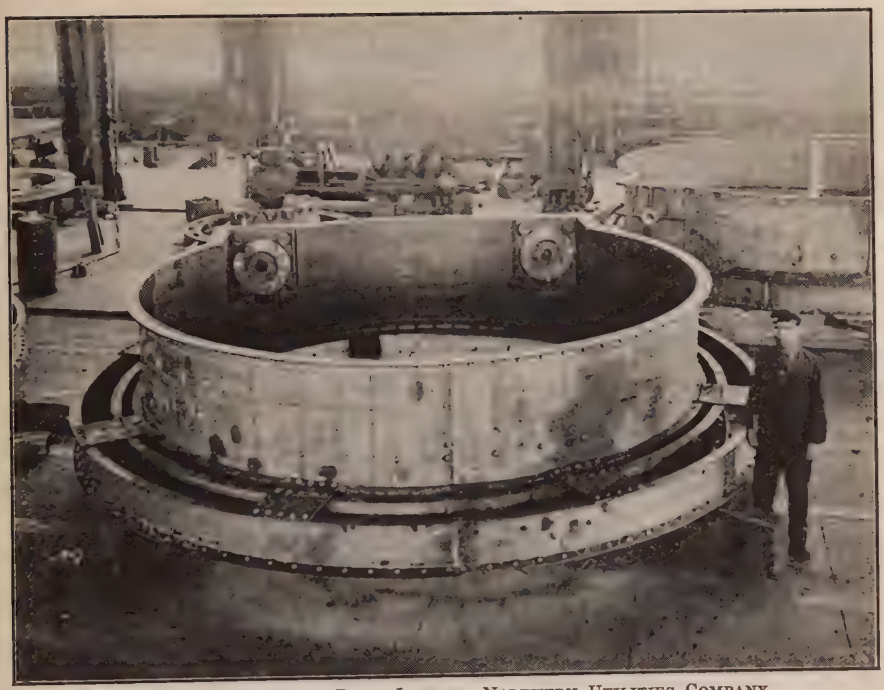


Fig. 24.—Foundation Ring, Illinois Northern Utilities Company.

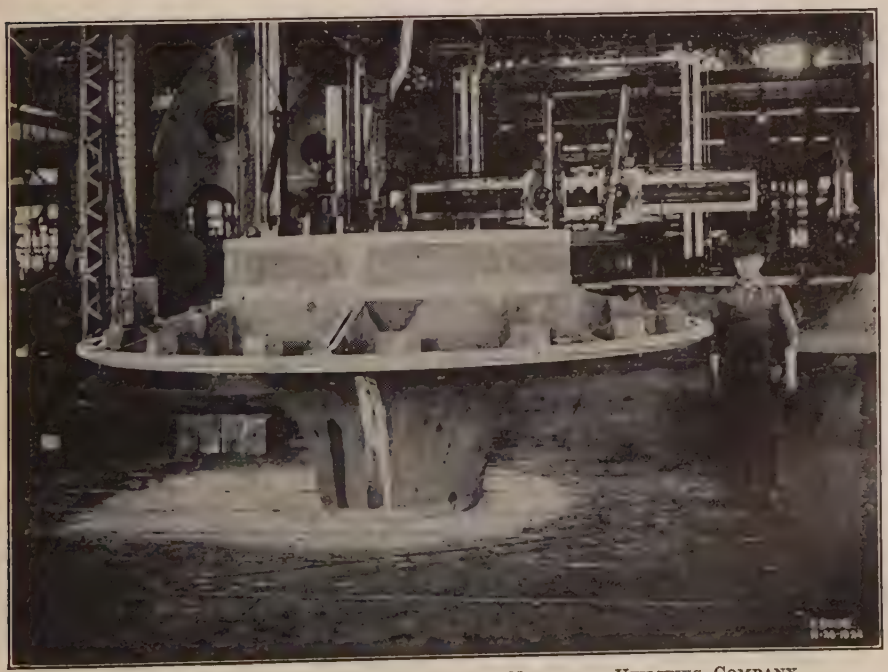


Fig. 25.—Turbine Head Cover, Illinois Northern Utilities Company.

Fig. 26.—Guide-Vanes, Illinois Northern Utilities Company.

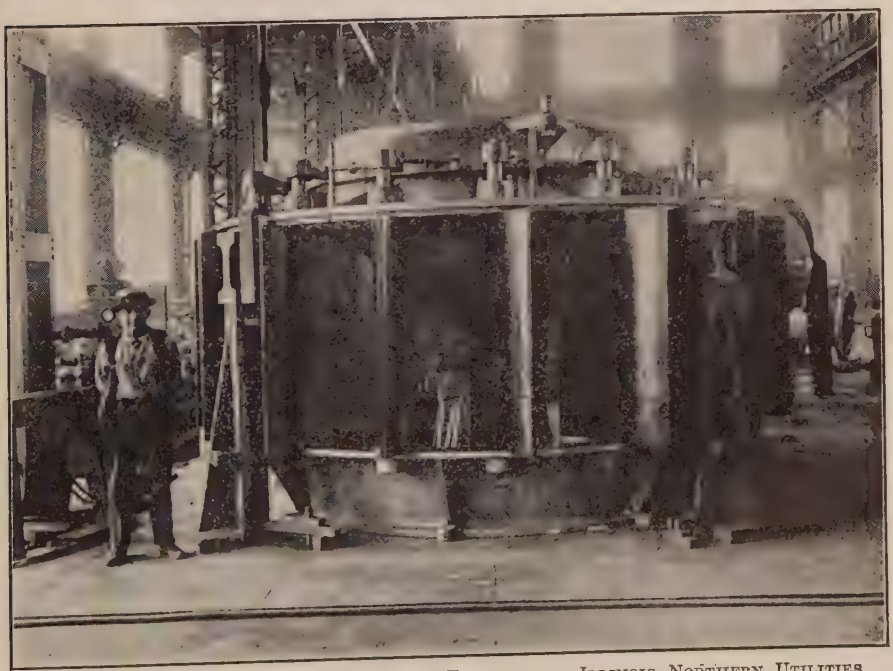


Fig. 27.—Partial Assembly of 800-h.p. Turbine for Illinois Northern Utilities Company.

of 800 h.p., under a head of 8 ft., operating at a speed of 80 rev. per min. This plant is of unusual interest because of the low head which has been utilized. The specific speed is 168 (748 metric). Every effort was made to keep the cost of installation to a minimum. Separate stay-vanes have been used, bolted at the top to the foundation ring. This foundation ring is built up of plates and structural shapes and is continuous with a plate-steel pit liner carrying the operating cylinders as shown in Fig. 24. Fig. 25 shows the turbine head cover. Fig. 26 shows the guide-vanes, which are built up of plate steel with cast-steel stems. Fig. 27 shows the turbine partly assembled in the shop. As in all the other units described in this paper, the draft-tubes are of the spreading type and in this installation are constructed of plate steel. In all the later installations, the draft-tubes have been of the continuous annular type with central cores continuous in contour with the runner hub.

Conclusion

It may be stated that although as a result of experience with this new form of turbine, certain minor defects have developed which have required for their correction some modifications in design, the many major difficulties which had been predicted, such as serious corrosion, vibration, instability, imperfect regulation, failure to develop the power corresponding to the head, or refusal to run under low head, have not occurred.

HIGH-SPEED SUCTION TURBINES

By Forrest Nagler,* M. Am. Soc. C. E.

Synopsis

This paper is intended to outline in a general way the experience to date with the high-speed, axial-flow type of hydraulic turbine runner for low heads. A brief comment as to the origin of this type is given, together with a statement as to the number of installations, total horse-power involved, etc. Comment is given as to the operating features that have been observed, the heads to which this type has been applied, and the present commercial range of its application.

Definition

The high-speed axial-flow type of hydraulic turbine runner, called variously in the United States the "suction runner" or "propeller runner", is generally representative of a type wherein the flow within the runner itself is in a direction mainly parallel to or "cylindrical" about the axis of rotation. (See Fig. 28.)

This is in contrast to the usual types where the flow is concentric about, but transverse to, the axis of rotation as in the Francis types (Fig. 29 left), or at right angles to the axis, but not concentric about it, as in the "tangential impulse", or "Pelton" type (Fig. 29 extreme right). The particular design for which the writer has been responsible is further characterized by a small number of blades (usually and preferably four), as contrasted with the fifteen to thirty prevalent in present conventional forms (see Fig. 29), and by the absence of a rim around the tips of the blades. In addition, there is a reduction in the width of the blade so that for the higher speed conditions there is no overlap, this being essential to highest efficiency when high characteristic speeds are reached. It is with the type thus defined, that this paper has to deal.

Historical

The place which this type of runner holds historically, particularly with reference to all other types, is shown in Fig. 31. In plotting this diagram, every obtainable performance figure has been used for the earlier forms. The writer believes this diagram to be still correct, although a paper† presented before the Society would indicate otherwise.

In that most excellent paper Messrs. Safford and Hamilton made the following statement,‡ under the heading, "The Propeller Type of Runner:"

"* * * wheels of almost exactly the same type were used many years ago, and that their high speed was appreciated. * * * Not only did Truax

* Hydr. Engr., Allis-Chalmers Mfg.. Co., Milwaukee, Wis.

† "The American Mixed-Flow Turbine and Its Setting", by Arthur T. Safford, M. Am. Soc. C. E., and Edward Pierce Hamilton, Jun. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1237.

‡ *Loc. cit.*, pp. 1263–1264.

FIG. 28.—TYPICAL HIGH-SPEED SUCTION RUNNER (1916). COMPARE WITH FRANCIS WHEEL OF ABOUT ONE-EIGHTH CHARACTERISTIC SPEED.

FIG. 29.—TYPES OF WATER WHEELS FOR HEADS OF 2 000 FT. TO 8 FT.

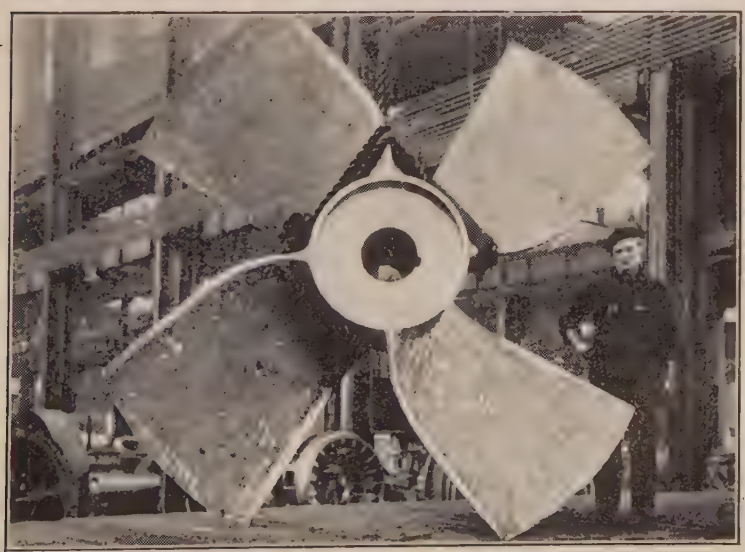


FIG. 30.—NAGLER 116-IN. HIGH-SPEED RUNNER, CAST STEEL IN ONE PIECE.

appreciate what he had, but he also knew how he got it, which was unusual at this time. * * * Assuming that the measurement of the revolutions per minute and of the water was correct, and, furthermore, assuming an efficiency of 60%, which seems fair and reasonable in the light of the performance of similar wheels of the same period, a specific speed of 125 is obtained. If the efficiency has been only 40%, the specific speed would still be more than 100."

Lest this statement from so well known an authority give rise to a misapprehension as to the number of years of experience behind this type of runner, the following comments are thought to be in order. They are offered for the purpose of avoiding too rapid an application of this type to conditions for which experience has not yet proven it economical.

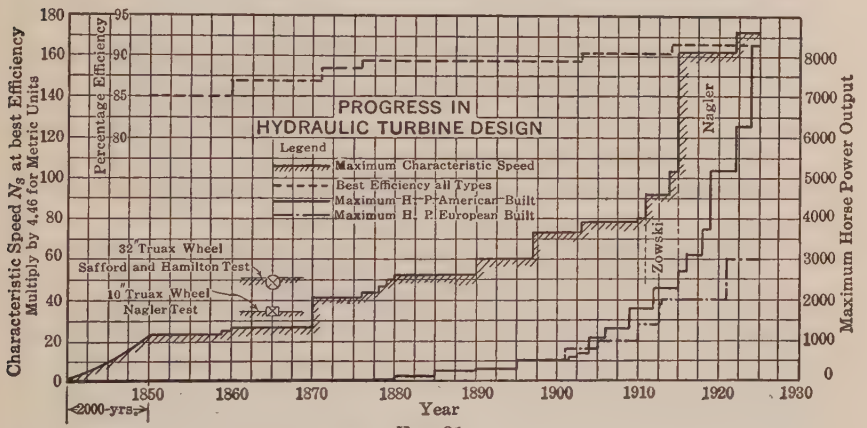


FIG. 31.

The writer possessed 10-in. wheels of the Truax and Austin (similar to the Truax) types; he made tests of these wheels and gave their results in detail in discussion* of the paper by Messrs. Safford and Hamilton. Specific speeds of 35 and 23 were obtained, using settings duplicating as nearly as possible those shown in the original Truax and Austin illustrations. Messrs. Safford and Hamilton took most vigorous exception to these tests; they secured a 32-in. Truax wheel and tested it, without finding specific speeds in excess of 60, even with a draft-tube added. The results are shown in Table 2.

TABLE 2.—COMPARISON OF TRUAX AND NAGLER WHEELS.

Wheel test.	Efficiency, percentage.	Specific speed (English units).
Original Truax advertisement	60 (assumed)	125 (calculated)
Original Truax advertisement	40 (assumed)	100 (calculated)
10-in. Truax, Nagler test	38 (by test)	35 (by test)
32-in. Truax, Safford and Hamilton test	50 (by test)	51 (by test)
32-in. Truax plus draft-tube, etc., Safford and Hamilton test	61 (by test)	60 (by test)
Nagler's original 34-in. runner, 1915	87 (by test)	150 (by test)

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1310.

Replotting this comparative data on a specific speed (N_s) basis, gives the curves shown on Fig. 32(a). Another basis is given in Fig. 32(b), in which the use of unit horse-power exaggerates the comparison even as the speed basis suppresses it. The N_s basis is the fairer contrast.

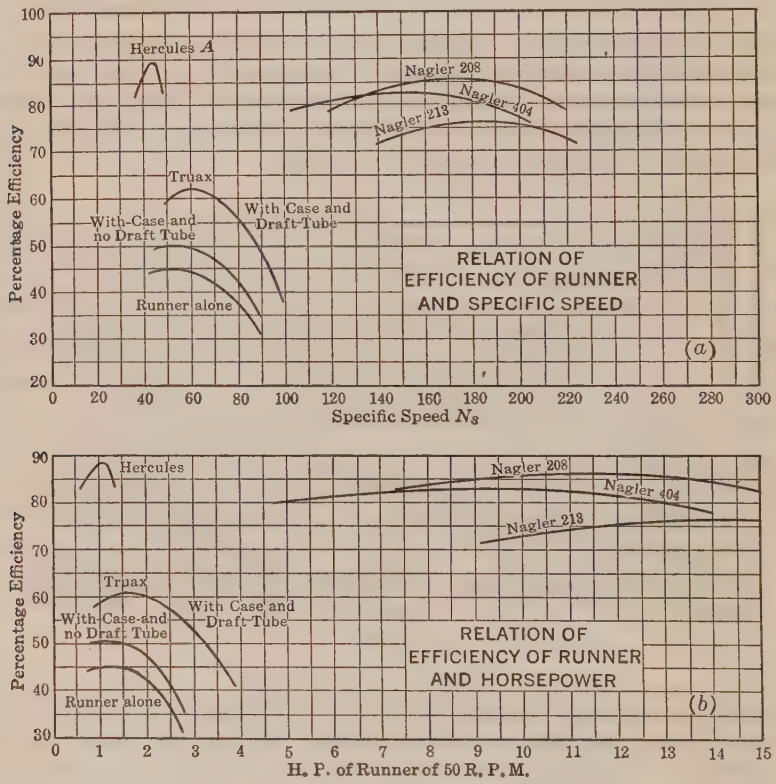


FIG. 32.

This comparison, particularly as it includes tests with additions in the form of guide-cases and draft-tubes built in the light of present knowledge, seems to the writer to show conclusively that there is some decided difference between a type of wheel which tests only to 60% efficiency and a specific speed of 61, and the modern suction type as here presented which reaches efficiencies of 90%, and specific speeds from 150 to 200, or about triple that of the earlier type.

The summary diagram, Fig. 60*, of Messrs. Safford and Hamilton, is entirely misleading. In the first place, the curve labeled "Truax" should be labeled "Truax 1876 plus Safford and Hamilton's Guide-Case and Draft-Tube, 1922". Even neglecting this point, the Truax wheel with a specific speed of 60 at the best efficiency point, represents more closely the class including the Hercules, which ranges in specific speed from 40 to 70, than it does the writer's

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1349.

wheel with its specific speed of 150 to 200. The Truax curve projected for over-speed extends up to a specific speed of less than 100, but it should be pointed out that the wheel which the writer contends is different, extends similarly up to 250.

Performance

The writer's experimental work on this type of runner as applied to hydraulic turbines was started in 1913. The initial plant (Fig. 28) was built in 1916, the initial Holyoke test (No. 2 521) was made in January, 1917, and the first publicity was given to this type in 1919.* At that time a total of 17 units had been built, 9 of which had been tested. To date more than 100 of these units have been built, 90 of which have a total combined capacity of 90 680 h.p. The operation of every plant has been watched most carefully and tests have been made wherever possible. The sizes have ranged from a diameter of 30 in. up to one of 156 in. The horse-power per unit has ranged from 100 up to slightly less than 5 000. The heads have varied from 22 in. up to 35 ft. and, on test plants, up to 65 ft., these high-head tests dating from 1920. The specific speeds have varied from 90 to 250, English system (400 to 1 100 metric). Holyoke tests have not been made as frequently as is customary with Francis units. The highest Holyoke efficiency reached was on a runner designed in 1920 and tested in 1923, showing a maximum efficiency of 89.2 and a specific speed at best efficiency of 175.

In general, plants have been operated with remarkably little difficulty, having an over-all continuity or reliability factor not noticeably different from that of the Francis type of unit, as far as mechanical features are concerned, and being superior to the Francis type from a standpoint of freedom from troubles due to ice, trash, and débris, either within the turbine or in the trash racks. There has been no replacement of any runner due to mechanical difficulties, nor a single instance of pitting, although practically every installation of any considerable head has been carefully watched for this trouble.

A typical performance curve is shown in Fig. 33, wherein it will be noted that a maximum hydraulic turbine efficiency of 91% is reached in the field. The runner of this particular unit is shown in Fig. 30.

Characteristics and Limitations

Practically all installations have followed the general practice conventional for the Francis type of runners, as concerns the guide-case, flume, or spiral casing. On the discharge side, practically every installation has made use of the White hydraucone regainer,† an efficient diffuser being essential by reason of the higher discharge velocity from this type of runner and the consequent necessity for as efficient a regain as possible.

One of the principal characteristics of this type of runner is its lower efficiency at partial loads, as contrasted with those of lower speed, such as the Francis type. For example, the efficiency at half load of a high-speed runner having a characteristic speed of 175 is about 66 per cent. The corre-

* "A New Type of Hydraulic Turbine Runner for Low Head Installations", by Forrest Nagler, M. Am. Soc. C. E., *Transactions*, Am. Soc. Mech. Engrs., Vol. 41 (1919), p. 829.

† *Transactions*, Am. Soc. Mech. Engrs., Vol. 43 (1921), p. 255.

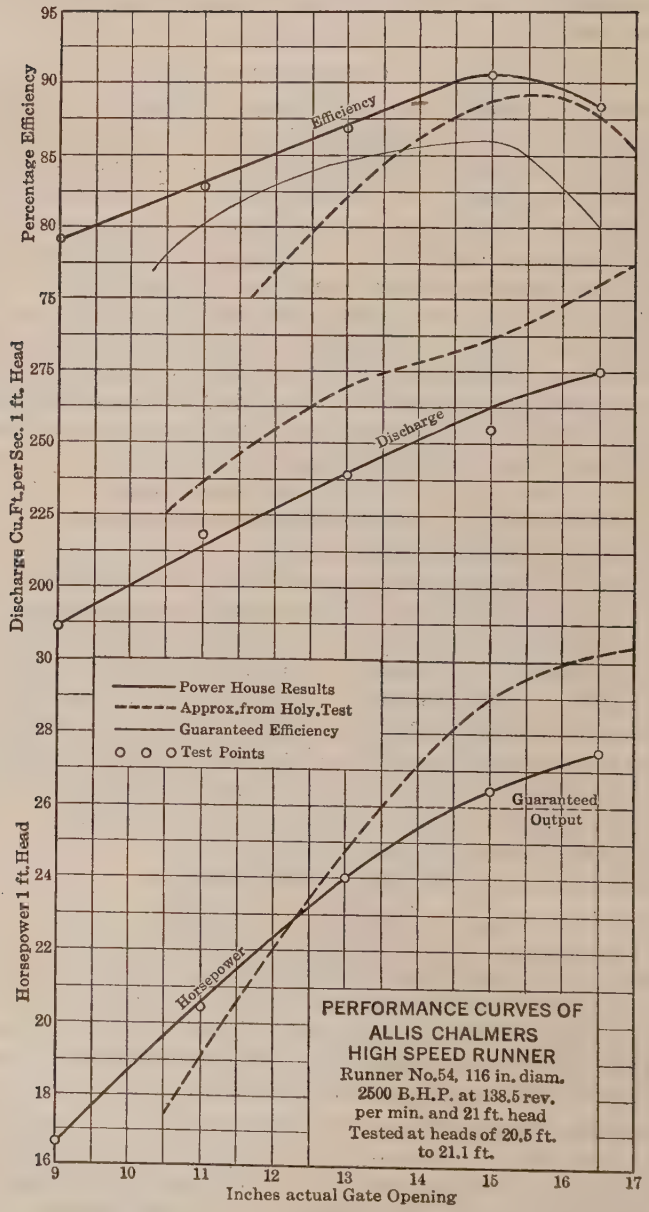

Percentage Efficiency
Discharge Cu.Ft.per Sec. 1 ft. Head
Horsepower 1 ft. Head
Inches actual Gate Opening
Efficiency
Discharge
Horsepower
Guaranteed Output
Power House Results
Approx.from Holy.Test
Guaranteed Efficiency
Test Points
PERFORMANCE CURVES OF
ALLIS CHALMERS
HIGH SPEED RUNNER
Runner No.54, 116 in. diam.
2500 B.H.P. at 138.5 rev.
per min. and 21 ft. head
Tested at heads of 20.5 ft.
to 21.1 ft.

FIG. 33.

FIG. 34.—EXAMPLES OF MODERN HIGH-SPEED SUCTION RUNNERS.

FIG. 35.—LARGEST HIGH-SPEED SUCTION RUNNER EVER INSTALLED FOR LOW HEADS.

sponding efficiency at half load of a Francis runner having a specific speed of 100 is approximately 75 per cent. Francis runners of low capacity, for specific speeds of 40 or 50, have reached as high as 90% efficiency at half load, but these runners have not the characteristics that will permit of their application to low heads and, consequently, the comparison is a technical and not a practical one. In actual service, the efficiency at part gate is little or no handicap, except in plants where there may be a small number of units. It is not a considerable factor when there are more than two units in the station, because under ordinary working conditions, the operation may be maintained at or near the point of best efficiency, by intermittently shifting the load or by drawing on the pondage. This possibility combined with the fact that the over-all operating efficiency of the high-speed unit under low-head conditions (including the generator) is usually superior to that of a Francis unit, makes this type economical regardless of the saving in initial cost. Figs. 30, 34 and 35 show runners typical of both horizontal and vertical installations. The contrast in Fig. 34 between high-speed suction runners and, in the background, the Francis reaction runner which they have virtually displaced for low heads, is clear. The runner shown in Fig. 35, built for Henry Ford and Son, Incorporated, Green Island development, is 156 in. in diameter, 17 000 lb. in weight, and cast in four sections. It operates at 80 rev. per min. under a head of 13 ft.; its discharge capacity is about 85% that of the Keokuk runners, which weigh about 179 000 lb. each and measure more than 17 ft. in outside diameter.

One of the features encountered earliest in designing and experimenting with this type of runner is the possibility of varying its capacity widely without materially affecting its efficiency. It is simply a question of flattening the blade angles. European practice has apparently taken advantage of this fact and carried the application of this principle even so far as to make the blades movable under control of the governor, either simultaneously with or separately from the similar control of the guide-vane. Hydraulically, there is no question as to the results; mechanically, such a construction may come well within the standards of European practice, but the writer does not believe that it will ever become popular under American standards of simplicity, ruggedness, and high costs of labor. The need for such a construction does not enter into consideration when there are many units in the station. Governors are becoming less and less a factor, and the writer believes they will ultimately cease to enter into hydraulic turbine design to any appreciable extent, their ultimate elimination being due to the fact that hydro-electric systems will be largely interconnected with steam power where regulation is inherently less harmful to structures and to economy. It is believed, however, that this feature of variable capacity will be utilized, not so much for its advantages in increasing part-load efficiency, as for the possibilities it presents in increasing the capacity under conditions of flood or low head. This feature has been utilized in numerous installations already built, construction details being used corresponding to the adjustment of pitch on screw-pumps, which has long been the regular practice in the United States.

It has been a fairly well-defined policy of the Allis-Chalmers Manufacturing Company not to apply the suction type of runner to heads materially in

excess of 30 ft., this being an arbitrary limit, although it is supported by experimental data as well as by theoretical considerations. By departing from the pure suction type of runner with its minimum blade surface and by using blades of larger projected area, the limiting head can undoubtedly be increased, but it is the writer's belief and experience that the considerable increase of projected area, either by reason of wider blades or a larger number of blades, is accompanied by a very decided reduction of efficiency. It will undoubtedly be found that for certain installations, a definite gain in some other direction may counterbalance this particular detrimental feature and probably as experience grows, this general type will be applied to higher and higher heads and to larger and larger capacities. At present, it is not believed that it is commercially advisable or sound from an engineering standpoint to apply it to heads or capacities far outside the range of proven practice. A consistent adherence to this policy has enabled the achievement of an operating record wherein there has been no single shut-down of a unit of as much as 24-hours duration, due to conditions traceable to this type of runner.

Governor regulation has not been as perfect as that obtained with corresponding types of Francis units, but this is principally attributable to the fact that, with the higher speeds in question, the factor, $W R^2$, of the generator has been relatively lower. The regulation experienced has not differed from that obtained with Francis units by an amount unexplainable by this reduction of regulating constant.

The high-speed suction type of runner, due to its higher and sometimes critical velocities, is not conducive to as quiet a running machine as the Francis runner, perhaps in exactly the same way that the flow conditions in a Francis runner are not conducive to as smooth operation as is possible with an overshot wheel. Nothing, however, of a nature that would make it uncommercial has been experienced with this type when it is used under conditions to which it is inherently adapted, that is, relatively low head and large capacity.

HIGH SPECIFIC SPEED HYDRAULIC TURBINES IN THEIR BEARING ON THE PROPORTIONING OF THE NUMBER OF UNITS IN LOW-HEAD HYDRO-ELECTRIC PLANTS

GEORGE A. JESSOP,* ASSOC. M. AM. SOC. C. E.

The writer will indicate briefly the comparative efficiencies which can be obtained by using medium-speed wheels (of which the specific speed, N_s, is equal to about 75) and high-speed wheels (N_s equal to about 175). The efficiencies shown throughout this paper are based on model runners of the size usually tested at the Holyoke flume, that is, about 39 sec-ft. discharge at 1-ft. head. Under good conditions, it is possible to obtain better results with larger wheels in place. The data and information contained herein are obtained from tests made in the Holyoke flume and in the specially designed and constructed testing flume of the S. Morgan Smith Company.

The medium-speed runner used in these tests is of the Francis type and of normal design. The high-speed runner is a design first developed and used by The Theodore Bell Company, of Switzerland, and now being further developed and improved jointly by the Engineering Staffs of The Bell Company and the S. Morgan Smith Company. It is distinctive in that it has fewer blades than the usual high-speed wheel, and the total blade area is exceptionally large. The hub is also large, thus reducing the usual overhang of the buckets and providing a large cross-sectional area where each bucket joins the hub. An inspection of the curves will show that this wheel has its peak efficiency at a load of about 90% comparing very favorably with the slower speed runner in this respect. This is a desirable feature, because the turbine can be operated at its most economical point with a liberal allowance of power for regulating purposes, and, furthermore, this design assists in obtaining high part-load efficiency.

For very low heads (30 to 40 ft. and less), the comparison given herewith is possibly somewhat less favorable to the high-speed wheel than it should be, because usually for these heads, if a Francis runner is used, it will be of a specific speed of about 100 to 110, and will have a slightly lower peak efficiency and considerably lower efficiency at one-half to three-quarter load, than a wheel with N_s equal to about 75. Furthermore, this being a comparison of turbines only, the increased efficiency of the high-speed generator is not taken into account.

In this paper it will be impossible to present a full and complete discussion of this question, but a comparison will be given of the results that can be obtained with plants containing from two to eight units of the medium and high-speed types, together with some averages that it is hoped will be interesting and helpful.

Fig. 36 shows the comparison between medium and high-speed wheels in a 2-unit plant. Up to 50% load, of course, only one unit is in operation.

* Hydr. Engr., S. Morgan Smith Co., York, Pa.

Above 50% load, the curve is the resultant or combined efficiency of the two units. For the medium-speed wheels, when operating between 50 and 75% load, a higher efficiency can be obtained by running one wheel at maximum efficiency and supplying the remainder of the demanded power by running the second unit at the required gate-opening, than by dividing the load equally between the two units. The efficiency to be gained by the unequal distribution of load is shown by the full and dotted lines (lower left, Fig. 36). The high-speed wheels show maximum efficiency when the load is equally divided. In order to determine whether it is economical to divide the load equally or unequally between two or more units, it is necessary to calculate each plant separately, using the curves which apply. On the left of Fig. 36 curves showing the average weighted efficiency have been plotted, with the station operating from 100% load to any other load between 0 and 50%, based on the load varying at a constant rate.

The weighted efficiency for a given range of load is simply the total output in horse-power hours divided by the total input in water horse-power hours, the latter being actually computed from the discharge curves, which are not shown on the diagrams. In computing the average weighted efficiency for any given range of output, it is obviously necessary to use a definite load curve, and for purposes of comparison only a load curve has been used which varies from the minimum to the maximum output at a constant rate.

For example, if the load varies from 40 to 100%, the average weighted efficiency is about 89.5% for the 75 N_s wheel and about 84.5% for the 175 N_s wheel. If the load varies from 0 to 100%, the average weighted efficiency is about 87% for the 75 N_s wheel and about 80% for the 175 N_s wheel.

To obtain the correct efficiencies for any given station, the weighted averages must be computed according to the load curve which applies to the station. It should be noted that for most stations the comparison would be more favorable to the high-speed wheel than shown herein, because the plant would be operated for a greater length of time at high load than at low load.

The dotted curves are simply the arithmetical averages of the efficiency and are plotted to show that, at least for approximate preliminary purposes, where the load varies from about 25 to 100%, it is sufficiently accurate, perhaps, to use the more easily determined arithmetical average rather than the weighted average.

The possible exception to this statement is the high-speed turbine for the 2-unit plant, where the difference between the numerical and weighted efficiency is material. For the curves under discussion (Fig. 36), the numerical efficiency is always less than the weighted efficiency, but for the range where the curves are shown to be coincident, the differences are too small to be shown with the scale used.

Fig. 37 shows the same comparisons for a 4-unit plant, Fig. 38 for a 6-unit plant, and Fig. 39 for an 8-unit plant. It will be noted that with

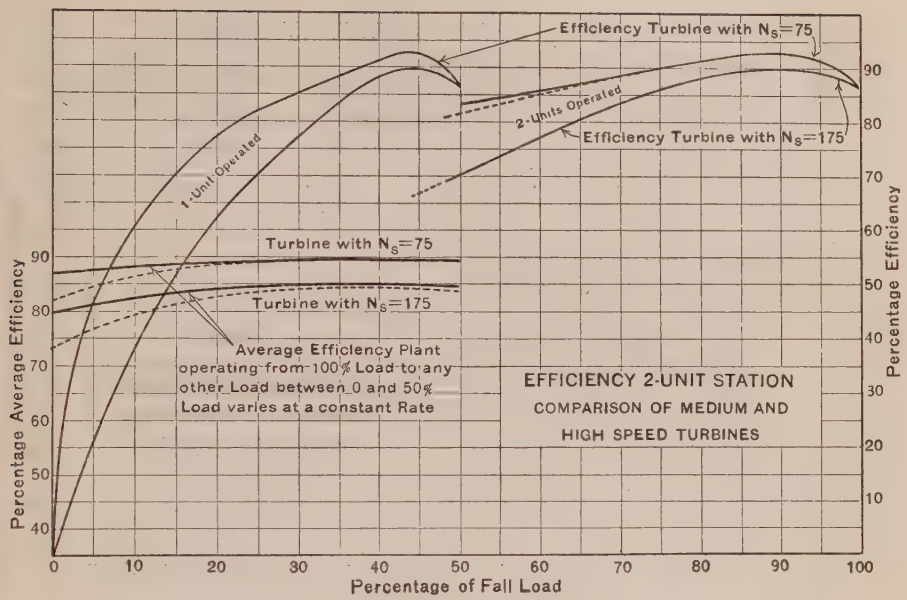

Efficiency Turbine with N_S=75
2-Units Operated
Efficiency Turbine with N_S=175
1-Unit Operated
Turbine with N_S=75
Turbine with N_S=175
Average Efficiency Plant operating from 100% Load to any other Load between 0 and 50% Load varies at a constant Rate
EFFICIENCY 2-UNIT STATION
COMPARISON OF MEDIUM AND HIGH SPEED TURBINES
Percentage Average Efficiency
Percentage Efficiency
Percentage of Fall Load

FIG. 36.

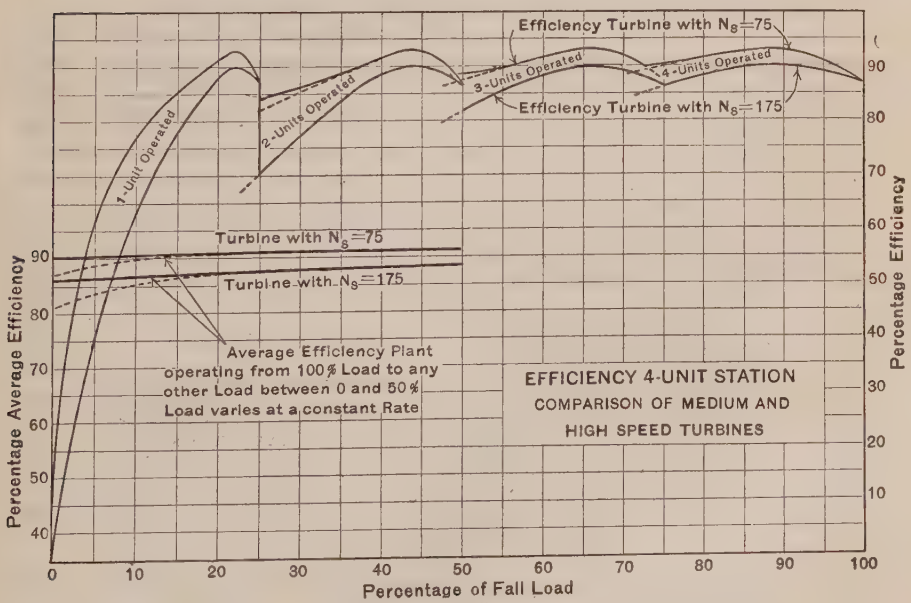

Efficiency Turbine with N_S=75
3-Units Operated
4-Units Operated
2-Units Operated
Efficiency Turbine with N_S=175
1-Unit Operated
Turbine with N_S=75
Turbine with N_S=175
Average Efficiency Plant operating from 100% Load to any other Load between 0 and 50% Load varies at a constant Rate
EFFICIENCY 4-UNIT STATION
COMPARISON OF MEDIUM AND HIGH SPEED TURBINES
Percentage Average Efficiency
Percentage Efficiency
Percentage of Fall Load

FIG. 37.

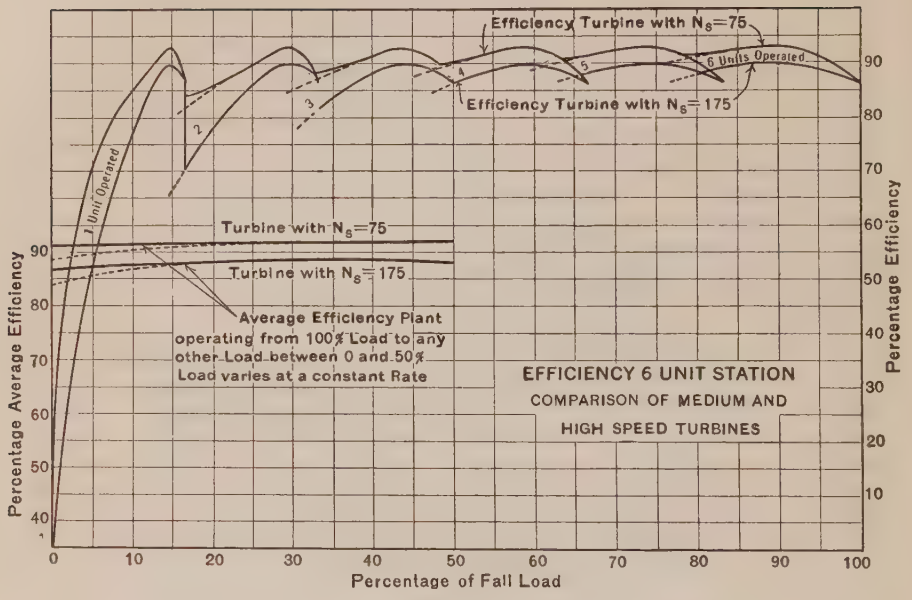

Efficiency Turbine with N_S=75
6 Units Operated
Efficiency Turbine with N_S=175
1 Unit Operated
Turbine with N_S=75
Turbine with N_S=175
Average Efficiency Plant operating from 100% Load to any other Load between 0 and 50% Load varies at a constant Rate
EFFICIENCY 6 UNIT STATION
COMPARISON OF MEDIUM AND HIGH SPEED TURBINES
Percentage Average Efficiency
Percentage Efficiency
Percentage of Full Load

FIG. 38.

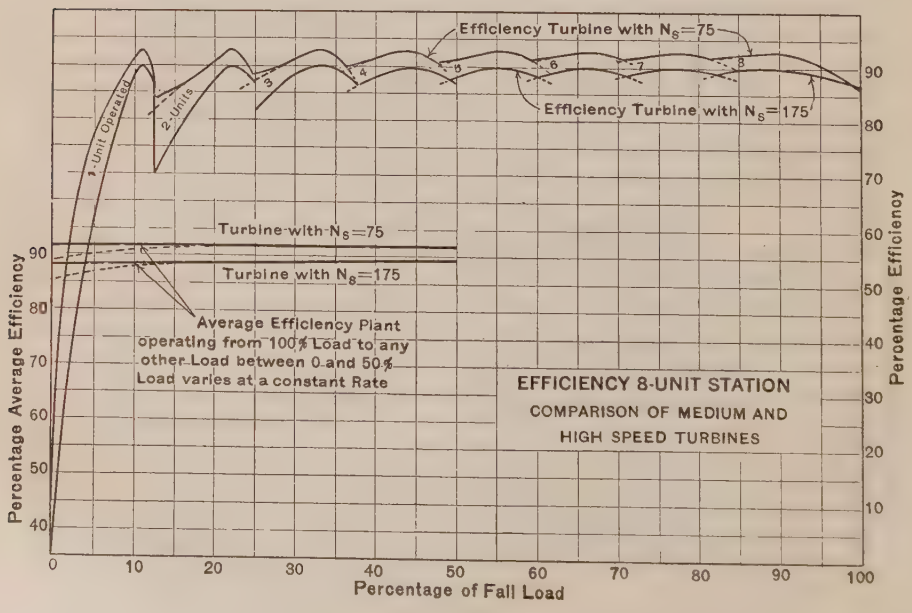

Efficiency Turbine with N_S=75
1 Unit Operated
2 Units
Efficiency Turbine with N_S=175
Turbine with N_S=75
Turbine with N_S=175
Average Efficiency Plant operating from 100% Load to any other Load between 0 and 50% Load varies at a constant Rate
EFFICIENCY 8-UNIT STATION
COMPARISON OF MEDIUM AND HIGH SPEED TURBINES
Percentage Average Efficiency
Percentage Efficiency
Percentage of Full Load

FIG. 39.

a 6 or 8-unit plant, medium speed, and an output equal to 50% or more of the maximum capacity, there is very little advantage in dividing the load other than equally between the units.

Fig. 40 shows how the average efficiency increases with the number of units. The full lines are plotted using the averages for ranges of load starting with full load and decreasing at a constant rate to any percentage between 25 and 50, the efficiency being practically constant for any range between these limits. The dotted lines are plotted using the averages for a range of load from full output to zero. The average efficiency for any number of units, with the range of load varying from full load to any point between 25% and zero, can easily be approximated.

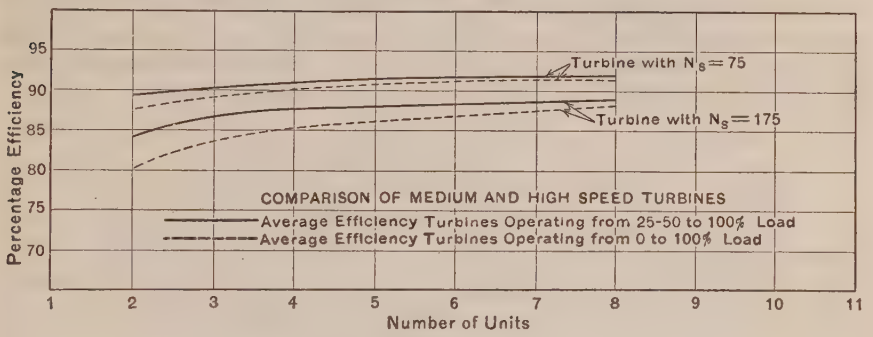


FIG. 40.

It is not intended that these curves be used to determine the average efficiency of medium or high-speed wheels; rather, the averages should be computed for each individual case. The curves simply indicate how the wheels compare in general. It must be considered that if the station is operated a greater length of time at nearly full capacity than at the low capacity, the average efficiencies will be increased over the values shown, and the averages of the high-speed wheels will increase more rapidly than those of the low-speed wheels.

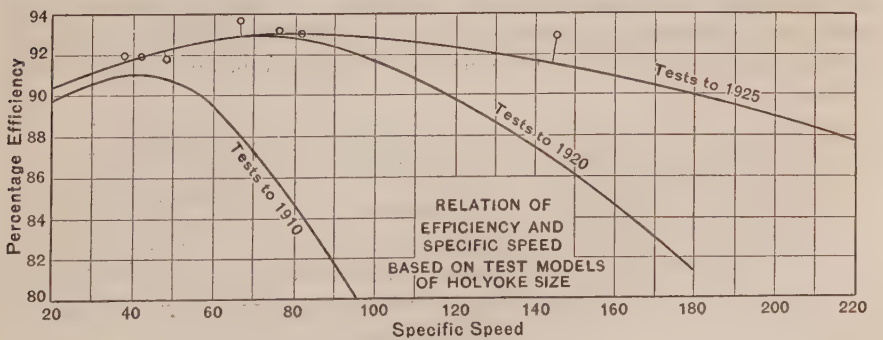


FIG. 41.

Fig. 41 shows the "efficiency-specific speed" curves for 1910, 1920, and 1925, the efficiencies being based on results which can be obtained in the Holyoke flume, on wheels of the size usually tested. Undoubtedly, it is

possible to obtain somewhat better results on large wheels, with very good settings. It is interesting to note that little improvement has been made since 1910 in low specific speed wheels, and that the improvement since 1920 is entirely in wheels with specific speeds of 80 and higher. This comparison emphasizes the reasons for present interest in wheels of high specific speed.

The comparative average efficiencies, as shown in this paper, may be considered, perhaps, somewhat unfavorable to the high-speed wheel. To bring the average efficiency of the high-speed wheel to a point more nearly equal to that of the low-speed wheel, it is necessary to increase the efficiency of the former. An inspection of Fig. 41 will show that this can be done by using high-speed wheels of a specific speed somewhat less than those selected. Not only would this give a substantial increase in the peak efficiency, but the part-load efficiencies are increased to even a greater extent.

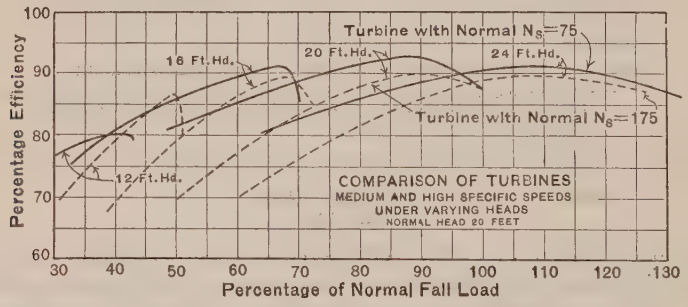


Fig. 42.

No attempt has been made to show a comparison of the average efficiencies under variable head conditions. Fig. 42 shows that the high-speed wheel has a better range at variable speed or variable head than the medium-speed turbine. The power and efficiency of the high-speed wheel hold up particularly well under low-head conditions. It follows, therefore, that where the percentage of head variation is considerable, the comparison between the two wheels will be somewhat more favorable to the high-speed wheel.

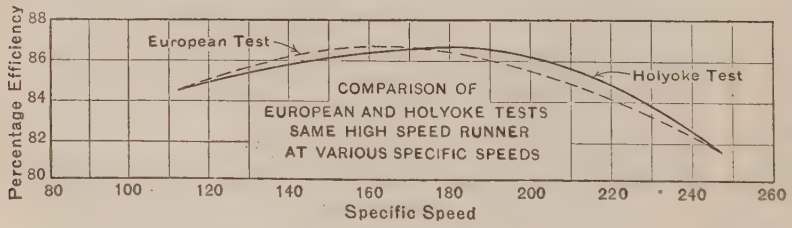


Fig. 43.

The comparison of European and Holyoke tests is a matter of general interest. Foreign engineers have always claimed that results obtained at Holyoke have been 2 or 3% higher than the correct actual. The writer has

recently had occasion to compare a Holyoke with a European test, using the same runner, draft-tube and wheel-case, and under the same head. Fig. 43 shows a comparison on the basis of the efficiency plotted against the specific speed. The difference between the tests is at no place equal to 1 per cent.

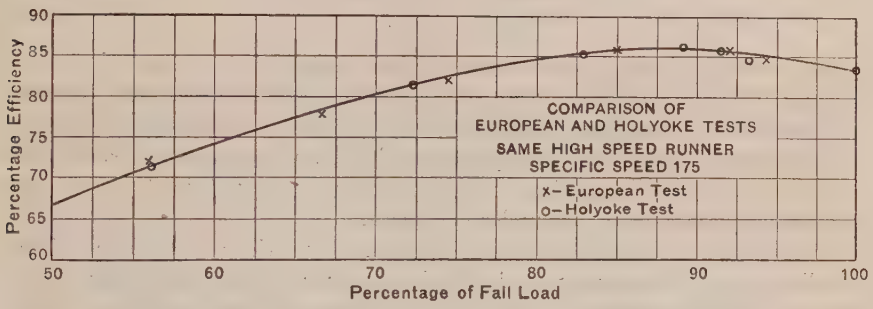


FIG. 44.

To show how the tests compare at the same speed, Fig. 44 has been plotted. The runner is about 19 in. in discharge diameter, and the efficiency differential between this size and one of normal size for the Holyoke flume is about + 3½ per cent.

SOME APPLICATIONS OF THE PROPELLER TYPE WATER TURBINE IN EUROPE

By Charles C. Egbert,* Esq.

The records of the United States Patent Office for the last fifty years will show that parallel to the cases of the Francis and Pelton wheels, pioneer work was done in America on water turbines of the propeller type. However, as the commercial demand during that period was for comparatively slow-speed machines, the propeller turbine was apparently neglected in favor of the so-called Francis type for use under low heads.

The writer has no record of early European inventions of turbines of the propeller type. Reference may be made, however, to the old Jonval type turbine which was occasionally built with rimless runners, with the blades strongly bent, or curved, to suit the slow speed for which such turbines were designed.

It was not until about 1913 that the publication of the studies and experiments of Professor Viktor Kaplan, of Brünn, drew serious attention to the possibilities of the propeller type turbine. This has resulted in its development both in Europe and America.

The commercial application of the propeller turbine in Europe was undoubtedly retarded by the World War; nevertheless, noteworthy progress has been made. This is indicated by the number of power plants in which various adaptations of wheels of the propeller type have been installed and the number of companies building that type. Furthermore, designs have been highly developed by intensive study by a number of manufacturers, coupled with much experimental work in numerous well-appointed testing flumes.

This paper makes no attempt to cover the entire subject of the application of the propeller turbine in Europe, and only gives brief descriptions of a few developments selected from data readily available.

The first to be referred to is the Lilla Edet Station (Fig. 1 and Figs. 45 to 48) built by the Swedish Government. This station contains three vertical shaft units, each of a nominal capacity of 10 000 h.p., at 62.5 rev. per min., under a normal head of 21.3 ft. (6.5 m.).

One of the turbine units is of the Kaplan type with a propeller wheel, 19 ft. (5.8 m.) in diameter, over the tips of four horizontally disposed blades (Figs. 46 and 48). It was built by the Verkstaden in Kristinehamn, Sweden. A most interesting feature of this unit is the use of movable runner blades which are pivoted in the runner hub so that the angle of the blades can be varied in accordance with the gate-opening and discharge of the turbine, thus insuring a flatter efficiency curve. The guaranteed output of this turbine under a head of 21.3 ft. (6.5 m.) is 11 200 h.p. (metric), from which a specific speed of about 144 ft-lb. units (637 metric) may be computed.

* Cons. Engr., Niagara Falls, N. Y.

The other two turbines in the Lilla Edet Station are of the Lawaczeck type (see Figs. 1 and 3), each having one runner with eight diagonally disposed blades, 19.6 ft. (6 m.) over the tips. These two units were built by the A-B Finshattan. The guaranteed output of each of the Lawaczeck turbines under a head of 21.3 ft. (6.5 m.), is 10 400 h.p. (metric), from which a specific speed of about 139 ft-lb. units (616 metric) is computed.

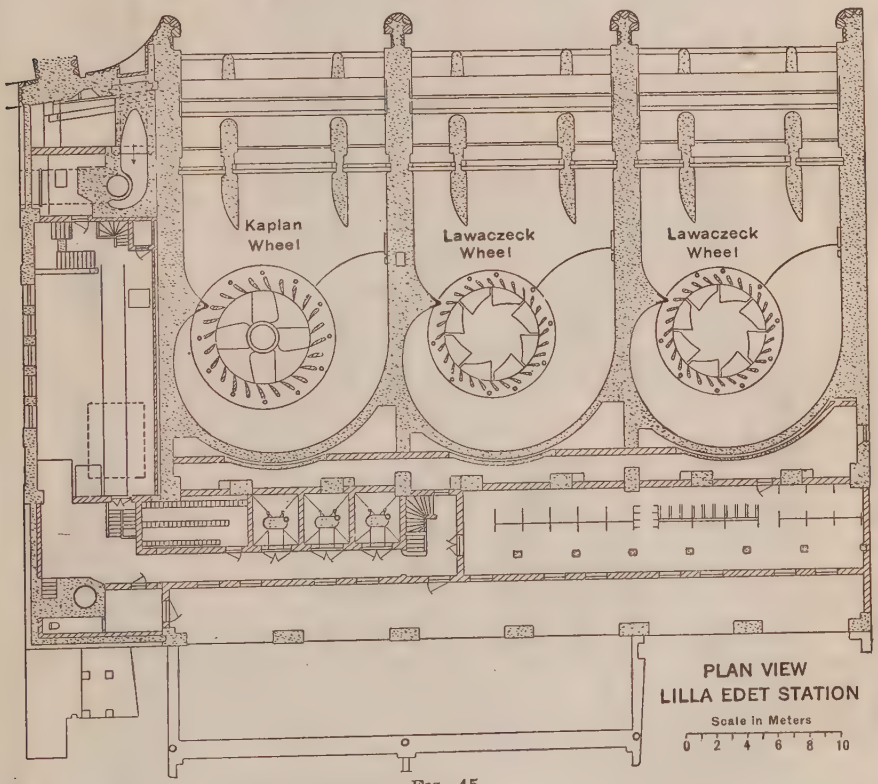


FIG. 45.

The final designs of the Lilla Edet turbines, including wheel-cases and draft-tubes, were evolved after much experimentation in testing flumes, using model runners up to 3.27 ft. (1 m.) in diameter. Various well-known forms of draft-tubes were tested before the decision to use the elbow type was reached. Fig. 4 gives the performance of the turbines at Lilla Edet.

Tables 1 and 3 show the relative capacities of some large single-runner turbines, which have been installed in different countries.

Another installation of the propeller type turbine is that in the Matte Central Station of the City of Bern, Switzerland, built by Theodore Bell and Company of Kriens, Switzerland (Fig. 56). Although this is a very small installation, the form of the runner makes it of particular interest.

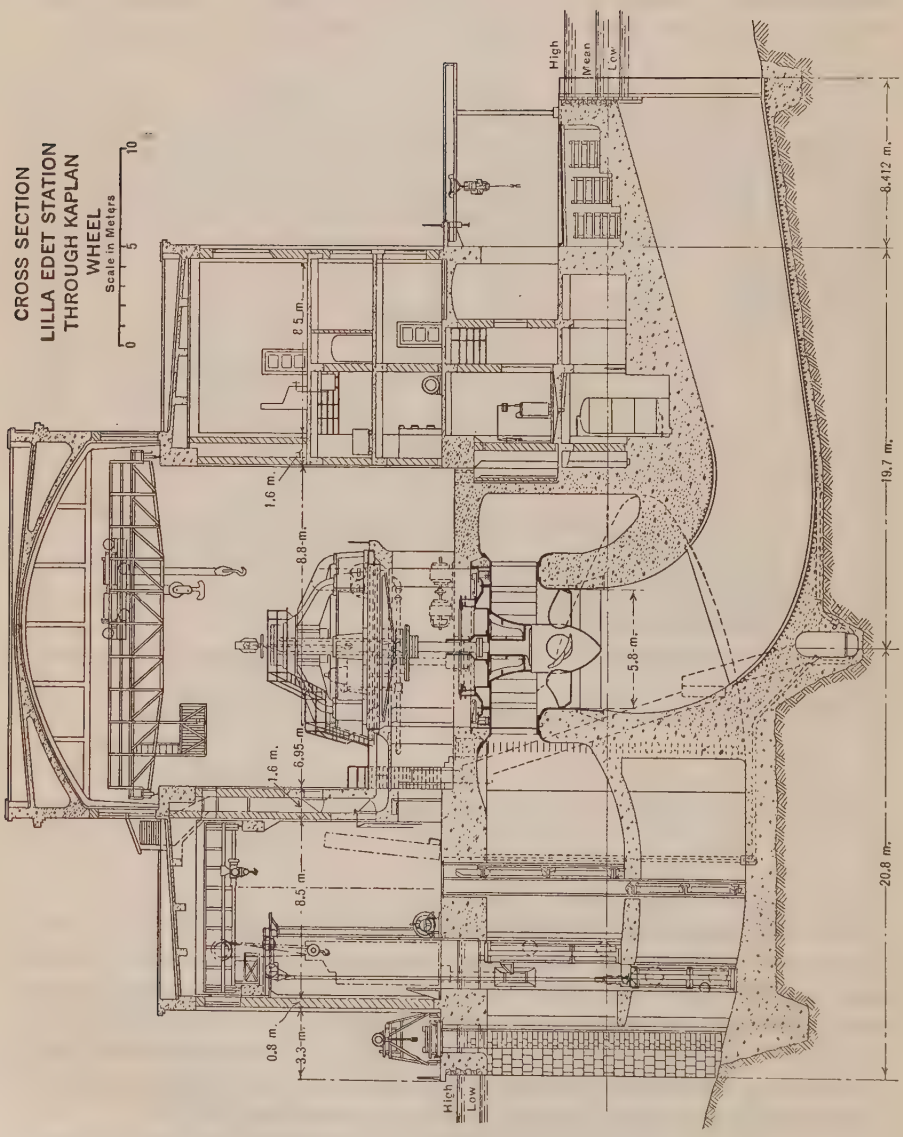

CROSS SECTION
LILLA EDET STATION
THROUGH KAPLAN
WHEEL
Scale in Meters
0
5
10
High
Mean
Low
8.412 m.
19.7 m.
20.8 m.
5.8 m.
1.6 m.
8.8 m.
6.95 m.
1.6 m.
8.5 m.
0.8 m.
3.3 m.
High
Low

FIG. 46.

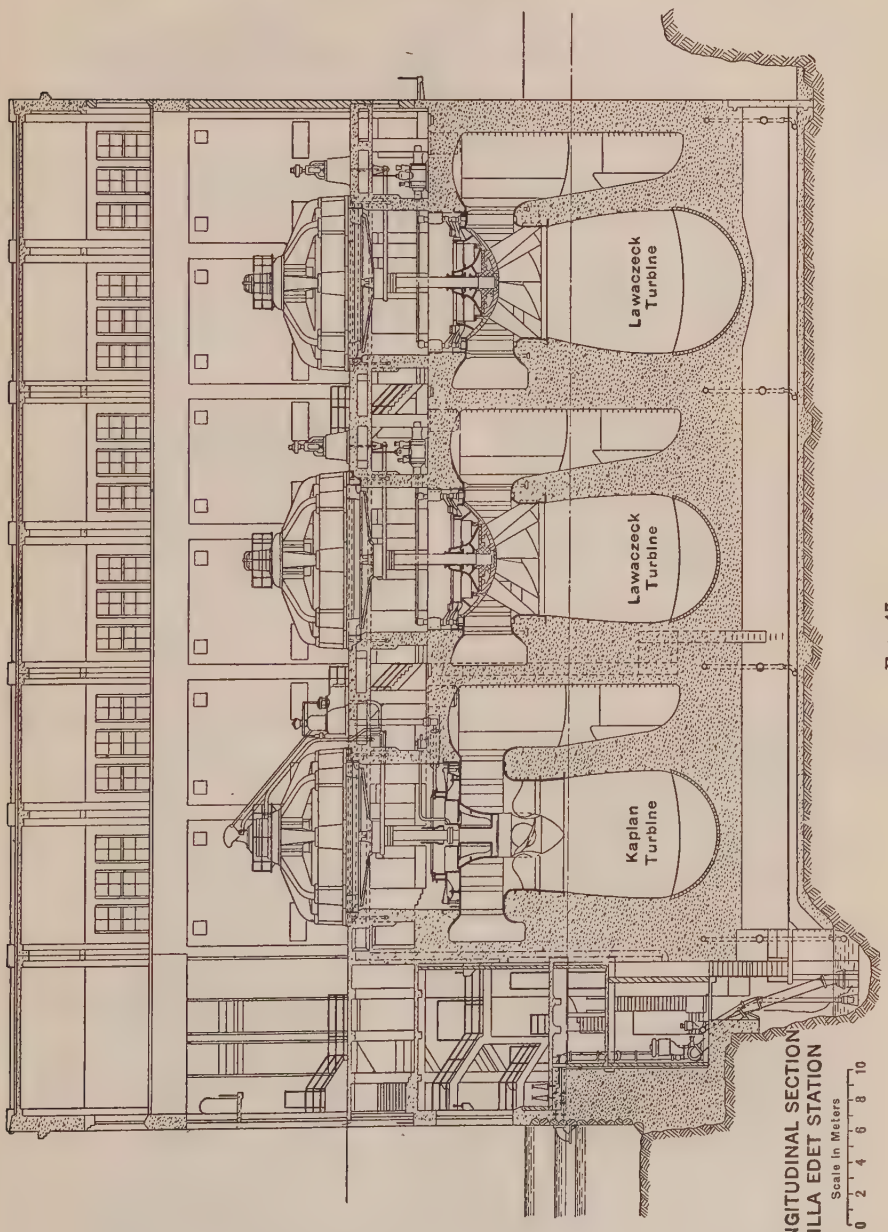

Lawaczeck Turbine
Lawaczeck Turbine
Kaplan Turbine
LONGITUDINAL SECTION
LILLA EDET STATION
Scale in Meters
0 2 4 6 8 10

FIG. 47.

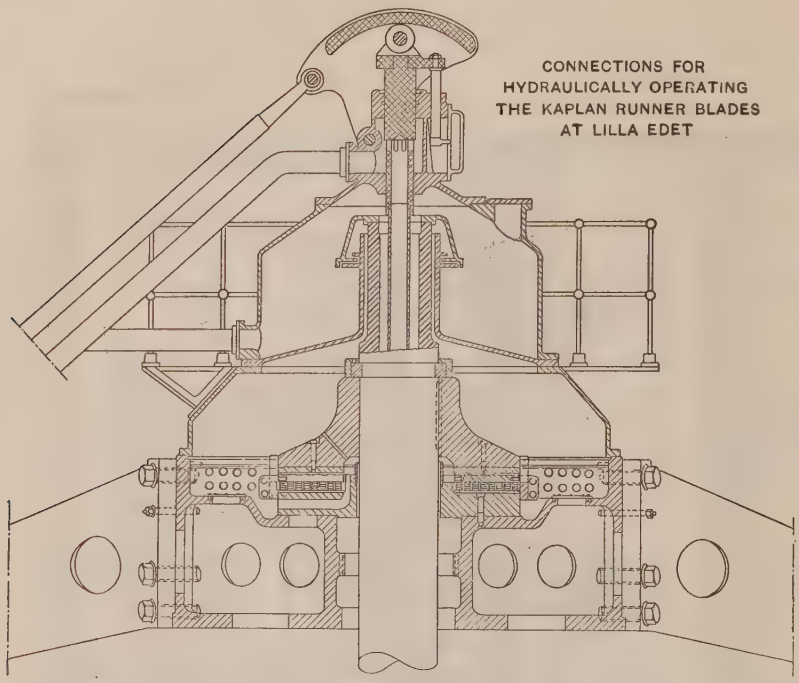


FIG. 48.

The turbine was installed under the following guaranties, for operation under a head of 11.25 ft. (3.45 m.):

Speed, in revolutions per minute	200	250
Output, in horse-power	290	323
Percentage of efficiency	80	78

TABLE 3.—COMPARISON OF SOME LARGE SINGLE-RUNNER WATER TURBINE UNITS.

Power plant.	Head, in feet.	Generator output, horse-power.	Speed, in revolutions per minute.	Discharge (approximate), in second-feet.	Type of turbine.
Mississippi River Power Company	32	10 000	57.7	3 230	Francis
Cedar Rapids, Canada	30	10 800	54.3	3 730	"
Parahyba, Brazil	104	32 500	125.0	3 120	"
Niagara Falls Power Company	213	70 000	107.0	8 250	"
Isle Maligne, Canada	110	45 000	112.5	4 100	"
Manitoba Power Company, Canada	56	28 000	138.5	5 100	Propeller
La Gabelle, Canada	60	30 000	120.0	5 100	"

From this, the specific speed may be computed as about 165 ft-lb. units (725 metric), at 200 rev. per min., and 216 ft-lb. units (955 metric), at 250 rev. per min.

The form of the runner is interesting; it is a two-bladed screw type, each blade covering about 180° of the circumference. The final design of

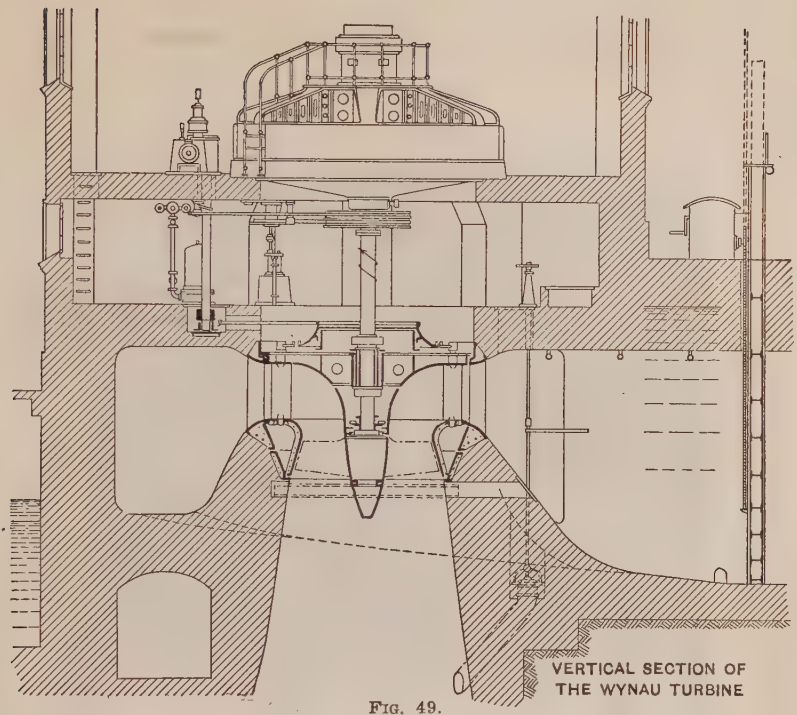

VERTICAL SECTION OF
THE WYNAU TURBINE

FIG. 49.

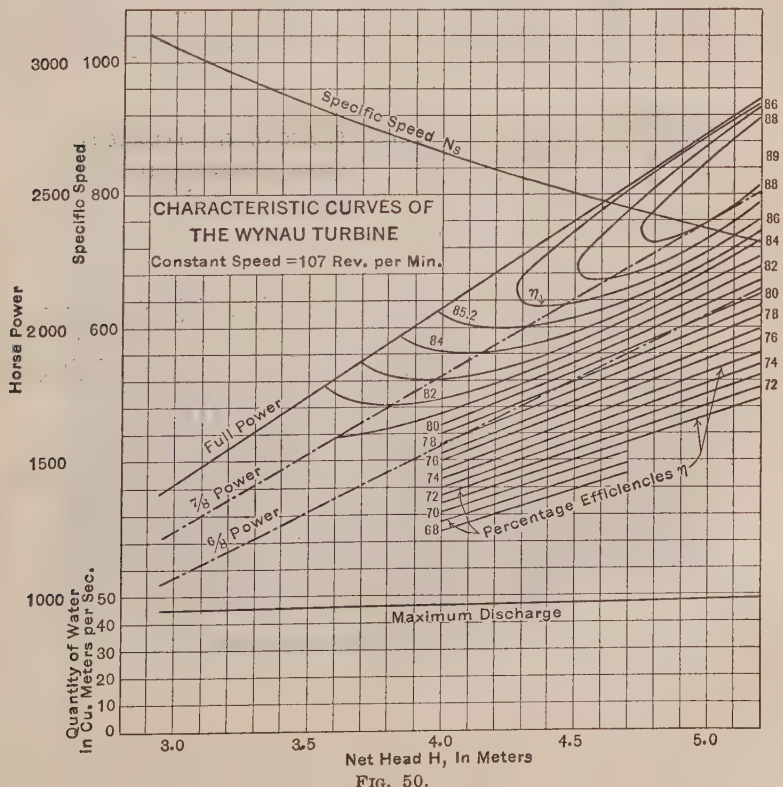

CHARACTERISTIC CURVES OF
THE WYNAU TURBINE
Constant Speed =107 Rev. per Min.
Specific Speed Ns
Specific Speed
Horse Power
Full Power
7/8 Power
6/8 Power
Maximum Discharge
Percentage Efficiencies η
Quantity of Water in Cu. Meters per Sec.
Net Head H, In Meters
3000
2500
2000
1500
1000
800
600
3.0
3.5
4.0
4.5
5.0

FIG. 50.

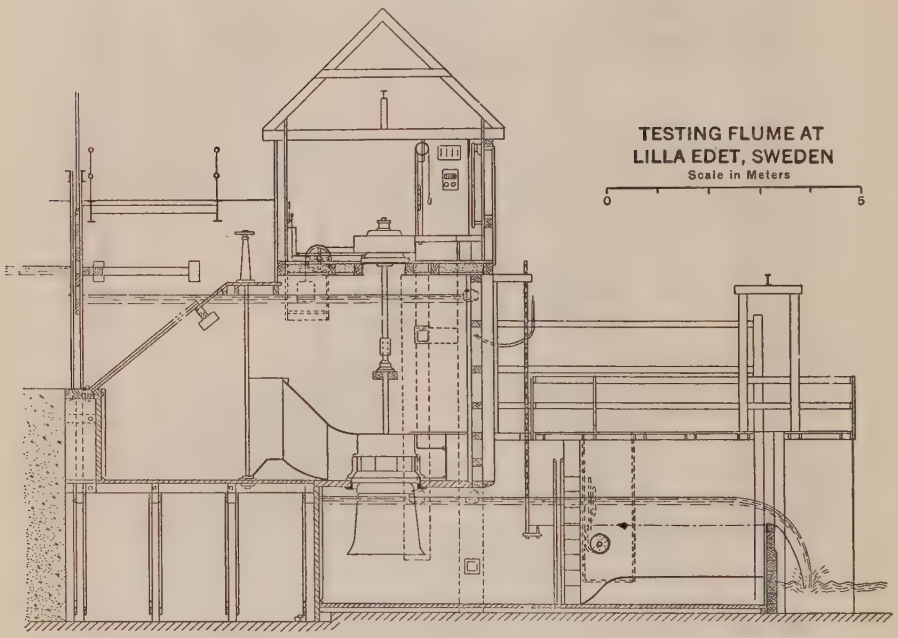

TESTING FLUME AT
LILLA EDET, SWEDEN
Scale in Meters
0
5

FIG. 51.

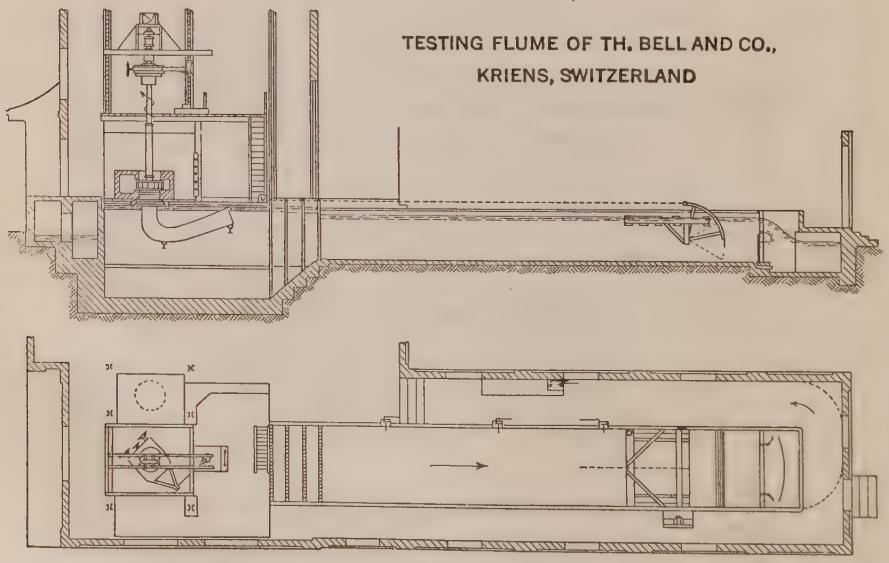

TESTING FLUME OF TH. BELL AND CO.,
KRIENS, SWITZERLAND

FIG. 52.

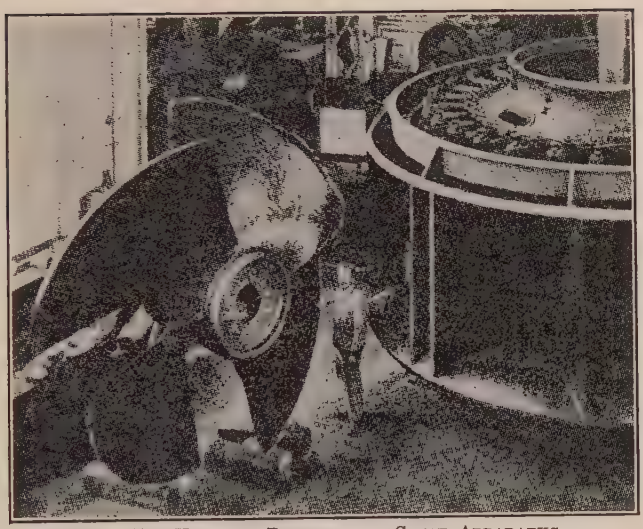


FIG. 53.—VIEW OF RUNNER AND GUIDE APPARATUS OF THE WYNAU TURBINE.

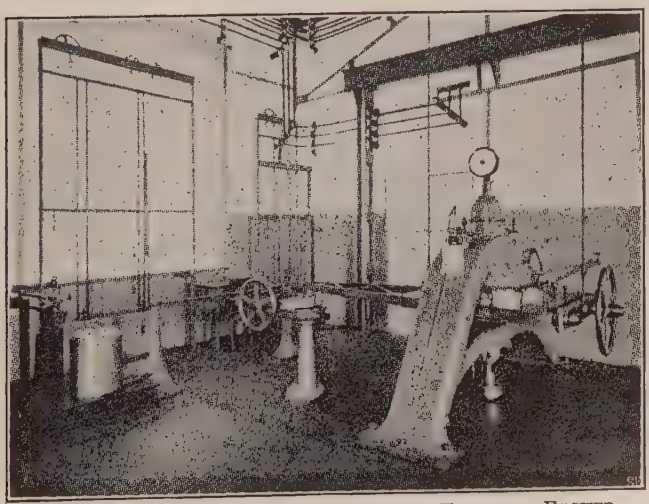
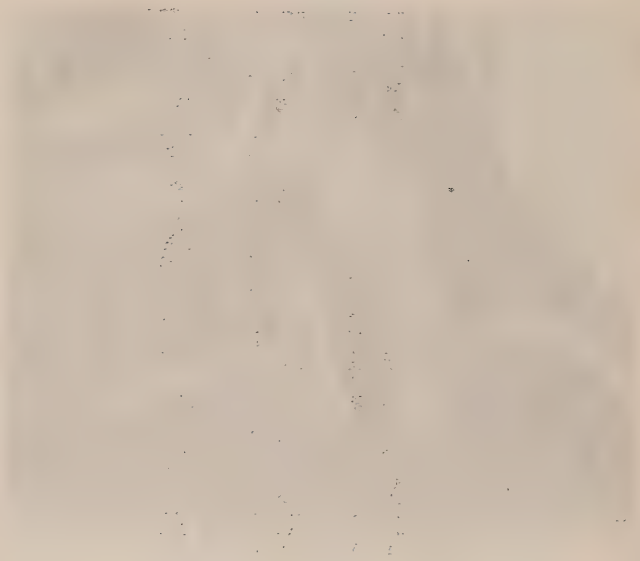
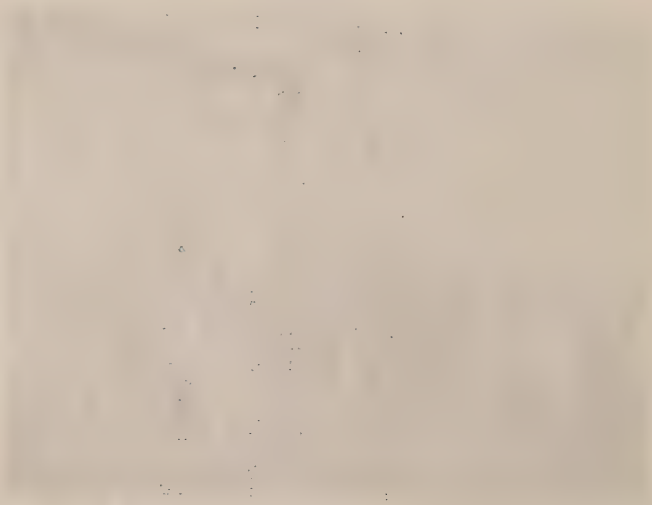
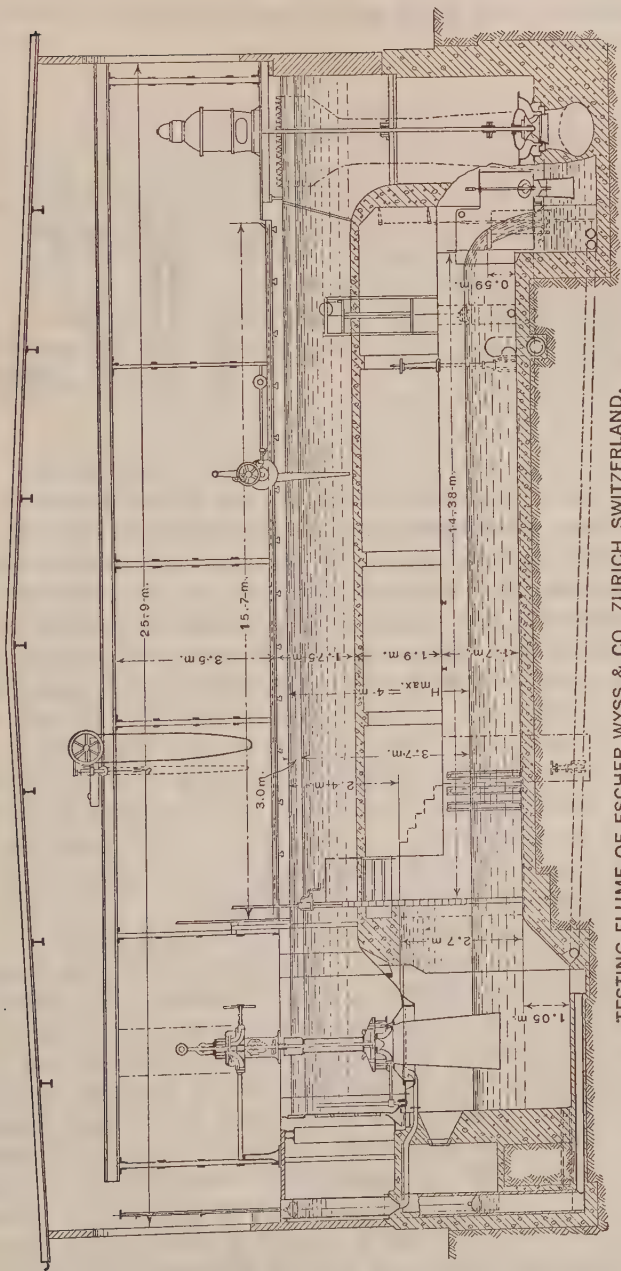


FIG. 54.—OPERATING ROOM FOR TESTING FLUME OF ESCHER WYSS AND COMPANY, ZURICH, SWITZERLAND.







TESTING FLUME OF ESCHER WYSS & CO., ZURICH, SWITZERLAND.
LONGITUDINAL SECTION
FIG. 55.

the unit followed extensive experiments in the testing flume of the manufacturer.

The construction of the Matte turbine and its efficiency curve are shown by Figs. 8, 10, and 56.

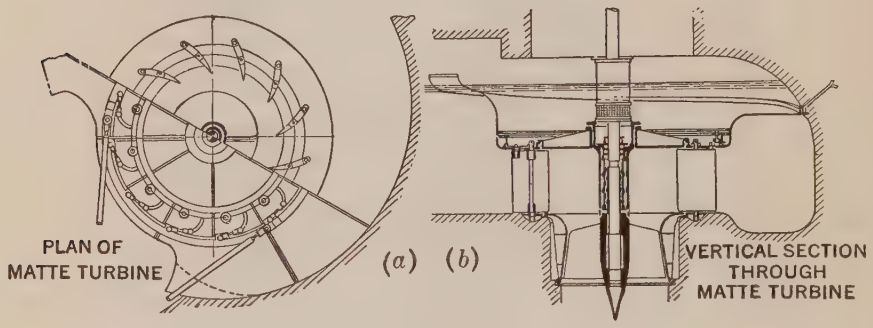


FIG. 56.

One of the latest installations is that of the Wynau Electricity Works on the Aare River, in Switzerland. This installation includes two units, each of which has a nominal capacity of 2 700 h.p. and operates at 107 rev. per min. under a variable head of 8.2 ft. (2.5 m.) to 17 ft. (5.2 m.). These units were furnished by the Ateliers de Construction Mecaniques de Vevey. Acceptance tests of these turbines indicate a maximum efficiency of 89%, with a specific speed of 165 ft-lb. units (728 metric), when operating under a head of 17 ft. (5.2 m.). Under a head of 8.2 ft. (2.5 m.), at 85% efficiency, the specific speed is given as 195 ft-lb. units (862 metric).

The design of the turbine runner was developed with the assistance of tests of models 15 in. (380 mm.) in diameter. The generator coupled to the Wynau turbine is designed for a runway speed of 2.7 times the normal speed of 107 rev. per min. In Figs. 49, 50, and 53 are shown details of the construction and performance of the Wynau turbine.

The writer particularly regrets that he is not in a position to present the developments made by Escher Wyss and Company, of Zurich, Switzerland, which have followed very remarkable results obtained in its testing flume. Fig. 7 shows one of this Company's propeller type runners having six blades.

A reluctance on the part of European turbine manufacturers and engineers to publish the results of their findings in respect to propeller type turbines, has been noted. This, perhaps, may be explained by the rapid development accomplished, leaving little opportunity to pause and review the results.

In the descriptions given, references to the testing of model runners in the development of the turbine designs have been made, which testing, it is easy to believe, is very important in respect to the propeller type runner. It appears, therefore, not without the scope of this paper to show Figs. 51, 52, 54, and 55, illustrating some of the flumes used in testing models of the turbine runners referred to by the writer.

PROPORTIONING OF UNITS IN LOW-HEAD PLANTS

By John P. Hogan,* M. Am. Soc. C. E.

In a paper delivered before the American Society of Civil Engineers on October 4, 1922,† the writer indicated certain tendencies in water power development, the more important of which were developments in storage and interconnection and a tendency toward increase in the size of units.

Increased interconnection has accomplished two important results: First, by broadening the market, it has made it possible to consider the distribution in the general market of the output of the large continuous powers of boundary streams; and, second, it has permitted the selective development for peak purposes of plants on interior streams, by removing the need for flexibility in any individual plant.

Storage on interior streams, now in process of construction or proposed, will permit not only the selective development of individual plants but will make possible the unified development of entire streams for peak power capacity.

If it is assumed that the average present installation on the interior streams is about equal to the average flow and that such a stream may be completely regulated so that the minimum flow will be brought up to the average flow, a development of this stream for 8-hour peak power will require three times the present installation. The development of regulated streams for peak power capacity requires that the time element between plants be eliminated, if possible, in order that the peak capacity in all the plants may be synchronous with the peak demand. This requires a continuous chain of plants with sufficient capacity in each pond to correct the time of travel of the water from the preceding pond. In order to make the chain continuous it is necessary on the average stream to develop relatively low-head plants which might not otherwise be economical.

In the plans for development of boundary streams a number of relatively low-head plants have been proposed. At the same time the uniformity of both the continuous and regulated flows insures the operation of larger units at full capacity and best efficiency. There is a rapidly increasing demand, therefore, for larger capacity units operating under relatively low heads.

In proportioning units for a plant, the controlling factors in determining the number of units, assuming load regulation to be provided from other sources, are: Maximum head and variation in head; character of the stream flow; speed of units; efficiency; and velocity of approach and discharge of water.

The writer does not propose to discuss in detail the variations in head and flow as they represent special problems to be considered and determined for each particular case.

For ordinary service on a fluctuating flow and varying load the Francis wheel with its relatively flat efficiency curve at varying flows is the ideal. The

* (Parsons, Klapp, Brinckerhoff & Douglas), New York, N. Y.

† "Present Tendencies of Water Power Development in New York State", *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 732.

limit of size in Francis wheels for low heads is soon reached, due to decrease of speed in proportion to the increase in size.

For a 30-ft. head the normal Francis runner operating at full efficiency should have a specific speed of about 90. For a 30-ft. head and specific speed of 90 the characteristics of normal Francis wheels of various sizes would be approximately as shown in Table 4.

TABLE 4.

Quantity of water, in cubic feet per second.	Revolutions per minute.	Electrical horse-power.
500	170	1 330
1 000	125	2 660
2 000	85	5 300
4 000	55	10 800

At the largest size shown, the size and cost of the wheel and generator become almost prohibitive. It will be readily seen therefore that speed fixes a definite limit to the economic size of Francis wheels for low-head developments. Various experiments have been made with special designs and modifications of Francis wheels to exceed the normal specific speeds, but, in the writer's opinion, they do not promise a solution of the problem.

As has been stated, designers and manufacturers, both in the United States and abroad, have put at the disposal of the constructor and operator special types of wheels to meet this situation, which heretofore have been called propeller wheels. These wheels have somewhat different designs and different spheres of usefulness, but they have the common object and result of increasing the specific speed at a slight loss in maximum efficiency.

The result is increase in speed of wheel and generator for a given quantity of water and consequent decrease in cost of wheel and generator or a decrease in the number of units, all resulting in power-house savings. A normal propeller wheel operating on a 30-ft. head will probably take 4 000 cu. ft. per sec. of water at 85 rev. per min. and therefore would be roughly equivalent to the 2 000 cu. ft. per sec. Francis wheel operating at the same speed although the propeller wheel will develop nearly twice the power.

It would appear also that the limit of size of units for low heads has been substantially doubled, if 50 rev. per min. is considered to be a reasonable minimum speed for both types. It is true that all the so-called propeller wheels, except the type with movable blades, have increasingly lower relative efficiencies at part gates than the Francis wheels. This is of no importance in continuous plants of large flow where the number of units is so great that sufficient flexibility can be obtained by combinations of units. On stream-flow plants, on the other hand, it is probable that at low flows the combinations of propeller wheels of large flow might give low efficiency and capacity at the most critical periods and that economy will dictate the use of two types of wheels of which the majority will be propellers and the minority either Francis wheels, or special wheels having characteristics similar to those of the

Kaplan wheel. This has already been done abroad for a low-head plant, as at Lilla Edet, Sweden.

In peak capacity plants it will be easier to arrive at a compromise in the number of units which at the same time will permit a large wheel and efficiency at relatively small daily flows, since it will be possible to operate for a relatively short time on a concentrated flow.

Other characteristics of the propeller type which should be considered are that they give better efficiencies at fluctuating heads and that they must be set much lower than wheels of the Francis type.

This leaves only the velocity of approach in the penstock and velocity of discharge to the tail-race to be considered. The limitations imposed by the propeller type are very similar to those of the Francis type and in low-head plants of large flow these limitations are already presenting serious problems in the design and construction of waterways.

Already, under some conditions, the provisions of adequate water passages are of more influence in limiting the size of the units than the characteristics of the wheels, and with increasing use of wheels of the propeller type in large units, the influence of these hydraulic limitations will increase. They may even be finally responsible for placing a definite limit to the increase in size of units for plants of large flow and relatively low head.

DISCUSSION

EDWARD P. HAMILTON,* ASSOC. M. AM. SOC. C. E.—The modern propeller wheel, as developed by Mr. Nagler, has many advantages over the Francis type for certain conditions. This is clearly evidenced by the large number of installations of the propeller type during the last eight years. It must be gratifying to Mr. Nagler to realize how readily what appears to be a most radical change in wheel design has come into common use.

In his paper Mr. Nagler takes exception to statement† by Arthur T. Safford, M. Am. Soc. C. E., and the speaker, that, "a wheel of the high-speed propeller type was patented and on the market fifty years ago", and that the "Green Mountain" wheel patented by Jacob Truax in July, 1860, was the forerunner of the modern propeller wheel. It was intended to show that the "Green Mountain" wheel bears much the same relation to the propeller runner designed by Mr. Nagler that the early Francis, Swain, or Hercules runners do to the modern high-speed reaction turbine, such as the Leffel "Z" model, designed by Professor Zowski.

Fig. 57 (*a*) shows the Francis runner of 1847 alongside the Zowski runner of 1913, Fig. 57(*b*), while Fig. 58(*a*) shows the Truax "Green Mountain" runner of 1860 beside the Nagler runner of 1915 Fig. 58(*b*). A glance at these photographs would seem to make it evident that the Nagler runner of 1915 would recognize the "Green Mountain" runner of 1860 as its forebear much more readily than the Zowski runner of 1913 would recognize its ancestor in the Francis runner of 1847.

The main structural difference between the Truax and the Nagler runner is the band which surrounds the former. Remove this and the wheels look very much alike. Fig. 59, taken from Emerson's "Hydraulics" published in Willimansett, Mass., in 1892, shows the Barber wheel, which was of the propeller type and had no band. Several other wheels of the Sixties and Seventies, such as the Blackstone and the Milliken, were built without bands around the runner. The Barber wheel is most interesting because, although the hub is large and the speed consequently low, the projections of its buckets and hub do not completely cover the area circumscribed by the bucket tips. In other words, the buckets fail to meet or overlap in the same way as those of a marine propeller or a Nagler runner.

Fig. 60 also taken from Emerson's "Hydraulics", shows the "Economy" wheel, made in Indianapolis, Ind. The two propeller runners have no bands. Note, also, the comparatively flat angle of the blades.

Fig. 61, taken from Bélidor's "Architecture Hydraulique", published in Paris in 1737, shows a type of wheel very common in those days in Southern France on the River Garonne and its tributaries. It was used on low falls, and was set in a masonry scroll of very much the same proportions as the wooden ones used a hundred years ago in America. Note the marked narrow-

* Asst. on Eng. Staff, Proprietors of Locks and Canals on Merrimac River, Lowell, Mass.

† "The American Mixed-Flow Turbine and Its Setting", *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1237.

FIG. 57(*a*).—ORIGINAL FRANCIS RUNNER.

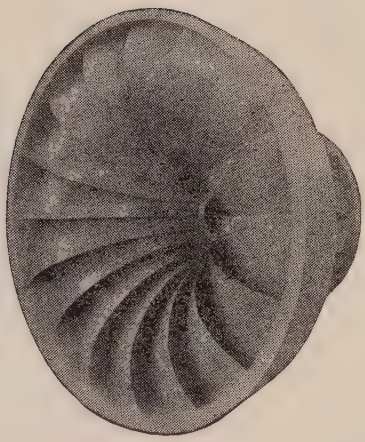


FIG. 57(*b*).—MODERN LEFFEL "*Z*" RUNNER.

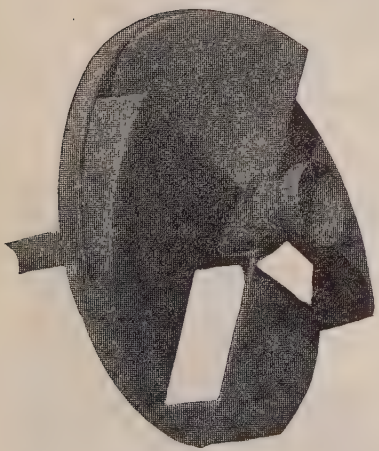


FIG. 58(*a*).—GREEN MOUNTAIN RUNNER

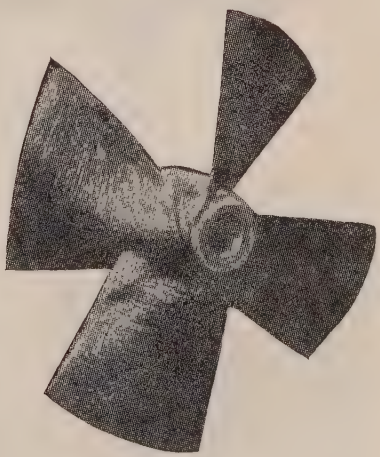


FIG. 58(*b*).—NAGLER RUNNER.

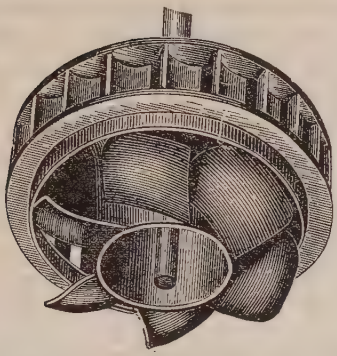


FIG. 59.—THE BARBER WHEEL.

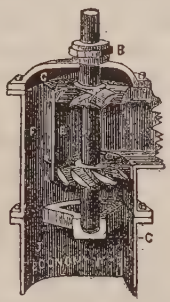


FIG. 60.—THE "ECONOMY" WATER-WHEEL.

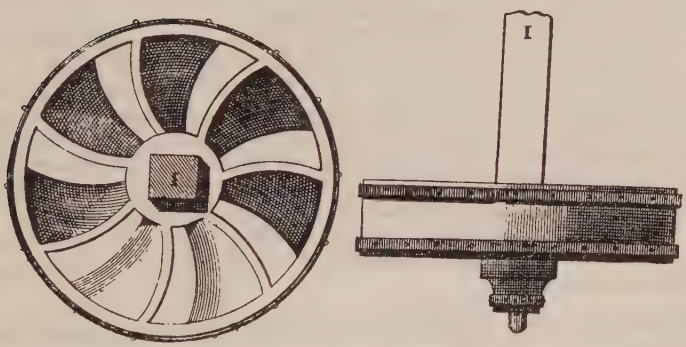


FIG. 61.—FRENCH TUB WHEEL RUNNER.

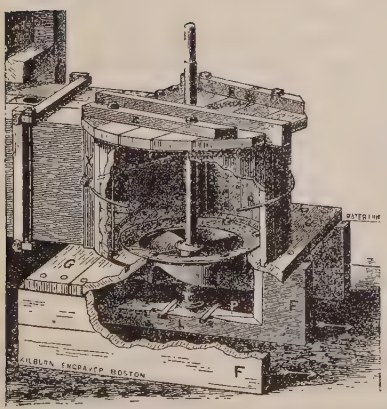


FIG. 62.—TRUAX WHEEL IN WOODEN CASE.

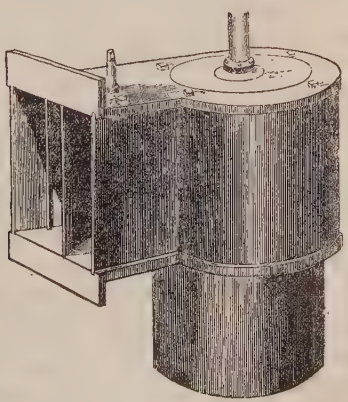


FIG. 63.—TRUAX WHEEL IN IRON CASE.

ness of the seven blades and their failure to overlap, leaving one-quarter or more of the total runner area entirely open in an axial direction.

Fig. 62, taken from Truax's 1879 circular, shows the wheel and the form of casing that he recommended. He advised that his wheels be set in wooden rather than iron spiral casings, which, he said, cost more and were less adaptable to varying conditions.

The following extracts from Jacob Truax's "Green Mountain" circular of 1876 show a striking resemblance between the claims made for the "Green Mountain" wheel and those made for the modern propeller wheel:

"To Mill Owners.—Please do not hesitate, for I am offering you the only wheel that is adaptable for the low and common falls, and selling a wheel of the same power, using as little water, for less money than any other wheel now in use.

"I am having the most gratifying assurance from my patrons of the unequaled efficiency or superiority of this wheel over all other wheels, for common or low falls.

"This wheel has been in practical use over fifteen years, and has proved to be one of the best water wheels now in use."

"* * * the Green Mountain Water Wheel is a great money saver, for mill-owners that have low falls. One-half of the money that is laid out on a 5-ft. fall by Leffel and Risdon, or nearly all other wheel manufacturers, is saved by the use of the Green Mountain Wheel, and one-fourth on a fall of 9 to 10 ft. There is no material difference in the use of water for the same power on the same falls, but the capacity of the Green Mountain Wheel is such that much less diameter wheel is necessary to use the same cubic feet of water per minute. It being so much smaller it rotates much faster, and is impeded but little by back-water compared with other wheels. It will not wear out or break or clog with anchor ice or flood trash. Its smallness is the cause of its cheapness."

There follows a list of thirty-three installations of "Green Mountain" wheels on falls of between 4 and 10 ft., the average being 6.7 ft., after which the circular continues:

"These are a few of the low falls that my wheel is working on, and the falls are sometimes reduced one-half, sometimes two-thirds by reason of low water, or back-water caused by high water, or by being dammed up by ice below the mills, or tide-water; and fair business has been done with some of the above wheels when the fall was not more than 2 ft. They have been run when the head-water was 12 ft. above the wheel and the back-water 10 ft., leaving but 2-ft. fall, the wheel not being retarded except the loss of fall. This result is not attained by any other wheel in the world.

"It is a well established principle among scientific men that velocity doubles every four times the pressure, whether it be steam, air or water. This being a well settled fact, it will be seen, in the absence of any additional pressure, the velocity in a rotating body may be doubled by making the incline at the mean diameter four times as long as it is deep, and having thus doubled the velocity by this mechanical construction, it is plain that a water wheel of equal diameter may be made to revolve 200, 300, or 400 per minute, instead of 100 per minute, on the same fall. This is no secret, it is a plain, simple principle, and ought to be understood by every wheel builder. If the wheel builder understands this little principle or rule, he will not wonder, but say Eureka. The fact that a 4-ft. wheel may rotate 200 per minute on a fall of 6 ft. will be an easy problem to him. This principle understood, and he will not exclaim that he does not see why one wheel of the same size may run

slow and the other fast, the pressure and quantity drawn being equal. The power and high speed of the Green Mountain wheel is the application and use of this principle. It is constructed in accordance with this principle, and the use of this principle is the grand secret of the double and quadruple motion, and double and triple power, of other wheels of its size. There are none of the Green Mountain wheels that have incline planes at the mean circumference of 4 to 1, but I have used these figures to illustrate the rule, and the fact.

* * * *

"When the Green Mountain wheel was first introduced, other wheel builders in this locality treated me and my wheel with contempt, but notwithstanding this I with energy pursued my calling, put the wheel into use, kept the hammer of logic pounding, and have succeeded in driving my opponents quite out of the field. Many of them now acknowledge the usefulness and superiority of the Green Mountain Water Wheel, saying that if a man has a low fall, and fails with the Green Mountain wheel they may stop trying. For this compliment I return my sincere thanks."

The following is from an 1879 supplement to the Truax circular:

"It has double the power of the average of the other wheels of its size, and triple the motion, and free from the hindrances caused by ice and flood trash, and can, as a rule, be placed in a position ready for work for 50% of the cost of the average of other good wheels."

These statements placed alongside various extracts which might be made from the articles on Mr. Nagler's runner would show that Mr. Truax and Mr. Nagler had both worked along the same lines and had both attained a striking success in securing speeds far in excess of other contemporary types.

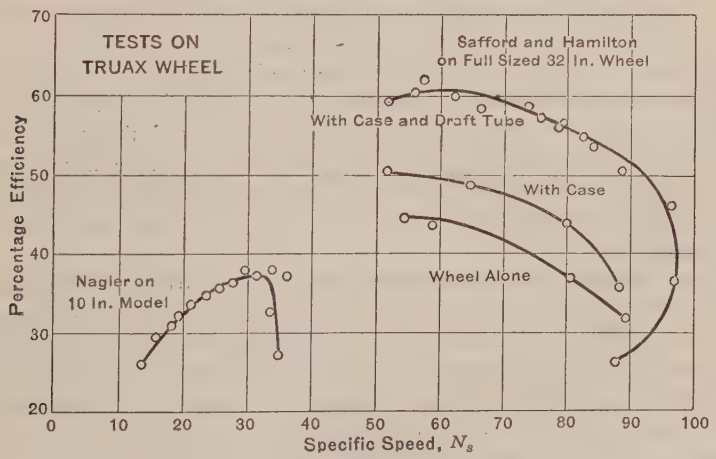


FIG. 64.

Coming now to the question brought up by Mr. Nagler as to the actual power and speed of the Truax runner, the tests made by Mr. Nagler on his 10-in. model and the results he gives in his paper are most unfair to the Green Mountain wheel. The tests made by Mr. Nagler were on a 10-in. model without any scroll-case or draft-tube and under a head of 12.90 ft. The Truax wheel tested in Lowell, Mass., was an actual 32-in. runner that had been in use

for many years in a mill in Northern New York. The results of the two tests are entirely different, as shown in Fig. 64. It is very interesting to note that the uppermost of the speaker's curves reverses and turns back toward the origin in exactly the same manner as the propeller-wheel performance curves shown by Professor Moody.

In the closing discussion* of their paper on "The American Mixed Flow Turbine" Mr. Safford and the speaker pointed out the unfairness of the Nagler tests, on account of the head used and of the omission of the casing.

Mr. Nagler feels that the use of a scroll-case and a draft-tube by Mr. Safford and the speaker made an essential addition to the Truax wheel. The Truax patent of 1860 specified both. The present casing was built to conform to the dimensions for a 32-in. wheel given in the 1876 circular from which was taken Fig. 63, showing the wheel in an iron scroll-case with a draft-tube attached. The draft-tube shown is not flaring, but to quote again from the paper† by Mr. Safford and the speaker:

"It is interesting to note that 'W. W. T.,' writing of the 'Green Mountain' propeller wheel, realized that, with a wheel of this type, a draft-tube would be of greatly increased value. He says, 'If Mr. Boyden would put his diffuser on one of these wheels, we would promise him a gain of more than 3 per cent. It would be more nearly 50.'"

The draft-tube used was far from ideal, and it actually added only about 19% to the power.

Mr. Nagler's Table 2 requires certain changes in order to be correct. He credits the Truax wheel with a maximum of 61% efficiency and 60 specific speed. It attained a maximum specific speed of 98 in the test made by the speaker and Mr. Safford. This was not reached at the point of maximum efficiency, but at a considerably lower point (37%); at a speed of 97, the efficiency was 46 per cent. It is interesting to note that while Mr. Nagler tabulates the values of efficiency and specific speed for the Truax for the same point, he uses two different points for his own wheel giving high values for both. In his original wheel he secured a specific speed of 129 at 87% efficiency, rather than the 150 of Table 2. Of course, his wheel has been greatly improved since 1915.

Fig. 65 is a copy of Mr. Nagler's Fig. 32(*a*) with a few changes and additions. The test on the 10-in. model Truax has been added, and also tests on the original Francis, the Swain, and a modern Leffel "Z" wheel as designed by Professor Zowski. All curves are for full gate. Both the Leffel and Nagler wheels show a considerably better efficiency at lower gates, but the specific speed becomes less. An inspection of these curves will show that the Leffel "Z" wheel shows a greater advance in specific speed beyond the original Francis wheel than the Nagler does beyond the Truax wheel.

It is granted that the efficiency of the Truax wheel was low. It must be remembered, however, that its efficiency compared very favorably with that of its contemporary wheels. By 1860 a great many small crude turbines were beginning to come into use, but the usual wheels in the little mills in the

* *Transactions*, Am. Soc. C. E., Vol. LXXXIV (1922), p. 1346.
† *Loc. cit.*, p. 1286.

northeastern part of the United States were the "flutter" and Rose wheels in saw-mills, and tub or scroll wheels in grist-mills and other small shops. Some undershot and overshot wheels were used, of course, but ice conditions were such as greatly to restrict their use. The "flutter" wheel probably gave an efficiency of from 15 to 25%, the Rose wheel considerably more, perhaps even as high as 60%, while the tub and scroll wheels ran from about 20 to 40 per cent. In the fall of 1923, in connection with some work at Calais, Me., the speaker ran a test on an extremely good example of tub wheel. An efficiency of 31% was obtained on the jack-shaft, equal to about 36% on the wheel-shaft.

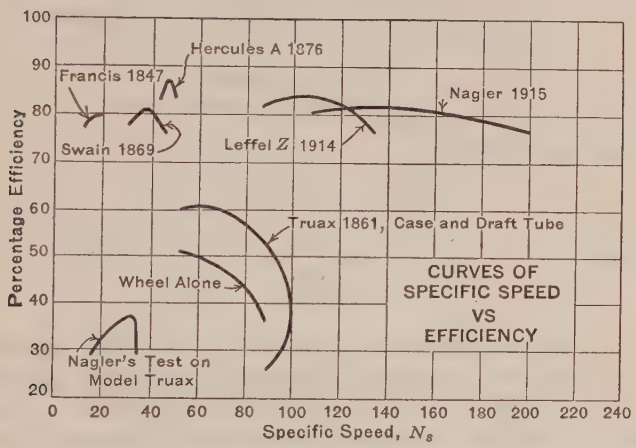


FIG. 65.

Until the latter part of the Nineteenth Century the efficiency of a water-wheel was of comparatively little importance; power was what was required. At the time the Truax wheel appeared, aside from a few tests in Philadelphia, Pa., in Lowell, and in France, the testing of water-wheels was almost unknown. It was not until about 1870 that systematic testing began.

The Truax wheel delivered the required power at a high wheel speed. Its maximum specific speed of 98 was delivered at 37% efficiency, but it gave a specific speed of 90 at somewhat more than 50% efficiency, an efficiency which exceeded that of the great majority of its contemporary wheels. Testing the wheel without the flaring draft-tube, as undoubtedly most of the Truax wheels were set, still gives a specific speed of 80 at 43% efficiency.

The water-wheels of the Sixties were not operated at their point of best efficiency. The average miller or millwright hardly would have known the meaning of the word. The wheels were operated to get the power. The maximum power shown in the test made by Mr. Safford and the speaker was developed at a specific speed of about 80 with the draft-tube and 72 without it. Unquestionably many of the wheels, particularly when under back-water conditions, were operated at still higher specific speeds.

The Truax wheel of 1861 undoubtedly had a high specific speed for its time, and this certainly was not exceeded until after 1910. The speaker feels justified in stating that a specific speed of 90 was attained, and that this speed was

attained at an efficiency which was more than ample to justify the commercial use of this wheel. Accordingly, Mr. Nagler's chart of specific speed (Fig. 32), should be modified as shown in Fig. 66.

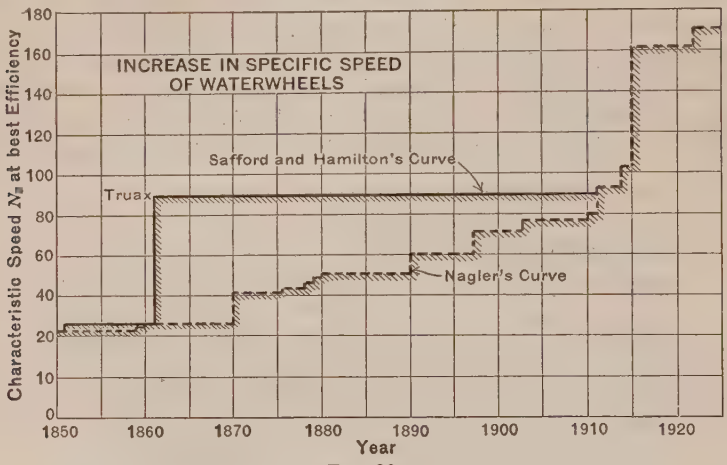


FIG. 66.

Mr. Nagler has developed a crude wheel of sixty years ago into a commercial type of wheel that satisfies present-day requirements of efficiency and mechanical construction, and great credit is due him for this. The speaker's only contention is that Mr. Nagler has not invented a new type of water-wheel, but that he has improved an existing type.

H. K. BARROWS,* M. AM. SOC. C. E.—The speaker has been much interested in the papers presented before the Power Division; the data, particularly for high-speed turbines, furnish information of value in the study of this type of wheel.

For comparison with wheels of lower specific speed the speaker has prepared Fig. 67, on which is plotted specific speed, $N_s = \dfrac{N\sqrt{HP}}{h^{\frac{5}{4}}}$, against the power coefficient of the wheel. The latter is P_u in the equation,

$$HP = P_u D^2 h^{\frac{3}{2}}$$

or it is the power delivered by a 1-in. wheel under a head of 1 ft.

Curve *A*, Fig. 67, is plotted from data of a large number of American reaction wheels of different makes. It is an average curve, hence individual wheels will depart somewhat from it in both directions.

Curve *B* is an approximate curve based on the data given by Mr. Orrok in Table 1 (numbering the wheels for convenience from 1 to 8, inclusive), and by Mr. Nagler in Figs. 6 and 8 (the latter numbered 9 and 10, respectively, on Fig. 67). The point numbered 11 is for the Nagler runner at the Plainwell, Mich., plant.

* Prof. of Hydr. Eng., Mass. Inst. Tech.; Cons. Engr., Boston, Mass.

At first glance it would appear that the propeller type of wheel delivered much less power for a given diameter and head than the ordinary reaction wheel. This is due, however, largely to the different manner in which the wheel diameters are measured. With the propeller type the nominal wheel diameter is approximately equal to the outlet diameter. With the ordinary reaction wheel the nominal diameter is measured at the entrance and is considerably less in amount than the outlet diameter. If outlet diameters are used in plotting the N_s-P_u curve for reaction wheels, as shown in Curve *C*, Fig. 67, it will be seen that the two types of wheels are not greatly inconsistent in this respect. For specific speeds approximately 100 to 110, the propeller type still shows a somewhat less power coefficient than the ordinary reaction wheel.

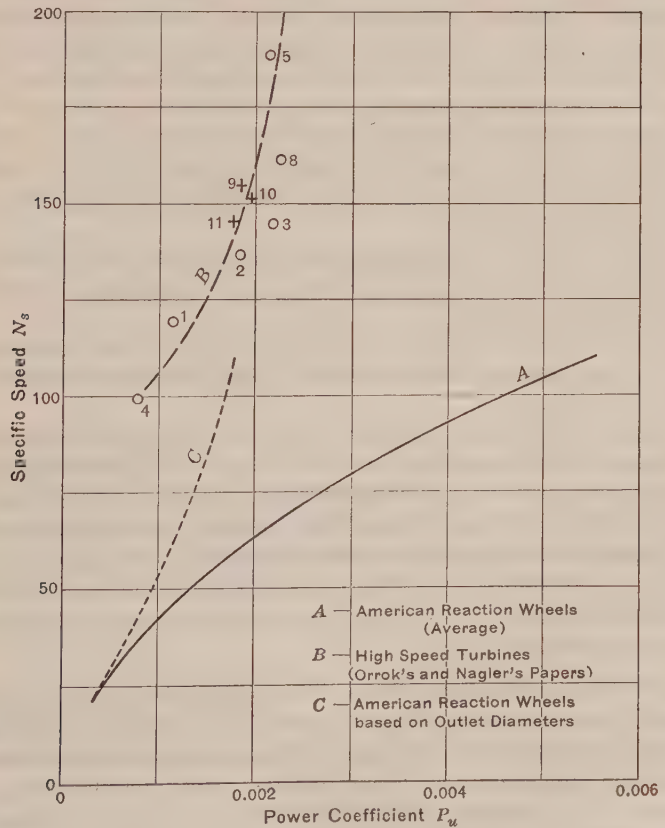


FIG. 67.—RELATION OF SPECIFIC SPEED AND POWER COEFFICIENT.

Considering the turbine as a form of orifice, it will be seen that for a given area at the outlet there should be a similar discharge and power for any given head (at, say, full gate) provided efficiencies are the same. As the propeller type is usually about 5% less in efficiency than the ordinary reaction wheel the power coefficient of the former is bound to be somewhat less.

The relatively sharp peak of the load-efficiency curve of the propeller type of wheel renders its use less advantageous where the wheel must frequently run at part load, as is usually the case. Although under some circumstances it may be practicable to run a propeller type of wheel fairly steadily at best gate, the speaker agrees that this is likely to be difficult of attainment with the ordinary hydro-electric plant.

W. M. White,* M. Am. Soc. C. E.—The papers contained in this Symposium on the important subject of high specific speed hydraulic turbines, have been very comprehensive. The speaker had the pleasure of visiting the Lilla-Edet plant to which reference has been made and was greatly impressed with the tremendous size of the conduits leading the water to and from the wheels. The construction has been considerably delayed due to labor troubles. The Verkstaden Works in Kristinehamn, Sweden, the builders of the turbines for this plant, extended an invitation to the speaker to visit the works and he had an opportunity of seeing the turbines in course of construction. There is more need for the movable vane runner in Europe than in America, because in this country such a network of power lines exists that nearly every plant constructed can be operated to its full capacity as long as water is available.

Mr. Moody has given a formula which throws additional light on an interesting and important subject.

Mr. Jessop's paper has conformed more exactly to Mr. Orrok's suggestions as to scope than any of the others and the speaker is sure that his presentation will be of value.

Mr. Nagler has done valuable work and has made a real contribution to the art. His type of runner enables the development of lower heads than would otherwise be possible.

The high-speed runner, as designed by Mr. Moody, seems to be a type which lies in the intermediate zone between the "Z"-type of Leffel wheel and the four-bladed "Nagler" wheel. It is to be hoped that the Engineering Profession will encourage further development along these lines by giving protection to those making inventions and pursuing development work. The only way a manufacturer has of recovering expenditures made for developments such as have been discussed in these papers is by having the exclusive use of the devices thus developed for a reasonable time.

Lewis F. Moody,† Esq. (by letter).—As a supplement to the material contained in the Symposium, on "High Specific Speed Hydraulic Turbines," the writer would call attention to a paper, which appeared shortly after the presentation of the Symposium, by Franz Staufer, of Munich, Germany, entitled "Influences on the Efficiency of Water Turbines."‡ A brief abstract of this paper by C. Reindl has also been published.§

Referring to the writer's discussion of the variation of efficiency with size of turbine, it is interesting to note that Mr. Staufer has taken up this

* Mgr. and Chf. Engr., Allis-Chalmers Mfg. Co., Milwaukee, Wis.

† Cons. Engr., I. P. Morris Dept., The William Cramp & Sons Ship & Engine Bldg. Co., Philadelphia Pa.

‡ *Zeitschrift des Vereines deutscher Ingenieure*, March 28, 1925.

§ *Die Wasserkraft*, May 1, 1925.

same subject along similar lines, has developed a formula of the same form, and has arrived at the same numerical value of the exponent of the ratio of diameters appearing in the formula.

Mr. Staufer mentions the point noted by the writer that the change in efficiency of water turbines as affected by diameter and head, as first formulated by Camerer, was based on the formula of Biel. Instead of using the Biel formula, Mr. Staufer, proceeding along the same lines as the writer, has based his work upon the exponential expressions for the loss of head in fluid flow and using numerical values taken from the work of Reynolds, Schiller, Jakob, and Erk, has derived the formula:

$$e' = \left[1 - \left(1 - \frac{e}{e_m}\right)\left(\frac{D}{D'}\right)^{c_2}\left(\frac{H}{H'}\right)^{c_2{}^2}\right] e_m'$$

Staufer's notation being here modified to correspond to that used by the writer. Adopting a value of $\frac{1}{4}$ for the exponent, c_2, this formula then becomes:

$$e' = \left[1 - \left(1 - \frac{e}{e_m}\right)\left(\frac{D}{D'}\right)^{1/4}\left(\frac{H}{H'}\right)^{1/16}\right] e_m'$$

In this formula, e_m denotes the mechanical efficiency of the turbine, that is (expressed in percentage), 100 minus the percentage of loss in mechanical friction in bearings, etc.

It will be noted that the effect of change in diameter as derived by Mr. Staufer is the same as that proposed by the writer. A considerable number of comparative tests investigated by the writer indicate that the exponent of the ratio of heads should be even smaller than the figure of $\frac{1}{16}$ derived by Mr. Staufer. Unless the ratio of heads is very great the change in head is of only minor effect. Since the mechanical losses in most turbines of ordinary sizes are very small, it is believed that these losses need not be separated from the total loss, and it has been found that the simpler form of expression proposed on page 628 of the writer's paper can be applied satisfactorily to the total efficiency of the turbine with the value of the exponent as there stated, namely, $\frac{1}{4}$.

Mr. Staufer's paper also includes some interesting results of recent tests on a high-speed propeller type turbine.

FORREST NAGLER,* M. AM. SOC. C. E. (by letter).—In reviewing the papers and discussions, more or less confusion in the understanding of the subject of high-speed runners seems to be attributable to an assumption that the various names applied to this type are synonymous. The term "high-speed turbines" or "high specific speed turbines" is not definitely descriptive, nor does the term "propeller type", "axial flow type", "suction type", or any of the individual names attached to the type definitely fix a place in the specific speed scale. For example, high specific speed turbines may have a specific speed of 10 for impulse

* Hydr. Engr., Allis-Chalmers Mfg. Co., Milwaukee, Wis.

turbines, 100 for Francis turbines, or 200 for suction turbines. Similarly, a Moody runner may aparently have a specific speed of 90 or 190. A propeller type turbine or an axial flow turbine may apparently have a similar range of specific speed. The suction type turbine is likewise open to criticism, but it is the most definite term, because it describes a certain characteristic which enters not at all, or to a negligible extent, in conventional forms of turbines; and, furthermore, the name signifies a characteristic which gives an indication of its specific speed and design limits, the latter being based on barometric limits.

The writer has no solution to offer for the confusion arising from the broadness of the terms used to identify the various runners. It is desired, however, to point out that a propeller turbine does not necessarily indicate a high specific speed turbine, nor does a suction turbine necessarily cover a type having a specific speed of less than about 140. Naturally, the design features of any one of these types merge gradually into those of another type, without a definite line of demarcation. The foregoing discussion is offered because of the very great need of definitions to permit avoidance of generalizations that are technically incorrect.

The various papers, particularly those by Messrs. Orrok, Moody, Egbert, and Jessop, should go far toward improving the understanding of American engineers as to the new runner types, their range of characteristics, and their limits of application. Mr. Moody's paper contributes one valuable tool much needed by all those concerned with the construction or operation of water power plants, namely, his simplification of formulas for the variation of turbine efficiency with the variation of size. Previous comparisons, if at all practical, have been largely empirical and the need of a simplification of the intricate and clumsy algebraic expression of the laws has been very great. It would be well if Mr. Moody's expression for the variation of efficiency with the size of units could be embodied in present test codes, not so much by reason of its peculiar application to test code uses, but because that is probably the most practical place for its inclusion as a reference of interest. It has a certain utility in testing, on account of the growing tendency toward careful comparison between model tests and power plant tests.

The runners which Mr. Moody shows are very interesting. The newness of the art of building this type of wheel is indicated by the fact that Mr. Moody's experience is apparently not in accord with that of the writer, who, for example, has not been able to secure as high an efficiency at the higher specific speeds, that is, of more than 150, with the large projected blade area shown in Mr. Moody's design. Conversely, all the large projected-area runners have been very definitely improved in efficiency as their areas were reduced. Not only is this tendency logical, from considerations of wetted surface, the reduction of which has been largely responsible for the development of specific speeds in excess of 125, but the fact is checked absolutely by experience with marine propellers, where the initial perfection of steamship drive resulted from reduction of the blade area. Similar tendencies are found in the development of windmills, sails, electric fans, and blowers. Its parallel is lacking in the case of the steam turbine only, because the primary consideration of steam

turbine design is that of keeping the speed low, rather than high, and because friction loss is to a greater or less extent regained thermally.

Mr. Hamilton is agreeably persistent, but the main basis of difference in opinion is found in the writer's previous comments as to definitions of types. Had there been any positive knowledge of how to secure high specific speed or had specific speeds even remotely approaching 100 ever actually been obtained, there seems to be hardly a reasonable doubt but that the principles involved would have entered into commercial practice, at least to some extent, without waiting 40 or 50 years. Mr. Hamilton's revision of the specific speed curve after 1861, as given in Fig. 66, is not in accordance with established practice and even the inclusion of a single point approaching a specific speed of 100 between 1860 and 1870 would still effect the curve but slightly, because that point did not enter into subsequent practice.

The writer does not take exception to Mr. Hamilton's statement that Mr. Truax knew what he had and knew how he got it. Exception is taken only to the assumption that he had any particularly high specific speed. The original statement by Messrs. Hamilton and Safford in their paper on "The American Mixed-Flow Turbine and Its Setting",* credited Truax with a specific speed of 125. The writer's tests showed less than 50 and Mr. Hamilton's tests only 50 or 60 at the point of best efficiency, with this point so low as to result almost inevitably in the wheel being discarded. The Barber wheel could be set so that, with the proper draft-tube, it might show specific speeds of more than 100. It is doubtful whether either the Truax or Austin could ever do so. Their principal lack is in their adherence to the helical blade form as contrasted to the curved form that is so essential to producing the suction principle on which high specific speed depends. This deficiency is exactly comparable in an aeroplane to the sacrifice of, roughly, one-half its lifting power which would result from using a flat wing in place of a curved or ærofoil section. The quoted statement of aviation engineers to the effect that, with sufficient power, one could fly a plank, has no particular counterpart in runner design, however, and nothing that can be applied to the Truax runner as built can bring it into the class which Mr. Hamilton wishes to place it.

The writer is rather surprised at Mr. Hamilton's statement that "until the latter part of the Nineteenth Century the efficiency of a water wheel was of comparatively little importance". Francis' efficiency of 80% in 1847, and Swain's, of 80% in 1869, according to Mr. Hamilton's diagram, Fig. 65, as well as Risdon's, who certainly was not asleep prior to 1873, offer interesting comments on this point.

Mr. Hamilton criticizes the writer for using a specific speed of 150 for his original high-speed runner, stating that it should be 129, rather than 150. Referring to the original paper entitled "A New Type of Hydraulic-Turbine Runner,"† it will be found that two runners are given, one of which had a specific speed of 154 and the other of 125, as tested at Holyoke several years previously. The writer has never used anything but the point of best effi-

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 1237.

† *Transactions*, Am. Soc. C. Mech. Engrs., Vol. 41 (1919), p. 829.

ciency in comparing runners. If Mr. Hamilton wishes to make tests on the writer's high-speed runner as he has done on the Truax, it is a question of comparing a speed in excess of 200 with one below 100, these being the relative maximum points obtainable. The fact of "W. W. T.", according to Mr. Hamilton, stating that if a diffusing draft-tube should be added with the Truax wheels there would be a gain, does not justify Mr. Hamilton in so adding a draft-tube and crediting it back to Truax as of 1861. The writer has found no instance of any specific speed ever having been obtained in excess of 60 at the point of best efficiency, or 90 at the maximum possible obtainable point, as far back as Mr. Hamilton outlines. If he wishes to include a point as high as 90, as shown in Fig. 66, the basic curve should likewise be raised to cover the maximum obtainable specific speed, disregarding efficiency entirely for the various runners which made up the basic curve.

In Fig. 67, Mr. Barrows contributes a very interesting comparison of wheels. He correctly analyzes Curve *A*, in that it is based on an uncomparable method of measurement. As a matter of fact Curve *A* could be readily moved 30 or 40% to the right, simply by taking the runners on which it was based and cutting back their inlet. Similarly, the high-speed runner curve could be moved 100% to the right by taking a mean radius in place of the outside diameter. As a matter of fact Curves *C* and *B* are truly comparative. The writer does not quite agree with Mr. Barrows that the difference in the position of the curves, which means difference in power coefficient, is attributable to a difference in efficiency. The two turbines may be forms of orifices, but they may differ widely in their coefficients. Centrifugal force opposes the flow in the one and does not oppose it in the other. This gives one of the most positive and inherent obstacles to obtaining a large power coefficient with a Francis type with its more or less radial inflow.

This consideration of power coefficient brings up a very interesting problem —one which the writer has frequently presented to students on this subject. In its simplest elements it may be stated thus: What is the maximum theoretical power that can be developed from an orifice 1 in. in diameter under 1-ft. head?

Mr. Jessop's paper should serve to owners of power plants as an excellent example of what can be accomplished by proper operation. Probably one of the greatest wastes to-day is due to a lack of appreciation as to what it means to regulate units over a wide range of gate-opening. The curves shown, particularly those of Figs. 36 to 41, illustrate this operating feature splendidly, although the writer would point out that it is not fair to compare a Francis turbine of 75 specific speed with a high-speed turbine of 175 specific speed, as they would not be used for the same plant. The former is adapted to heads above 50 ft. and the latter to heads below 20 ft. If a Francis runner of 110 specific speed is compared with a high-speed runner of 150 specific speed, the contrast would be more appropriate and the difference less exaggerated.

The writer is rather surprised at Mr. Jessop's conclusion that identical units should be operated at varying loads for best total economy. Most cer-

tainly this is not the case except when the load is such that the gates of one of them may be closed entirely. The general law seems to be that with any type of identical units having efficiency curves that are convex upward, all units carrying load should be operated at the same load. Unfortunately, Mr. Jessop's system of operation is only too frequently encountered in practice, due to the fact that it is not particularly easy to measure the relatively small gain for moderate shifting of load with the usual type of board instruments.

Mr. Jessop's comparison of European and Holyoke tests is excellent and has long been needed, not so much for the information of American engineers as for answering a more or less prevalent belief among European engineers to the effect that American test methods are not correct. The greatest progress can be obtained only by co-operation and anything that will make for mutual confidence between the two groups of engineers cannot help but be advantageous. Hence, the value of Mr. Jessop's Fig. 44.

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INSTITUTED 1852

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Paper No. 1583

THE ENGINEER AS A RAILROAD EXECUTIVE*

By Julius Kruttschnitt,† Esq.

To judge correctly of the qualifications of a man for the position of "Railroad Executive", a comprehensive survey of the duties devolving on the chief executive officer of a railroad under present-day conditions and of the difficulties which he has to overcome, must be made. Not only must the problems that require solution be considered but the limitations imposed by law on the revenues of railroads and the extent of control of their expenses that the executive is permitted to exercise. It seems appropriate, therefore, to introduce a quite full outline of the problems that confront the responsible managing officer, to show the extent to which so far he has solved them, and to what extent they remain unsolved, and then to show how the engineer has risen to new responsibilities, and what recognition stockholders and directors of railroads have accorded to his fitness to administer their properties.

Fundamentally, a railroad is a huge manufacturing plant designed to convert the energy locked up in fuel into work for transporting persons and property on specially designed roadways. The energy conversion is by means of individually operated locomotive units, and the success of the entire plant, measured by the spread between production costs of output, measured in passenger-miles and ton-miles, and in selling prices, depends on the efficient working of each individual unit.

To attain the maximum or even moderate success in creating a margin between the cost and sales prices of output, the necessary executive and administrative work can best be done by men specially trained in those branches of knowledge that contribute most to the efficient operation of railroads, on the degree of which depends success or failure; and the most important duty of the chief executive is to select his official staff so as to attain the best results.

* Address before the New York Section of the American Society of Civil Engineers, in Joint Meeting with the New York Sections of the American Institute of Electrical Engineers, the American Society of Mechanical Engineers, the American Institute of Mining and Metallurgical Engineers, and the New York Electrical Society, March 18, 1925.

† Chairman, Executive Committee, Southern Pacific Co., New York, N. Y. Mr. Kruttschnitt died on June 15, 1925.

To pass intelligently on the qualifications of his staff and their recommendations that may reach him—in other words, to avoid being a mere "rubber stamp" to approve whatever is laid before him—requires a fair knowledge of the functions of each department head by the chief executive.

At the outset, the speaker will explain the causes that have brought about the epochal changes in the conditions of the railroads in the first quarter of the Twentieth Century, conditions that have obstructed successful operation to an extent never theretofore encountered, and then show how American talent has adapted the railroads to these new conditions and thus far has averted bankruptcy and ruin.

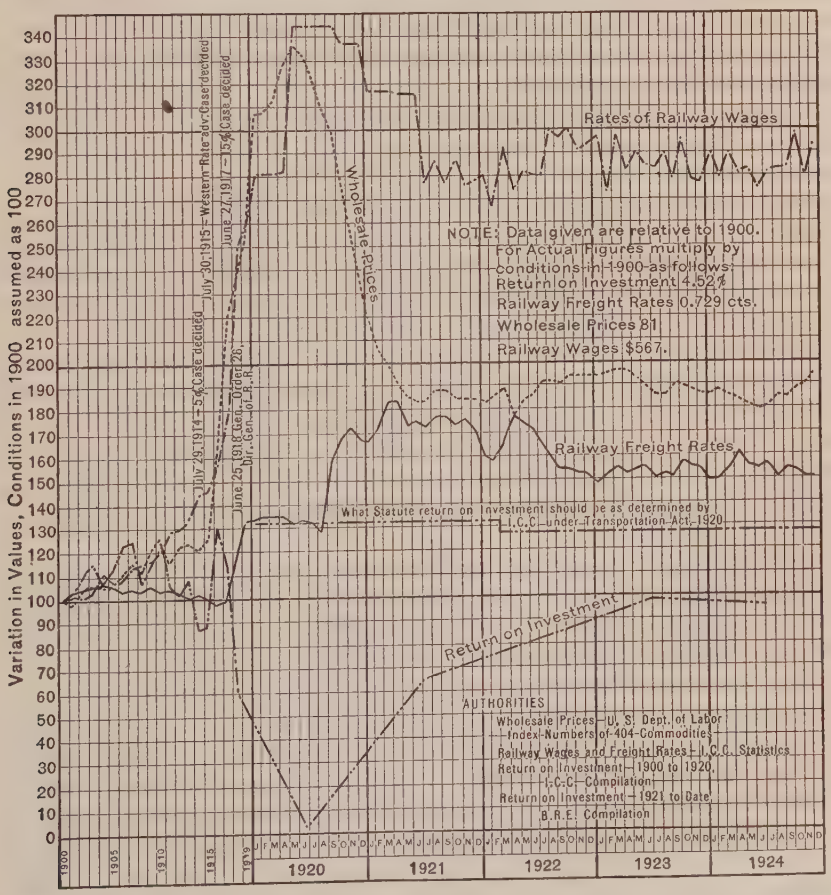

FIG. 1.—RELATION OF PRICES, RAILWAY WAGES, FREIGHT RATES, AND RETURNS ON INVESTMENT FOR TWENTY-FIVE YEARS COMPARED WITH DATA FOR 1900.

From Fig. 1 it will be observed that the rise of prices of all commodities throughout the country, started by the outbreak of the World War in the summer of 1914, culminated in the United States in May, 1920, when the index number of wholesale prices showed an increase of 236% above the

corresponding number in 1900. Starting from the base index number—taken as 100 in 1900—there was a slight fall in 1901, but in 1902 prices were 4% above those of 1900 and, with some reversals, the rise was continuous to the outbreak of the World War, when they were 20% higher than in 1900. At the date of America's entry into the war, the index number of wholesale prices was 120% above that of 1900. The fall from the peak of 236% over 1900 in 1920 was rapid to 83% above 1900 in the middle of 1921, since which time there have been fluctuations leading to the figure of 94% above 1900 in December, 1924.

During the 17 years from 1900 to 1917, the average railway freight rate in the United States remained nearly stationary. True, there were fluctuations that affected the average but never sufficiently to raise it over 5 to 7% above 1900, so that when the United States entered the World War the average freight rate was almost exactly the same as it was 17 years theretofore, but in 17 years average prices had risen 120% and wages of railway employees had risen 77 per cent. At the beginning of the century, 25 years ago, the average operating ratio for all the roads of the United States was 65 per cent. Under Government operation it rose in 1920 to 94.3% and for the twelve months of 1924, after four years of private operation, it fell to 76.14 per cent. These data are quoted to show how the spread between revenues and expenses has shrunk, and how narrow a margin exists at the present time out of which to pay interest on obligations and dividends on stocks.

It will be observed that the percentage return on investment that was 4.52 in 1900 substantially vanished in 1920 on the return of the railroads to their owners by the Government, and never since that time has equalled the 1900 return. For 1924 it was only 96% thereof, or 4.35%; since then it has fallen still farther below the statutory return prescribed by Congress as reasonable and allowable, 6% from March, 1920, to March, 1922, and 5.75% from March, 1922, to date.

The figures quoted show the wide difference in the kind of managing talent required on railroads now and 25 years ago. At the end of the Nineteenth Century, 35 cents net out of every dollar could be turned into the Treasury, while at the present time only 24 cents, or 32% less, can be turned in. It requires the closest kind of management and the services of very much more highly trained executives to meet all money requirements other than mere operating expenses with only 24 cents out of every dollar. To obtain the most work out of fuel, the largest material item of operating expenses, taxes the efforts of substantially every department on a railroad, and to obtain the best results requires the highest talent in a great many different branches of engineering. The course of Civil Engineering in leading universities embraces courses in the allied sciences of Mechanical and Electrical Engineering, and the important principles of physics, chemistry, geology, and economics, so that the executive who has enjoyed the training afforded in a civil engineering course is peculiarly well fitted for co-ordinating and guiding the numerous operating activities on railroads and for securing the maximum output of salable products at minimum costs. An analysis of operations that produce transportation units will show that the engineer who has been trained

to secure desired ends in the most practical, efficient, and least expensive way can best contribute to the success of a railroad enterprise, and will indicate the directions in which he should prepare himself in college for executive duties, and will correspondingly indicate to the owners of railroads, other things being equal, the qualifications to seek in managers.

To repeat—viewing the railroad as a manufacturing plant designed for the sole purpose of creating the maximum of transportation units out of the minimum of fuel units, the greatest efficiency will be attained by that railroad the executives of which co-ordinate best all departments and concentrate their activities on increasing output, from the sale of which the railroad must secure its revenues.

Enumerating some of the activities of a railroad will show how numerous are the demands made on a chief executive, and at the same time will indicate to a certain extent how the problems that confront him have been partly solved or are still demanding study for their solution.

I.—Executive Department

The wise executive will spare no expense in establishing an Accounting Department to keep track of earnings and expenses; to record the history of operations; to act as general inspectors; to expose poor results as prerequisites to their correction; to work up promptly cost data of every important item of operating expense and as promptly to put them in the hands of all concerned. While the experience of the men who make the decisions will usually keep them from going far wrong, yet in matters involving millions of dollars in both capital and operating expenses, a railroad officer should not be satisfied unless he has definite figures on which to base decisions. The record traffic handled with record efficiency in 1917 was the response of the railroads to the monthly data sheets first issued by the Railroads' War Board; these are now issued in even more detail by the Interstate Commerce Commission and the Bureau of Railway Economics. Fuel accounts kept with individual engineers and firemen and locomotives furnish railroads with data from which to decide definitely what type of locomotive is the most economical for their use, and whether the expense of applying and maintaining some of the many fuel-saving devices is warranted.

"The location of a railroad is giving it its Constitution. It may be sick almost unto death with accidents of construction and management, but with a good Constitution it will ultimately recover." The board of directors, advised by the chief executive, must pronounce on the location of the railroad between its termini; it allots the amount of money to be spent on construction; fixes the grade and curve systems, because the ideally perfect railroad, between given points, should be absolutely straight and of uniform grade, that is, devoid of undulations in alignment or profile. The expense of operation will increase in proportion to the grade and curve resistance involved in departures from the ideal line, and the extent of departure must be authorized by the board, on which devolves the duty of providing the money for construction. Whenever a ton weight is lifted 1 ft., an amount of work equivalent to moving it 333 ft. over a straight, level track is spent; and every time a ton weight of

train is forced through 100 ft. of 1° curve, the work done in deflecting it from a straight to a curved path is equivalent to moving the same ton over 16⅔ ft. of straight, level track. The equated length of the old line of the Central Pacific between Ogden and Lucin, Utah, compared with the new Lucin Cut-Off line is as 280 to 129 miles; 1 lb. of fuel on the new line moves as much tonnage as 2.2 lb. on the old one.

The Executive Department must determine the justifiable outlay to reduce grades and curvatures on existing lines, as obviously the expenditure necessary to obtain a straight line between two points without any undulations in grade might be commercially prohibitive. It is, therefore, of the greatest importance for the commercial success of a railroad to balance the money cost of approaching as nearly as possible ideal conditions with the expected benefits.

Motive power can often be economized by expensive physical improvements, such as installing automatic block signals, re-designing terminals, modernizing roundhouses so as to utilize the residual heat from incoming locomotives to raise steam on outgoing ones, and increasing the number and lengths of sidings to save delays on the line. On the chief executive rests the responsibility of balancing the interest, maintenance, and depreciation costs on large capital expenditures against possible savings, and of providing the necessary capital if the investment is justified.

II.—Traffic Department

The Traffic Department is the sales department of the railroad, and its contract specifications as to loads in cars and character of service exert a profound influence on net earnings. It requires 41% less power to haul 50 tons of lading in one car than in two cars of the same class and weight, and 53% more power to haul a given load at 40 than at 20 miles per hour.

III.—Maintenance of Way Department

The resistance to traction of locomotives and cars varies between wide limits with the condition of the track. The train resistance figure commonly used of 6 lb. per ton (2 000 lb.) at low speed assumes track in good condition. It may very easily be two or three times that amount on track laid with light rail, poorly maintained, in bad surface, and with low joints, etc. The heavier the rail and the better the ballast, the easier it is to maintain good alignment and surface, the less will be the deflection under loads, and the less will be the work required to force the wheels out of the depression caused by weak and yielding joints.

IV.—Maintenance of Equipment Department

About one-fifth of all locomotive coal (in 1921 this amounted to 25 000 000 tons) was consumed when the locomotive was not doing useful work: in firing up, in waiting for trains, in standing on sidings and passing tracks, in delays at terminals, drawing fire, and waiting at ash-pits. This consumption is not productive, nor is it all waste. By re-designing terminals and increasing the lengths and numbers of sidings the waste can be minimized. Ingenious graphs on which obstructions to traffic are likened to obstructions to the flow of water

in incrusted pipes have proved of great assistance to indicate how to increase the capacity of the line and defer double-tracking.

In designing locomotives, the features exercising the greatest influence on fuel efficiency are: Careful and expert proportioning of engines, boilers, fireboxes, grates, flues, draft appliances, so as to produce by their combination the most perfect device to extract from fuels their full heat contents. Barely 8% of the energy in fuels is now extracted and converted into work.

Although it is customary to speak of the waste of fuel in the average locomotive, the extremely unfavorable conditions under which it is required to work must not be forgotten. Space limitations have thus far prevented the successful use of condensing engines, and this has deprived the locomotive of the advantage of 12 to 13 lb. additional effective cylinder pressure. It is driven through all kinds of weather at high speed, so that radiation losses are excessive. To meet the demands made on its boiler for steam, explosive draft must be used, which expels much unburned fuel from the firebox; the resultant effect of all this is evidenced in low thermal efficiency.

With the increase in the power of locomotives and the use of higher steam pressures, mathematical talent instead of "rule of thumb" has been called on to compute stresses and to proportion parts. Questions as to the adoption of alleged improvements are difficult to decide, as they involve first cost, interest, maintenance and depreciation, their possible effect on engine failures, and the idle time of locomotives while undergoing repairs. Sometimes economy of operation is sacrificed to facilitate adjustment and repairs, as when outside valve gears replaced the original Stephenson link motion.

The practice of buying locomotives regardless of their adaptability to special service, as was done during Federal control, simply because they could be built in quantity at some reduction in cost, is recognized by competent judges as the poorest kind of economy.

The brake thermal efficiency—or the percentage of energy in the fuel transformed into work—for various engines is as follows:

Reciprocating, condensing, stationary steam engines (boilers excluded)	19.0%
Steam turbines	19.2%
Average of eight large power plants (boilers included)	13.3%
Non-condensing steam locomotives at maximum capacity, equipped with proven fuel-economizing devices.	7.3 to 8.1%

Some of the problems that have been closely studied and have been solved to a greater or less extent are:

Brick Arches.—Brick arches built into the firebox mix the entering air thoroughly with the combustible gases rising from the fire and save on some types of locomotives about 12 per cent.

Superheating.—The advantages resulting from the use of superheated steam are due mainly to (*a*) prevention of cylinder condensation; and (*b*) larger volume of steam per pound.

Since the cylinder takes the same volume of steam per stroke for the same cut-off, it is obvious that substantially less pounds of steam will be used for the same work done. Superheating may save 20% of the fuel.

Feed-Water Heaters.—Feed-water heaters utilize waste heat to raise the temperature of the feed water, thereby relieving the fuel in the firebox from heating the cold feed water from an average of 60° Fahr. up to 220° Fahr. and saving about 10 to 14% of the fuel.

The combined average effect of these three devices in saving fuel on passenger and freight locomotives of various types and speeds in actual service is:

Arch	9 %
Superheater	20 %
Feed-water heater	10 %
Combined	34.5%

These figures vary widely for different locomotives, speeds, and classes of service.

The savings claimed for the various fuel-saving devices are gross figures. The cost of application is quite large, and in some cases not justified by the age or obsolescence of the locomotive; in others, it is beyond the financial ability of railroads to apply them; hence care must be taken to balance the cost of interest, maintenance, and depreciation against the value of the fuel saved. The net saving so determined—not the gross—will show whether or not the money to provide the appliances can be providently expended. Experience shows that thus tested the gross saving of superheaters is reduced from 20 to about 17.4%; of feed-water heaters from 10 to about 6 per cent.

Boosters.—Under normal conditions a locomotive can haul as much, or even more, load than it can start. The booster consists of a pair of independent engines coupled to the trailer wheels of locomotives, thus utilizing weight not otherwise used for traction, in starting and quickly accelerating trains and in reducing violent shocks in starting, which are so hard on draft gear and the temper of passengers. As its power cuts out automatically at a speed of 10 miles per hour, it uses steam at low speeds when the boiler can over-supply the main cylinders. It is, therefore, an indirect but valuable fuel saver through increased locomotive capacity. The booster can add about 10% to the starting drawbar pull of heavy freight and as much as 25 to 35% to the starting drawbar pull of lighter passenger locomotives.

Feed-Water Purifiers.—After selecting the softest natural feed waters, such others as may have to be used should be chemically treated to reduce objectionable impurities to a minimum. A conservative estimate of the effect of scale $\frac{1}{16}$ in. thick on flues is to increase fuel consumption 10 per cent. The Hungarian State Railroads on 3 000 locomotives use a mechanical feed-water purifier mounted on top of the boiler. The water on entering the purifier is subjected to full boiler pressure and temperature, and deposits its scale in and on this adjunct to the boiler instead of in the boiler and on the flues. With reduction of scale there is a direct saving in fuel consumption, while a reduction in frequency of boiler washings saves considerable time and fuel.

Without the purifier, locomotives were washed on an average every 5 days; with the purifier, the required washing was once every 54 days.

Lubrication.—All journal bearings must be carefully lubricated to reduce friction; so must the center and side-bearings of cars, the friction of which prevents the adjustment of trucks on entering and leaving curves and thereby, in the aggregate, causes a heavy increase in the resistance to traction that the locomotive must overcome.

Wear on all wheels of locomotives, tenders, and cars must be watched. If the treads are worn hollow or flanges worn vertical, the areas in contact with the rails when cars are oscillating or rounding curves will be increased and the friction correspondingly increased. A heavy freight locomotive, tender, caboose, and sixty freight cars are carried on more than 500 wheels; a slight increase in the friction on a few of them only would, in the aggregate, create a heavy drag on the locomotive.

Flange Lubrication.—Flange lubrication reduces friction between rail and flanges and, therefore, has a substantial effect in reducing fuel consumption. On two roads on which flange lubrication was used, locomotive tires showed average mileages between turnings of 33 000 and 44 000, respectively, compared with 16 000 and 23 000 without such lubrication. Similarly the wear on rails in curves showed a reduction of as high as 87 per cent. On one road of excessive curvature and grade the life of locomotive tires was increased 375% with flange lubrication.

Fuel Specifications.—To insure full heat value the specifications for fuels purchased should cover their value in British thermal units.

Maximum Loads in Vehicles.—Freight cars should be fully loaded as far as may be. The resistance of a train of empty freight cars may run up to 8 lb. per ton; that of cars with average loads to 6 lb. per ton, while that of heavily loaded cars, of total weight of 60 to 70 tons, may be as low as 4 lb. per ton. The load, in other words, should be carried in as few cars as possible.

Reduction of Dead Weight.—As the expenditure of fuel in hauling a ton is the same whether or not it is paying freight, it is evident that the smaller the percentage of non-paying or dead weight to total weight moved, the smaller will be the cost of hauling the paying freight.

The United States Railroad Administration's 50-ton, single-sheathed box car weighs 47 200 lb.; the American Railway Association's 50-ton box-car design weighs 42 400 lb. Thus, the excess weight of the United States Railroad Administration's car is 4 800 lb., or 11.3 per cent.

Based on 1923 data, the saving in fuel by eliminating this excess weight, is $9.77 per car per year, or $4.10 per ton, which, capitalized at 5%, shows that an added initial expense of $82 in building a car is justified to reduce its weight 1 ton.

As far as practicable all weight in a locomotive above what is needed for friction to develop traction should be eliminated.

Boiler design to suit the class of service and the character of the fuel requires considerable flexibility in this principle.

Stimulation of Interest and Co-Operation in Saving Fuel.—Assuming that locomotives are properly maintained, in order to obtain the best results from burning fuel it is necessary to inspire the interest and secure the co-operation of the men on the foot-boards of 65 000 traveling power plants. How these aids are attained will appear from a brief of the practice on a large system covering many States and operated under the most varied topographical and climatic conditions:

1.—In addition to the inspection at each terminal of engines on their arrival and again before departure to insure the correction of all defects before the engine begins its trip, very thorough monthly inspections are made of all parts of the engines, boiler and appliances, draft arrangements, etc.

2.—Classified statistics supply a complete history of operation. To be of most use they are issued immediately after the occurrences they record:

(*a*) A fuel bureau receives the report of all train and locomotive movements within two days. Ton-miles, fuel consumed per 1 000 ton-miles, tons per train for each individual locomotive and each individual engineer and fireman are then computed, and the results put into the superintendent's hands three to six days after the movement, so that shortcomings can be detected and quickly corrected.

(*b*) Monthly statements of individual performance of engineers, firemen, and locomotives are displayed on bulletin boards.

(*c*) A sheet recapitulating results by divisions is issued monthly.

3.—On each division a fuel committee composed of four engineers, four firemen, one conductor and one brakeman, serves for six months and holds meetings bi-monthly, the superintendent acting as chairman. The meetings are well advertised and are attended by the superintendent's staff and employees from all related departments, who offer suggestions freely. Experts on operating methods and appliances deliver addresses. Motion pictures illustrating actual road conditions are exhibited. Minutes of meetings are widely distributed and published in an employees' magazine.

4.—To create emulation between divisions, a red silk banner is awarded quarterly to divisions showing the greatest improvement in the previous year.

5.—Gold-plated cap badges are awarded quarterly on each division to the engineer and fireman showing the best fuel record in through and local freight and passenger service.

6.—Each superintendent publishes the names of five engineers and five firemen having the best fuel records for the month in through and local freight, passenger, and yard service on a roll of honor, which is posted in roundhouses and printed in an employees' magazine. A letter of commendation is sent to each man whose name appears on the roll.

7.—The company sends yearly, at its expense, thirty-eight men, consisting of an engineer and a fireman from each division with the best fuel records, as its representatives to the Annual Convention of the International Railway Fuel Association.

As a result of the policy described and of many fuel-saving devices, the railroad moved 50% more gross ton-miles per pound of fuel in 1924 than in 1913.

V.—Transportation Department

About 20% of all locomotive fuel is consumed when the locomotive is not doing useful work.

(*a*) Avoidable losses include fuel used in firing up, unconsumed fuel drawn from fireboxes, from blowing off and cooling down at ends of runs, and losses incurred while waiting for water, sand, and other supplies on roundhouse tracks. These can be very greatly lessened by reducing the frequency of these occasions, that is, by lengthening locomotive runs. It is not uncommon now to run freight locomotives over two or three freight divisions, and passenger locomotives over four before cooling them. Tables 1 and 2 give some examples of modern practice.

TABLE 1.—Passenger Runs of 575 Miles and More.

Road.	Limits of run.	Mileage.
Southern Pacific	Los Angeles, Calif., to El Paso, Tex.	815
Missouri, Kansas and Texas	Parsons, Kans., to San Antonio, Tex.	678
Union Pacific	Kansas City, Kans., to Denver, Colo.	640
Southern Pacific	San Antonio to El Paso, Tex.	620
Atchison, Topeka and Santa Fe.	Winslow, Ariz., to Los Angeles, Calif.	602
Union Pacific	Denver, Colo., to Ogden, Utah	577
Southern Pacific	New Orleans, La., to San Antonio, Tex	575

Four locomotives only are used by the Southern Pacific to haul passenger trains from New Orleans to San Francisco, 2 490 miles.

TABLE 2.—Freight Runs of 350 Miles and More.

Road.	Limits of run.	Mileage.
Southern Pacific	El Paso to Del Rio, Tex.	453
Southern Pacific	Sparks to Carlin, Nev.	387
Southern Pacific	Houston, Tex., to Algiers, La.	362
Southern Pacific	San Antonio to Fort Worth, Tex.	352

(*b*) Good despatching will reduce the number of needless stops, and careful planning will avoid fuel waste from locomotives standing in sidings because of poor meeting points, and from idle time at terminals waiting for trains.

To stop a freight train of 66 cars weighing with its locomotive and tender 6 204 000 lb. and moving at a speed of 40 miles per hour, and to regain that speed requires a consumption of 380 lb. of coal.

(*c*) The economy of moving tonnage in as few cars and trains as possible and of reducing empty mileage is obvious.

Much idle time and fuel can be saved by carefully designed engine terminal facilities, such as hot-water boiler-washing plants, well-located supply tracks, etc., and above all ample shops and modern tools to make repairs quickly. Additional shops and tools provided at El Paso on the Southern Pacific Lines have reduced the average time in shops of locomotives 25%, the effect of which is to add 33% to the number of locomotives assigned to that Division. On a division operated with Consolidation locomotives where water is exceptionally good, 16 000 lb. of fuel is saved per locomotive per annum by the installation of a hot-water boiler-washing plant. Such improved facilities, while requiring large capital outlay, will increase the productive time of locomotives.

The burning of fuel in large terminals and shops should be centralized as far as practicable in order to eliminate machinery starting and stopping losses. Through free use of individual electric motors on large tools and groups of small tools the waste of running long lines of shafting and of operating tools needlessly when only one or two on the line are used, can be eliminated and substantial economies can be effected in the 16 000 000 tons of fuel consumed on railroads in shops, stations, etc. In a large railroad shop 20% of the power was saved by eliminating steam-pipe losses and 13% more by abandoning long lines of shafting and many belts through substituting electric for steam power. The changes represented a saving of $120 000 per year.

In locations where electrical energy can be purchased at lower cost than it can be produced by the railroad company, opportunities for substantial fuel savings are presented.

It is encouraging that the improvement in the work obtained from a pound of fuel is constant, but it is disappointing to reflect that European engineers are responsible for most of the improvements now in use on American locomotives. This is the case with the improved outside valve gear, superheaters and feed-water heaters, some details of which, however, have been designed by American engineers, but nevertheless it is mortifying to admit that the principles underlying these devices were not first used in the United States. The reason of course is that heretofore fuel has been so cheap and easily obtained that there was little inducement to pay much attention to its conservation; conditions have radically changed in the last few years, however.

The two principles affecting most profoundly the efficiency of stationary steam engines are compounding and condensing. Compounding alone may increase the efficiency of an engine 13%, and condensing alone may increase it as much as 30%, that is, condensing alone would increase the efficiency of fuel consumption of locomotives nearly as much as the brick arch, superheater, and feed-water heater combined, which as already shown amounts to 34.5 per cent. Compounding, which was used in the United States at one time much more than at present, will certainly come into use again; so that the combined effect of all the devices described, with compounding and condensing superadded, would be:

Brick arch when used has reduced fuel consumption to..	91.0%
The superheater when added has reduced it to.....	72.8%

The feed-water heater when added, to	65.5%
Using condensing engine may reduce it to	45.9%
Compounding may reduce it to	39.9%

European railroad engineers are fully alive to the advantages of using a condensing engine on the locomotive, as evidenced by descriptions that have appeared recently in the technical press.

Turbine Locomotives.—The Swiss Federal Railways* is testing a 10-wheel type locomotive, with a superheater, driven by a condensing steam turbine, located on locomotive frames in front of the smokebox. It is claimed that tests thus far show reduction in fuel consumption of 25% compared with compound locomotives of the usual construction, and very smooth running at high speeds on account of the reduction of heavy reciprocating parts.

The Swedish State Railway has also built a turbine locomotive.† It is claimed that this locomotive, known as a Ljungstrom locomotive, shows a fuel consumption of about 50% of that of a Pacific type locomotive, which it replaced.

The London and Northwestern Railroad has built an experimental locomotive driven by a condensing turbine which operates a generator, and the power is applied to the driving wheels through four electric motors of 275 h.p. each.

Electric Locomotives.—Where electric current can be generated from water power, trains can be moved very cheaply, but where the current is generated in a steam plant the advantages are much diminished. In eight large steam-electric plants studied in 1919 by the United States Geological Survey,‡ 12.2% of the energy in coal is converted into electric energy; transmitting this electric energy to the motors of an electric locomotive reduces the available power to 9.9% and the output is further reduced in the motors to 8.4% of the total original energy in the coal. The corresponding efficiency of a modern steam locomotive with arches, superheater, and feed-water heater is 8.1% under favorable conditions. While under existing conditions the advantage in favor of the electric locomotive may be more than that indicated, the spread is hardly enough to tempt capital to assume the expenses of interest, depreciation, tax, and maintenance that would have to be incurred in changing from steam power to electricity generated in a steam plant. The soundness of this conclusion was demonstrated by studies made of the Sierra Nevada and other heavy grades on the Southern Pacific, where trains are lifted as high as 6 854 ft. in 86 miles. Assuming the current to be generated in a steam station, the interest, taxes, and depreciation on additional net plant required were found to be more than three times as great as the estimated savings to be obtained from electric operations.

Other considerations, such as smoke prevention and the increased capacity on heavy grades, may influence the problem, but the speaker recalls no instance of a change being made solely to save fuel where a steam-generated current was used. The steam locomotive is by no means as obsolescent as its

* *Railway Review*, April 16, 1921.

† *Loc. cit.*, August 12, 1922.

‡ *Professional Paper No. 123*, U. S. Geological Survey.

critics contend. For example, a fuel consumption of 100 lb. per 1 000 gross ton-miles by electric locomotives is given* as applicable "to conditions universally obtaining on regular profiles." Data published by the Bureau of Railway Economics in September, 1921, show that the fuel consumption on 24 000 selected miles of steam railroad averaged but slightly more than 100 lb. per 1 000 ton-miles. Embraced in this mileage were the New York Central, Illinois Central, Chesapeake and Ohio and St. Louis Southwestern Railroads. On 3 142 miles, or 60% of Southern Pacific main-line mileage, the fuel consumption in the same month was approximately 100 lb., while on the Salt Lake Division, 543 miles, the fuel record in October, 1921, averaged only 91 lb. per 1 000 gross ton-miles, for all locomotives. During the entire month of October, 1924, sixty-two Southern Pacific freight engineers held records of having moved 1 000 gross ton-miles with a fuel consumption running between 48 and 59 lb. per 1 000 ton-miles, under "conditions universally obtaining on regular profiles," or but little more than one-half of what was considered a reasonable fuel consumption for electrically moved trains.

These are every-day performances; in the case of the Swedish experimental locomotive, and in records of electrically operated trains, however, the data relate either to experimental runs or to performances under exceptional conditions.

The performance of the modern steam locomotive in every-day service without favorable stage setting is shown in the records of some American roads. On one of them, 26 superheated Consolidation locomotives showed in daily service on a run with long grades of 21 ft. per mile consumptions of coal ranging from 55.8 to 71.7 lb. per 1 000 gross ton-miles, with an average of 63.5 lb. Still more creditable to the steam locomotives and the men who drive them are the records of individual runners. Engineer A in two trips with trains of 2 809 tons consumed 48.7 lb. per 1 000 gross ton-miles; Engineer P in eight trips with trains of 2 863 tons consumed 58.2 lb.; Engineer R in fourteen trips with trains of 2 411 tons consumed 58.3 lb.

Internal Combustion Engines.—These engines vary widely in thermal efficiency and weights per unit, as follows:

	Weight per brake horse-power, in pounds.	Percentage of brake thermal efficiency.
Automobiles	15	19.0
Diesel four-cycle	370	35.0
Diesel two-cycle	250	30.0
Airplanes	1.04	...

The use of the automobile engine, except perhaps for short runs under exceptional conditions with light vehicles, is prohibitive on railroads, because although it is two and one-third times as efficient as the steam locomotive the cost of its fuel is ten times as great.

The Diesel engine, however, shows the highest thermal efficiency of any known engine. As compared with the most modern locomotive, which transforms only about 8.1% of the heat energy of fuel into work, the Diesel engine

* *Journal*, Am. Inst. Elec. Engrs., March, 1920.

transforms 35%, or more than four times as much. Its fuel (crude oil, commonly called Diesel oil) is not converted into gas as in gasoline engines, but is burned in the cylinder in the form of spray, and the expansive force of the products of combustion provides the motive power. Like the gas engine, however, it cannot start itself, but must use some form of variable speed transmission to start trains; the best way thus far devised consists of electric transmission, but the combined engine and generator plants have proved so heavy as to bar their use for heavy traction. Because of its high thermal efficiency, the Diesel engine offers opportunity to make great savings, and its future is very hopeful. A locomotive of this type now being built for the Southern Pacific Lines by the Baldwin Locomotive Works seems to have overcome these handicaps and will soon be ready for trial.

The designs of an Italian Diesel locomotive show the following:

Weight in operation, approximately	117 tons
Tractive effort at 18½ miles per hour	19 840 lb.

The locomotive is started by highly compressed air stored in tanks and fed to cylinders at the front and rear, and when sufficient speed has been attained to start the Diesel engines, the compressed air is cut off. The air tanks are recharged by a connection with the main engines. The estimated fuel saving over a steam locomotive is 66⅔ per cent.

This design shows progress in adapting the Diesel engines to locomotives of serviceable commercial sizes, particularly in solving the problem of weight, as the weight of a steam locomotive and tender to develop about 20 000 lb. of tractive effort is 150 tons.

The hope of the railroad executive for conserving fuel lies in:

1.—Substituting hydro-electric power for steam, as the best steam locomotives can now fully equal the efficiency of electric locomotives using current generated in steam power stations.

2.—Substituting compound condensing engines for the simple engines now used. From progress made in Europe, the solution of this problem appears encouraging.

3.—The discovery of a cheap, high-gravity fuel that can be used in some engine of the type used in automobiles and airplanes.

4.—Reduction of the weights of Diesel engines sufficiently to permit their use as locomotive engines and the development of a satisfactory variable speed transmission which will weigh much less than an electric generator and motor.

A Diesel electric locomotive built in Germany for the Russian Government shows an over-all efficiency between 21 and 27.4 per cent. A steam locomotive tested at the same laboratory two days later averaged a total efficiency from 7.6 to 8.67 per cent. The fuel consumption of the Diesel electric locomotive was about one-third that of the steam locomotive.

In a special report published in 1923, the National Industrial Conference Board appointed a committee to determine among other things:

1.—Do the industries in the United States need more or fewer engineers than the number now being graduated from engineering schools and colleges?

2.—What kind of men do the industries require from the engineering schools and colleges, and what should be the nature of their education?

The report states:

"The expansion of industry has been accompanied by vastly increased complexity of industrial operations, and the grouping of greater numbers of workers in corporate units has placed upon employers an increased social responsibility toward their employees and toward the life and affairs of the community. All of these changes have created problems of human, social and political relationships in which industry and society as a whole have become increasingly dependent upon trained technical and administrative leaders."

Solutions to these problems have been sought by railroad executives in the establishment of pensions for superannuated and incapacitated workers; in hospital departments, to furnish hospital service on the payment of nominal fees; by the establishment of group life insurance plans whereby employees, regardless of age and without physical examination, receive the benefits of life insurance at approximately one-third the cost of the current commercial rates; and by the establishment of rest and reading rooms at terminals where no opportunities exist for relaxation after working hours.

Continuing the report says: "Of the 41 600 000 persons who were gainfully employed in 1920, according to the United States Census for that year, less than 4% planned the activities and directed the energies of the whole working force." The same census figures show that since 1870 the number of administrators, supervisors, and technical experts has rapidly increased. In 1870, the number of persons engaged in administration and supervision was 170 000, or 1.25% of those gainfully occupied. In 1890, the corresponding data were 450 000, or 1.74%; in 1920, 1 510 000, or 3.6 per cent.

An examination of the personal records of the Chairmen and Presidents of the railroads of the United States shows that in 1922, of 32 Chairmen, 11, or 34%, had risen from the Engineering Department, and of 171 Presidents, 33, or 19%, had risen from the ranks of engineers. Based as they are on data taken from census reports, the conclusions of the National Industrial Conference Board show increasing demands for the services of trained men, and furnish the most reliable and valuable information obtainable on the matters under consideration.

As to schools, the report says:

"It is the business of the schools to train young men into fertile and exact thinkers, guided by common sense, who have a thorough knowledge of natural laws and of the means for utilizing natural forces for the advantage of men and the advancement of civilization. In other words, it is the business of engineering schools to produce, not finished engineers, but young men with a great capacity for becoming engineers, the goal being attained by the graduates only after years of development in the school of life."

Although a good technical expert may not develop into a good executive, nevertheless successful administrators should have certain characteristics, such as, a good grasp of the fundamentals of science; the ability to think logically and quantitatively; exactness of method and power of analysis; the habit of looking forward; sound economic theories; an unlimited capacity to

learn; pronounced firmness combined with high sense of fairness and charity needed to control men; and the ability to write and speak clearly and correctly.

F. B. Jewett, Vice-President of the American Telephone and Telegraph Company and President of the Bell Telephone Laboratories, Incorporated, expresses these views:

"The industry which I am connected with is a highly technical one which is based on physics and chemistry and mathematics; but so are many of the big industries in our country and the more I see of them the more convinced do I become that the directors of these industries in the future are going to be drawn more and more from men who have a thorough grounding in the fundamentals on which the industries are based. In the old days it was not so, but each day we go on now, it becomes more clearly apparent that because the industries are technical, because they are based on the fundamental science, they must be guided by men who have a thorough appreciation of the fundamentals because business is hardly more than mixing the dollar or cents with physics and chemistry and mathematics. Since engineering schools are the place where we expect to turn out men who are going to guide technical industries, we should turn out men who know the principles of the fundamental sciences; who know the method of combining fundamental sciences with the dollar and cents proposition and know English in a way to enable them to express clearly what they know, so that others may profit by it, because it does none of us very much good to be ever so wise if we cannot express our ideas to others.

"It seems to me that if the engineering schools of the future are going to perform their function for the industries they have got to get closer to the proposition of teaching and really inculcating the fundamentals. Let the matter of trying to make executives go, if you will. The men of executive ability will come to the fore in the general run of things, I think, provided the Lord gave them the right human traits and provided they have a thorough grounding in the fundamentals."

E. M. Herr, President of the Westinghouse Electric and Manufacturing Company, says:

"I have been very much impressed with the discussion and especially by the points brought out by the last speaker. I heartily endorse his view of the importance of the fundamentals and of the idea of patient plodding in the minds of men who are going into industry. There is no royal road, to my mind, to a place of commanding importance in industry, or in any other business, and I do not believe that it is the function of educational institutions to try to teach young men to become captains of industry and administrators, or occupy very great and important executive positions."

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"While there is general agreement on this view that one of the leading essentials in education in relation to industry is an adequate training in and knowledge of fundamentals, there is wide variation in the meaning of the term 'fundamentals.' From the point of view of the industrialist, the term 'fundamentals' means a general groundwork of useful knowledge but, more especially, a training in the power to use this knowledge in effective thought and action.

"The fundamentals of engineering training may be stated as follows:

(*a*) Mathematics;

(*b*) The important principles of physics (mechanics, heat, light, sound, electricity, magnetism), chemistry, biology and geology, the interrelations of natural phenomena and the application of these principles to practical problems;

(*c*) The principles of economics and their application to industry and commerce;
(*d*) The principles that govern the relations between people, not only as applied to managers and men, but also as applied to governments and society;
(*e*) The history of nations;
(*f*) The art of clear and correct expression in speaking, writing, and drawing.

"The relative emphasis to be placed on each of these fundamentals is a matter to be determined for each engineering course."

In emphasizing the importance of training, one must studiously avoid withering the aspirations and stifling the ambition of the large numbers of men in railroad and industrial service who have not had the benefit of overmuch school or any college preparation. These men outnumber vastly the high school and college men, and to their credit be it said that the 65% of Chairmen and 81% of Presidents remaining after taking account of those who have risen from the ranks of engineers have been drawn largely from the men who have had none of the benefits of higher education. Without mentioning them, the names of a number of the ablest and most distinguished railroad Chairmen and Presidents in the United States, who have by sheer ability and pluck risen from lowly ranks and have actually built their own careers and builded admirably well, suggest themselves.

Again quoting from the report to the National Industrial Conference Board:

"Sympathetic guidance of both the college man and the non-college man presupposes a careful evaluation of the work of all employees and assurance to them that talent and effective work will be known to and appreciated by the management. Educators report a growing feeling among undergraduates that in most large corporations a young employee often is lost in a department and for many months may not come under the observation of the leading men of the organization. A large public utility company, the two largest manufacturers of electrical apparatus, and a few other large corporations which have unusual and enlightened personnel departments, in many cases are getting the pick of the young talent, because of their attention to this problem."

"These are some of the ways in which industry can supplement the work of the educators," and profiting by their example thus can the present generation help those who are to "carry on" when it shall have passed.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1584

MULTIPLE-ARCH DAM AT GEM LAKE ON RUSH CREEK, CALIFORNIA*

By Fred O. Dolson† and Walter L. Huber,‡ Members, Am. Soc. C. E.

With Discussion by Messrs. J. Y. Jewett, L. R. Jorgensen, Walter H. Wheeler, C. A. P. Turner, Thomas H. Wiggin, C. H. Howell, E. W. Lane, E. J. Waugh, Alfred D. Flinn, Luigi Luiggi, Thaddeus Merriman, M. M. O'Shaughnessy, J. B. Lippincott, Oren Reed, B. F. Jakobsen, J. D. Galloway, I. Oesterblom, B. C. Collier, Fred A. Noetzli, F. W. Scheidenhelm, Charles W. Comstock, George W. Howson, and Fred O. Dolson and Walter L. Huber.

Synopsis

This paper is submitted for the purpose of calling the attention of engineers to the necessity of giving serious consideration to the action of frost when designing dams for use in cold climates. The writers believe deterioration from frost action will occur in any thin concrete structure subjected to water pressure and extreme cold unless the concrete can be made 100% water-proof (or water-tight).

The results of frost action, therefore, have been pointed out in this paper, together with the expensive repairs necessary to safeguard the structure, and it is hoped the discussion will develop some positive and permanent method of water-proofing concrete.

Introduction

Much has been written in recent years concerning multiple-arch dams, but the writers believe that all the story is not yet told, and it is with the

* Presented at the meeting of October 7, 1925.
† Vice-Pres. and Asst. Gen. Mgr., Southern Sierras Power Co., Riverside, Calif.
‡ Cons. Engr., San Francisco, Calif.

hope of placing before the Engineering Profession another valuable chapter that this contribution is offered. Much of the material heretofore presented has dealt wholly, or almost entirely, with mathematical analyses. It is not intended to attempt an addition to the store of material available along these lines. Indeed, a mathematical analysis and a complete description of the structure which is the subject of this paper has already appeared in the publications of the Society.*

Gem Dam was built in 1915 and 1916, and at the time embodied all the best practice in the design of multiple-arch dams, as is evidenced by its designer, L. R. Jorgensen, M. Am. Soc. C. E., having been awarded the Norman Medal by the Society in 1917 for his paper entitled, "Multiple-Arch Dams on Rush Creek, California".* Eight years of subsequent operation of this structure, during which time the concrete has partly disintegrated by freezing, has afforded opportunities for observations and has necessitated remedial measures which, it is believed, cannot fail to be of interest.

Descriptions of engineering structures which have failed to function as intended are often very instructive. It is hoped this one will prove to be as interesting to those members of the profession who are concerned with similar structures as it has been to the several engineers who have had to do with remedial methods.

Selection of Multiple-Arch Type

Gem Dam, on the head-waters of Rush Creek in Mono County, California, is well up on the eastern slope of the Sierra Nevada Mountains at an elevation of 9 050 ft. (water-surface level) and in one of the most remote localities in the West. All materials necessary for its construction except lumber, which could be cut locally, and rock, must be moved long distances and under difficult conditions. Cement had to be shipped from the place of manufacture by broad-gauge railroad 336 miles, transhipped to a narrow-gauge railroad, and hauled 84 miles farther; then hauled over a sandy desert road, using engines or motor trucks of the caterpillar type, for 70 miles, to the power-house below the dam. Here, it was reloaded on tram cars and raised more than 1 250 ft. vertically on a 4 826-ft. tramway to Agnew Lake, where it was rehandled, loaded on barges, and towed across the lake to be again rehandled and raised an additional height of 550 ft. by another tramway, being finally placed on the dam site at a total cost, even under 1915 conditions, of $7.50 per bbl. In the high altitude of this site, the season during which construction work can be advantageously conducted is necessarily short, often only four months and seldom longer than five months.

The dam site is one of exposed, hard, blue limestone and igneous granite, still showing in many places the polish placed on it in the Glacial Age by the glacier which formerly extended down from the higher Sierra. No earth and very little sand is found in this locality.

Physical conditions governing the original construction of Gem Dam had much to do with the selection of the multiple-arch type. Materials for an

* *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 850.

earth dam were not available. A masonry dam was thought, at the time, to have certain advantages over a rock-fill type. Because of the excessive cost of materials, the quantities required for a masonry dam of the gravity type had to be avoided. These considerations led to the selection of the multiple-arch type.

Description

Gem Dam was constructed in 1915 and 1916 at the outlet of Lower Gem Lake. The reservoir created by it has a storage capacity of 17 216 acre-ft., and is supplied by the run-off from a water-shed of 21.3 sq. miles, ranging in elevation from 9 000 ft. at the dam to 13 090 ft. at Mt. Lyell. The dam is composed of sixteen complete arches, of 40-ft. span, between centers of buttresses, and two fractional arches at the ends. The length at the crest is 688 ft. The maximum height of any individual arch is 84 ft., and the vertical distance from the crest to the deepest point in the foundation is 112 ft.

The arches vary in thickness from 1.0 ft. at the crest to 3.95 ft. at the deepest point, and the up-stream face is inclined at an angle of 50° from the horizontal. The buttresses vary in thickness from 1.85 ft. at their tops to 4.25 ft. at the deepest point, and are strengthened by counterforts, varying in total width from 4.5 ft. at their tops, which are 15 ft. below the crest of the dam, to 11.0 ft. at the deepest point. Two sets of double, horizontal, braced 12 by 30-in. struts extend between the buttresses, one set 15 ft. and one set 45 ft. below the crest. Spillway openings, which can be fitted with stop planks, have been left near the crest of the dam in the two southern arches, the five openings, 6 ft. long by 2 ft. deep, in the end arch being at Elevation 9 050, and the eight openings, 6 ft. long by 2 ft. deep, in the next arch at Elevation 9 048. A steel pipe line leads directly from the dam to the power-house, where a static head of 1 807 ft. is obtained.

This general description, together with the plans, Figs. 1, 2, and 3, and the photographic views, Figs. 4, 5, and 6, should give the reader a fair idea of the structure as it existed from its completion in 1916 until its repair in 1924. A more complete description, including details of reinforcement and an analysis of stresses, is given in Mr. Jorgensen's paper, previously mentioned. A review of this analysis will show that the stresses are within the limits prescribed by good engineering practice and that the design is more conservative than that of other multiple-arch dams which have been constructed to greater heights.

History of Operation

During the eight years of operation of the structure, very few cracks occurred in it. A horizontal crack developed in each of the arches immediately below its crest, due undoubtedly to the fact that the deflection of the walk along the crest, which acted as an arch between the tops of the buttresses, was less than the deflection of the thinner arches of the dam immediately below. This crack introduced no element of weakness and, because of its location and

its very slight width, was not evident except to the critical observer. Its existence simply permitted the arches of the dam to resist all water pressure unaided, a function for which they were designed.

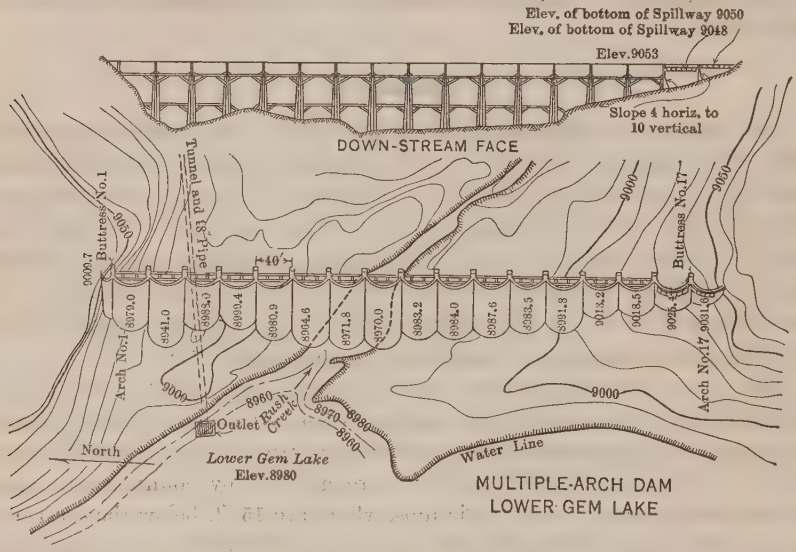


FIG. 1.

Very small diagonal cracks developed near the up-stream toes of three of the buttresses. They were similar to cracks which have developed in the buttresses of practically all the multiple-arch dams which have been constructed to date. The usual practice has been to provide but little reinforcement in buttresses or to omit it. It is believed that the introduction of a comparatively small amount of reinforcement would have eliminated these cracks in every instance.

Gem Dam has withstood some rather severe tests. In September, 1918, a very heavy rain on the upper water-shed occurred when the spillways were partly blocked with flash-boards, and before the flash-boards could be removed, the entire structure had been overtopped to a depth of 4 in. This overtopping lasted for 8 hours, but not to the maximum depth throughout. Although it was not designed as an overflow dam, no damage resulted.

When the structure was constructed, the up-stream faces of the arches of Gem Dam were covered with a 1 : 2 plaster coat of cement mortar, applied with a cement gun. This coat varied in thickness from $\frac{1}{4}$ in. at the top to $\frac{3}{4}$ in. at the bottom.

At first, the leakage was very small. Mr. Jorgensen* thus describes it:

"Some of the arches on the Gem Lake Dam are absolutely tight, some of them sweat in places, and a few drip in places. A few small springs have formed behind the dam, and a trickle of water comes under one arch; but, all told, the total leakage is very small."

* *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 906.

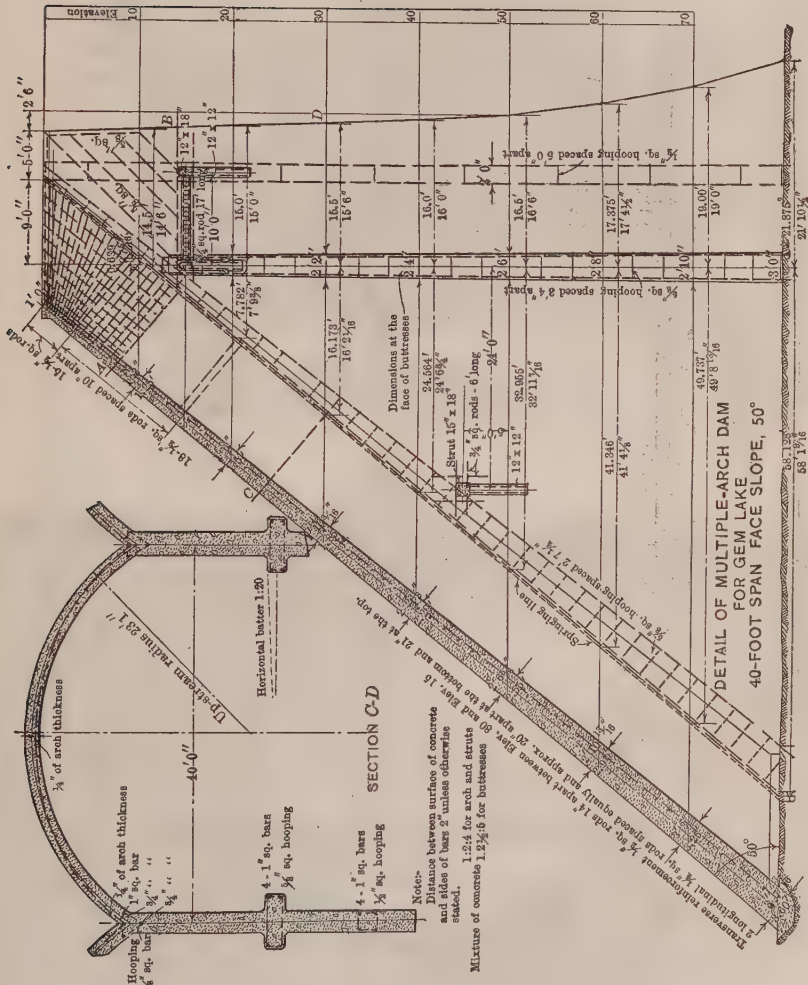

DETAIL OF MULTIPLE-ARCH DAM
FOR GEM LAKE
40-FOOT SPAN FACE SLOPE, 50°
SECTION C-D
Elevation
Dimensions at the face of buttresses
Horizontal batter 1:20
Up-stream radius 23' 1"
Springing line
Note:-
Distance between surface of concrete and sides of bars 2" unless otherwise stated.
Mixture of concrete 1:2:4 for arch and struts 1:2½:5 for buttresses

FIG. 2.

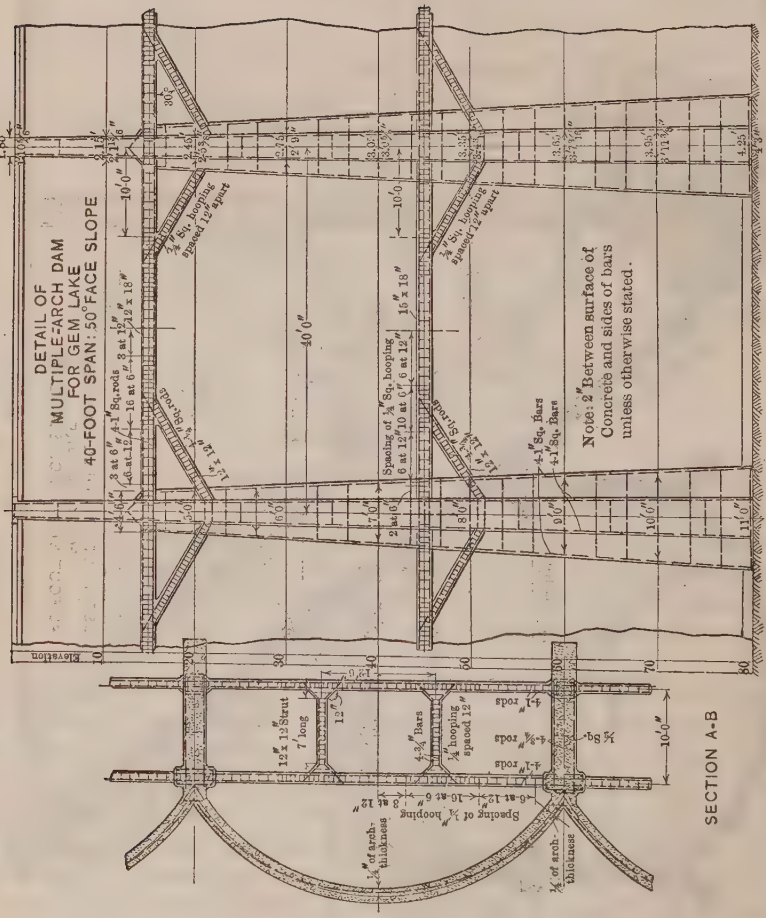

DETAIL OF
MULTIPLE-ARCH DAM
FOR GEM LAKE
40-FOOT SPAN: 50° FACE SLOPE
Note: 2" Between surface of
Concrete and sides of bars
unless otherwise stated.
SECTION A-B
Elevation

FIG. 3.

FIG. 4.—GENERAL VIEW OF GEM DAM.

FIG. 5.—DETAILED VIEW OF REAR FACE OF ARCH NO. 3, GEM DAM.

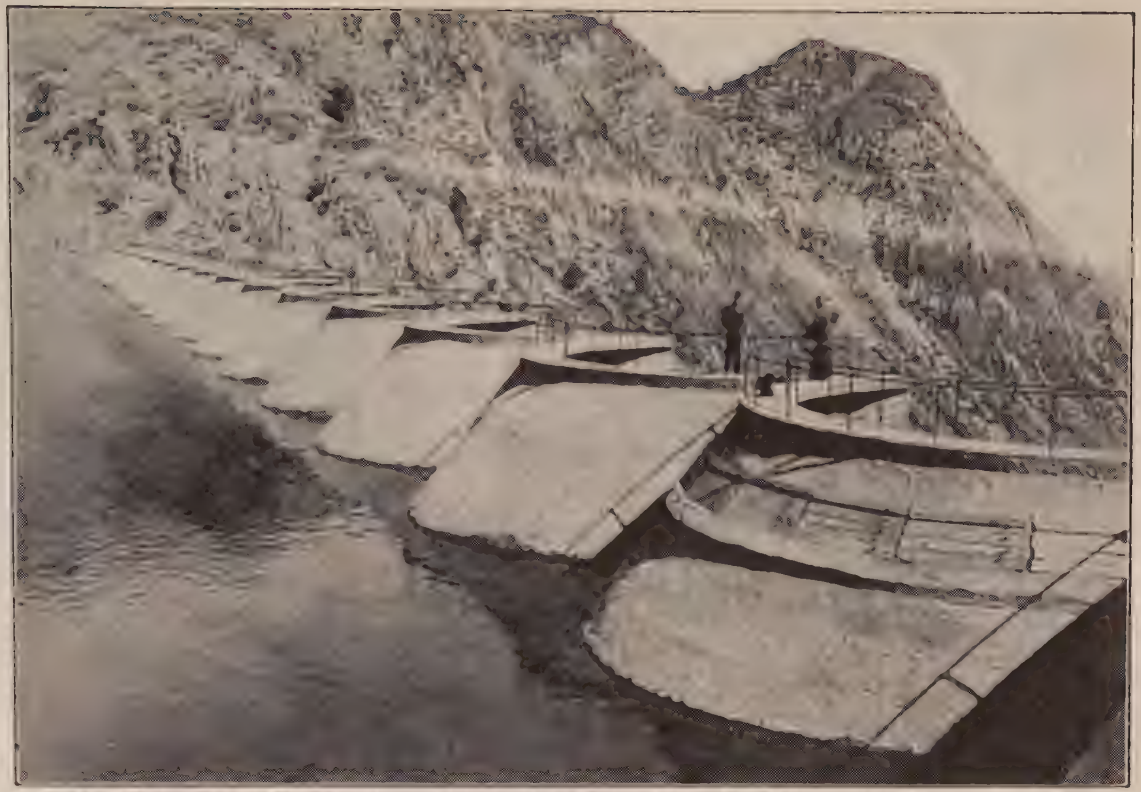


FIG. 5.—WATER FACE OF GEM DAM AFTER COMPLETION OF GUNITE COAT. NOTE OPERATION OF SPILLWAY WITH STOP PLANKS IN PLACE.

It remained small for two years after the completion of the dam and the filling of the reservoir, and during this time no change in the structure was apparent. During the third year some leakage developed and a white deposit was noticeable on the rear faces of the arches. At first, this condition was not considered serious, as a small amount of leakage through a concrete structure is not necessarily a cause for alarm. After chemical analyses, it was concluded that the white deposit was laitance which was being washed out of construction joints.

During the next two years serious disintegration started on the face of the dam, apparently due to the concrete freezing in thin layers on the face and disintegrating by frost action. This action which was slow at first, increased very rapidly with each additional year of age. It was originally believed that the damage had resulted from ice which formed on the surface of the water in the reservoir to a thickness of 30 or 40 in. This ice adhered to the surface of the dam and, as the water was lowered and the ice thus left overhanging, it would tear loose, presumably taking the gunite coating with it. However, this theory was partly disproved by cutting a hole through the ice near the dam and examining the water face immediately below. It was found that, for 8 or 10 ft. under the reservoir surface, this face was covered with a thin layer of ice, thus proving that the freezing was through the thin concrete section of the dam and that the ice on the surface of the reservoir was not the entire cause of the damage.

It was believed that the condition could be remedied if the water face of the structure could be effectively and permanently water-proofed. A careful and detailed study of possible methods of accomplishing this result was made. This study included laboratory tests by J. Y. Jewett, Assoc. M. Am. Soc. C. E., of twelve brands of water-proofing compounds, all of which failed to meet the requirements of the tests.*

The study of water-proofing compounds led to the selection of the so-called "Ironite" treatment. Before applying this treatment, all concrete that showed any signs of disintegration was chipped from the face of the dam. This proved to be quite a task. In places, the concrete over large areas was chipped to a depth of as much as 8 in. Concrete was very carefully replaced in these areas to insure a good bond, and the ends or corners of such areas were chipped out square in an attempt to preserve arch action. After the application of the "Ironite" treatment, the dam was practically water-tight for a short time. However, the "Ironite" coating soon developed hair-line cracks, probably due to expansion and contraction of the arches, and thereafter, with the return of low temperatures, deterioration began again. It was then apparent that more heroic remedial measures must be adopted.

Condition of Dam in 1924

A careful examination showed the upper portion of all arches, extending down approximately 30 ft. from their tops, to be in perfect condition and undoubtedly as strong as at any time in their history. Likewise, a section

* See p. 737.

of the bottom of each arch was little affected. In the intervening middle belt, the concrete had become a dead, inert material with little strength. All concrete in the buttresses was in perfect condition.

Holes were dug into the affected parts of the arches from their rear faces. A blow from a pick produced no ringing effect. Instead, the effect was such as would result from striking hard clay. The concrete was easily displaced, and, with little effort, holes were dug beyond the surface of the reinforcing steel to depths of approximately 18 in. (see Fig. 7), as far, in fact, as it was believed to be safe to carry such exploration, considering that at the time some water was in storage in the reservoir.

A review of conditions convinced all the engineers who made investigations that the damage was due to water from the reservoir percolating into the concrete and there freezing. Temperatures as low as — 25° Fahr. had been recorded during winter months. A review of records showed that when extremely low temperatures had occurred, the water level, in the natural operation of the reservoir, had always been drawn down 30 ft. or more from the crest of the dam, which accounted for the good condition of the concrete in the upper portion of the arches. The flood stages of Rush Creek, which are due almost entirely to melting snow, occur in May, June, and July; consequently, the reservoir is drawn down before the low temperatures of the following winter occur. The buttresses, not being subjected to water pressure at any point, were unaffected. Those parts of the bottoms of the arches which were but little affected, were undoubtedly protected from extreme low temperatures by a snow blanket drifted against their rear faces. This conclusion is substantiated by a similar experience of The Nevada California Power Company with a simple intake dam on Bishop Creek built of thin sections which were similarly affected by low temperatures until the sections back of the concrete water face were filled with earth.

Because of the developments just described, it becomes of particular interest to inquire into the history and quality of the concrete which was affected. As noted by Mr. Jorgensen,* the arches were of 1 : 2 : 4 concrete, the actual ratio of sand to cement being varied with the size of the aggregates, but in all cases 1½ bbl. of cement per cu. yd. of aggregates were used. The rock was crushed in a gyratory crusher and separated into three sizes by a revolving screen having 1½, ¾, and ¼-in. meshes. The rejects from the screen went into a jaw crusher set to give a maximum size of 2 in. A sand deposit along the shore of Gem Lake was used. Mr. Jorgensen describes the sand, as follows:

"This sand was first pumped, and later shoveled, from the lake, and transported to a storage pile near the mixing plant. This lake sand, which contained 3½% of clay and 1% of dirt, was mixed with the sand from the rock crusher (all particles being less than ¼ in. in diameter) in the proportion of about three-fourths of lake sand to one-fourth of crushed rock sand. This gave a very good combination, both as to strength and water-tightness."

Compression tests on 6-in. cylinder specimens, made as the work progressed, showed an average strength of approximately 900 lb. per sq. in. for crushing

* *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 879.

FIG. 7.—EXPLORATION IN REAR FACE OF GEM DAM EXPOSING REINFORCEMENT.

at the age of 14 days. Concrete was distributed to the different arches and buttresses with two-wheeled push carts and short chutes with the minimum opportunity for separation.

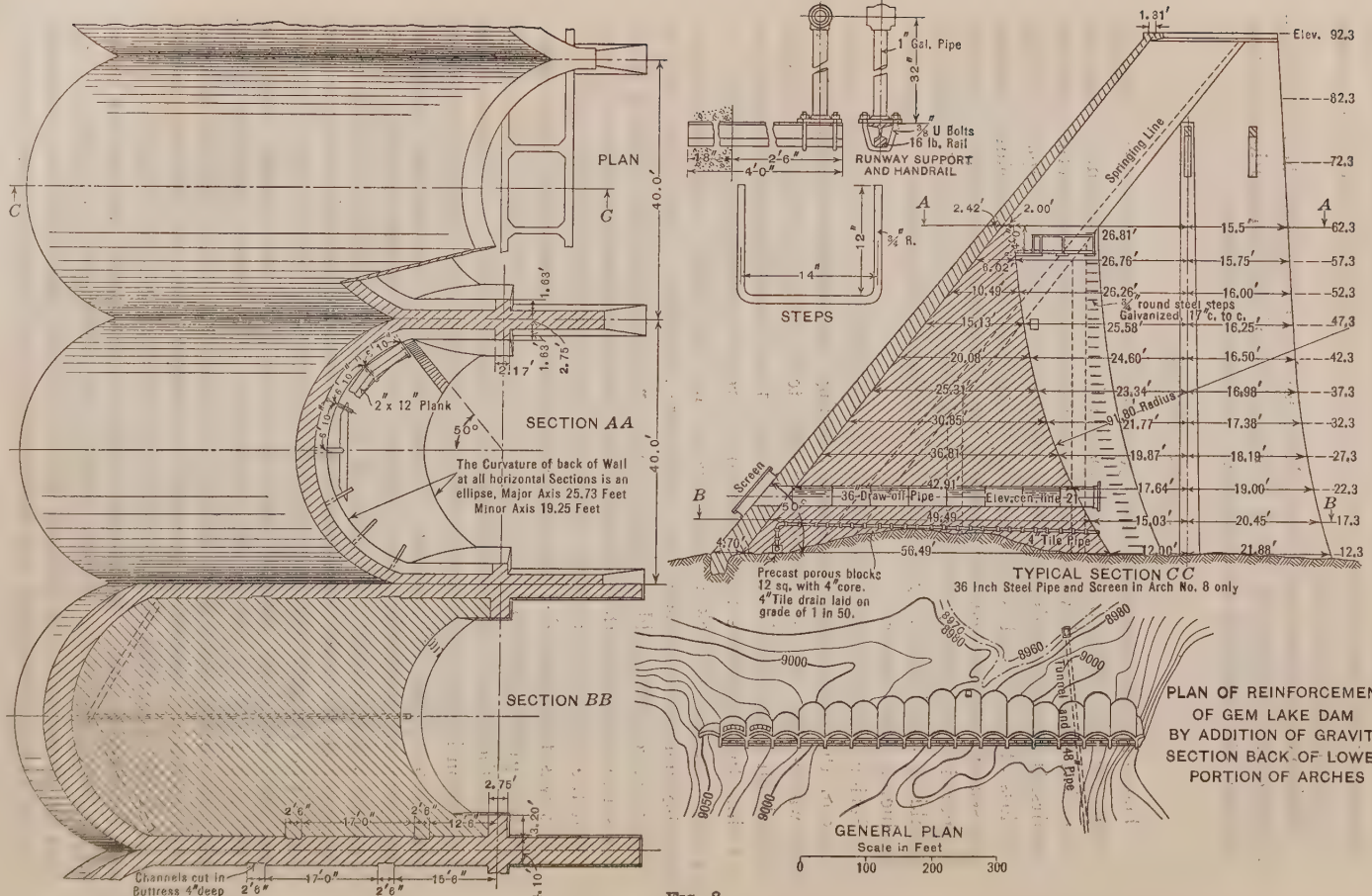
Before construction was begun, the design was reviewed by the J. G. White Engineering Corporation, and a representative of this Corporation also later reviewed and favorably commented on the work in the field.

Remedial Methods

The conditions described had developed by the spring of 1924. The preceding winter had been one of almost record low precipitation. It was evident that the run-off would not be sufficient to fill the reservoir and that repairs, if carried out during this one field season, could be completed without the loss of any stored water. On the other hand, repairs extending through two seasons might necessitate wasting considerable water and thus result in a financial loss which should be directly added to the cost of repairs. It should be noted that 1 acre-ft. of water stored in this reservoir is capable of producing 1 200 kw-hr.

Serious consideration was given to the problem of providing a rock-fill back of the concrete structure and utilizing the existing concrete arches as the water-tight face of the rock-fill dam to be thus constructed. Although a rock-fill dam, with its ready drainage, is very desirable at a location such as this, two objections to this method of repair were at once evident. The great quantity of rock necessary could not be placed in a single season, possibly not in two seasons, but, even more important, it was hardly probable that such rock-fill could be placed by any practicable methods so that it would not settle away from the concrete arches and allow them to continue to withstand the full pressure of the water until after they had actually failed.

It was finally decided that the best method of repair would be to pour a concrete gravity section back of each of the arches, extending it up to within 30 ft. of their tops. The section adopted is shown in Fig. 8. To have carried the section higher would have required greater quantities than could have been poured in a single field season. The upper 30 ft. of the existing arches and the buttresses supporting them appeared to be in perfect condition and to need no reinforcement. To clear up all doubts concerning the condition of the upper 30 ft. of the existing arches, they were drilled and tested for hardness. Two samples were cut from what appeared to be the poorest sections and were tested for strength in compression. The tests showed results of 1 880 and 2 170 lb. per sq. in., respectively. The normal operation of the reservoir during eight years had not subjected this part of the dam to water pressure during seasons of extremely low temperature, and it is not probable that future operating conditions will require that they be so exposed. It is not improbable that a horizontal crack will develop in the existing arches opposite the top of the gravity section, due to difference in rigidity of the two types, but it is not believed that any serious results will be occasioned. The two types will simply act independently and without serious leakage.



PLAN OF REINFORCEMENT OF GEM LAKE DAM BY ADDITION OF GRAVITY SECTION BACK OF LOWER PORTION OF ARCHES

FIG. 8.

It will be noted that the section added back of the arches is not only heavier than standard profiles of gravity dams of similar height, but that, with the added pressure due to the reservoir being filled 30 ft. above the top of the gravity section, the resultant falls well within the middle third of the base, and the maximum pressure at the down-stream toe does not exceed 5 tons per sq. ft. (70 lb. per sq. in.). This is a very light pressure for the excellent hard rock foundation. In addition to the stability afforded by the gravity section, it is necessarily arched in plan, following the 40-ft. arches of the structure already existing. The section was also carried back to secure an excellent bond against the counterforts of the buttresses. It is not believed that stresses can be induced in the gravity section by pressures on the upper 30 ft. of the dam, because such pressures will be transmitted direct to the buttresses.

Drains of precast porous blocks 12 in. square with 4-in. cores were placed at the up-stream toe of the gravity section, and thus at the base of the rear faces of the arches of the former multiple-arch dam and also along the springing lines of the arches. These drains were connected to 4-in. tile-drains laid on a slope of 1 in 50.

Construction

The gravity section was constructed by Dwight P. Robinson and Company, Inc., E. C. Macy, M. Am. Soc. C. E., being in personal charge. The work was begun in June, 1924, and approximately two months of the short available field season was required for the installation of plant and equipment, consisting of rock crusher, aerial cableway, inclined tramway, bunkers, decking, etc.

The tramway from the power-house at Silver Lake to a boat landing at Agnew Lake, constructed in 1915, and previously mentioned, was again utilized after extensive repairs. This tram was constructed of 36-in. gauge track, and both 25-lb. and 16-lb. rails were used, the total length being 4 826 ft., the maximum grade, 68%, and the vertical rise more than 1 250 ft., with many curves and several trestles. A small flat car, 4½ by 8 ft., was used to carry materials. The ¾-in. steel hauling cable was attached to a single drum hoist driven by a 100-h.p. electric motor. All construction equipment, tools, camp supplies, lumber, and cement, was hauled up this incline; in fact, all material used except sand and rock. The usual load per trip was 5 tons. The maximum load, the jaw of the rock crusher, was 10 tons. The most troublesome load was the motor launch, 30 ft. long with 8-ft. beam, which had to be carried on two flat cars and shifted several times while in transit because of small clearances. In handling regular loads, such as cement, a round trip was ordinarily made in 50 min., including time for loading and unloading.

Material delivered at the boat landing on Agnew Lake was loaded on a barge (18 by 24 ft. by 3 ft. deep; capacity, 400 sacks of cement, or approximately 20 tons) and towed across Agnew Lake by a launch driven by an 8-h.p. motor.

From the upper end of Agnew Lake, a double-track, 36-in. gauge, incline railway was constructed to the concreting plant situated above the south end of Gem Dam. Because of the precipitous rock slopes, it was necessary to

support this tramway on a trestle throughout its length of 1 715 lin. ft. The grades varied from a minimum of 10% to a maximum of 70%, and the total elevation surmounted was almost 600 ft. The tracks were straight with the exception of one curve, having a 40-ft. radius and a 48° deflection angle, located near the top. This tram not only carried the loads from the lower tramway, but also all the rock and sand for concrete. Only the southerly track was used for hauling sand and rock, although the tracks were located so that either could have been used. Sand and rock were hauled in a steel trip car of 4 cu. yd. capacity, which, when loaded, weighed approximately 7 tons. The northerly track, which was used for the transportation of cement, lumber, and camp supplies, was fitted with a small flat car similar to that used on the lower tram. Each car was hauled by a ¾-in. steel cable attached to a single drum hoist operated by a 100-h.p. electric motor. The motors were equipped with two speed controllers, a slow starting speed and a hauling speed of 400 ft. per min. The rock car was able to make approximately 4 round trips per hour. It is estimated that 26 428 tons of materials were hauled by this double incline during the short field season.

It was hoped that suitable rock could be found near and above the elevation of the top of Gem Dam, but final examinations showed the most suitable rock available for both the crushed rock aggregate and for making sand, to be a deposit of broken granite located below the dam and across Agnew Lake. Laboratory tests of samples of this rock showed it to be entirely satisfactory. To transport this rock across Agnew Lake, a cableway was constructed from the rock pit to the crushing plant which was at the foot of the upper incline. The rock was excavated and conveyed to the loading bunker by a 1¾-cu. yd. Bagley grader operated by a double drum hoist equipped with a 75-h.p. electric motor and ⅞-in. hauling and haulback lines.

The overhead cableway between the rock pit and the rock-crushing plant required a span between supports of 950 lin. ft., the span between anchors being 1 250 lin. ft. The anchor at the end above the crushing plant comprised a set of four 2-in. eye-bolts grouted in holes drilled in a rock ledge about 300 ft. back of the plant. A single-sheave block was laced by ¾-in. wire rope to each of these eye-bolts, and a 4-sheave block was laced to another single-sheave block acting as a thimble for attaching the carrying cable. The 4-sheave block was about 50 ft. from the eye-bolts and was laced continuously through each of these single-sheave blocks with the loose end of the cable clipped back on itself. Whenever an adjustment for height was necessary, this loose end was attached to the hauling line for the cableway car and any slack was taken up by power.

The anchor back of the rock pit consisted of a "deadman" and head-frame of round timbers back-filled over with about 100 tons of rock. The dead end of the cable was clipped around a 36-in. log and carried up through the head-frame, where a 6-sheave block allowed adjustments for height. The hauling hoist was on the mountain side to the rear of the rock-crushing plant and directly under the carrying cable. It was designed to operate the car at a line speed of 500 ft. per min. The haulback hoist was set in the head-frame

Fig. 9.—Partial View of Equipment and Camp, Showing Operation of Inclined Double Tramway, Bunkers, Decking, Etc.

FIG. 10.—FORM WORK FOR ARCHES AND CONCRETE SPOUT FOR CONVEYING CONCRETE FROM OVERHEAD DECK.

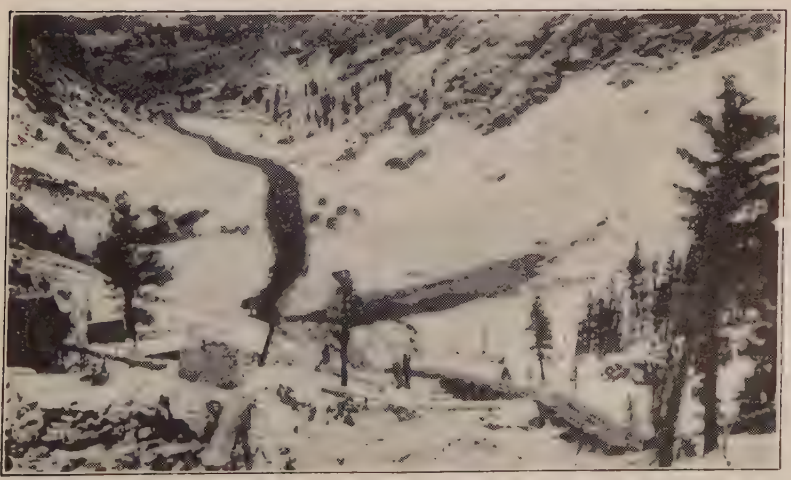
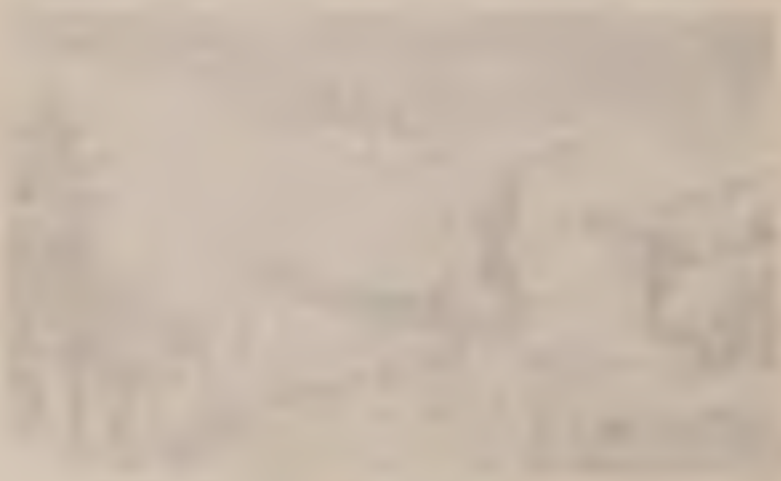


FIG. 11.—VIEW OF AGNEW LAKE, SHOWING LANES KEPT OPEN THROUGH ICE FOR PASSAGE OF LAUNCH AND BARGE.



of the rock-pit anchor. The erection of the original carrying cable, which was 2¼ in. in diameter, was a difficult task requiring the laying of the cable across the lake by utilizing the barge and motor, attaching the end at the rock-pit anchor, and pulling it up to position by use of the hauling hoist and rope. The original cable stranded soon after it was put in operation and was replaced by a 1½-in. Langlay, steel-core cable, which was much more easily handled. Pontoons across the lake, constructed by securing in position beneath the carrying cable a number of small logs held in position by ⅜-in. wires, prevented the hauling lines from dropping down in the lake and catching under rocks.

The concrete mixing plant was located above the south end of Gem Dam. From this point a trestle and deck was constructed over the top of the dam to serve as a platform on which to push the concrete in dump cars of the Kopple type to the point of pouring. Fig. 9 is a view of part of the equipment and camp, showing the inclined double tramway, bunkers, decking, etc. The concrete was dumped through platform hoppers into elephant trunk chuting 16 in. in diameter and conveyed to place in the forms. Owing to the use of large aggregate, it was difficult to handle concrete which showed a slump of less than 2½ in. without clogging the chutes, even with the steep slopes used. A concrete inspector made frequent slump tests and attempted to keep all the concrete of such consistency as to show a little less than a 3-in. slump. A cement bag shaker was utilized and 1 sack of cement was recovered from each 58.4 sacks shaken.

A total quantity of 12 004 cu. yd. of concrete was added to the dam and 16 425 bbl. of cement, or 1.368 bbl. per cu. yd. of concrete, were used. The approximate proportions of the mix were 1 part cement, 3 parts sand, 3 parts crushed stone (¼ in. to 1½ in.) and 1.84 parts cobbles. Compressive tests of concrete samples showed an average strength of 2 400 lb. per sq. in. at an age of 28 days, and 3 440 lb. per sq. in. at an age of 3 months.

Weather conditions permitted continuance of the work somewhat longer than usual, and the last concrete was poured on November 15, 1924. Fig. 10 shows the form work for the arches and the concrete spout for conveying concrete from the overhead deck. Fig. 11 is a view of Agnew Lake showing the lane kept open through the ice for the passage of the launch and barge.

Conclusions

To prevent deterioration of concrete subjected to water pressure and extreme low temperatures, it is necessary that impervious concrete be obtained. The slightest penetration of water will be followed by deterioration, further penetration, and further deterioration.

Plaster coats, such as "Ironite", might be satisfactory except for hair-line cracks which will be developed on any thin arch section by movement due both to temperature changes and to loading and unloading the arches.

Under the difficulties imposed at this site and similar sites in very remote high mountain areas, it is doubtful whether, under conditions of the present day, impervious concrete can be obtained. Therefore, structures composed of

thin sections of concrete should be protected from extremely low temperatures. For such localities, the writers believe that a dam of the rock-fill type has many advantages over dams of other types.

ACKNOWLEDGMENT

In addition to the writers, those who reviewed conditions and assisted in planning the reconstruction include Messrs. R. G. Manifold and the late Charles Oscar Poole, M. Am. Soc. C. E., Consulting Engineers, in whose office the plans for reconstruction were made, and C. W. Comstock, M. Am. Soc. C. E. The construction work was under the general charge of E. J. Waugh, M. Am. Soc. C. E., Construction Engineer of The Nevada California Power Company. As previously noted, this work was performed by Dwight P. Robinson and Company, Inc., with E. C. Macy, M. Am. Soc. C. E., in personal charge.

DISCUSSION

J. Y. Jewett,* Assoc. M. Am. Soc. C. E. (by letter).—In the early summer of 1923, the writer was called on to examine the structure referred to in this paper, and make a report on certain features of concrete construction related thereto. The repairs, involving use of the "Ironite" process, were at that time under way. Later, as mentioned in the paper, he was asked to make some tests of water-proofing materials, with a view to finding a suitable method of treatment for water-proofing the up-stream face of the dam, a brief description of which tests may be of interest.

The first series of these tests was confined principally to bituminous materials. Engineers of the Company felt that the solution of the problem might lie in the use of some substance having the flexible and elastic properties of this type of material, in contrast to rigid materials of the Portland cement mortar type which readily crack and allow the entrance of water under the severe temperature changes of that locality. The main points to be determined were: Durability under the prevailing climatic conditions; resistance to water pressure; bonding with the concrete surface; and penetration into the pores of the concrete under pressure.

Manufacturers of water-proofing materials of this type were asked to furnish samples of products which they could recommend as being especially resistant to the range of temperature involved. The samples received were classed in three groups, as follows:

(*a*) Solid material, to be melted and applied hot.
(*b*) Liquid material, to be applied with a brush as a paint.
(*c*) Material in mastic or putty form, to be applied with a trowel.

For practically all the samples submitted, the use of a light asphaltic paint as a primer coat was prescribed, and this was generally furnished with the sample.

In tests for resistance to water pressure, a pipe apparatus connected with the city system was used, on which a pressure of 55 to 60 lb. per sq. in. was available. The test specimens were 2 in. thick and 5 in. in diameter, coated on top with the material to be tested. These specimens were bolted between plates, with gaskets to make a tight joint, leaving an area, 3 in. in diameter, in the center exposed to water pressure, with provision for catching and measuring any water coming through. To determine absorption under pressure and penetration into the pores of the concrete, specimens 6 in. in diameter and 4 in. high, entirely coated with the material to be tested, were immersed in a closed tank under the same pressure. The effect of variation of temperature was observed by heating to 110° Fahr., and by exposure in cold storage at slightly below 0° Fahr.

All the materials except a light paint in Group (*b*) formed a coating which was water-tight under the pipe-pressure test. The tank apparatus did not prove to be adaptable to the purpose for which it was used on account of the unbalanced condition caused by difference between air in the pores of

* Testing Engr., City of San Diego, San Diego, Calif.

the concrete at atmospheric pressure and the higher pressure maintained in the tank. This produced rupture of the enclosing membrane, and caused it to puff out in small bubbles and pull away from the surface of the concrete. In no case was there any penetration into the pores of the concrete.

Under the conditions prevailing at this dam, application of the materials of Group (*a*) would be difficult. The hot melted material cools very quickly on striking the cold surface of the concrete, and tends to stick to the tool with which it is being applied, and to pull away from the surface in a sheet. The materials of Group (*c*), although possessing spongy, elastic properties, were in general too soft and easily abraded to form a satisfactory coating, and some of them became softer under water than in the air. Under an increase of temperature up to 110° Fahr. a few of the samples remained unchanged, but the general effect was to soften the material, although not sufficiently to cause flow or change of shape. The specimens placed in cold storage were recently removed after one year's continuous exposure, and were practically unchanged in condition, the coatings showing no sign of cracking or peeling. To show the effect of varying conditions of temperature and moisture as existing at the structure, field exposure tests would be needed.

The conclusion is that if material of this type is used, it should be in paint form, of the class included in Group (*b*). Some of the paints of this group are of heavy consistency, about as thick as can be readily handled with a brush, and dry with a hard, smooth, tenacious coating. Among the several samples having these characteristics, a difference in hardness and tenacity under water immersion was noticed. Trial of a combination coating of such a paint over the mastics of Group (*c*) indicated that, while retaining the advantages of the former, some benefit would also accrue from the elastic properties of the latter.

A second series of tests was made on samples of general water-proofing compounds of the type intended to be applied as surface coatings in paint form. These materials, although showing properties which might be useful in certain classes of work, did not develop results under water pressure that would recommend them for the purpose in question. This series included several brands of the so-called water-proof, plastic, Portland cements now on the market. The plasticity of these products is doubtless of benefit as an aid to workability, in some classes of work, but they did not show water-proofing qualities in these tests equal to those of straight Portland cement. A marked feature of these cements was their low strength in compressive tests of concrete, as compared with straight Portland cement.

To the writer, in view of the results outlined, the problem seems to narrow down to a choice between two methods of treatment:

First.—The use of bituminous paint of the type described. This, although an unusual form of treatment for a structure of this kind, gives indications of furnishing an effective water-proof coating at a relatively low cost. If, however, it should need renewal from time to time, as may be the case, the cost would be a continuing item.

Second.—The use of a Portland cement mortar or concrete coating of high density, reinforced against temperature effects. Although the original cement-

gun coating did not prove to be satisfactory, the usual product of this process is well known as a material of high density, and it should be possible to apply such a coating in this case so as to obtain advantage of this feature; or the procedure could be carried a step further, taking the form of a coating of reinforced concrete, similar to that of which the walls of the concrete ships are built. Without entering into discussion of this type of ship in general, it may be noted that they have shown the possibility of concrete construction in thin walls, of high density. As an alternative to the use of Portland cement in this proposed mortar or concrete coating, the use of a quick-hardening, high-strength cement of the newly developed "alumina" type deserves consideration.

With reference to the discussion by the authors of the concrete in the original structure, it was evidently considered that the work was of a good grade of concrete construction, and that the gunite facing could be relied upon for water-tightness. Tests of the concrete, however, as reported* by L. R. Jorgensen, M. Am. Soc. C. E., and as obtained on the specimens taken from the structure in 1924, show a low compressive strength for the mix used. One feature contributing to this, judging from the writer's observation, was the presence in the rock used as aggregate—which was obtained from material near the site of the work—of thin, flat, smooth-surfaced pieces, which would have somewhat the same effect in concrete as mica in a sand mortar. The authors, in describing the new construction, refer to the selection of a more suitable rock at some distance from the dam site.

Experience with this structure indicates that concrete to be used under similar conditions should be of what may be termed a super-excellent grade. This raises the question of the adequacy of the ordinary contract system (with its natural emphasis on speed of operation and profit for the contractor) to meet such a condition. From the point of view of a testing engineer, who, from his study of materials, perhaps realizes more fully than others the possibilities inherent in concrete as a construction material, a more effective method would be to place such work directly in charge of an engineer familiar with the problem and give him a free hand to obtain the results desired. This is subject, of course, to the limitation that there be no waste or needless expense; but it does leave such a man free to devote his energies to obtaining these results, rather than to expend them in more or less futile inspection efforts under the contract system.

L. R. Jorgensen,† M. Am. Soc. C. E. (by letter).—The distintegration of the middle zone of the arches in the Gem Lake Dam, has naturally given great concern to the writer, as he was responsible for the design and construction of this dam nine years ago; he has therefore been seeking earnestly for a true explanation of this rapid deterioration.

As concrete consists simply of rock, sand, cement, and water, the problem is limited to the investigation of these four materials, aside from the mixing and general workmanship. At the time of the construction of the dam and

* "Multiple-Arch Dams on Rush Creek, California", *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 850.

† Cons. Engr., Constant Angle Arch Dam Co., San Francisco, Calif.

also as late as June, 1923, these materials were analyzed and tested frequently. The following notes on the tests are taken from a report made by the Noble Inspection Bureau (now Abbot A. Hanks, Incorporated, San Francisco, Calif.):

"Water from Gem Lake	Parts per Million
Sulphates (as Na_2SO_4)	None
Chlorides (as NaCl)	2.90
Carbon dioxide (CO_2)	2.20

"As the sulphates, chlorides, and carbon dioxide were present in such a very small amount, it was not necessary to make further analysis in connection with the investigation of deposit on the concrete. This water is unusually pure."

Relative to crushed rock, the report states:

"We [Noble Inspection Bureau] treated the sample of crushed rock to determine its solubility in water. The results indicate that the calcium carbonate deposit on the down-stream surface of the dam does not originate from this rock."

The sand used was found along the shore of the natural Gem Lake, which was lowered about 15 ft. when excavating for the dam foundation. Approximately, one-half this sand required washing in order to eliminate excesses of clay and vegetable matter. Both mortar briquettes and compression specimens were made of the local sand and of standard Ottawa sand, and the local sand was washed whenever it gave results below the standard in compression and below 90% of the standard in tension. Three kinds of sand were used: A semi-silicious, a basaltic, and a granite sand from the rock crusher. The latter was mixed with the sand from the two natural deposits.

The original structure was built by the Duncanson Harrelson Company, of San Francisco, which Company had had large experience in concrete work and built a good structure. The water contents were kept to a minimum for reinforced water-tight concrete. It is true that the crushing strength of the concrete was not quite as high as ordinarily would be expected for the richness of the mix used. There are two reasons for this: First, concrete in reinforced arches which has to be water-tight can not be deposited as dry as required for maximum strength. In a gravity dam where men can work directly on the newly deposited concrete a slump of 3 in. can be used, but for reinforced concrete in forms, where the concrete can only be reached with long-handled tools, a slump of approximately 5 in. must be used to obtain water-tight concrete. Secondly, the shape of the crushed rock was somewhat elongated. The stresses used in the design were conservative, and therefore the structure was never over-stressed due to the water load.

When the dam was first subjected to water pressure it was remarkably tight. The first time the writer observed anything wrong was about three years after completion. A very heavy white deposit consisting of practically pure calcium carbonate was noticed on the down-stream face 30 ft. below the crest. It is almost impossible entirely to avoid this white deposit on the down-stream face of the dam, at least in moist spots, but in this case the deposit was too heavy to be harmless and kept increasing.

The small white deposits often seen on the down-stream face of dams originating merely from the leaching of the free lime in the cement, or from whatever laitance there may be in the construction joints, are harmless, but when these materials have leached out and appear on the down-stream side in the form of calcium carbonate, the process should stop and nothing further should happen. In the case of the Gem Lake Dam the process never stopped; it seems that in the zone subject to water pressure the cement itself broke down during the coldest weather. Where else could the large deposit come from?

The cement had passed the usual test. It might, however, have been slightly underburned without being noticed, causing it to be at the point of unstability when exposed to the severe cold. Under ordinary conditions the concrete undoubtedly would have kept its strength, but under the severe climatic conditions imposed on it in the middle zone, it deteriorated and lost its entire strength.

The deterioration started at the construction joints, as they were naturally the weakest points in the structure, but if the leaching had only been confined to whatever laitance or free lime there might have been in the joints, the process would soon have stopped. Fig. 7 shows clearly that the deterioration of the concrete was not confined to the joints. A zone 30 ft. high, beginning 30 ft. below the crest and extending all the way across the structure, was subject to heavy frost while the water level fluctuated between the two levels. In this zone the concrete is practically uniformly deteriorated, whereas above and below it is still in good condition.

If the sand had been the cause of the poor showing of the concrete, it could be expected that there would have been good and bad spots of the material, as three kinds of sand were used, sometimes washed and sometimes unwashed.

An analysis of the deposit on the Gem Lake Dam shows the following:

	Percentage
Silica (SiO_2)	0.76
Iron (Fe)	0.05
Alumina (Al_2O_3)	0.39
Calcium carbonate ($CaCO_3$)	97.44
Magnesium carbonate ($MgCO_3$)	0.11
Total	98.75

showing that the deposit is practically all calcium carbonate.

Inasmuch as the deposit continued to accumulate, it was due, in the writer's opinion, to the cement breaking down gradually and the lime in the cement being leached out continually. The time of actual breaking down was during the coldest weather.

The Agnew Lake Dam, built of practically the same material on the same stream, but 500 ft. lower in elevation, is still in first-class condition. The water level in this reservoir, however, is not appreciably lowered during the cold season, and snowdrifts protect the down-stream face to a great extent from the extreme cold weather.

Three years ago (1922) the writer acted as Consulting Engineer on a multiple-arch dam then being built in Northern Sweden on the River Luleaa, 50 miles north of the Arctic Circle. Two dams of a design similar to the Gem Lake Dam were constructed at a location where at times the temperature drops to 50° Fahr. below zero. Both dams have been in use for two years; they are being carefully watched for frost action, but so far none has appeared.

These dams are provided with a back-fill of earth on the down-stream side for a short distance up. The back-fill was put in to protect the lower arch portion against the very low temperature and consequent contraction near the footing where the arches are fixed to the rock. The upper portion of the down-stream face is still uncovered and, therefore, open to inspection. The water level fluctuates during the cold season.

On recent designs of multiple-arch dams, the writer has favorably considered arches of 50 ft. span, instead of 40 ft. A greater thickness of arch is necessary, but this should make it possible to keep the construction joints tight, just by careful workmanship, without plastering the up-stream side. When an arch is so thin that heavy frost can penetrate the concrete and cause freezing on the water side below the water level in the reservoir, it can hardly be expected that a plaster face will have a long life; the adhesion is seldom 100%, and reliance for water-tightness therefore should be placed as much as possible in the concrete itself.

Walter H. Wheeler,* M. Am. Soc. C. E. (by letter).—The description of the Gem Lake Dam is interesting, but the disintegration of the concrete in the structure by frost can be readily explained.

In the description of the aggregates two facts are mentioned which would account for the difficulty. One of these facts is that the lake sand used in the concrete contained 3½% of clay and 1% of dirt, and the other that the screenings from the rock crusher were mixed with the lake sand in the proportion of 1:3. Presumably, the blue limestone, referred to as the country rock at the dam site, was used in the concrete without removing the dust which resulted from the crushing operation. It is the writer's experience that concrete made of aggregates which contain clay and dirt will not resist frost action, and also that limestone dust is very detrimental to the durability of concrete, also that concrete made with certain varieties of limestone aggregate will be quickly disintegrated by frost. The writer has observed many examples of the disintegration (by frost) of concrete in which similar aggregates have been used, and also many examples of concrete made with clean durable aggregates, which have resisted the frost under equally trying conditions. He does not hesitate, therefore, to assign at least a part of the trouble on the Gem Lake Dam to the aggregates used in the concrete. If concrete is to be exposed to frost the aggregates must be carefully selected with that possibility in mind.

There may also be other contributory causes which are not so apparent from the published description.

* Designing and Cons. Engr., Minneapolis, Minn.

C. A. P. Turner,* M. Am. Soc. C. E. (by letter).—This paper should prove of much value to the profession in calling pointed attention to that particular kind of concrete which will not withstand frost action, namely, concrete compounded of sand as fine aggregate containing 3½% of clay and 1% of dirt. The broad conclusion that concrete disintegrates in like manner when made with suitable aggregate is not warranted from this example. Good initial resistance to compression of sample specimens is too often mistaken as a determinant of frost resistance although in no wise related thereto.

For purposes of discussion, the concrete utilized in the construction of dams may be classified as (*a*) structural concrete; and (*b*) mass concrete. Hollow dams (such as the original Gem Lake design), flat slab dams, and the like represent Class (*a*), while the mass concrete of the gravity type present examples of Class (*b*).

That frost disintegration occurs in Class (*a*) concrete because of the relative thinness of the section, but does not occur in Class (*b*) concrete because of its greater thickness, is apparently the deductions of the authors from the single experience described and discussed.

In certain slabs of reinforced concrete dams much thinner than the Gem Lake section and exposed at times to temperatures of 50° below zero, or lower, no material disintegration by frost has occurred in twelve years or more, whereas in some structures of the gravity type with far thicker sections, in which care had not been used to secure clean aggregate, the writer has observed disintegration by frost.

The importance of the question at issue lies in the fact that the structural concrete dam may commonly be safely built at a much lower cost than a dam of the mass concrete type. Many irrigation projects are rendered commercially feasible by using the economic dam structure whereas they would be lacking in feasibility with the more expensive gravity types. The disintegration of mass concrete, or the thinner section of concrete, by frost appears to the writer as one of progressive action which may render even the massive structure unsafe within a period shorter than its expected life. Hence, the need of care in the selection of concrete materials whether in the bridge pier or dam structure exposed to the elements. The additional first cost of washing sand and securing sound aggregate is always small compared to the cost of repairs when proper care has not been exercised.

Thomas H. Wiggin,† M. Am. Soc. C. E. (by letter).—There is at present under construction from the writer's designs a multiple-arch dam about 300 ft. long and 33 ft. maximum height, and the subject is, therefore, of great interest to him.

Many concrete structures, including dams, are deteriorating with the aid of moisture and frost and it is generally impossible to fix the cause with certainty. The authors call attention for the first time in a pointed way to the fact that the immersed side of a thin exposed wall of concrete is in the same danger from the formation of ice crystals in its pores under the influence of

* Cons. Engr., Minneapolis, Minn.
† Cons. Engr., New York, N. Y.

cold coming through from the dry side of the structure as the side not immersed of a thick, leaky wall.

Deterioration was expected and occurred in the mortar lining of uncovered steel pipe, but here the exposure was severe as only a thin plate of steel intervened between the outside cold and the saturated mortar lining only 2 in. thick.

Filter roofs, 6 in. or more in thickness, covered with about 2 ft. of earth, are often wet with condensed moisture and sometimes permit thin ice to be formed on the water below, showing that the air which touches them is, at least, as cold as the unfrozen water against a thin dam. The writer has never noticed deterioration in the lower surface of such filters. None of these filters was in a region of extreme cold.

In the United States some concrete stand-pipes have deteriorated outside from leakage and freezing, but the writer does not recall deterioration inside although the concrete was porous and had to be water-proofed on the inside to prevent further damage outside. The French have constructed some of these stand-pipes that seem to have no imperfections from leakage or frost. These stand-pipes, again, are not in semi-Arctic regions; also, there is more or less circulation of water.

The Stony River Dam in West Virginia is a hollow dam of the Ambursen type. It is at an elevation of about 3 000 ft. and experiences very severe winters compared with most places in the East. Ice forms on the inner faces of the hollow. Deterioration has occurred on the water face, although mostly high up in the zone of fluctuating water levels.

The writer has been advising on the repair of a gravity concrete dam. The concrete has deteriorated both on the water face and on the down-stream face. Disintegration down stream is apparently due to leakage and freezing; that up stream, similarly, is apparently due to saturation and freezing. The concrete is about 1:3:5 and rather poor. Several hundred thousand dollars will be expended in repairs. The safety of the dam has not yet been endangered by deterioration. The interior is sound. This is only one of many dams that have shown signs of surface disintegration. Only good stone masonry seems exempt, and that needs repointing in some cases.

It is not agreeable to contemplate that in spite of all care, concrete structures so often fail to endure. The writer started thirty years ago with the feeling that the place for concrete was underground. Unfortunately, stone for stone masonry is not often available at a reasonable price and the advent of concrete was the beginning of the decline in the supply of stone masons. Engineers are often forced to use concrete where its expectation of life may be comparatively limited, or at least doubtful, because stone would cost so much more that the concrete structure could be often renewed at a less total capital cost. "Hope springs eternal" also that vigilance will produce concrete of such quality that it will be among the structures exempt from major deterioration.

Among those who are engaged in research on cement and concrete are some who lay the deterioration of concrete to the use of too little cement. In the Gem Lake Dam only 1.5 bbl. per cu. yd. were used. Perhaps 1.75 or 2 bbl.

would have shown different results. Some old structures with even less cement are still without blemish, it is true, but this may have been due to extraordinarily good sand and proportioning.

The writer chose a multiple-arch dam for the case mentioned at the beginning of his discussion because rock foundation was found near the surface and other conditions made it by considerable the cheapest type, except possibly a slab dam. About 1.6 bbl. of cement per cu. yd. are being used in the piers and 1.75 bbl. in the arch which is only 1 ft. thick. Even with this thickness the stress is only about 150 lb. per sq. in. from water pressure alone and it is still (October, 1925) very low, with scarcely any tension, with extreme conditions of temperature. It is a flexible structure and no joints need slip to provide for temperature changes. It was hoped to make the dam tight by the excellence of the concrete, but in addition a complete water-proofing of asphalted fabric and asphalt, covered with mastic for protection against ice, is to be provided. The circulation of water in the reservoir will be fairly vigorous and the climate is that of the New York region, that is, generally free from prolonged intense cold.

In their struggles to avoid one set of difficulties, human beings are prone to run into others just as serious. Nature will not be altogether circumvented. An earth dam with generous slopes, paved water slope, grassed down-stream slope, masonry core, and well-drained toe comes as near to being a part of the landscape in permanence as can be produced by man, yet it requires a generous masonry spillway for its protection and perhaps fails by overtopping when the record storm for the region occurs. The masonry dam even partly disintegrated may safely withstand overtopping at such a time.

C. H. Howell,* M. Am. Soc. C. E. (by letter).—This paper is an exceedingly instructive one. The writer believes with the authors that "all the story is not yet told" concerning multiple-arch dams; he would also include all flat slab dams.

The smaller volume of concrete contained in these hollow types, as compared with ordinary gravity and arch sections, has often led to their erection at sites utterly unsuited to them. To expose thin concrete slabs or thin arches to high water pressures and low temperatures is inviting ultimate failure. It is a misuse of types of dams which have a definite, although rather limited, use.

Like all innovations, these hollow types of dams have their over-enthusiastic supporters who can see no merit in the older types and strenuously insist on multiple-arch or flat slab designs for every site. The history of the Gem Lake Dam will probably diminish some of this enthusiasm. The writer knows of one other dam, a flat slab structure, which is deteriorating in the same manner as the Gem Lake Dam. Both these dams were erected in climates that are too cold.

E. W. Lane,† M. Am. Soc. C. E. (by letter).—Since many dams of the multiple-arch and Ambursen types have not been seriously affected by frost action, the trouble at the Gem Lake Dam does not seem to be inherent in dams

* Engr., U. S. Bureau of Reclamation, Denver, Colo.

† Lafayette, Ind.

with thin concrete walls. It does serve, however, to emphasize a possible source of weakness, and the necessity of much more than ordinary precautions in the construction of such dams, especially in cold climates.

A few years ago, the writer inspected a large gravity dam in which the concrete was disintegrating in a manner very similar to that at Gem Lake. The dam was constructed by competent engineers, but within a few years the concrete began to disintegrate and, when inspected, had scaled off in places to a depth of as much as 6 in. and could readily be dug with a knife for some distance from the surface, being about as resistant as compact gravel. The writer was informed that the cause of the trouble was the limestone aggregate. This had been brought from a considerable distance, as the local stone was unsuitable. It had passed all the ordinary tests, but an examination to determine the cause of the disintegration had revealed that the limestone was filled with microscopic veins of shale which broke down on exposure to the weather. Had this material been used in a dam with thin sections, it would certainly have necessitated reconstruction.

The fact that the buttresses of the Gem Lake Dam were not affected strongly points to the water as being an element in the cause of disintegration, but this does not prove that it was not the quality of the aggregate that was the primary cause of the difficulty. In his description of the construction of this dam, Mr. Jorgensen* states that, in the lower part, limestone from the outlet tunnel was used for the aggregate. Is it not possible that the limestone aggregate was the cause of the trouble, and that the reason the upper part was not affected was because it was constructed with granite aggregate?

The writer believes that the materials entering into the construction of a multiple-arch or buttressed dam should be subjected to a much more critical examination than for a dam of gravity section. He would suggest subjecting the aggregate and moulded specimens of concrete to a considerable number of alternate freezings and thawings. This should not be difficult with the small mechanical refrigerators which are coming rapidly into use. In northern climates it could be readily done in winter by natural means.

The danger resulting from disintegration of multiple-arch dams was called to the writer's attention by an inspection which he made of a dam of this type in Central New York. This dam (Fig. 12) consisted of three vertical arches supported by buttresses, flanked by an ogee gravity spillway section at one end and a gravity section abutment at the other. The concrete had disintegrated badly in places (Fig. 13), especially at the springing line of the arches and at a crack (probably caused by temperature) which extended down the crown of each of the three arches. The construction joints were also seats of attack. The disintegration was not confined to the arches, but included also the buttresses and the ogee spillway section. The writer understands that this dam has been repaired, but is not aware what method was used.

E. J. Waugh,† M. Am. Soc. C. E. (by letter).—In any discussion involving disintegration in concrete the first question that naturally arises is what was

* *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 868.

† Constr. Engr., The Nevada-California Power Co. and Southern Sierras Power Co., Riverside, Calif.

FIG. 12.—MULTIPLE-ARCH DAM IN CENTRAL NEW YORK, DOWN-STREAM FACE, SHOWING DISINTEGRATION.

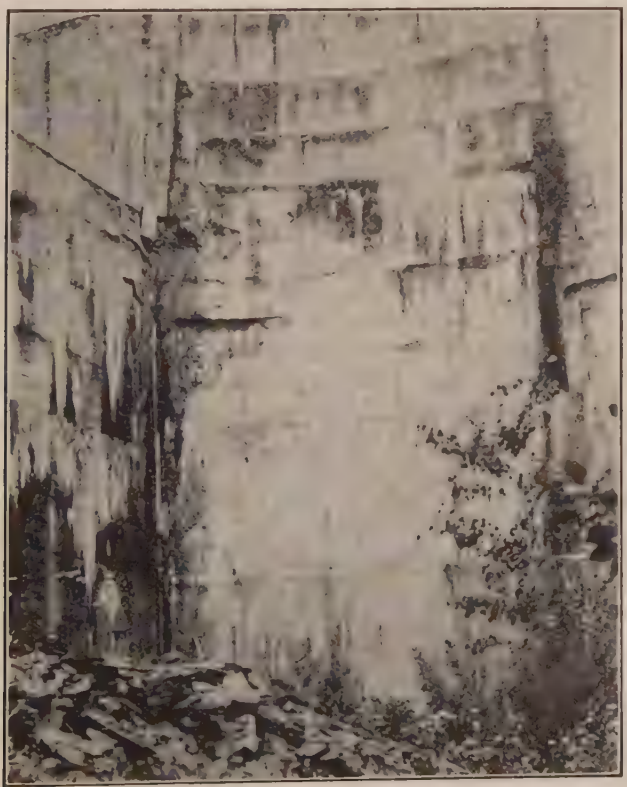


FIG. 13.—DETAIL OF DOWN-STREAM FACE, MULTIPLE-ARCH DAM (LEFT BAY IN FIG. 12), SHOWING EXTENT OF SPALLING.

the quality of the concrete before deterioration commenced. If the up-stream face had been absolutely water-tight and had remained so, it may be assumed that the original concrete in the Gem Lake Dam would have given satisfactory service, as no deterioration appears in the buttresses. Also, the arches, where not subjected to freezing when filled with water, are not seriously affected as to structural strength, as is evidenced by compressive test samples taken in 1924.

The failure of the gunite coating to absolutely prevent moisture from getting in back of the top skin was the primary cause of the trouble. During the long season of freezing temperatures, extending from September or October to April or May, any moisture admitted would freeze and in expanding would either loosen the material at the face or tend to enlarge the pores throughout the mass. This operation repeated itself each time the weather warmed enough to thaw. After seven winters, slabs of gunite coating of all sizes had been loosened; that this was not due to faulty bond of the gunite is proved in part by the fact that often a thin layer of concrete adhered to the gunite when it was forced loose from the arch.

The quality of the concrete and the character of the bonding at construction joints has a direct bearing on the rapidity of disintegration, but it is doubtful whether any concrete would resist such freezing conditions indefinitely. It is inconceivable that any concrete will prevent the penetration of moisture. With temperatures that will cause ice to form under water on the reservoir face of the wall, any saturation of the wall whatever will cause damage. Either some facing must be provided that will be water-tight to the extent of allowing no moisture to reach the concrete or else the dam should be back-filled or otherwise insulated against cold. In the original construction a water-tight metal up-stream form would be the most satisfactory solution. It could be asphalt dipped, or otherwise treated, so that it would not rust against the concrete and it could be repainted when necessary on the up-stream face and maintained in water-tight condition at comparatively little cost. Such a form could be designed with expansion joints. In this way the concrete in the arches could be kept dry and the full strength would remain so long as the up-stream metal facing were maintained water-tight. It would be impracticable to carry the metal form below the ground line, but at this point it would be a simple matter to fully protect it by back-fill and still leave the structure open for observation.

It is possible that metal is still the best protection for the Gem Lake Dam, but it is much more difficult to place than it would have been in the original construction. Some substance that has the qualities of adhesion and elasticity and can be sprayed or brushed may solve the difficulty; field tests are now being made with certain bituminous compounds to determine the feasibility of their use.

Alfred D. Flinn,* M. Am. Soc. C. E.—There is danger in over-emphasizing one notable failure. It is creditable to engineers who have knowledge of failures to report them for the benefit of the profession; but it would also

* Secy., United Eng. Society, and Director, Eng. Foundation, New York, N. Y.

be of service if the Society, through a small committee of inquiry, or an existing committee, would set down against this failure the record of the thin concrete dams under as great or even greater exposure that have stood successfully for years. There are many such dams in the United States. The importance of presenting such countervailing information is not slight. Engineering Foundation might be asked to collect the data for the Society.

Dams are among the structures that are supervised by certain States. Some supervising officials are not well informed about the theory or the practice of the construction of dams. Their responsibility is to make sure that structures which they approve do not fail. The cost put on the owner, or the limitations on the engineers in charge of design and construction, are only secondary to the officials.

Endeavors of American engineers to build successful dams at moderate cost are being watched closely by engineers in other countries. Recent correspondence with an Italian engineer indicates that some American experiences have been misinterpreted. He has been strongly opposing the use in his country of any thin dams. American engineers owe it to themselves to answer the questions which have been raised. These questions may be grouped under the heads: Sufficiency of design, suitability of materials, efficiency of construction methods, and adequacy of maintenance.

For the case in hand: What is known about the materials put into the concrete which failed? What were the chemical and physical characters of the water, cement, sand, stone, and gravel? What attention was given to the proper placing and curing of this concrete for the particular purpose and conditions of this dam?

In some instances is not concrete being used with about the degree of intelligence that would use window-weight iron or alloy steel indifferently as materials for tools for cutting metal? In other words, is not concrete too often designed without sufficient regard for the purposes and conditions of the structures into which it is to be put, and with still less discrimination in supervising its mixing, placing, and curing? Concrete for simple foundations, or for structures which will be wholly protected from the weather or active destroying agents, may be different from concrete suitable for thin hydraulic structures under pressure and exposed to severe weather. This fact is so obvious that slowness in carrying it to the necessary limit in practical operations is hard to understand. Probably the tardiness to realize the relation between cause and effect has been due partly to the fact that the results of some wrongdoing in concrete construction do not appear for several years and partly to the fact that the men who design and construct often do not see their structures again and, therefore, do not learn the lessons which develop during years of maintenance. However, notable progress has been made.

In engineering textbooks, reports, journals, and specifications it is commonly implied that Portland cement is uniform; that cement from any source may be used just like cement from any other source; but experience in testing and using cement shows that, instead of being uniform, cement is a various material and that some cements are not well suited to certain special services,

for example, hydraulic structures. It is encouraging to know that a fundamental study of hydraulic cements has recently been undertaken at the U. S. Bureau of Standards with the co-operation of the Portland Cement Association, and that a number of engineers have been giving thought to this branch of the subject. In this connection reference should be made to the paper by William G. Atwood and A. A. Johnson, Members, Am. Soc. C. E., entitled "Disintegration of Cement in Sea Water".*

Engineering Foundation is aiding the Committee on Arch Dam Investigation, made up of members of the Society. Its experiments on the Stevenson Creek Test Dam, near Fresno, Calif., and on concrete used in that dam, and its observations on existing and proposed dams, should add knowledge that would be helpful to engineers designing and constructing dams of several types. This project has already been described.†

Luigi Luiggi,‡ Hon. M. Am. Soc. C. E.—This is a most valuable paper. Greater information is to be obtained from structures in a critical condition or from disasters than from those that are physically successful. Engineers, like physicians and surgeons, to become experts must operate on bad cases.

The paper emphasizes three important factors in a successful work, that is, design, materials, and supervision during construction. Dams, even in the most favorable circumstances, are difficult structures to build; and often the design has been considered more as a calculation of stresses than as a construction problem. Indeed, the design is an exceedingly important factor, for a badly designed dam is destined sooner or later to fail; but it is not sufficient merely to design the dam well—it is necessary to choose the materials best adapted to the conditions of the locality of the work. In particular, the cement must be of the highest class and be used in quantities substantially in excess of the theoretical proportions to allow for losses and deterioration.

It is easy to test cement in a laboratory and from the test to specify the exact proportion; but in the field, at a distance from the cement factory and lacking good material, it is necessary to adopt precautions that only practice can suggest. Consider the case of the Gem Lake Dam. The cement traveled over a normal railway, a narrow-gauge railway, then across a lake, and finally up a mountain side. It thus became very expensive; and when a material is costly there is always the temptation to use it sparingly. That is bad economy. Instead, more than the computed quantity should have been used to counterbalance the inevitable local losses: The wind blows some of it away, some of it sticks in the mixer and in the chutes. The best practice is to use 5 to 10% more than the theoretical quantity or even more than this under especially difficult conditions. It is bad economy to make weak concrete; this is dearly paid for later in the cost of repairs.

If the builders of the Gem Lake Dam had been more generous as to the cement, the concrete would have been less porous, and all the heavy expenditure to remedy this condition would have been avoided. As an adjunct to

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 204.

† *Proceedings*, Am. Soc. C. E., October, 1925, Society Affairs, p. 340.

‡ Hon. Pres., State Council of Public Works; Insp. Gen. of Civ. Engrs.; Prof. of Hydr. Eng., Univ. of Rome; Member, International Technical Committee for the Suez Canal; Senator, Rome, Italy.

good design, a proper proportioning, a particularly good cement, and an excess of it, are indispensable for hydraulic work that must not be porous.

A third necessary precaution is a thoughtful and conscientious supervision of the work. To impress these three points more deeply on the minds of engineers—especially younger engineers—the speaker will instance a multiple-arch dam which, in height, design, and general dimensions, was practically a reproduction of the Gem Lake structure. The failure of this dam in the Valley of Gleno, in Italy, high up in the Alps of Bergamo, caused an immense loss of lives and tremendous damage. This dam was built about 6 000 ft. above sea level in a very cold climate and with difficult transportation.

As to design the Gleno Dam, although not excellent, was fairly good; but the materials were rather bad and the supervision even worse. The work was begun during the World War. Cement was scarce and costly because all building materials were going to the front. Workmen were scarce—either older than 50 or younger than 18 years of age—and unskilled. Supervision was almost completely lacking. The work was mainly under the supervision of a manufacturer, a self-made man, exceptionally clever as a cotton weaver, but with little experience in construction work. Under him were only ordinary workmen in charge of the construction. The work was carried out trusting to Providence.

During that time and for a few years after the war, Italy had a very inefficient Government; Socialists, not to say Communists, easily imposed on the Government officials, who were powerless. This employer was permitted, therefore, to proceed, carrying out this dangerous work without any technically competent man to advise him. Although Government officials occasionally went to inspect the progress and reported that the work should be stopped, should be partly demolished and rebuilt, still nothing was done to effectuate this. Further, because coal was extremely expensive and difficult to obtain and because hydro-electric power was the only remedy for the lack of coal, water was allowed to impound behind the dam, to produce as soon as possible the electric energy which, in turn, would keep the cotton mill busy and provide cloth for soldiers and work for the men at home. When the war was over, little was done to remedy this state of affairs.

As the dam was leaking in many places, some inefficient repairs were made, but these did not cure the consumptive condition of the whole structure. Then came a rather heavy storm, almost a cloudburst; the spillway was not large enough to discharge all the surplus water, which rose behind and nearly overtopped the dam. It was only a question of minutes, as described by an eye witness, before the dam began to crack and suddenly burst. This happened during the night. In the valley below several hundred people were killed and damage estimated at 150 000 000 lire was done. Many of the people were drowned in their beds.

As a result of an inquest into the cause and responsibility for this terrible disaster, some men were imprisoned and now have to answer for the disgraceful condition; but no one can resuscitate the drowned, nor compensate for the property destroyed. It is more to the point to search out the reasons for failure and thus avoid the mistakes made in the Gleno structure.

Although its design was questionable, with excellent materials and good workmanship, conscientiously controlled, the structure might have stood; but the defectiveness of the cement, the porosity of the concrete, and the badly built masonry which allowed water to percolate, were the main causes of the failure. In addition, there was an underlying mistake in the foundation, namely, a large mass of masonry which blocked up the bottom of the gorge and formed a platform on which the dam rested.

So poor was the cement mortar in this block, that it allowed water to pour out through the masonry. Worse still, it was not founded by means of proper steps cut out of the rock, but was laid on top of the natural surface which had been rubbed smooth by glacial action. No cut-off trench was provided to prevent leakage and consequent uplift. The only precautions taken were to grout iron bars into the rock where it was too smooth and thus, one might almost say, to nail the dam to the foundation. As a result of all these dangerous factors, the percolating water must have exerted an uplift, and when this occurred the masonry must have moved slightly, the iron bars lost their grip, and the whole structure slid down to destruction over a part of the dam where these treacherous foundations existed. Thus, the disaster was due primarily to lack of proper and skilled supervision.

State authorities should always insist on the advice and inspection of competent engineers. The author states that "for such localities [the Gem Lake Dam], a dam of the rock-fill type has many advantages over dams of other types." The speaker fully concurs in these conclusions; it is certain that in the case of the Gleno Dam, if a rock-fill structure had been built—which during the war could have been effected more easily than the multiple-arch dam—the disaster would have been avoided.

Regarding the use of preparations like "Ironite", the speaker's experience of more than forty-six years convinces him that it is impossible to construct masonry and make it absolutely impermeable under a head of more than 40 or 50 ft. Not even a coating of neat cement, applied with a cement gun, will resist the percolation of water, because, besides stresses due to water pressure and change of temperature, there is another important effect, often disregarded, namely, the slight imperceptible movement of the crust of the earth to which the structure is subject almost daily. On a large scale these movements become earthquakes; but the very small daily tremors, noted only by seismography, cause the masonry to develop small hair cracks, which enlarge little by little and allow the water to percolate. In addition, the filling and lowering of the lake causes an elastic movement of the dam and of the foundation, which produces slight stresses that result in hair cracks. The great dam of the Murrumbidgee River in Australia is an illustration of this condition. The structure is about 240 ft. in height and the enormous pressure of the water causes actual movements in the foundation, as have been clearly recorded by seismographs set up in that locality.

Although it is possible to make masonry water-tight for some time by using special compounds, these infinitesimal movements, as years pass, will cause the masonry to crack. The speaker believes there is only one way of

making the surface impermeable—at least as nearly so as possible—and that is by covering the up-stream face with a membrane of sheets of bitumen, now commercially available in sizes about 4 ft. wide by 10 to 20 ft. long and ⅛ in. in thickness. These should be made to overlap and should be fused together with hot tools to form a plastic, continuous, and absolutely impermeable coating over the masonry. To protect this sheet from summer heat, floating logs, and blocks of ice, a revetment of masonry 1 to 3 ft. in thickness must be provided.

The durability of bitumen is attested by archeologists who have found in Nineveh sewers made with brick laid in bitumen which is still impermeable after 3 000 years. The speaker's belief that the Gem Lake Dam could have been properly protected in this manner, is strengthened by the instance of a rock-fill dam in the Alps, the Devero Dam. The up-stream face of this dam has a sheet bitumen coating protected by masonry 1½ ft. in thickness at the top, increasing to nearly 3 ft. at the bottom. After twenty-five years this dam shows no sign of leakage.

Thaddeus Merriman,* M. Am. Soc. C. E.—The buttresses and the arches of the Gem Lake Dam were both made of the same concrete. In the buttresses this concrete endured, while in the arches it failed. The paper seems to demonstrate that concrete which is serviceable in one exposure is not necessarily serviceable in another.

Mr. Flinn has made a fair statement. Much good concrete has been placed, but there has also been some of doubtful quality. All concrete which disintegrates is not bad because the cement with which it was made was poor. Many of the troubles with concrete seem to be due to the fact that those who make it do not fully appreciate all the conditions which the process of concrete making involves. The speaker is free to admit, after many years of study, that this great problem still seems to be far from final solution.

On September 16, 1925, the speaker mixed a Portland cement with 43% by weight of water and poured a part of the mixture into one bottle and a second part into a second similar bottle. Forty-eight hours thereafter the first bottle was broken and the cement specimen so formed has since been exposed to the air. Twenty-one days later, on October 7, 1925, the second bottle was broken, and both specimens are here for examination. The first is soft and chalky, while the second is hard and dense. These specimens appear as if they had been made of entirely different materials, but, in fact, they are merely a demonstration of a difference in curing conditions. This is a simple experiment and one which all may profitably try.

The first of these specimens was treated in about the same way that most concrete is treated. It was put into a form, the form was pulled off at an early age, and Nature took its course. The specimen immediately dried out, leaving the hydrations uncompleted. In the second specimen, however, the conditions were such that the hydrations went on to practical completion and the resulting texture indicates a strong, hard, and durable material. Concrete so cured would be a wonderful compound.

* Chf. Engr., Board of Water Supply, City of New York, New York, N. Y.

Between the texture and the quality of these two specimens there are many intermediate kinds of texture and quality. Under one condition of curing, one quality of product is realized; under other conditions, other qualities result. Some are good, some are indifferent, and some may be decidedly bad. So it is that one structure is very good, while another in the course of a few years may show serious disintegration. The quality of a concrete is not only a function of the conditions under which it was cured, but it is also dependent on the conditions of subsequent exposure. A thoroughly cured concrete, all other conditions being the same, will resist almost any exposure. A partly cured concrete, depending on the conditions of exposure, may or may not prove to be satisfactory.

The curing of concrete, and particularly that in thin sections, is, in large measure, dependent on the weather and moisture conditions during the first five or six days following its placement. If the humidity be low, nearly all the water necessary for the completion of the reactions will be lost, while, if the air be moist or water supplied, the hydration of the cement will proceed in an orderly manner. It has been claimed, if the process of hydration be interrupted, and if water be supplied at a later time, that the hydrations will resume themselves. No proof of this supposition has been adduced. Such evidence as the speaker has been able to develop is to the contrary. If the first specimen described is given plenty of water at any time after it has been dried out, it will never take on a texture resembling the second.

In the case of the Gem Lake Dam, all the conditions for rapid drying out were favorable. The high altitude, and the consequent low humidity, did its fatal work in withdrawing from those thin sections the moisture which was the very life blood of the cement. For weeks, and possibly months, that cement product was athirst. It was like the first specimen previously mentioned—spongy, chalky, and unstable. It could not resist the solvent effect of the water, and its constituents were leached out. On the other hand, that same imperfect cement product, in the buttresses, served its intended purpose. Freezing was not the primary cause of this disintegration. Frost merely hastened the visible evidences of deterioration. The quality of a cement product is always dependent on the maintenance of an environment in which the cement reactions may complete themselves. Good concrete can be realized only if it is kept for several days after making, under such conditions as will insure for the cement an opportunity to develop its latent possibilities. These conditions are clearly indicated by the bottle experiment which has been outlined.

In many cases the treatment accorded to concrete after placing is analogous to that of abandoning a two-day infant to its fate. On State highways it has been learned that curing is one of the most important single operations in the entire process of concrete making. In many other situations, however, concrete after placing is allowed to shift for itself and then, as in a post-mortem investigation, the causes of disintegration are vainly sought.

The problem of durable concrete is the problem of durable cement. With a given cement the best results are not possible of attainment unless that

cement be used with understanding skill of workmanship. Cements differ among themselves and results which can be obtained with one cement are impossible with another. The task which confronts us is twofold: First, we must learn to differentiate cements with respect to the quality of the hydration products they are capable of producing; and, second, we must learn how to use a cement of given quality so as to secure the results which any particular situation demands.

At present, none of these matters is susceptible of standardization. Standards of measure are useful as means of convenience and of comparison. Standards of quality and of methods operate to suppress individual initiative. Standards imply averages, and if quality of product be a consideration they tend toward mediocrity.

In this great matter of concrete and of cement we cannot rest content with averages and with mediocrity. Until the uttermost of the problem is within the grasp of our understanding we must do no less than seek for the very best within the present possibility of attainment.

M. M. O'SHAUGHNESSY,* M. AM. SOC. C. E. (by letter).—The authors are to be commended for their frank discussion on the repair of the buttressed arch dam in the high Sierras near the State of Nevada, south of Lake Tahoe, California. No estimate has been furnished as to the resultant losses sustained by The Nevada-California Power Company due to the two years failure of this dam to function in storing water, which losses must have amounted to far in excess of the cost of initially building a properly designed dam of a type suited to the conditions and available materials. It is hoped they will furnish these data before closing the discussion, so that a frank statement of all conditions may be available to engineers interested in this type of dam.

The building of this dam followed the conventional design for this type of structure at the date of its construction (1916), and the failure is not primarily due to theoretical design features. Its designer, after describing it in a paper† before the Society, deservedly received the Norman Medal. He was retained by the writer in 1917 to aid Assistant Engineers McIntosh, Nishkian, and the late Mr. Wood in designing the Lake Eleanor Dam, 70 ft. high, 1 200 ft. long, on the Hetch Hetchy Project. Further studies on the design for this type of dam resulted in several beneficial modifications being incorporated into the Lake Eleanor Dam,‡ which is a structural and economic success.

The true principle controlling dam design and construction was tersely elucidated by Mr. George L. Dillman,§ namely, that there must be only one impervious surface next the water, and that this face must be adequately supported by solid masonry, buttresses, rock, or earth-fill, depending on the locality and the type of dam.

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* City Engr., San Francisco, Calif.

† "Multiple-Arch Dams on Rush Creek, California," by L. R. Jorgensen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 850.

‡ *Engineering News-Record*, September 4, 1919, p. 455.

§ *Transactions*, Am. Soc C. E., Vol. LXXV (1912), p. 52.

The term "multiple arch" as applied to this type of dam is a misnomer; there are so-called "multiple-arch" dams in India 120 years old, and the name is meaningless as to design. This particular type of dam survives because of the buttresses, and the principles and reason for its selection were first described (after a structure built in Oregon) in 1902 by Mr. Dillman,* and due credit should be given him for his initiative in recommending it.

Overtopping the Gem Lake Dam with 4 in. of water is not a very severe test. The Upper Otay Dam (a single arch), in San Diego County, 84 ft. high, built by laymen without engineering skill, was topped to a depth of 30 in. without injury. However, an enlarged spillway now prevents a recurrence of this happening.

The sections are so thin in the buttressed arch type of dam that it is essential to have the concrete of such density that it will be water-proof, and this can be accomplished only by a careful selection and proportioning of the materials, avoiding an excess of water in mixing, and thoroughly spading the concrete after it has been deposited in the forms.

The writer's impression of the value of alleged water-proofing compounds on concrete surfaces subjected to water pressure is in entire agreement with the experience of the authors; that is, these compounds have no worth except to their manufacturers and salesmen.

The concrete in the Gem Lake Dam failed because it was defective in that it was porous, thus allowing the water to penetrate it, and, with the heavy frosts and ice formation, permitting destructive disintegration to set in.

Why was the concrete porous? It may have been due to an excess of granite, mica, dust, and clay in the sand, or defective cement; or, as Mr. Jewett states:

> "* * * the presence in the rock used as aggregate—which was obtained from material near the site of the work—[cheaply, of course, which feature should not be considered in dam construction] of thin, flat, smooth-surfaced pieces, which would have somewhat the same effect in concrete as mica in a sand mortar."

This class of rock prevents the proper interlocking of the remainder of the rock in the mass, and has a tendency to reduce the density of the concrete; and it is in such a situation that the experienced man, with judgment of materials, is needed fully as much as the designing engineer or the testing laboratory in determining the character and suitability of the materials to be used in the structure.

In building the 396 000-cu. yd. concrete dam at Hetch Hetchy, the engineers had quite a controversy with the contractors on their initial attempt to use a readily accessible, cheaply obtained sand. Exhaustive tests were made on samples of concrete and briquettes from this sand. They were found to be defective and failed to show the proper strength and hardness characteristics under time tests. The chemist's analysis subsequently disclosed the fact that the sands were heavily coated with a black compound which was largely tannic acid (from a dense grove of oak trees) and which caused the trouble.

* "A Proposed New Type of Masonry Dam," by George L. Dillman, *Transactions*, Am. Soc. C. E., Vol. XLIX (1902), p. 94.

Three miles up stream from the dam, at the junction of Tuolumne River and Rancheria Creek, there was found a bank (250 000 cu. yd.) of excellent sand which, after some protest on the contractor's part (because of the necessity of building a 3-mile narrow-gauge railroad to reach it), was used with very satisfactory results.

Evidently, Mr. Jewett's experience as a testing engineer indicates a doubt as to the value of the ordinary contract system for dam construction, since the tendency is to slight this class of work because of the contractor's desire for speed of operation and profit; this agrees in general with the writer's ideas. It may be mentioned in this connection that Lake Eleanor, Early Intake, Cherry Diversion, and Priest (145 ft. high) Dams were not built by contract, but in a very satisfactory manner under the direct charge of the resident engineers; and the O'Shaughnessy Dam at Hetch Hetchy was built by contractors who faithfully and promptly complied in every respect with the wishes of the resident construction engineer, who was assisted by a staff of twenty men, inspecting and testing.

In December, 1923, the Gleno Dam (buttressed arch type) failed in Italy, drowning 500 people and damaging property below to the extent of $5 000 000. No doubt, inadequate preparation of the foundations, together with defective lime masonry and concrete, put in without engineering control by the overzealous contractor in critical parts of the structure near the base, without Government supervision, caused the accident.

There is now a barrage of literary experts for contractors and damage claimants mailing reports back and forth on the causes of and responsibility for this failure, with one certain result, namely, that the Italian Government is allowing no more questionable dams to be erected by any one, and has commissioned skilled Government engineers to inspect all materials and supervise all designs, so that there will be no repetition of the Gleno Dam disaster.

It is a pleasure to note that an engineer of extensive construction experience, such as Mr. Huber, endorses the virtues of a rock-fill dam for this type of location, with its abundance of cheap rock, thereby saving the expensive hauling of cement.

The Morena rock-fill dam, built under the writer's direction, survived the cloudbursts of 1916 in San Diego County, California, when the Lower Otay Dam failed, and when every county and railroad bridge in that community was destroyed by floods.

The idea of the rock-fill type of dam is traveling eastward, crossing the Mississippi River, as witness the Dix River Dam, 280 ft. high, in a limestone formation, in Kentucky, built for the Middle West Utilities Company by L. F. Harza, M. Am. Soc. C. E., to conserve water for power.

Although time and the financial exigency in 1917-18 compelled the City of San Francisco to build the buttressed arch Eleanor Dam (forming an 8 000 000 000-gal. reservoir at a cost of less than $300 000), definite plans are now being made for practically scrapping this structure, and substituting the ultimate dam of the rock-fill type at this point. This rock-fill dam will be below the present dam, using the 70-ft. concrete buttressed arch for the lower

part of the up-stream toe or water-tight skin, and carefully placing and thoroughly grouting rock in layers, with spalls behind the arches to give them adequate support and so prevent their failure.

The writer would strongly urge all engineers first to heed the careful words of Mr. Dillman on design; next to examine exhaustively all the materials in the vicinity of any proposed dam that might enter into the structure, eliminating any cheap untried methods of experimenters and scrutinizing carefully vendors of special cheap types; then to select a type of structure that is best suited to the location and service required, regardless of the cost; and then to build so that it will not fail—not like Lord Macaulay's idea of the New Zealand tourist who, to illustrate the instability of things, was to stand on a broken arch of London Bridge and sketch the ruins of St. Paul's (London Bridge and St. Paul's are now being underpinned due to defective foundations in both structures, and the work is being done by British engineers).

J. B. Lippincott,* M. Am. Soc. C. E. (by letter).—The authors are to be commended for writing so frank a discussion of the failure of the Gem Lake Dam due to the freezing and thawing of the concrete on the outer side of the arches below the water levels in the reservoir. The use of this type of dam in regions subject to severe frost has long been a subject of conjecture among Western engineers. Although a thin shell of concrete may be made water-tight against percolation, it is not possible by any process known to the writer to prevent it from absorbing water when in contact with it. The Gem Lake Dam has shown that this is an unsatisfactory condition, in locations of extreme cold.

The multiple-arch dam because of the economy of its construction has such merit that its future use in many localities is assured. This type of structure, however, has its limitations which enthusiasts, in the past, have sometimes overlooked. Most of the dams of this type have had buttresses of plain concrete without reinforcement.

The writer has had occasion to examine the buttresses of five of these dams and in each has found one or more diagonal cracks in some of the buttresses, starting from the junction with the arch approximately two-thirds down from the top of the dam, and running in a line generally normal to the water face to an intersection of the foundation. These cracks do not extend into the reinforced arch rings. In one dam this type of crack developed prior to the time that the arch rings were built on the buttresses. In another instance the cracks are reported to have appeared after the arch rings were placed, but before the reservoir was filled. They are believed by the writer to be due to shrinkage caused by either the drying of the concrete or by temperature changes, because they open wider in winter and partly close in summer.

It is almost impossible to set up a triangular slab of concrete, say, 100 ft. high, with a base length of 100 ft. firmly anchored into bed-rock without having shrinkage cracks occur. This shrinkage and cracking of flat concrete pavements at intervals of 30 to 50 ft. is common. These cracks do not occur

* Cons. Hydr. Engr., Los Angeles, Calif.

in all the buttresses of the dams examined, but usually in a number of the higher ones. Some cracks of this kind have been noticed, however, in buttresses that are not more than 30 ft. high. In every multiple-arch dam having an age of, say, two years, that has been examined by the writer, such cracks have been found, including one having reinforcement.

It is believed that these cracks do not indicate a failure or any serious weakness of the structure, but they are disturbing and lead to discussion and comment. They distinctly suggest the necessity of thoroughly reinforcing the buttress walls of multiple-arch dams in order to minimize such conditions. This paper does not mention any such occurrence in the Gem Lake Dam, but as attention has been called by the authors to some of the limitations in this type of structure, it is deemed appropriate to call attention also to an improvement of past practice.

Oren Reed,* Jun. Am. Soc. C. E. (by letter).—The writer has followed the fortune of the Gem Lake Dam with a great deal of interest. The Company with which he is associated has plans for two regulating dams on the North Fork of Kings River on the west side of the Sierras, at elevations of 6 550 ft. and 8 170 ft. The failure of the Gem Lake and other dams leaves a doubt in the minds of engineers as to the permanency of such structures when subject to the action of frost and ice. On the other hand, Lake Eleanor Dam,† 4 660 ft. above sea-level, of the multiple-arch type, has withstood frost action without damage, and several dams of the Ambursen type have been built in Canada and the northern part of the United States with slight damage from frost.‡ The Southern California Edison Company is constructing a multiple-arch dam at Florence Lake on the San Joaquin River, at an elevation of about 7 200 ft.

One wonders whether the primary cause of the failure of Gem Lake Dam was frost action or whether frost action was made possible by a weak porous concrete. It would have been of interest if the authors had stated whether the Agnew Lake Dam had also been damaged by frost, since it is only 550 ft. lower in elevation.

Since information as to the destruction of concrete by acidic and even pure water is very meager, some discussion on that important subject may be of value. While in Europe in 1922-23 the writer was much impressed by the general interest among engineers in this subject. Many tests of concrete, as used in the construction of dams, have been made in Germany, Denmark, and Norway during the last few years.§ It was found that some dams had been failures, but as a rule these failures had been of a purely local character and could be laid to poor workmanship and frost, and a few to poor sand, etc. In other cases the weakening had been more general, especially where water had an opportunity to work in porous concrete. There are several factors which were thought to have contributed to these failures.

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* Asst. Designing Engr., San Joaquin Light & Power Corporation, Fresno, Calif.

† *Engineering News-Record*, Vol. 95, July 20, 1925.

‡ *Loc. cit.*, September 3, 1925.

§ Ing. Bonde, *Teknisk Ukeblad*, No. 14, April 3, 1925, and Ing. Groner, *Teknisk Ukeblad*, No. 10, April 17, 1925.

1.—Sand.—Sand containing dirt, humus or humic acids was found to make poor concrete. Concrete made with humus-holding sand has a comparatively low early strength and must be kept moist while hardening in order to gain its normal strength. These humus materials can be disposed of by heating, and, in part, by washing. The sand used should be clean, sharp, and well-graded. Whenever crushed artificial sand is available or can be produced cheaply enough it should be used.

2.—Cement.—In many cases it was found that the cement was at fault. In almost all concrete construction only pure Portland cement is used. This is composed of about 63% lime (CaO), about 20% silicic acid (SiO_2), about 6% of clay and smaller parts of iron oxide (Fe_2O_3), magnesia (MgO), sulphuric acid anhydrate (SO_3), etc. In hardening after hydration lime is freed, which, together with water, forms lime hydrate ($Ca(OH)_2$); this comprises about 15% by weight of the cement's mass. Lime hydrate is soluble in water and is washed out of the structure by leakage water, and, under the action of the carbonic acid in the air, becomes carbonate of lime. This is the white coat to be found on the structure. By the dissolution of lime hydrate the concrete becomes more porous and the voids are filled with leakage water. When freezing takes place, the sand grains are further loosened. This dissolution of lime takes place most easily when the mixture is lean and when humic acid is present in the water or in the sand. Dissolution of the lime may be prevented by adding to the concrete mixture a material which, reacting with the free lime, forms an unsoluble compound, as a silicate of lime. For this purpose the Germans use trass. This is a pale yellow or gray volcanic rock found in the valley of the Eifel in the Lower Rhine. It is a wholly neutral material, but with lime undergoes a chemical combination, forming an insoluble silicate of lime. Finely ground brick clay has also been used with good results.

A concrete may be made water-tight by using a rich cement mortar, but this is a costly method. The French alumina cement, which is especially rich in clay (Al_2O_3), containing 36%, is very durable against acids.

3.—Water.—Water for concrete must be clean and free from organic matter. From tests it has been proven that a plastic concrete gives better results than a dry mixture. A plastic mixture gives a less porous concrete.

4.—Workmanship.—The fourth factor in concrete placing is workmanship. This must be high grade to insure the proper mixing and placing. Poor workmen can produce poor results with the best materials.

The natural lakes and meadows on the west slope of the Sierra Nevada Mountains are, in effect, marshes. To a great extent they are of glacial origin. In many cases large tracts of meadow or marshy ground are covered by water when a dam is constructed. Many tests, especially in Germany, have been made on these marsh waters.

Most marshes contain more or less sulphur compounds, in the form of gypsum, sulphuric pyrite, or even free sulphuric acid. It is well known that sulphuric acid and sulphate-holding water attack concrete by combining with the lime to form gypsum, which is soluble. Destruction by these sulphuric acid combinations occurs not only to new but to old concrete.

Furthermore, great quantities of carbonic acid are found in marshes, which combines with the lime of concrete to form soluble salts. The free lime is

thus seen to be the main point of attack and, to prevent failure of the concrete, the lime must be fixed. Tests on marsh water have given varying results but concrete made with it has become weakened sooner or later in nearly all cases. As a general rule the acids which may be present in a reservoir will be in a very dilute state and will not be effective but, where there are large marshy tracts near the dam, and especially with low-water level in the reservoir, the acids will no doubt have a certain effect on the durability of the concrete.

It is not only the sulphate and acid-holding water which damages concrete. Under certain circumstances, pure water, on account of its high dissolving power, may act equally as effectively as weak acids. The water from the granite regions near the crest of the Sierras will often be nearly pure.

In general, concrete tests are concerned only with strength without regard to density, but in dams density is very important. A dense concrete made of the best materials prevents attack both from weak acids and from pure water. A porous concrete has many points of attack for mechanical and chemical action.

Many multiple-arch and flat-deck dams have been successful in Norway and Sweden. One of the most important of these is Holmevand Dam, A/S Saudefaldene, in South Norway. This is a flat-deck dam of the Ambursen type, and is 740 m. (2 400 ft.) above sea level. Fjergeng Dam, in Central Norway, north of Trondhjem, is also a flat-deck dam. As a protection against frost action, it was provided with a vertical insulating wall between the buttresses on the down-stream side. At the Porjus Dam, in Sweden, just above the Arctic Circle, there are concrete arch spillways 648 ft. in length. The remainder of the dam consists of earth embankments faced with stone. The up-stream side of the crest of the concrete dam is inclined, in order to force the ice to break up and be pushed over the crest. A stone-protected dirt fill, furthermore, has been placed in front of the dam to insure against leakage, as in dams of the Intze type. This spillway structure is a modified Ambursen dam and consists of concrete arches between concrete buttresses. The arches in front, against which the fill rests, have a curved exterior, while those forming the crest of the dam and the down-stream face are covered by a flat concrete deck paved with stone set in the mortar.

When it is guarded against, frost action of itself has no great effect on concrete in dams, but as a secondary factor it can do great damage.

B. F. JAKOBSEN,* M. AM. SOC. C. E. (by letter).—It would be hazardous in the extreme to conclude anything from the behavior of a single structure, and more especially since other similar structures subject to equally severe conditions, have shown no sign of deterioration.†

It would seem to the writer that some experiments on concrete made up of the material used in the dam and on other concrete, by subjecting it to

* Engr. in Chg. of Dams, Los Angeles County Flood Control Dist., San Fernando, Calif.

† See letter by M. M. O'Shaughnessy, M. Am. Soc. C. E., in *Engineering News-Record*, July 30, 1925, p. 194, stating that the Lake Eleanor multiple-arch dam located at an elevation of 4 660 ft. shows no sign of deterioration; also a letter by Spencer W. Stewart, M. Am. Soc. C. E., in *Engineering News-Record*, September 3, 1925, p. 396. Mr. Stewart states that a record of about seventy-five Ambursen dams built in Canada, in the Northwest, and in the New England States, shows that only three dams have deteriorated. Many of them are subject to minimum temperatures of between 40 and 50° below zero Fahr.; the oldest of these dams was built 22 years ago.

water pressure and freezing, could not have failed to throw some light on the subject; they might have furnished some explanation as to why this structure has deteriorated while other structures have not. The mere statement that a structure has deteriorated, while another has not, is of no great value to the engineer, who needs to know the why and wherefore—the *modus operandi*, so to speak—so that he can guard against a repetition.

The authors state that the gravity section is heavier than "standard profiles" of gravity dams. There are gravity dams in existence with base widths of from 64% to in excess of 90% of the height. What is meant by "standard profile"? A dam with a base width of 64% has no provision for uplift; but that can hardly be termed standard. In Italy and Switzerland, where there are Government provisions governing the construction and design of gravity dams, uplift must be included in the design and this was also true in France until recently. The new French regulations do not prescribe uplift, if the dam is provided with proper drainage, not only at the foundation, but throughout the structure, since a construction joint may be quite as weak as the joint at the foundation. The new French regulations, however, prescribe such moderate maximum stresses, that for a dam of any magnitude it is the same as if uplift were included. When a base width in excess of 81% is obtained it is quite immaterial whether this is accomplished by considering uplift or by lowering the maximum permissible stress.

The authors also state:

"It is not believed that stresses can be induced in the gravity section by pressures on the upper 30 ft. of the dam, because such pressures will be transmitted direct to the buttresses."

This may be an important question, however, and therefore well worth investigating, rather than leaving it to an offhand assumption. The multiple-arch dam is a much more elastic structure than a gravity dam and the writer's guess would be that the gravity dam might share to a considerable extent the load on the upper 30 ft. of the multiple-arch dam. Since a gravity dam—even the full section having a base width in excess of 81%—has a rather small factor of safety, an additional load applied at the crest and for which no allowance has been made might be a serious matter.

The writer wonders if any provisions were made to prevent the closing of the drain-pipes from the foundation by frost and what the factor of safety of the gravity dam would be in case these drains should become clogged by ice, or in any other manner, and the dam be subjected to uplift?

In the writer's opinion it is of no importance whether the maximum stress is 70 lb. per sq. in., as stated by the authors, or 170 lb. per sq. in., for this does not affect the safety factor in the least. The question is whether tension can be produced in the up-stream face when the dam is affected by adverse conditions.

J. D. Galloway,* M. Am. Soc. C. E. (by letter).—There seems to be no question but that the Gem Lake Dam was designed by Mr. Jorgensen in accordance with standard practice and that it was well built. The character

* Cons. Engr., San Francisco, Calif.

of the contractors would lead to the latter inference. All the usual tests of cement and sand were carried out.

The progressive failure of the structure is clearly presented by the authors. Over the entire length of the dam, in the zone covered by the winter fluctuations of the water surface, the concrete was disintegrated and the dam had to be rebuilt as described. The value of the paper lies in the description of the failure.

To make the proper interpretation of this or any other failure of an engineering structure is seemingly more difficult than to describe the failure. It is of interest to read the various explanations given by different engineers. Practically all point to the quality of the concrete as the reason for the failure, the idea being that it was the internal composition of that material that caused the failure and not some cause independent of the concrete. Mr. Jorgensen thinks the cement itself broke down during the coldest weather. The question immediately arises as to why the cement in the concrete above the highest water did not break down. This concrete was subjected to lower temperatures than that in contact with water on one side. The Agnew Lake Dam did not fail and there the water was not lowered during the winter.

The evidence seems to the writer to be most plain and the explanation given by the authors correct. The disintegration of the concrete was due to the freezing of the water which had entered into the concrete from the reservoir. The force developed when water freezes is enormous and will easily disrupt concrete. In fact, the same agency is responsible for the wearing down of dense rock masses such as native granite in regions subject to temperatures below freezing. In the present case the thin concrete of the arches permitted the water in the reservoir to be frozen into the form of a layer of ice on the water face, and in addition allowed to freeze whatever water may have been in the voids and hair cracks of the concrete. It is conceivable that cracks would develop in the concrete when on the water face the temperature was that of the water of the reservoir and on the air face the temperature was that of the air, admittedly far below freezing.

The breaking down of concrete by freezing in dams and other structures is not unknown—in fact, is fairly common. Evidences of this have come under the writer's notice and several instances are cited in this discussion. It is unnecessary to look for the fault in the concrete—the cause is found in the type of dam and the operation of the reservoir. It should be noted that in this region the low-water season may occur in the winter when all ordinary springs and streams are frozen. It is then necessary to draw on the reservoir. During a winter, the water surface will alternately rise and fall.

With the conclusions of Mr. Flinn the writer desires to differ in every respect. The danger in engineering is not to over-emphasize a failure of this type; the contrary is the case. Usually, the effort is directed toward minimizing, or covering up, or explaining away the failure. It is to the credit of The Nevada-California Power Company that it permitted the authors to set forth in detail the failure as it occurred. In most cases the endeavor of the company manager is to suppress all knowledge of such a failure. It is partly by failures of this type that the Engineering Profession

advances. The Gem Lake Dam was a test to destruction of a full-sized dam under operating conditions—a test of the most valuable kind. Tests to destruction are always the best, especially if the progress of the test is observed. No others can yield such valuable results in the form of knowledge. This is the type of test to be carried out on the Stevenson Experimental Dam near Fresno, Calif.

Again, little if any information would be gained by counting the dams that have failed and those that have stood. Engineering is not a matter of majorities, one way or the other. For example, there was a time in the United States and elsewhere, when the compression members of metal bridges were made of cast iron. Many of them held together, but some did not. It took the crowning disaster at Ashtabula, Ohio, to teach engineers that cast iron was an improper material for the vital members of a bridge. Notwithstanding the numerous bridges of that type then standing, engineers eliminated cast iron. Too much emphasis cannot be given the Gem Lake failure. It teaches that under such severe climatic and operating conditions, the multiple-arch dam with its relatively thin concrete, should not be used.

The writer also wishes to call attention to what is believed to be an unfortunate tendency among engineers and is exemplified in the present instance. As soon as a concrete structure fails, a number of commentators feel that it is necessary to rush to the defense of concrete as such, and endeavor to explain the failure by citing some unusual defect. In this case, some think that the small percentage of clay and silt explains the whole matter. Possibly so, but the evidence is all to the contrary. As a rule, a small percentage of silt renders the concrete more water-tight. Comparison is made between the structure that failed and some ideal concrete that is sometimes obtained in the laboratory and rarely if ever in the structure as built. Engineering structures are built of the materials at hand. There are regions in the West, of volcanic or granite formation, where it is impossible to obtain anything but a medium quality of sand or rock. Construction, even under the best of supervision, is always subject to conditions that make the resulting structure far short of the ideal. This imperfect structure is the one that engineers have to deal with and that sometimes fails—not the ideal one for which everything is perfect. Although always striving for the very best, the Engineering Profession should design in the light of information obtained in actual practice as to just what degree of perfection may be realized in the structures built under ordinary conditions.

Mr. Jorgensen undoubtedly feels chagrined over the failure of the dam. The writer believes this to be largely unnecessary. His dam was well designed in accordance with good practice, and it was reasonably well built. It resisted the stresses of the water, but failed gradually under severe climatic conditions coupled with certain operating conditions not usual in many cases.

The value of the failure at Gem Lake will lie largely with those who interpret it correctly. To those who see in it merely another example of a defective type of concrete, to be added to the thousands of such examples already existing, the failure will be of no value. To those engineers who

find in the combination of thin concrete arches, severe climate, and rather unusual operating conditions an explanation of the failure, the lesson will be of the utmost value. Had Mr. Jorgensen had before him a similar example when designing the Gem Lake Dam it is conceivable that he would have adopted another type. Therein lies the value of the lesson of this failure.

I. OESTERBLOM,* M. AM. SOC. C. E. (by letter).—This paper is unusually interesting and raises again a question that is causing anxiety all over the world. Slow disintegration of concrete, substantially as described at the Gem Lake Dam, seems to cause even more trouble and anxiety in the tropics. Naturally, one is, therefore, led to ask: Is it really the cold that does the damage? This is usually assumed—it seems so obvious; but the obvious is not always correct and is very often far from correct.

In the case under discussion it may be the cause in part, but there also seems to be evidence of other causes. Under the heading, "History of Operation" (page 715), it is stated that: "During the eight years of operation of the structure, very few cracks occurred in it." This does not seem to be the behavior of cold weather which acts on entrapped water with a great deal of violence and speed. The force is evidently of a slower nature, but is none the less very destructive.

There is further evidence that it was not frost that caused the essential part of the destruction. Referring to the first paragraph under the heading, "Condition of Dam in 1924" (page 723), it is stated that in the intervening middle belt, the concrete had become dead and inert, but that a section at the bottom of each arch was little affected. If frost had been the cause, would not the effect have been general?

The subsequently recorded opinion of investigating engineers would seem to be inconsistent. If there was sufficient porosity to permit percolation, the destruction, by necessity, would have taken place during the first winter after construction, but this Mr. Jorgensen, himself, is able to disprove by very competent evidence—his own records—long before any trouble was anticipated. The paper also states that "during this time [two years] no change in the structure was apparent." The concrete evidently was most excellent and of the highest consistency.

Everything seems to point to a more insidious force than mere frost and water. The writer suggests the supplementary compressive forces due to the colloidal expansion of the silica gels that make up the cement.

It would be interesting to know where the cracks really occurred from the viewpoint of stress calculation based on such expansion, in addition to the usual effects of gravity and temperature. Such calculations, compared with the actual records of the cracks, would really be conclusive.

Colloidal expansion is also due to presence of water but in very small quantities only and in such a state, ordinarily, that frost would have no direct effect. Even in the best of concrete this water is present and it is only necessary that the external water should vary—as it did in this case—in order to set up a change in colloidal absorption and subsequent variation of internal

* Care, Truscon Steel Co., Youngstown, Ohio.

stress. It is well known how destructive such variation of stress is; it is also known how slowly it works and that a catastrophe, due to such conditions, sometimes may come without warning and most unexpectedly if special precautions are not taken.

It would seem that the destruction recorded at the Gem Lake Dam is an ideal example of how a most excellent piece of construction is being slowly destroyed by colloidal action. At the same time it offers another opportunity for the engineer to take a decided step forward in technical progress.

Destruction similar to that recorded is general in the tropics even to such extent that in certain parts competent engineers refuse to use concrete for permanent construction. The basis there for the change in colloidal absorption and subsequent stress variation is the monsoonal change, with some months very dry and others very wet. A comparison between conditions and effects in cold and warm climates is interesting, and the matter has been discussed.*

B. C. Collier,† M. Am. Soc. C. E. (by letter).—The writer has read with great interest the various points brought out in the paper, the discussion regarding the failure of the concrete in Gem Lake Dam to withstand the effects of cold or other disintegrating agents, and especially the statement in the Synopsis (page 713) relative to the necessity of providing a "positive and permanent method of water-proofing concrete."

For several years he has been an earnest advocate of the necessity of proper selection of aggregates in order to insure the life of concrete structures, especially when subjected to water conditions. The statements made by several discussors, to the effect that a very probable cause of the failure was the use of limestone as the aggregate, are fully in accord with his experience. For a number of years he has believed that concrete will not withstand the continued action of water pressures especially when it is in thin slab sections. Examination of a number of reservoir and similar linings seems to prove that the continued passing of minute quantities of water through the pores of the concrete has the effect of a gradual attrition of the cement particles, and the consequent breaking down of the structure.

This discussion has also been of especial interest to the writer, because he was asked, during the period of construction of this dam, to pass opinion on the thickness of the gunite covering to be applied to the up-stream face as a water-proofing medium. At that time knowledge regarding such details was very limited, but the suggestion offered to the effect that at least 1 in. of reinforced gunite should be used, was not accepted as advisable. According to the paper a very thin coat ($\frac{3}{4}$ in. at the bottom and $\frac{1}{4}$ in. at the top) was applied without any reinforcement.

It is, therefore, of particular interest to note what the probable result might have been had a thicker coating of gunite with proper reinforcement been applied. All statements made are to the effect that for a period of almost three years no seepage through the dam was noted, but that subsequently this seepage occurred and gradually increased in volume. In a structure such as this the

* *Proceedings*, Institution of Engrs. (India), Yearly Meeting, Delhi, 1925.

† Pres. Cement Gun Company, Inc., Allentown, Pa.

alternate rise and fall of the water naturally causes severe stresses and a tendency to movement in the up-stream surface. Without question, this stress, continued over a period of time, caused the development of checks or cracks in the thin shell of unreinforced gunite, thereby ultimately defeating its original purpose, and allowing the passage of the water through the dense surface covering to the porous concrete beneath. Although, as stated, at the time at which this work was done little knowledge was extant regarding methods of water-proofing with gunite, later experiences have indicated that had a heavier coat of properly reinforced gunite been used, it would probably have prevented the seepage of the water and the consequent disintegration of the concrete whether due to faulty sand, faulty aggregate, porosity, or frost action.

It is interesting to note the suggestion made by some of the discussors that a coating of asphalt on the up-stream face would have resulted in the saving of the structure. In view of the statements recently made to the writer by several engineers in charge of large water-works activities, to the effect that not only is asphalt, when subjected to such conditions, not water-proof, but also that the life of a membrane coating under water does not exceed 15 or 20 years, doubt naturally arises as to the efficacy of such treatment. An engineer of wide experience in testing work has recently stated that he had definitely proven that asphalt would absorb moisture.

The writer has long been an ardent believer in the adequacy of Portland cement, when it is properly applied, as a resistant to water action. In order to produce this resistance it is necessary to provide a density of application similar to that of gunite. In addition, however, a broad experience in inspection of such structures as this had indicated the necessity of properly reinforcing the gunite slab in order to eliminate the breaking down by "weave" and constant movement; or to insure against the development of checking or cracking from internal stresses. Such authorities as F. E. Turneaure, M. Am. Soc. C. E., have clearly shown that cement structures should always be reinforced against the initial "setting up" stresses, and, in accordance therewith, the writer is now recommending that, for gunite slabs for water-proofing, reinforcement of not less than 0.3 of 1% should always be used, and that preferably, under such severe conditions as those at Gem Lake, the percentage should be as high as 0.5. This percentage of reinforcement placed equally in both directions will also provide an almost positive protection against cracks that will arise from expansion and contraction. With such reinforcement, and with a thickness of approximately 2 in. of gunite, experience has shown that the positively water-proof conditions specified by the authors as being necessary will be obtained.

Fred A. Noetzli,* Assoc. M. Am. Soc. C. E. (by letter).—It is well known that certain concretes are subject to deterioration from frost and other actions when they contain a considerable amount of moisture. The deterioration of concrete in sea water is a striking example of this phenomenon. In this case the physical effect of the expansion of the salt crystals which, under certain

* Cons. Hydr. Engr., Los Angeles, Calif.

conditions, may form in the pores of porous concrete in sea water is increased by chemical action. In hydraulic structures subjected to frost conditions the particles of water contained in the concrete will expand considerably when this water is transformed into ice, and thus tend to spall the concrete or weaken it otherwise. In passing from the liquid to the solid state, water expands with a force equal to about 150 tons per sq. ft. One cubic inch of water at 32° Fahr. occupies 1.09 cu. in. in the form of ice at 32° Fahr., which is equivalent in expansion to about one-tenth of the original volume. Porous concrete is, therefore, apt to suffer considerably from frost action. Dense concrete is known to be practically immune against it.

The authors point out that serious deterioration has taken place in the arches of the Gem Lake Dam where the concrete was subjected to water pressure at the time of low temperatures. On the other hand, M. M. O'Shaughnessy, M. Am. Soc. C. E., has stated* that in the Lake Eleanor multiple-arch dam, built in 1918, at an elevation of 4 660 ft., "there are no indications of wear and tear on this concrete structure, and the location is subject to severe frosts without any noticeable depreciation whatever in the concrete"; further, that, in his opinion, "the failure of the Gem Lake Dam is due not to design of structure, but to defective granite dust and materials being incorporated in the sloppy overwatered concrete during construction."

The writer can testify from personal knowledge that, also, the Bear Valley multiple-arch dam, located at an elevation of 6 700 ft., shows very little, if any, deterioration from frost action.

The Gem Lake Dam was built in 1915 and 1916. The Bear Valley and Lake Eleanor multiple-arch dams were constructed in 1912 and 1918, respectively. The Gem Lake Dam suffered from frost action, the other two dams are apparently as strong as at any time in their history, although the climatic conditions are similar at all three sites.

What was wrong, then, with the concrete of the Gem Lake Dam? The authors state that the upper 30 ft. of the existing arches and the buttresses supporting them appeared to be in perfect condition. Only that part of the arches which was below the reservoir level during the severe winter conditions was damaged. The temperature variations in the upper arches and the buttresses were at least as great as for the lower arches, and probably greater. It was, therefore, apparently not the low temperature itself that damaged the lower arches, but evidently the water which had been filtered into the concrete and which expanded when it was transformed into ice in freezing weather. Similar deterioration of concrete has been observed in many structures exposed to water and frost, in canal head-works as well as in some gravity dams many feet in thickness. In most of these cases, deterioration could easily be traced to porous concrete, or unsuitable aggregate, or both.

For the construction of the Gem Lake Dam, a lake sand, containing 3½% of clay and 1% of dirt, was mixed with the sand from the rock crusher in the proportion of about 75% of lake sand to 25% of crushed rock sand. It is not stated whether or not this sand was properly graded. The fact that compression tests on specimens made during construction showed an average

* In a letter to the Editor of *Engineering News-Record*, July 30, 1925.

crushing strength of the 1:2:4 concrete of only 900 lb. per sq. in. at the age of 14 days would indicate that the quality of the sand used was not at all what it should have been for this type of structure. Chemical decomposition of certain ingredients of the granite dust of the "crusher run" may also have been one of the reasons for the troubles with the dam.

The authors deserve credit for emphasizing the necessity of seriously considering frost action when designing dams for cold climates. The method of repair adopted for the Gem Lake Dam is about as radical and expensive as could be devised, but it may have been necessary in view of the advanced state of deterioration.

The writer believes that a satisfactory water-proofing of the arches, as originally attempted by the owners of the dam, could have been obtained in the following manner: At the time when the concrete of the arches showed signs of deterioration, the weak spots, leaky construction joints, etc., should have been cut out as far as practicable and re-filled, for instance, by means of a cement gun. Then a four-ply water-proof membrane of a Malthoid or Tartex product might have been applied in successive layers on the up-stream face of the arches, and a protection slab of concrete, from 12 to 18 in. in thickness, cast against the membrane. The four-ply water-proof membrane would have protected the old arches from infiltrations of water and thus have stopped further deterioration. The concrete protection slab, besides preventing injury to the membrane, would have added materially to the safety of the dam. A dense concrete such as was used successfully for the Lake Eleanor multiple-arch dam, referred to previously, would have insured the permanency of the protection slab. The cost of this concrete slab would have been relatively small as no forms would have been required for the under side of the slab. The water-tight membrane of the character described would have cost about 15 cents per sq. ft. in place. The quantity of concrete for a 12-in. protection slab from the bed-rock up to an elevation of 30 ft. below the crest of the dam, would have been approximately 1 250 cu. yd. This compares with the 12 000 cu. yd. used for the method of repair adopted by the owners of the dam. The multiple-arch type of dam has the advantage of offering easy accessibility to all parts of the structure. Repairs may be made, therefore, before the danger point has been reached.

In what follows, the writer wishes to make a few suggestions which may be helpful for avoiding in future thin dams some of the difficulties that have arisen in connection with the Gem Lake Dam.

The writer agrees with the authors that an impervious concrete will prevent the deterioration of concrete subjected to water pressure and extreme low temperatures. A careful selection and the proper grading of sand and gravel or crushed rock will go a long way toward accomplishing this end. A somewhat richer mixture than the standard 1 : 2 : 4 proportion, say, about 1 : 1.8 : 3.5, will furnish a dense and strong concrete at a comparatively small increase in cost per cubic yard. Naturally, proper care in mixing and placing the concrete is also of great importance.

The arches of multiple-arch dams should be designed so that for shrinkage, rib-shortening, and maximum variations of temperature the tension stresses

in the arches are safely within the tensile strength of the concrete. A liberal quantity of reinforcing steel should be added for additional safety. Special attention should be given to the preparation of construction joints between successive horizontal layers of concrete. Leakage is most likely to occur in joints that have not been properly cleaned of laitance and foreign matter. Attack by frost would inevitably start sooner or later in such joints, and the deterioration of the concrete would continue progressively. A 4-in. strip of galvanized iron, placed from 3 to 4 in. from the up-stream side of the arches, across all horizontal as well as vertical construction joints, will prevent leakage through the joints.

In important structures the entire up-stream face of the arches should be covered with a layer of properly reinforced gunite from 1 in. to 2 in. thick. Thin layers of unreinforced gunite, such as was used on the Gem Lake Dam, are not effective very long.

In spite of the partial disintegration of the Gem Lake Dam, the writer firmly believes that safe and permanent structures of the thin arch and multiple-arch type may be built in locations subject to severe weather conditions. A proper design of the arches, suitable, well-graded concrete materials, and careful mixing and placing of the concrete will result in structures which will be safe, water-tight, and permanent. A considerable number of such dams were built years ago and are satisfactory in every way. A few others, like the Gem Lake Dam, have given trouble for reasons which are fairly well understood by this time and can easily be avoided in future constructions.

A multiple-arch dam that will be 160 ft. high and more than ½ mile long is now (December, 1925) under construction in the high Sierras at an elevation of 7 200 ft. The multiple-arch type was selected for this structure after careful investigation of the probable causes that may have been responsible for the troubles on Gem Lake Dam.

The authors seem to favor dams of the rock-fill type for locations where extremely low temperatures may occur. Such rock-fill dams naturally would have to depend for water-tightness on a concrete core wall or on a concrete face slab. A loose rock-fill is relatively porous and permits of more or less free air circulation, including "chimney effect", that is, allowing the warmer and lighter air in the interior of the mass to rise and escape at the top of the dam, thus drawing in the cold outside air through the lower porous parts of the fill. The thin slab of a concrete core or face wall of such a rock-fill dam, therefore, would undoubtedly be subjected to some frost action, especially near the water line, which, in case of porous or otherwise unsuitable concrete, sooner or later would unquestionably lead to progressive deterioration of the concrete. Inasmuch as there would be no simple way of observing such deterioration of the most vital part of a rock-fill dam, a "blow-out" of a weak spot and a possible failure of the structure might occur in time.

According to the authors the Gem Lake Dam was overtopped during a severe flood to a depth of about 3 in. along the entire crest of the dam. Such conditions might have been disastrous for a rock-fill dam.

If on Gem Lake a rock-fill dam had been constructed, and if for the core or face wall the same kind of aggregate had been used with the same class of

workmanship as for the multiple-arch dam, it is possible that the authors would have had occasion to describe the failure of a rock-fill dam instead of giving an interesting account of the successful repairs of a multiple-arch dam.

F. W. Scheidenhelm,* M. Am. Soc. C. E.—The discussion by Senator Luiggi is of exceeding interest to the speaker.

Senator Luiggi is of the land which presumably was the first to develop cement, namely, the Roman or puzzolan cements. From much that has been said and written (not by Senator Luiggi) one would infer that Roman cements, or more particularly the structures in which they were used, are everlasting. If so, an important art has been lost. However, the speaker suspects that the long life actually enjoyed by certain Roman puzzolan cement-bound structures is due to the fact that, except for the northern part, Italy is not subject to a rigorous climate. Freezing either does not occur or is of minor extent. In fact, the speaker's personal observation on the occasions of two visits to Italy lead him to believe that, under similar climatic conditions, Roman cement-bound structures behaved much like Portland cement-bound structures of the present age. In neither case does the cement seem to be perfect. Even Italy, as well as America, has its problems as to the durability of hydraulic structures.

A Fundamental Question

The Gem Lake Dam is a hollow concrete structure which, by reason of disintegration of its deck, has failed to perform its intended function. Long before the presentation of this interesting paper there was raised in the speaker's mind the fundamental question: Is a hollow dam of concrete an appropriate engineering structure, particularly for cold climates? This question carries with it the premise that the concrete is of a kind which is practicable of attainment under the present state of the art. The question applies to hollow concrete dams of both the multiple-arch and the reinforced concrete slab-deck types. Study of this question has led the speaker into a fairly broad investigation and the occasion seems opportune for briefly stating the results and presenting his deductions.

At the outset it may be stated broadly that the investigation has left the speaker optimistic as to the propriety of the use of concrete in hydraulic structures which are exposed to severe freezing and thawing and, until there is further and contrary evidence available, his answer to the foregoing question is definitely, even though qualifiedly, in the affirmative.

Gem Lake Dam Disintegration

First consider briefly the conditions obtaining in the case of the Gem Lake Dam. The design involves relatively thin arches. The thickness was apparently from about 16 to about 34 in. within the areas of the arched decks where disintegration occurred. However, relatively thin decks, whether in the form of arches or reinforced concrete flat slabs, constitute an essential feature of hollow concrete dams. If the decks were not relatively thin, then

* Cons. Engr. (Mead & Scheidenhelm), New York, N. Y.

hollow concrete dams would lose a considerable proportion of any economy which they may offer. It is fair to state that the design of the Gem Lake Dam follows the general lines accepted as structurally satisfactory for dams of its type.

As to the concrete of which the dam was originally built, the facts available from published information and private inquiry are none too detailed nor reassuring. Among other things it is not unlikely that the sand was considerably too fine. The coarse aggregate is questioned by certain interested engineers, but the information available is not such as to cause the speaker to attach any serious blame to it. The cement was presumably of a quality equal to the average of standard Portland cements and subject only to the qualifications that there is still much to be learned about cement in general and that undoubtedly there are some steps still to be taken for the improvement of the product.

The proportions in which the arch concrete is reported to have been mixed, namely, 1 part of cement to 6 of aggregate, might be adequate if the other factors of the manufacture of the concrete in point were satisfactory. However, it seems not unlikely that the fineness of the sand would have warranted a somewhat greater cement content. Moreover, a greater cement content would have been desirable, if indeed not imperative, in case the water-cement ratio was relatively high. As to the latter, the speaker does not doubt that in conformity with general practice at the time of the construction of the Gem Lake Dam (1916) the water content was too great. (If the builders of the Gem Lake Dam erred in this respect, let it be noted that many others, including the speaker, have made the same error.)

The speaker has been unable to learn that any particular attention was paid to the adequate curing of the concrete and agrees thoroughly with Mr. Merriman that conditions were most unfavorable for proper curing. Although impermeability and permanence of concrete are not direct functions of strength, nevertheless, it is pertinent to note that the 14-day strength of specimens of concrete tested during the original construction was only about 900 lb. per sq. in. in compression. That fact in itself was none too good an omen.

The outstanding characteristic of the disintegration of the concrete is that apparently the arch mortar has tended to change from the original texture and strength to a texture and strength approximating those of hard clay, or perhaps even chalk. The disintegration is reported to have been limited to the deck arches, the buttresses remaining in good condition. Although presumably there was a tendency to require greater workability and, hence, a wetter mix for the arch concrete than for the buttresses, yet this possibility and the disintegration of the arch do not necessarily indicate that the arch concrete was inferior to the buttress concrete—in fact, the buttress concrete is reported to have had a somewhat lower cement content. It seems clear only that the arch concrete was not of sufficiently good quality to withstand the severe physical conditions to which it was subjected. It is likely that if the buttress concrete were subjected to similar physical conditions for an equal length of time similar disintegration would result.

As pertinent to the causes of the disintegration and the possible means for preventing similar disintegration in other structures, it is to be noted that apparently the disintegration occurred only below the winter water surface of the reservoir. The reservoir is reported to have been drawn down regularly for the winter period to a level about 30 ft. lower than the "pool-full" level. Correspondingly, the upper limit of the disintegration is said to have been about 30 ft. below the top. It is, therefore, a reasonable deduction that percolating water from the reservoir was a prerequisite of the disintegration.

On the other hand, the authors report that "a section of the bottom of each arch was little affected." Clearly the absence of disintegration in the lower part of the arched decks cannot be attributed to the absence of percolating water. It is true that the lower part of each arched deck is thicker than that directly above, but such a difference in thickness is too gradual and too minor in amount to afford the explanation. Instead the speaker is strongly inclined to agree with the authors in attributing that better condition to the protection afforded by the drifts of snow which lodge against the lower parts of the decks.* Precipitation appears to be plentiful and must be in the form of snow even before the temperature becomes very low.

The absence of disintegration in the lower parts of the decks indicates that percolation of reservoir water through the concrete is not necessarily, or at least not rapidly, harmful provided there is no severe freezing at the under side of the deck. (Whether in the case of even the best quality of concrete permitting percolation of water such percolation might not *per se* cause some disintegration in the course of a number of years is another question.)

The speaker inclines to the view that the disintegration of the arched decks at Gem Lake was due to the combination of inferior concrete, percolating water, and severe freezing alternating with thawing, and that, in the absence of any one of these three unfavorable factors, disintegration would probably not have occurred. In fact, the number of unfavorable factors may practically be reduced to two, for if the concrete were impermeable percolation could not have occurred. It is of special significance that the down-stream faces of those parts of the arched decks which suffered damage were at no time of the year protected by structural or natural (snow) enclosure against severe freezing.

Successful Unenclosed Hollow Concrete Dams

Engineers may well take cognizance of the fact that there are in existence a number of hollow concrete dams, both of the multiple-arch and the reinforced concrete slab-deck types, which, despite their locations in regions of rigorous climate involving alternate freezing and thawing, have continued to perform their intended functions and which have suffered no disintegration or, if there has been any disintegration, such disintegration has been too insignificant to record. This fact indicates strongly that such difficulties as have been encountered with hollow concrete dams have not been due primarily

* Charles W. Comstock, M. Am. Soc. C. E., has informed the speaker that he was told by members of the local operating staff of the Power Company that the near-by 30-ft. Agnew Lake Dam becomes completely covered with snow on its down-stream side.

to the thin decks, whether in the form of arches or of reinforced concrete plates. Also, it shows that concrete can be produced which, in the accepted sense, is permanent and can successfully withstand the rigors of such climates.

Agnew Lake Dam, California.—Of existing hollow concrete dams which are performing their intended functions the two which have the most direct bearing on the Gem Lake case are the Agnew Lake Dam (the mate of the Gem Lake Dam) and the Lake Eleanor Dam. The Agnew Lake Dam was built by the same company, by the same construction organization, and at the same time as the Gem Lake structure, so that it, too, has had a life of more than nine years.* It is situated on the same stream, at an elevation only about 500 ft. lower; hence, both dams are subject to practically the same climatic conditions. The Agnew Lake Dam, however, is only 30 ft. high and is in a small gorge, so that its crest length is only 280 ft. In consequence, the dam is reported to be snowed in every winter, with the snow on the down-stream side extending to the very top of the structure. Mr. Jorgensen, the designer of both dams, states that Agnew Reservoir remains practically full throughout the winter.

The design of the Agnew Lake Dam is the same as that of the Gem Lake Dam. In view of the low height of the Agnew Lake Dam, therefore, the thicknesses of arch decks subject to hydrostatic pressure from the reservoir during the winter are nowhere so great as in the cases of those decks of the Gem Lake Dam which have suffered disintegration.

The concrete of the Agnew Lake structure is practically the same as that at Gem Lake. The coarse aggregate was taken from Gem Lake to Agnew Lake. The sand apparently involved the only difference from the Gem Lake construction, as about one-half of it came from Gem Lake and the remainder from Agnew Lake itself. Hence in spots, at least, even the sand must have been the same as at Gem Lake; on the other hand, Mr. Jorgensen was of the impression that the local Agnew Lake sand was of the poorer quality. As regards water and cement contents and the method of making the concrete, the reasonable assumption is that there was no difference between the two structures.

Nevertheless, competent engineer observers agree that the Agnew Lake Dam is in first-class condition, having suffered no disintegration. Why, then, the difference? Presumably the unfavorable conditions as regards quality of concrete, percolation of water, and severity of climate are essentially the same. The one marked difference appears to be that involved in the complete snow covering, thus obviating severe freezing. The speaker agrees with Mr. Jorgensen in attributing the good condition of the Agnew Lake Dam to its snow cover protecting the down-stream faces of the arched decks.

Lake Eleanor Dam, California.—The Lake Eleanor Dam, of the City of San Francisco, located on a tributary of the Tuolumne River, is likewise of the multiple-arch type and is of approximately the same height, namely, 70 ft. gross against the 80 ft. gross height of the Gem Lake Dam. Lake Eleanor

* "Multiple-Arch Dams on Rush Creek, California," L. R. Jorgensen, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), p. 850.

Dam is on the western slope of the Sierra Nevadas at an altitude of only about 4 650 ft. above sea level, whereas the Gem Lake and Agnew Lake Dams are on the eastern slope of the same mountains at elevations about 4 000 ft. higher. Nevertheless, the Lake Eleanor Dam, too, is subject to the most important and severe element of climatological conditions—alternate freezing and thawing; in fact, the conditions at the Lake Eleanor site are, not unlikely, more severe in that, by reason of the lower altitude, the cycle of freezing and thawing may occur more frequently within a given time.

The panels of Lake Eleanor Dam between centers of buttresses are 40 ft., just as in the case of Gem Lake. Its length is approximately 1 260 ft. It was built in 1917-18 and the same engineer had an important part in the design of each of the two structures. Although there are certain differences in the thickness of members, these are relatively minor; it is fair to state that structurally the two dams are essentially alike. Furthermore, the likeness extends to the conditions of operation, for in each case the water surface of the reservoir is reported to be comparatively low in winter. Possibly there is somewhat more fluctuation of water surface during the winter season in the Lake Eleanor Reservoir than in the Gem Lake structure. As in the case of the latter, the Lake Eleanor Dam has no spillway apron, nor any other type of enclosure of the space between buttresses. Its spillway is in a solid gravity part of the dam.

That there has been no disintegration has been stated* by Mr. O'Shaughnessy, according to whose report, the present multiple-arch dam at the Lake Eleanor site is in due course to be displaced by a rock-fill dam. The ultimate dam is to be between 150 and 175 ft. high.† Apparently, the designers consider a multiple-arch dam—or a buttressed arch dam, in the terminology which they properly prefer—as being either unsafe or uneconomical for so great a height as that ultimately intended for the Lake Eleanor site. Nevertheless, the arched decks of the existing dam are intended to be utilized and relied on to furnish the lower 70 ft. of water-tight up-stream diaphragm for the ultimate, high, rock-fill dam. Evidently, those in responsible charge entertain no doubts as to the durability of this particular multiple-arch dam. Mr. O'Shaughnessy characterizes the Lake Eleanor Dam as being a structural, as well as an economic, success.

Aziscohos Dam, Maine.—A successful unenclosed hollow concrete dam which has had a fairly long life is the Aziscohos Dam, built in 1910-11, on one of the tributaries of the Androscoggin River in Maine. It is about 75 ft. high and the hollow concrete part is about 500 ft. long, of which one-half is spillway. The spillway portion has an apron extending down about two-thirds the way from the spillway crest to the foundation rock, but not a sufficient distance to have a material effect on temperature conditions in that part of the dam as compared with the bulkhead part, where there is not even a partial enclosure on the down-stream side.

The deck is reinforced and has a flat surface on the up-stream side, but is arched only on the down-stream side. The minimum thickness of the deck

* *Engineering News-Record*, July 30, 1925, p. 194.

† *Loc. cit.*, September 4, 1919, p. 466.

slab at the crown of the arches is apparently about 30 in. and the maximum about 36 in. The deck concrete is said to be of 1 : 2.5 : 3 proportions. A detailed description of the dam has been published.*

On the occasion of an inspection of the Aziscohos Dam in June, 1922, the only disintegration observed by the speaker was in the cases of two reinforced concrete brace-beams in the outlet bay, exposed to water and freezing. This good condition is confirmed by the observation of George C. Danforth, M. Am. Soc. C. E., who visited the structure in 1923. Admittedly twelve years is not a long life for a dam; on the other hand, in practically every case of serious disintegration of concrete the trouble has made itself known within ten years.

Eugenia Falls Development Dam, Ontario, Canada.—A hollow concrete dam exposed to temperature conditions approximating those of Gem Lake Dam is that of the Eugenia Falls development of the Hydro-Electric Power Commission of Ontario. This dam is of the Ambursen type, with the conventional reinforced concrete flat deck slab. It is situated about 70 miles northwest of Toronto, Ont., Canada, in latitude 44° 20′ North, where it is subject to a maximum annual variation of temperature of about 125° Fahr., ranging from 35° below zero to 90° above. The structure was completed in 1914, and thus has been in service for more than 11 years. It is about 50 ft. high and the hollow concrete part is about 1 250 ft. in length.

This part of the structure is entirely of bulkhead section, that is, open on the down-stream side, except a relatively short spillway the apron of which extends so short a distance below the spillway crest as to have negligible enclosing effect. During the winter the reservoir surface fluctuates throughout the upper one-third of the height of the dam. In this part of the structure the reinforced concrete deck slabs are about 16 in. thick. The deck concrete is of 1 : 2 : 4 proportion, the minimum thickness being 12 in.

In the main the foregoing information regarding the Eugenia Falls Dam was furnished by the Chief Engineer of the Commission, F. A. Gaby, M. Am. Soc. C. E. Mr. Gaby has further stated that recent inspection showed that the lower faces of the deck slabs have not disintegrated. A minor amount of flaking is said to be in evidence in the upper part of the up-stream face of the deck at some of the construction joints, this being attributed to the fact that the upper parts of many of the deck slabs were poured during the unfavorable temperatures of December. From the performance of this structure Mr. Gaby infers that concrete with proper proportioning and good mixing, so as to result in a dense product, will successfully withstand rigorous conditions of cold.

Enclosed Hollow Concrete Dams

One need not be surprised that there are various instances of hollow concrete dams (generally of the Ambursen type) which are completely enclosed along the down-stream faces of the buttresses and which have suffered no disintegration of concrete. In general, the enclosure results from the use of a complete spillway apron. An example of such structures, which has been visited in person by the speaker, is the Warrior Ridge Dam of the Penn Central Light and Power Company, on the Frankstown Branch of the Juniata River in Penn-

* *Engineering News*, March 9, 1911, pp. 288-291.

sylvania. This structure, about 30 ft. high and about 450 ft. long, is entirely of spillway section. It was constructed in 1906, thus having about 20 years service to its credit. The speaker's earlier observation as to absence of disintegration has been confirmed more recently by several other engineers.

Other Cases of Disintegration

To mention several other instructive cases where disintegration of concrete in dams has occurred, the first is an intake dam,* on Bishop Creek, in California, likewise on the eastern slope of the Sierra Nevadas, which suffered some disintegration on the down-stream face of the relatively thin section of the dam. This disintegration is reported to have been stopped by an earth back-fill against the down-stream face, thus preventing extreme temperature variations at the surface of the concrete.

Stony River Dam, West Virginia.—A case with which the speaker has had considerable personal contact is that of the hollow, reinforced concrete dam of the Ambursen type, on Stony River in West Virginia.† This dam is situated in the northern part of the State at an altitude 3 400 ft. above sea level, and is subject to severe freezing weather. It was constructed in 1912-13, failed in part in January, 1914, by the undermining of a too shallow cut-off wall, and was reconstructed in 1914-15, the reconstruction being under the supervision of the speaker.

In the concrete of both the original work and the reconstruction local sandstone was utilized for the coarse aggregate and likewise, by crushing and rolling, for the sand. In both cases the concrete was placed wet, one might say "slushy". The deck slabs were of 1 : 2 : 4 concrete. In no respect had the failure of the dam been due to the quality of the concrete; in fact, the good appearance of the original concrete in the spring of 1914 led to the decision to continue the use of local aggregates. The only excuse for the wet concrete lies in the fact that at that time such was the generally accepted practice.

As part of the reconstruction certain structural modifications were made. That part of the dam which had failed had been of open bulkhead section, but this was replaced by means of a second spillway, which, like the original spillway, has a completely enclosing apron on the down-stream side. Furthermore, in order to prevent freezing of the foundation drainage system, installed throughout the entire dam as part of the reconstruction, a concrete housing was built in all the previously open bulkhead bays, except only the bays of lesser height near the ends of the dam.‡ This housing consists of a curtain-wall of concrete, 12 in. thick, extending for about two-thirds of the height of the dam, together with a roof, 7 in. thick, extending horizontally from the top of the curtain-wall up stream to a junction with the deck.

The water-level fluctuates considerably. There is no fixed regimen which applies one winter after another.

During the first six years of the life of the original concrete, that is, until 1919, practically no evidence of disintegration was discernible. An inspection

* See p. 724.

† *Transactions*, Am. Soc. C. E., Vol. LXXXI (1917), pp. 907-1100.

‡ *Loc. cit.*, p. 955, Plate XII.

in the autumn of 1919, however, revealed some spalling of concrete on the upstream face of one of the original deck slabs near the center of the dam. It is presumed that the spalling originated during the severe winter of 1917-18, for which period a minimum temperature at the dam of 36° below zero Fahr., was reported. At the time of this inspection the spalling extended to a depth of about 5 in., the total thickness of the deck slab at this point being about 14 in. The spalling extended across the entire width of the deck slab (about 13 ft.); this fact, together with the further fact that this slab was the only one which had spalled, indicated that the disintegration may have been due to improper treatment of the concrete in that particular area. In 1921 the disintegrated slab was patched by superimposing a reinforced concrete slab. However, water penetrated between the original and superimposed slabs and the disintegration of the original slab continued, until finally the concrete crumbled and spalled on the lower or down-stream face of the slab in the same region, exposing the reinforcing steel.

In the meantime the two adjacent deck slabs were showing an increasing seepage through the concrete, thus casting doubt on the earlier indication that the slab first showing disintegration had been subject to an isolated instance of faulty workmanship. It is likely, however, that this particular part of the original deck was made of concrete in which, as compared with the remainder of the work, there was aggravated some error in making the concrete, as, for instance, excessive water-cement ratio or improper curing. The slabs in point were undoubtedly cast at a time when there was no freezing and possibly even during hot weather. Further repair work was done during 1924.

The area affected comprises only about one-fourth of 1% of the total deck area of the dam. It is important to note that there is no record of any disintegration below the roof of the housing (which applies to about three-fifths of the bulkhead sections of the dam), and, similarly, that there appears to be no disintegration whatsoever within either the old or the new spillway, and, therefore, completely enclosed, sections of the structure.

The fact that in the new spillway section there has been no disintegration, whether of deck, apron, or buttresses, is the more noteworthy because there has been considerable disintegration in the new spillway mat or channel flooring and also in the concrete placed during the reconstruction to reinforce the footings in that part of the dam which had not failed. This disintegration of the newer concrete was most marked on horizontal, or only slightly inclined, surfaces. The newer concrete which has suffered disintegration was undoubtedly made with too much water and too little cement, especially considering the quality of the aggregate used, and much of the concrete was placed during the winter. However, as compared with the new concrete which disintegrated, the concrete of the new spillway similarly contained too much water; the cement content for the spillway was greater only in the deck and apron ($1 : 2\frac{1}{4} : 3\frac{3}{4}$), but not in the buttresses, which, like the spillway channel mat, were of $1 : 2\frac{3}{4} : 4\frac{1}{4}$ mix.

The new deck concrete is not superior to that of the original decks. The former are somewhat thicker at the same elevations, but the speaker believes that it is primarily because of the complete enclosure that the new decks

withstood successfully the severe winter of 1917-18 and appear still to be in excellent condition.

In considering possible causes of the disintegration of the small area of deck slab in the original part of the dam, the question arises whether the difficulty may not have focused at and been due to the ice sheet covering the reservoir in winter. The water surface of the reservoir may actually have been at or slightly above the disintegrated area during the winter of 1917-18; moreover, the fact that the disintegration was first noticeable on the up-stream surface implies that there may have been some plucking action on the part of the ice, thus causing the observed spalling. However, other facts indicate that the ice sheet is not primarily responsible, but rather that it may have accentuated and made obvious a disintegration which had an earlier and more direct cause, namely, percolation of reservoir water through the deck slab and consequent freezing of the slab while in a saturated condition and unprotected on the down-stream side. One fact bearing on this matter is that the water level is by no means fixed seasonably; another is the experience in the case of the Gem Lake Dam where the disintegration extended for many feet below the winter water level of the reservoir.

Limitation of Disintegration to Unenclosed Dams.—The speaker is familiar with or has reports on disintegration of concrete in various other hollow concrete dams, both of the multiple-arch and reinforced concrete deck-slab types. However, persistent inquiry on his part has not revealed to date a single instance where disintegration of concrete has occurred in the decks of hollow concrete dams in which the down-stream faces of the decks are protected against severe freezing temperatures. He would not have been surprised to learn of some cases where obviously poor concrete had disintegrated even under such protected conditions, for, at best, a relatively thin slab subject to percolation from reservoir water is functioning under severe handicaps. The apparently complete absence of disintegration in enclosed parts of hollow concrete dams thus seems all the more remarkable.

Time and space do not warrant consideration of the disintegration of concrete which has taken place in altogether too many solid concrete dams, but it is worthy of note that such disintegration has occurred primarily in regions where there is severe freezing and in parts of dams which are fully exposed to the prevailing atmospheric conditions. Incidentally, the disintegration which has occurred in such dams appears to be greater as the surfaces approximate the horizontal.

Tentative Conclusion

In the light of the foregoing considerations, the speaker has reached the tentative conclusion that hollow concrete dams may be constructed and relied on to be relatively permanent, provided:

1.—The design is good and the proportions of the members liberal, especially as regards thickness of concrete protecting any reinforcing steel.

2.—The concrete is made of proper materials and in a proper manner, in accordance with present knowledge of the art. This involves capable, conscientious, rigid, and authoritative inspection.

3.—The dams are enclosed by means of diaphragms or heat-insulating walls located at or near the down-stream faces of the buttresses.

This tentative conclusion has been reached with full recognition of the fact that hollow concrete structures of necessity can tolerate disintegration to a less extent than solid concrete dams. The point is that disintegration is believed to be avoidable.

The speaker is sufficiently optimistic to believe that hollow concrete dams can be built in freezing climates successfully even without the enclosure. However, in view of the present status of the art of concrete construction and the contingencies which at best enter into any concrete work, he believes it is not conservative to rely entirely on obtaining the proper quality of concrete.

The most important quality to be attained in the deck concrete is, of course, water-tightness and this, in turn, is directly dependent on the density of the concrete itself. Without presuming to have any final knowledge on the point, the speaker doubts the practicability of obtaining in dams reliably water-tight concrete decks by means of surface or membrane coverings. Acknowledging that, in general, such coverings must be of some benefit, he prefers to place his main reliance on properly attained density of the concrete itself.

As to the character of the recommended enclosure, it should be noted that the form of the enclosure in the case of the Stony River Dam was governed by a different consideration. If this point of protecting the entire down-stream surface of the decks had been in mind, the enclosure would undoubtedly have been carried to the full height of the dam. A fact of importance developed in the functioning of the Stony River Dam is that within the enclosure the temperature does not fall materially below the freezing point. For instance, during the winter of 1915-16, the lowest temperature within the enclosed part of the dam was + 26° Fahr., whereas the lowest outdoor temperature was — 12° Fahr. Under these conditions no serious ice formation took place within the enclosure. Evidently, the radiation of heat from the reservoir water and from the foundation material into the enclosed space to a large extent offsets the radiation of heat through the enclosing diaphragm from the enclosed space into the outside air.

The down-stream insulating wall may be of concrete, either single or, better still, double; in the latter case, it affords an insulating air space within the wall. Hollow tile also seems appropriate for the purpose. The tile might be coated with stucco or gunite if a more pleasing appearance is desired. Incidentally, such a diaphragm, especially if of concrete, serves as a brace between buttresses.

The water-works reservoir dam, at Coatesville, Pa., of the Ambursen type and constructed under the supervision of Alexander Potter, Assoc. M. Am. Soc. C. E., in 1916-17, is understood to have its non-spilling concrete sections enclosed by an approximately vertical diaphragm of No. 28, four-rib "Hyrib" (a form of metal lathing), covered by a 1-in. thickness of plaster. However, it is further understood that such an enclosure is solely for architectural effect. The speaker also notes with interest Mr. Reed's statement

that one of the more recent Norwegian dams, the so-called Fjergeng Dam, is enclosed by an insulating wall on the down-stream side.

Miscellaneous

The speaker will await with much interest the result of the long time test by Nature as to the quality of the concrete used in the Gem Lake reconstruction work, especially in view of the somewhat low cement content. It would be of interest to know in what direction the diagonal cracks extend, that are mentioned by the authors as having occurred near the up-stream toes of three of the buttresses. It appears not unlikely that these cracks are of the same nature as those mentioned by Mr. Lippincott.

Charles W. Comstock,* M. Am. Soc. C. E.—The dominant note in the preceding discussion seems to be one of condemnation of the concrete used in the original construction. From this opinion the speaker dissents.

Records of concrete tests made during construction are not at hand, but the speaker has had access to them and has studied them carefully. To the best of his recollection compression tests on 6 by 12-in. cylinders at 28 days gave upward of 1 900 lb. per sq. in. Generally this would be regarded as satisfactory.

The speaker made an examination of Gem Lake and Agnew Lake Dams in May, 1924. The exposed portions of both dams, arches and buttresses, were carefully gone over with pick and hammer. Holes were drilled or dug at various places where indications were unsatisfactory. Except for the belt, about 30 ft. wide, of the arches of Gem Lake Dam, which has been described in the paper, the concrete was satisfactory in appearance and perfectly sound, ringing clearly when struck with a hammer. The buttresses at Gem Lake were in perfect condition. There was no spalling or other evidence of deterioration. Agnew Lake Dam was apparently in perfect condition throughout.

The speaker was not connected with the original construction of these dams and has no personal reason for defending the methods or materials used, but all the evidence indicates that the work done and the results obtained were as good as possible with the materials available.

Mr. Flinn has suggested that concrete materials should be selected with greater care than is usually bestowed upon them, that they should be carefully analyzed and scientifically proportioned. While the soundness of this suggestion will not be questioned, and it is certain that the best results cannot otherwise be obtained, it should be remembered that the constructor is nearly always limited by considerations of cost to the sand and coarse aggregate afforded by the surrounding territory. Cement is purchased under standard specifications with which all established brands comply. Defective concrete is rarely due to poor cement, although sometimes to improper manipulation.

In the speaker's opinion unsatisfactory concrete results more often from poor sand than from any other one cause, but in this material the constructor frequently has little or no choice. The alternative is to use the best that is available and be content with concrete of quality inferior to the ideal.

* Engr., Dwight P. Robinson & Co., Inc., New York, N. Y.

At the time of the reinforcement of Gem Lake Dam the sandpits used in the original construction were entirely submerged and inaccessible. The nearest natural sand would have had to be hauled 16 miles and then drawn up the inclines and barged across Agnew Lake, as was done with cement and lumber. The cost was prohibitive. It was necessary to grind the country rock to obtain sand. Whether the new concrete will be superior to the old remains to be seen.

A word as to Agnew Lake Dam—it is about 550 ft. lower than Gem Lake and was built to afford storage for one or two seasons prior to the completion of Gem Lake Dam. Since the completion of the upper dam water stands almost constantly at spillway level in Agnew Lake, thus protecting the water face against freezing on that side. Agnew Lake Dam is only a little more than 30 ft. high, and snowdrifts pile up to its full height on the lower side every winter. This affords ample protection against the freezing which has damaged a part of the Gem Lake structure, and accounts for the present sound condition of the lower dam.

After a careful study of both these dams the speaker was unable to find any other explanation of the peculiar nature and location of the damage than that given in the paper. He does not believe that the facts square with any assumption of poor concrete in the original construction. Whether it would have been better to have chosen an entirely different type of dam in the first place is another question.

George W. Howson,* Assoc. M. Am. Soc. C. E. (by letter).—This very frank discussion of the failure of the concrete in the arches of the Gem Lake Dam is extremely interesting to those engaged in the design and construction of dams, and carries a warning to those building thin exposed walls subject to water pressure. Many theories may be advanced as to the cause of the failure, but all are without conclusive proof. The most logical conclusions have been reached by the authors that moisture found its way into the concrete and there froze, causing disintegration. They rightfully conclude that field construction conditions make extremely difficult, if not impossible, a concrete absolutely impervious to the slightest penetration of moisture. It is gratifying to note that they conclude that the rock-fill type has many advantages over other types of dams in such a locality as Gem Lake. The writer believes that the virtues of the rock-fill dam extend further than to localities such as Gem Lake. It adapts itself to certain foundations, such as loose rock or cemented gravel in stream beds, requiring a minimum of excavation compared with masonry dams. There is a tendency toward the popularity of the rock-fill dam, probably due to the increasing familiarity of engineers with it. The writer has knowledge of five large proposed dams, all of the rock-fill type.

The rock-fill dam is not susceptible to such a refined degree of analysis as the multiple-arch, and owing to the very nature of the structure it is not necessary that it should be. The design is largely a matter of experience and not of refined computation.

* Res. Engr., Dix River Dam Project, Burgin, Ky.

Recently, the writer has been in charge of the construction of the super-rock-fill dam on the Dix River in Kentucky, designed by L. F. Harza, M. Am. Soc. C. E. This dam is 275 ft. high above the stream bed and 296 ft. above the foundation excavations. It contains 1 800 000 cu. yd. of material and is by far the master effort yet attempted in this type of construction.

When properly designed the rock-fill dam has, for a water-tight skin spread over the exposed up-stream face, a layer of reinforced concrete which, in the larger dams, generally varies from a maximum of 2 ft. to a minimum of 6 in. in thickness. In this respect the rock-fill type has something in common with the multiple-arch; both rely on a thin slab of concrete for water-tightness. In case of the rock-fill dam the concrete slab is protected from the elements on the lower side by the mass of the rock-fill over which it is spread and on which it relies for support. In the multiple-arch dam the thin arches are exposed on the lower side to the elements, and it is believed to be this exposure at Gem Lake that caused the failure.

The large rock-fill dam must have a concrete water-skin of a quality equal to, if not better than, the arches of a concrete structure. It must be as nearly impervious as possible and, in addition, be able to develop great strength to meet indeterminate stresses due to movements in the rock-fill itself. By proper construction of the body of the dam, movements of the fill may be partly controlled and reduced in amount, but nevertheless they are present in a greater or less degree, depending on the intelligence, guided by experience, used in the supervision of the construction of the fill. Once the apron of concrete is poured against the fill it must conform to the movements or take stresses sufficient to overcome them.

No concrete apron spread over the face of a large rock-fill dam would be able to resist major movements of the dam, nor is it ever intended that it should in a well-designed apron; but the apron may be so designed and so constructed that it can overcome tendencies to small local movements. This is a matter of the fine combination of proper design and good supervision so necessary in dam construction.

FRED O. DOLSON,* AND WALTER L. HUBER,† MEMBERS, AM. SOC. C. E. (by letter).—The writers' conclusion that a type of dam other than the multiple arch should be selected for a site such as that at Gem Lake is well demonstrated by the great diversity of opinion, as to causes of deterioration and as to remedies, displayed in the discussions. The wide interest shown by this full discussion is, to them, a source of satisfaction.

The positiveness and the wide diversity of opinion regarding the quality of concrete in the original structure would be amusing if it were not to some extent a reflection upon the profession. It is interesting to note that criticism of materials and workmanship of the original structure is offered by those who have not made a personal examination, while others, as, for instance, Mr. Comstock, who made a careful inspection of the structure previous to its repair, defend both and state that the evidence indicates that the work

* Vice-Pres. and Asst. Gen. Mgr., Southern Sierras Power Company, Riverside, Calif.
† Cons. Engr., San Francisco, Calif.

done and the results were as good as could be obtained with the materials available. Many points brought out in the discussion are worthy of further consideration.

Mr. Jorgensen contributes some interesting notes from the laboratory which made the analyses and tests of materials used in the original construction, as well as some interesting facts relating to that construction. He refers to some similar dams in Northern Sweden where low temperatures are encountered, and states that no damage has occurred in two years, although he admits that these dams are partly back-filled with earth as a protection against low temperatures.

In personal correspondence Mr. Jorgensen states that for each of the dams the back-fill behind the arch section covers that part which remains below the water level during cold weather. The writers are in entire accord with his conclusion that under severe freezing conditions it can hardly be expected that a plaster face applied to a thin concrete section will have a long life.

Mr. Jewett has offered some of the results of the many tests he made for The Nevada California Power Company in an effort to find a coating which would afford a water-tight face and, at the same time, would offer some degree of permanence. The defects of many so-called "water-proofing" compounds are intimated if not actually expressed. It will be noted that Mr. Jewett has, wisely in the writers' opinion, avoided the use of any trade names.

As an example of incorrect conclusions, Mr. Wheeler and Mr. Lane attribute the disintegration of the concrete to the presence of limestone and limestone dust. It is interesting to note that, in the lower part of the dam where the limestone (mentioned by Mr. Jorgensen as coming from the outlet tunnel) was used, no disintegration has occurred, whereas in that part of the dam which has disintegrated this limestone was not used.

Mr. Turner mentions certain concrete dams, much thinner than the Gem Lake Dam, and exposed at times to temperatures of 50° Fahr. below zero, but he does not state where these structures are located and at what season the water stored behind them is drawn down; consequently, the writers have been unable to make a comparison.

Mr. Wiggin credits the writers with calling attention for the first time in a pointed way to the fact that the immersed side of a thin exposed wall of concrete is in the same danger from the formation of ice crystals in its pores, under the influence of cold coming through from the dry side of the structure, as the side not immersed in a thick, leaky wall.

The writers note Mr. Howell's agreement with their conclusions and his reference to another dam of thin section of the flat-slab type which is deteriorating in the same manner.

Mr. Waugh, who is thoroughly familiar with all conditions at Gem Lake Dam, having assisted in its original construction as well as having had charge of its repair, attributes the primary cause of the disintegration to the failure of the gunite coating to prevent moisture from getting in back of the top

skin. His conclusions are substantially in accord with those of the writers, that the slightest penetration of water under such climatic conditions will be followed by deterioration. In the paper doubt was expressed as to whether, under conditions imposed for construction at such sites, impervious concrete can be obtained under present-day conditions. After several seasons of hard-earned experience in carrying the responsibility of construction in this difficult location, Mr. Waugh is even more positive that "it is inconceivable that any concrete will prevent the penetration of moisture."

Mr. Flinn's suggestion that Engineering Foundation should collect data concerning the behavior of other thin section dams is a valuable one, providing such data are complete and include temperature records, raising and lowering of water surfaces, etc. The writers feel that they have not overemphasized one example of deterioration. They have placed a notable example before the profession as completely as they have been able to present it and, through the discussion which has followed, as well as from other sources, have learned that this is by no means an isolated instance. In their opinion the serious tendency has too often been to observe complete silence in the case of sad experiences. Such experience is a rightful heritage of the Engineering Profession. Examples of similar structures that have stood (and the writers do not believe such success will be found in similar locations) will not afford a sense of security to the designer contemplating a new structure of this type to withstand similar conditions. To use the terse expression of another discussor, "engineering is not a matter of majorities".

Senator Luiggi gives some interesting observations on the Gleno Dam, a multiple-arch dam in Italy, which failed but not from causes such as the deterioration at Gem Lake Dam. The writers are glad to note his concurrence in their opinion that for such a locality as that at Gem Dam a rock-fill dam has many advantages over other types. Senator Luiggi questions the use of preparations such as "Ironite", or neat cement applied with a cement gun, in which view the writers concur. The effectiveness of these coatings is impaired by the expansion and contraction of the arches of the dam. He suggests methods of water-proofing the up-stream face of the dam; but since the methods suggested would necessitate the loss of at least one season's storage the cost is thus rendered greater than that of the method chosen.

The contribution of Mr. Merriman is, in the opinion of the writers, one of the most valuable and constructive of all those submitted because he points out the difficulty, not only of making good concrete up to the point of placing it, but the danger in not properly caring for concrete while it is curing. Neglect of concrete during curing occurs in the great majority of cases. Mr. Merriman is to be commended for bringing this out so pointedly.

Mr. O'Shaughnessy makes inquiry concerning the resultant losses sustained by the failure of the dam to function during certain seasons. During 1923, when an effort was made to water-proof the face of the structure, the loss was 10 000 acre-ft. of storage—a heavy loss when it is realized that at this locality this quantity amounts to approximately 12 000 000 potential kw-hr. The probability of having to face a similar, or even greater, loss in 1924 was

one of the deciding factors in favor of the method of repair chosen, which was carried out entirely from the down-stream side with no loss of storage. Mr. O'Shaughnessy states that in an arch dam it is necessary to have concrete of such density that it will be water-proof. In the opinion of the writers, concrete which will not absorb moisture to any degree whatever does not exist. Therefore, for climatic and operating conditions, such as obtain at Gem Lake Dam, it is absolutely necessary that the impervious surface be provided next to the water, as required by the principles of Mr. George L. Dillman which were referred to by Mr. O'Shaughnessy. If this face is provided and is permanent, it should be immaterial whether the concrete back of it is porous, so long as this concrete is able to withstand the stresses imposed upon it as a structural element of the dam. The writers are glad to note that Mr. O'Shaughnessy, with his extensive experience, commends a rock-fill type for a location such as that at Gem Lake Dam. It is also interesting to note that Mr. O'Shaughnessy plans ultimately to replace the buttressed arch dam at Lake Eleanor with one of a rock-fill type.

Mr. Lippincott very clearly expresses an idea, which the writers have endeavored to make plain, in his statement:

"Although a thin shell of concrete may be made water-tight against percolation, it is not possible by any process known to the writer to prevent it from absorbing water when in contact with it."

Observations of diagonal cracks in the buttresses of several multiple-arch dams have been made by the writers and they are in entire accord with Mr. Lippincott's views that these buttresses should be reinforced. These diagonal cracks, although not an important consideration in Gem Lake Dam, did exist near the up-stream toes of three of the buttresses. They are described and comments are made on their prevalence in other multiple-arch dams under "History of Operation" (page 716), in the writers' paper.

Mr. Reed makes inquiry concerning the conditions at Agnew Lake Dam, a smaller but similar multiple-arch dam on the same stream approximately 550 ft. lower in elevation. The concrete in Agnew Lake Dam has not disintegrated, but a small amount of leakage has developed. The gunite surface was removed in 1925 and various types of water-proofing compounds are now being tried on the up-stream face. In the opinion of the writers, deterioration of the concrete in Agnew Lake Dam has not occurred because some protection from extremely low temperatures is afforded by the snow with which the entire down-stream face is usually covered before the extreme cold weather occurs. In fact, the arches are banked full of snow at such times. The protection by such banking is apparent from the fact that a valve enclosed in a light board shaft does not freeze. Mr. Reed cites instances of multiple-arch and flat-deck dams that have been successful in Norway and Sweden, but indicates that protection against low temperatures has been provided. In the opinion of the writers such protection is essential.

Mr. Jakobsen opens his discussion with a statement that it would be hazardous in the extreme to conclude anything from the behavior of a single structure and more especially since other similar structures, subject to equally

severe conditions, have shown no sign of deterioration. His references to such structures are very meager indeed. It would be interesting to know the exact location of each and to have exact climatic and operating data for each. Mr. Jakobsen apparently has missed the statement in the "Introduction" (page 714) to the writers' paper to the effect that a large amount of mathematical discussion concerning multiple-arch dams had already been presented, and that it was not intended to include any addition to such material. The paper was a discussion of a failure and a method of repair, not of the theory of design. He can rest assured, however, that thorough analyses of the new section and of its effect on the remaining original structure were made, not only by the engineers of The Nevada California Power Company, but by the several consulting engineers who were employed and that no important considerations were left to off-hand assumption as he seems to infer.

Mr. Oesterblom seems to have missed some of the salient points of the paper. It is easy to understand why the middle belt of the arches was affected and why the section at the bottom, as well as at the top, was little affected. The up-stream faces of the lower parts of the arches are generally submerged to considerable depth in water, which insures somewhat warmer temperature. At the same time the backs of these arches are protected from extreme low temperature by the early snows drifting against them, thus forming a blanket; also, the arches themselves are much thicker at this point, thus affording less opportunity for the extreme cold to chill the material completely. These conditions, combined with the fact that the upper sections, from which water is always drawn before the advent of cold weather, do not show deterioration, seem to afford ample proof that frost action is the controlling cause.

Mr. Collier offers a conjecture on what results would have been obtained had a thicker coating of gunite with reinforcing been applied. He states, in closing, that with a thickness of approximately 2 in. of reinforced gunite, experience has shown that positive water-proof conditions will be obtained. It would be interesting to know the number of years during which such conditions have obtained on any structure and what conditions of expansion and contraction, or of deflection, occur in the operation of the structure. The writers believe that a gunite face of these specifications would have prevented percolation of water at first, but that over a period of years the gunite would have absorbed water and would have disintegrated to such an extent that water would have been admitted to the concrete back of it, unless it was replaced before this occurred.

With much of the discussion of Mr. Noetzli the writers disagree entirely. Mr. Noetzli refers to the Lake Eleanor Dam as being an entirely successful structure, but apparently he has not noted that Mr. O'Shaughnessy, from his discussion, plans ultimately to replace this structure with a rock-fill dam. He refers to the Bear Valley Dam as being a similar structure with similar climatic conditions. At the Bear Valley Dam the reservoir is not regulated for hydro-electric power development and is ordinarily not drawn down during the winter. At this location temperatures do not range as low as at Gem Lake, nor do long sustained extreme cold spells occur. Climatic conditions are not

comparable. After referring to the adopted method of repair as being as radical and expensive as could be devised, he launches off with confidence into advocacy of a method that was considered and discarded by the several engineers who studied this problem for the owners. His method is not believed to be feasible, even if considered only with relation to this single structure. However, Mr. Noetzli has completely missed an important practical point. Gem Lake Reservoir is now an important and necessary part of a great operating hydro-electric system. Any method of repairing Gem Lake Dam that would render the reservoir useless for a storage season, or even a part of such season, must prove expensive, due to loss of revenue; Mr. Noetzli's proposed method would cause such a loss. The method adopted avoided this great economic loss and thus it has a large differential in its favor. Mr. Noetzli does not consider that a loose rock-fill, which is relatively porous, will materially protect a concrete core wall or a concrete face slab against low temperatures. The writers know the facts to be otherwise. Proof can be secured by taking temperatures in tunnels through rock-fill dams where it will be found that at a distance of about 60 ft. in from the tunnel portal water will not freeze. For example, Mr. Dolson had occasion to construct a tunnel 157 ft. in length through the Skagway Dam near Victor, Colo.; later, he used this tunnel for the storage of foodstuffs and found that 60 ft. in from the tunnel portal no freezing occurred.

The several examples of hollow concrete dams described by Mr. Scheidenhelm are interesting, but those particular structures which are enclosed by a deck on the down-stream face are not comparable.

The writers wish to express their appreciation of the very clear statement from Mr. Galloway, interpreting the lessons from Gem Lake Dam which they have offered to the profession. He has most clearly and concisely pointed out the value which may be gained from a proper interpretation of an engineering failure.

A dam of the thin multiple-arch type, if it is to resist the conditions existing at sites such as at Gem Lake, must depend on a permanently impervious face; but, under present-day conditions, the writers know of no practical means by which such permanent imperviousness can be obtained nor has the discussion disclosed one. They, therefore, adhere to the opinion already expressed that for this or similar locations a dam of the rock-fill type has many advantages over other types.

In closing, the writers express their appreciation to the officers of The Nevada California Power Company for their courtesy and their frankness in granting permission to publish a full account of the investigation and repair of this structure. It has been a source of satisfaction to note expressions of appreciation of the Company's attitude from many of those who have contributed to the discussion.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1585

EXCESS CONDEMNATION IN CITY PLANNING

A SYMPOSIUM*

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* Presented at the meeting of the City Planning Division, New York, N. Y., January 22, 1925.

THE PRESENT STATUS OF EXCESS CONDEMNATION IN THE UNITED STATES

By Frank B. Williams,* Esq.

Excess condemnation is the technical remedy for a legal technicality, and being the outgrowth of bad logic, is difficult to explain clearly. It has been defined as the taking by eminent domain (and, therefore, without the consent of the owner) of more land than is needed for a public improvement. The practical man realizes that the development of the land outside the lines of the given improvement, so as to harmonize with the improvement within them, is essential to the attainment of the maximum usefulness of that improvement. In short, he knows that the outside land—not in fact "excess"—is needed for the improvement. This, the judges in the United States, apparently have not been able to realize. The result is that the Courts in this country have held as unconstitutional several statutes of excess condemnation, and in a few States constitutional amendments have been passed authorizing it.

There are two main purposes for which excess condemnation, as it may for convenience be called, can be used with advantage in public enterprises, as follows:

1.—It may be used for the profit to be obtained by the re-sale of the "excess" land.

2.—It may be used for the purpose of controlling the development of this "excess" land so that it and the main improvement may best be utilized each in connection with the other and both render the greatest service to the community.

A few illustrations may not be out of place. Excess condemnation is usually advocated either for the laying out of a new business street, or the widening of an existing one, in a neighborhood in the center of the city where the present developments are unsuitable or unsanitary and of low value; or for the construction of a thoroughfare in the outlying part of the town where the land is relatively unimproved. In these cases one of three results may occur, as follows:

First.—The neighboring land may be increased in value, in which case it would be good business, if there is sufficient capital and the matter can be handled with skill, to take this land and sell it to pay in part for the street. Failure to follow this course increases the cost of this and similar enterprises, to the public loss. In this age of keen competition no private business could long neglect such incidental economies and escape bankruptcy.

Second.—There may be parts or remnants of lots on one or both sides of the new street which are too small for independent development and which also shut off land from it and, therefore, from proper improvement. If the uniting of the remnant and the land in the rear is left to private initiative, considerable time may elapse during which proper buildings will

* Attorney-at-Law, New York, N. Y.

not be erected on the street, or cheap ones will be constructed, with a loss, in either case, to the city in taxable values and to the community in the lessened economic service of the land and the street. Often the tone of a street is in this way permanently lowered, the ugly temporary structures giving it a character which it never afterward outgrows.

Third.—Even if the contiguous land is not cut into remnants, the use to which it is put may lessen the value of the thoroughfare. A boulevard with ugly houses along it, is not the beautiful boulevard for which the city has spent its money so freely. A view which the boulevard was planned to exhibit may be cut off by a row of houses, whereas the erection of detached structures, with ample space between, would have left it open and the land would have sold as well if not better. A much needed industrial street with the adjoining land cut into small irregular lots is not capable of use for enterprises requiring large sites. The desired conditions may often be obtained by the proper subdivision and planning of the contiguous land, and its re-sale for private use without any legal restriction on it, because conditions suited to the street having once been created may be relied on, as a rule, to continue. Of course, legal restrictions in favor of the city may be placed on the property if deemed wise; and, when suitable, such restrictions often enhance the value of the property affected, as real estate dealers and owners have found by experience.

The thoroughfare is not the only improvement with which excess condemnation may be used; indeed it may be used as a part of most public works. Thus, a new city hall properly located and constructed will cause neighboring land values to increase; and the public by proper business management could secure the profit which its enterprise has created, condemning and re-selling the land for that purpose. A beautifully designed city hall is not the structure of beauty it should be unless it has an appropriate setting. Often the cheapest and indeed the only feasible way of obtaining some measure of setting is by the control of the architecture of near-by buildings. In similar ways the construction of parks may be made a source of income to offset in part their cost, their beauty and utility being increased by the control of the neighborhood which only excess condemnation will give.

Excess condemnation, and the various expressions used for it in foreign countries, are terms of modern origin; indeed the concept itself is the product of a legalistic age. Excess condemnation in fact, however, has been practiced very generally throughout the past. A monarch, not hampered by the division of powers, has found it quite natural to take abundant land for an enterprise he had undertaken, carrying it out as seems wise, and selling any surplus or giving it away with any necessary conditions attached to insure the completeness of the work. All will remember the humble citizen in one of Molière's plays, who, suddenly and enormously enriched, took a Master of Philosophy as an instructor, only to discover to his amazement that for forty years he had been speaking prose without knowing it. Thus, the administrators of former ages used excess condemnation with entire ease before, technically, such a thing existed. It is to excess condemnation in this sense that we owe the Place des Vosges in Paris, France, and many other beautiful and useful squares and streets in Europe.

Excess condemnation in the modern meaning of the expression seems to be French in origin, and is to be found in most foreign countries. It has been extensively practiced in England, usually for the profit that could be recouped to pay in part for the main improvement. It was provided for and considerably used in the State of New York under a statute enacted in 1812, but was held to be unconstitutional by the New York Courts in 1834, and virtually ceased to be practiced until revived by statutes in Massachusetts and Ohio in 1904 and subsequently in other States. These statutes were so criticized by the Courts that little use was made of them and constitutional amendments for the purpose were passed, first in Massachusetts (1911), and then in Ohio (1912), Wisconsin (1912), New York (1913), and Rhode Island (1916).

Whatever doubt there may have been with regard to the validity of the statutes of excess condemnation, under the State Constitutions as interpreted by the State Courts, there can be no doubt with regard to the validity of the constitutional amendments, which can be challenged only on the ground that they are contrary to the Constitution of the United States as construed by the Supreme Court. This may be asserted with confidence in spite of the fact that no case involving excess condemnation has ever come before the Supreme Court. There is an impression that the State Courts are often more narrow and technical than the United States Supreme Court. Be that as it may, this is not the reason for the confidence so generally felt by students of the subject. Land taken by excess condemnation, like land appropriated by condemnation proper, is acquired by eminent domain, which can be invoked only for a public purpose. In deciding what is a public purpose the Courts are guided both by history, which is past usage, and by current custom, which is history in the making. Local conditions also have great influence. In a country as large as the United States, past history, present opinion and custom, existing physical conditions, and surrounding circumstances, differ greatly in its widely separated sections. It cannot be assumed, therefore, that there is any one standard that can be set for the entire country, in accordance with which such questions can be decided; the locality must be studied and the question settled in the light of what it reveals. For instance, the Supreme Court has held that irrigation is a public purpose in a dry State like California, whereas it might not be so in the Eastern States. In so deciding, the United States Supreme Court followed the decision of the Supreme Court of California. The Supreme Court gives great weight in the decision of such matters to State laws and constitutional provisions as interpreted by the highest Court of the State in question, considering them to be most persuasive evidence of local conditions; so much so that in no case has the United States Court held invalid, under the National Constitution, a taking of property by eminent domain under a State law upheld by the highest Court of that State.

Excess condemnation has been utilized with success abroad, especially in England. In this country, it has been used comparatively little, in spite of the fact that the need of it, in numberless instances, is clear.

One of the reasons excess condemnation is not more used in the United States is that in many instances the procedure provided for it is not practical. City planners, in various parts of the country, although fully realizing the

advantages of excess condemnation in an enterprise, cannot see any way under existing laws to make it a success. A study of the laws of excess condemnation throughout the country, in consultation with city attorneys and engineers, would be a great service to the cause of city planning.

A second reason excess condemnation is so little resorted to in this country is the undue conservatism of city attorneys. They have an excessive reluctance to apply methods that are not already in general use, and thereby sacrifice the progress that can come only from improvement in method. The writer is also at times tempted to think that this comes in part from an unwillingness to do the work involved in any change in their usual routine.

A third reason excess condemnation is so little used in the solution of problems urgently calling for it, is the debt limit and the financial condition of cities. To take land several times in excess of that needed for the work within the narrow lines of the principal improvement calls for the outlay of many times the capital required for the main improvement; and, with many public enterprises urgently calling for funds, and a rigid debt limit almost if not quite reached, it seems impossible to get the necessary capital for the many years during which it will be tied up. A new thoroughfare, or similar undertaking, can be begun with a small expenditure, the cost being assessed against those especially benefited, to be paid by them in due course. In addition, there is the conservatism of city administrators who find it so much easier to do the work in the old way, even if thereby many of its advantages are sacrificed. There is also the unwillingness still existing among the mass of the voters, to the giving of authority to officials to enter into what is, in some of its aspects, a business transaction involving some speculative elements and the possibility of favoritism and corruption.

A fourth reason excess condemnation is not appreciated is a lack of knowledge of the advantages of city planning and the evils of unregulated growth. In the United States excess condemnation is not needed as it is abroad for the recoupment of expense to be obtained by it; in this country these advantages can be obtained by imposing local assessments on those especially benefited. In some States constitutional provisions forbid betterment assessment entirely or to a considerable extent. These constitutions can be changed, and in other States there are precedents for such amendments that have been tried and found to be sound. In some States the present statutes do not provide, adequately at least, for local assessment. Adequate legislation can be passed, with comparative ease, along lines adopted with success elsewhere. In many States the machinery exists and is constantly in use. Excess condemnation is needed for the harmonious planning and subsequent development of the land within and just outside, the lines of the main improvement; and for this there is no substitute for excess condemnation. In this country the law is often the impediment to progress. In city planning, there is the decision rendered in 1920 of the case of Windsor *vs.* Whitney by the highest Court in Connecticut. It is the so-called practical man who lags behind; and in this, as in so many other matters, the engineer who is at once a theorist and a man of affairs, has the privilege and the duty to lead the way and to receive as his reward the satisfaction of knowing that he has performed a public service.

THE USE OF EXCESS CONDEMNATION IN THE OPENING, WIDENING, AND EXTENSION OF STREETS

By A. L. Vedder,* Assoc. M. Am. Soc. C. E.

Cities have always found it necessary to open, widen, and extend streets. During the last few years, since the development of city planning, there has been more of this activity than formerly.

When these openings, widenings, and extensions are made in developed parts of cities, irregular pieces are often left and the cost of the part taken may be as great as that for the whole lot, the claim being sustained that "the damage to the part remaining equals the value of the whole property." Sometimes the parts of lots left are not sufficient to insure a proper development.

Excess condemnation has been suggested as a cure for these ills. The writer will give the law developed and instances where it has been found to be advantageous in Rochester, N. Y.

Law.—In 1913 a new paragraph was added to Section 7 of Article 1 of the Constitution of the State of New York, which provided that the Legislature might authorize cities to take "excess condemnation". In 1921 the City Charter was amended in accordance with this constitutional amendment, the Charter, as amended, reading as follows:

"*Sec. 2.—Corporate Powers.*—The city has power to receive by gift, grant, device, bequest, purchase, or condemnation proceedings, and to hold, lease, and convey such personal estate and such real estate within or without the limits of the city as the purposes of the corporation may require; to make and lay such restrictions, negative easements, or amenities, or to receive such covenants with respect to real property within or without the limits of the city as the purposes of the corporation may require; to take more land and property than is needed for actual construction in the laying out, widening, extending or relocating parks, public places, highways or streets, provided, however, that the additional land and property so taken shall be no more than sufficient to form suitable building sites abutting on such park, public place or street; to contract and be contracted with; to sue and defend, and to be sued in any Court; to make, have, use and alter at pleasure a common seal; to have and exercise all other rights and privileges conferred upon it by law or necessary to carry out its corporate functions and duties.

The following form of ordinance was then adopted:

"By Alderman.....

"Final Ordinance No.....

(Title.)

"The Common Council of the City of Rochester do ordain and determine that the following improvement is necessary and shall be made, to wit;

"Sec. 1. The Common Council hereby declares its intention to acquire the fee of the following described lands, which it deems necessary for municipal purposes, to wit:

(Description of lands actually taken for street purposes.)

"Sec. 2. The Common Council hereby declares its intention to acquire the following territory in fee absolute, and declares that it deems the same neces-

* Deputy Supt. of City Planning, Rochester, N. Y.

sary for municipal purposes, to wit: For street purposes; that it determines the same to be not needed for actual construction in laying out, widening, extending or relocating of said street, and determines that same is no more additional land and property than is sufficient to form suitable building sites abutting on said street.

(Description of land taken in excess.)

"Sec. 3. The whole expense of acquiring the land above described in Sections 1 and 2, estimated at.....(the estimate of the maximum cost per front foot to the property to be assessed therefor being.....; the estimate of the minimum cost per foot being.....) to be and become due and payable in.....annual installments and assessed upon.....

(Description of assessed territory.)

"Sec. 4. The Commissioner of Public Works is hereby directed to purchase said lands hereinbefore described at a price approved by the Board of Estimate and Apportionment, and in case that said Commissioner is unable to purchase said land at a price approved by said Board, the Corporation Counsel is hereby directed to institute condemnation proceedings for the acquirement of same.

"Sec. 5. This ordinance shall take effect immediately."

All sections of this ordinance, except the second, are similar to former ordinances for the opening, widening, and extending of streets, and conform to the requirements of the City Charter. This form was later modified to allow the separate assessment of Sections 1 and 2.

Ordinances were adopted and excess condemnation was used for the following purposes:

Example 1.—To Develop a Section in Conformity with a City Plan.—In 1919, the municipal authorities made a plan connecting four "dead-end" streets over a tract of vacant farm land in a moderately priced residential section. (See Fig. 1.) In 1920, an ordinance opening these streets was passed. The farm was owned by an elderly widow who asked an exorbitant price for the land required for streets, and who desired to subdivide the property in a manner different from that proposed.

If the land was to be acquired by condemnation, it was probable that a high price would be awarded by the Commissioners. The former ordinance was repealed and, in 1921, the first "excess" ordinance was passed, the land being acquired by negotiation at a rather high price.

A subdivision was surveyed and staked, and all but one lot advertised and sold at public auction, according to the provision of the City Charter. The price of the property was $17 224; the receipts from sales amounted to $16 750; the lot retained by the city is valued at $2 000; thus making a profit of about $1 500, besides opening the streets as planned without cost.

This is the only case which is near consummation and about which a financial statement can be made.

Example 2.—Reducing Cost of Opening.—A street, 80 ft. wide, connecting two existing streets in the commercial section of the city, was opened. This is part of a plan for a main thoroughfare extending from the extreme eastern to the extreme western limits of the city. This extension crossed lots diagonally and left the parts of lots not taken in an unsalable condition. (See Fig. 2.) Sufficient land was taken in excess to make good business sites on the

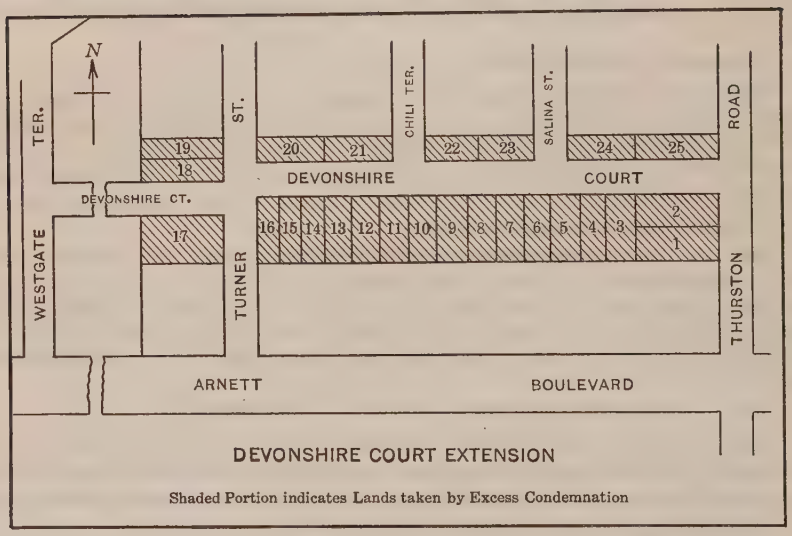

N
TER.
ST.
CHILI TER.
SALINA ST.
ROAD
19
18
20
21
22
23
24
25
DEVONSHIRE
COURT
DEVONSHIRE CT.
17
16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
WESTGATE
TURNER
THURSTON
ARNETT
BOULEVARD
DEVONSHIRE COURT EXTENSION
Shaded Portion indicates Lands taken by Excess Condemnation

FIG. 1.

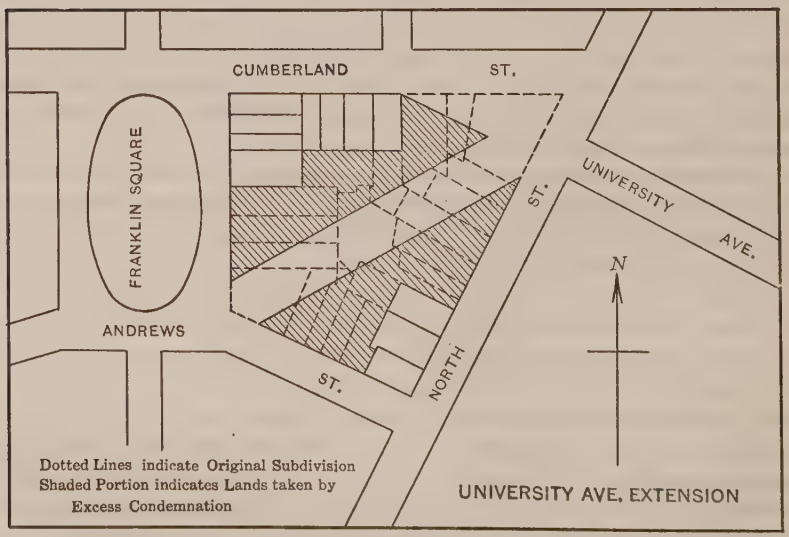

CUMBERLAND
ST.
FRANKLIN SQUARE
ANDREWS
ST.
NORTH
ST.
UNIVERSITY
AVE.
N
Dotted Lines indicate Original Subdivision
Shaded Portion indicates Lands taken by
Excess Condemnation
UNIVERSITY AVE. EXTENSION

FIG. 2.

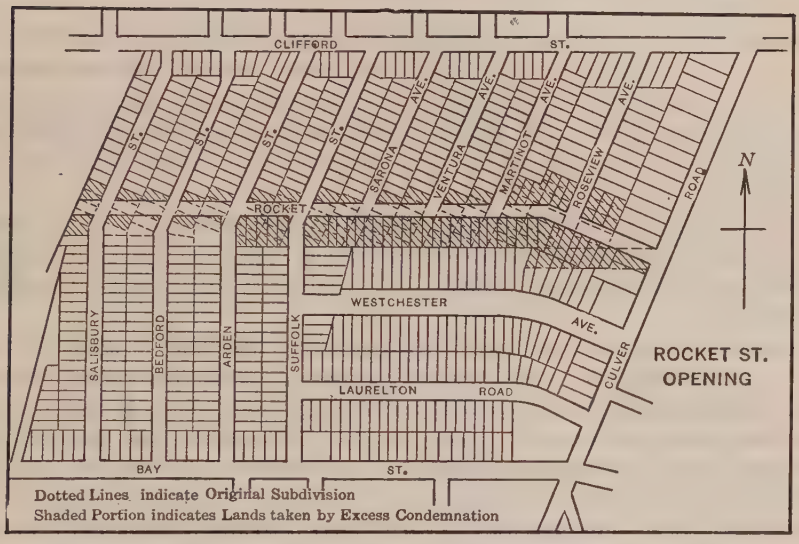

CLIFFORD
ST.
ROCKET
SARONA AVE.
VENTURA AVE.
MARTINOT AVE.
ROSEVIEW AVE.
ROAD
N
WESTCHESTER
AVE.
SALISBURY
BEDFORD
ARDEN
SUFFOLK
CULVER
ROCKET ST.
OPENING
LAURELTON
ROAD
BAY
ST.
Dotted Lines indicate Original Subdivision
Shaded Portion indicates Lands taken by Excess Condemnation

FIG. 3.

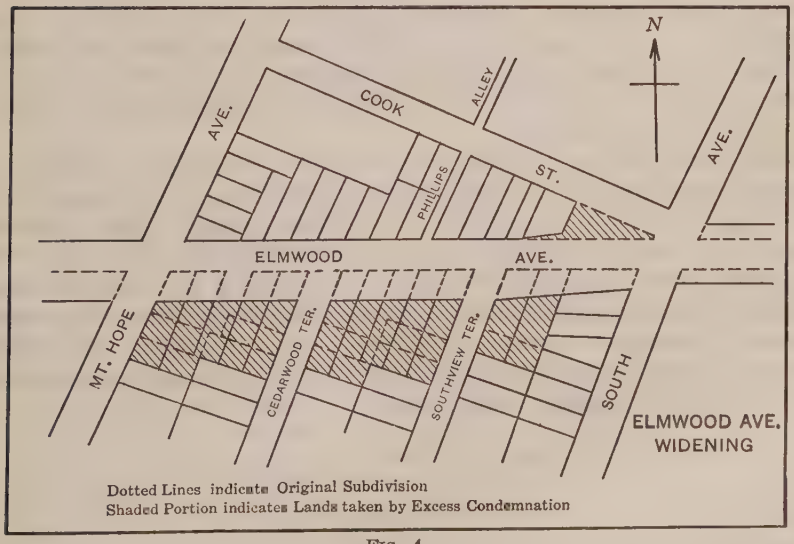

N
AVE.
COOK
ALLEY
ST.
AVE.
PHILLIPS
ELMWOOD
AVE.
MT. HOPE
CEDARWOOD TER.
SOUTHVIEW TER.
SOUTH
ELMWOOD AVE.
WIDENING
Dotted Lines indicate Original Subdivision
Shaded Portion indicates Lands taken by Excess Condemnation

FIG. 4.

adjoining streets. A new pavement is being laid, after which these business sites will be sold. The surrounding property having doubled in value since the property was purchased, the financial outcome of this transaction does not seem to be in doubt.

Example 3.—The Re-Arrangement of Excess Lands to Insure a Better Development.—A secondary thoroughfare was planned through old subdivisions, the lots of which were all separately owned but had not been developed from lack of sewer facilities. (See Fig. 3.) This street cut the lots diagonally and intersected "dead-end" streets, which made the use of excess condemnation necessary.

A large part of these lands was acquired by condemnation. All the land required has been obtained and a re-subdivision is being planned. As there are several survey adjustments involved, this subdivision will not be completed until later.

Example 4.—To Reduce Cost of Widening by the Use of Rear Lots in Place of Lots Fronting on the Street Widened.—A section of a street, part of a circumferential boulevard system, was widened from 49½ to 100 ft., all the land being taken off one side. (See Fig. 4.) This reduced the depth of the lots fronting on this street sufficiently to make them unfit for later development. These lots were also higher priced than those fronting on side streets extending off the boulevard in this subdivision. Recourse was again had to excess condemnation and not only the remainder of the lots fronting on the main thoroughfare, but also sufficient of those fronting on the side streets, were taken to give lots of proper depth. This area has been re-subdivided and staked.

Other Cases.—In numerous other widenings and extensions, the entire lot has been taken where only a part was necessary for the improvement under consideration. In every case the cost of the necessary part would have been as much as the entire lot. The intention is to dispose of the unrequired part to the abutting property owner. Any money received from such sales will reduce the cost of the improvement.

Observation.—A city, by placing restrictions regulating the setback and character of buildings on the land taken, can make "excess condemnation" of great value in the re-arrangement of abutting property when widening, extending, or opening streets in the development of the city plan. The increase in price of property caused by the publicity necessary to obtain ordinances make the cost of land taken by the city higher than if purchased privately. This applies to either negotiations or condemnations. Nevertheless, if used judiciously, "excess condemnation" is of advantage to the orderly development and re-arrangement of a city.

THE NEED AND THE SCOPE OF EXCESS CONDEMNATION

By Charles Wellford Leavitt,* M. Am. Soc. C. E.

Excess condemnation! It sounds harsh and gives the feeling that it conveys no promise of benevolence. By definition it is "The lawful power, by the exercise of the right of Eminent Domain, to obtain land in a greater area than that physically necessary for a purposed public improvement, although incidental to it." The writer will discuss several reasons why excess condemnation is most necessary in present-day building of cities, and also the extent to which it should be carried, and the results of its proper use.

In opening up a new street, it is necessary:

1—To prevent waste of property.
2—To conserve to the City the greatest possible return on ratable frontage.
3—To conserve to the City its share of the increment due to the improvement.
4—To plan so that adjacent land may be replotted into lots of reasonable and useful size.
5—To protect adjoining owners from assessments unwarranted by the probable use of their lands.

Figs. 5 and 6 are views which illustrate the many small triangular areas left in private ownership by the extension of Seventh Avenue to Varick Street, New York, N. Y. About 1918, this extension was cut diagonally across a number of streets to Varick Street to form a much-needed traffic artery. Unless combined with other property, these small parcels are capable of very limited usefulness to the owner, and yet, in order to conserve to the City its needed return on frontage and the necessary increment due to the improvement created, they must be made to yield tax return. As shown, the parcels are occupied by gasoline stations and small one-story buildings. Another popular use for such land adjoining much used thoroughfares is the advertising billboard, of greater commercial value than artistic, decorative, or educational merit.

It undoubtedly will be many years before private negotiation will succeed in re-arranging property lines so that a reasonable and useful depth and frontage can be obtained on this fine new street. Instances have been cited where nearly sixty years elapsed from the time the streets were widened until the small odd-shaped lots caused by the improvements were incorporated with adjoining properties, the small parcels remaining, in the meantime, of little use and of small return. The effect on the neighborhood containing them was stultifying, if not directly degrading, and the City as well as the individual

* Civ. and Landscape Engr. (Charles Wellford Leavitt & Son), New York, N. Y.

owners suffered rather than gained by the improvement which should have meant increased instead of depreciated values.

Figs. 7, 8, and 9 illustrate the viciousness of widening a street without contingent or excess condemnation. The narrow strips of privately owned land between the new street line and the abutting ownership, in some cases only 3 or 4 ft. wide, are too narrow to be improved by buildings—not even the odorous gasoline station can use them, and the only, but excessively apparent, use to which they can be put is that marvellous blighter of artistic hopes—the sign-board.

Public interest is affected not only indirectly, but very directly, in proportion as the expense of the improved street does not bring to the City the return in ratables that it should. Both the property owner and the City are sufferers, because the property owner is assessed for benefits which he can not use, and the City cannot reasonably assess for taxes on frontage incapable of development. Many instances might be cited similar to those illustrated, not only in New York, but elsewhere as well.

Through the kindness of Charles B. Ball, M. Am. Soc. C. E., Figs. 10 and 11 are presented to illustrate the necessity for excess or remnant condemnation where diagonal streets are opened through plotted and built-up districts. The view, Fig. 12, of the unfortunate condition of a narrow lot left after widening Michigan Avenue, Chicago, Ill., is a forcible illustration of the necessity of exercising excess condemnation in such cases. Fig. 13 shows Michigan Avenue during the progress of widening and extension.

To illustrate the use of excess condemnation as exercised at Portland, Ore., the City and School District No. 1 joined forces in acquiring a site for the U. S. Grant High School and also land for adjoining playground space and for park purposes. A total of 30 acres was purchased at a price of $99 124.70, or an average of $3 304 per acre for the raw land.

The profit obtained from this transaction is, as follows:

Gross receipts, 44 lots at $1 000, average		$44 000
Expenses:		
Bonded improvements	$24 800	
Brokerage, 12½% of $44 000	5 500	
Title insurance at $15 per lot	660	
Miscellaneous	440	$31 400
Profit, or refund to general fund		$12 600

The writer is not prepared to recommend this illustration of excess condemnation as he feels that it is carrying the idea beyond the limits of justification and is a trespass on the proper field of private enterprise.

An extension of Sixth Avenue, New York (Fig. 14), has been approved The result of this plan, if carried out as shown, will be a repetition of what happened in Lafayette Street, New York, which was widened in 1903 and made a main traffic artery. Fig. 15 shows this street in 1915, still undeveloped in a manner warranted by the expense of the improvement of widening; build-

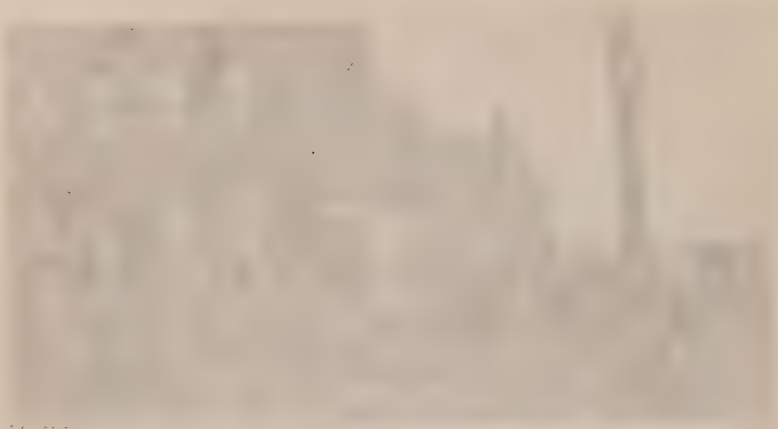
FIG. 5.—VIEW OF TRIANGULAR AREA LEFT IN PRIVATE OWNERSHIP BY SEVENTH AVENUE EXTENSION, NEW YORK, N. Y.

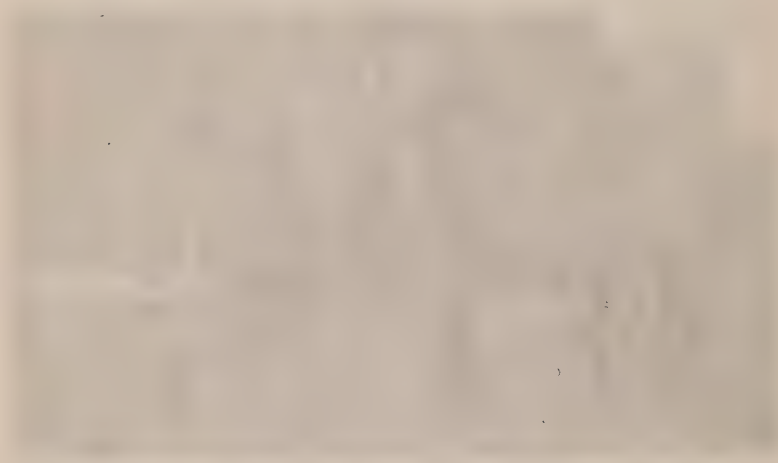
FIG. 6.—VIEW OF TRIANGULAR AREA LEFT IN PRIVATE OWNERSHIP BY SEVENTH AVENUE EXTENSION, NEW YORK, N. Y.

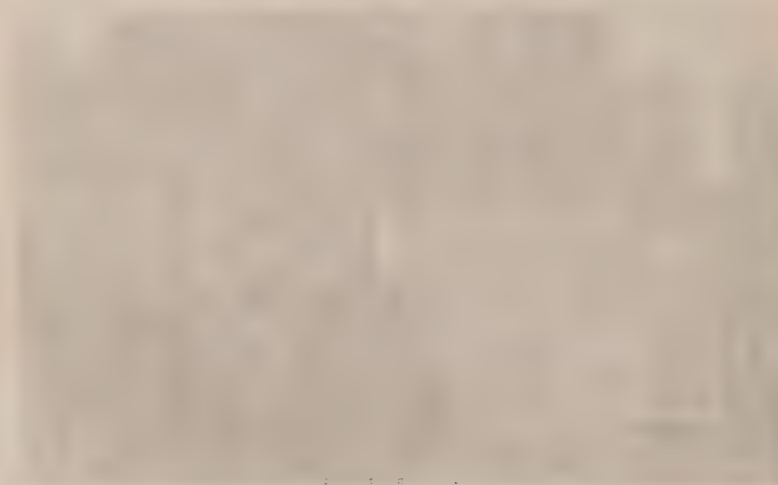
FIG. 7.—VIEW OF NARROW STRIP OF PRIVATELY OWNED LAND BETWEEN NEW STREET LINE AND ABUTTING OWNERSHIP.

FIG. 8.—NARROW STRIP OF PRIVATELY OWNED LAND LEFT AFTER STREET WIDENING.

FIG. 9.—VIEW OF NARROW STRIP OF PRIVATELY OWNED LAND BETWEEN NEW STREET LINE AND ABUTTING PROPERTY.







ings suitable for the location could not be constructed on lots 20 ft. wide and 23 ft. deep; or on a sliver of land approximately 70 ft. long, with a depth of 3 ft. on one end and 23 ft. on the other! The expedient of excess condemnation should have been resorted to, and the whole block acquired for the City, in order that it could have been re-subdivided into useful lots and sold under suitable restrictions.

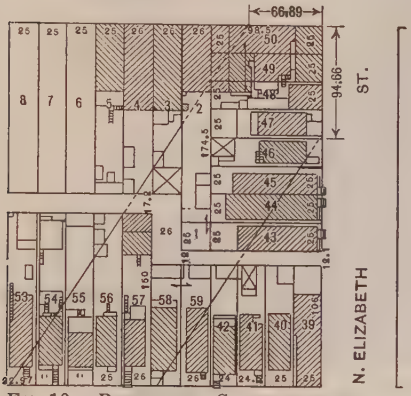


FIG. 10.—PERMANENT CONDEMNATIONS ON EXTENSION OF OGDEN AVENUE, CHICAGO, ILL.

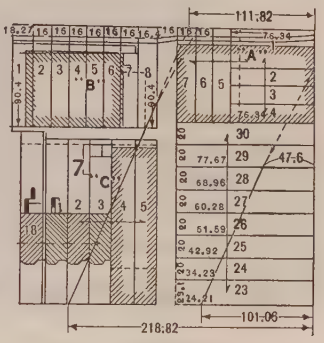


FIG. 11.—PERMANENT CONDEMNATIONS ON EXTENSION OF OGDEN AVENUE, CHICAGO, ILL.

It does not seem too strong a statement to label as a moral and economic crime, such loss to the City and to property owners. It is apparent how utterly useless small, isolated parcels of land become when left in private ownership. There is so much that can be done with the adjacent property if they are taken over by the City, and re-planned for proper development.

Fig. 16 shows the type of buildings which should be capable of construction anywhere along the Seventh Avenue Extension or on Varick Street, New York. The Grand Concourse in The Bronx is also an illustration of a fine avenue developed with suitable buildings made possible because of a land subdivision that provided lots of sufficient depth and frontage on the Parkway.

The cities of the United States are hesitating and backward about using excess condemnation. In Canada it is different; excess condemnation is exercised whenever it is deemed necessary for the public good. To illustrate, reference is made to Montreal, Que. The following figures are of interest in connection with the development of the George Etienne Cartier Square in Montreal:

Land purchased	164 504 sq. ft.
Land used as street, lots, and square	82 426 " "
Land sold	82 078 " "
Total purchase price	$82 252
Net proceeds of sales	99 032
Profit	$16 780

In other words the City acquired a park which, together with the abutting street area, was about 2 acres in extent; and the profit to the City was $16 000.

The City of Montreal has also opened one street and one boulevard with the following results:

St. Lawrence Boulevard Opening:	
Land taken	102 002 sq. ft.
Land used for street purposes	49 910 " "
Land sold	53 092 sq. ft.
Total purchase price	$690 570
Net returns from sales	716 194
Profit	$25 624
Cartier Street Opening:	
Land purchased	130 817 sq. ft.
Land used as street	55 637 " "
Land sold	75 180 sq. ft.
Total purchase price	$99 626
Net proceeds of sale	112 443
Profit	$12 817

This method has been so successful that the Bonaparte Boulevard has been carried forward in the same manner, namely, by condemning more property than was needed, and re-selling the remainder.

In London, England, the cost of opening the following streets was reduced by the re-sale of surplus property to the extent noted:

Carrick Street	72%
Southward Street	67%
Queen Victoria Street	53%
Northumberland Avenue	$600 000 profit
Kingsway	87%

These figures were taken from the Thirty-first and Thirty-second Annual Reports of the City Park Association of Philadelphia, Pa., furnished through the courtesy of Mr. Andrew Wright Crawford.

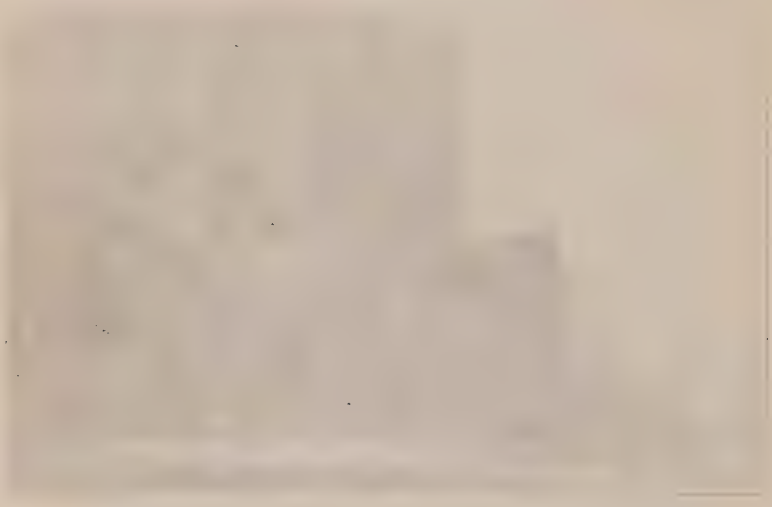
The following is quoted from these reports:

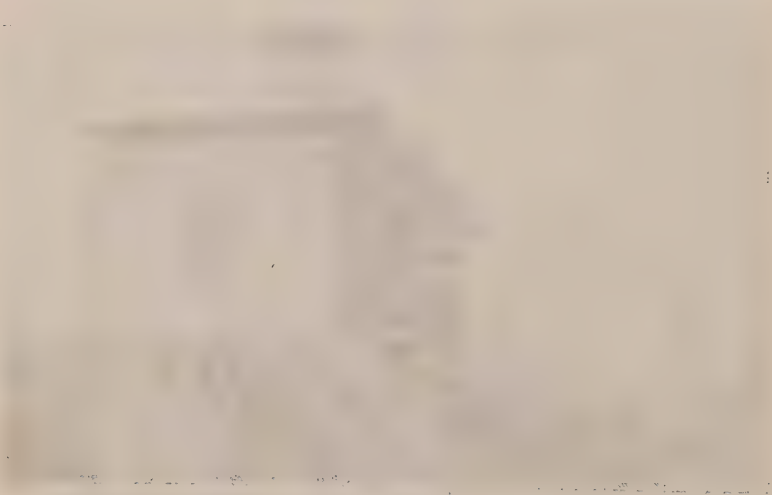
"A recalcitrant owner might refuse to sell, except at an exhorbitant price, or even refuse to sell altogether.

"The power of excess condemnation, in addition to the power of excess purchase, which the City already has, is thus necessary to prevent the citizens from failing to get adequate results from such an improvement and to prevent the City from being mulcted. The City usually has to pay as much as an entire property is worth, where it takes half of it or more; that being so, it ought in justice to be able to condemn the entire property. In addition to this reason for excess condemnation there is the consideration, that, when this power is exercised, the owner gets absolutely all that he is entitled to. He gets the full fair value of his property at the time it is taken; and, indeed, he often gets a high value for it. When the public takes property and a

FIG. 12.—WIDENING AND EXTENSION, MICHIGAN AVENUE, CHICAGO, ILL.

FIG. 13.—WIDENING AND EXTENSION, MICHIGAN AVENUE, CHICAGO, ILL.





jury passes upon it, the property owner is not the one who suffers; he is usually fully compensated. Even at that, most property owners want to keep as much as possible of their property, where the City only needs a portion; they realize, that if they do keep it, they will reap a harvest; because, if the City takes a third of the property for such an improvement as the Parkway, it is safe to say the City will pay at least half the value of the entire lot, and the remaining two-thirds will be worth more than the original whole lot was worth; and in the course of a few years it may double in value, owing to the improvement."

The reasons for requiring excess condemnation for street have been stated fully and illustrated. The scope of excess condemnation at the time of construction or improvement should include not only land contiguous to new and improved streets, but also land bordering on and in the immediate vicinity of highways, parks, parkways, public and semi-public building sites, sewerage systems, trunk lines, and aqueducts.

The shaded portions represent areas of building to be condemned and razed. The present streets will be included in the new avenue, which will be not less than 100 feet wide at any point. At Spring and Canal streets the avenue's width will exceed 200 feet.

FIG. 14.—SIXTH AVENUE EXTENSION, NEW YORK, N. Y.

For highways, parks, and parkways, the taking over, by the City, or other political body interested, of more property than is necessary for the actual construction contemplated would eliminate as far as possible contingent damage collectable from the City or other political units. It removes from any private owner the damages of unfair benefits assessments. It also gives to the authorities a control, not securable in other ways, over the use of land which, if deemed advisable, can be held as open spaces for the use of future generations.

Esthetics has been called the science of the suitable embellishment of practical things, of which practical things, public buildings and their sites are by no means the least. The use of the land surrounding public buildings is, in general, a subject on which in America evidently little real creative thought has been put. The establishment and the preservation of fitting environments for governmental buildings are possible only by securing not merely the immediate grounds but a goodly portion of the land contiguous, the replotting of all not needed for immediate surroundings, and the selling of lots with suitable restrictions as to size and development. Excess condemnation must be resorted to, if private purchase at a reasonable figure cannot be made.

At Camden, N. J., it was recently proposed to select a site for a future civic center. After much deliberation a location was found which, although of somewhat greater area than might be required strictly for the public buildings in the group, was procurable in a single parcel. The City bought this land in total at private sale for an economical price, and now is in the position of being able to use as much of it for public purposes as may be required. The remainder can be sold under such restrictions as to preserve the environment of the civic center in a fitting manner. Also, as soon as the civic center is realized, the property adjoining will increase in value and ratables to an amount calculable only on consideration of the probable growth of the city in size and importance. What Camden has done for her future through private purchase, other cities, not so fortunate, must do by excess condemnation.

In the case of sewerage trunk lines and aqueducts near a built-up community, excess condemnation eliminates the expense of contingent damages—a cost for which the municipality has nothing material to show—and in its stead furnishes land possible for future parks; or if parks are not deemed necessary at these points, land capable of sale at a price approximate to the cost of the improvement. In open country, the cost of constructing private crossings may equal or exceed the value of the contiguous land.

Fig. 17 illustrates a case in which an aqueduct passed through a farm. Owing to the fact that the farm was cut in two, contingent damage would probably amount to the full worth of the farm. Fig. 18 shows what could have resulted if the farm had been condemned, for probably an equal sum of money, taken over by the municipality, and then subdivided. The re-sale figures for the lots would have been greater than the purchase price, the municipality and the land owners near-by would have been benefited, and the aqueduct, instead of being a detriment, for which the city had to pay damages, would be regarded as a benefit. Most communities constantly are face to face with their ever-present bogey, the debt limit, and find themselves unable, in the event of other needed expenditures, to avail themselves of the benefit of excess condemnation because of the nearly approached debt limit. Land is one of the most tangible of assets, and it seems logical that a community should be enabled to bond lands acquired under the exercise of excess condemnation, which are in excess of the true physical requirements of the purposed improvement, and that such a debt should be considered as outside the community on which limitations are placed.

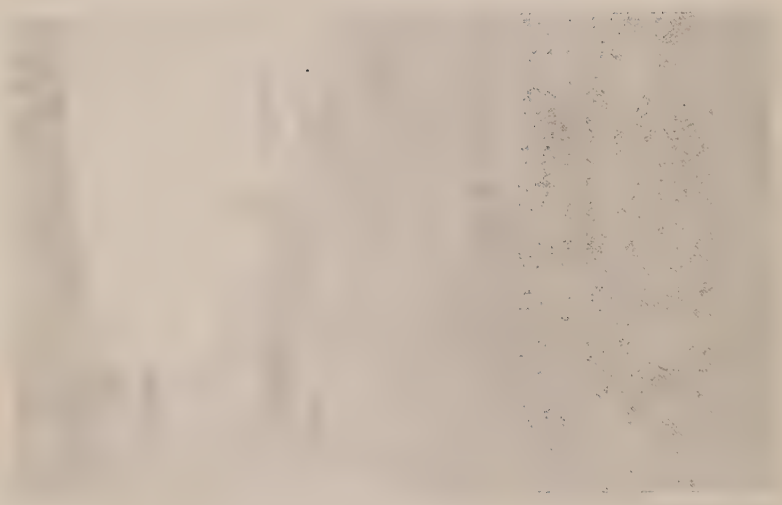
This was proposed to the people of Michigan in a constitutional amendment, as follows:

"Bonds may be issued to supply the funds to pay in whole or in part for the excess property so appropriated, but such bonds shall be a lien only on the property so acquired and they shall not be included in any limitation of the bonded indebtedness of such municipality."

It is hoped that this idea will be realized in many places and that such indebtedness will be regarded as perfectly justifiable.

FIG. 15.—WEST SIDE OF LAFAYETTE STREET, NEW YORK, N. Y., TWELVE YEARS AFTER BEING WIDENED.

FIG. 16.—VIEW ILLUSTRATING SUITABLE BUILDINGS MADE POSSIBLE BY PROPER LAND DEVELOPMENT.



Remnant condemnation impresses one as an expression which conveys the thought of possession by the municipality of under-sized parcels of property. This would be highly proper procedure, from the viewpoint of the land owner, who would be left with possession of the small useless piece. What real use, however, would such a piece be to a city, unless the City could combine it with other property and make a good lot? Ownership by the municipality of these remnants prevents the holding up of these narrow frontages for exorbitant prices, but one should go further than that, and be sure that the

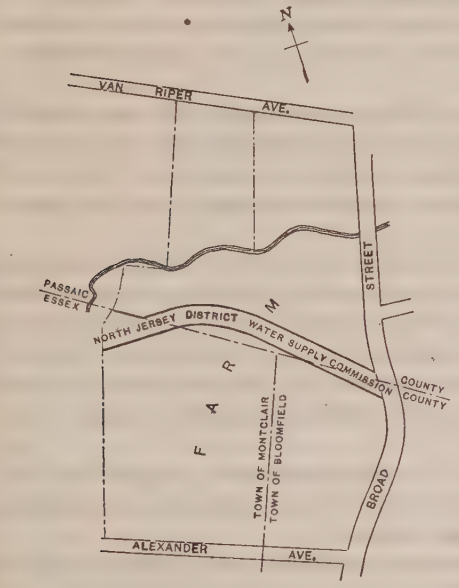


FIG. 17.

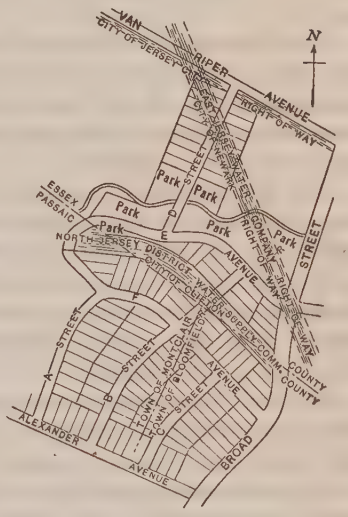


FIG. 18.

City gets its share of the profit for making the costly improvement which caused the small lots. To be safe in securing this share, the City could not take the poor end of the bargain. Remnant condemnation, with the meaning stated, is a poor half-way measure. It remains for all those interested in city planning and allied matters to educate the public authorities and the public in general, to realize the meaning and benefits of excess condemnation. How it works to the benefit of the average citizen, and enthusiastically to those in the real estate business, is self-evident because public economics mean lower taxes; the man who pays the taxes prefers to live where taxes are low, and the real estate man finds his best market there.

EXCESS CONDEMNATION IN MASSACHUSETTS

By Frederic H. Fay,* M. Am. Soc. C. E.

Synopsis

The story of excess condemnation in Massachusetts, as in most States, is a story of small accomplishment so far as the actual exercise of this power is concerned. In certain directions, however, Massachusetts has been a pioneer in formulating principles and establishing the right to use excess condemnation.

The Massachusetts Remnant Act of 1904 was the first legislation passed by any State applying the principles of excess condemnation to the taking of remnants of lots of which only portions are actually required for purposes of street improvements. This was followed shortly afterward by similar legislation in several other States.

The Massachusetts Constitutional Amendment of 1911, which is broader in scope than the Remnant Act of 1904, and permits the taking in fee of land sufficient to provide suitable building lots on both sides of a projected street or highway improvement, was the first recognition of the principle of excess condemnation for street improvement purposes in the Constitution of any State. This, too, was soon followed by constitutional amendments of like character in other States.

By the "Homestead Amendment" to the Constitution, adopted in 1915, Massachusetts has assumed the right to go into the business of providing homes for her citizens through State action.

In Massachusetts the principle has been established and upheld of applying excess condemnation to the taking of restrictive easements in private property for the public good.

These and other incidents in the story of excess condemnation in Massachusetts are given in some detail in this paper.

Excess Condemnation Applied to Land Reclamation

About seventy-five years ago, the State of Massachusetts undertook extensive work of land reclamation by a method which, although perhaps not excess condemnation in the strict interpretation of that term, is at least on the border line. This project was the reclamation of tidal flats along the Charles River, which now constitute the well-known Back Bay District of Boston. Although in most tide-water States, ownership of tidal flats rests in the State up to the high-water line, in the State of Massachusetts as well as in the State of Maine, owners of water-front property own the tidal flats to low-water mark provided the low-water mark is not more than 100 rods from the shore. Title to all flats beyond the 100-rod limit rests in the Commonwealth. This private ownership of tidal flats results from an ordinance passed

* (Fay, Spofford & Thorndike), Boston, Mass.

in 1641 under authority of the Colonial Charter of the Massachusetts Bay Colony.

The extensive tidal flats which have since been developed as the Back Bay Section of Boston were separated from the main "tidal-estuary" portion of the Charles River about a century ago by the building of a tide-mill dam on what is now the line of Beacon Street. Prior to 1850, the population of the City of Boston and of the adjoining Town of Roxbury had grown to such extent that the drainage from these communities into this tidal-flat basin had created a nuisance. Shortly after 1850, the Commonwealth undertook the reclamation of these flats as a measure of sanitation. As the simplest method of procedure, the State acquired title to the area by right of eminent domain. The flats were filled and proper drainage was provided. Suitable streets and public spaces were laid out, certain portions of the lands were granted by the Commonwealth to educational institutions, and the remaining lots were sold to private parties. The reclamation of this considerable territory was of benefit not only to the inhabitants of Boston and neighboring communities in the abolition of a nuisance, but the net result financially was a substantial profit to the State. Since that time other tidal flats along different sections of the water-front of Boston Harbor have been and are being similarly reclaimed by State agency.

Viewed from the standpoint of the abolition of a nuisance the reclamation of the Back Bay flats was not excess condemnation for the reason that the State took by eminent domain only such land as was necessary to fill in order to get rid of the nuisance. Considered as a land-reclamation project it is to be noted that the State took by eminent domain the fee in private property, filled the land, laid out streets, etc., and then in the case of a considerable portion of this land sold the fee to private parties other than the original owners. Possibly this project might have been carried out by condemning merely easements in the privately owned flats. The taking of the fee was certainly a more direct and satisfactory method, as it permitted the replotting of the land according to a well-considered plan. Probably this materially enhanced the value of the properties which were sold.

Excess Condemnation of Lot Remnants

Early Special Legislation.—The State of Massachusetts long ago passed certain special legislation dealing with the problem of remnants of land resulting from the laying out of street improvements. In 1865, an Act relating to the widening of Oliver Street, Boston (Chapter 159) was passed, which provided that any person owning an estate, part of which is taken for street uses may, "at any time before the estimation of damages * * * elect to surrender all of said estate to the city of Boston." Any remaining portion of an estate not used may be sold and "the net proceeds thereof shall be applied toward the expense of said widening and grading." This was not excess condemnation, as the City had no discretionary power; the choice rested with the owner of the land whether or not the whole of his estate should be taken.

In 1866, another Act was passed by the Massachusetts Legislature (Chapter 174) entitled "an act concerning the laying out, altering, widening and im-

proving the streets of Boston." Although the former Act of 1865 applied only to one particular street, this Act of 1866 was of broader scope and was intended to be general in its application as far as street improvements in Boston were concerned. Like the Special Act of 1865, this Act gave the owner of an estate the choice of surrendering the whole estate instead of the part to be taken for street improvements, provided his decision were made prior to the award of damages. Should he elect to surrender his entire estate, however, the City was not compelled to take it but could do so "if said Board of Aldermen shall then adjudge that public convenience and necessity require the taking of such estate." This may be considered excess taking by mutual consent, and it is not excess condemnation.

Massachusetts Commission on Excess Condemnation.—About twenty-five years ago the subject of excess condemnation became of much interest in Massachusetts. In 1903, the Legislature passed a Resolve (Resolves, Chapter 86) "to provide for the appointment of a committee to consider the matter of making public improvements under a more extensive exercise of the right of eminent domain than is now authorized by the constitution and statutes." The Special Commission created under this Resolve went abroad and made extended investigations of the workings of excess condemnation in Europe. It reported* the results of its investigation with its recommendations to the Legislature of 1904. The Commission submitted with its report a draft of a proposed statute embodying its recommendations in part and providing for the condemnation and replotting of remnants. Dr. Cushman, in his admirable book,† which is the first work on this subject to be published in the English language, states:

"There are five features of this proposed law which are worthy of note. First, it gave cities the power to condemn unusable remnants of land. In the second place, it authorized the city to offer such a remnant to the owner of the adjoining parcel with which the public authorities deemed it wise to unite the remnant at a price fixed by an elaborate system of appraisal. Third, if this offer was not accepted the city might condemn the whole or any portion of the property of such adjoining owner. Fourth, all the surplus land taken under the provisions of the Act should be sold at public auction. Fifth, the owner of a remnant the area of which was not more than one thousand square feet might compel the city to purchase it."

A sixth point, not mentioned by Dr. Cushman, is also of interest, namely, the draft provided that after the filing in Court of the plan and petition of a proposed improvement no damages should be awarded for any building thereafter erected upon the land in question; this feature had been used in connection with street improvements in London, England, to prevent buildings being erected or improvements to buildings being made for the sole purpose of enhancing the damage award collected from the City, but its constitutionality in America may be seriously questioned.

Dr. Cushman states further of the statute proposed by the Special Commission:

* For Report, see House Document 288, and for Supplemental Report, House Document 1096.

† "Excess Condemnation", by Robert Eugene Cushman (National Municipal League Series), 1917, p. 62.

"Such a law would allow a municipality to condemn not merely remnants of land but enough additional land to make possible an adequate replotting of such remnants into suitable building lots. The Legislature was unwilling to go to this extent, however, and contented itself with passing, in 1904, an act allowing cities to condemn merely the remnants of land left by the construction of an improvement."

The Massachusetts Remnant Act of 1904.—The Remnant Act of 1904 (Chapter 443) is an elaborate Act of thirty sections and defines procedure in minute detail. Section 2 of the Act is most significant and reads as follows:

"The Commonwealth, or any city in the Commonwealth so far as the territory within its limits is concerned, may in the manner hereinafter set forth, take in fee by right of eminent domain the whole of any estate, part of which is actually required for the laying out, alteration or location by it of any public work, if the remnant left after taking such part would from its size or shape be unsuited for the erection of suitable and appropriate buildings, and if public convenience and necessity require such taking."

The term "public work" in the Act is defined as meaning any public highway, square, open space, public playground, or park. As Dr. Cushman states:*

"It is important to note that this law does not confer upon the city or State unlimited discretion in the taking of remnants. The land which may be condemned in excess of actual need is only that which is not suitable for independent development. In other words, no broad power seems to be given to condemn all estates, any part of which, however small, may be needed for actual public use."

The Massachusetts Remnant Act of 1904 was the first important Act in the United States providing for the excess condemnation of remnants of land. It was followed by similar Acts in Ohio, Virginia, Connecticut, Pennsylvania, Maryland, Wisconsin, and New York. Thus, in this field of excess condemnation, Massachusetts was a pioneer. The constitutionality of the Massachusetts Remnant Act of 1904 was upheld by an opinion of the Supreme Court of the State, rendered March 4, 1910 (Opinions of Justices, 204 Mass., 616 *et seq.*). In passing on the constitutionality of certain other legislation then pending in the Legislature, the Massachusetts Supreme Court had occasion to refer to the Remnant Act of 1904, as follows:

"We are asked whether it would make any difference if the proposed statute contained provisions like those of the statute of 1904, chapter 443, section 6. In our opinion, given to the House of Representatives, we intimated that this statute is constitutional. In our judgment it goes to the very verge of constitutionality. The grounds on which we are inclined to sustain it have little relevancy to the stated purpose of the unusual provisions of the proposed statute. They are: First, that there can be no taking outside the location of the public work except of a remnant of an estate a part of which is actually required for the laying out, alteration, or location of the public work, and then only if the remnant left after taking such part would, from its size or shape, be 'unsuited for the erection of suitable and appropriate buildings'—in other words, only when there is a remnant that is too small or too ill shaped to be of any practical value for the use to which valuable land is commonly put; and, secondly, that such a remnant can be taken only upon an adjudication that public convenience and necessity require the taking. Unless it can be said that public convenience and necessity never can require the taking of

* "Excess Condemnation", p. 63.

such a remnant the statute cannot be declared unconstitutional. While it is plain that a city or town cannot take land outside a public work for speculative purposes, we can conceive of a remnant of an estate, a part of which is necessarily taken, which remnant is so small, or of such shape and of so little value, that the taking of it in the interest of economy or utility, or in some other public interest, may be fairly incidental and reasonably necessary in connection with the taking of land for the public work."

The authority conferred by the Massachusetts Remnant Act of 1904 was seldom used. The only instance known to the writer was in the City of Springfield, where it was utilized on five occasions both for new street layouts and for the widening of existing streets and is understood to have been found of advantage to the city and protective to property interests.

The 1904 Remnant Act was repealed by the Legislature in 1918 (Chapter 257, Section 203 of Acts of 1918), partly because it has been little used and more particularly because under the Constitutional Amendment of 1911 all the powers of the 1904 Act and much broader powers of excess condemnation could be conferred at any time by special legislation.

Constitutional Amendment of 1911 Applying Excess Condemnation to Street Improvements

Proceedings Leading to Proposal of Constitutional Amendment.—The limited power incurred by the Remnant Act of 1904 was brought out as the result of a report of the Joint Board on Metropolitan Improvements made to the Legislature in 1910. This Joint Board, consisting of four State Commissions, created by legislative resolve (Chapter 113, Resolves of 1909) to consider the question of needed improvements in Metropolitan Boston, advocated the building of an important new thoroughfare through the down-town portion of the city. This thoroughfare would not only provide an important artery for surface traffic, but would also be utilized as a route for a railroad tunnel, connecting the North Station and the South Station, the two passenger terminals of Boston. The building of this thoroughfare which would cut through closely built-up territory, would necessitate the taking of the whole or portions of more than 150 parcels of property. If the City were to proceed under the Remnant Act of 1904 and condemn in addition to the land actually required for street purposes only such lot remnants as were not suitable for the erection of proper buildings, it was felt that the results would by no means insure suitable development along the new street. The Joint Board was of the opinion that this was a case in which it seemed desirable to exercise broader powers of excess condemnation and to take, in addition to the remnants which might be condemned under the 1904 Act, other parcels which could be combined with the remnants and the whole replotted into suitable building lots.

The Joint Board reported:

"A thoroughfare such as suggested will fall far short of its full usefulness if it furnishes simply a convenient route for traffic. It cannot entirely fulfill commercial requirements unless individuals and corporations desiring to erect important business and office buildings or warehouses and manufacturing plants can secure suitable lots thereon."

Commenting on the unsuitability of remnants remaining after takings under the Remnant Act of 1904, the Board reports further:

"If, however, takings in addition to these remnants can be made wherever needed to provide areas sufficient for the construction of buildings of large size, then the street will more quickly and surely become an important business thoroughfare, and the commercial and industrial growth and prosperity of the city will be promoted in a larger and more adequate way."

In accordance with the recommendation of the Joint Board, legislation was proposed in 1910 to broaden the powers of excess condemnation conferred by the Remnant Act of 1904 so as to make possible more extensive property takings for the new thoroughfare. In accordance with Massachusetts practice the Legislature sought and secured from the Supreme Court an advance opinion as to the constitutionality of the proposed measure, if it became a law. The Court held that the measure proposed would be unconstitutional, although by indirection it upheld the Remnant Act of 1904, which the Court believed went to the very limits of constitutionality. This decision has been reported* as follows:

"39. Opinion of the Justices, 204 Mass., 607, 91 N. E. 405, 27 L. R. A. (N. S.)—483. The House of Representatives passed an order requiring the opinion of the Justices of the Supreme Judicial Court upon the following question (in substance): If the commercial interests and general prosperity of the inhabitants of the Commonwealth and particularly the city of Boston are dependent upon the existence in that city of facilities for the transaction of foreign and domestic trade and commerce, and chief among such facilities is a broad thoroughfare with adequate sites upon it for warehouses and other buildings, and no such street exists, and if such a street should be laid out in the ordinary way the adjoining land would be left divided into parcels of such unsuitable size and shape, that the proper facilities could not be furnished by the mere laying out of a street, but could only be secured by the concentration through the exercise of eminent domain of estates abutting thereon into parcels of suitable size and shape, is it within the constitutional power of the Legislature to authorize the taking by the city of land for such a thoroughfare and of so much land on both sides thereof as may be reasonably necessary to furnish the proper facilities and with a view to subsequent use of such land by private individuals in such manner as to secure the public interests referred to?

"The answer of the Court was in part as follows:

" 'The question seems to relate particularly to the power of the Legislature to take and use land outside of the proposed thoroughfare, for purposes which have no direct relation to the construction or use of the street for travel. It is presented upon the hypothesis that the desired facilities for the profitable use of the land can be secured only by the obliteration in whole or in part of the present lines of individual ownership along the street, the concentration, through the exercise of the power of eminent domain, of these abutting estates, in parcels of suitable size and shape, and the development or use of such parcels for warehouses, mercantile establishments and other buildings suited to the demands of trade and commerce. The question is whether such land can be taken with a view to the subsequent use of it by private individuals, under conveyances, leases or agreements which shall embody suitable contracts for the construction on the land of buildings adapted to use in domestic and foreign trade and commerce, and for the use, management, and control of the lands and buildings in such manner as to secure and promote such trade and

* "The Law of Eminent Domain", by Philip Nichols, 1917, p. 179.

commerce. The proposed legislation to which the inquiry relates, necessarily would contemplate action by the city in the procurement, management and control of land along a street within the city, for no other purpose than to induce and promote a use of it by merchants or traders. It would contemplate a taking of private property in the exercise of the right of eminent domain, and an expenditure of money to pay for it and fit it for occupation. It is a rule of law universally recognized in this country that neither of these things can be done unless the taking or expenditure is for a public use. This has been stated so often, and the principles on which it is founded have been considered so fully that it is unnecessary to discuss it or to cite authorities. The only question about which there is a possibility of doubt is whether the proposed use of the land outside of the thoroughfare is a public use. It is plain that a use of the property to obtain the possible income or profit that might enure to the city from the ownership and control of it would not be a public use. The city cannot be authorized to take the property of a private owner for such a purpose, nor can the city tax its inhabitants to obtain money for such a use. It could as well tax them to raise money to carry on any other private business with a hope of gain. Such proceedings are entirely outside the functions of a state or of any subdivision of a state. It is equally true and indubitable that a management and use of such property to promote the interests of merchants and traders who might occupy it, and to furnish better facilities for doing business and making profits, would not be a public but a private use of the real estate. An affirmative answer to this question would make it possible for the city to take the home of a resident near the line of the thoroughfare, or the shop of a humble tradesman, and compel him to give up his property and go elsewhere, for no other reason than that, in the opinion of the authorities of the city, some other use of the land would be more profitable and therefore would better promote the prosperity of the citizens generally. We know of no case in which the exercise of the right of eminent domain or the expenditure of public money has been justified on such grounds. * * * We answer the question in the negative.'"

Constitutional Amendment of 1911.—Finding in the case of the proposed law for the construction of a new thoroughfare between the North and South Stations, that the exercise of broad powers of excess condemnation was not possible under the Constitution of Massachusetts as it then existed, the Legislature of 1910 proposed a constitutional amendment as follows:

"Article ten of part one of the constitution is hereby amended by adding to it the following words: The Legislature may by special acts for the purpose of laying out, widening or relocating highways or streets, authorize the taking in fee by the Commonwealth, or by a county, city or town, of more land and property than are needed for the actual construction of such highway or street: *provided, however,* that the land and property authorized to be taken are specified in the act and are no more in extent than would be sufficient for suitable building lots on both sides of such highway or street, and after so much of the land or property has been appropriated for such highway or street as is needed therefor, may authorize the sale of the remainder for value with or without suitable restrictions."*

In Massachusetts a constitutional amendment must be passed by two successive Legislatures and then ratified by the people. Accordingly, the amendment quoted, passed by the Legislature of 1910, was again passed by the Legislature of 1911 and was ratified by the people at the election on November 7, 1911.

* Now Article XXXIX of the Constitution of Massachusetts.

It is to be noted that the Constitutional Amendment of 1911 relates solely to the laying out, widening, or relocating of highways or streets. Also, that the powers of excess condemnation made possible by this amendment may only be exercised through special acts of the Legislature.

Massachusetts was the first State to adopt a constitutional amendment of this character. It was followed by amendments in Ohio and Wisconsin in 1912, in New York in 1913, and in Rhode Island in 1916.

Special Legislation Under Provisions of 1911 Constitutional Amendment.—Several Special Legislative Acts have been passed under this constitutional amendment although the project for the proposed thoroughfare between the North and South Stations in Boston was not carried out. Among the Special Acts may be noted the following:

Acts of 1912, Chapter 186, authorizing the City of Worcester to take land for the widening of Belmont Street.
Acts of 1913, Chapter 169, relating to highway improvements in the City of Brockton.
Acts of 1913, Chapter 201, authorizing land takings by the City of Worcester in connection with the extension of Madison Street.
Acts of 1913, Chapter 326, authorizing the City of Worcester to take certain lands near Washington Square and to sell part of the same.
Acts of 1913, Chapter 703, providing for the widening of Bridge Street in Salem.

As far as the writer is aware, the authority to use excess condemnation, conferred by these several Special Acts, has in no instance been exercised.

It is to be noted that the constitutional amendment permits special legislation to be passed authorizing the sale of remaining land with or without restrictions. The permission to sell remaining land with restrictions was granted in some but not all of the Acts enumerated. There seems to be some doubt whether a city could impose restrictions unless expressly authorized so to do by legislation or by the State Constitution.

The only Special Act permitting the application of excess condemnation, which has been passed with relation to street improvements in the City of Boston, is that relating to the Stuart Street Improvement (Special Acts of 1917, Chapter 329, as amended by Special Acts of 1918, Chapter 118; Acts of 1920, Chapter 312; Acts of 1920, Chapter 465; Acts of 1921, Chapter 407). The Stuart Street Extension was the first stage in a comprehensive plan for a "western artery" proposed by the City Planning Board in 1917 to provide an adequate street outlet for the down-town business section of the city. Under the original Act the excess condemnation plan was to be used and the Act specified the parcels of land to be taken. The City was authorized to issue bonds outside the debt limit to the amount of $4 000 000, but was directed to enter into agreements with, or take guaranties from, property owners so that the total expense to the City would not exceed $250 000. This requirement was due to the fact that the Park Square Real Estate Trust, which had extensive property holdings in the vicinity, gave assurance at the time the legislation was passed that it would take over the excess land on terms such that the net expense to the City would not exceed the amount stipulated in

the Act. The original Act was approved May 14, 1917, shortly after the United States entered the World War. Under war conditions it was deemed inexpedient to proceed with the Stuart Street Improvement and the Special Act of 1918 extended the earlier legislation by providing that the 1917 Act "shall take effect upon its acceptance by the Mayor and City Council of the City of Boston at any time before the expiration of one year after the termination of the present war, as defined by Federal authority." The Park Square Real Estate Trust made an offer to the City to take over the properties in excess of the actual requirements for the street on terms such that the estimated net cost to the City would not exceed the limit of $250 000 prescribed by law. This offer held until February 1, 1920. Meantime, the City Council had not accepted the 1917 Act and doubt was expressed as to the constitutionality of its excess condemnation provision which had never been tried in so large and important a project. The City Council being apparently hostile to the undertaking, no agreement was reached with the Park Square Real Estate Trust, and the excess condemnation proposal fell through. In 1920, further legislation was passed authorizing the Stuart Street Improvement without the excess condemnation feature, and the street has since been constructed without the exercise of excess condemnation.

One other proposal for the extensive application, under the 1911 Constitutional Amendment, of the principle of excess condemnation in Boston is worthy of notice. This was the Bill (Senate No. 268) introduced in the Legislature of 1920 for the North End Improvement. The section of Boston known as the North End is one of the oldest parts of the city and two centuries ago was the leading residential section. Originally, its house lots were large and the street system which was gradually developed resulted in irregular city blocks of large size. During the latter part of the Nineteenth Century the North End became a tenement house district, all available land was built upon, and many tenements were constructed in the interiors of these blocks without suitable street access and without adequate provision for light, air, and recreation. Thus, a population of great density, in certain blocks exceeding 500 persons per acre, came to be housed under unwholesome conditions. The remedy proposed was the cutting of new streets through the larger and more congested blocks and the widening and improvement of existing streets and alleys. To carry out the improvement properly, and secure suitable replotting of the land, the proposed legislation provided for a liberal amount of excess condemnation. The measure was strongly endorsed by public and semi-public organizations, was passed to be engrossed by the Senate, but was finally defeated in the House of Representatives. Thus was lost another opportunity to apply the excess condemnation principle in Boston on a large scale and to a major improvement which could not be carried out adequately by other means.

Although Massachusetts by its Constitutional Amendment of 1911 has made possible the application of excess condemnation to street improvements, this authority has not yet been exercised.

Excess Condemnation for Park Purposes Declared Unconstitutional

One instance has occurred in Massachusetts where the attempt to apply the principles of excess condemnation to land takings for park purposes has been declared unconstitutional. In 1912 the Legislature passed an Act (Chapter 715) "to make Salisbury Beach a public reservation and to establish the Salisbury Beach Reservation Commission." This Act authorized the Commission which it created to take by eminent domain any or all of certain shore lands in the Town of Salisbury within certain wide limits which were defined in the Act. The Act provided (Section 10) that the Commission "may sell or lease any lands or rights in land taken or acquired by it, which are not needed as a public reservation for the use, exercise, and recreation of the inhabitants of the Commonwealth, with or without restrictions as to its use as it may deem advisable." The Act was of such broad scope that the powers of the Commission to condemn and re-sell land were practically unlimited. Its constitutionality was brought in question before the Supreme Court in 1913. In its decision (Opinions of Justices, 215 Mass. 371), the Court held the Act unconstitutional on the ground that it was "an attempt to authorize the exercise of the right of eminent domain in part for a private purpose, and because it is impossible to separate the valid from the invalid portions of the statute," and, further, that "the interest of the public that the people should be well housed although proper to be considered in the exercise of the police power, is not a proper subject for the exercise of eminent domain." Under the takings contemplated an existing seaside resort with a number of summer cottages would have been included in the reservation.

According to Chief Justice Rugg, who wrote the decision:

"The underlying objection is that the main end of legislation for this purpose is a private utility rather than the general good. While incidentally it may be an advantage to the public that private persons prosper, if the essential character of the transaction in its direct object is private benefit to individuals, the purpose is not public. * * * Legislation which is designed or which is so framed that it may be utilized to accomplish the ultimate result of placing property in the hands of one individual for private enjoyment after it has been taken from another individual avowedly for a public purpose is unconstitutional."

In the opinion of the Court the power conferred on the Commission was too broad. It is implied in the decision that if the power to take land and sell it had been restricted as to time, or by "changed conditions", or to the sale of remnants, that part of the Act might be considered as constitutional.

It is to be noted that this opinion of the Supreme Court of Massachusetts was rendered in 1913, prior to the adoption of the Homestead Amendment to the Constitution in 1915, which does permit the Commonwealth to condemn land for housing purposes, and, after improvement, to sell it for the purpose of relieving congestion of population and providing homes for citizens.

The "Homestead" Constitutional Amendment of 1915

By 1911, prior to the shortage of homes resulting from the World War, the question of proper and adequate housing of its citizens received the attention of the Massachusetts Legislature. In that year the Massachusetts

Homestead Commission was created by legislative enactment (Acts of 1911, Chapter 607). The Commission was directed to make an investigation and report and to recommend legislation "embodying a plan and the method of carrying it out whereby, with the assistance of the Commonwealth, homesteads or small houses and plots of ground may be acquired by mechanics, factory employees, laborers, and others, in the suburbs of cities and towns." In its report to the Legislature in 1912 the Commission recommended that part of the unclaimed deposits of savings banks which had been called into the State Treasury should be loaned to the Commission for the purpose of providing workingmen's homes as contemplated in the Legislative Act of 1911.

Although the Homestead Commission Bill, as it was called, was pending in the Legislature in 1912, that body requested the opinion of the Supreme Court on the constitutionality of the measure. The Court held that the legislation proposed would not be constitutional. The main questions decided by this opinion were that the purpose of the bill was to enable the State Commission to do private business, utilizing public funds, therefore, it was unconstitutional; that the Commonwealth could not raise money for such purposes by taxation; and that money from unknown owners deposited in the State Treasury by savings banks was held in trust by the Commonwealth, subject to the claims of the owners at any time (Opinions of Justices 211 Mass. 624).

After the Supreme Court declared the proposition unconstitutional the Legislature further instructed the Homestead Commission (Chapter 714, Acts of 1912) to:

"Continue its investigation of the need of providing homesteads for the people of the Commonwealth and its study of plans already in operation or contemplated elsewhere for housing wage-earners * * * and * * * recommend such legislation as in its judgment will tend to increase the supply of wholesome homes for the people."

The Commission made exhaustive studies of governmental aid to housing in twenty-seven foreign countries, including Australasia, the United Kingdom, Continental Europe, and parts of South America and of South Africa. As a result of its studies, the Commission concluded that "if there is to be created a sufficient supply of wholesome homes within the means of the ordinary wage earners, cities and towns must be built according to well considered plans." Accordingly, two bills were recommended to the Legislature of 1913, one providing for the establishment of local planning boards by the cities and towns of Massachusetts and the other for studies by the Commission of questions of town and city planning and for the Commission's aid to cities and towns in planning work. These bills were passed as Chapter 494 and Chapter 595, respectively, of the Acts of 1913.

The first of these two Acts made City Planning Boards mandatory in every city of the Commonwealth and in every town having a population of more than 10 000 and marked the beginning of organized official city planning work in Massachusetts. This was the first mandatory city planning legislation to be passed by any State.

Being unable to find a way to provide homes for working people through governmental aid under the provisions of the State Constitution as it then

existed, a constitutional amendment was passed by two successive Legislatures, in 1914 and 1915, and ratified by the people on November 2, 1915.

The Homestead Amendment, as it is called, is as follows:

"The General Court shall have power to authorize the Commonwealth to take land and to hold, improve, sub-divide, build upon and sell the same, for the purpose of relieving congestion of population and providing homes for citizens: *provided, however,* that this amendment shall not be deemed to authorize the sale of such land or buildings at less than the cost thereof."

Two Legislative Acts have been passed upon recommendation of the Homestead Commission under the provisions of the Homestead Constitutional Amendment of 1915. One Act (Chapter 185, General Acts of 1916) permits cities to maintain schools of agriculture and horticulture. This was passed as an educational measure, as the Homestead Commission thought that:

"Great waste would result, and possibly danger to the homestead movement, if many persons, inexperienced in the care and management of the soil, were put in possession of 'small houses and plots of ground', as contemplated in the Act under which the Homestead Commission was created."

The other Act (Chapter 310 of the Acts of 1917) authorized the Homestead Commission to provide homesteads for citizens, and carried a small appropriation of $50 000 for inaugurating the work. A site of about 7 acres was purchased by the Commonwealth in the City of Lowell and a number of houses were built at moderate cost. These workingmen's homes are being sold on a long-term basis at prices ranging from $2 400 to $3 100 each. This marks the entry of the State of Massachusetts into the business of housing. Although the housing experiment at Lowell is working out satisfactorily and the State will, in due time, recover its investment, there has as yet been no disposition on the part of the State to embark upon another housing development.

As in the case of the reclamation by the State of tidal flats, noted previously, it is a question whether the "Homestead Amendment" falls under excess condemnation as strictly interpreted.

Excess Condemnation Applied to the Taking of Easements in Property

In 1893, the Legislature passed an Act (Chapter 462) granting to cities and towns authority to establish building lines parallel to, and not more than 25 ft. from, a highway or street and to prohibit the erection of buildings within such restricted area. The right was given owners to recover damages in the same manner as in the taking of land for a highway.

Supplementing this legislation of 1893, an Act was passed in 1896 (Chapter 313) granting further power to cities and towns to limit the heights of buildings on parkways and boulevards, the Act reserving to owners the right to recover damages by reason of such restrictions.

Following the precedent thus set by this general legislation, the Legislature in 1898 passed a Special Act (Chapter 452) limiting the height of buildings on or near Copley Square in Boston, this Act also reserving to owners the right to recover damages by reason of the restrictions imposed. The Special Act of 1898 was contested and its constitutionality upheld by a decision of the Supreme Court in 1899 (Attorney General *vs.* Williams, 174 Mass. 476).

The Court held the Act to be constitutional because:

"The statute gives rights in the nature of an easement over lands facing Copley Square, which easement is annexed to the Square for the benefit of the public, for whose use and enjoyment Copley Square was laid out; and * * * these rights are similar in their nature to rights in highways, in great ponds, and in the navigable waters of the Commonwealth. * * * In all respects the statute is in accordance with the laws regulating the taking of property by right of eminent domain, if the Legislature properly could determine that the preservation or improvement of the park in this particular was for a public use."

In regard to limitation of buildings around Copley Square as relating to public use, the Court was of the opinion that:

"If the Legislature, for the benefit of the public, was seeking to promote the beauty and attractiveness of a public park in the capital of the Commonwealth, and to prevent unreasonable encroachments upon the light and air which it had previously received, we cannot say that the law-making power might not determine that this was a matter of such public interest as to call for an expenditure of public money, and to justify the taking of private property. * * * It hardly would be contended that the same reasons which justify the taking of land for a public park do not also justify the expenditure of money to make the park attractive and educational to those whose tastes are being formed and whose love of beauty is being cultivated."

Conclusion

Massachusetts, in 1904, was the first State to apply the principle of excess condemnation to the taking of lot remnants in connection with street improvements. Massachusetts, in 1911, was the first to provide through amendment to its Constitution, authority for the broader exercise of excess condemnation in connection with street improvements so as to secure "suitable building lots on both sides of such highway or street." In Massachusetts, the principle has been established that parkways and boulevards may be protected by the taking of easements in abutting property, restricting the construction of buildings thereon. Thus, Massachusetts has laid the foundation for the protection and the adequate development of street improvements through the reasonable, although limited, exercise of excess condemnation. Except for the taking of easements and the condemnation of fee in lot remnants, these powers have not yet been utilized, however, and no project for street improvements involving the broad application of excess condemnation has been put into execution in Massachusetts.

In two other directions, which strictly speaking may not be interpreted as excess condemnation, but which are close to the border line, Massachusetts has carried out public improvements by State agency. These are the reclamation and development of tidal flats, work which has been going on for seventy-five years; and the provision of homes for its citizens, made possible by constitutional amendment, and for which thus far one small project has been built.

DISCUSSION

HORACE H. SEARS,* M. AM. SOC. C. E.—The speaker proposes to bring to the attention of engineers certain facts which relate to matters of design and construction where excess condemnation forms a part of eminent domain proceedings discussing the several matters under various headings, as follows:

Municipal Use of Power of Excess Condemnation.
Public Utility Use of Excess Condemnation.
Excess Condemnation Compared with Zoning Laws and with Police Power.
Financial Considerations: (*a*) Special Assessment; and (*b*) Issuing Securities.

The municipal engineer should have a clear understanding of these topics as his work of preparing the preliminary plans is one of the first matters of procedure. Given a valid law under which to proceed, the engineer plans his improvement district; he describes it definitely by metes and bounds, and thus takes the initial step. The laying out of a proposed street on an official map has been held not to be a taking to divest or impair the owner of the title. (See McQuillin Mun. Corp., Em. Dom. Sec. 1471; citing N. Y. C. R. Co. *v.* State, 55 N. Y. S. 685; 37 App. D. 42; affmd. 165 N. Y. 658; and 59 N. E. 1130; also, similar holding in *re* Phila. Pkway, 250 Pa. 257; but see case of Daley *v.* St. Paul, 7 Minn. 390, as limiting the act of filing plans, etc.)

The municipal engineer in co-operation with the City Attorney, should prepare the plans and the description characterizing the construction work proposed. The "public use" which forms the basis for eminent domain proceedings must be shown by engineering description in specifications and plans as a necessary part of proposed excess condemnation proceedings. The importance of excess condemnation is an engineering factor which the lawyer and legislator need to have defined and described for their use by civil engineers.

The proper legal status or the position of the law (valid law) cannot be insisted upon too much. That must come first. As frequently the engineering fees are to be paid from funds obtained from the ultimate financing of this project, the engineer should be certain that there is a valid law and not merely a favorable opinion of counsel. If the engineer can evolve his designs with the assistance of additional land secured by excess condemnation, it may make a material saving in cost over restricting his design to the rigid lines permitted under eminent domain powers. The engineer should make sure that there is a valid law before risking his recompense or reputation. If the attorney's procedure goes wrong, he should have known better; but any mistake will rise up to plague the engineer who has been identified with invalid municipal proceedings.

MUNICIPAL USE OF POWER OF EXCESS CONDEMNATION

This filing of plans (under a valid law) with a proper description (metes and bounds) for purposes of publication, will be incorporated by the City Attorney

* Cons. Engr. and Attorney-at-Law, New York, N. Y.

in the first legal procedure as a Notice of Intention. The engineer should follow this step and check up his part of this very important preliminary as to plans and description. The plans in general will be concerned with a taking of private property under eminent domain proceedings, with which excess condemnation is an integral part. For reasons set forth in the Symposium, it is highly desirable that the engineer should have the most economical area for his design, as he most certainly will be limited to the minimum expenditure for construction purposes.

This matter of economy of design and lowest cost of construction is so related to "public use and necessity" as to give an added argument for the use of excess condemnation. Where a design for construction can show material savings by the use of excess condemnation it may well be argued before the Court that boundaries limiting the area needed for construction are to be determined by "cost" and not by the minimum limits within which skillfully designed construction may be completed; but this is a fair question for the Court to decide and more properly belongs to a discussion of the legal phases of excess condemnation.*

Excess condemnation has received special attention from legislative action as to streets and parks. It should be noted that special legislative action is limited and does not give powers outside or apart from itself. There is also the need for engineers to consider whether the State Constitution permits this legislative action of a special kind. Many legislative enactments have been held invalid when tested by Court decisions. Frequently, a so-called "friendly suit" is brought to test a particular situation where untested laws are the basis for special assessment procedure.

The need of excess condemnation in connection with city planning applies to the entire schedule of public improvements wherever the power of eminent domain is exercised. Streets and parks usually receive attention, but water, gas, sewer, and service mains also require consideration when the matter of economy in engineering design calls for additional land adjacent to the minimum area which present methods under eminent domain allow the engineer to use.

Decisions on municipal uses of the power of excess condemnation are very limited. These decisions have been summarized,† as follows:

Unfavorable Decisions.—

(*A*) The "public use" for which private property may be taken by eminent domain means "use by the public", which is invalid when the city plans to dispose of the excess property later. (Penn. Mut. L. I. Co. *vs.* Phila., 242, Penn. State, 47.)

(*B*) Without relying on the "use-by-the-public" rule, the promotion of commercial and industrial development (that is, the creation of business thoroughfares) is not a purpose which is "public use" in the sense of justifying the condemnation of land or the use of taxation. (Opin. Just., 204, Mass., 607; 209 Mass., 616.)

* See "Financial Considerations," p. 1866.

† "Excess Condemnation," Robert Eugene Cushman, 1917, pp. 305–306.

(*C*) The actual result of excess condemnation, whatever may be the motive back of its use, is the taking of the property of one man by eminent domain and transferring it to another. This is violative of the spirit if not of the letter of the Constitution, etc. (The Matter of Albany St. 11 Wend., 148; Dunn *vs.* City Council of Charleston, Harper's Law Rept. (S. C.), 189.)

Judicial Approval of Excess Condemnation.—

(*D*) A taking by excess condemnation involves the taking of private property by eminent domain proceedings for educational and esthetic purposes constituting a "public use". (Penn. Mut. L. I. Co. *vs.* Phila., 22 Penn. Dist. Repts. 195.)

(*E*) The condemnation of remnants of land is an exercise of the right of eminent domain for a use necessary and incidental to a public purpose. (Opin. Just., 204 Mass., 616.)

To the engineer engaged in the preparation of plans for municipal improvements, the foregoing may seem confusing and very unsatisfactory; but these legal distinctions must be understood as controlling in the States where they were rendered. Mr. Williams* states the most reliable rule as to the valid features by which engineers may work, as follows:

"Whatever doubt there may have been with regard to the validity of the statutes of excess condemnation, under the State Constitutions as interpreted by the State Courts, there can be no doubt with regard to the validity of the constitutional amendments, which can be challenged only on the ground that they are contrary to the Constitution of the United States as construed by the Supreme Court."

The need and scope of the municipal use of excess condemnation may therefore be as extensive as that of exercising the power of eminent domain, as will be developed under "Public Utility Use of Excess Condemnation".

Public Utility Use of Excess Condemnation

This phase of city planning is entitled to similar considerations of a valid law as a basis for procedure as already discussed.

A separate consideration is needed for those cases in which the city acts as a corporation to furnish water, gas, or electric service—and sometimes transportation—to the public as an incident of its charter rights. The bona-fide public service corporation is the other basis for special consideration in exercising the sovereign right of eminent domain and its attendant powers of excess condemnation.

Relative to special legislative enactment, take the Rochester, N. Y., case cited by Mr. Vedder (page 796):

"*Sec. 2.—Corporate Powers.*—The city has power * * * to take more land and property than is needed for actual construction in the laying out, widening, extending or relocating parks, public places, highways or streets, provided, however, that the additional land and property so taken shall be no more than sufficient to form suitable building sites abutting on such park, public place or street; * * *."

Assume that the engineer has under consideration the design for a trunk-line sewer, requiring the exercise of the power of eminent domain and the

* See, p. 794; see, also, Williams, "The Law of City Planning and Zoning," Chapter III.

use of a street which must be widened in connection with this improvement. Can this special statute amendment of the Rochester City Charter of 1921, made in accordance with revised constitutional provisions to authorize "excess condemnation", be considered as a valid law for this sewer construction? Will this statute relating to city powers be strictly construed, or can several additional matters be saddled on a proposed street improvement, the said street improvement being clearly within the statutory powers?

It is not proposed to discuss here the legality of the legislative enactment mentioned. A word of caution may be in order, as all legislative enactments, until approved by subsequent Court decisions, are of doubtful value; hence, the so-called friendly suit, which most quickly determines specific points of legality with respect to a particular construction procedure.

The point under discussion is that of saddling an otherwise perfectly valid street proceeding with additional construction work. If excess condemnation is valid for necessary street widening, it would seem to follow that the engineer may introduce at the same time other construction as water, gas, electric, and even a street railway, the total cost of all this construction to be carried by the same bond issue that is the source of funds. This shows the imperative need for excess condemnation accompanying eminent domain proceedings and for a constitutional provision in every State, which will permit (and thus leave unquestioned) the validity of engineering designs based on the properly safeguarded use of excess condemnation.

Public utilities are entitled to utilize the powers of eminent domain. The attendant use of excess condemnation is of practical importance to the engineer when designing construction within city limits as well as outside them. Once the door to the validity of excess condemnation for specific use of streets and parks is opened through constitutional amendment, the same treatment must logically follow with respect to public utilities. The use of excess condemnation can be based on the cost of construction as well as on the area. The area necessary for "public use" has been the basis for eminent domain proceedings. The cost of construction is of the essence of "public use", and is frequently a greater deciding factor than area when public improvements are considered. Excess condemnation is a logical development of the use of eminent domain. It is an agency incidental to the proper "public use" of eminent domain. Why should not the principle of "agency" as recognized by the Courts be considered here. That what one may do himself, he may do through the agency of another? Apply this principle to eminent domain and there results "what the State authorizes as permissible under eminent domain, may be exercised by excess condemnation proceedings." There is constitutional authority for eminent domain—the time has arrived to secure constitutional authority for this very necessary agency of eminent domain—the power of excess condemnation.

Excess Condemnation Compared with Zoning Laws and with Police Power

The topics of zoning and police power may be properly considered together as of interest to the engineer in his program of municipal improvements wherein excess condemnation has its part. Zoning laws should be understood

as forming a special feature of the police powers which the State delegates by implication and by express authority. The city charter derives its express authority from the State; but this express authority carries with it certain implied (unwritten) powers which are reasonably incidental to those powers specifically set forth. One of the most important of these powers is that which restricts absolute free personal action so that the liberty of all may be assured. This assurance of liberty in a shape that will result in the greatest good to the greatest number is a form of police power and is concerned with public safety as much as the matter of morals and health is incidental to public safety.

Zoning laws have recently come into use and have been upheld as a valid exercise of the police power. Claims have been made that the power of eminent domain was being exercised and that compensation was due an owner where damages resulted. An example of a police law of special legislative design affecting New York, N. Y., is the so-called "Rent Laws" which have undoubtedly, in some cases, arbitrarily impaired contracts made between landlords and tenants. Even this extreme and valid exercise of the police power is of interest to engineers. However, it is the zoning law that the engineer should study. The application of the principles of zoning by the engineer when the plans for sub-division of territory are in their formative stage, offers great possibilities, which are independent of the use of excess condemnation powers.*

The selection of zoning laws in developing new areas gives partial assurance that in the future excess condemnation will not be as necessary as formerly. The engineer should clearly understand the fundamentals which distinguish zoning from excess condemnation; and also the relation which each of the following terms bears to the other, namely, excess condemnation, eminent domain, zoning, and police power. This is of importance to the municipal engineer as his preliminary designs are intimately concerned with one or all. The legal viewpoint on these subjects is quite different from the practical application which the engineer must make of them.

The building laws are typical of special police power ordinances passed by a city, within its charter powers. The restricting of areas for special purposes so as to safeguard and promote the public health have generally been held valid, but many instances have occurred where zoning laws have been held invalid. The engineer, therefore, is interested in knowing whether the procedure under which he works is a law for zoning or for excess condemnation.†

The use of zoning laws under the police powers should be clearly understood as to their legal and practical distinction. The powers of excess condemnation are more closely related to the powers of eminent domain. Each of these last two powers is found where there is a condemnation without the owner's consent (a taking of realty and the incidents connected therewith) and a payment to the owner for his property thus taken by virtue of the sovereign right to take for "public use".

* "The Law of Zoning", by H. S. Swan, and "Zoning", by Edward M. Bassett, National Municipal League, New York.

† "Zoning and the Courts in Texas," *National Municipal Review*, June, 1924.

Financial Considerations

Remnants of realty and gore strips of unusable area form a costly part of the use of excess condemnation. Mr. Vedder gives a condensed statement of the condemnation of excess area and the sale of part. This feature has been discussed in certain cases already cited. (See 11 Wend. N. Y. 148; Harper's Law. Rep. (S. C.) 189; 204 Mass. 607; 242 Penn. St. 47.) The available decisions on excess condemnation treat this practice as unconstitutional in the absence of express constitutional amendment making such use valid.

Cases where the right of exercising the power of eminent domain requires a right of way as an easement over the land of another and the boundaries are clearly defined, illustrate the application of the principle of "minimum area for a public use" and clearly of "reasonable necessity". It came within the province of the civil engineer to determine and describe the area for a highway —its length and breadth—and to file an accepted plan. When embankments became necessary and the foot of the slope (or top of the cut) made an increased width necessary—as compared with the width of actual traveled way—this constituted in reality the first departure from the "eminent domain" area actually needed for the traveled portion of the highway. The excess spread for the slope was excess area if it be conceded that a vertical wall retaining the traveled width of the roadway could have been erected.

As a matter of practice many engineers have sanctioned the additional expense of building a retaining wall to avoid condemning a remnant or gore piece of land because the owner would secure damages for his injury out of all proportion to the proposed use. Eminent domain proceedings would have been allowed but their excessive cost justified the engineer in neglecting the minimum construction cost in favor of keeping off additional land. It is submitted that, the principle of eminent domain being admittedly the seizure without consent and payment therefor to the owner, the same principle should obtain in the taking of excess land without consent and payment therefor, the use of the excess land being based on financial considerations within the scope of the engineer to determine; and dependent for its supporting facts on the engineer submitting to the same tribunal that passes on eminent domain proceedings the cost data and valuations which form the basis for the valid eminent domain procedure.

This "financial" element of the cost data is the criterion and not the area. "What does the public pay?" is as pertinent to decide as, "What does the public acquire in land area?" The public is burdened for years with a bonded debt, long after a suitable use is made of the excess area. Let the engineer approach this need of excess condemnation as he would the plans for an earth fill for a highway; let him file his preliminary plans for eminent domain based on the "cost" of the proposed project, and make the condemned area include all that is reasonably incidental to a "public use" and to the lowest total cost to the taxpayer. The engineer's estimate will include, of necessity, future transactions in land, but it also includes costs of future construction and the purchase of materials. Land is merely an additional element to the customary items of labor and materials. The cost of land, labor, and materials should be

recognized by the Court as equally applicable in determining the reasonably necessary cost to the public. This is submitted to engineers, because they constitute the proper source of initiating proceedings. They will have to work in harmony with the City Counsel.

Changed conditions demand changed methods. A change in the law has been refused, therefore it is necessary to change the method of applying valid laws. This matter of applying new methods will not meet with favorable reception in the older cities, especially where decisions have been handed down which have rejected the method of reducing costs by selling "remnants" of land resulting from excess condemnation proceedings. There is a possibility of using "cost to the public" in those jurisdictions where constitutional amendments have allowed excess condemnation proceedings.

Given a constitutional amendment allowing the use of excess condemnation proceedings, the next step is to limit its application. By showing on the procedure plans filed for publication the total area affected by eminent domain plus an excess area which is "reasonably necessary", a rational claim would be placed before the tribunal which will consider the objections of owners and citizens.

Financial considerations also enter into the use of excess condemnation by way of paying the cost, initial and final, of any given municipal improvement. The engineer is interested in this feature of the "source of funds", be it the general city fund, the special fund created by ordinance, or the special assessment tax on the property benefited. This opens a field for discussion which is here arbitrarily confined to (*a*) special assessment financing and (*b*) issuing securities.

Engineers who are not acquainted with methods of financing the projects on which they somewhat blindly follow political activities may well be careful. The reputation of an engineer is in part related to the success or failure of the projects on which he works. Failure of a splendid engineering project may frequently be laid to the fact that it was not financed properly or to invalid proceedings—that is, invalid from the start and not detected until some months after the project is in full swing. Special assessment proceedings are not difficult—merely routine—but frequently mistakes are made by ignorant public officials. The engineer should verify these proceedings as his initial fees and his ultimate recompense are often dependent on their validity. Given a valid law under which to use the power of excess condemnation, the construction work will frequently follow procedure known as "special assessment".

Special Assessment.—The details of special assessment procedure do not come within the scope of this discussion, except as it is based on excess condemnation, which calls for financing by a district specially benefited. The Southern and Western States are more inclined to handle special assessment district procedure than the Eastern cities. Whenever possible, the engineer should have the general credit of the city as an underlying promise on all special assessment financing. When securities are issued which are merely a lien on the abutting property of a special assessment district, a greater price must be paid for the use of money, as these securities are considered of inferior

quality compared with those that have the general credit of a city supporting them. (See Fla. Laws, 1923, Chapter 9298 (Special Assessment Bonds as General Obligations of the Municipalities).)

Issuing Securities.—The engineer should have a good understanding of the auditing and financing of the projects which he initiates. Given the organized engineer office of to-day, with subordinates capable of handling routine engineering matters, the chief engineer should study the related legal and financial questions which may make or break his project.

The securities that must be issued to pay for the excess condemnation procedure give one cause for thought. Shall the entire cost of the project—direct costs, eminent domain cost, and excess condemnation costs—be treated separately? If the entire budget can be handled as one item it will certainly simplify matters. Then comes the very usual and vital question—does an invalid excess condemnation item, which has been grouped with valid items, invalidate the entire procedure? This is a point worth considering. The answer depends on the project and all the surrounding circumstances.

The total amount involved is some indication of the cost of handling excess condemnation. Anticipated profits from realty transactions are not always realized; no more than where a private operator in real estate is concerned. If the municipal operations in the sale of remnants from excess condemnation proceedings fare no better than the average results from municipal operation of public utilities, there will be little for the profit side of the ledger. Municipal activities had best be restricted to acting as an agency of the State. At the same time, the new, rapidly growing communities of the South and West have a legitimate use for municipal ownership of public utilities, because no one else will help activities that have still to demonstrate their value. Thus excess condemnation proceedings are most likely to be favorably received in those States where the Courts have not taken a decided stand against their validity. The full faith and obligation of the city should be pledged to support the securities issued to finance excess condemnation. The engineer should see that this is done in every case. The type of security, whether notes or bonds, will depend on so many factors that only the character of the lien supporting the loan can be mentioned. The very highest kind of financial obligation should be given by the city to support excess condemnation financing.

To the engineer is given an opportunity to study excess condemnation procedure as affecting his designs and as the proper auxiliary of eminent domain proceedings. The consideration of the legal, engineering, and financial elements only serves to emphasize the importance of this subject and to indicate the large amount of educational work which must be done before all the States will grant proper constitutional amendments placing excess condemnation on a par with eminent domain proceedings. The economic use of remnant land for the furtherance of the public welfare provides an opportunity for the engineer to assist in the education of the public concerning the powers of excess condemnation as a saving in cost to them on numerous improvements.

FRANK O. WHITNEY,* M. AM. SOC. C. E.—The subject of excess condemnation has been very carefully studied by the Street Laying Out Department of the City of Boston. Before it was enacted, the Massachusetts law was expected to open a way to secure much benefit esthetically, and, in some cases, to become a source of much profit. Since the enactment of the law making it possible to take excess quantities, its application on a large scale has been considered in numerous cases of proposed thoroughfares.

Generally, it has been deemed to be impracticable on account of the great value of the properties concerned, of their present complete development, and of the necessity of placing high values on the property to be sold after the condemnation. In only one instance has the Legislature been asked to give its approval to the use of the law, namely, the laying out of Stuart Street in the Back Bay. Although the Legislature complied, the street was laid out and constructed under the general statutes, as the City Council failed to accept the Special Act.

This is one instance where excess condemnation might have proved beneficial, as the street (about a mile in length) was laid out over run-down or stagnant property the value of which before the improvement had been assessed at from $3 to $12 per sq. ft., and was either vacant land or occupied by ancient and cheap buildings. The improvement has resulted in a greatly increased value to abutting property, as evidenced by recent sales averaging from $20 to $25 per sq. ft. As the City of Boston is allowed to assess betterments wherever sustained up to the cost of the improvement, the seeming loss of opportunity in this case was not as great as it would at first seem, the cost ($3 100 000) being offset by a betterment assessment of $2 700 000.

Several propositions for an adequate thoroughfare connecting the North and South Stations have been suggested and estimated; as yet these propositions have not materialized. Excess condemnation has been considered in these estimates, but in each case on account of the high value of the properties involved, it has not been used, as the remaining land would have had to be sold for three times its present value to provide a profit equal to the cost.

In the case of a large improvement a considerable amount of property would be temporarily untaxable. The carrying of this real estate would require large capital with heavy interest, and also involve the city in an immense real estate business.

The Massachusetts law necessitating a special act of the Legislature in each case of the application is very inconvenient, as comparatively few cases arise when the Legislature is in session. The general assessment laws provide ample means for such compensation as is necessary in most instances.

Excess condemnation may be of great benefit in special cases, but should not be expected to provide a universal remedy for all the ills involved in laying out streets. If the City of Boston could have a law that would allow the taking of small remainders at will, much good would ensue, as abutters could then be protected from the avarice of small owners, and unsightly indentations would be avoided.

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* Cons. Engr., Boston, Mass.

ANDREW WRIGHT CRAWFORD,* ESQ.—The principle of remnant condemnation, which is restricted excess condemnation, was applied in the case of Fairmount Park in Philadelphia, Pa., about 1870 and proved very valuable.

In the case of the King's Highway, in London, where a street 100 ft. wide was extended through a closely built-up area, the cost of the entire outlay for the right of way and for the abutting property was about $30 000 000. None of the abutting property was resold; it was leased on 100-year improvement leases. Capitalized on the basis of 4%, the entire cost has been recouped, according to the latest information.

It is necessary to "think through" in such matters, in the case of this London example to the end of 100 years, when the municipal corporation will own the enormously valuable property on both sides.

One of the greatest advantages of excess condemnation is that in the re-sale of the excess property, or in the leases of such excess property, on a 100-year improvement lease, it may be provided that whatever buildings are erected must be approved as to design by the Municipal Art Commission. In the long run that will be a very powerful factor in beautifying the city, especially on the wider and longer streets.

Excess condemnation ought frequently to be used in the case of remnants fronting on streets. Mr. Leavitt instanced a small strip, 3 to 20 ft. in width, running along the street for some distance. The trouble there was in the lack of education of the engineer. There is no reason why any street intended to be about 100 ft. wide, should be exactly 99 ft. 12 in. wide all the way through. For a block, it might be 97 ft., or 103 ft., or even 110 ft. It is well enough to have straight streets, but as to their boundaries, that is absurd. In the case of small remnants that are parallel or slanting at a slight angle to the street, the real trouble is with the inflexibility of the particular engineer who was not sensible enough to vary his street somewhat from block to block, or within the block.

HAROLD M. LEWIS,† M. AM. SOC. C. E. (by letter).—The advantages of excess condemnation have been very well summarized by Mr. Leavitt. It may seem strange that a policy which has so many arguments in its favor and is generally admitted by students of city planning to be equitable and advantageous should meet with so much local opposition when attempts are made to utilize it. A brief summary and analysis of the reasons for such opposition may be of interest. The principal ones can be stated as follows:

1.—The term, "excess condemnation", which is an unfortunate one tending to arouse immediate opposition.
2.—Objection to any change in old and well-tried methods.
3.—An increased first cost of improvements.
4.—Reluctance of local property owners to relinquish the opportunity of making large speculative profits from holdings immediately adjacent to the improvement.

There are probably no two words the use of which is more apt to have the effect of waving a red flag in the face of public opinion than "excess" and "con-

* Philadelphia, Pa.
† Executive Engr., Regional Plan of New York and Its Environs, New York, N. Y.

demnation". The former implies that more property is to be taken than is necessary and the latter implies a confiscation of private rights. The term, "Remnant Act", used in Massachusetts, was much less inclined to arouse opposition, but the acquiring of remnants did not go far enough to bring about the best results. The writer does not attempt to suggest a better term, but if one could be found which would refer only to the taking of remnants and a re-subdivision of abutting property, it would certainly be advantageous. Provisions for purchase or condemnation of adjacent parcels necessary to make an effective re-subdivision of land fronting on the new street or park improvement properly belong in the text rather than in the title of the legislation.

Any change in customary methods will always meet with opposition but this can be overcome through courageous perseverance by progressive citizens, if they are properly enlightened by the professional men studying the problems.

A method of decreasing the first cost of bonding the excess lands acquired and exempting such bonds from the debt limit has been suggested by Mr. Leavitt. There is no doubt that such bonds would prove safe investments, as in many cases the re-sale of the land outside the limits of the improvement should provide a substantial profit. By this means a fund might be created which could be utilized to assist in financing other similar improvements needed in the city.

Some street-widening improvements result mostly in local benefit but any large scale improvement is greatly to the advantage of the whole city and raises land values several blocks distant from the actual improvement. If the project is the cutting through of a new and important traffic or transportation thoroughfare its influence may be felt to still greater distances. It is obvious, therefore, that many property owners and the city at large could profit by improvements which may be held up by the selfish ambitions of those few whose property might be taken, although they would receive a fair value in return. The majority should rule.

M. W. Weir,* M. Am. Soc. C. E. (by letter).—In excess condemnation the element of speculation looms large with the profit going to the deserving, that is, to the community in which an improvement requiring condemnation takes place. No doubt some will think that such a proceeding comes dangerously near to the acceptance of some of the political theories of several decades ago when the single tax was being discussed.

Others will look upon excess condemnation as a public usurpation of individual rights and the public ownership question will again be brought up for discussion, because public ownership of utilities will be confused with the temporary public ownership of property corollary to a public utility, such as a street. There is no ground tenable for questioning the policy of subjugating the rights of the individual, no matter what they are, to the public necessity, benefit, and welfare.

Whether or not a discarded political theory is revived is less a question than whether there is value in the project. Applying excess condemnation has a double object, namely, to eliminate an improbable individual speculation

* Cons. Engr., New York, N. Y.

and to conserve to the community that accretion of values which the community as a whole creates when a public improvement is completed.

There never has been displayed an organization of individuals as parcel property owners in a community, with sufficient public spirit or sufficient trust in the worthiness of each other to carry on property improvements involving the re-plotting of a number of small holdings to fit the new lines created by the necessary public improvement affecting those properties abutting and adjacent. Few individuals have the financial resources necessary to carry out such a program nor the courage to attempt it. Yet such a public improvement will have only partly succeeded if it leaves ragged outlines and uneconomical building units in its wake, as in the cases of Lafayette Street and the Seventh Avenue Extension, New York City, cited by Mr. Leavitt and the Flatbush Avenue Extension in Brooklyn.

In these three cases, certain communities or districts of the city have been the sufferers to the advantage of other districts lying to the north and to the south; all are glaring examples of the economic loss to the city at large in its failure to clean up the path and save to itself the increment of valuations of its own creation, now lying waste and impossible of individual development through many years.

It would even seem reasonable that adjoining owners had good grounds on which to sue for contingent damages because they could not share in the accretion of value which should be theirs and for which they had paid a share in the taxes and assessments.

Sins of commission are seized upon with avidity by political parties for campaign material; the misappropriation of petty hundreds of dollars receives glaring double headers in the newspapers and the story continues with gloating and glee, but the obscure sin of omission involving lost millions in assessed valuations, in such instances as Lafayette Street and the Seventh Avenue Extension, pass by unnoticed by political parties and unheralded by the press.

Some may say that with political corruption a constant evil, how much greater the opportunity for corruption in applying excess condemnation in public improvements. This may be true, but no one stops raising wheat and more wheat because rats in the granaries cause losses. Not long ago there appeared an article written by a manufacturer on the subject of city planning. Among other things it was stated that engineers, as a rule, permit themselves to become the victims of concentration and, therefore, to fall into narrowness of vision, failing to take a broad interest in public affairs.

This is not the first time such an accusation against engineers has been made, nor is it an unwarranted or undeserved statement. It is a regrettable truth that, singly, collectively, or as a professional class, engineers do not take the interest in public affairs that they should. Why is it; is it fear of criticism? Physical dangers galore have been encountered by engineers. It cannot be possible that they lack moral courage. Then let them prove that public questions, such as excess condemnation, have received their attention and let them demand that the public authorities put into practice those theories which engineers have discussed, weighed, and found worthy. In like manner, let them fearlessly condemn such sins of omission wherever they may be found.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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Paper No. 1586

THE IMPROVED VENTURI FLUME*

By Ralph L. Parshall,† Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. H. B. Muckleston, E. W. Lane, E. S. Lindley, C. E. Carter, Julian Hinds, E. A. Jacob, J. C. Stevens, Carl Rohwer, and Ralph L. Parshall.

Synopsis

The device herein described is a promising structure for measuring the rate of flow of either small or large quantities of water. It has no moving parts, and is simple and inexpensive. It utilizes the principle of an hydraulic control and sacrifices little head. Tests indicate an accuracy of about 5 per cent.

Requirements to Be Met.—The measurement of flowing water in irrigation practice often requires that little loss of head be sacrificed by the measuring device. Because of the relatively large loss of head, the standard overfall weir is, in many cases, objectionable. Although when operating under standard conditions, it is one of the most accurate of measuring devices for practical field installation, it is not always possible to maintain these standard conditions, such as proper bottom contraction, velocity of approach, and aeration of nappe. The submerged orifice has not met with entire success because of the uncertainty of the coefficient of discharge, velocity of approach,

Note.—The Special Committee on Irrigation Hydraulics selected the subject of "Measuring Irrigation Deliveries" as one of ten for study and research. This paper was submitted to the Committee by its author, and the Committee recommended its publication. (See Progress Report of the Committee, *Proceedings*, Am. Soc. C. E., March, 1925, Society Affairs, p. 137).

* Published in *Proceedings*, Am. Soc. C. E., September, 1925.

† Irrig. Engr., U. S. Dept. of Agriculture, Fort Collins, Colo.

and constancy of condition. The ordinary rating flume often becomes inaccurate within a short time.

To meet the objections found in other measuring devices now in common use, the present Improved Venturi Flume has been developed. The improvements over the old Venturi flume consist in the reduction of the convergence in the inlet section, lengthening of the throat section, change of divergence of the outlet section, and depressing the floor in the throat section.* These changes have improved the flow conditions, reduced the effect of submergence, and simplified the operation by reducing the number of gauges necessary to determine the discharge.

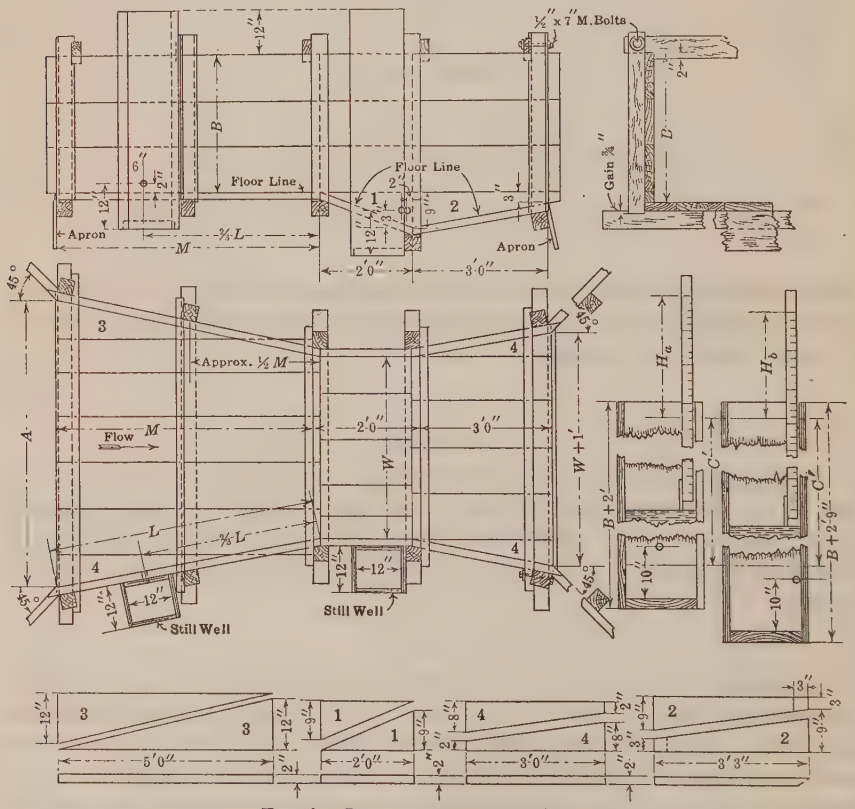


FIG. 1.—IMPROVED VENTURI FLUME.

Type of Structure.—This flume (Fig. 1) has three major parts, a converging inlet, a parallel-sided throat, and a diverging outlet. All the sidewalls are vertical. The inlet has a level floor between the sides, each side converging at a rate of 1 ft. in 5 ft. of length. The floor of the throat slopes downward with a vertical fall of 9 in. in a horizontal distance of 2 ft. The outlet floor slopes upward at the rate of 6 in. in 3 ft., and each wall diverges

* "The Venturi Flume," by V. M. Cone, *Journal of Agricultural Research*, Vol. IX, No. 4, 1917; "The Venturi Flume," by R. L. Parshall and Carl Rohwer, Associate Members, Am. Soc. C. E., *Bulletin 265*, Colorado Agricultural Experiment Station, 1921.

at the rate of 1 ft. in 6 ft. of length. The crest is the down-stream end of the level floor, and the crest length or size of flume is the distance, in feet, between the vertical walls of the throat. The lengths of the throat and the outlet of the flume are 2 and 3 ft., respectively, for flumes of all sizes from 1 ft. to 8 ft. The side of the inlet is made longer as the width of the flume increases, according to the arbitrary rule, $\frac{W}{2} + 4$, in which, W is the width of the flume, or crest length, in feet.

There are two general conditions of flow, namely, "free flow" when the elevation of the water surface at the throat gauge, or H_b, is lower than the crest, and "submerged flow", when the elevation of the water at the throat gauge is higher than the crest. The upper head, H_a, is observed at a point two-thirds the distance from the crest to the upper end of the inlet, along the side-wall, whereas H_b, the throat head, is observed at a point 3 in. vertically and 2 in. horizontally up stream from the lowest point of the depressed throat floor. Both the upper head, H_a, and the throat head, H_b, are referred to the crest level as the datum. Under certain conditions of flow the throat reading may be negative, as shown in Fig. 2.

Laboratory Tests.—Calibrations were made on flumes having crest lengths of 1, 2, 3, 4, 6, and 8 ft., with a minimum flow of 0.3 sec-ft. through the 1-ft. flume, and a maximum of 62.5 sec-ft. through the 6-ft. flume, the supply not being available to exceed this maximum through the 8-ft. flume. The discharge in second-feet through this device, under free-flow conditions, and under submerged conditions when the down-stream head does not exceed 70% of the upper head, is very closely approximated by the empirical formula:

$$Q = 4\ W\ H_a^{1.522\ W^{0.026}}$$

in which,

Q = discharge, in second-feet;
W = width of flume or crest length, in feet; and
H_a = upper head, in feet.

The accuracy of this device under these conditions is believed to be sufficient for practical field purposes. Table 1 shows the distribution of errors resulting from the comparison of the observed discharges with the results computed from the empirical formula. In determining the errors and their signs, the observed values are taken as the basis of comparison. Table 1 is based on the complete free-flow series of tests made on the six sizes of flumes studied, except one test which is believed to be in error* and the submerged flow tests for submergence up to 71% of the upper head. Submergence, as herein mentioned, means that the water surface, as indicated by the throat gauge, or H_b, is positive or above the crest level. For conditions of flow, as shown in Figs. 3 and 4, the degree of submergence, as indicated by the ratio, $H_b : H_a$, is somewhat less than the actual submergence would be if the water surface in the outlet were considered.

*Test No. 6512 on the 3-ft. flume, in which, $H_a = 0.705$ ft.; $H_b = 0.264$ ft.; percentage of submergence = 0; observed quantity = 8.33 sec-ft.; computed quantity = 6.94 sec-ft.; percentage of error = 16.7.

TABLE 1.—Showing Number of Observations and Distribution of Errors, Free-Flow and Submerged Flow up to 71 Per Cent. of Upper Head. Improved Venturi Flume.

Size of flume, in feet.	Total observations.	Percentage of observations under 5% error.	Percentage of Error.										
			0-0.9	1.0-1.9	2.0-2.9	3.0-3.9	4.0-4.9	5.0-5.9	6.0-6.9	7.0-7.9	8.0-8.9	9.0-9.9	10.0-10.9	11.0-11.9
			Number of Observations.*										
1	32	75	+ 6 — 9	+ 5 — 1	+ 3 — 0	+ 1 — 0	+ 3 — 0	+ 2 — 0	+ 0 — 0	+ 1 — 0	+ 0 — 0	+ 0 — 0	+ 0 — 0	+ 1 — 0
2	31	81	+ 6 — 4	+ 5 — 8	+ 2 — 0	+ 4 — 0	+ 1 — 0	+ 0 — 0	+ 0 — 0	+ 1 — 0	....	....	....	....
3	33	94	+ 5 —12	+ 4 — 6	+ 2 — 2	+ 1 — 0	+ 1 — 0	....	....	....	....	....	....	....
4	19	95	+ 3 — 6	+ 4 — 4	+ 0 — 1	+ 0 — 1	....	....	....	....	....	....	....	....
6	14	100	+ 2 — 1	+ 1 — 6	+ 0 — 4	....	....	....	....	....	....	....	....	....
8	23	96	+ 5 —12	+ 1 — 3	+ 1 — 0	+ 0 — 0	+ 0 — 0	+ 0 — 0	+ 0 — 0	+ 0 — 0	+ 1 — 0	....	....	....

* The figures preceded by plus or minus signs show the number of tests, within the limits indicated, in which the computed values are, respectively, greater or less than the observed values.

FIG. 2.—FREE-FLOW DISCHARGE, 6-FOOT IMPROVED VENTURI FLUME.

FIG. 3.—FREE-FLOW DISCHARGE, 6-FOOT IMPROVED VENTURI FLUME, WITH APPROXIMATELY 75 PER CENT. SUBMERGENCE.

FIG. 4.—EXPERIMENTAL 3-FOOT IMPROVED VENTURI FLUME AT WELLINGTON, COLO.

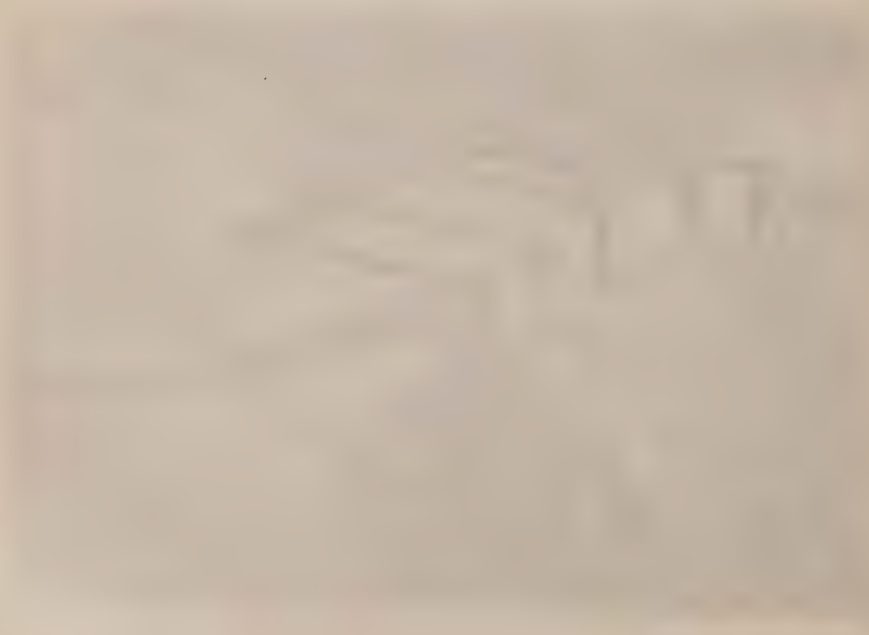


Table 2 shows the percentage of deviation of the calculated free flow from the observed discharge, where the degree of submergence varies from 69 to 79 per cent. It will be noted in this table that the agreement of calculated and observed values increases with the head, as well as with the width of flume. The increased velocity of the water in the throat section, together with the action of the depressed floor, causes a hydraulic jump to be formed immediately below in the diverging outlet of the flume, as shown in Figs. 3 and 4. This combination of increased velocity and depressed floor has the effect of resisting the submergence to a point of about seven-tenths of the upper head, H_a. It will be observed in Table 2 that for the 1-ft. flume and small flow the velocity is not sufficient to overcome the resistance of the water held in the depressed section, or, in other words, the point of measuring the throat head, H_b, is too far down stream to be within the range of the draw-down of the water surface for these small heads.

For the most satisfactory operation of this device, it is recommended that the flume operate under conditions in which the submergence does not exceed 70 per cent. Under such conditions only one observation of depth is necessary to determine the rate of flow. For submergence greater than 70 to 75%, the discharge is then a function of both the upper head, H_a, and the throat head, H_b. Laboratory tests show that when the submergence is greater than about 98%, no great dependence can be placed on the measurement.

Field Tests.—After completing the calibrations of the various sized flumes, outside field installations were made to test the device under practical conditions. At Wellington, Colo., a small lateral was selected having a flat grade, where the stream of water carried some sand and silt. In Fig. 4 is shown an experimental 3-ft. improved Venturi flume in this lateral, where the upper head, H_a, is 0.86 ft., and the throat head, H_b, 0.63 ft., with a difference of 0.23 ft., the ratio, $H_b : H_a$, being 73 per cent. The discharge under this condition is in accordance with the free-flow law, and the rate of flow can be assumed to be a function only of the upper head, H_a, thus giving a discharge of 9.48 sec-ft. A standard weir will not operate successfully in this lateral, first, because of the deposit which would occur in the weir box; and, second, because of the small head available. This discharge of 9.48 sec-ft. over a 3-ft. rectangular weir would require a head of exactly 1 ft., and to this possibly an extra 0.10 ft. should be added to assure a complete aeration, making a total loss of head of about 1.1 ft. By careful measurements with an engineer's level, the loss of head through the improved Venturi flume, under the conditions as stated, was only 0.19 ft.*

Under conditions where sand and silt occur, it has been observed that if a difference in head of 0.05 to 0.10 ft. can be maintained for a moderate upper head, no deposit will accumulate in the flume even where excessive quantities of sediment are carried in the stream.

Velocity of approach seems to have little or no effect on the rate of discharge for free-flow conditions.

* Note approximately similar conditions of heads for the 3-ft. flume, Table 2.

TABLE 2.—Comparison of Calculated Free-Flow with Observed Discharge for Submergence Varying from 69 to 79 Per Cent.

Test No.	H_a, in feet.	H_b, in feet.	Percentage of submergence.	Observed discharge, in second-feet.	Calculated discharge, in second feet.	Difference.	Percentage of deviation.
			1-Foot Flume.				
6707	0.200	0.141	70.5	0.31	0.35	0.04	12.9
6703	0.398	0.276	69.3	0.91	0.98	0.07	7.7
6667	0.600	0.424	70.7	1.74	1.84	0.10	5.7
6699	0.599	0.448	74.8	1.71	1.84	0.13	7.6
6695	0.801	0.568	70.9	2.72	2.85	0.13	4.8
6694	0.802	0.605	75.4	2.69	2.86	0.17	6.3
6710	1.002	0.708	70.7	3.86	4.01	0.15	3.9
6689	1.001	0.759	75.8	3.78	4.01	0.23	6.1
6686	1.202	0.857	71.3	5.10	5.29	0.19	3.7
6675	1.603	1.200	74.9	7.73	8.20	0.47	6.1
			2-Foot Flume.				
6645	0.201	0.140	69.7	0.62	0.66	0.04	6.5
6646	0.201	0.154	76.6	0.61	0.66	0.05	8.2
6637	0.402	0.281	69.9	1.87	1.95	0.08	4.3
6638	0.402	0.298	74.1	1.82	1.95	0.13	7.1
6631	0.598	0.421	70.4	3.49	3.60	0.11	3.2
6632	0.602	0.456	75.7	3.38	3.64	0.26	7.7
6606	1.002	0.707	70.6	7.89	8.02	0.13	1.7
6607	1.000	0.768	76.8	7.70	8.00	0.30	3.9
6612	1.602	1.127	70.3	16.51	16.61	0.10	0.6
6613	1.593	1.217	76.4	15.89	16.47	0.58	3.7
			3-Foot Flume.				
6579	0.199	0.140	70.4	0.92	0.96	0.04	4.3
6575	0.298	0.221	74.2	1.71	1.80	0.09	5.3
6528	0.399	0.298	74.7	2.75	2.85	0.10	3.6
6520	0.503	0.379	75.4	3.93	4.09	0.16	4.1
6534	0.595	0.446	75.0	5.13	5.32	0.19	3.7
6556	0.600	0.466	77.7	5.13	5.39	0.26	5.1
6514	0.699	0.499	71.4	6.74	6.85	0.11	1.6
6553	0.900	0.647	71.9	10.09	10.17	0 08	0.8
6543	0.996	0.710	71.3	11.84	11.93	0.09	0.8
6561	0.999	0.769	77.0	11.92	11.98	0.06	0.5
6563	1.000	0.754	75.4	11.81	12.00	0.19	1.6
6544	1.004	0.799	79.6	11.79	12.08	0.29	2.5
6572	1.200	0.836	69.7	15.99	15.96	—0.03	0.2
6411	2.039	1.509	74.0	35.23	36.62	1.39	3.9
6408	2.358	1.697	72.0	45.66	45.98	0.32	0.7
			4-Foot Flume.				
6399	1.014	0.791	78.0	15.09	16.35	1.26	8.4
6395	1.339	0.977	73.0	24.55	25.36	0.81	3.3
6373	1.702	1.293	76.0	35.44	37.03	1.59	4.5
6382	2.008	1.475	73.5	46.31	48.07	1.76	3.8
			6-Foot Flume.				
6359	0.900	0.678	75.3	20.11	20.29	0.18	0.9
6353	1.143	0.803	70.3	30.04	29.71	—0.33	1.1
6354	1.171	0.905	77.3	29.94	30.87	0.93	3.1
6347	1.396	1.065	76.3	40.20	40.86	0.66	1.6
6339	1.562	1.120	71.7	49.51	48.88	—0.63	1.3
6340	1.617	1.273	78.7	50.07	51.66	1.59	3.2
			8-Foot Flume.				
6313	1.064	0.774	72.7	35.08	35.36	0.28	0.8
6304	1.246	0.871	69.9	45.12	45.57	0.45	1.0
6305	1.270	0.937	73.8	45.23	46.97	1.74	3.8

Pertinent Features.—From the standpoint of measuring irrigation water, the improved Venturi flume has the following desirable features:

1.—It is sufficiently accurate for practical purposes.
2.—It operates in sand or silt-laden streams without trouble.
3.—It operates successfully with relatively small loss of head.
4.—It is able to withstand a high degree of submergence.
5.—It maintains constancy of condition.
6.—It possesses simplicity of operation.
7.—There are no moving parts.

DISCUSSION

H. B. Muckleston,* M. Am. Soc. C. E. (by letter).—The measuring device described in this paper should hardly be called a Venturi flume, at least, as long as only one head is measured and the formula on page 843 is applied. Under these conditions the device is a weir with the bottom contraction suppressed.

The ordinary form of weir can hardly be called a satisfactory instrument for measuring irrigation water. Theoretically, perhaps, a weir is a very accurate device, but the theory implies a great many things which are seldom obtainable under ordinary irrigation conditions. It is not to be expected that any one device will be applicable under all conditions. Much depends on the system of charging for the water. When the charge is for the actual quantity delivered, some kind of integrating mechanism is necessary; and such a mechanism, applicable to any device which really measures the rate of flow, is apt to be very expensive. Under such conditions, a device which actually measures the quantity and delivers it in batches is cheaper. When the charge is for the irrigable area covered, whether or not actually irrigated, time-rate devices are the cheapest and the simplest. The main object, in such conditions, is to satisfy the user that he is getting his share of the supply. Much depends on whether each user must have his individual measuring-box. When water is rotated among a group, group measurement is all that is really necessary. The quantity delivered to the unit then depends on the time of flow.

Considering only time-rate devices, a satisfactory measuring instrument must meet a number of conflicting requirements. It must be cheap, nearly "fool-proof", immune to silt troubles, easily read, and fairly accurate; it should hold its accuracy under considerable abuse, and it should require a minimum of lost head. The weir meets the first and fourth requirements very well, but its virtues stop there. The Venturi flume, as first described by Mr. Cone and afterward by the author, came more nearly to meeting all the conditions than any other with which the writer was acquainted. It is possible that the new device will do even better.

One of the main objections to the weir was the loss of head which it involved. This might be several inches or even a foot or more. One consequence was that the user developed a strong tendency to back up the water and drown the weir. The original Venturi flume was fairly accurate even when deeply submerged, and the use of a plug, as described by Mr. H. K. Smith, extended the accuracy over a wider range. The new device gives promise of sufficient accuracy, and it should be nearly proof against abuse; but it should be noted that, as long as it must work under the condition of free flow, the possibility of the user drowning the flume is present. The indications from a single head measurement will then be inaccurate, possibly greatly inaccurate. The device promises to be a cheap one.

As long as other factors are so indefinite, third decimal accuracy is not required in measuring devices. Under any conditions, 6% should be close enough; less than 3% would be splitting hairs.

* Cons. Engr., Vancouver, B. C., Canada.

The rather formidable looking formula on page 843 is easily plotted as an alignment nomograph, Fig. 5. The lines for Q and H_a are simple loga-

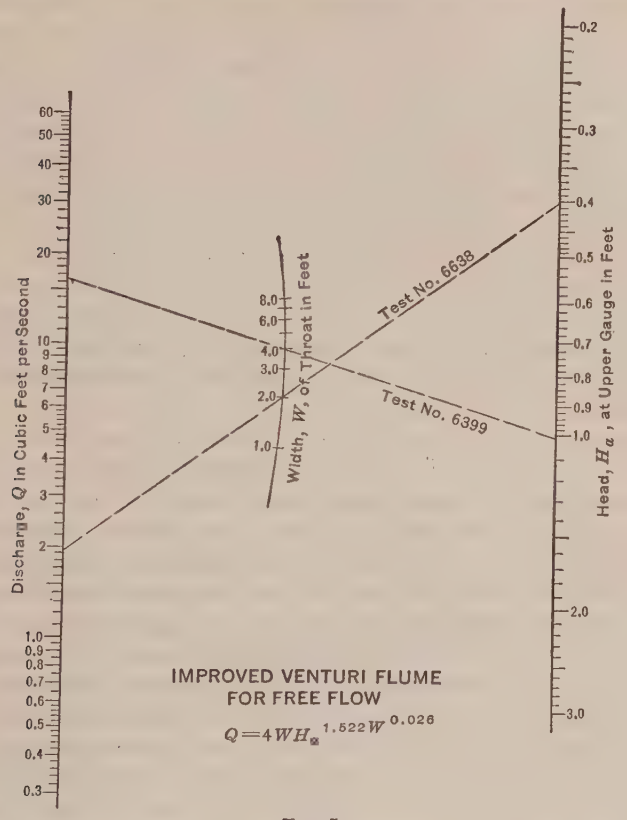


FIG. 5.

rithmic scales to any convenient pattern. The line for W can be established by cross-intersections, using the calculated discharges from Table 2. No calculations are needed.

E. W. LANE,* M. AM. SOC. C. E. (by letter).—In his paper, "Experiments on the Flow of Water Through Contractions in an Open Channel",† the writer pointed out that the experiments for developing the Venturi flume for measurement of irrigation water dealt only with conditions of discharge under which readings of both up-stream and throat gauges were necessary to determine discharges, and that for a large range of conditions the discharge could be determined from readings on the up-stream gauge only, the advantage of which had been overlooked. The flume described by the author seems to be a recognition of this advantage, as the improvement claimed over the Venturi type

* Lafayette, Ind.

† *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1201.

seems to be principally that the discharge over a larger range of conditions can be determined from readings of one gauge.

The writer knows of no experiments to show over what range of conditions the Venturi flume will discharge under what may be called "single gauge conditions", but the indications of his experiments are that for a flume with gradual expansion it is likely to be considerable. Perhaps with the loss of hydraulic efficiency which the adaptation of this device to practical irrigation conditions necessitated, the range over which it would operate under "single gauge conditions" was reduced, and, therefore, the new device is better.

Since small loss of head is one of the advantages claimed for the improved flume, it is to be regretted that the experimental results do not include the total loss of head due to the flume. A value of 0.19 ft., with a discharge of 9.48 sec-ft. through a 3-ft. flume, is the only one mentioned. Run No. 3 of the writer's experiments with a Venturi flume at practically the same discharge per foot of width and operating under "single gauge conditions" showed a loss of 0.12 ft.

To the writer the most interesting feature of the new flume is the principle on which it acts. It is called an "improved Venturi flume", but it is not a Venturi flume at all, since the principle on which it acts over the greater part of its range and from which it derives its superiority over the original type is not the same as in the true Venturi flume or Venturi meter. The new flume is really a trough with a free drop at the end and a device for converting the resulting velocity of the water back into head. This conversion is accomplished by causing the water to flow up an inclined plane, the velocity being expended in actually lifting the mass of water to a higher elevation. There is nothing new about this in principle, but the practical application is rare. The trough part of the flume is not essentially different from a type of measuring device developed just before the Venturi flume in which the water flowed through a trough with a free, or nearly free, fall at the lower end and had a discharge varying practically as the three-halves power of the head. The effect of the converging sides of the trough section seems to be to increase the velocity through the narrow section, thus insuring, over a greater range, velocities higher than the critical value, and, therefore, a flow through the opening independent of back-water conditions. Although the author does not seem to have appreciated that a new principle is involved there is no reason why this flume may not be named for the originator and the name Venturi dropped.

The term, "free flow", as used by the author is confusing if not incorrect. Any condition in which water at the crest of the weir is flowing at a higher than critical velocity is a condition of free flow, since the flow is independent of the tail-water level. This may occur with the water levels indicated by the throat gauge either higher or lower than the crest of the weir. The author seems to recognize this in his title to Fig. 3, "Free-Flow Discharge, 6-Foot Venturi Flume, with Approximately 75 Per Cent. Submergence", which is obviously contradictory according to his definitions of "free flow" and "submerged flow".

E. S. LINDLEY,* M. AM. SOC. C. E. (by letter).—The first mention of this device quoted by the author, and the first in America of which the writer is aware, is the paper by Mr. V. M. Cone, published in 1917.† In the Panjab, use of the device was initiated by the late Messrs. Stoddard and Harvey of the Irrigation Department some time before 1914.

The object with which the device was primarily developed in the Panjab, was control; that is, to secure a constant relation between discharge and water level above the device, with only a small drop of level through it. Calibration and precise knowledge of the discharge passing were of course also important.

The writer wished to extend the use of the device and use it as a Venturi flume, that is, to measure discharge by means of gauge readings at the throat as well as up stream. In correspondence, Clemens Herschel, Past-President, Am. Soc. C. E., recorded the opinion that this was not really practicable; the writer soon came to agree with him. The attempt to develop the Venturi flume was still proceeding in America. To avoid the confusion that arose over applying that name to the "control section" use of the device, the name, "Standing Wave Flume", was originated instead. It would appear that part of the improvement described by Mr. Parshall is to omit the use of the throat gauge except to show when there is a drop of water surface sufficient to avoid damping the maximum discharge.

It is to be remarked that the principles involved are the same whether the "control section" be produced by constricting the width of the channel, or by raising a hump or broad-crested weir on the bed, or by both in combination.

The workers at this form of device are divided into two schools. One school starts with a design of weir or flume that is convenient to build, and that is an effective control, and then calibrates it experimentally. The other school argues that the well-known broad-crest weir formula generally ascribed to Unwin entails a definite physical condition of flow, that any departure from the rates of discharge given by that formula indicates a state of uncertainty of flow, and that the best course is to determine the limits of form that the work must satisfy in order to discharge according to that formula. The former school produces a new formula (as does this paper) for every new shape tested; the latter claims that each such formula is uncertain of correct application outside the range of the tests on which it is based, and that a structure of any size can be built which will discharge correctly.

Unfortunately, those who have done the most work on this device in India have not placed their facts and views on record. The writer has made an attempt to fill the gap in the course of a technical paper;‡ a certain amount of data are also to be found more accessibly in a discussion that he contributed to another paper that bears on the subject.§ He does not propose to repeat the whole matter here, but only to outline the main points, leaving those who are sufficiently interested to read the original.

The limits of shape have been ascertained, which a broad-crest weir must satisfy to discharge according to the broad-crest formula. Data are quoted

* Executive Engr., Indian Public Works Dept., Panjab Irrig. Branch, Care, Messrs. Thos. Cook & Son, London, England.

† *Journal of Agricultural Research*, Vol. IX, No. 4, 1917.

‡ "Canal Falls and Their Use as Meters," Panjab Eng. Congress, 1925.

§ "Measurements of Discharge Over a Rock-Faced Dam", *Minutes of Proceedings*, Inst. C. E., Paper No. 4515, 1924.

from structures built for discharges up to 8 000 cu. ft. per sec. and up to 6½-ft. depth on the crest. Subject to a few limitations and conditions that will be obvious to practical hydraulicians, and in no way impair the practical utility of the works, they are probably correct within about 1 per cent. Those not built to effect any recovery of head are independent of fluctuations of down-stream levels as long as the drop does not fall short of one-third of the depth on the crest. Those built to work with less head are thus independent as long as the drop does not fall short of one-sixth of the depth on the crest for small structures, while the fraction may be shown to be even smaller for larger works.

A rational weir formula is also given (due to Mr. E. S. Crump of the Panjab Irrigation Department), which indicates the manner in which many weirs and flumes discharge more than the broad-crest formula.

Such weirs and flumes on a small scale, and orifices similarly made to discharge as if they had free falls, are now being largely used in Indian irrigation to eliminate manual control and secure automatically correct distribution of irrigation supplies. This subject is fully dealt with in another technical paper by the writer,* which also gives a full bibliography of the subject.

In Mr. Parshall's paper, an accuracy of 5% is claimed for the device described. Greater accuracy is not likely to be achieved as long as angles are allowed in the sides and floor. These relatively loose limits also probably cover a certain amount of dependence on the fluctuation of down-stream levels. In the only observation of which the necessary data are reported, the limit for independence occurred with a 0.19-ft. fall for a 0.86-ft. depth on crest, a fraction of two-ninths, or only little better than the Panjab form without attempting to recover the head. The writer considers the position for taking the throat head unsatisfactory; in fact, the essence of objections to a Venturi flume is that throat gauges are unreliable unless the fall is quite small. The reasons for this are fairly obvious even from theory.

The exponent of W in the formula, given on page 843 as 0.026, would appear to be a misprint; judged by a few checks of calculations in Table 2, it would seem that the exponent of H_a in the formula should be nearer 1.6 than 1.52, and that the exponent of W is actually 1.0.

The coefficient, 4.0, in the formula is surprising. Mr. Crump's rational formula, previously alluded to, indicates a maximum possible coefficient of 4.18. In some tests with small weirs intended to give the greatest possible coefficients, the writer was unable to find one greater than 3.9, and that seems to be the largest value given by the late Hamilton Smith, M. Am. Soc. C. E.

A paper of this kind is less convincing than it should be, when no account is given of the means and manner of testing, and only a few of the observed data are included.

C. E. Carter,† M. Am. Soc. C. E. (by letter).—A means of measuring accurately the flow of water in irrigation canals and other open channels has long been needed and that fact is well appreciated by those who have ever had anything to do with the delivery of certain definite quantities of water to individual consumers.

* "The Application of Modules in Irrigation", Panjab Eng. Congress, 1923

† Asst. Engr., Metcalf & Eddy, Boston, Mass.

TABLE 3.—COMPARISON OF CALCULATED FREE FLOW WITH OBSERVED DISCHARGE FOR SUBMERGENCY FROM 69 TO 79 PER CENT.

(Based on formula, $Q = 3.90\ W\ H_a^{1.58}$.)

Test No.	H_a, in feet.	H_b, in feet.	Percentage of submergence.	Observed discharge, in cubic feet per second.	Calculated discharge, Carter formula.	Difference, in cubic feet per second.	Percentage of deviation by Carter formula.	Percentage of deviation by Parshall formula.
				1-FOOT FLUME.				
6707	0.200	0.141	70.5	0.31	0.31	0.00	0.00	12.9
6703	0.398	0.276	69.3	0.91	0.91	0.00	0.0	7.7
6667	0.600	0.424	70.7	1.74	1.74	0.00	0.0	5.7
6699	0.599	0.448	74.8	1.71	1.72	0.01	0.6	7.6
6695	0.801	0.568	70.9	2.72	2.74	0.02	0.8	4.8
6694	0.802	0.605	75.4	2.69	2.75	0.06	2.2	6.3
6710	1.002	0.708	70.7	3.86	3.91	0.05	1.3	3.9
6689	1.001	0.759	75.8	3.78	3.91	0.13	3.4	6.1
6686	1.202	0.857	71.3	5.10	5.22	0.12	2.4	3.7
6675	1.603	1.200	74.9	7.73	8.22	0.49	6.3	6.1
				2-FOOT FLUME.				
6645	0.201	0.140	69.7	0.62	0.62	0.00	0.0	6.5
6646	0.201	0.154	76.6	0.61	0.62	0.01	1.6	8.2
6637	0.402	0.281	69.9	1.87	1.85	−0.02	−1.1	4.3
6638	0.402	0.298	74.1	1.82	1.85	0.03	1.6	7.1
6631	0.598	0.421	70.4	3.49	3.46	−0.03	−0.9	3.2
6632	0.602	0.456	75.7	3.38	3.50	0.12	3.6	7.7
6606	1.002	0.707	70.6	7.89	7.82	−0.07	−0.9	1.7
6607	1.000	0.768	76.8	7.70	7.80	0.10	1.3	3.9
6612	1.602	1.127	70.3	16.51	16.42	−0.09	−0.5	0.6
6613	1.593	1.217	76.4	15.89	16.28	0.39	2.5	3.7
				3-FOOT FLUME.				
6579	0.199	0.140	70.4	0.92	0.93	0.01	1.1	4.3
6575	0.298	0.221	74.2	1.71	1.73	0.02	1.2	5.3
6528	0.399	0.298	74.7	2.75	2.74	−0.01	−0.4	3.6
6520	0.503	0.379	75.4	3.93	3.94	0.01	0.3	4.1
6534	0.595	0.446	75.0	5.13	5.15	0.02	0.4	3.7
6556	0.600	0.466	77.7	5.13	5.22	0.09	1.7	5.1
6514	0.699	0.499	71.4	6.74	6.64	−0.10	−1.5	1.6
6553	0.900	0.647	71.9	10.09	9.91	−0.18	−1.7	0.8
6543	0.996	0.710	71.3	11.84	11.63	−0.11	−0.9	0.8
6561	0.999	0.769	77.0	11.92	11.68	−0.24	−1.2	0.5
6563	1.000	0.754	75.4	11.81	11.70	−0.11	−0.9	1.6
6544	1.004	0.799	79.6	11.79	11.77	−0.02	−0.2	2.5
6572	1.200	0.836	69.7	15.99	15.60	−0.39	−2.4	−0.2
6411	2.039	1.509	74.0	35.23	36.06	0.83	2.4	3.9
6408	2.358	1.697	72.0	45.66	45.38	−0.28	−0.6	0.7
				4-FOOT FLUME.				
6399	1.014	0.791	78.0	15.09	15.95	0.86	5.7	8.4
6395	1.339	0.977	73.0	24.55	24.74	0.19	0.8	3.3
6373	1.702	1.293	76.0	35.44	36.14	0.70	2.0	4.5
6382	2.008	1.475	73.5	46.31	46.93	0.62	1.3	3.8
				6-FOOT FLUME.				
6359	0.900	0.678	75.3	20.11	19.81	−0.30	−1.5	0.9
6353	1.143	0.803	70.3	30.04	28.90	−1.14	−3.8	−1.1
6354	1.171	0.905	77.3	29.94	30.03	0.09	0.3	3.1
6347	1.396	1.065	76.3	40.20	39.64	+0.58	+1.4	1.6
6339	1.562	1.120	71.7	49.51	47.34	+2.17	−4.4	−1.3
6340	1.617	1.273	78.7	50.07	50.00	−0.07	−0.1	3.2

TABLE 3.—(*Continued.*)

Test No.	H_a, in feet.	H_b, in feet.	Percentage of submergence.	Observed discharge, in cubic feet per second.	Calculated discharge, Carter formula.	Difference, in cubic feet per second.	Percentage of deviation by Carter formula.	Percentage of deviation by Parshall formula.
				8-Foot Flume.				
6313	1.064	0.774	72.7	35.08	34.44	—0.64	—1.8	0.8
6304	1.246	0.871	69.9	45.12	44.15	—0.97	—2.1	1.0
6305	1.270	0.937	73.8	45.23	45.41	0.18	0.4	3.8

Mr. Parshall's paper opens the way for making such measurements of flow within a reasonable degree of accuracy, and it is to be hoped that future experiments will extend the scope of those presented to include wider channels, greater depth of flow, and larger rates of discharge.

Believing that the author's formula for discharge,

$$Q = 4\ W\ H_a^{1.522\ W^{0.026}}$$

would be in a more workable form if the function of W could be eliminated from the exponent of H_a, the data presented have been studied with this in view and the formula,

$$Q = 3.90\ W\ H_a^{1.58}$$

has been deduced. The writer believes that this formula gives results that are fully as close to the actual discharges as that of the author for the sizes of flumes and quantities of discharge shown.

Using this formula, Table 2 has been recomputed as shown in Table 3. Only ten items show deviations greater than those by the author's formula, of which only one varies more than 5% from the actual quantity discharged. The remaining thirty-eight items show deviations less than these by the author's formula.

A comparison of the percentages of deviation shows (Table 4) that 75% of the computed quantities, using the writer's formula, are within 2% of the observed quantities and that 96% vary less than 5% from the observed quantities.

TABLE 4.—Comparison of Deviations from Observed Discharges.

Percentage of deviation.	Percentage of Deviation.	
	Carter formula.	Parshall formula.
1 or less	42	19
2 " "	75	31
3 " "	88	33
4 " "	94	60
5 " "	96	71

Although the 1-ft., 2-ft., and 3-ft. flumes show discharges under heads as low as $H_a = 0.20$ ft., the 4-ft., 6-ft., and 8-ft. flumes do not show discharges for

heads lower than $H_a = 1.0$ ft. In order to compare the two formulas for heads lower than 1.0 ft., controlling computations have been made, giving the results shown in Table 5.

TABLE 5.—COMPARISON OF FORMULAS FOR LOW HEADS ON THE 4-FOOT, 6-FOOT, AND 8-FOOT FLUMES.

Width of flume, in feet. (1)	H_a, in feet. (2)	Computed Discharge, in Cubic Feet per Second. Parshall formula. (3)	Carter formula. (4)	Difference. (5)	Percentage of variation, $\left(\frac{\text{Column (5)}}{\text{Column (3)}} 100.\right)$ (6)
4	0.20	1.27	1.24	—0.03	—2.5
	0.60	7.15	6.96	—0.19	—2.6
	1.00	16.00	15.60	—0.40	—2.5
6	0.20	1.84	1.86	0.02	1.1
	0.60	10.63	10.44	—0.17	—1.6
	1.00	24.00	23.40	—0.60	—2.5
8	0.20	2.41	2.48	—0.07	2.9
	0.60	14.09	13.92	—0.17	—1.2
	1.00	32.00	31.20	—0.80	—2.5

A study based on the complete record of experiments for all the flumes might show that a slight change in the constant or exponent could be used to advantage, but, from the foregoing, the writer would expect that the formula,

$$Q = 3.90\ W\ H_a^{1.58}$$

would give results well within 5% of actual discharges for all sizes of flumes and heads within the scope of the experiments.

JULIAN HINDS,* M. AM. SOC. C. E. (by letter).—This paper is an important contribution to engineering knowledge on the subject of water measurement. The open Venturi flume has come to be recognized as a valuable device because of its freedom from interference by silt accumulations and because of the small head required for its operation.

As originally contemplated, the Venturi flume requires the reading of two gauges, as in the case of the closed Venturi tube. Because of the variable areas of the water prism at points of measurement, diagrams and tables for the original device are more complicated than for the closed Venturi meter. It is now proposed to eliminate one of the gauges and to simplify the utilization of the device, putting it on an equality with the sharp-crested weir so far as reading the discharge is concerned, without seriously interfering with the advantages of the original type.

The writer is in full accord with the proposal to develop a one-gauge Venturi flume, and is of the opinion that the type of structure tested by the author can be satisfactorily developed.

However, it is believed that the proposed details can be improved. The discharge formula presented by the author is entirely empirical, and is applicable only to structures of the precise form tested. By slightly altering the structural details it is believed that the device can be made to operate in accordance with a rational theory. The writer considers this desirable, al-

* Engr., U. S. Bureau of Reclamation, Denver, Colo.

though the resulting formula may be as difficult to use as the empirical formula. The proposed flume takes partial advantage of the controlling effect of a point of critical depth in open-channel flow. By taking full advantage of this effect the flow formula can be rationalized.

If a flow of water is required to pass through the flume shown in Fig. 6, the floor of the flume being level from *A* to *D* and moderately steep down stream from *E*, then, at some point between *D* and *E*, flow will take place under a minimum energy head and the depth will be critical. The critical depth will occur at the point where the slope of the bottom of the channel is just sufficient to overcome friction due to flow at critical depth, which point is called the "control point." The corresponding cross-section of the canal may be called the "control section."

The theory of the critical depth and the point of control has been thoroughly discussed in recent years before the Society and in technical publications, and need not be referred to here further than to call attention to the fact that it assumes normal hydrostatic pressure throughout the depth of the stream. At points of irregularity, such as sharp corners in the floor or sides of the channel, or at the outfall end of a flume discharging freely into the air, the hydrostatic pressure in the stream departs from normal, and the energy equation is correspondingly disturbed. Usually the critical depth is not situated exactly at such points, although it is often assumed to be.

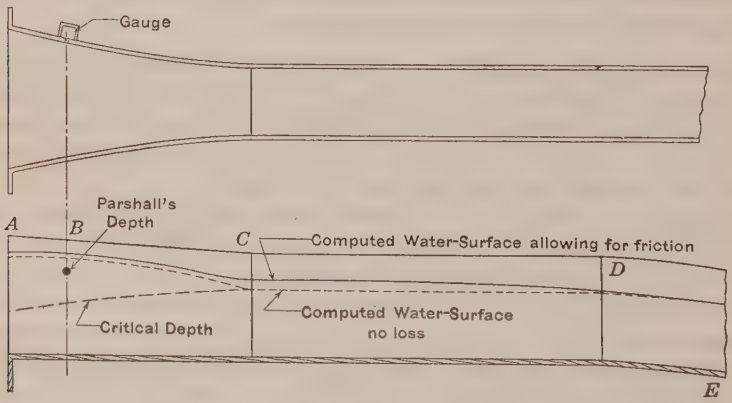


FIG. 6.—ELONGATED CONTROL-SECTION METER. CURVED INTERSECTIONS.

If irregularities of this kind are avoided, a point of control always exists where an open channel changes, within certain limits, from a comparatively flat slope to a comparatively steep slope. At the point of control many of the uncertain factors normally affecting flow, such as unknown friction coefficients, velocity of approach, etc., are eliminated, and the depth bears a definite relation to the discharge. Thus, by introducing into a canal a proper change in grade, a point of definite stage for any given flow is established. It is not necessary that the steep slope to the right of *E* in Fig. 6 continue for any great distance, provided the elevation of the back-water below,

after allowing for recovery of velocity head, is low enough to prevent submergence of the point of control.

So far as the writer knows, no attempt has been made to utilize the directly measured critical depth for determining discharge, although the relation between this depth and the flow is simple. This is probably because the exact location of the control is likely to be uncertain. The depth near the critical point is subject to rapid variation and there is a lack of confidence in measurements taken at the theoretical point of control. A flume such as that shown in Fig. 6, 2.00 ft. wide, will flow 2.00 ft. deep at the point of control with a discharge of 32.08 sec-ft. With Kutter's $n = 0.013$, the friction slope for the critical depth is approximately 0.008. The friction loss in the first 0.25 ft. above the control, therefore, will be about 0.002 ft. To build up the energy head needed to compensate for this loss requires an increase in depth of approximately 0.05 ft. Therefore, if the floor of the flume is level above the control, the water surface will rise more than 0.5 in. in the first 3 in. of channel, due to the flatness of the energy curve near the point of critical depth. For shorter distances the rate of rise is even more pronounced, and a slight error in the selection of the point of measurement will appreciably affect the measured depth. A curved bottom for a short distance above the control tends to compensate for the disadvantageous shape of the energy curve, and it may eventually be found possible, with a proper type of structure, to use the measured critical depth for the determination of discharge. Such a plan would have the distinct advantage of being free from the influence of changing approach conditions.

With the gauge located in the inlet section as advocated by the author, the theoretical relation between depth and discharge may be computed from Bernoulli's theorem, making allowance for friction and transition losses between the point of measurement and the control. If the length of the structure is great, or if the transition is imperfect, the uncertainty as to the total value of the losses is likely to be considerably greater than if the depth is measured at the point of control. However, the effect on the computed flow of an error in determining the loss is less marked with the gauge in the inlet. The gauge should be set at a point where the channel will always be swept clean by the velocity, to avoid interference by accumulations of silt.

Two water surface curves are shown in Fig. 6, one for frictionless flow and one allowing for friction due to Kutter's $n = 0.013$. Transition losses are disregarded. The divergence of the two lines indicates that with carefully rounded corners of the type shown, and with a long radius vertical curve from D to E, the head may be safely measured at the control.

The gauge height for the assumed flow, computed from the author's formula, is shown at A (Fig. 6), and is appreciably lower than that computed from the control theory. This discrepancy is explained by a study of the flume shown in Fig. 7, which is identical with that of Fig. 6, except that all curves in the side-walls and bottom are omitted. The inlet is identical with that proposed by the author for a 24-in. flume. Because of the angular change in the slope at I, the stream will leave, or tend to leave, the bottom. As a result there will be a reduction in the hydrostatic pressure, and an alteration

of flow conditions, preventing the formation of a control at *I*. Passing up stream from *I*, normal hydrostatic conditions are no doubt gradually approached, and if the flume is sufficiently long a control will exist at some point. If a control does exist, the flow may be determined from a gauge reading at *G*, the only effect of the irregularity at *I* being the introduction of further uncertainty as to the head lost in friction between the gauge and the point of control. The contractions at *H* (Fig. 7) introduce eddy losses of unknown value which probably affect the gauge reading at *G* more seriously than do the irregularities at *I*.

The throat section, or level, parallel part of the flume in Fig. 7, is purposely made long to avoid the possibility of overlapping the effects of disturbances at the two ends. If the distance, *H I*, is gradually reduced the point of control will approach the disturbed region below *H* and will eventually pass above *H*. An entirely new set of conditions will thus be set up, and if the inlet transition is blunt, weir discharge will be approached. If the taper of the inlet is gradual a control probably will be established a short distance up stream from the junction between the inlet and the throat. The control thus coming at a point where the flume is wider than in the throat, the relation of the gauge reading at *G* to the flow will be altered.

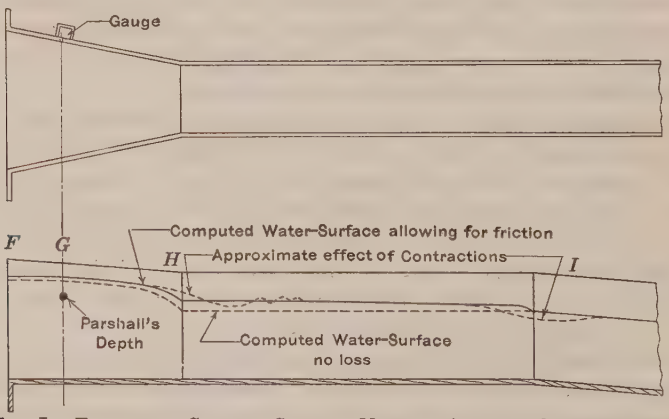


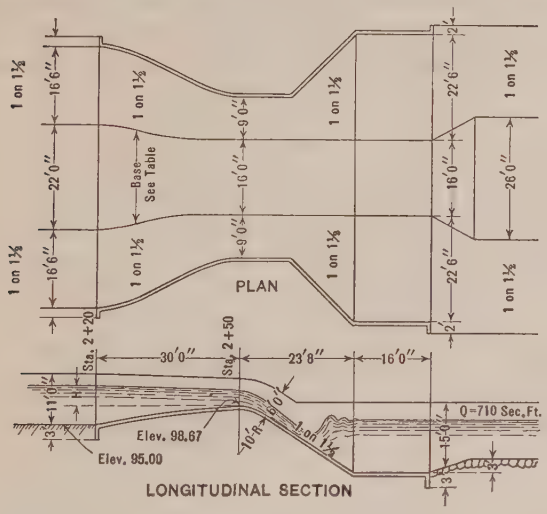
FIG. 7.—ELONGATED CONTROL-SECTION METER. ANGULAR INTERSECTIONS.

This is believed to be the situation existing in the author's device. The throat of the structure is not lengthened, as claimed, but is shortened to a zero length by the vertical expansion beginning immediately below the end of the converging inlet.

The distance from the end of the transition up stream to the point of control is probably influenced by the angle of break in the grade line. The relation between the depth at the gauge and the discharge depends on the width of the channel at the point of control, which is dependent on the location of the control and on the rate of flare of the inlet. The author's formula is, therefore, applicable only to flumes constructed in strict conformity with the structures tested.

By making the floor of the throat section of the flume level, extending the throat sufficiently to avoid overlapping the effects of angularity at the two ends, and providing sufficient fall to prevent flooding the throat, the device can be made to follow the theoretical law for flow through a point of critical depth. This alteration brings the structure back to the form of the original Venturi flume, which the writer believes to be a better structure fundamentally than the one now proposed.

The U. S. Bureau of Reclamation has developed a type of measuring device designed to operate on the control-section principle. To facilitate construction in concrete, without the use of forms, the throat and approach walls are given a 1 on 1.5 slope. A discussion* by the writer regarding this structure and its operation was published in 1920. The bearing of Venturi flume tests made at Cornell University on the control-section theory was discussed at the same time.



BASE DATA

Sta.	Elev.	Width
2+20	95.00	22.0
+22	95.25	21.9
+24	95.60	21.6
+26	96.08	21.0
+28	96.60	20.0
+30	97.00	18.8
+32	97.30	17.8
+34	97.57	17.0
+36	97.84	16.5
+38	98.04	16.1
+40	98.24	16.0
+42	98.41	16.0
+44	98.53	16.0
+46	98.61	16.0
+48	98.66	16.0
+50	98.67	16.0

CONTROL SECTION
C-G CUTOFF CANAL

FIG. 8.

The first experiments by the U. S. Bureau of Reclamation on the trapezoidal control section were made at a structure on the C-G Cut-Off Canal on the Klamath Project (Fig. 8), in Oregon, and have been described in a paper† by J. L. Savage, M. Am. Soc. C. E. A number of additional measurements have been made at this structure. All the ratings made to date are shown on Fig. 9. The discharge measurements were made with a current meter. Field measurements were made by Allen L. Darr and H. K. Smith, Assistant Engineers, U. S. Bureau of Reclamation. The agreement between the current-meter discharges and the theoretical discharge of the control section is good. A single test made on a similar structure on the main canal of the King Hill Project, in Idaho, showed practically exact agreement with the theoretical discharge.

* "Venturi Flume Data Throws Light upon Control Weir," *Engineering News-Record*, December 23, 1920, p. 1223.

† "Discharge Measurements for Control-Section Weir," *Engineering News-Record*, May 15, 1924, p. 842.

In the fall of 1923, a 24-in. trapezoidal control-section structure was installed at the Hydraulic Laboratory of the Bureau of Reclamation, at Boise, Idaho, and a series of tests were run by W. G. Steward, Assistant Engineer, U. S. Bureau of Reclamation. The structure was built of wood, the inlet being warped to conform as nearly as practicable to the lines of the standard concrete device. Timber construction was adopted to facilitate remodeling during the progress of the tests. It is not contemplated that wood will be used for permanent installations of the warped type. Details of the structure as installed are shown in Fig. 10. Results of the ratings are shown in Table 6 and Fig. 11. The position of the scales and gauges and the hydraulics of a typical run are shown on Fig. 6.

Gauge readings were made with hook gauges in stilling-wells, located as shown on Fig. 12. The flat scale at the upper end of the inlet structure was countersunk flush with the surface of the wall. The structure was constructed and maintained accurate to the theoretical dimensions within 0.01 ft. The discharges were measured volumetrically in carefully calibrated concrete-lined measuring tanks.

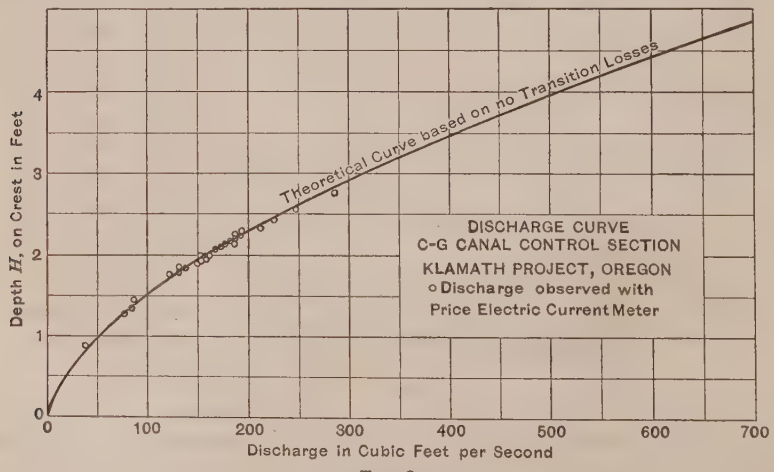


FIG. 9.

In order to determine the influence on the gauge height of the throat length and of curvature at the down-stream end of the throat, the parallel part of the structure was made removable, and removable curved "throat pads" (Fig. 10) were provided at the junction of the throat with the outlet transition. Throat lengths from zero to 3 ft. were tried, both with and without the throat pad. Eighty-seven separate observations were made. The data obtained are not sufficient to establish with precision the laws of flow for a trapezoidal control-section, but they indicate substantial conformity with theory to depths approximately equal to the throat length.

During the progress of the tests a slight erosion of the banks occurred above the up-stream head-wall, causing a disturbance opposite Gauge No. 1 and the scale. To prevent this condition the side-walls were extended as indicated

by dotted lines on the plan view, Fig. 10. These extensions were maintained in place from Series 5, Run 2, to Series 11, Run 6. For Series 12 to 15, inclusive, the extensions were removed and the banks carefully rip-rapped.

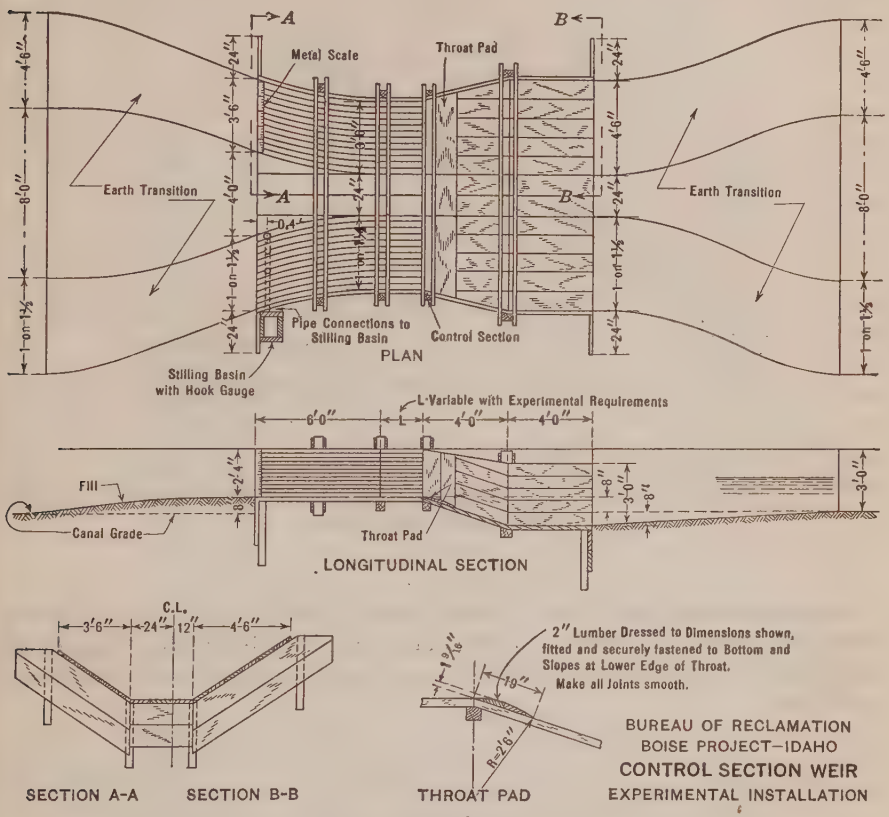


FIG. 10.

The theoretical discharge curve shown in Fig. 11 is based on the assumption of no entrance or friction loss, and is determined by making the gauge depth for a given flow such that the energy gradient at the gauge is equal to the minimum energy gradient at the control.

The general conformity between observed and computed discharges indicates that friction and inlet losses are small. Increased friction in the long throat partly explains the apparent reduction in discharge for Series 15. The computed theoretical discharges shown in Table 6 were scaled from an enlargement of the theoretical curve shown in Fig. 11. All notes shown under the heading, "Remarks", in Table 6 are copied from the original field notes.

In addition to the data tabulated, Gauges Nos. 2 and 3 were read for all runs. These readings are not shown in Table 6. Scale No. 1 was also read for the majority of the runs, and, in general, gave results reasonably close to the theoretical. The scale was originally installed as shown in plan in Fig. 10. For this position of the scale the readings were not the same on the two edges,

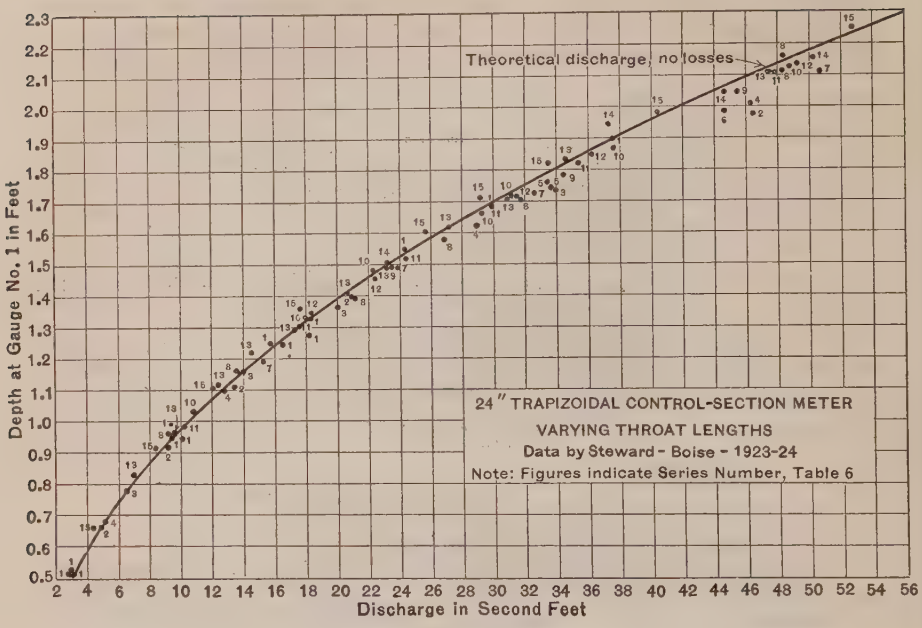


FIG. 11.

which caused some confusion. Scales located at other points were tried out later, but yielded no results of particular value. The impression was gained that a scale, properly set, is sufficiently accurate for field use for irrigation purposes.

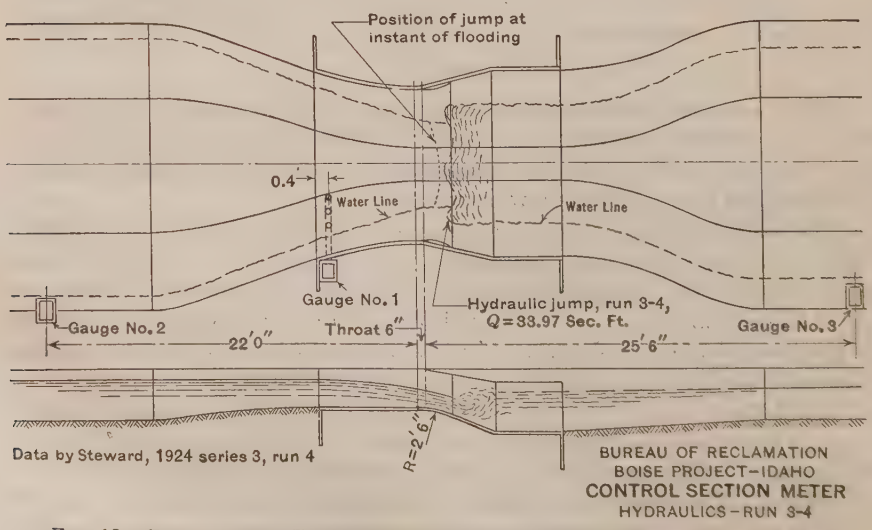


FIG. 12.—TYPICAL RUN, EXPERIMENTAL CONTROL SECTION METER, BOISE, IDAHO.

TABLE 6.—Observed and Computed Data, 24-Inch Trapezoidal Control-Section Meter, Boise, Idaho.

Series.	Run.	Gauge.	Observed discharge, in second-feet.	Computed discharge, in second-feet.	Error, in second-feet.	Error, percentage.	Throat length, in inches.	Pad.	Remarks.
1	1	0.517	3.11	3.16	−0.05	− 1.6	24	No	
	2	0.521	3.19	3.21	−0.02	− 0.6	24	"	
	3	0.525	3.15	3.28	−0.13	− 4.0	24	"	Slightly flooded
	4	0.949	10.06	9.41	+0.65	+ 6.9	24	"	Questionable
	5	0.952	9.55	9.45	+0.10	+ 1.1	24	"	
	6	0.954	9.43	9.51	−0.08	− 0.8	24	"	Slightly flooded
	7	0.997	9.49	10.36	−0.87	− 8.4	24	"	Flooded
	8	1.250	15.74	16.12	−0.38	− 2.4	24	"	
	9	1.252	16.55	16.18	+0.37	+ 2.3	24	"	
	10	1.278	18.28	16.85	+1.43	+ 8.5	24	"	Unsettled
	11	1.326	18.26	18.18	+0.08	+ 0.4	24	"	
	12	1.548	24.34	24.69	−0.35	− 1.4	24	"	
	13	1.690	29.94	29.48	+0.46	+ 1.6	24	"	
	14	1.900	37.65	37.61	+0.04	+ 0.1	24	"	
2	1	0.672	4.97	5.01	−0.04	− 0.8	None	No	
	2	0.923	9.22	8.93	+0.29	+ 3.3	"	"	
	3	1.116	13.46	12.90	+0.56	+ 4.3	"	"	
	4	1.408	22.73	20.42	+2.31	+11.3	"	"	
	5	1.979	46.50	40.85	+5.65	+13.8	"	"	
3	1	0.782	6.57	6.60	−0.03	− 0.5	None	Yes	
	2	1.160	14.05	13.92	+0.13	+ 0.9	"	"	
	3	1.367	20.10	19.27	+0.83	+ 4.3	"	"	Flooded
	4	1.745	33.97	31.42	+2.55	+ 8.1	"	"	
4	1	0.688	5.16	5.26	−0.10	− 1.9	6	No	
	2	1.102	12.78	12.63	+0.15	+ 1.2	6	"	
	3	1.627	29.03	27.30	+1.73	+ 6.4	6	"	
	4	2.009	46.25	41.90	+4.35	+10.4	6	"	
5	1	1.750	33.78	31.73	+2.05	+ 6.5	6	No	
	2	1.762	33.46	32.15	+1.31	+ 4.1	6	"	
6	1	1.989	44.70	41.28	+3.42	+ 8.3	12	No	
7	1	2.115	50.65	47.00	+3.65	+ 7.8	12	Yes	
	2	1.728	32.60	30.85	+1.75	+ 5.7	12	"	
	3	1.492	23.91	22.85	+1.06	+ 4.6	12	"	
	4	1.200	15.21	14.90	+0.31	+ 2.1	12	"	
8	1	0.957	9.44	9.58	−0.14	− 1.5	18	No	
	2	1.162	13.86	13.95	−0.09	− 0.6	18	"	
	3	1.401	20.87	20.28	+0.59	+ 2.9	18	"	
	4	1.583	26.86	25.83	+1.03	+ 4.0	18	"	
	5	1.710	31.63	30.20	+1.43	+ 4.7	18	"	
	6	2.156	48.45	48.92	−0.47	− 1.0	18	"	
	7	2.114	48.20	46.95	+1.25	+ 2.7	18	"	
9	1	1.497	23.44	22.95	+0.49	+ 2.1	18	Yes	
	2	1.790	34.72	33.25	+1.47	+ 4.4	18	"	
	3	2.048	45.50	43.95	+1.55	+ 3.5	18	"	
10	1	1.666	29.28	28.60	+0.68	+ 2.4	24	No	
	2	2.127	48.80	47.55	+1.25	+ 2.6	24	"	
	3	1.030	10.93	11.05	−0.12	− 1.1	24	"	
	4	1.325	18.03	18.15	−0.12	− 0.7	24	"	
	5	1.483	22.37	22.60	−0.23	− 1.0	24	"	
	6	1.723	31.10	30.70	+0.40	+ 1.3	24	"	
	7	1.876	37.66	36.61	+1.05	+ 2.9	24	"	
	8	2.156	50.09	49.00	+1.09	+ 2.2	24	"	
11	1	0.988	10.30	10.19	+0.11	+ 1.1	24	Yes	
	2	1.307	17.68	17.62	+0.06	+ 0.3	24	"	
	3	1.523	24.41	23.90	+0.51	+ 2.1	24	"	
	4	1.690	29.93	29.45	+0.48	+ 1.6	24	"	
	5	1.823	35.43	34.58	+0.85	+ 2.5	24	"	
	6	2.108	47.93	46.65	+1.28	+ 2.7	24	"	

TABLE 6.—(*Continued*).

Series.	Run.	Gauge.	Observed discharge, in second-feet.	Computed discharge, in second-feet.	Error, in second-feet.	Error, percentage.	Throat length, in inches.	Pad.	Remarks.
12	1	1.345	18.37	18.68	−0.31	− 1.7	24	Yes	
	2	1.465	22.43	22.02	+0.41	+ 1.9	24	"	
	3	1.719	31.55	30.48	+1.07	+ 3.5	24	"	
	4	1.852	36.25	35.70	+0.55	+ 1.5	24	"	
	5	2.136	49.21	48.00	+1.21	+ 2.5	24	"	
13	1	0.962	9.67	9.67	0.00	0.0	24	Yes	
	2	0.833	7.12	7.40	−0.28	− 3.8	24	"	
	3	1.113	12.47	12.84	−0.37	− 2.9	24	"	
	4	1.222	14.54	15.45	−0.91	− 5.9	24	"	
	5	1.293	17.04	17.28	−0.24	− 1.4	24	"	
	6	1.415	20.72	20.65	+0.07	+ 0.3	24	"	
	7	1.497	23.32	22.95	+0.37	+ 1.6	24	"	
	8	1.622	27.21	27.18	+0.03	+ 0.1	24	"	
	9	1.713	31.05	30.30	+0.75	+ 2.5	24	"	
	10	1.828	34.62	34.75	−0.13	− 0.4	24	"	
	11	2.113	47.66	46.92	+0.74	+ 1.6	24	"	
14	1	2.153	50.23	48.84	+1.39	+ 2.8	24	No	
	2	2.050	44.63	44.04	+0.59	+ 1.3	24	"	
	3	1.948	37.30	39.60	−2.30	− 5.8	24	"	Questionable
	4	1.506	23.25	23.28	−0.03	− 0.1	24	"	
15	1	0.664	4.48	4.94	−0.46	− 9.4	36	No	
	2	0.923	8.41	8.94	−0.53	− 5.9	36	"	
	3	1.106	12.08	12.70	−0.62	− 4.9	36	"	
	4	1.364	17.77	19.20	−1.43	− 7.4	36	"	
	5	1.605	25.68	26.58	−0.90	− 3.4	36	"	
	6	1.711	29.24	30.24	−1.00	− 3.3	36	"	
	7	1.822	33.57	34.55	−0.98	− 2.8	36	"	
	8	1.978	40.58	40.80	−0.22	− 0.5	36	"	
	9	2.245	52.85	53.35	−0.50	− 0.9	36	"	

Fig. 12 shows the hydraulic conditions existing at the time of Run 4. Series 3, with a discharge of 33.97 sec.-ft. The condition shown is typical for the structure. A standing wave was maintained below the throat for all non-submerged flows. By means of a check located down stream from Gauge No. 3, back-water could be caused to flood the control. The point of flooding was determined by slowly increasing the depth at Gauge No. 3, with constant flow, and observing the point at which the depth at Gauge No. 1 began to increase. This observation was made for 18 runs. The results are given in Table 7. The percentage of submergence at the point of flooding varied from 63 to 85, the average being 77. Only three runs indicated a reduction in flow below a submergence of 74 per cent.

While making the submergence test, the standing wave was closely watched to determine, if possible, the visual evidence of the change from free to submerged flow. Submergence appears to begin when the wave reaches approximately the location of the control. The control in Fig. 12 might be expected to come at about the point shown for the up-stream position of the jump. Run 8, Series 10, was made with a 24-in. throat without a throat pad, and, following the theory outlined in the first part of this discussion, the control might be expected to be a short distance above the down-stream end of the throat. Flooding began when the wave had moved 6 in. back into the throat. Run 1, Series 3, was made with the throat pad, but with a zero throat length. Flood-

FIG. 13.—CONTROL-SECTION METER AT BOISE, IDAHO, SHOWING HYDRAULIC JUMP.

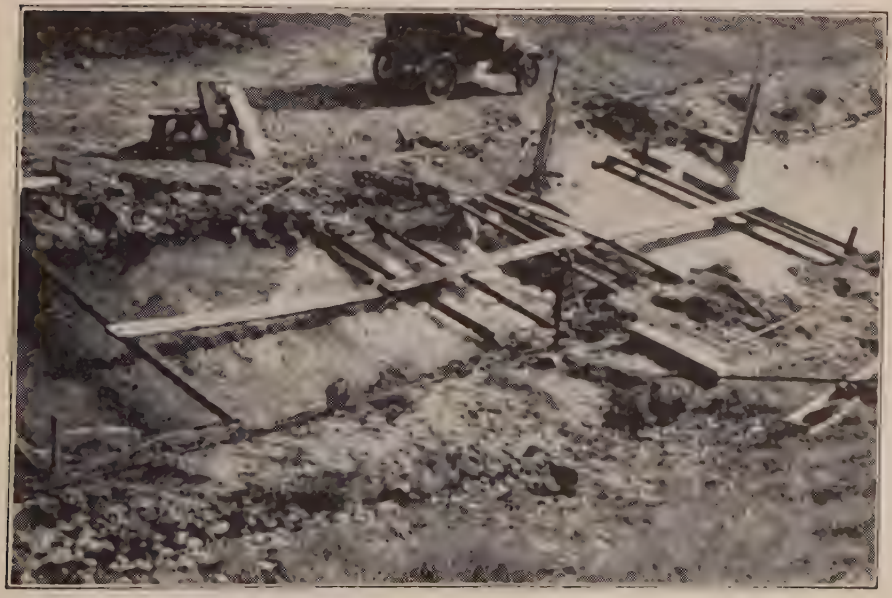
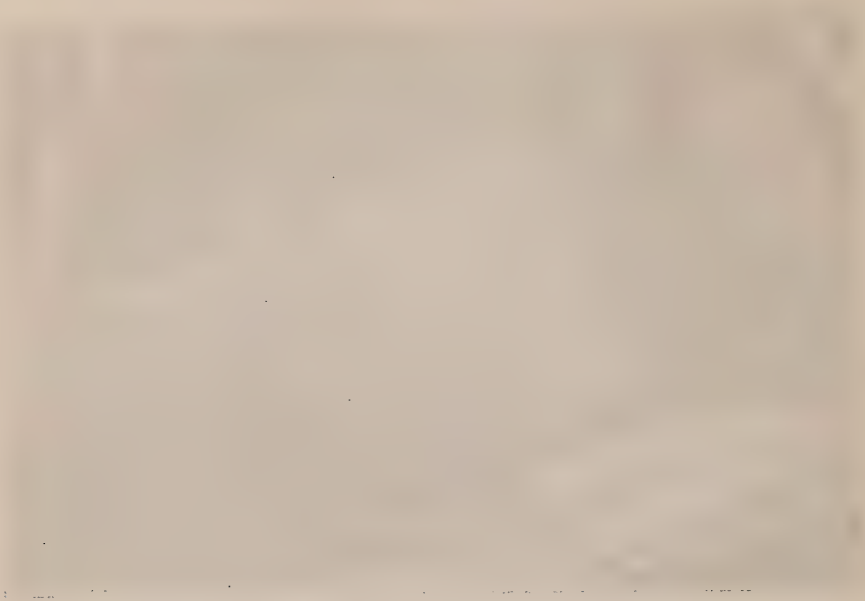


FIG. 14.—CONTROL-SECTION WEIR AT BOISE, IDAHO, RUNNING AT FULL CAPACITY.



ing began with the wave slightly farther up stream than the position shown for flooding in Fig. 12. The characteristic appearance of the wave just before submergence is shown in Fig. 13, and a view of the control-section weir running at full capacity is shown in Fig. 14. After submergence the wave usually changes to a series of ripples, which may be practically as rough as the last stage of the wave, but which have a distinctly different appearance. It is believed that with a little development the change from submerged to non-submerged flow can be made clearly visible.

TABLE 7.—SUBMERGENCE REQUIRED FOR FLOODING, 24-INCH TRAPEZOIDAL CONTROL-SECTION METER, BOISE, IDAHO.

Series.	Run.	Reading at Gauge No. 1, in feet.	Reading at Gauge No. 3,* in feet.	Percentage of submergence.*
1	3	0.525	0.420	80
	6	0.954	0.606	63
	12	1.548	1.380	89
	13	1.690	1.475	87
	14	1.900	1.63	86
2	1	0.672	0.45	67
	2	0.923	0.71	77
	3	1.116	0.83	74
	4	1.408	1.093	78
	5	1.979	1.66	84
3	1	0.782	0.655	84
	2	1.160	0.880	72
	4	1.745	1.30	75
4	1	0.688	0.413	60
	2	1.102	0.820	74
8	5	1.710	1.28	75
	6	2.156	1.72	80
10	8	2.156	1.75	81
Average.....	..	.....	.....	77

* At instant of flooding.

No attempt was made to rate the structure for submerged flow, since under such conditions the device becomes a true Venturi flume, for which many ratings have been made.

Conclusions.—It is believed that the data submitted in this discussion justify the following tentative conclusions:

1.—A Venturi flume, properly set, can be made to act as a control and indicate flow from a single gauge reading in accordance with a fully rational law.

2.—For all practical purposes, especially in irrigation work, the coefficient of discharge for such a structure may be taken as unity.

3.—For practical purposes, the throat length of a structure similar to that shown in Fig. 10 should not be less than the greatest depth expected at the gauge. For best results, a throat length equal to twice the depth is desirable.

4.—The necessary throat length may be somewhat reduced by carefully rounding the corners at the junction of the throat and the outlet.

5.—For carefully rounded approaches no entrance loss need be expected.

6.—Friction losses may be ignored unless the structure is long or extreme precision is required.

7.—The Venturi principle is not utilized and the device should not be called a Venturi flume. Control-section meter is suggested as a more appropriate name.

E. A. JACOB,* M. AM. SOC. C. E. (by letter).—The improved Venturi flume is a type of measuring device that deserves to be used more generally on irrigation canals. However, there are practical limitations to its adoption.

Some time ago, the writer installed a test Venturi flume in a canal having a grade of 0.0003, and carrying about 60 cu. ft. per sec. of water. The canal was in clay which would withstand a velocity of 3 to 4 ft. per sec. The flume itself operated well, but the canal below it was inadequate for the velocity of the outlet stream, about 5 ft. per sec. The flume remained in the canal a few days, and then was taken out.

Venturi flumes may be used only in canals where the material will not erode under the force of the current flowing from the flume; otherwise, the canal must be lined with concrete or other stable material for some distance below the flume. If this outflow velocity is kept low, that is, 3 to 4 ft. per sec., the throat of the flume must have a water cross-section relatively large as compared with that of the canal. This would produce a high percentage of submergence which would make the formula proposed by the author unreliable if not entirely erroneous.

In Utah, most of the canals have moss growths which, in July and August, greatly decrease their capacities and produce back-water effects that would make it necessary to take into account both the throat head, H_b, and the upper head, H_a. It would seem that more experiments showing the effects of variable back-water conditions with a constant discharge should be made, and a formula developed for such conditions involving both heads.

J. C. STEVENS,† M. AM. SOC. C. E. (by letter).—The Special Committee on Irrigation Hydraulics feels that the importance of proper measurement of deliveries in irrigation work deserves especial emphasis. The pay-for-what-is-consumed principle, if generally adopted, will eliminate many of the perplexing problems of operators and managers. It will reduce waste, eliminate the "water hog", and conserve the available supply to such an extent that in many instances substantial increases in the irrigated acreage may be possible. The chief difficulty in the way of putting such a system into general practice has been the lack of practicable metering devices, such as the one described by Mr. Parshall.

This is an excellent start in the right direction. It offers a means of metering the flow without sacrificing much head—a most desirable feature in flat countries. The writer, however, believes it can be improved and simplified to the point where the flow through it can be calculated from hydraulic principles.

Any device that requires the recording of two heads does not meet the requirements of simplicity. The improved Venturi flume requires only one observation of head to the point of about 70% submergence and, therefore, should be considered as a practical metering device up to but not beyond that limit.

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* Engr., Provo Reservoir Co., Provo, Utah.

† Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

The writer suggested to the Committee the use of a straight-sided, level-bottomed flume with a hump on the bed over which the water would have to flow at critical depth. In order to recover as much of the velocity head as possible the use of a second hump below the first was also considered, the purpose being to force the water from low stage below the critical to the high stage above without appreciable impact or eddy loss. A flume of this type was constructed and some experiments have been made on it at the Fort Collins Hydraulic Laboratory by Mr. Parshall.

Although the results of these tests have not been fully analyzed it appears that the second hump is superfluous. Its function is to raise the water to the high stage without a hydraulic jump, but this can occur for only one rate of flow. The intention in design was that this rate should be the maximum and that for all lesser rates of flow a hydraulic jump would occur. The eddy loss in a low jump is practically negligible.* Thus, for a depth after the jump 25% greater than that before, the energy loss is only 0.17% and for a 50% increase in depth the loss is only 1.2 per cent.

Plans are being made, therefore, for laboratory tests on such a flume with a single hump in the bottom somewhat along the lines shown in Fig. 15. Julian Hinds, M. Am. Soc. C. E., suggested to the Committee the desirability of measuring the head at the critical depth. If this can be effected in a practical way all question of velocity of approach, silting, weeds, etc., will be eliminated, and the formula for flow will reduce to the simple theoretical form:

$$Q = L \sqrt{g}\, d^{\frac{3}{2}} \qquad (1)$$

in which, Q is the flow, in second-feet; L is the length of crest of the hump or the width of flume at the point of critical flow; and d is the critical depth. A coefficient may have to be introduced, since theory does not take account of variable velocities in the plane of critical flow.

Theoretically, the critical depth should remain at the same point on a hump of this kind for all rates of flow. Neglecting friction, the following relation is obvious at any point in the flume:

$$y + z + h = E \qquad (2)$$

in which, z is the height of the hump above the flume bed; y is the depth of water; h, the velocity head; and E, the energy head. Differentiating,

$$dy + dz + dh = 0 \qquad (3)$$

since E is constant but, in a unit width of a rectangular channel,

$$h = \frac{V^2}{2g} = \frac{Q^2}{2gy^2}$$

hence,

$$dh = -\frac{Q^2}{g y^3}\, dy$$

* See "Determining the Energy Loss in the Hydraulic Jump," by J. C. Stevens, M. Am. Soc. C. E., *Engineering News-Record*, June 4, 1925.

but,*

$$\frac{Q^2}{g} = d^3$$

hence,

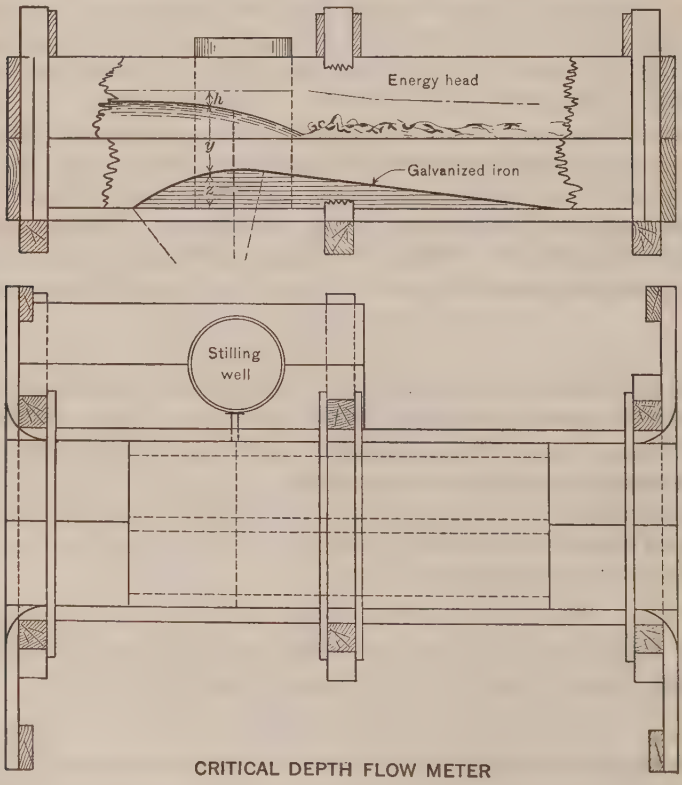
$$d\,h = -\left(\frac{d}{y}\right)^3 d\,y$$

which, substituted in Equation (3), gives,

$$d\,z + d\,y\left[1 - \left(\frac{d}{y}\right)^3\right] = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

At the point of critical flow, $d = y$, and Equation (4) reduces to:

$$d\,z = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$



CRITICAL DEPTH FLOW METER

FIG. 15.

This can only occur at the crown of the hump where a tangent to the curve of the hump is parallel to the bed of the flume. It would appear from theoretical considerations, therefore, that the critical depth can be measured at one point for all flows.

*At critical flow the velocity head is one-half the depth, that is, $\frac{V^2}{2g} = \frac{1}{2}d$, also, $V^2 = \frac{Q^2}{d^2}$; hence, $\frac{Q^2}{g} = d^3$.

It is the writer's opinion that a metering flume of the type shown in Fig. 15 will retain control of the flow under a submergence considerably in excess of 70 per cent.

The term, "Venturi flume", is not a happy expression for the device described by the author, nor for the one suggested herein. The term, "Critical Depth Flow Meter", is more to the point and is offered as a substitute.

CARL ROHWER,* Assoc. M. AM. SOC. C. E. (by letter).—Mr. Parshall lays particular stress on the effect of submergence. According to his report, the degree of submergence is immaterial for amounts of less than 70 per cent. Compared with the weir this is unusual, as shown by Table 8,† which is a comparison between the effect of submergence on a 4-ft. rectangular weir and a 4-ft. improved Venturi flume. Table 8 gives for different percentages of submergence the ratio of the submerged flow discharge to the free flow discharge. The percentages of submergence for both the weir and the Venturi flume are based on heads referred to the crest level.

On account of the nature of the data it was not possible to obtain similar percentages of submergence for both the weir and the Venturi flume,

TABLE 8.—A COMPARISON OF THE EFFECT OF SUBMERGENCE ON A 4-FOOT RECTANGULAR WEIR AND A 4-FOOT IMPROVED VENTURI FLUME.

Test No.	Upper head, in feet.	Lower head, in feet.	Submergence ratio.	Discharge, in second-feet.	Free flow discharge, in second-feet.	Discharge ratio.
4-FOOT RECTANGULAR WEIR						
1858	0.400	0.300	0.750	2.155	3.316	0.650
1860	0.400	0.200	0.500	2.761	3.316	0.833
1863	0.400	0.100	0.250	3.128	3.316	0.943
1868	0.400	0.000	0.000	3.333	3.316	1.005
1913	0.600	0.500	0.833	3.351	6.000	0.558
1877	0.600	0.400	0.667	4.406	6.000	0.734
1833	0.600	0.300	0.500	5.046	6.000	0.841
1856	0.600	0.200	0.333	5.502	6.000	0.917
1881	0.600	0.100	0.167	5.853	6.000	0.976
1891	0.800	0.700	0.875	4.686	9.161	0.512
1893	0.800	0.600	0.750	6.167	9.161	0.673
1894	0.800	0.500	0.625	7.069	9.161	0.772
1904	0.800	0.400	0.500	7.790	9.161	0.850
1929	0.800	0.300	0.375	8.341	9.161	0.911
1898	0.800	0.200	0.250	8.885	9.161	0.970
1852	1.000	0.900	0.900	5.952	12.705	0.468
1849	1.000	0.800	0.800	8.016	12.705	0.631
1835	1.000	0.700	0.700	9.177	12.705	0.722
1839	1.000	0.600	0.600	9.550	12.705	0.752
1854	1.000	0.500	0.500	10.913	12.705	0.859
1840	1.000	0.400	0.400	11.534	12.705	0.908
1862	1.000	0.300	0.300	12.041	12.705	0.948
1922	1.300	1.200	0.923	7.936	18.71	0.424
1924	1.300	1.100	0.846	10.514	18.71	0.562
1927	1.300	1.000	0.770	12.301	18.71	0.657
1917	1.300	0.900	0.682	13.799	18.71	0.738
1931	1.300	0.800	0.615	14.702	18.71	0.786
1925	1.300	0.700	0.539	15.546	18.71	0.831

* Assoc. Irrig. Engr., Div. of Agri. Eng., U. S. Dept. of Agriculture, Colorado Experiment Station, Fort Collins, Colo.

† Prepared from data of the Office of Irrigation Investigations, Div. of Agricultural Eng., Bureau of Public Roads, U. S. Dept. of Agriculture, co-operating with the Colorado Agricultural Experiment Station.

TABLE 8.—(*Continued.*)

4-Foot Improved Venturi Flume.

6405	0.598	0.338	0.557	6.98	7.11	0.982
6406	0.792	0.761	0.961	7.02	11.07	0.634
6407	1.155	1.146	0.992	7.02	20.08	0.350
6398	0.966	0.599	0.620	14.94	15.15	0.986
6399	1.014	0.791	0.780	15.13	16.35	0.925
6400	1.248	1.180	0.946	15.43	22.69	0.680
6401	1.735	1.708	0.984	15.76	38.17	0.413
6395	1.339	0.977	0.730	24.55	25.86	0.968
6390	1.340	0.828	0.618	25.60	25.89	1.008
6394	1.421	1.177	0.828	24.55	27.85	0.882
6393	1.624	1.508	0.929	24.40	34.40	0.709
6392	2.079	2.020	0.972	24.98	50.78	0.492
6372	1.644	1.070	0.651	35.66	35.05	1.017
6373	1.702	1.295	0.761	35.40	37.03	0.956
6374	1.804	1.519	0.842	35.16	40.59	0.866
6375	2.088	1.955	0.936	38.72	51.13	0.659
6376	2.441	2.384	0.977	29.54	65.42	0.452
6381	1.978	1.286	0.652	47.00	46.75	1.005
6380	1.994	1.300	0.652	47.80	47.55	1.005
6379	2.001	1.309	0.654	47.92	47.81	1.002
6382	2.008	1.475	0.735	46.32	48.07	0.964
6383	2.082	1.688	0.811	45.67	50.90	0.897
6384	2.280	2.064	0.905	43.42	58.74	0.739

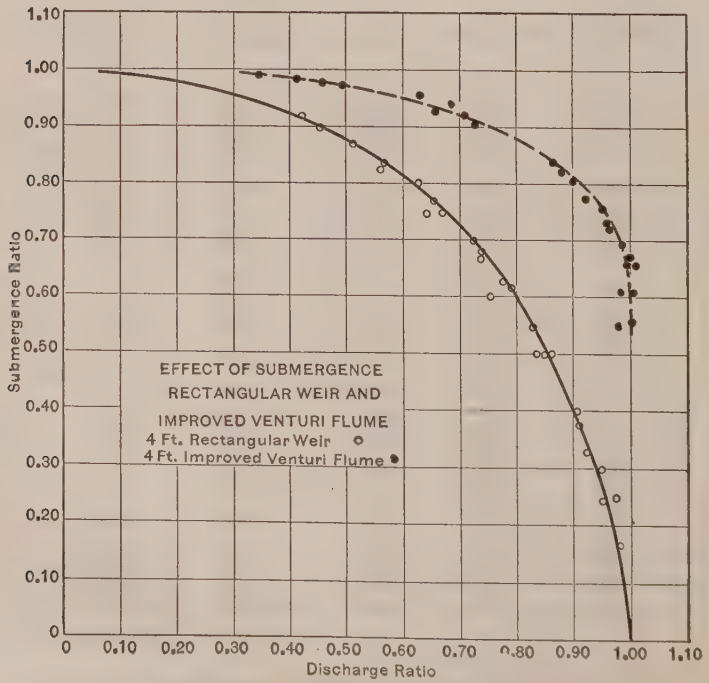


Fig. 16.

and in order to make direct comparisons possible, Fig. 16 was prepared. A study of the diagram shows that for submergences below 70%, there is little effect on the discharge of the Venturi flume, while the discharge from the weir is materially reduced. For submergences from 70 to 90%, the discharge ratios for both the weir and the Venturi flume decrease at about the same rate, but for values greater than 90% the effect is greater on the Venturi flume.

The fact that submergence does not affect the flow of the improved Venturi flume for ratios less than approximately 70% is to be expected. It has been shown* that at the control point, which in this case is at the crest, the depth cannot drop below two-thirds of the total head at the upper gauge point. In other words, submergence does not affect the discharge until the down-stream depth is sufficient to cause the depth at the control point to exceed two-thirds of the total head at the upper gauge point. For the improved Venturi flume, this apparently occurs when the down-stream head, as measured by Mr. Parshall, exceeds 70% of the depth at the upper gauge.

RALPH L. PARSHALL,† ASSOC. M. AM. SOC. C. E. (by letter).—When experiments were first made in the spring of 1915 on a type of measuring device having a converging and diverging section joined by a short throat section, with floor level throughout, this device was called the Venturi flume, because in plan it greatly resembled the converging and diverging sections as found in the Venturi meter, although in the flume the angles of convergence and divergence were equal. However, in this new device the water was not under pressure. As in the Venturi meter, two heads were observed, one in the converging and one in the throat section. Later, when experiments were conducted on the improved type of this device in which the angles of convergence and divergence were not equal and a depression in the throat section was introduced, this modified form was first called "the hydraulic jump flume"; but because of its similarity to the original type, both in design and section, it was thought advisable to designate it merely as the "improved Venturi flume."

For conditions of flow in which the discharge is a function only of the upper head and width of the throat, the law is not similar to that of the Venturi principle, but the device may be considered as a broad-crested weir without contractions. However, the additions of the depressed section at the throat introduces other conditions which modify its action. The improved Venturi flume, when under submergence of more than 70 to 75%, operates under the Venturi principle. It would seem, therefore, that it may or may not be a Venturi flume, according to the condition of flow.

From a theoretical standpoint, when the depth of water below the crest becomes positive, the flow is regarded as being submerged. As the depth below increases and approaches that of the upper depth, the percentage increases, approaching a limit when both these values are equal. Theoretically, under this condition the flow would be zero. For small differences or a high degree of

* "The Hydraulic Jump, in Open Channel Flow at High Velocity," by Karl R. Kennison, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 346, and "Flow of Water Through Contractions," by E. W. Lane, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1168.

† Irrig. Engr., U. S. Dept. of Agriculture, Fort Collins, Colo.

submergence, when based upon readings taken as indicated in the plan and elevation of the improved Venturi flume, it has been found that a considerable discharge results when these heads are practically identical. The real effective head, however, is not considered because the points where the observations were taken do not give the true difference causing the velocity of the stream.

As pointed out by Mr. Lane, the flow is theoretically submerged when the elevation of the water surface in the throat is higher than the crest elevation, but because the rate of discharge is not affected by this submergence until the ratio of heads approaches about 0.7 and because it follows the law of free flow, this range of conditions is called free flow and is a function only of the upper head and width of throat. When the upper head, H_a, is approximately 0.5 ft., the velocity of the water through the throat is sufficient to clear the throat section of the agitated condition, and the stream, through this section, is smooth and of relatively high velocity. When this condition is once established, the discharge may be reduced materially and yet this smooth condition of flow persists until the resistance, due to the upward inclined surface of the converging section, is sufficient to overbalance the momentum of the flowing stream. At this point the flow breaks and runs back into the throat. It is found that the law of flow is not changed where the stream passing the throat section is smooth or where the hydraulic jump is formed in the throat section, providing the ratio of the throat head to the upper head does not exceed approximately 0.7. Where the ratio of the throat head to the upper head exceeds this value of 0.7 the condition of flow is assumed to be submerged.

In his discussion, Mr. Carter has developed a formula based on the original data limited in submergence from 69 to 79 per cent. The total free-flow data were not all presented in Table 2. The data given were for the purpose of showing merely the agreement of computed and observed discharges for these limits of submergence.

In the determination of the free-flow discharge formula,

$$Q = 4\ W\ H^{1.522\ W^{0.026}}$$

observations were considered only where the degree of submergence was not greater than 69.9 per cent. These data were obtained both at the Bellvue and Fort Collins laboratories for discharges ranging from 0.3 sec-ft. to approximately 80 sec-ft., the size of the flumes being 1, 2, 3, 4, 6, and 8 ft. These data, taken by various observers, were plotted logarithmically, giving the mathematical relation as stated.

The free-flow series consisted of 162 tests, of which 2 were rejected. The remaining 160 tests formed the basis for the determination of the free-flow expression previously stated. Of this number it was found that in the comparison of the computed and observed discharges, 155 tests, or 97%, agreed within $\pm$ 3 per cent.

It is not claimed for the improved Venturi flume that its accuracy is superior to that of a standard weir, but from the writer's experience in the field with standard weirs, he would be inclined to think that the flume is as

accurate as the ordinary weir under field conditions. This would be especially true where much sand or silt is carried in the stream.

The calibration of this type of measuring device of various sizes included an extensive series under submerged flow conditions. It appears from the study of these data that although the submerged flow does not give as close agreement as the free-flow condition, it may be assumed to be within practical limits. Of a total of 229 submerged flow tests, in which the upper head did not exceed 2.52 ft. and the degree of submergence ranged from 70 to 95%, 1 test was rejected. Of the remaining 228 tests, it was found that in the comparison of the computed and observed discharges, 194 tests, or 85%, agreed within $\pm$ 5 per cent. The tests in which the submergence was greater than 95% were inconsistent and erratic and were not included in the computations.

In 1924 preliminary tests were conducted for the purpose of ascertaining the effects of introducing a parabolic curved surface as a floor in the throat section. These tests seem to indicate that an increase in the degree of submergence could be realized before it materially affected the free-flow discharge. The introduction of a warped surface, especially where structures are to be built by workmen of little mechanical experience, leads to the conclusion that the inclined plane floor surface in the throat section would be better, even if the effect of the degree of submergence would be slightly less. Tests were also made on a small-sized improved Venturi flume with a trapezoidal section.

It has been found from the practical application of the improved Venturi flume in the field, that protection is necessary below the structure when the stream is being discharged under free-flow condition. The exit velocities are relatively high, and unless the bottom and sides of the channel are protected, erosion will result. A wire net, filled with stone, anchored to the lower sill of the structure, has been found to be effective in preventing excessive erosion.

It would seem that the flume shown in Fig. 6 may be impracticable because of the high velocities that would exist at the exit end, *E;* even when the section is expanded as in the improved Venturi flume, protection is frequently necessary. The measurement of the critical depth would, perhaps, be uncertain as well as difficult to obtain because of the doubt as to its location. To base the discharge on this one reading may not be altogether satisfactory, even if the simple relation between discharge and depth exists from theory. The structure shown in Fig. 6, from a practical standpoint, would be unduly expensive because of its great length, and it might be that an undisturbed flow in this section could not be depended upon. Too much weight should not be given to the advantage of its single gauge reading because submergence is always a possibility, and in that case it is necessary to have both the up-stream and throat-gauge readings to determine the actual discharge.

The type of flume shown in Fig. 15 will, in all probability, measure successfully under certain conditions but it is doubtful whether the head could be measured accurately at the critical depth. Experiments would be necessary to verify the statement that this device is immune from silting troubles. For small flows it would be expected that there would be an accumulation of deposit

immediately up stream from the hump, and as the velocity increases this deposit may be washed out, thus changing the velocity of approach or contractions. Practical formulas of the type,

$$Q = L\, g^{\frac{1}{2}}\, D^{\frac{3}{2}}$$

will hold only when the critical depth is accurately determined. Since this point is unknown, it is not apparent how a formula of this type can apply without modification.

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Paper No. 1587

SIDE CHANNEL SPILLWAYS: HYDRAULIC THEORY, ECONOMIC FACTORS, AND EXPERIMENTAL DETERMINATION OF LOSSES*

By Julian Hinds,† M. Am. Soc. C. E.

With Discussion by Messrs. C. H. Howell, H. B. Muckleston, I. Gutmann, A. J. Wiley, Ivan E. Houk, J. C. Stevens, and Julian Hinds.

Synopsis

The design of a side channel spillway involves certain special hydraulic and economic factors which, it is thought, have not been previously discussed. On account of the disturbed conditions of flow, there are large internal frictional losses, the relative amounts of which are largely dependent on the magnitude and form of the individual installations. Flow can be computed by Bernoulli's theorem, using an experimentally determined coefficient of loss, similar to that of friction in ordinary flow, but such a coefficient would not be constant, and would need to be determined separately for many different sets of conditions. This situation can be avoided by utilizing the law of conservation of linear momentum for determining the flow. Formulas required for this purpose are developed.

The question of the proper location of the spillway on the ground is discussed, and a systematic method of determining the most economic design is suggested.

Although the fundamental principles utilized are well established in physics, the question arose as to the propriety of applying these principles to

Note.—The subject of this paper is one of ten selected by the Special Committee on Irrigation Hydraulics for study and research. The Committee, after studying the paper, has recommended its publication. (See Progress Report of the Committee, *Proceedings*, Am. Soc. C. E., March, 1925, Society Affairs, p. 137).

* Published in *Proceedings*, Am. Soc. C. E., September, 1925.

† Engr., Bureau of Reclamation, Denver, Colo.

the conditions under discussion. A series of experiments was planned and carried out for the purpose of establishing the correctness of the assumption used. The results of experiments at the Arrowrock Spillway, and of laboratory experiments at Bellvue, Colo., are given in the paper.

The two most important conclusions reached are, as follows:

1.—Bernoulli's theorem is not conveniently applicable, because of lack of uniformity in the coefficient of loss.

2.—The law of the conservation of linear momentum is directly applicable without an experimental coefficient, subject only to a small correction for swell in volume due to entrained air and unequal distribution of velocities.

Introduction

The proper design of a spillway of the type shown in Fig. 1 involves a number of more or less novel problems, which, although simple if taken sepa-

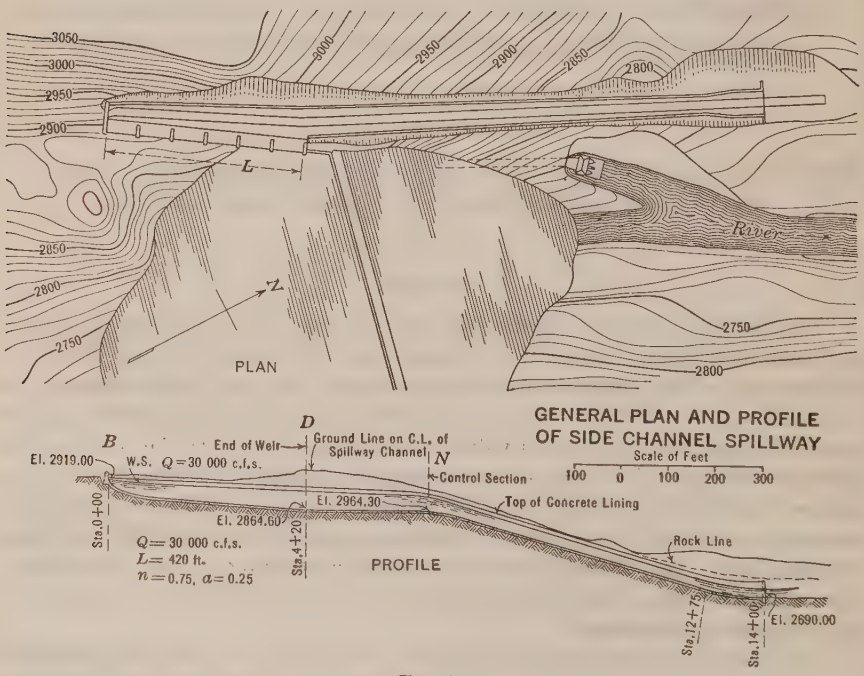


Fig. 1.

rately, form a complicated whole. The ultimate economy of construction depends on the form and dimensions of the spillway channel, its location, both in plan and elevation, its hydraulic properties, and other factors. The hydraulics presents a special problem, due to loss by shock and other irregularities caused by the constant addition to the flow in the channel of the water coming in over the spillway crest. Although this type of structure is very inefficient hydraulically, physical conditions sometimes make its use desirable.

It is best suited to spillway discharges of moderate quantities. The determination of the quantity of flood discharge to be provided for is beyond the scope of this paper. It will be assumed that the required spillway capacity is known. The depth of flow over the crest has an important influence on the cost of both the spillway and the dam. Generally, and within certain limits, the cost of the spillway channel is reduced by a short crest with a deep overflow, but if the spillway is of the open or uncontrolled type the cost of the dam will be increased, due to the excessive free-board required. If spillway gates are provided, their cost increases with their height. The factors involved in this part of the problem are too many and too varied to permit of a general solution. The most practical method of solving this problem is to assume a number of depths of overflow and prepare preliminary designs and cost estimates for the spillway, dam, and other related features.

Accuracy of Computations.—The quantity of flow to be provided for in the design of a spillway can be only roughly approximated at best, and extreme precision in computation is not justified. As the preparation of this paper progressed, arithmetical simplifications were developed, and the method of procedure finally recommended differs from that used in some of the typical designs and the reduction of experimental data. However, all computations are based on the same fundamental theories, and sufficient checking has been done to show that the various methods of procedure yield results in sufficiently close agreement for all practical purposes.

Units of Measure.—Although the equations deduced are independent of any particular system of measurements, units of 1 ft. and 1 sec. will be used to avoid confusion of language. The weight of 1 cu. ft. of water will be taken as a unit of force, to eliminate the necessity of multiplying all forces and momenta by 62.5 to convert them into pounds.

Symbols.—The letters and characters used in the formulas are, as follows:

A = area of water prism.
a = arbitrary coefficient of x in velocity equations.
b = inflow per foot length of weir crest.
d = depth of water in channel.
dM, dx, dV, etc. = derivatives of M, x, V, etc.
g = acceleration due to gravity.
H = head on crest (in weir formulas).
$H = d + y$ (in channel-flow formulas).
h_v = velocity head.
M = momentum.
M_u = momentum at up stream of two adjacent sections.
M_d = momentum at down stream of two adjacent sections.
$\Delta M = M_d - M_u$, or change in momentum over a short length, Δx.
n = arbitrary exponent of x in velocity equations.
Q = discharge, in cubic feet per second.
Q_1 = discharge at upper of two adjacent sections.
Q_2 = discharge at lower of two adjacent sections.
T = width of channel at water surface.

t = time, in seconds.
V = velocity, in feet per second.
V_1 = velocity at upper of two adjacent sections.
V_2 = velocity at lower of two adjacent sections.
$\Delta V = V_2 - V_1$, or change in velocity in a short distance, Δx.
x = distance along axis of channel.
Δx = distance between consecutive cross-sections of the channel.
y = ordinate to the water surface curve in the channel.

The Hydraulic Theory

Referring to Fig. 1 it will be noted that the flow moves over the crest in a direction approximately at right angles to the axis of the spillway channel. At the moment it comes in contact with the body of the water already in the channel, the incoming water has an appreciable velocity in a vertical plane normal to the channel axis. This transverse velocity is of no assistance in moving the water along the spillway channel.

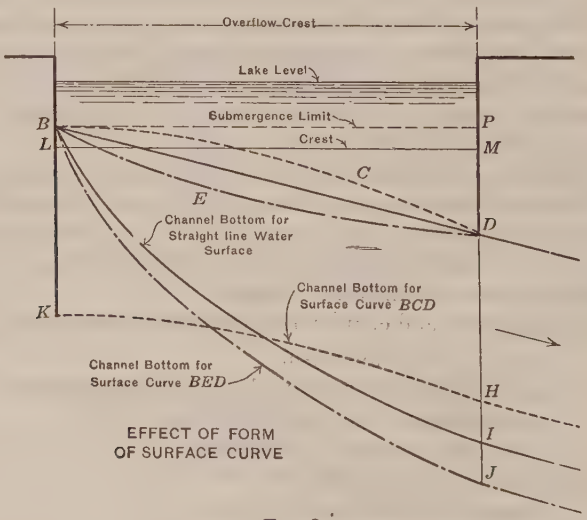


Fig. 2.

The force producing axial motion in the channel is derived from the falling of the particles of water along the water surface curve. The useful fall of any particle is measured by the drop in the water surface below its elevation at the point at which the particle entered the stream. Fig. 2 represents diagrammatically a section along the center line of a side-channel spillway. The water surface is assumed to fall along some curve from the point, B, at the upper end of the channel to a point, D, opposite the down-stream end of the crest. Each particle coming into the channel contributes to the production of velocity and the overcoming of resistances between B and D an amount of energy represented by its effective fall, or the drop in water surface from the point at which it enters, to D. Thus, a particle coming in at B will have an

effective fall equal to $P\,D$, whereas one coming in at D will have no effective fall. The total applied energy down to the point, D, is equal to that produced by the entire flow falling through the average drop for all the particles. If the surface curve from B to D is a straight line, and if the inflow per foot of crest is uniform, the average fall will be one-half the total fall, $P\,D$. If the curve is convexed, upward, as $B\,C\,D$, the average fall will be greater than $\frac{1}{2}\,P\,D$, and if the curve is concaved, the proportion will be less than $\frac{1}{2}\,P\,D$.

Only a part of the average fall is available for the production of velocity head at D. The remainder is used to overcome frictional and impact resistances. The frictional resistances are relatively small and may be neglected or estimated by methods used for other cases of variable flow. Impact is of greater importance.

Application of the Laws of Motion.—Water flowing in a channel must conform to each of the two laws of motion—the law of the conservation of energy and the law of the conservation of linear momentum. The first law is commonly expressed in hydraulics in the form of Bernoulli's theorem, which states that the velocity head plus the static head at any point is equal to the velocity head plus the static head at any other point, plus or minus the intervening "losses." The word "loss" in this sense refers to the transformation of kinetic or static energy into some less available form, such as heat. The energy is not actually lost in the sense of being destroyed. The law of the conservation of linear momentum requires that the momentum of any system of particles can be changed only by the application of an external force, the amount of change being proportionate to the magnitude and duration of the force. This law is commonly expressed in the form: "Force is equal to mass times acceleration." In hydraulics, it is more convenient to make the equivalent statement that "force is equal to rate of change of momentum with respect to time."

Neither of these laws is inherently superior to the other, each having its particular field of usefulness. The results obtained from the proper application of the two laws always correspond. In special cases one or the other may be more readily applicable.

A good example of the special usefulness of the momentum theory is afforded in calculations for the hydraulic jump in a straight uniform channel, where violent surges cause heat losses which make the use of Bernoulli's theorem impracticable. However, the external forces acting on the water, in the direction of flow, are readily isolated, and the amount of deceleration can be computed with precision. The only force to be approximated is the friction between the water and the channel, and this is usually small. After flow conditions below the jump are computed from the momentum equation the loss of head can be found from Bernoulli's theorem. A loss of head, in such a case, is unavoidable, and is caused by the fact that after the fast moving particles coming into the jump and the slow moving particles below the jump collide they must eventually move away with equal velocities.

Necessity for Loss of Head.—The fact that a loss of head is necessary in such a case may be illustrated by the collision of two simple bodies, such as

two billiard balls. In order to eliminate consideration of external forces the balls may be assumed to roll or slide without friction over a horizontal surface. It will also be advantageous first to assume the two balls to move in the same straight line. At the instant of first contact the centers of the two balls are moving toward each other. The materials of the balls at the point of contact immediately begin to be compressed. This compression results in a reaction which accelerates the forward ball and retards the rear ball. The compression increases until the velocities of the two balls are identical, at which time considerable energy has been expended in the compression of the balls. If the balls are perfectly elastic this energy is available, through the resilience of the balls, to force them apart again. If the balls are entirely inelastic no force will be available to separate them, and they will move together with identical velocities. As a result, there will be a reduction in the combined kinetic energy of the two balls equivalent to the elastic energy of the perfectly elastic balls at the instant of greatest compression. The important conclusion results that if all the energy of impact is absorbed the two balls will move away together and, conversely, if the balls are to move away together all the energy of impact must be absorbed.

Momentum and Energy Laws Both Required.—If the elasticity is perfect, the relative velocity of the two balls after impact will be exactly the same as before impact, whereas if they are perfectly inelastic the resultant relative velocity will be zero. With intermediate degrees of elasticity the resultant relative velocity will be determined by the proportion of the energy of impact absorbed.

The individual action of the balls after impact may be determined only by the application of both the laws of conservation of energy and of momentum. There is only one combination of individual velocities that will satisfy both the energy and momentum requirements, and that combination is controlled by the energy absorbed in impact. That part of the energy of impact not absorbed is utilized in the production of relative velocity between the balls after impact, that is, the velocity of one ball with respect to the other. The average velocity can be determined from the equation of momentum, and if the relative velocity is known, the motion is completely determined. If the relative velocity is zero, the actual and average velocities are identical, and in this special case only, the factor representing the energy equation may be dispensed with, the law of the conservation of momentum affording a complete solution.

Impinging Streams of Water.—When two streams of water flowing at different velocities come together and flow away down a common conduit, the two systems of particles eventually intermingle and the particles move with approximately equal individual velocities. Consequently, the energy of impact must be lost, or changed into some form not readily available for the production of velocity. As in the simpler case of the impinging balls, flow for this special condition is completely determined from the momentum equation. This is because the momentum always precisely determines the average velocity, which, in this instance, is also the actual velocity.

The term "average velocity" should not be confused with the "mean velocity" as ordinarily used in hydraulic computations. Average velocity, as here used, is the arithmetical or algebraic average of the velocities of all the particles. The mean velocity as ordinarily used is the average velocity over the cross-section of the channel taken by areas. Unless the velocity is uniform over the entire cross-section of the channel at a given point, these two terms are not identical. Where the velocities vary, more particles pass through an area of high velocity than through an equal area of low velocity, and the average velocity is greater than the mean. The assumption that the velocity is uniform over a given cross-section introduces an error into all ordinary hydraulic computations, as this assumption is never strictly true. This factor is relatively of greater importance for side-channel spillways than for other more usual types of structures, because velocity variations are more pronounced. However, the relation of the actual and average velocities is too complicated to be given practical consideration, and the usual assumption of uniformly distributed velocities will be made.

The effect of this approximation on the hydraulics of impinging streams is interesting. As previously stated, the average velocity is completely and accurately determined from the momentum equation. If the velocity is uniform over the cross-section there will be no relative velocity between particles and all the energy of impact will be lost. If the velocity is not uniform over the section there will be some relative velocity, and a part of the energy of impact will not be lost. However, as shown, the average velocity is greater than the mean velocity, and water-prism areas and depths computed on the basis of the known average velocity will be too small. Thus, although for a given average velocity, the theoretical impact loss computed on the basis of a uniform distribution of velocities is greater than the actual impact loss, nevertheless, the theoretical depth computed on the same basis is smaller than the actual depth.

Hydraulic Formulas for Side-Channel Flow.—A condition of impact and shock loss exists at every point along a side-channel spillway from the beginning to the end of a spillway crest, as from B to D in Fig. 2. Flow in such a case is completely determined by making the momentum after impact equal to that before impact plus any acceleration due to external forces, subject only to the approximations mentioned previously. The same result may be obtained by placing the energy after impact equal to that before impact minus all the energy lost in impact. However, equating the momenta will be found to be more convenient.

Consider conditions at two consecutive sections, a small distance, Δx, apart, somewhere between B and D, Fig. 2. If the weir discharge per unit length is b, the inflow between the two sections will be $b \Delta x$. The velocity and the discharge at the up-stream section may be designated by V and Q, respectively, and at the down-stream section by $V + \Delta V$ and $Q + b \Delta x$. The momenta at the two sections, therefore, will be:

Up stream,

$$M_u = \frac{Q V}{g} \qquad (1)$$

Down stream,

$$M_a = \frac{Q + b\,\Delta x}{g}(V + \Delta V) \ldots\ldots (2)$$

Subtracting Equation (1) from Equation (2):

$$\Delta M = \frac{Q\,\Delta V}{g} + \frac{b\,\Delta x}{g}(V + \Delta V) \ldots\ldots (3)$$

As Δx approaches the limit zero, ΔV becomes infinitesimal as compared to V, and disappears, giving the differential equation:

$$\frac{d\,M}{d\,x} = \frac{Q}{g}\frac{d\,V}{d\,x} + V\,\frac{b}{g} \ldots\ldots (4)$$

The rate of change of momentum with respect to time being V times the rate of change with respect to x, we may write:

$$\frac{d\,M}{d\,t} = V\,\frac{Q}{g}\frac{d\,V}{d\,x} + \frac{b\,V^2}{g} \ldots\ldots (5)$$

The rate of change of momentum with respect to time is equal to the accelerating force, which in this case is the component of the weight of the water acting down the water-surface slope. If the discussion is confined to the horizontal part of the velocity and if x is taken as the horizontal distance along the channel, the accelerating force will be $Q\,\frac{d\,y}{d\,x}$, in which, $\frac{d\,y}{d\,x}$ is the tangent of the water-surface slope. Substituting this force for $\frac{d\,M}{d\,t}$ in Equation (5), and reducing, gives:

$$\frac{d\,y}{d\,x} = \frac{V}{g}\frac{d\,V}{d\,x} + \frac{b\,V^2}{Q\,g} \ldots\ldots (6)$$

Integrating,

$$y = \frac{1}{g}\int_0^x \left(V\,\frac{d\,V}{d\,x} + \frac{b}{Q}\,V^2\right) d\,x \ldots\ldots (7)$$

in which, y is the ordinate to the water-surface curve.

Special Formula for Simple New Designs.—If the relations of Q and V to x are known in a given case Equation (7) can be integrated and the form of the surface curve determined. These relations depend on the form and dimensions of the channel, and will be algebraically complicated unless the channel is purposely designed to make them simple. In preparing a new design there appears to be no objection to choosing a form of channel that will simplify the computations, or, what is the same thing, choosing the hydraulics and computing the dimensions of the structure to correspond.

The inflow per foot of spillway crest for the purpose of design will generally be uniform, and the total discharge at any section x distant from the upper end of the crest will be given by an equation of the form:

$$Q = bx \ldots\ldots (8)$$

An equation of the exponential type will be found convenient for expressing the velocity relation, and by properly choosing constants such an equation

can be made to conform to a wide range of physical requirements. The following form is suggested:

$$V = ax^n \quad \ldots\ldots\ldots\

In many cases it will be necessary to line the spillway channel with concrete. Other things being constant, the cost of lining, which is an important item, is least when the bottom width is such that the wetted perimeter is a minimum. With steep side slopes this will require an average width of water prism equal approximately to twice the depth of the water. The bottom of the channel may be made somewhat narrower without greatly increasing the amount of lining, but the cost of lining should be considered in the final selection of channel width.

Figs. 3 and 4 are drawn to represent that part of the channel down stream from the crest structure, but the same principles apply to the part of the channel opposite the crest.

With a movable crest of the drum type an additional factor is introduced, due to the necessity for supporting the crest on a concrete base. It will be found advantageous to set the channel back into the hill a certain distance to reduce the quantity of concrete required for this purpose. The distance, z, in Fig. 5, should be chosen to make the combined cost of the shaded part of the concrete and the shaded part of the excavation a minimum. After a tentative bottom profile has been determined, the selection of which will be discussed later, the correct value of z can be calculated for a number of cross-sections, and a theoretical plan of the structure plotted on a contour map of the site. Unless the topography is unusually regular, it will not be possible to fit a practicable structure to the computed points with accuracy, but a location approximating the computed locations may be selected for trial.

After the cross-section of the channel has been selected, the profile is controlled by the values of a and n. Assuming a specified drop in the water surface from B to D, the effect of varying n is illustrated roughly in Fig. 2. If $n = 0.5$ the surface curve will be straight and the channel bottom will follow some such line as $B\,I$. If n is greater than 0.5 the water-surface curve will be convexed upward, as $B\,C\,D$. If n is exactly 1.0 the surface curve will be a parabola and the bottom will be a parallel curve, as $K\,H$. If n lies between 0.5 and 1.0 the bottom line will start at B, but if the value of n is nearly unity, it will drop rapidly nearly to the line, $K\,H$, which line it will cross up stream from H. If n exceeds unity, the bottom line theoretically drops to an infinite depth at the upper end of the channel, then rises rapidly approximately to the line, $K\,H$, which it crosses before reaching H. These statements can be readily proved by making a few trial computations from hypothetical data.

The effect on the profile, of varying both n and a, is shown in Fig. 6. If the value of n is known or assumed, the greatest economy in excavation at any point is obtained by making a such that the total drop from the submergence limit at B to the bottom of the channel at the point considered, expressed as $y + d$, is a minimum. Choosing H to represent $y + d$, the minimum value can be found by an equation, derived as follows:

$$H = d + y \qquad (12)$$

$$= d + \left(\frac{n+1}{n}\right) h\,v \text{ (from Equation (11))} \qquad (13)$$

As $h\,v$ is equal to $\frac{V^2}{2\,g}$ and V^2 is equal to $\frac{Q^2}{A^2}$:

$$H = d + \left(\frac{n+1}{n}\right)\frac{V^2}{2\,g}$$

$$= d + \left(\frac{n+1}{n}\right)\frac{Q^2}{2\,g\,A^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

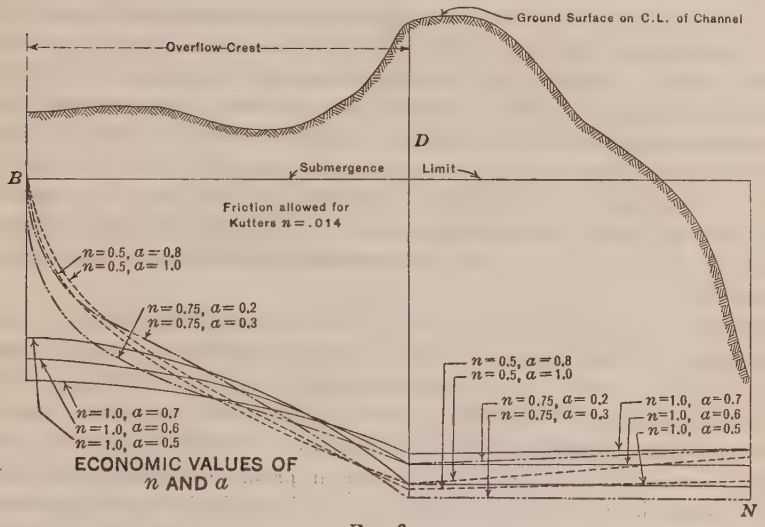


FIG. 6.

Differentiating with respect to d:

$$\frac{d\,H}{d\,d} = 1 - \left(\frac{n+1}{n}\right)\frac{Q^2}{g\,A^3}\,\frac{d\,A}{d\,d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Making $\frac{d\,H}{d\,d}$ equal to zero, the minimum value of H is found to obtain when:

$$g\,A^3 = \left(\frac{n+1}{n}\right) Q^2\,\frac{d\,A}{d\,d} \quad \ldots\ldots\ldots\ldots\ldots\ldots (16)$$

The factor, $\frac{d\,A}{d\,d}$, is equal to T, the top width of the water prism. Inserting this value and recalling that $\frac{Q^2}{2\,g\,A^2} = h\,v$, Equation (16) simplifies to:

$$h\,v = \frac{n}{n+1}\,\frac{A}{2\,T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

or, from Equation (11):

$$y = \frac{A}{2\,T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Having chosen a value for either a or n, the corresponding value of the other required to give the greatest economy at a specified point in a given channel, can be found from these equations.

The correct combination of values for a and n to give the greatest economy for the entire structure is not so readily found. A general solution involves the equation of the surface of the hillside which can not be practically expressed algebraically. The problem is best solved by trial.

Anticipating the results of the study, it may be stated that n is a much more constant factor than a, and is more easily guessed. A start may be made, therefore, with an assumed value of n. Having selected the trial value, choose some station along the channel where the excavation is unavoidably heavy and assume that this section will be a controlling factor in the cost of the spillway. Usually, as a safety precaution, the channel must be set well into the hill opposite the end of the dam, with the result that the excavation is heavy near the point, D, Fig. 6. Below some point, N, the channel need be set into the rock only as far as may be required to afford secure sidewalls. The section at D, therefore, influences the economy of construction more than any other single section, and the maximum economy of excavation for a given value of n, will be approximated if $d + y$ is made a minimum at D.

Therefore, unless special conditions indicate that the controlling section is elsewhere, find the economic value of y for the point, D, from Equations (17) and (18) by trial* and the corresponding value of a from Equation (10). The velocity and the water-surface curve may then be found from Equations (9) and (10), or Equation (11). Knowing the velocity and the form of the channel, the depth of flow and the bottom profile at points above D are readily obtained. Below D it is economical to have the bottom of the channel as high as possible without interfering with flow above D. Below N (Figs. 1 and 6), the channel will usually be steep and flow will occur at a velocity in excess of the critical velocity.

The velocity at D, determined as outlined, will be less than the critical. A "control", or point of critical flow, therefore, may be provided at N or between D and N. The control may be anywhere within this reach, but should be a sufficient distance below D to permit the necessary change in velocity to take place gradually. In the examples illustrated, the control is assumed to be at N. Having located the control, the hydraulic computations may be completed and the profile platted down stream from B to the point, N. The grade from D to N may be level, or even reversed.

Comparative Estimates.—As soon as a bottom profile has been completed the principles of Fig. 5 may be applied to several sections of the spillway crest, and the resulting channel platted on a topographic map of the site. A cost estimate for the spillway may then be prepared, which will locate one point on one of the curves in the left-hand part, (a), of Fig. 7. The correctness of the first trial value of a should be checked by assuming additional trial values, with the same value of n, and making more cost estimates. The result will give a number of points from which one of the curves in Fig. 7

* Assume a trial depth, d, and compute the area, A, and the top width, T. From Equation (17), find $h\,v$ and the corresponding velocity, V. Then, if $A\,V$ is equal to the required discharge, the assumed depth is correct. Otherwise, assume a new depth and repeat the process, continuing until an agreement is secured. Having thus arrived at the economic depth, the corresponding value of y is found from Equation (18).

(*a*) may be drawn. The lowest point on this curve will give the correct value of *a* for use with the assumed value of *n*. A new value of *n* should then be assumed, and the process repeated, platting other curves as shown in the diagram, until it is clear that the most suitable combination of *a* and *n* has been found. The curve in Fig. 7 (*b*) is obtained by platting the low points of the curves in Fig. 7 (*a*) against their respective values of *n*.

In the example given it is seen that 0.75 is the best value for *n*, the corresponding value for *a* being approximately 0.25. This value of *a* is somewhat larger than the value corresponding to the value of *y* computed for least depth at *D*. In all usual cases, with a lined channel, this will be true, and trial values of *a* should never be less than are required to give the minimum value of *H* at *D* unless the estimates show smaller values to be necessary.

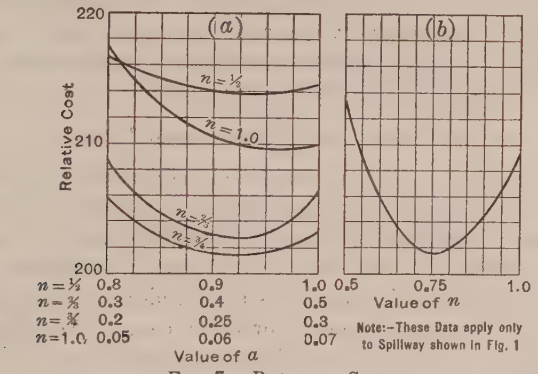


FIG. 7.—RELATIVE COSTS.

Submergence of Crest.—An important matter not yet discussed is the amount the weir can be submerged at *B* without seriously reducing the discharge. It has been the general practice heretofore to assume that Herschel's coefficients for ordinary submerged weirs are approximately applicable. The accuracy of this assumption will be discussed subsequently in connection with the experimental data. The economic importance of this point is apparent from the fact that the bottom profile is obtained by measuring $d + y$ downward from the permissible submergence line. In Figs. 1 and 6 the submergence is assumed to be such that the inflow will not be reduced, that is, according to Herschel, about 0.13 of the head on the crest. This small allowable submergence is chosen in these examples principally to simplify the theoretical discussion by making the inflow per foot constant. According to experiments, submergence reduces the discharge only slowly up to a fairly considerable depth. Furthermore, in a spillway the back-water falls away below the point, *B*. The crest may be submerged at *B* by one-half or two-thirds of the head without seriously reducing the total discharge. Equation (8) and Equations (10) to (18), inclusive, are not strictly applicable to such a partly submerged condition, as the inflow is variable. However, they apply with sufficient accuracy to any ordinary case, and the results may be checked by methods to be given subsequently. It may be desirable sometimes to

assume a heavy submergence, increasing the depth of spill to compensate for the reduced discharge factor. The limit to which this process may be carried can be determined by cost estimates.

Allowance for Swell.—In estimating the free-board required to prevent slopping over the edges of the channel or the top of the lining, allowance must be made for turbulence and "swell" due to entrained air and unequal distribution of velocities. As previously pointed out the average velocity is always greater than the mean velocity, which fact is ignored in the computation of depths and areas. Consequently, the actual depth may be expected to exceed the computed depth by an amount depending on the extent of longitudinal eddies. The volume will be further increased by air drawn into the stream by the infalling water. It is believed that the only information available on this subject is that given subsequently in connection with the experimental data. The average bulking observed for a small model was about 4% with a variation from zero to about 10 per cent. Observations on full-sized structures under normal flow conditions are needed.

Computations for a Restricted Design

Hydraulic Formulas.—The information already given is intended for use in the preparation of new designs not hampered by special limiting conditions. The same fundamental hydraulic principles may be applied to the analysis of an existing channel, or to the design of a channel which must conform to certain prescribed dimensions and grades, or where inflow is non-uniform due to excessive submergence or other cause. Equation (6) can be rewritten to make it applicable, approximately, to finite values of Δx. Equation (3) divided by Δx gives:

$$\frac{\Delta M}{\Delta x} = \frac{Q \Delta V}{g \Delta x} + \frac{b}{g}(V + \Delta V) \quad \ldots\ldots(19)$$

Multiplying by the average velocity, $V + \frac{1}{2}\Delta V$,

$$\frac{\Delta M}{\Delta t} = \frac{Q}{g}\left(V + \frac{1}{2}\Delta V\right)\frac{\Delta V}{\Delta x} + \frac{b}{g}(V + \Delta V)\left(V + \frac{1}{2}\Delta V\right) \quad \ldots\ldots(20)$$

As $\frac{\Delta M}{\Delta t}$ = accelerating force = $\frac{\Delta y}{\Delta x}$ times average discharge, Equation (20) becomes:

$$\frac{\Delta y}{\Delta x}\left(Q + \frac{1}{2}\Delta Q\right) = \frac{Q}{g}\left(V + \frac{1}{2}\Delta V\right)\frac{\Delta V}{\Delta x} + \frac{b}{g}(V + \Delta V)\left(V + \frac{1}{2}\Delta V\right) \quad \ldots\ldots(21)$$

From which,

$$\Delta y = \frac{Q}{g}\left(\frac{V + \frac{1}{2}\Delta V}{Q + \frac{1}{2}\Delta Q}\right)\left[\frac{\Delta V}{\Delta x} + \frac{b}{Q}(V + \Delta V)\right]\Delta x \quad \ldots\ldots(22)$$

or:

$$\Delta y = \frac{Q_1}{g}\frac{(V_1 + V_2)}{(Q_1 + Q_2)}\left[\Delta V + \frac{b V_2 \Delta x}{Q_1}\right] \quad \ldots\ldots(23)$$

or:

$$\Delta y = \frac{Q_2 (V_1 + V_2)}{g (Q_1 + Q_2)} \left[\Delta V + \frac{b V_1 \Delta x}{Q_2} \right] \quad \ldots\ldots\ldots\ldots\ldots (24)$$

in which, Q_1 and V_1 are the discharge and velocity at the up-stream end of the reach, and Q_2 and V_2 are the same functions at the down-stream end. It is interesting to note that when b is zero, $Q_1 = Q_2$ and Equations (22), (23), and (24) reduce to $\Delta y = \frac{V_2^2 - V_1^2}{2g}$, which is the energy equation for ordinary variable flow, neglecting friction. Equations (22), (23), and (24) are identical, except as to form, and may be used interchangeably, as found convenient.

Location of Control.—These equations are not as cumbersome as might at first appear, but before they can be applied, a starting place, at which the velocity is known, must be found. Such a starting place will be located at a point of control, where the depth passes through the critical point, from above to below.

If the channel slope between D and N, Fig. 1, is insufficient to support flow at the critical depth, and below N, more than sufficient, the control will be located at N, and the computation of the hydraulics may be started from that point. If the slope between D and N is greater than is required to support flow at the critical depth, the control will come at D, or at some point above. Up stream from D there is, in addition to the force of friction, a resisting force due to impact. This force has the same effect on the formation of a control as a flattening out of the grade. If the slope of the channel is insufficient to overcome losses and maintain flow at the critical depth immediately above D, then the control will be at D. If the slope immediately above D is more than sufficient for maintaining critical flow, then the control will come at some point farther up stream, where the slope becomes insufficient for this purpose.

The actual location of a control above D is complicated by the fact that the critical depth, the impact resistance slope for critical depth, and the discharge are all variable. If the equation of the critical velocity can be written, the slope can be found from Equation (6), but this can seldom be done. A suggested method of attacking the problem is illustrated in Fig. 8 and Tables 1, 2, and 3. First, compute the critical velocities and discharges corresponding to a number of depths, as shown in Table 1. The hydraulic radii may also be computed and recorded for use in estimating friction losses. From Table 1 values of the critical depths and velocities may be taken, by interpolation, for use in Table 2, which is self-explanatory and gives in Column (13) the drop in water surface necessary to maintain flow at the critical depth throughout the full length of the channel. Starting from an arbitrary water-surface elevation at some station, a profile of the channel, for critical flow at all points, can be platted, as shown by dashed lines in Fig. 8. A tangent parallel to the bottom of the actual channel may then be drawn to the resulting bottom line of the critical depth channel, and the point of control

will be located at the point of tangency. Referring to Fig. 8, it is evident that the slope required to maintain critical flow to the left of Station 1 + 64 is greater than the actual slope, and to the right it is less, which is the condition necessary to the formation of a control. If more than one point of tangency is possible, that one giving the lowest position of the tangent will be likely to control. It is possible to have two or more control points with hydraulic jumps between.

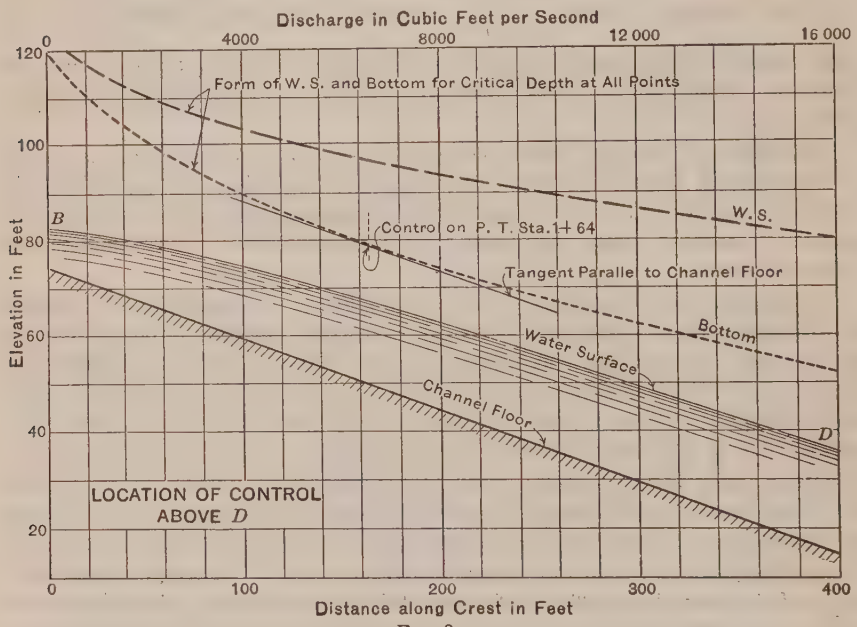


FIG. 8.

The Back-Water Curve.—Having located a control, the back-water curve may be calculated both ways, as illustrated in Table 3, and as platted in the lower part of Fig. 8. Equation (23) is used in the upper part of Table 3 and Equation (24) in the lower part, in order to take advantage of as many constant terms as possible. The accuracy of the computations will depend on the length and number of subdivisions assumed. The number of divisions used in the examples shown could be increased to advantage.

EXPERIMENTS UPON A MODEL STRUCTURE AT BELLVUE, COLO.

In the latter part of December, 1923, the U. S. Bureau of Reclamation constructed a model spillway at the Bellvue Laboratory of the U. S. Department of Agriculture, near Fort Collins, Colo., and conducted a series of experiments for the purpose of establishing, if possible, the correctness of the foregoing theory. The experiments were made by the writer under the direction of J. L. Savage, M. Am. Soc. C. E., Designing Engineer of the Bureau of Reclamation. A large part of the observational work was done by Ivan E. Houk, M. Am. Soc. C. E., and Samuel Judd, Engineer, Bureau of Reclama-

tion. Valuable assistance in operating the laboratory was given by R. L. Parshall, Assoc. M. Am. Soc. C. E., Irrigation Engineer, U. S. Department of Agriculture.

TABLE 1.—COMPUTATIONS FOR CRITICAL DEPTHS. FOR CHANNEL IN FIG. 8.

(Bottom width, 10 ft.; side slopes, $\frac{1}{2}$: 1)

Depth, in feet, d.	Area, in square feet, A.	Top width, in feet, T.	Velocity head, $\frac{A}{2T}$	Critical velocity, in feet per second.	Discharge, Q.	Hydraulic radius, in feet.
2	22	12	0.92	7.68	169	1.52
4	48	14	1.71	10.49	504	2.53
6	78	16	2.44	12.52	978	3.33
8	112	18	3.11	14.15	1 585	4.01
10	150	20	3.75	15.53	2 330	4.63
12	192	22	4.36	16.75	3 216	5.22
14	238	24	4.96	17.86	4 252	5.76
16	288	26	5.54	18.88	5 440	6.29
18	342	28	6.11	19.82	6 780	6.82
20	400	30	6.67	20.71	8 284	7.31
22	462	32	7.22	21.55	9 960	7.81
24	528	34	7.76	22.34	11 800	8.29
26	598	36	8.31	23.12	13 820	8.77
28	672	38	8.84	23.84	16 020	9.26

TABLE 2.—COMPUTATIONS FOR LOCATING CONTROL. FOR CHANNEL IN FIG. 8.

(Bottom width, 10 ft.; side slopes, $\frac{1}{2}$: 1.)

$$\Delta y = \frac{Q_1 (V_1 + V_2)}{g (Q_1 + Q_2)} \left\{ \Delta V + \frac{b V_2 \Delta x}{Q_1} \right\}$$

x	Δx	Q	Q_1+Q_2	d_c	V_c	V_1+V_2	ΔV	$\frac{b V_2 \Delta x}{Q_1}$	$\Delta V+\frac{b V_2+\Delta x}{Q_1}$	Δy	h_f*	$\Delta y+h_f$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
0	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	0.03	.....
10	10	400	400	3.4	10.0	10.0	10.0	.....	.....	.....	.....	.....
25	15	1 000	1 400	6.2	12.5	22.5	2.5	18.8	21.3	4.25	0.04	4.29
50	25	2 000	3 000	9.2	14.9	27.4	2.4	14.9	17.3	4.91	0.08	4.99
100	50	4 000	6 000	13.5	17.6	32.5	2.7	17.6	19.3	6 77	0.15	6.92
150	50	6 000	10 000	16.9	19.3	36.9	1.7	9.6	11.3	5.21	0.15	5.36
200	50	8 000	14 000	19.7	20.6	39.9	1 3	6.8	8.1	4.33	0.15	4.48
250	50	10 000	18 000	22.1	21.6	42.2	1.0	5.4	6.4	3.74	0.15	3.89
300	50	12 000	22 000	24.2	22.4	44.0	0.8	4.5	5.3	3.30	0.15	3.45
350	50	14 000	26 000	26.2	23.2	45.6	0.8	3.8	4.6	3.04	0.15	3.19
400	50	16 000	30 000	28.0	23.8	47.0	0.6	3.4	4.0	2.73	0.15	2.88
.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	38.28	1.17	39.45

*h_f = friction loss, computation not shown.

The general arrangement and principal dimensions of the Bellvue Laboratory and the model spillway are shown in Fig. 9. The intake to the flume is adjacent to the head-works of the Jackson Ditch, and is controlled by a single 4 by 4-ft. sliding gate. Two 2 by 2-ft. waste-gates are available for additional control, and an 8-in. pipe line, with a gate-valve near the weir pool, may be utilized for the final adjustment of flow.

TABLE 3.—COMPUTATIONS FOR BACK-WATER CURVE.
FOR CHANNEL IN FIG. 8.
(Bottom width, 10 ft.; side slopes, $\frac{1}{2}$: 1.)

x.	Δx.	Bottom elevation.	Trial Δy.	Surface elevation.	Depth, d.	Area, A.	Discharge, Q.	Velocity, V.	$Q_1 + Q_2$.	$\frac{Q_1}{g(Q_1+Q_2)}$.	$V_1 + V_2$.	ΔV.	$\frac{b V_2 \Delta x}{Q_1}$	$\Delta V + \frac{b V_2 \Delta x}{Q_1}$	Δy_m*	h_f.	$\Delta y_m + h_f$	Error.	Notes.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
164		49.10	....	66.80	17.70	333.6	6 560	19.70	.....	.....	.....	....	.....	.....	....	....	....	....	.....
100	64	58.70	7.90	74.70	16.00	288.0	4 000	13.89	10 560	0.0118	33.59	5.81	12.61	18.42	7.29	0.15	7.44	0.46	
			7.10	73.90	15.20	267.5	.....	14.95	.....	.....	34.65	4.75	.....	17.36	7.09	....	7.24	0.14	
			7.27	74.07	15.37	271.8	....	14.72	.....	.....	34.42	4.98	.....	17.59	7.13	....	7.28	0.01	O. K.
50	50	66.20	5.00	79.07	12.87	211.5	2 000	9.46	6 000	0.0104	24.18	5.26	14.72	19.98	5.02	0.07	5.09	0.09	
			5.08	79.15	12.95	213.5	.....	9.37	.....	.....	24.09	5.35	.....	20.07	5.01	....	5.08	0.00	O. K.
25	25	69.95	1.80	80.95	11.00	170.5	1 000	5.86	3 000	0.0104	15.23	3.51	9.37	12.88	2.03	0.02	2.05	0.25	
			2.06	81.21	11.26	176.0	.....	5.68	.....	.....	15.05	3.69	.....	13.06	2.04	...	2.06	0.01	O. K.
10	15	72.20	0.59	81.80	9.60	142.1	400	2.82	1 400	0.0089	8.50	2.86	8.52	11.38	0.86	0.01	0.87	0.29	
			0.87	82.08	9.88	147.6	.....	2.71	.....	.....	8.39	2.97	.....	11.49	0.86	....	0.87	0.00	O. K.
0	10	73.70	0.23	82.31	8.61	Δy assumed $= 2 h_v$ at $x = 10$													

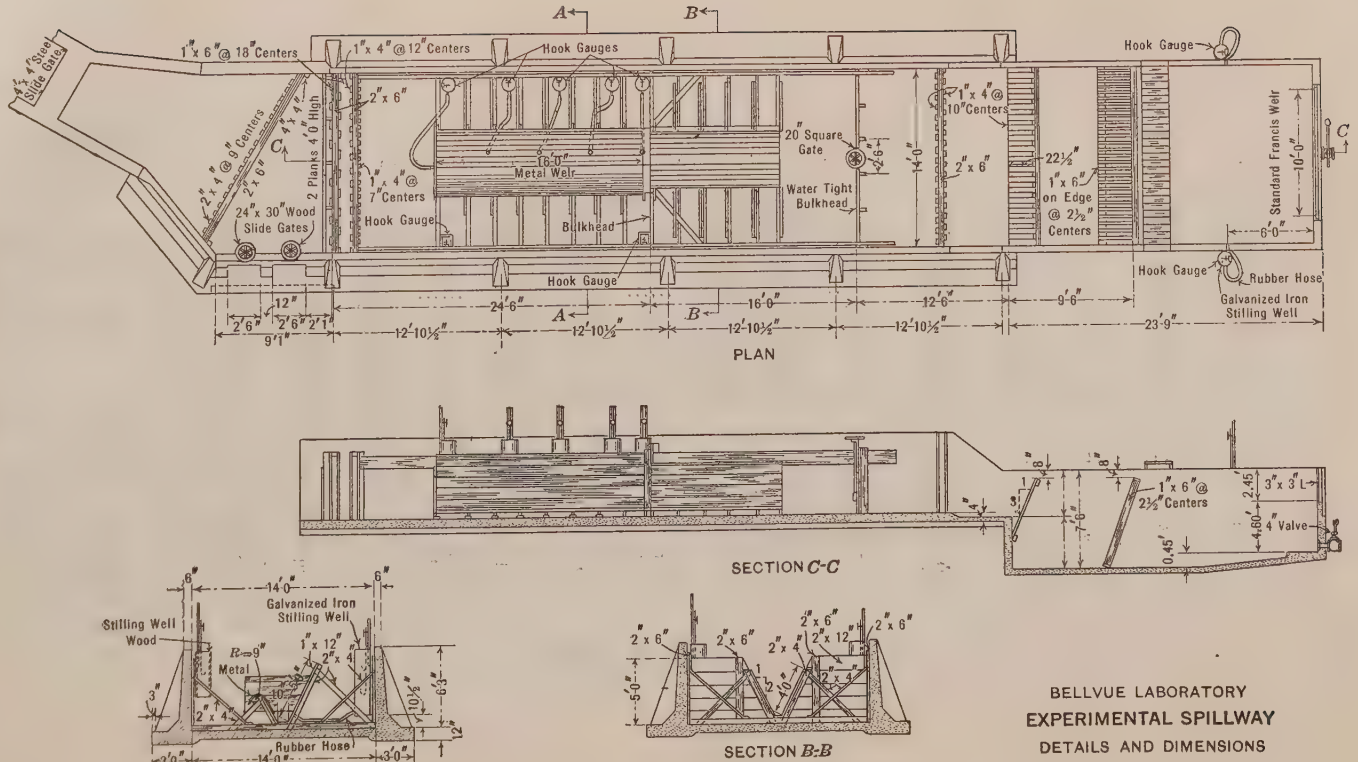
*$\Delta y = \frac{Q_1 (V_1 + V_2)}{g (Q_1 + Q_2)} \left\{ \Delta V + \frac{b V_2 \Delta X}{Q_1} \right\}$ = Column (11) × Column (12) × Column (15).....(24)

TABLE 3.—(*Continued.*)

x.	Δx.	Bottom elevation.	Trial Δy.	Surface elevation.	Depth, d.	Area, A.	Discharge, Q.	Velocity, V.	$Q_1 + Q_2$.	$\frac{Q_2}{g(Q_1+Q_2)}$.	$V_1 + V_2$.	ΔV.	$\frac{b V_1 \Delta x}{Q_2}$	$\Delta V + \frac{b V_1 \Delta x}{Q_2}$	Δy_m*	h_f†	$\Delta y_m + h_f$	Error.	Notes.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
164		49.10	....	66.80	17.70	333.6	6 560	19.70	.....	.....	.....	....	.....	.....	....	....	....	....	.....
200	36	43.70	4.10	62.70	19.00	370.5	8 000	21.59	14 560	0.0171	41.29	1.89	3.55	5.44	3.82	0.12	3.94	0.16	
			4.50	62.30	18.60	359.0	.....	22.27	.....	.....	41.97	2.57	.....	6.12	4.40	....	4.52	0.02	
			4.45	62.35	18.65	360.4	.....	22.20	.....	.....	41.90	2.50	.....	6.05	4.34	....	4.46	0.01	O. K.
250	50	36.20	6.15	56.20	20.00	400.0	10 000	25.00	18 000	0.0173	47.20	2.80	4.44	7.24	5.92	0.18	6.10	0.05	
			6.22	56.18	19.98	397.9	.....	25.13	.....	.....	47.33	2.93	.....	7.37	6.03	....	6.21	0.01	O. K.
300	50	28.70	6.63	49.50	20.80	424.3	12 000	28.29	22 000	0.0170	53.42	3.16	4.19	7.35	6.68	0.24	6.92	0.29	
			6.40	49.73	21.03	431.4	.....	27.81	.....	.....	52.94	2.68	.....	6.87	6.18	....	6.42	0.02	
			6.39	49.74	21.04	431.7	.....	27.79	.....	.....	52.92	2.66	.....	6.85	6.16	....	6.40	0.01	O. K.
350	50	21.20	6.51	43.23	22.03	463.0	14 000	30.24	26 000	0.0168	58.03	2.45	3.97	6.42	6.24	0.30	6.54	0.03	
			6.48	43.26	22.06	463.9	.....	30.18	.....	.....	57.97	2.39	.....	6.36	6.18	....	6.48	0.00	O. K.
400	50	13.50	6.73	36.53	23.03	495.5	16 000	32.29	30 000	0.0166	62.47	2.11	3.78	5.89	6.10	0.35	6.45	0.28	
			6.92	36.34	22.84	489.2	.....	32.71	.....	.....	62.89	2.53	.....	6.31	6.58	....	6.93	0.01	O. K.

*$\Delta y = \frac{Q_2 (V_1 + V_2)}{g (Q_1 + Q_2)} \left\{ \Delta V + \frac{b V_1 \Delta X}{Q_2} \right\}$ = Column (11) × Column (12) × Column (15).....(25)

†h_f = friction head, computations not shown.



BELLVUE LABORATORY
EXPERIMENTAL SPILLWAY
DETAILS AND DIMENSIONS

FIG. 9.

A 10-ft. Francis weir for measuring the discharge and baffles for stilling the flow are located as shown. Fig. 10 is a general view of the laboratory and Fig. 11 shows the spillway structure.

The spillway crest was of sheet iron rounded to a 9-in. radius, and was approximately 16 ft. long. The trough was of 1-in. planed boards carefully joined on the edges and made water-tight with fiber cement. The leakage was negligible.

That part of the structure carrying the rounded crest, corresponding to the part, *B D*, of Fig. 1, was located above a tight bulkhead across the test flume. In order to simulate conditions from *D* to *N*, Fig. 1, a short length of channel was provided below the end of the spillway crest. This part of the channel was located below the tight bulkhead. A second stop-plank bulkhead below the end of the channel was available for controlling the submergence at the outlet. A 2½ by 2½-ft. slide-gate in the center of this bulkhead made it possible to control the back-water accurately.

The quantity of water which could be passed over the spillway was limited by the flow in the river and by the height at which the flow could be delivered into the flume. Flash-boards were used on top of the diversion dam to increase the head. On account of the lateness of the season, the river discharge was affected by ice, and a steady flow was difficult to obtain.

Hook-gauges for reading depths in the spillway channel were placed in metal stilling-wells anchored to the concrete walls of the test flume. Water-tight connections from the spillway trough to the gauge-wells were made by short lengths of garden hose. Nipples for the hose connections were screwed into holes bored partly through the wooden sides of the trough, reinforcing cleats being used where required. Smaller holes were bored through to the inside surface to permit the passage of water. Five trough-gauges were used, one at each end of the crest and three at equally spaced intermediate points. The up-stream gauge was connected to the end of the channel, on the center line, 8 in. above the floor. The other gauges were connected through the sides, 8 in. above the floor, as shown in Fig. 9. Two additional hook-gauges set in wooden stilling-boxes on the wall of the test flume were used for reading the head on the spillway crest.

Heads on the 10-ft. sharp-crested, rectangular weir were taken with two hook-gauges in wells outside the weir box, as shown on Fig. 9. Discharges were read from a large scale curve platted by Mr. Parshall, and are undoubtedly more exact than would be required for the present purpose.

Observations.—When the structure was put into operation it appeared that a sufficient amount of air was being entrained to reduce materially the weight of the flowing water. It was thought, therefore, that the water depths measured in the stilling-wells might not furnish accurate information as to the actual depth of flow in the channel, and it was decided to take direct level-rod measurements of the surface profile. On account of the disturbed condition of flow, it was not expected that these readings would be accurate, but later they were found to be consistent and reliable.

The conditions shown in Figs. 12 and 13 and Figs. 14 and 15 are typical, respectively, for non-submerged and submerged flows. The level-rod was held

opposite each gauge connection and at such elevation that, as far as could be judged, it was in and out of the water equal intervals of time. Generally, two readings were taken for each station, one on top of the boil shown in the photographs, and one in the depression on the crest side of the channel. In some cases, where the flow was less regular, extra readings were taken near the middle of each section. Level-rod readings were also depended on for determining the back-water level in the pool into which the channel discharged.

The program for making the observations was arranged to eliminate, as far as practicable, personal equation and chance. Tests with varying discharges were run on three settings of the trough, two with a flat bottom slope and one with a considerable grade, as shown on the profiles, Figs. 16, 17, and 18. Fifteen separate runs were made. Preparation for each run was made by setting the head-gates and by-passes to approximate the flow required.

As soon as the depth in the gauge-boxes at the Francis weir became constant, it was assumed that steady flow had been established. The level-rod profile of the water surface was then taken as rapidly as possible, after which the rodman beginning at the up-stream end read all the channel gauges, the Francis weir gauges, and the down-stream and up-stream spillway crest gauges in turn, making three complete circuits. The levelman recorded all readings. The observers then changed duties and repeated the series, reading the gauges first and ending with the level-rod readings. The level of the water in the submergence pool was read only once, after all other readings were completed. Frost in the connections to the stilling-wells for the spillway crest gave considerable trouble, and these gauges could not always be read. The gauge-wells for the measuring weir were outside the structure and were more accessible for clearing out ice. It was necessary each morning to drill the ice out of the connections and to thaw out the wells with hot water or by building fires around them.

Comparative Water-Surface Profiles.—The test was planned primarily as a check on a previously devised theory, and the most advantageous method of presenting the results seems to be in a series of profiles showing measured and computed water surfaces. (See Figs. 16, 17, and 18.) The high, low, and average water surfaces, determined from level-rod readings, are shown. Where available, the results of the trough-gauge readings are also platted. The down-stream part of the channel being on a level grade, the location of the control was not difficult. Where a control existed, it was found theoretically and actually to be close to the outfall end of the channel. In Runs Nos. 3, 9, 10, 12, 14, and 15, the submergence at the outfall was such that the critical velocity was not attained at any point and no control existed. In these cases, the measured water surface in the lower end of the channel was taken as a starting point.

Apparent Swell.—The computed profile generally is of the same form as the average observed profile, lying slightly below it. As noted in the theoretical discussion, it was expected that the measured depths would exceed the computed depths on account of entrained air and swell due to the discrepancy between mean and average velocities. Many factors may influence the amount

FIG. 10.—VIEW OF BELLVUE, COLO., LABORATORY.

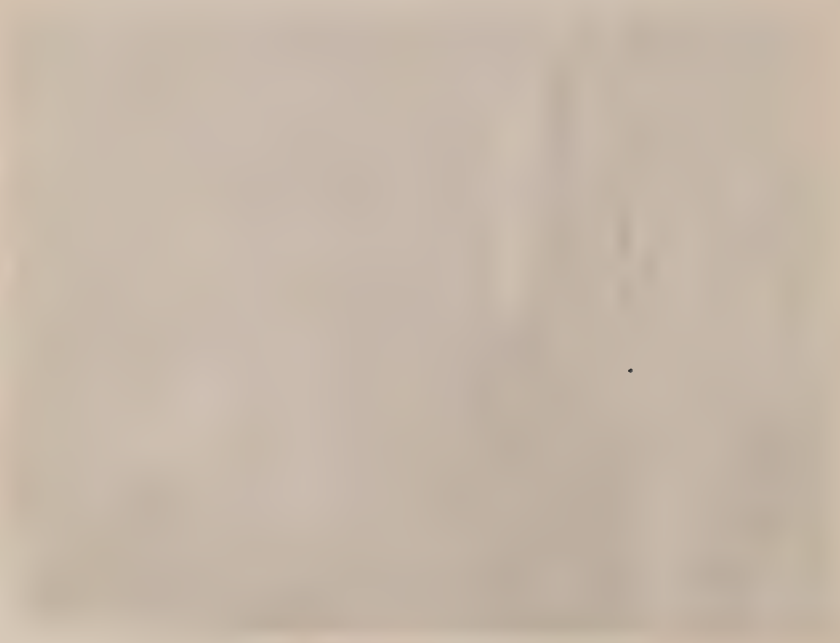
FIG. 11.—SPILLWAY STRUCTURE, SETTING NO. 1.

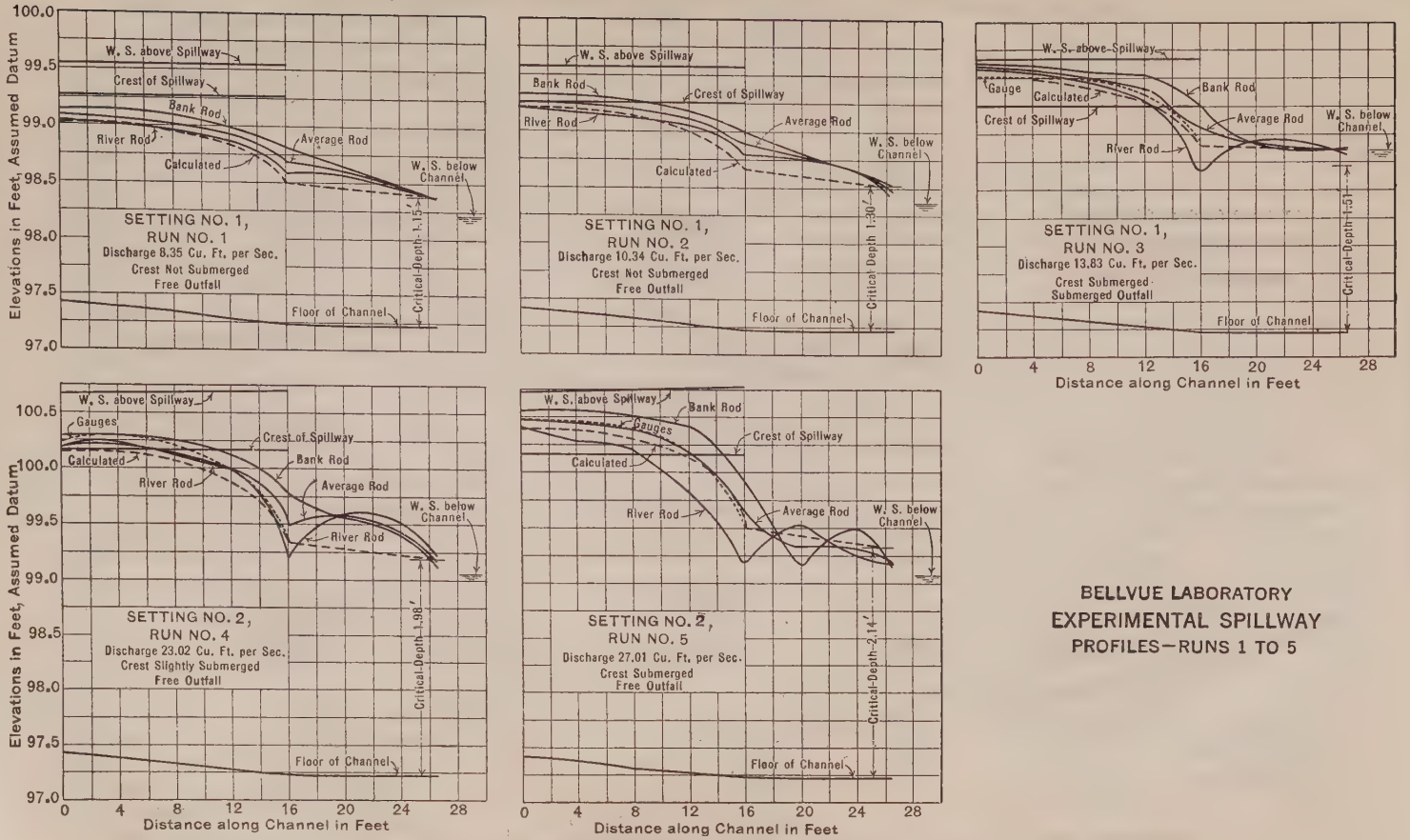
FIG. 12.—RUN NO. 1. DISCHARGE, 6.25 SECOND-FEET. TYPICAL FOR NON-SUBMERGED FLOW.

FIG. 13.—RUN NO. 8. DISCHARGE, 22.82 SECOND-FEET. NON-SUBMERGED FLOW.

FIG. 14.—RUN NO. 5. DISCHARGE, 13.89 SECOND-FEET. TYPICAL FOR SUBMERGED FLOW.

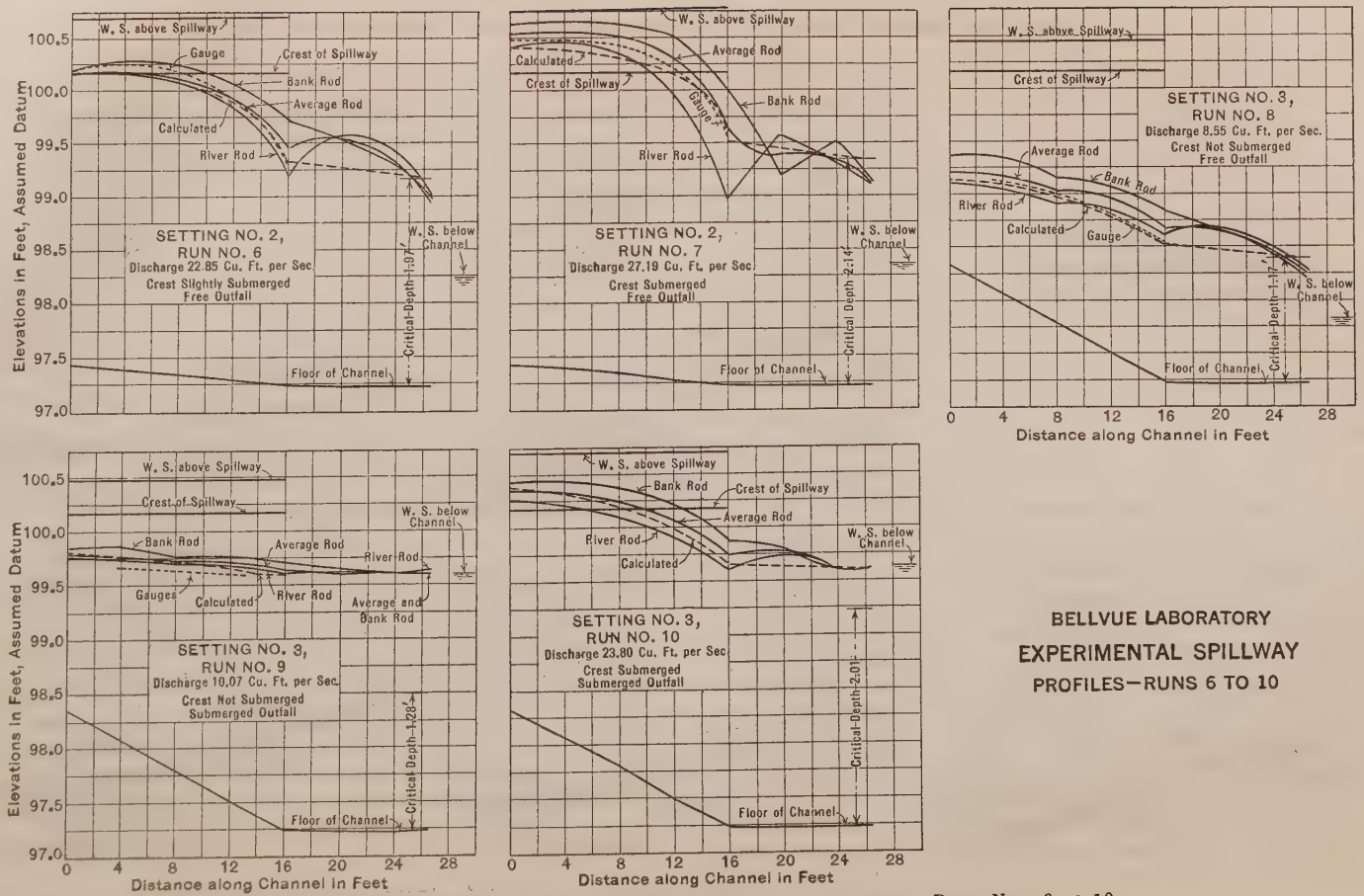
FIG. 15.—RUN NO. 7. DISCHARGE, 27.19 SECOND-FEET. SUBMERGED FLOW.





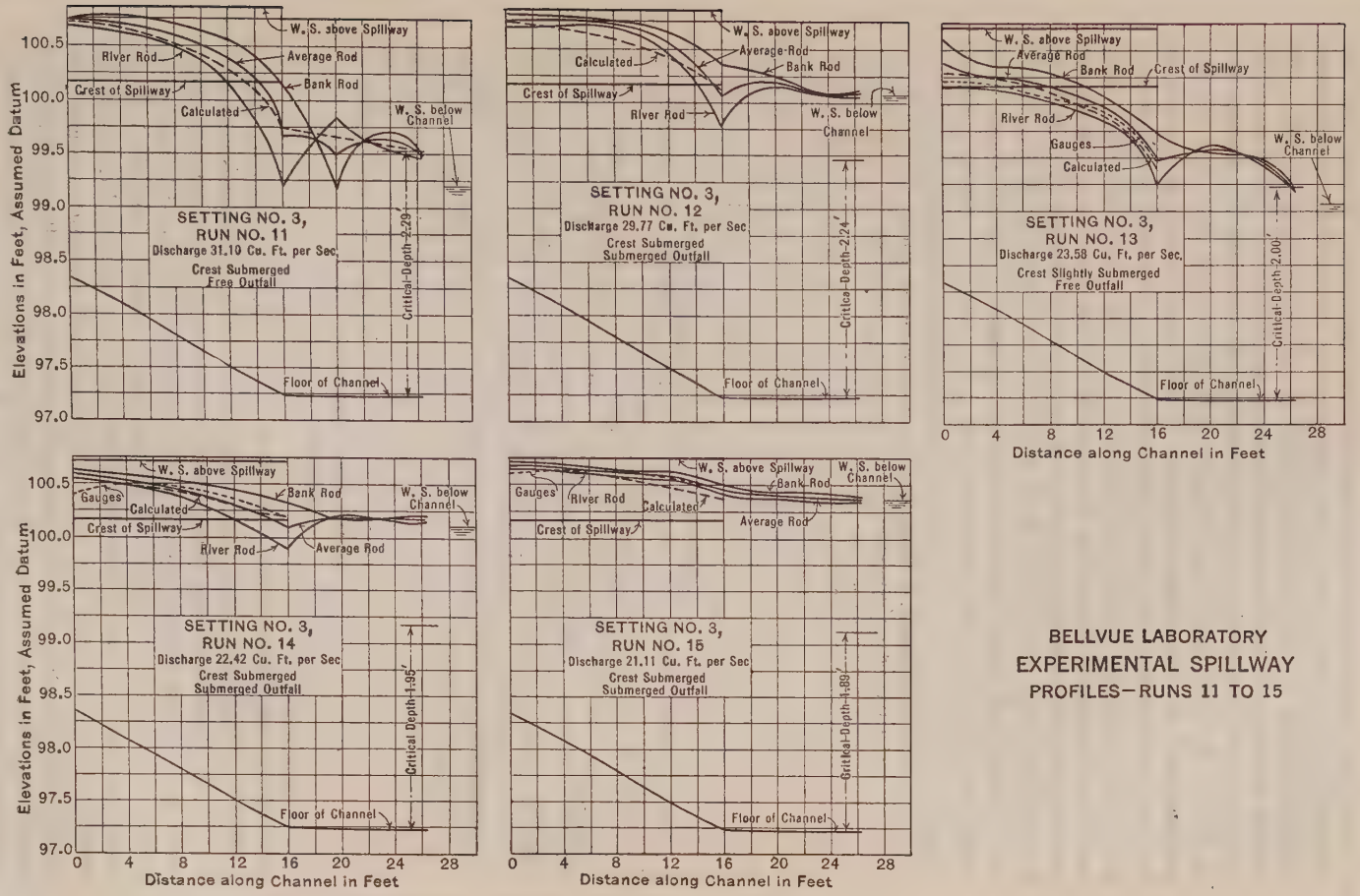
BELLVUE LABORATORY
EXPERIMENTAL SPILLWAY
PROFILES—RUNS 1 TO 5

FIG. 16.—EXPERIMENTAL SPILLWAY, BELLVUE LABORATORY, PROFILES OF RUNS NOS. 1 TO 5.



BELLVUE LABORATORY
EXPERIMENTAL SPILLWAY
PROFILES—RUNS 6 TO 10

FIG. 17.—EXPERIMENTAL SPILLWAY, BELLVUE LABORATORY, PROFILES OF RUNS NOS. 6 TO 10.



BELLVUE LABORATORY
EXPERIMENTAL SPILLWAY
PROFILES—RUNS 11 TO 15

FIG. 18.—EXPERIMENTAL SPILLWAY, BELLVUE LABORATORY, PROFILES OF RUNS NOS. 11 TO 15.

of swell. The volume of air taken into the stream probably increases with the quantity and fall of the inflow. The proportion of this air held in suspension probably depends on the volume and velocity of the channel stream. The velocity swell is probably influenced by the same conditions. The data available are neither sufficiently accurate nor sufficiently extensive for developing a law for the determination of the increase in volume due to these causes. The apparent swell for the fifteen runs made at Bellvue is shown graphically in Fig. 19. The points on the graphs were determined by comparing the channel area for the computed depth with that for the average depth from rod readings. Small uncertainties in the rod readings render the results erratic, but there is a general tendency to substantiate the expected results. For example, the weir crest is unsubmerged in Runs Nos. 1 and 8, and the channel outfall is free, allowing high velocities and small storage volume. As might be expected,

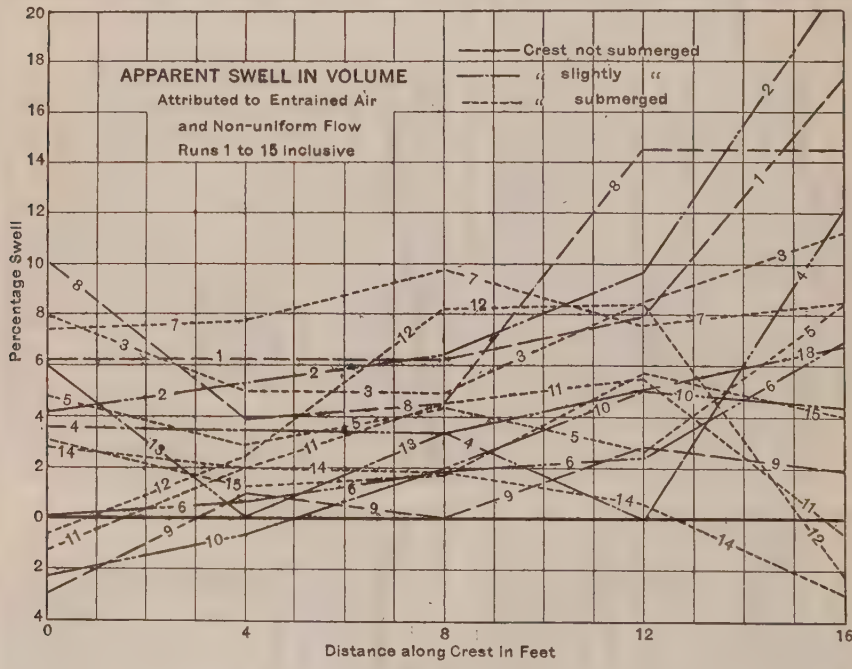


FIG. 19.

the swell is comparatively high. In Runs Nos. 2, 4, 6, and 13, the crest is slightly submerged and the outfall is free. At Station 16, Runs Nos. 2 and 4 show fairly high air contents. Otherwise, these four runs show only a moderate swell. Runs Nos. 5, 7, and 11 show a heavily submerged crest with free outfall. Runs Nos. 5 and 11 show moderate and consistent swells. Run No. 7 is consistent, but higher than might be expected. The crest in Run No. 9 is unsubmerged, but the outfall is obstructed so that channel velocities are low. The swell is small. In Run No. 10, both the crest and the outfall are slightly submerged, and the swell is small. Runs Nos. 3, 12, 14, and 15 are fairly heavily submerged, both at the crest and the outfall, and, in general, show small or

moderate swells. Run No. 13, which probably approximates usual design conditions as closely as any of the runs, shows an average swell of about 5 per cent. The effect of the scale of the structure on the swell is undetermined, and the test results should be applied to actual design with caution.

A Peculiar Air Condition.—An interesting phenomenon was discovered in studying air conditions. A "rope" of air varying in size, but about 2 in. in diameter, was formed approximately at the gravity axis of the water prism, and remained there throughout all the tests, except perhaps in cases of deep submergence. This "rope" ended squarely against the up-stream end of the channel. It increased slightly in diameter and became less distinct toward the down-stream end. If broken up with a hoe, it immediately reformed.

Discharge and Submergence Coefficients for Crest.—It was not anticipated in planning the tests that any valuable data would be obtained on the discharge coefficient of the rounded metal crest, and no special attempt was made to obtain favorable approach conditions. Nevertheless, the results roughly check, in an interesting way, a theory of compound weir discharge previously proposed. According to Bazin's measurements* the lower nappe of the discharge over a sharp-crested weir rises above the crest elevation by an amount equal to 0.112 times the head on the crest. If a crest having the exact shape of the nappe is put under the falling sheet, it may be assumed that the discharge will not be altered. However, the summit of the new crest will be above the original crest level by the previously mentioned percentage of the head on the crest. If for convenience it is desired to express the discharge in terms of the depth on the new crest, the discharge coefficient must be multiplied by $\left(\frac{1}{1-0.112}\right)^{\frac{3}{2}} = 1.195$. This factor should apply to the coefficient of any discharge formula in which the head appears as the three-halves power, and is theoretically strictly applicable only to a perfectly shaped crest. The crest can be shaped to fit only one flow. For smaller flows, the conditions of a flat-topped weir are approached, and the discharge coefficient decreases. As the depth approaches zero, the crest becomes relatively very flat and the weir tends to act as a control section. The discharge over a control is:

$$Q = \frac{2}{3} H^{\frac{3}{2}} \sqrt{\frac{2}{3} g} = 3.09 H^{\frac{3}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (25)$$

The coefficient in Equation (25) is 0.93 that of the Francis coefficient, 3.33, for suppressed sharp-crested weirs.

In estimating the flow over compound weirs, the use of multipliers to be applied to results from tables for sharp-crested weirs is very convenient. The highest multiplier, if the flow does not clear the crest, is 1.195, and the lowest for a very flat crest, with a rounded approach, is 0.93. Usually, the flat type weir is subject to a transition loss and the multiplier can be less than 0.93 if the approach is not properly rounded. Multipliers for a number of crest forms have been published by the U. S. Bureau of Reclamation.† The highest

* See Parker's "Control of Water", p. 400.

† "Hydraulic and Excavation Tables", p. 96.

value shown is 1.169. Crest multipliers for the test structure are platted in Fig. 20 (*a*) over depths as abscissa, no allowance being made for submergence. The figures to the left of the hyphen at each point designate the number of the run and those to the right show the percentage of submergence at Station 0 on the basis of the computed depths. It will be noted that the points for zero submergence fall quite regularly along a curve joining the theoretical points at the two extremes. The submerged points are distributed in a fairly regular way, but are insufficient in number to permit the derivation of a law governing the effect of submergence.

It was expected that the law of submerged discharge would differ materially from that for a sharp-crested weir. In order to test this assumption the theoretical discharges were recomputed, making allowance for the submergence due to the computed water-surface profile in the channel, in accordance with Herschel's coefficients.* The results are shown in Fig. 20 (*b*). The figures designate the number of the runs. The points follow a definite line, indicating that the multipliers drop away from the theoretical maximum toward the control section value as the head on the crest decreases. There is no characteristic difference between submerged and unsubmerged runs. Run No. 15 is the only one that is seriously erratic. All multipliers are based on Bazin's formula for suppressed weirs, with allowance for velocity of approach, except for the lower theoretical point. Bazin's formula does not extend to a zero depth. Until more complete data are available, it appears that a multiplier of 1.15 may be safely used for a correctly shaped crest, and that Herschel's coefficients for submergence apply with sufficient accuracy to spillway conditions.

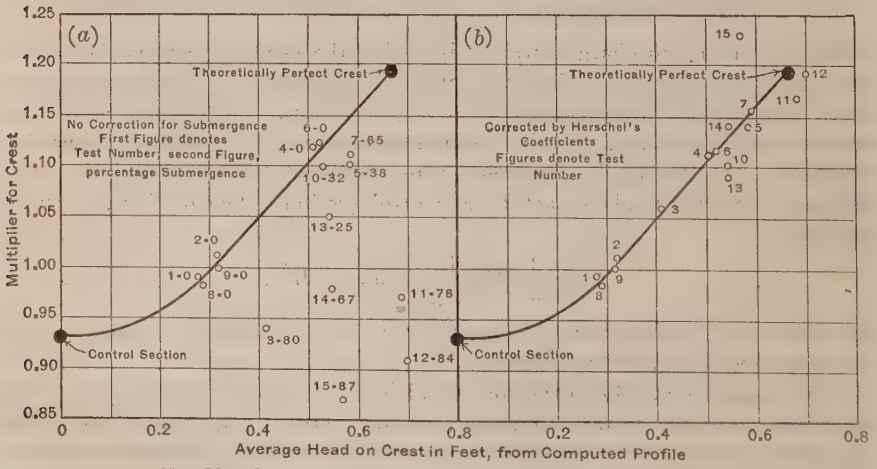


FIG. 20.—CREST MULTIPLIER AND EFFECT OF SUBMERGENCE.

Characteristic runs are shown on Figs. 21, 22, 23, and 24. The effect of submergence on the turbulence of flow is illustrated in Figs. 21 and 23, Runs 13 and 15, respectively. Conditions in these two views are approximately the same, except as to submergence at the outfall.

* See Merriman's "Treatise on Hydraulics."

Fig. 21.—Run No. 13. Discharge, 23.58 Second-Feet. Practically No Submergence.

Fig. 22.—Run No. 14. Discharge, 22.42 Second-Feet. Moderately Submerged Flow.

Fig. 23.—Run No. 11. Discharge, 21.31 Second-Feet. Heavily Submerged.

Fig. 24.—Run No. 12. Discharge, 29.77 Second-Feet. Showing Cross-Flow Below Crest for Certain Stages.

Experiments at Arrowrock Dam

The 1923 Overflow.—A spillway structure of adequate capacity is rarely required to carry the full discharge for which it is designed. For that reason tests on full-sized installations are difficult to obtain. In the spring of 1923, the Arrowrock Reservoir, on the Boise Irrigation Project, in Idaho, overtopped the spillway by about 10 000 sec-ft. The crest of this spillway is a movable type and might have been lowered to obtain full spillway discharge. However, such a discharge is in excess of any flood ever recorded in the river, even before the construction of the reservoir, and would be considered more or less of a calamity. The designed capacity is 40 000 sec-ft., or about four times the 1923 overflow. The crest gates were not fully lowered, but the water spilled over them in a partly raised position. The resulting conditions were very different from those contemplated in the foregoing discussion, especially in regard to height of fall, turbulence, and air content. However, it was considered desirable to make such rough observations as were possible with simple equipment.

Preparations and Equipment.—The work was in charge of W. G. Steward, Assistant Engineer, U. S. Bureau of Reclamation. Measurements on two different dates were made of the water surface profile in the channel, of the draw-down to the weir, of the upper nappe of the overfalling sheet, and of the surface velocities in the spillway channel.

The general arrangement and dimensions of the spillway are shown in Fig. 25. The crest is divided by piers into six sections, each controlled by a 62 by 6-ft. drum-gate. As shown on the diagram the gate crests were approximately 1 ft. below their maximum height at the time the tests were made.

Before flood stage was reached the gauges were painted on the side of the channel opposite the crest to measure any flow to a depth of 12 ft. The gauges were located half-way between Piers 1 and 2, 2 and 3, opposite Piers 3, 4, 5, 6, and 7, and at 100-ft. intervals below Pier 7. Opposite Piers 1 and 2 there were heavy deposits of sand and gravel, washed in from the hill above, which prevented the painting of the gauges at these points. Wires were stretched across the channel from Piers 1, 2, 3, 4, 6, and 7 for the purpose of running out a trolley to which was attached a weight that could be lowered by a cord to determine the distance to the water surface. A transit was used with the weight and the results were satisfactory, but the work was slow. The wave action on the land bank of the channel was measured partly with a transit and partly by gauge readings. One of the gauges was completely submerged and the others were covered part of the time. The elevation of the water surface on the reservoir side of the channel was determined by a weight suspended over the parapet wall, making allowance for the inclination of suspension. The curvature of the water surface above the weir crest was measured down from the tops of the piers, and the curvature of the nappe of the falling water was measured by an extension lever with a movable weight suspended near the end.

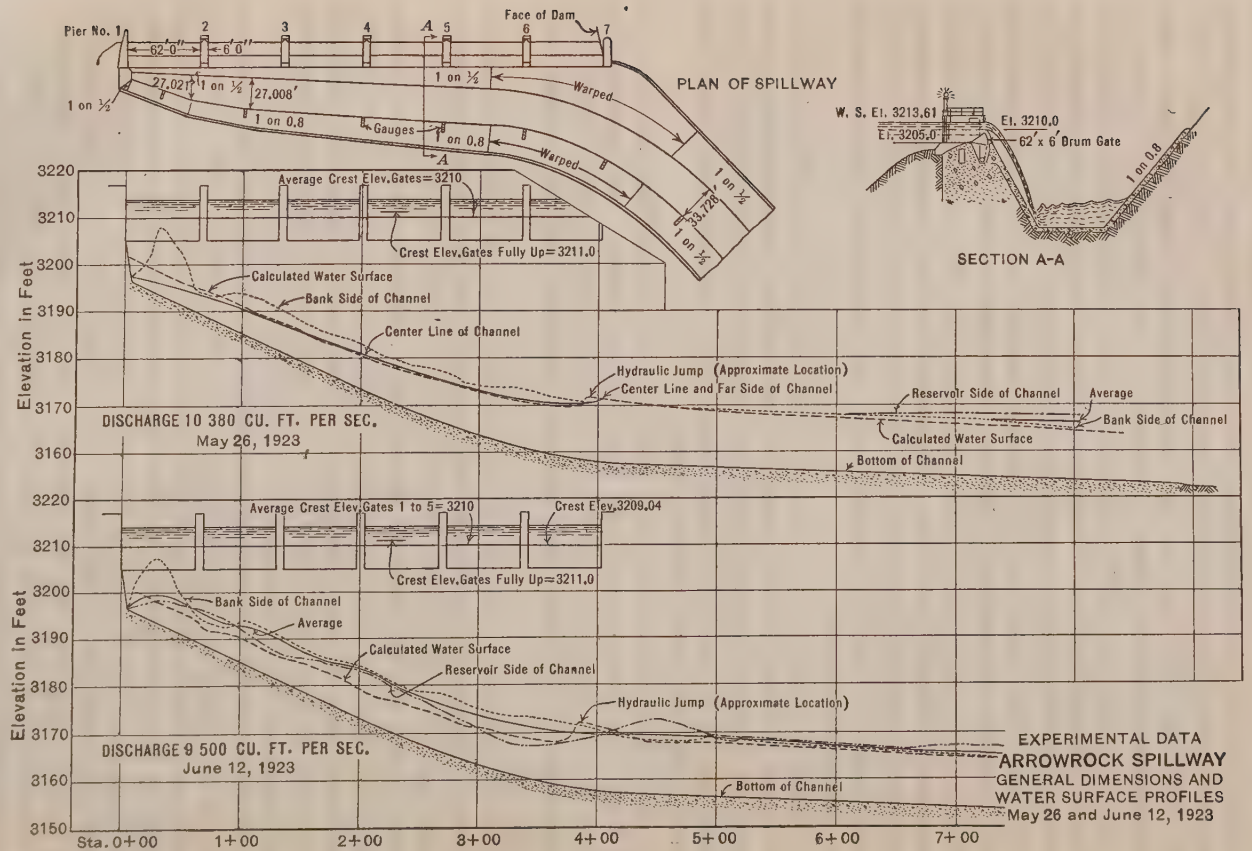

Pier No. 1
2
3
4
A
5
6
7
Face of Dam
62-0
6-0
27.021'
1 on ½
27.008'
1 on 0.8
Gauges
Warped
1 on ½
35.728'
PLAN OF SPILLWAY
W. S. El. 3213.61
El. 3205.0
El. 3210.0
62' x 6' Drum Gate
1 on 0.8
SECTION A-A
Average Crest Elev. Gates=3210
Crest Elev. Gates Fully Up=3211.0
Calculated Water Surface
Bank Side of Channel
Center Line of Channel
Hydraulic Jump (Approximate Location)
Center Line and Far Side of Channel
Reservoir Side of Channel
Average
Bank Side of Channel
Calculated Water Surface
Bottom of Channel
DISCHARGE 10 380 CU. FT. PER SEC.
May 26, 1923
Elevation in Feet
3220
3210
3200
3190
3180
3170
3160
Average Crest Elev. Gates 1 to 5=3210
Crest Elev. 3209.04
Crest Elev. Gates Fully Up=3211.0
Bank Side of Channel
Average
Calculated Water Surface
Reservoir Side of Channel
Hydraulic Jump (Approximate Location)
DISCHARGE 9 500 CU. FT. PER SEC.
June 12, 1923
Bottom of Channel
3150
Sta. 0+00
1+00
2+00
3+00
4+00
5+00
6+00
7+00
EXPERIMENTAL DATA
ARROWROCK SPILLWAY
GENERAL DIMENSIONS AND
WATER SURFACE PROFILES
May 26 and June 12, 1923

FIG. 25.

The elevation of the movable crests was found by measuring down from the tops of the piers. The velocity of the water in the channel was determined by surface floats consisting of pieces of driftwood or bundles of kindling. The velocity of approach to the weir crest was measured with a current meter.

All measurements were made by Mr. Steward, and a report giving in detail the methods used is on file in the office of the Chief Engineer of the U. S. Bureau of Reclamation at Denver, Colo.

Comparative Profiles.—The observed water surface profiles, in comparison with the computed depths, are shown on Fig. 25. The theoretical profiles shown were not calculated from Equations (23) and (24), but from a previously devised more complicated formula based on the same theory. On May 26, 1923, the water surfaces in the channel were measured along the land bank and along the center line. The agreement between the observed center-line profile and the computed profile is very close; closer, in fact, than would be expected in view of the extreme turbulence. On June 12, 1923, the agreement is not so close, the observed profile coming well above the computed one at all points above the down-stream end of the crest. Below the end of the crest, the computed and observed depths agree quite closely for both sets of observations.

There is an apparent discrepancy in the observed depth for the two dates. The discharge on May 26 was 10 380 sec-ft., and that on June 12, 9 500 sec-ft., yet the observed depths were greater on the latter date. It is probable that the second set of measurements is more reliable. According to Mr. Steward, in the first set the water-surface elevations in the channel were obtained by lowering the weight from the cross-wire to what appeared to be the average surface elevation and then taking the angle to the weight with a transit. There was, he states, a possibility of error in this method due to the continual splashing and wave motion which caused the weight to swing if lowered too far. The weight was more or less hidden by the spray. The second set of measurements was made more carefully, advantage being taken of the experience gained in the first trial. The distance and angle to the weight were measured while the weight was at a sufficient height above the water surface to be out of the spray. The weight was then lowered to what appeared to be the mean elevation of the water surface by paying out additional cord, the length of cord paid out being measured. It will be realized that great accuracy was not possible in either case. An idea of the turbulent conditions of flow is afforded by Figs. 26, 27, and 28.

In computing the theoretical water surface profiles it was necessary to proceed as in the case illustrated in Fig. 8. The control points were near the upper end of the channel. An interesting theoretical situation appears to exist near the down-stream end of the crest in both experiments. It is not possible to follow the flow through to the end of the inflow section without a hydraulic jump. The theoretical jump apparently lies within the observed water prism. There are many complicating influences at this part of the channel, and any attempt to make accurate computations appears useless. Among other things the horizontal curvature of the spillway channel gives the inflow a component velocity in the direction of the channel, and, furthermore, a complete admixture of the last incoming water with channel flow

probably is not accomplished for some distance. The approximate locations of the theoretical jumps are shown on Fig. 25.

Apparent Swell.—At the time the tests were planned it was thought that by observing velocities of flow in the channel an estimate of the volume of entrained air could be obtained. The only available method of measuring these velocities was by means of surface floats. The velocity being variable, it was necessary to time the floats at numerous points along the channel. The number of observers was not sufficient to permit the timing of a single float at all required points. The time of passage from the upper end of the channel to each of the various stations was determined separately by different floats, and the velocities for the various reaches were found by taking differences of these time intervals. No great dependence can be placed on the results. Even if the surface velocity along a given line were definitely known, it would give very little idea of the actual average velocity. A comparison of the mean velocity computed from observed depths with the surface velocity from floats gave apparent air contents of from —4% to +130% of the volume of water. The variations were erratic.

The results are shown in Table 4, as is also the swell obtained from a comparison of the observed and computed water prism areas. It is believed that the percentages of swell shown are higher than need be provided for in design.

TABLE 4.—Air Content or Swell, Arrowrock Spillway.

(Ratio of Air, or Swell, to Theoretical Water Volume.)

Station* numbers	From Observed Surface Velocities		From Computed Depths	
	May 26, 1923	June 12, 1923	May 26, 1923	June 12, 1923
1	....	....	....	....
2	—0.04	0.56	....	0.45
3	0.21	0.46	0.02	0.57
4	0.34	0.44	0.09	0.54
5	0.27	0.38	0.05	0.43
6	0.19	0.46	....	0.33
7	0.08	0.50	....	....
8	0.09	0.07	....	....
9	0.04	0.19	....	....
10	0.04	0.32	....	....
11	0.01	0.29	....	....

* Numbers of observation stations, not distances. (See Fig. 29.) Values show very rough approximations.

Approach Velocity to Crest.—The observed surface velocities and the approach velocities to the weir crest are shown on Fig. 29. The approach velocity measurements were taken on June 12, 1923, from Pier 5. The velocity of approach is affected by the contour of the reservoir floor near the crest and by the position of a log boom up stream from the crest. It is not the same for all stations along the crest.

Shape of Upper Nappe.—An interesting result of these observations is shown on Fig. 30. One of the simplest problems in experimental hydraulics would appear to be the determination of the path of a jet over a sharp-crested

FIG. 26.—ARROWROCK SPILLWAY, JUNE 12, 1923, LOOKING DOWN INTO CHANNEL FROM PIER 1.

FIG. 27.—ARROWROCK SPILLWAY, JUNE 12, 1923, LOOKING UP CHANNEL FROM DOWN-STREAM END.

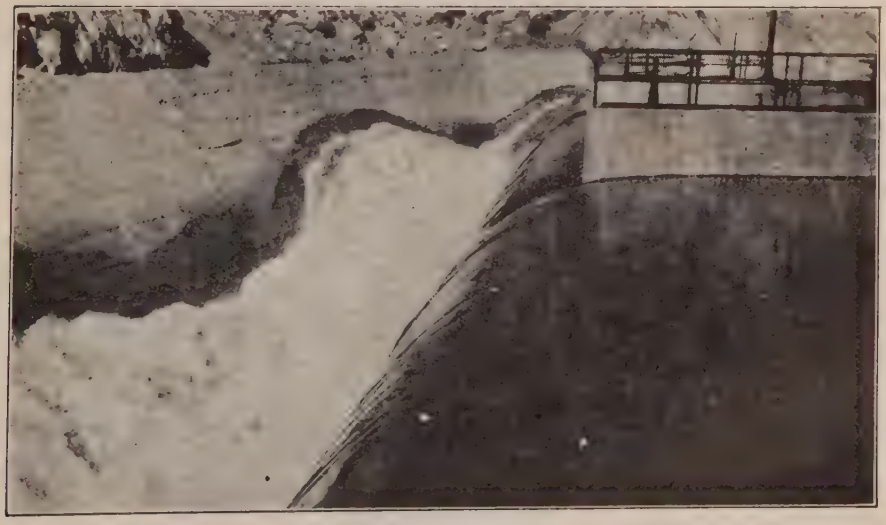


FIG. 28.—ARROWROCK SPILLWAY, JUNE 12, 1923, LOOKING UP STREAM FROM PIER 4.

weir, yet practically nothing toward this end has been done. Measurements out to a limited distance were made by Bazin, and various writers have theorized about the subject, generally using these measurements as a basis, with the widely varying results shown in the diagram. The trajectories shown are not all based on identical assumptions, and in order to plat some of them it was necessary to make certain interpretations which may not agree with the original intentions of the various authors. The majority of

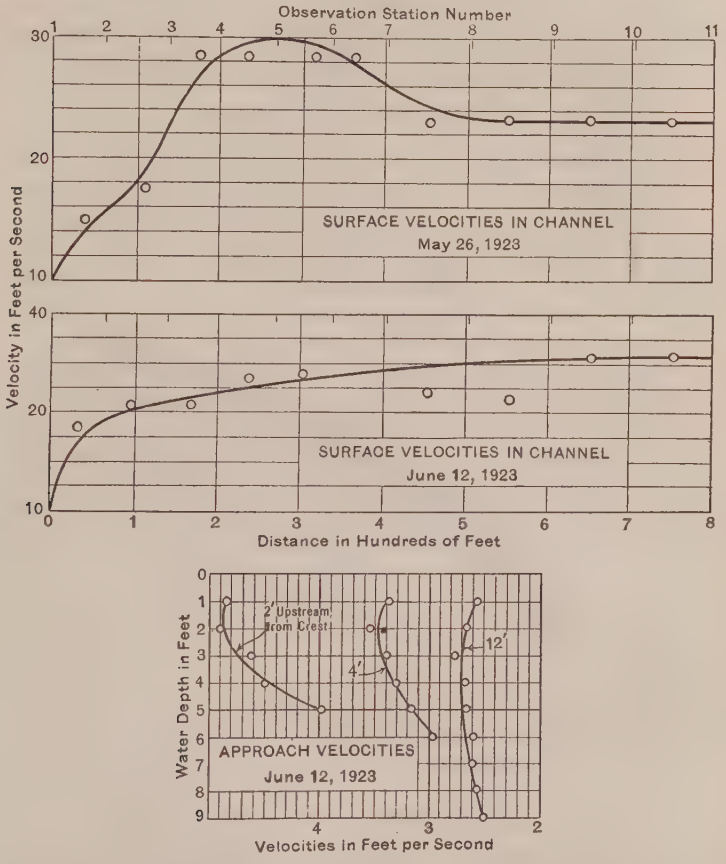


FIG. 29.—EXPERIMENTAL DATA, ARROWROCK SPILLWAY. SURFACE AND APPROACH VELOCITIES.

the curves are based on an assumed sharp crest with a vertical up-stream face. These curves are presented, not as a criticism of any of the theories proposed, but to show the lack of definite information on the subject. Mr. Steward was fortunate enough to obtain measurements of the upper nappe for both measured flows at Arrowrock. The observed nappe for June 12 is shown superimposed upon the theoretical curves for comparison. In order to avoid too great confusion the observed nappe for May 26 is platted separately.

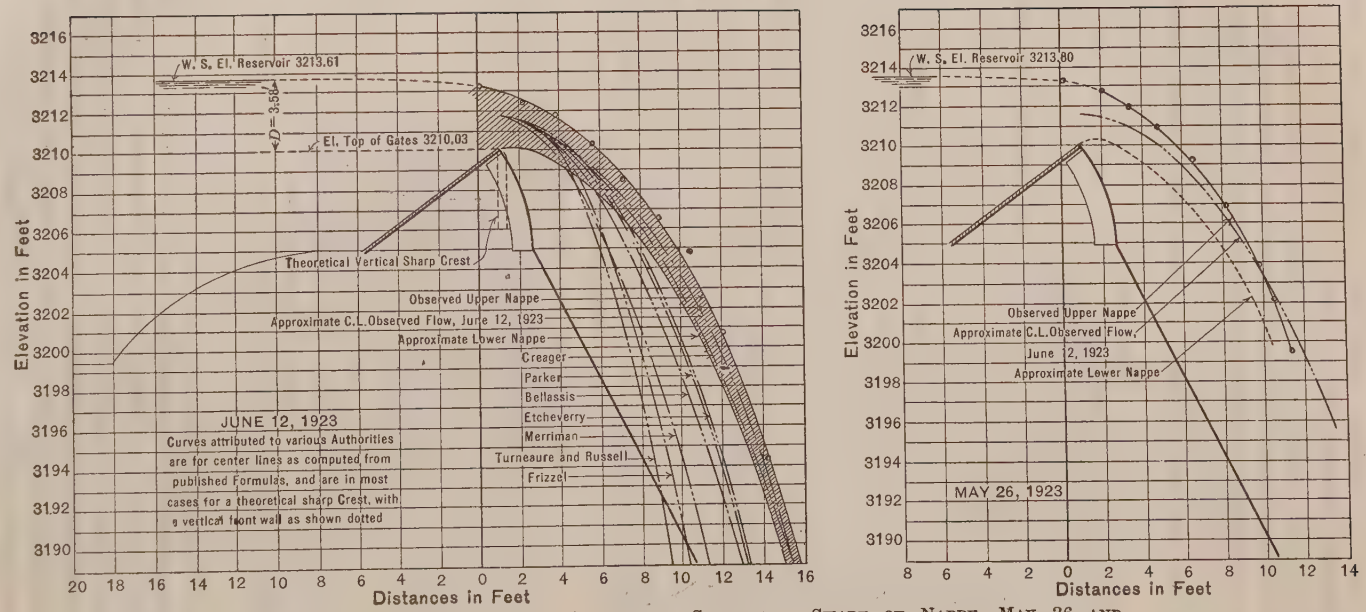


FIG. 30.—EXPERIMENTAL DATA, ARROWROCK SPILLWAY. SHAPE OF NAPPE, MAY 26 AND JUNE 12, 1923.

Although the discharge on May 26 was greater than that on June 12, the trajectory for the latter date is the flatter of the two. The first observation was made at Pier 3 and the second at Pier 5, where the velocity of approach appeared to be greater. Mr. Steward thinks this accounts for the discrepancy between the two nappes. The observed points are indicated on Fig. 30 by circles. The lower nappe curves shown are estimated.

The experimental nappe for June 12 conforms closely to that proposed by Creager, the most conservative of all the theoretical maps. When it is remembered that the economy of an overflow dam is often largely influenced by the thickness required to prevent the jet from leaving its surface, the importance of pursuing this problem further will be appreciated.

Experiments by J. W. Ellms

A situation similar to side-channel spillway flow is found in the design of wash-water troughs for rapid sand filters. A discussion of this problem, by C. N. Miller, Assoc. M. Am. Soc. C. E., has been published.*

Mr. Miller develops a theoretical formula based on the energy equation, and containing an undetermined factor for what he calls friction. This factor, which is expressed as a percentage of the velocity head, includes the friction of the channel and the internal friction, or impact loss.

The mathematical discussion appears to be correct. In order to simplify the formula deduced, the assumption is made that the water surface curve is parabolic, making the average effective fall throughout the trough length equal to two-thirds of the total fall.

It is evident from the general discussion of the problem given in this paper that in many cases this assumption may be incorrect. However, it is probably applicable, within the necessary limits of accuracy, to the particular problems discussed by Mr. Miller.

Applying the theoretically derived equation to some experimental data obtained by Mr. Ellms at the Cincinnati, Ohio, Filtration Plant, a value of 0.75 of the velocity head was found for the friction or impact coefficient.

A coefficient found in this way is applicable only to the conditions under which the experiments were conducted, the loss by shock being dependent on the distribution of velocities throughout the length of the trough, rather than on the velocity at the outlet end.

Mr. Miller's formulas are designed for rectangular channels and are not directly applicable to the results obtained at Bellvue. However, the net loss can be found by subtracting the velocity head from the average fall, which latter is obtained from the water-surface curve. The average ratio of impact and friction loss to velocity head at $x = 16$ ft., for the 15 runs at Bellvue, is about 0.375, or about one-half the value found at Cincinnati.

* "Water Purification," by J. W. Ellms, M. Am. Soc. C. E., Appendix B, 1917 Edition.

DISCUSSION

C. H. Howell,* M. Am. Soc. C. E. (by letter).—The writer believes Mr. Hinds has successfully solved a complicated hydraulic problem of considerable economic importance and scientific interest. To design a spillway properly is often more difficult than to design a dam. This is especially true with very large discharges. A rational method of analysis and design such as this paper presents will be valuable to engineers.

The confirmation of the theory by the Bellvue and Arrowrock experiments will increase the confidence of those using it. As the author states, this type of spillway is rather inefficient hydraulically and is best suited to moderate discharges. The writer's experience leads him to believe that large discharges, exceeding 20 000 to 25 000 sec-ft., can be handled much more economically by spillways with gates placed normal to the channel. The economy increases rapidly as the discharge increases above 25 000 sec-ft. With a gate or gates placed at the upper end of and normal to the channel, the velocity at the gate can be either maintained or increased by a proper slope. The large losses due to impact, inherent in the side channel spillway, are thus avoided.

The side channel spillway, however, lends itself well to the installation of segmental drum crests. These crests, as developed by the U. S. Bureau of Reclamation, are positive in their automatic action. With earth or loose rock dams a large part of the total spillway discharge should have automatic control. The value of the automatic feature of the drum crests may sometimes offset the inefficiency of the side channel spillway.

In designing the Guernsey Dam, on the North Platte River, the Bureau of Reclamation was confronted with the problem of discharging 80 000 sec-ft. past a gravel-fill dam about 100 ft. high. Automatic control for 30 000 sec-ft. was assumed. This is about the maximum discharge in 34 years of record. The side channel type of spillway was tried in various forms and abandoned on account of its excessive cost. The final designs contain some unusual features. The discharge of 30 000 sec-ft. is controlled by two automatic segmental steel drum crests, each 64 ft. long and 14.5 ft. high. These drums discharge into a vertical circular shaft 31 ft. in diameter which, in turn, discharges into a diversion tunnel 30 ft. in diameter. The method proposed by Mr. Hinds was used in designing this structure. This spillway is on the south side of the river. On the north side an open, concrete-lined channel, 34 ft. wide on the bottom and with ½ to 1 slopes, is controlled with one gate which is 50 ft. square in the clear and is the largest in the United States. It is exceeded materially in size, however, by some in Europe. This spillway will pass 50 000 sec-ft. without a raise in the reservoir water surface, and about 70 000 sec-ft. before the dam is overtopped.

H. B. Muckleston,† M. Am. Soc. C. E. (by letter).—The author has not pointed out the great influence which spiral flow in the channel may have on the magnitude of the swell ratio.

* Engr., U. S. Bureau of Reclamation, Denver, Colo.
† Cons. Engr., Vancouver, B. C., Canada.

The "peculiar air condition" noted on page 913 was to be anticipated. Owing to the manner in which the increment water is delivered to the channel, the water in the channel tends to flow in spiral stream lines; and any entrained air bubbles, being lighter than the water, naturally seek the region of least centrifugal force, like the oil in a rifled pipe line. Some of the air bubbles are constantly escaping from the free surface; but as more air is being added all along the line, the total air in suspension should remain sensibly constant or even increase somewhat. As soon as the air supply ceases, the "rope" dissipates quickly.

The "rope" is merely visible evidence of a condition which exists all through the mass of water in the channel so long as the flow is spiral. Below the center of the spiral the centripetal tendency due to spiral flow is assisted by the tendency of the bubbles to float; above the center, the two tendencies are opposed, and there will be a region where the resultant tendency is zero. Inside this region the resultant tendency is toward the center. Just as large bubbles rise faster in still water than small ones, so the large bubbles, in the case of spiral flow, find their way more quickly to the center, if inside the critical region, or escape from the free surface, if outside it, than small ones. It is probable that the critical region is farther from the center for large bubbles than for small ones; but, in any event, a condition of spiral flow, by retaining the larger bubbles which would otherwise escape quickly, is favorable to a high content of entrained air and a consequent high swell ratio.

It may be doubted if the effects of spiral flow can be calculated. They are likely to be the greater as the average width approaches the depth and the side slopes become steep. A wide shallow channel, besides being unfavorable to spiral flow, has a greater surface area from which air can escape. If the effects should prove objectionable in a constructed channel, they can be mitigated by building a few shallow ribs on the floor and far wall of the trough parallel to its axis. These ribs will also help in reducing the surges up the far wall.

The author's mathematical treatment of the problem is very neat and the resulting formulas will probably be ratified by experience as closely as engineers have learned to expect in similar treatment of other hydraulic phenomena. The premises do not cover all the conditions, so the formulas will not predict all the results. The calculated surface profile will probably take care of the impact losses, and a few per cent. added to the calculated areas will care for the friction and other losses. The swell due to spiral flow is likely to be high but can not be calculated, and is an additional reason for a generous factor of safety in fixing the dimensions.

I. GUTMANN,* ASSOC. M. AM. SOC. C. E. (by letter).—The following is a partial list of dams with spillways of the type discussed in Mr. Hinds' paper:

	Date of construction.		Date of construction.
Solingen, Germany	1902	Rochebut, France	1909
Queille, France	1904	Cold Springs, Oregon	1910
New Croton, New York City Water Supply	1906	Conconully, Washington	1910
Wachusett, Boston, Mass., Water Supply	1906	Croton Falls, New York City Water Supply	1910

* New York, N. Y.

	Date of construction.		Date of construction.
Sweetwater, San Diego, Calif., Water Supply	1911	Klingenberg, Bohemia	1914
Ashokan, New York City Water Supply	1912	Cedar Lake, Seattle, Wash., Water Supply	1915
Mauer, Germany	1912	Morena, San Diego, Calif., Water Supply	1915
Oakley, Idaho	1913	Arrowrock, Idaho	1915
Moehne, Germany	1913		

This list shows that the side channel type of spillway is extensively used in the United States as well as in Europe. In fact, it is usually the most appealing solution in cases where the dam site is in a canyon and a properly located saddle in the rim of the reservoir is not available.

Mr. Hinds' paper presents, as far as the writer knows, the first systematic analysis of this subject in American technical literature. The paper is confined principally to the analysis of one characteristic of flow in side channel spillways, namely, that the total discharge at any section is a linear function of the distance, x, from the head of the channel or, in mathematical terms, $Q = bx$.

It may be stated that the type of discharge expressed by this equation is not characteristic of side channel spillways only. In general, it applies also to the discharge of a drain passing through uniformly porous soil, an infiltration gallery through uniform gravel, etc., and by changing its sign to read, $Q = -bx$, it will give the equation of flow in a channel subject to uniform seepage losses.

Some of the specific and very perturbing problems in the design of side channel spillways are not discussed in Mr. Hinds' paper. The writer has in mind particularly the phenomena caused by the change in direction of flow of the over-falling nappe from normal to axial with reference to the weir. It is hoped that the observations and experiments described by Mr. Hinds will be extended to include studies of the transverse surface curve in the spillway channel, the extent of the effective area of the spillway channel, hydraulic jump, and other phenomena arising from the change in direction of flow.

A. J. Wiley,* M. Am. Soc. C. E. (by letter).—Mr. Hinds has given a complete and clear exposition of the hydraulics of the side channel spillways that have been used in a number of important dams.

Often the spillway is the most difficult part of a dam to design and one of the most expensive to construct. The side channel spillway, as Mr. Hinds correctly states, is best suited for moderate discharges, but it has been used for flood discharges of 75 000 sec-ft. Unless the protected channel is extended to deliver the water back into the river, the concentration of such a large volume in a comparatively small channel, with the resulting high velocities, may cause extensive erosion the deposits from which disturb the river flow below the dam. The channels of the Arrowrock and the Don Pedro Spillways both end a few hundred feet below the dam in a vertical drop over the unpro-

* Cons. Hydr. Engr., Boise, Idaho.

tected canyon slopes from which the water finds its own way back into the river.

The formation at Arrowrock at the end of the channel is a rather soft granite. The erosion was quite rapid at first and the resulting deposits caused a local rise in the river bed. It soon created a vertical fall at the end of the lined channel from which the discharge flows in an eroded channel which joins the river several hundred feet below the toe of the dam. The conditions are stabilized for ordinary flood conditions, and future erosions except for extreme flood discharges will be negligible.

At the Don Pedro Dam, where the drop from the reservoir surface to the river below the dam is about 275 ft., the rock is a very hard diorite with many surface seams. At first there was quite extensive erosion of the large blocks of seamy surface rock with a discharge of 15 000 sec-ft., the deposit of which in the river channel caused a rise of about 20 ft. in the back-water at the dam. In this case, as in the Arrowrock Dam, the conditions soon became stabilized and further erosion will be negligible for ordinary flood discharges.

The spillway of a storage dam discharges only a small part of the river flow, the major part being drawn off through the regulated discharge of the irrigation or power outlets; but it is necessary to make spillway provision for the maximum unregulated flood discharges.

The chief objection to the channel type of spillway is the difficulty of protecting the channel lining against upward pressure. Usually the channel is cut out of the solid rock, beginning, perhaps, 500 ft. above the dam and is about 20 ft. wide and 20 ft. deep. It gradually increases in dimensions until, when it passes the end of the dam, it is, in the case of the Don Pedro Dam, 70 ft. wide and 50 ft. deep. Concrete lining is necessary to avoid erosion at the high velocities obtaining, but the cost of the thickness required to resist the full upward pressure from the reservoir is prohibitive and dependence must be placed on under-drainage to carry off the water that finds its way through the seams in the rock, the quantity of which is uncertain. In the Don Pedro Spillway channel, a part of the floor was raised by the excess of the under-pressure over the relief provided. The concrete lining was replaced and about 450 bbl. of cement were used in pressure grouting the rock floor of the channel although it was from 20 to 50 ft. below the surface of the rock excavation.

In many cases the safest and cheapest spillway is directly over the top of the dam. This form is embodied in the design of the Melones constant radius arch dam, 200 ft. high above the river level. In many storage dams, an important source of revenue is found in the development of water power from the flow as regulated for irrigation. As this is "seasonal" power, it must be cheaply developed in view of its low market value. The lowest development cost usually results from building the power plant directly at the toe of the dam, as at the Don Pedro Dam. At the Exchequer Dam, where the river is raised 325 ft. above its old level, the power plant is built in the river channel at the down-stream toe. A modification of the side channel spillway is made by converting each end of the regular gravity type dam into an overflow section. These two overflow sections take the place of the single long lip, or weir, above

the dam which feeds the ordinary type of spillway channel. The discharge from each of the overflow sections of the dam is carried in a concrete-lined channel, symmetrically located on either side of the canyon, leaving the toe of the dam at the end of the overflow section and discharging into the river down stream from the power house.

Instead of determining the hydraulics of the spillway channel by the method outlined by Mr. Hinds, the writer has been accustomed to use a much less direct method; instead of adopting the most economical shape, he has used a broader and shallower section in which the velocities are relatively low and the danger of uplift under the floor is reduced.

Starting with a considerable submergence of the spillway lip at the upper end, assumed velocities and bottom widths are adopted, increasing by regular increments along the spillway section of the channel. Using assumed side slopes the depth at any point is calculated for the quantity of water at the point with the assumed velocity. The fall in water surface necessary to give the assumed velocity is found by considering that the velocity at the first of any two successive points is reduced at the second point to the same proportion of that at the first point as the discharge at the first point is to that at the second point. The head required to increase the velocity thus reduced up to the required velocity at the second point plus the sum of the velocity heads and friction heads of the first point plus the friction head between the two points then gives the total fall in water surface up to the second of the two points. To provide for the increase in volume by entrained air and for the losses caused by shock and eddies, an arbitrary addition of 25% is made to the required flood discharge.

Ivan E. Houk,* M. Am. Soc. C. E. (by letter).—The writer believes that Mr. Hinds' thorough and practical solution of the side channel spillway problem will be of great value to designing engineers throughout the world. So far as he knows, no adequate treatment of the problem has ever before been published, either in the United States or abroad, although some rather brief discussions of the subject have been given in certain foreign books on dams and reservoirs. That no comprehensive treatment has previously been published seems rather strange considering that this type of spillway is not uncommon and that the safety of the dam itself, especially one of the earth type, is contingent on the proper functioning of the spillway.

A few years ago the writer had occasion to inspect the remains of an earth dam the safety of which had been dependent on a side channel spillway. The length of the spillway crest was reasonable for the size of the drainage area involved, but the side channel was not deep enough to carry the water away from the spillway, despite the fact that considerable submergence could occur before the spillway discharge was reduced appreciably. This condition, together with the fact that little free-board had been provided, resulted in the overtopping of the dam and its complete failure. In this one instance, alone, several lives and several hundred thousand dollars worth of property might have

* Engr., U. S. Reclamation Service, Denver, Colo.

been saved if Mr. Hinds' method of analysis had been available for the designer's use.

The unusually high losses that occur in the side channel spillway are due to the impact of the spillway discharge with the flow in the channel, as Mr. Hinds explains. If the topographic conditions are such that the spillway discharge can be led directly away from the apron, into a gradually converging and curving channel, the direction of flow probably can be changed through an angle of 90°, from a direction normal to the spillway crest to a direction parallel with the spillway, with relatively small losses. The large impact losses will thus be avoided and probably some of the velocity head attained in falling over the spillway will be conserved. It is doubtful whether the loss caused by curvature in open channels ever amounts to 100% of the velocity head in the case of 90° bends. At any rate, that was the conclusion reached by the writer some years ago, after making careful experimental determinations of losses in curves in open channels carrying discharges of from 200 to 70 000 sec-ft. at velocities of from 2 to 10 ft. per sec. The controlling factor in spillway location, however, is not the magnitude of the losses in the spillway channel, but the total cost of an adequate structure. In many instances the cost of a side channel spillway, with proper provision for the large losses involved, will be less than that of any other type.

Figs. 16, 17, 18, and 25, which illustrate the experimental work at the Bellvue Laboratory and at the Arrowrock Dam, might have been improved by showing the actual platted water surface elevations. The photographs in the paper show that in several runs the water surface was comparatively rough, and, therefore, the reader may wonder how accurately the surface curves were determined and how widely the individual observations varied from the smooth curves shown. The writer knows that the curves were accurately determined and that the variation of the measured points from the curves was not excessive, inasmuch as it was his privilege to assist Mr. Hinds in the Bellvue experiments. The matter is simply mentioned for the benefit of those who may be interested.

The Engineering Profession undoubtedly will accept Mr. Hinds' solution of the side channel spillway problem, as it stands. The theory is based on one of the best established laws of physics; the results of the laboratory experiments at Bellvue show substantial agreement with the theoretical formulas; and the results of the field tests on the full-sized structure at the Arrowrock Dam remove all doubt as to whether the theoretical formulas found to be applicable to small-sized laboratory models will apply to full-sized structures in actual practice.

J. C. Stevens,* M. Am. Soc. C. E. (by letter).—This is a most excellent presentation of an intricate hydraulic problem. If the fundamental basis of the author's mathematical analysis be accepted, that the water overflowing the spillway crest has no kinetic energy in the direction of continued motion, then his conclusions must be accepted for they follow rigidly therefrom. The author states his hypothesis a little more generally in observing that when two streams

* Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

of water at different velocities come together and flow away together the energy of impact is lost.

In order to test the soundness of this hypothesis the Special Committee on Irrigation Hydraulics has arranged for certain experiments at the Hydraulic Laboratory at Berkeley, Calif., that are now (December, 1925) under way. These experiments are under the direction of B. A. Etcheverry, M. Am. Soc. C. E., and B. A. Vanleer, Assistant Professor of Mechanical Engineering, University of California, who is in immediate charge of the Laboratory. It is also hoped that other experiments with the same object in view will be conducted at the Hydraulic Laboratory of the Oregon Agricultural College at Corvallis, Ore., under the immediate direction of H. S. Rodgers, Professor of Hydraulics.

These experiments will endeavor to determine the head lost by impact and eddies when two streams of water intersect at right angles in pipes. Pipes are being used in preference to open conduits because greater heads can be secured and the water is free from entrained air.

The writer submits herewith the mathematical treatment of the hydraulics of intersecting pipes, based on the theory in the paper. If the experiments prove the correctness of the formulas herein the theory may be accepted as sound. Let two pipes flowing full under pressure intersect, as shown in Fig. 31.

Let d = inside diameter of the pipe;
a = area of pipe;
p = pressure head;
Q = flow;
v = mean velocity;
h = velocity head;
r = ratio of initial discharges $= \frac{Q_1}{Q_2}$;
z = ratio of areas, $\frac{A_2}{A_3}$; and
i = energy lost in eddies and impact, for brevity called the eddy loss.

All the quantities are expressed in consistent units; the subscripts in the equations denote the pipe involved.

The momentum of the water in Pipe (1) has no component in the final direction of flow. At the junction the pressures are equal, that is,

$$p_1 = p_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$

The change of pressure is equal to the change of momentum from Pipe (2) to Pipe (3), hence the relation,

$$(p_2 - p_3)\, a_3 = \frac{Q_1 + Q_2}{g}\, V_3 - \frac{Q_2}{g}\, V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (27)$$

which reduces to,

$$p_2 - p_3 = 2\,(h_3 - z\,h_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

The equation for lost energy is:

$$(Q_1 + Q_2)\, i = Q_1\,(p_1 + h_1) + Q_2\,(p_2 + h_2) - (Q_1 + Q_2)\,(p_3 + h_3) \ldots (29)$$

Substituting from Equation (28) in Equation (29) and reducing, the loss in terms of velocity heads becomes,

$$i = h_3 + h_2 \left(\frac{1}{r+1} - 2z \right) + h_1 \frac{r}{r+1} \quad \ldots\ldots\ldots\ldots (30)$$

When the pipe diameters are in the ratio 3, 4, and 5, that is, when, $a_1 + a_2 = a_3$, Equation (30) reduces to:

$$i = 2h\,(1 - z) = p_2 - p_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (31)$$

This is obvious since there is no change of mean velocity, the total change of pressure being used in supplying the eddy loss.

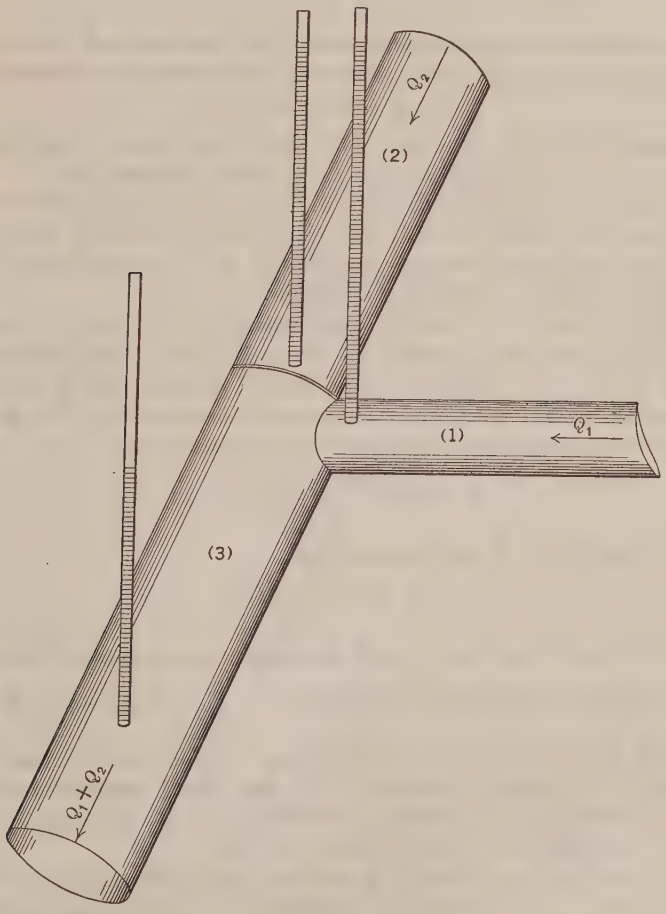


FIG. 31.

If $a_2 = a_3$, Equation (28) becomes:

$$p_2 - p_3 = 2\,(h_3 - h_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

that is, the change of pressure is twice the change of velocity head. In this case the eddy loss is:

$$i = h_3 - \frac{2\,r + 1}{r + 1}\,h_2 + \frac{r}{r + 1}\,h_1 \quad \ldots\ldots (33)$$

When Q_1 is zero, r becomes zero and Equation (30) reduces to:

$$i = \frac{(V_2 - V_3)^2}{2\,g} \quad \ldots\ldots (34)$$

which is the eddy loss in a sudden enlargement.

If Q_2 is zero, Equation (27) reduces to:

$$p_1 - p_3 = 2\,h_3 \quad \ldots\ldots (35)$$

and the eddy loss is:

$$i = h_1 + h_3 \quad \ldots\ldots (36)$$

which gives the loss in a right-angle bend. If Pipes (1) and (3) are of the same size the velocity is constant and the loss is twice the velocity head, or,

$$i = 2\,h = p_1 - p_3 \quad \ldots\ldots (37)$$

This is comparable with Equation (31) under conditions of no change in velocity, the entire drop of pressure going to supply the eddy loss.

These formulas do not take account of friction losses. In order to isolate the eddy losses the friction losses must be calculated separately and deducted from the drop in pressure between the junction and the point below it where the average pressure in the pipe is a maximum.

The writer knows of no reliable experiments on the losses in right-angle bends, although there are a number on curves in pipes. The experiments by Mr. W. H. Archer* show that the loss in sudden enlargements is very close to that indicated by the well-known formula, Equation (34). He found the actual losses to be more nearly represented by the formula:

$$i = 1.098\,\frac{(V_2 - V_3)^{1.919}}{2\,g} \quad \ldots\ldots (38)$$

and that the coefficient, B, in the expression,

$$i = B\,\frac{(V_2 - V_3)^2}{2\,g} \quad \ldots\ldots (39)$$

varied with entrance velocity, V_2, and the ratio of areas, from 0.75 to 1.2; and that, for velocities of 6 ft. and more, and for $z = \frac{1}{2}$, or less, the coefficient, B, is less than 1, showing that the actual loss in such cases is less than the theoretical. Mr. Archer attributes this to the "dead" water at the point of enlargement acting as a diffuser. Professor F. W. Medaugh, of Johns Hopkins University, takes† issue with him in this.

If the "dead" water, or eddies, in sudden pipe enlargement or in a right-angle bend, can act as a diffuser then the eddies in a side channel spillway can perform the same function and all the energy of impact is not lost as the author assumes. The Committee awaits the results of the experiments at Berkeley

* *Transactions*, Am. Soc. C. E., Vol. LXXVI, (1913), p. 999.

† *Engineering and Contracting*, January 10, 1923.

and Corvallis, previously mentioned, with interest, hoping that they will throw light on this fundamental principle.

In this connection the writer observed that curved metal vanes had been placed on the crest of the side channel spillway of the Sweetwater Dam near San Diego, Calif., the object being, of course, to give the overflowing water a down-stream direction and thus reduce eddy losses that would otherwise occur. It would be interesting to know whether any measurements have been made of the efficacy of these vanes.

Mention may be made of the tests on models of the side channel spillways of the Burrinjuck Dam* on Murrumbidgee River in New South Wales. These were originally designed to pass a flood of 40 000 sec-ft. While the dam was under construction in 1916 a flood of about 65 000 sec-ft. passed over the works. Because of this, models were built and tests made to ascertain whether increased capacity could be secured within the limits of practicable construction. As a result the south spillway was lengthened and then both spillways were deepened more than the original plans indicated, to take a flood corresponding to that of 1916 under a head of 10 ft. on the two spillways. In May, 1924, the greatest flood of record occurred. The water rose until the head on the spillways was 19.3 ft., the parapets of the dam being overtopped 3.3 ft.† The paper does not contain sufficient data to enable comparisons to be made of the model tests and the author's analysis.

The Committee also hopes to secure in due time actual tests on the side channel spillway of the Rimrock Dam of the U. S. Bureau of Reclamation, on Tieton River, Washington, and also on that of the Lake Fordyce Spillway of the Pacific Gas and Electric Company of California.

Julian Hinds,‡ M. Am. Soc. C. E. (by letter).—It is pleasing to note that the hydraulic principles proposed for the design of side channel spillways are generally accepted, with only a sufficient spirit of misgiving to insure the continued study of the problem. The explanations and improvements offered form a valuable addition to the paper.

The discussion by Mr. Muckleston of the "air rope", and his statements in regard to the effect of spiral flow, are interesting. From the statements made it might be inferred that the spiraling of the water around the axis of the channel, caused by the initial transverse velocity, is likely to increase the volume of flow materially. This probably should not be definitely accepted without proof. The statements in regard to the retention of air seem reasonable, and the spiral flow probably does tend to hold a part of the entrained air in the stream. However, it also promotes a rapid intermingling of particles, and tends to give a more nearly uniform axial velocity at any cross-section. A wide shallow channel would show a poor velocity distribution, which might more than offset the advantage resulting from the ready release of air. These spirals, or other equivalent trans-axial motions, are necessary for the temporary storage of the non-usable kinetic energy of the initial transverse motion,

* "The Spillways of Burrinjuck Dam," by Henry Harvey Dare, *Minutes of Proceedings*, Inst. C. E., Vol. CCXIV, Session 1921-22, Part II.

† *The Illustrated London News* of July 18, 1925, contains photographs of this flood.

‡ Engr., Bureau of Reclamation, Denver, Colo.

while it is being converted into heat. In a compact channel it is probable that the spiral action is more uniformly distributed to all the water particles, and is less violent. Under normal operating conditions there appears to be no tendency for the water to surge up the back wall of the channel.

The question raised is important and should be carefully considered in all future experimentation. Special attention should be given to "velocity swell" in the proposed laboratory tests in pipes. The fact that the mathematical treatment applies to the average velocity, and not to the mean velocity, as ordinarily computed, must be kept constantly in mind.

Mr. Wiley also advocates a wide shallow channel, for mechanical reasons. A deep gash through the foundation adjacent to the end of the dam is without question objectionable. With a deep channel the structure supporting the spillway crest, which separates the channel from the reservoir, assumes formidable proportions, and an elaborate drainage system is required to avoid destruction of the channel lining by uplift. Some advantage may also result from the lower velocities that prevail in a wide channel. In addition a wide thin sheet of water is likely to be less destructive at the point of final discharge than a concentrated jet. Considered from this point of view, the channel should always be made as wide as conditions permit. However, if the hillside is steep the cost of a wide channel is likely to exceed greatly the cost of the protective measures necessary to make a deep channel safe. The cost of a wide shallow channel at a location, such as that existing at the Tieton Dam, on the Yakima Project, Washington, for example, would be prohibitive.

Apparently the method used by Mr. Wiley for computing the hydraulics of the channel will give results essentially identical with those obtained by the writer.

The partial list of side channel spillways given by Mr. Gutmann is interesting. The Don Pedro Dam, in California, the Tieton Dam, in Washington, and the McKay Dam, in Oregon, are notable additions to the list.

Mr. Houk and Mr. Howell both offer alternative plans designed to escape the losses involved in the side channel spillway. Mr. Howell would use a deep, narrow, spillway notch, directed down stream, and controlled by a large Stoney gate. Mr. Howell's opinion that this is in all cases more economical than a side channel type is doubtless justified, and the advantages of a Stoney gate installation should be investigated wherever that type of control is considered adequate. The writer would not consider a Stoney gate spillway safe for the condition of a high earth dam in a heavily timbered region like that surrounding the Tieton Reservoir, for example.

Mr. Houk's suggestion that under favorable circumstances an advantage may be gained by curving the channel is believed to be sound. However, curvature of the main channel will not necessarily avoid impact loss, but by providing a means for reaction against the earth may make it possible to utilize a part of the energy of the transverse velocity.

It is possible that confusion sometimes results from the usual and convenient practice of speaking and thinking of eddies as causing losses of head. Under certain conditions it is impossible to maintain a balance between

kinetic energy and static head without violating either the laws of motion or the law of conservation of energy. The situation is met by converting a part of the kinetic energy into heat. This transfer can not occur instantly, and the condemned kinetic energy is stored temporarily in transverse motions which disappear as the water "warms up." The eddies are thus the result rather than the cause of the loss of head.

Mr. Stevens, near the end of his discussion, makes the following statement:

"If the 'dead' water, or eddies, in sudden pipe enlargement or in a right-angle bend, can act as a diffuser then the eddies in a side channel spillway can perform the same function and all the energy of impact is not lost as the author assumes."

There are many secondary hydraulic problems in connection with side channel spillways which have not been solved, and for which no data are available. As pointed out in the paper, and in several of the discussions, there are many factors that require further study. However, there is no question as to the existence and approximate magnitude of the impact losses. To question the fact that all the energy of impact is lost is equivalent to questioning the validity of Newton's laws of motion. The mathematical application of the laws of motion to the particles composing a stream of water is simple, but a physical conception of the resultant reactions is difficult. Perhaps some simple illustrative device will be discovered before the study of impact losses is finally dismissed.

Mr. Stevens' statement that his Equation (34) gives the eddy loss in a sudden enlargement, is not self-evident, and, so far as the writer knows, has never been rigidly proven. The so-called proofs usually given involve the assumption, or its equivalent, that the average unit pressure on the shoulder of the enlargement is equal to the unit pressure in the smaller pipe. The fact that under certain conditions the observed and computed losses do not agree is in all probability due to the inaccuracy of this assumption, rather than to any "diffusion" produced by "dead" water.

In discussing the testing of impact losses in pipes, Mr. Stevens proposes to combine the impact and sudden enlargement problems. This is believed to be undesirable. Either problem alone is sufficiently difficult. His Equation (27), and all others dependent thereon, are based on the assumption that the pressure against the shoulder of the enlargement is equal to the pressure in the incoming pipes, the strict correctness of which is not apparent. It is believed that the enlargement should be eliminated from the tests, unless it is desired to extend the study to both these effects, which are distinct. It is believed that the tests at Berkeley, Calif., mentioned by Mr. Stevens, contemplate a main line pipe of uniform diameter.

The interest shown in this paper is gratifying. It is hoped that other discussions of the problem will be prepared for publication after the results of laboratory tests now under way are made public.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1588

PERMISSIBLE CANAL VELOCITIES*

By Samuel Fortier† and Fred C. Scobey,‡ Members, Am. Soc. C. E.

With Discussion by Messrs. H. B. Muckleston, Elbert M. Chandler, Robert S. Stockton, R. A. Hart, R. A. Hill, Jerome H. Fertig, E. S. Lindley, Andrew Weiss, J. C. Stevens, Carl Rohwer, Ivan E. Houk, and Samuel Fortier and Fred C. Scobey.

Synopsis

The use of relative high velocities reduces the cost of canal per unit length. The limit of velocities permissible is somewhat less than velocities that will erode a canal bed. The determination of permissible velocities is not possible from data on transporting velocities or mere non-silting velocities. The presence or absence of colloidal matter is of prime importance. The opinions of irrigation engineers as determined from their experience are given, as well as deductions from available data and the final recommendations of the Special Committee on Irrigation Hydraulics.

Definitions.—Limiting velocity is taken to mean the maximum permissible value of the mean velocity. Although it is obvious that for any water prism the highest velocities are much in excess of the mean velocity, yet the only figure sufficiently tangible, under all conditions, for design and operation is the mean velocity.

Note.—This paper constitutes the Final Report of the Special Committee on Irrigation Hydraulics on this subject, reference to which is made in the Progress Report of that Committee for 1924, *Proceedings,* Am. Soc. C. E., March, 1925, Society Affairs, p. 137.

* Published in *Proceedings,* Am. Soc. C. E., September, 1925.

† Senior Irrig. Engr., Div. of Agricultural Eng., Bureau of Public Roads, U. S. Dept. of Agriculture, Berkeley, Calif.

‡ Senior Irrig. Engr., U. S. Dept. of Agriculture, Berkeley, Calif.

Silt is not considered as a specific gradation in soil sizes, but as a general term for all material brought into a canal by "muddy" waters. It is usually thought of, even in this sense, as different from sands and gravels. In other words, the "bottom load" is not included in the term as herein used.

Velocity and Canal Length.—As the discharge is the product of area and mean velocity, it follows that the area of the water prism, and, therefore, the approximate area of the channel, must vary inversely with the mean velocity. As the cost of construction per unit length depends on the area of the channel, it is usually desirable to make the mean velocity as high as the material will withstand. The cost of construction also varies with the length of the channel. By increasing the grade of the channel the velocity of water therein is increased and the water prism diminished, but the distance between the irrigable land and the diversion point is increased. If the point of diversion is fixed, a higher diversion dam will be required in order to command the same area of land. This phase of the question requires a comparative study for each location, whereas the purpose of this paper is to indicate maximum feasible mean velocities.

Non-Silting Velocities not Necessarily Scouring Velocities.—The subject of limiting velocities for open channels has usually been treated under a heading combining the silting and scouring of channels. It has been considered that for each depth of water there is a certain velocity below which silt will be deposited and above which silt will be eroded from the bed of the channel. In the opinion of the writers, two phenomena only slightly related are thus confused. The power of flowing water to maintain a movement of separate soil, sand, or gravel particles, has been confused with the power to break the bedding of a canal bottom and ravel off particles of that bed and transport them to places of lesser velocities where they may be deposited. In their opinion there is no sharp line of demarcation between the velocities that can no longer maintain silt in movement and those that will scour a canal bed. It is believed that there is a broad belt of velocities between these two "critical" velocities, within which silt already loosened or brought in through a head-gate will remain in suspension while the bed nevertheless will remain undisturbed as regards scour. It is easy to show the absurdity of accepting the laws of silting as giving the immediate answer to the laws of scour.

Kennedy's Formula.—Certain of the canals of India, served by heavily silt-laden streams, were known to have reached a condition of permanent channel régime, neither silting nor scouring. In 1895, Mr. R. G. Kennedy offered his well-known formula*:

$$V_0 = C\, d^{0.64}$$

in which, V_0 represents the "critical" mean velocity, in feet per second, of a water prism, d ft. in depth, and C is a coefficient. This so-called critical velocity is that for which there was no silting or scouring of the channel. The value of C is usually quoted as 0.84, but this coefficient was applicable for

* *Minutes of Proceedings*, Inst. C. E., Vol. CXIX (1895).

fine sand silt only; other coefficients for heavier materials range up to 1.09 for coarse silt or detritus from hard soils. Using the coefficient, 0.84, Table 1 may be developed.

TABLE 1.

Depth, in feet.	1.	2.	3.	4.	5.	6.	7.	8.	9.
Velocity, in feet per second.	0.84	1.30	1.70	2.04	2.35	2.64	2.92	3.18	3.43

A glance at Table 1 shows the fallacy of accepting these velocities as the lower limit of scouring velocities. Mean velocities of 2.5 to 3 ft. per sec. for general conditions are considered as most desirable in irrigation canals of the United States, yet Table 1 would indicate that a canal 6 ft. deep is the shallowest which would not show scour at the desirable velocities mentioned in that table. Yet even the velocities as tabulated are further modified, according to Bellasis,* who quotes rules attributed to Kennedy to the effect that:

"Near the hills [Northern India] where the bed is of shingle the velocity may exceed V_0. A few other soils will stand 1.1 V_0. In ordinary channels any excess over V_0 will give much trouble lower down. * * * If soil is very poor, especially if the depth of water is more than 6 or 7 ft., the velocity should be less than V_0, say, 0.9 V_0, so as not to cause falling in of the banks * * *."

A comparison between the figures in this quotation and the values of V_0 in Table 1 shows that the velocities resulting from such procedure would not be acceptable to engineers designing and operating canals in the United States. They would be considered much too low.

Dubuat and Gilbert on Transporting Velocities.—Reference to the earlier writings of accepted American authorities shows that they applied the findings of Dubuat to the determination of the permissible velocities for ditches and canals. These velocities are usually less than one-half the velocities now accepted by irrigation engineers. Dubuat made certain determinations of the transporting power of water. In the words of Gilbert "what he really investigated was chiefly competence for flume traction." The "competent bed velocity" as ascertained by Dubuat for various materials is given by Gilbert† as shown in Table 2.

TABLE 2.

	Velocity, in feet per second.
Potter's clay	0.27 to 0.35
Coarse angular sand	0.7 " 1.1
River gravel :	
Size of anise seed	0.35 " 0.53
Size of peas	0.62 " 0.7
Size of common beans	1.1 " 1.55
Rounded pebbles, 1 in. in diameter	2.1 " 3.2
Angular flints, size of hens' eggs	3.2 " 4.0

* "Irrigation Works", 1913, New York and Lond., p. 48.

† "The Transportation of Débris by Running Water", by G. K. Gilbert, *Professional Paper No. 86*, U. S. Geological Survey, Wash., 1914, p. 216.

Although the velocities along the bed of a flume, such as that used by Dubuat, would be less than the mean velocities, still the difference would not be sufficient to account for the great difference between the figures of Table 2 and those given by various authorities as determined from actual practice.

Material Easily Transported Is Difficult to Scour.—Additional evidence against confusing the movement of particles by flowing water with the scouring action of that same water is contained in the following: The finer the soil particles, the slower may be the velocities that will maintain the particles in suspension. In actual practice it is found, as noted in the suggestions given hereafter, that canal beds in stiff clays, classed by soil experts as being composed of particles of 0.005 mm., or less, in diameter, will withstand mean velocities approximating 4 ft. per sec., yet clear sand coarser than granulated sugar will be scoured at velocities of about one-half that, or 2 ft. per sec.

Colloidal Matter.—Some alluvial silts or loams will erode quite easily, when a canal is new, whereas some, of apparently the same size gradation, will not. The difference lies mostly in the presence or absence of effective colloidal matter. Colloids, as applied to soils, give the properties of plasticity, cohesion, toughness when wet, and hardness when dry, that are essential to an erosion-resisting soil. This point is so important that the writers quote extensively from that well-known authority on Western soils, Hilgard:*

"'Colloidal' Clay.—In connection with soils, clay may be defined, in the most general terms, as being the substance which imparts plasticity and adhesiveness to soils when wetted and kneaded, and which, when heated to redness, loses this property completely and permanently, becoming hard and coherent in proportion to the degree of heat to which it is exposed.

"In common life, however, the name is applied to the whole of any naturally occurring earth which on wetting and kneading assumes a reasonable degree of plasticity and adhesiveness. When the latter property becomes nearly or quite insensible, the earth is designated as a 'loam', more or less 'clayey' according to the amount of the pure, plastic and adhesive material associated with the mineral powders and sand that form the bulk of most soils. * * *

"It has of late been attempted to extend the meaning of this word (plasticity) to the behavior of all powders when wetted with water. But the adhesive plasticity of clay stands almost alone, in that (aside from contraction) it preserves in drying the form into which it may have been molded while wet, even when struck, whereas other powdery substances similarly treated at once collapse back into the original powder. The exclusive use of clay in modeling offers the typical example of plasticity as generally understood. The addition of any powdery substance, however fine, diminishes the plasticity of clay. * * *

"Causes of Plasticity.—In any case the property of plasticity and adhesiveness is restricted to the particles so fine that they fail to settle, in the course of 24 hours, through a column of pure water 8 in. high, while some are so extremely minute that they will not settle for many months, and even for several years. Such turbid 'clay water' may sometimes be found existing in nature, in moist, secluded places, for weeks after the subsidence of the

* "Soils," by E. W. Hilgard, N. Y., 1906, pp. 59 *et seq.*

overflows of rivers whose water is exceptionally free from dissolved mineral matter. * * *

"When separated from the water and dried, the jelly-like substance ('colloidal clay') shrinks as extravagantly as would so much boiled starch, into hard, shiny crusts or flakes, which when struck in mass are sometimes even resonant, and bear more resemblance to glue than to the clay of everyday life. Like glue, too, but much more quickly and tenaciously, the dried colloidal clay adheres to the tongue, so as to render the separation painful; when wetted it quickly bulges with great energy, and in a short time resumes its former jelly-like condition. When moistened with less water it assumes a highly plastic and adhesive condition, so that it is difficult to handle and almost as sure to soil the operator's hands as so much pitch."

With this understanding of soil colloids, it is easily seen that great changes may take place in a canal bed after a period of use under the influences of various types of waters. This explains the thought expressed in some of the answers to questionnaires, that old, well-seasoned canals will stand much higher velocities than new ones.

Effective Colloids.—The presence in the water of an electrolyte, such as calcium or carbon dioxide, will flocculate the colloids and partly or completely annul the property of creating cohesion. Therefore, effective colloids are those free from the influence of an electrolyte.

Types of Bedding.—The property of a canal bottom to withstand scour is due to beds of different types, of which the following are examples:

1.—Very fine silty or clayey particles so puddled together that there is distinct cohesion present; a homogeneous mass—the more colloidal matter present, the greater will be the cohesion.

2.—Graded materials, the finer filling the voids of the coarser; ranging from the finest silts to coarse gravel. Such a bed becomes very firmly compacted and will withstand velocities that would erode a bedding composed of any of the constituent particles alone.

3.—Non-cohesive particles, without grading as to sizes. The unit particles may be of "sugar" sand or of cobble-stone size; if free from colloidal silt or graded sizes a canal bed in such materials may come nearest to following what Gilbert terms the laws of "competent velocities."

Original Channel Material Usually Altered by Age.—A study of these types shows that the particular one to which any given canal will belong may or may not be determined at the time of construction. The type may not be determined by the materials excavated for the canal. It is finally determined only after years of use. Two examples will make this clear. The first is indicative of changes caused by the character of the water diverted from the river. The second example will develop under clear mountain waters carrying little or no silt, and is due to the character of the material through which the canals are excavated.

In the Imperial Valley of California many of the large laterals were excavated in a light sandy soil that would have scoured when the canals were new at a velocity of 2½ ft. per sec. The extremely fine silt particles brought in from the Colorado River prior to 1910 compacted and puddled

this sand and plastered it with a coat of slick colloidal mud, so that it would withstand velocities in excess of 5 ft. per sec.; yet the individual particles of this slick mud were so fine that they would remain in suspension at very low velocities, except when the water was "overloaded" with silt—in times of flood on the Colorado River in June and July. All experienced canal operators know the trick of holding muddy water above one check structure after another until the mud has painted over the sides and bottom of a new canal, reducing seepage losses and making the bed of the canal less susceptible to scour.

Many of the mountain valley canals of Colorado, Utah, and other Rocky Mountain States were constructed in loamy gravel soils. The present bottoms of these canals show little indication of loam. At the extreme edges, grass, roots, and low velocities combined have built up vertical banks of fine material, but the bottoms are of well-graded gravel, ranging from pea size to cobble-stones, but all so well compacted and fitted together that a high velocity is feasible, far above that which originally washed out all the loam particles until a gravel paving resulted.

Effect of Velocities on Plant Growth.—The problem of plant growth is an important one in the operation of irrigation canals and deserves special treatment. It is, however, influenced by factors other than canal velocities. More particularly these factors comprise water temperatures and turbidity and character of soil.

Few aquatic plants thrive in the absence of sunlight, therefore, canals carrying turbid waters are seldom bothered with plant growth. In clear-water channels some species thrive in velocities much greater than the channel soil will stand. Other species grow only in sluggish velocities. If, therefore, the highest permissible canal velocities as determined from the standpoint of erosion are provided, all that is possible will have been done to prevent trouble by plant growth as far as fixing velocities are concerned.

Difficulty of Securing Specific Data.—Thus far in this discussion the writers have endeavored to establish the fact that the determination of eroding velocities is a matter of empirical observation on erosion, not on silt deposition, to the time when a particle is broken off from the bed of the canal, the cohesion due to colloidal matter being overcome or the mechanical interkeying of various elements being disturbed. After that takes place, the data of Dubuat, Kennedy, Gilbert, and others may apply. The actual determination of these empirical data has never been attempted, as far as the writers are aware. That is, it has not been considered feasible to increase slowly the mean velocity in a canal of a definite material and thus determine the actual mean velocity at which scouring commenced. The only data available are the facts that certain canals have scoured under relatively high velocities. The magnitude of these high velocities is known from measurements of the canals, but the exact high velocity that started the scour has never been determined. Undoubtedly, a higher velocity is required to start scour than to continue it when once started. Likewise, it is known that certain canals do not scour even if the mean velocity is two to three times

those velocities suggested by Dubuat or Kennedy. It still remains to be determined how much higher a velocity this particular canal would stand before scour commenced.

Hence, at present, the best knowledge of scouring velocities comes from personal deductions as to the performance of individual canals and not from direct experimental work. With this understanding in mind, the Society's Special Committee on Irrigation Hydraulics submitted questionnaires to a number of irrigation engineers whose experience qualified them to form authoritative estimates of the maximum mean velocities allowable in canals of various materials. The questionnaire, as submitted, follows:

"Questionnaire on Permissible Canal Velocities

"This list of questions is submitted by the Special Committee on Irrigation Hydraulics of the American Society of Civil Engineers in order to ascertain the results of experience as to the permissible mean canal velocities in various types of soil under normal operating conditions.

"The Committee will appreciate the results of your experience and observation on this subject.

"1. Name?

"2. To what irrigation system or systems do your answers apply?

"3. What in your opinion is the maximum mean canal velocity permissible without eroding each type of soil according to the following classification:

	Ft. per sec.	Ft. per sec.
Alluvial silts		
Fine loam		
Fine sandy loam		
Fine sand		
Volcanic ash		
Coarse sand		
Fine gravel		
Coarse gravel		
Stiff clays		
Shales		

"4. If the soils with which you have had experience do not fall plainly in the above classification can you give such permissible velocities with a description of the soils to which they apply?

"5. Do you find that canal velocity has any marked effect on the growth of mosses and aquatic plants and, through them, on erosion?

"6. If you have made actual determinations of highest non-scouring velocities with mechanical analysis of soils, or if you have made an investigation of aquatic plant growth as affected by velocity of water the Committee will appreciate a copy of the results or reference to where they may be found, if published."

Opinions of Irrigation Engineers.—Constructive replies and suggestions were received from the following list of engineers, to whom the Committee acknowledges its indebtedness:

Herbert D. Newell, Assoc. M. Am. Soc. C. E., Superintendent, Klamath Project, U. S. Bureau of Reclamation, Klamath Falls, Ore.

C. N. Perry, M. Am. Soc. C. E., Consulting Engineer, Los Angeles, Calif.; formerly Chief Engineer, Imperial Irrigation District, California.

Fred D. Pyle, M. Am. Soc. C. E., Engineer Manager, Vista Irrigation District, Vista, Calif.; formerly Irrigation Engineer, Imperial Irrigation District; Irrigation Manager, North Platte Project; Project Manager, Uncompahgre Project of U. S. Bureau of Reclamation; and General Manager, Columbia Irrigation District, Washington.

J. L. Lytel, M. Am. Soc. C. E., Superintendent, Yakima Project, U. S. Bureau of Reclamation, Yakima, Wash.

W. H. Code, M. Am. Soc. C. E., Consulting Engineer, Los Angeles, Calif.

Jerome H. Fertig, M. Am. Soc. C. E., Los Angeles, Calif.; formerly Construction Engineer, Bitter Root Irrigation District, Montana; Construction Engineer, Uncompahgre Project, U. S. Bureau of Reclamation; and, later, Construction Engineer, Ochoco Irrigation District, Prineville, Ore.; Project Engineer, Grants Pass Irrigation District, Grants Pass, Ore.; and Engineer in charge of reconstruction of project, Bitter Root Irrigation District, Hamilton, Mont.

C. E. Atwood, Assoc. M. Am. Soc. C. E., Chief Engineer, The Valier Montana Land and Water Company, Valier, Mont.; formerly Chief Engineer, Valier Carey Act Irrigation Project, Valier, Mont.

M. E. Bunger, Engineer for various companies in Colorado; formerly Hydrographer and Deputy State Engineer, Colorado.

Charles Taylor, Hydrographer, Umatilla Project, U. S. Bureau of Reclamation.

C. C. Williams, Engineer, Wenatchee Reclamation District, Washington.

From these replies Table 3 has been compiled. Much of the value of the comments of the authorities noted would be lost if Table 3 alone were given, hence quotations and excerpts are also appended.

TABLE 3.—PERMISSIBLE VELOCITIES, IN FEET PER SECOND, ON TANGENTS AS SUGGESTED BY IRRIGATION ENGINEERS IN ANSWERS TO QUESTIONNAIRES.*

Material.	C. N. Perry.	F. D. Pyle.	J. L. Lytel.	J. H. Fertig.	C. E. Atwood.	M. E. Bunger.	C. C. Williams.
Alluvial silt	2¼-2¾	4.00	....	2.00	....	2.00	....
Fine loam	.....	2.75	2.00	2.00	1.80	2.50	....
Fine sandy loam	.....	3.00	2.00	1.50	2.50	2.50	2.00
Fine sand	.....	2.50	2.00	1.50	....	2.50	2.00–
Volcanic ash	.....	3.00	2.50	2.00	....	....	2.75
Coarse sand	.....	2.75	3.00	2.50	....	3.00	2.00
Fine gravel	.....	3.50	4.00	2.80	....	3.00	3.50
Coarse gravel	.....	5.00	6.00	3-5	4.00	....	....
Stiff clays	4½-5	3.75	.....	2.50	3.00	4.00	....
Shales	.....	6.00	7.00	2½-4	....	3-6	....
Materials not listed in questionnaire	.....	....	....	....	....	....	....
Ordinary sandy loam	.....	....	....	....	....	....	2.75
Heavy clay loam	.....	....	....	....	....	....	3.00

* A few of these engineers suggested velocities for curves ranging from 0.5 ft. to 1 ft. slower. It would not be practicable to change from one velocity to another as the canal went into and out of curves. It would be better to use one value for straight canals and a slightly lower one for sinuous ones.

Mr. Newell felt that the experience gained on the Klamath Project did not warrant an estimate of maximum velocities allowable, but submitted answers to questions in the questionnaire, as follows:

3. "There has been no trouble from erosion on any of the canals of the Project. Generally the velocities here used are comparatively low. The velocities listed are taken from the highest current-meter measurements of 1924.

4. "The soils encountered in canals of this Project are fine sand and a chalk that is intermediate between a stiff clay and a shale. Velocities as given below were in fine sand:

Canal.	Quantity, in second-feet.	Velocity, in feet per second.	Soil.
A	750	1.80	Fine sand
B	115	1.26	" "
C	531	1.35	" "
D	155	1.98	" "
E	18.6	1.10	" "
F	31.3	0.95	" "
C–G	249	1.03	" "
Div. Chan.	305	1.67	" "
C-4	92.3	1.62	" "

Mr. Pyle offers particularly valuable suggestions in the following:

"The velocities given are average only and are not proposed as proper to meet the various soil conditions without careful study. In general, an excess of grade controlled by checks is preferable to a flat grade, as the condition can be remedied in the first instance and cannot be in the second.

"Some soils are very light, mushy and easily eroded when new canals are being used for the first time and will set and toughen after several years' use so as to stand 50% more velocity than they would the first year. In fact, very few canals will stand velocities the first few years that they will after 10 years' use.

"Other soils such as volcanic ash and clays will erode gradually but almost constantly forming large holes in the bottom which create boils in the water. I filled a number of these while with the Columbia Irrigation District with sandy loam covered with about 3 in. of fine gravel.

"The nature of the silts carried by the water affects the erosion as some fine silts penetrate and toughen the bottom and sides of the canal, while some coarse silts tend to start erosion.

"The amount of silt carried affects the erosion as on the Imperial Irrigation District canals, where very little erosion takes place at times due to excess of silt which prevents the water from picking up more, while at other times as the water clears it will pick up silt previously deposited and commence eroding.

"In many instances a canal will run satisfactorily for a number of years without eroding, when something will occur to break the natural lining of the canal with the result that swirls and boils are formed which increase the erosion until considerable damage may result.

"Generally erosion commences on the outside of curves, around obstructions or where the current is deflected against the banks. In the last instance we are not always able to decide just why the current changes from one point to another. Back currents are very dangerous when once started and should be controlled as soon as possible."

Mr. Lytel submits a table showing the highest velocities found on some of the canals on the Yakima Project and brings out again the point that these may not be the limiting velocities. These figures are given in Table 5. He

then submits an estimate of limiting velocities included in Table 3, with the following explanatory remarks:

"We find that the velocities given afford very satisfactory service, but may be increased or decreased very considerably without damage to the banks or the depositing of silt. The Yakima River is comparatively free from silt, so a great deal of attention has not been given to the silt problem in connection with the designing of canal sections.

"The figures shown in Table 5 give the velocities at gauging stations on a number of the canals along the Yakima River to which we deliver water in bulk.

"All of the stations shown in Table 5 are on tangents. The velocities given are those which are found at our regular gauging stations on the canals and at the points given. Greater velocities in several cases would probably be possible without injury to the canal section. Lesser velocities in most of the cases would be possible without silting. We have no information as to the velocities on curves, since all our gauging stations are located on tangents. In any study of this kind it is practically certain that special gaugings are necessary. Our hydrographic work has been necessarily confined to the stream measurement work necessary for the establishment and maintenance of the various gauging stations.

"On account of the erosion of the canal banks in the main Sunnyside Canal, we have some silt in the water in the lower part of the system, but not in sufficient quantity to give trouble in handling the water or increase the expense of maintenance to any extent.

"On account of the lack of silt in the water of the Yakima River, this valley is not a very good place to secure information on velocities in canals built in different material with reference to the carrying or depositing of silt, but I thought such information as our records contain might be of some use."

Mr. Code is quoted as follows:

"The writer's experience in operating canals from the Salt River for a period of 10 years prior to the building of the Roosevelt and Granite Reef Dams taught him that with mean velocities materially under 2.5 ft. per sec. there would be deposits of silt on both sides and bottom of main canals and laterals, and all canals and main laterals built under his direction during this period were designed as far as practicable with hydraulic functions insuring approximately such maximum velocities during flood seasons. At other times there was little silt carried in the water and when the low water period occurred, our canals would be running small heads of clear water."

Mr. Atwood makes plain that the velocities given opposite his name in Table 3 are the highest found and may not be the highest attainable:

"The velocities given have been taken from our measuring stations where no erosion occurs and where the canals have given very satisfactory results from an operation and maintenance standpoint. We have not made a study of the maximum permissible velocities."

Pertinent remarks from the answers of Mr. Bunger are:

"If the water running through fine loam contains material in suspension that tends to deposit there will be no erosion until such a velocity is reached that no deposition will take place and I know of instances where that velocity was 3 ft. per sec. with no erosion.

"I have found on the inlet ditch to the Model Reservoir near Trinidad, Colo., where the water carries silt in suspension, that erosions take place in shale with a velocity of 6 ft. per sec. while in the outlet ditch where the water is clean no erosion takes place in the same character of shale with a velocity of 6 ft. per sec.

"The velocities I have given for the above soils are therefore based on a moderately clear water.

"I have made no actual determination of highest non-scouring velocities and soil analysis with a view of publishing them, but as Hydrographer and Deputy State Engineer in the State Engineer's office at Denver, Colo., I have rated hundreds of ditches located in all kinds of soils and know the velocities that cause some of them to scour and in others that do not, and as engineer and manager for different ditch systems, I have had occasion to measure velocities of water in various soils and shales and know at what velocities erosion takes place in them."

Mr. Taylor is quoted as follows:

"Fine to medium sand: The highest velocities in use through canals in this type of soil, shown by water measurements, are mean velocity, 1.7 ft. per sec. for Q equal to 45 sec-ft., and a mean, V, on tangents. The physical condition of the original soil was 60% passed through screen 74 mesh to the inch.

"At the above velocities erosion will show in places where submerged débris or weeds have lodged.

"Without having made actual tests, I think the above are the highest permissible mean velocities for the above type of soil.

"The coarse sand on the Umatilla Project through which some of the original canals were built, was so porous, and the losses were so great, they had to be lined. Some of this sand passed only 40% through screen 74 mesh to the inch."

Mr. Williams gives his experience with Wenatchee soils:

"The soil encountered on this project falls mainly into two general classifications: (*a*) heavy clay loam, and (b) sandy loam. The experience here has been that the first classification will stand velocity up to 3.0 ft. per sec., without serious erosion, and that the second classification erodes badly on curves at velocity of 2.0 ft. per sec. In stiff clays, coarse gravel, and shale, we have had no erosion. With very fine loam and silt, we have erosion to a considerable extent with velocity of 2 ft. per sec. The velocities on the project in unlined canal range from 2.0 to 3.0 and above observations are based solely on experience within these limits."

In addition to answers to the questionnaires, much excellent material is found in the writings of recognized authorities on irrigation engineering.

From the first great book on irrigation engineering by an American author, the late P. J. Flynn, M. Am. Soc. C. E.,* the writers quote at length (note his comment as to Dubuat and compare with the statements of the writers in the early paragraphs of this paper):

"If this slope is too great, the bed of the canal will be torn up, and the foundation of all bridges, drops and other works, will be endangered. The canal bed will be cut down and retrogression of levels take place, until the velocity of the water has adjusted itself to the cohesion of the material through which it flows. * * *

"The maximum mean velocity is not, however, so easily fixed. It must, in the first place, vary with the nature of the soil of the bed. A stony bed will stand a very considerable velocity, while a sandy bed will be disturbed if the velocity exceeds 3 ft. per sec. Some gravel beds will bear a high velocity. Good loam with not too much sand will bear a velocity of 4 ft. per sec. * * *

* "Irrigation Canals and Other Irrigation Works," by P. J. Flynn, San Francisco, 1892, p. 32.

"In computing the slope for the Ganges Canal, Sir Proby Cantley used the formula of Dubuat. This formula was often used at this time, but is now known to be unreliable, especially for large canals.

"It is better to give too great than too small a velocity, as, in the former case, measures can be adopted to protect the side slopes, or falls can be made in the canal and the longitudinal slope, and, therefore, the velocity reduced. In the latter case the deposition of silt will necessitate an annual clearance of the canal, at great expense, and the loss of ground along the canal banks on which to deposit the spoil."

In his chapter on maximum and minimum velocities, Etcheverry* recognizes the fact that the velocity necessary to start erosion is much higher than that required to maintain loosened particles in motion. After quoting other authorities he gives a tabulation (reproduced as Table 4) of suggested maximum mean velocities that does not permit of a direct place in Table 3, as the categories are different.

In speaking of farm ditches in particular (hence, those of shallow depth), Fortier† says:

"In fine sand or sediment a flat grade is required to prevent scouring. A mean velocity of 1 ft. per sec. is sufficient for such material. In hard gravel or hard clay or in a mixture of these, a velocity of 3 ft. per sec. can be used without eroding the bottom. In ordinary materials, ranging from sandy or gravelly loams to clay loams, a grade may safely be adopted which will produce a mean velocity of 2 to 2½ ft. per sec."

TABLE 4.—Maximum Mean Velocities Safe Against Erosion.

(By Etcheverry.)

Material.	Mean velocity, in feet per second.
Very light pure sand of quicksand character	0.75– 1.00
Very light loose sand	1.00– 1.50
Coarse sand or light sandy soil	1.50– 2.00
Average sandy soil	2.00– 2.50
Sandy loam	2.50– 2.75
Average loam, alluvial soil, volcanic ash soil	2.75– 3.00
Firm loam, clay loam	3.00– 3.75
Stiff clay soil, ordinary gravel soil	4.00– 5.00
Coarse gravel, cobbles, shingles	5.00– 6.00
Conglomerates, cemented gravel, soft slate, tough hard-pan, soft sedimentary rock	6.00– 8.00
Hard rock	10.00–15.00
Concrete	15.00–20.00

In order to show examples of relatively high velocities in irrigation channels, Table 5 has been compiled.

Deduction from Available Data.—After a careful study of all the data presented and in the absence of experimental data bearing on the subject, the Committee recommends the values given in Table 6 for maximum permissible mean velocities. The figures given are for canals with long tangents

* "Irrigation Practice and Engineering," Vol. II, "The Conveyance of Water," by B. A. Etcheverry, M. Am. Soc. C. E., New York, 1916, p. 57.

† "Use of Water in Irrigation," by Samuel Fortier, M. Am. Soc. C. E., New York, Second Edition, 1916, p. 33.

TABLE 5.—EXAMPLES OF CANALS CONVEYING WATER AT RELATIVELY HIGH VELOCITIES.

Authority.	Canal.	Location	Soil.	Flow, in second-feet.	Depth, in feet.	Mean velocity, in feet per second.	Remarks.
P. J. Flynn	Cavour	Italy				5	Over gravel bed.
"	Naviglio	"					
"	Grande	"				5 to 6	
"	Martesana	"				5 to 6	
"	Crappone	France				6.6	Istres Branch.
"	Baree Doab	India		3 000		5.0	Over gravel bed.
"	Del Norte	Colorado		2 400	5.5	5+	Over gravel drift and rock.
"	Eastern Jumna	India				1.93	Perfectly adjusted to a light, sandy soil.
"	" " "	"				2.28	Silt being deposited.
"	" " "	"	Clay, shingle and small boulders			5.47	
"	Ganges Canal	India	Sandy with admixture of clay			2.92	Bed silted.
"	" "	"	In stiff clay			4.12	Erosion trifling; no silting.
"	" "	"	In very light sand			2.50	
S. Fortier*	Middle Creek Canal	Montana				7.5	Bed washed clean of all earth and sediment.
"	Bear River Canal	Utah	Clayey loam	225		3.62	Channel covered coating of sediment, but free from vegetation.
F. C. Scobey†	Interstate Canal	Nebraska	In clay	830		4.75	
"	"	"	"	"		3.86	
"	"	"	"	"		4.66	Bed graded material, fine gravel to 6-in. radius.
"	Salt River Valley	Arizona	Graded gravel	131.3		3.12	Bed scoured clean, sides silted, slick.
"	Grand Canal	"	In firm sand and gravel.	161.7		2.72	
"	Empire Canal	Colorado	In clay loam	372.1		2.94	Now has deposit of clean sand in middle, silty mud at sides.
H. D. Newell‡	D. Klamath Project	Oregon	In fine sand	155		1.98	
Charles Taylor‡	Umatilla Project	"	Fine to medium sand.	260		2.4	60% of original soil passed a 74-mesh screen.
J. L. Lytel‡	Sunnyside Canal	Washington.	Coarse gravel	1 260	5.4	5.25	On tangent.
"	Mabton Canal	"	Fine gravel	120	4.2	2.85	On tangent.
"	Snipes Mt. Canal	"	Fine sand	125	4.2	1.95	On tangent.
"	Rocky Ford Canal	"	Fine sandy loam	100	4.3	1.95	On tangent.
"	Snipes Mt. Canal	"	Volcanic ash	28	2.1	1.85	On tangent.
"	Mills Power Ditch	"	Alluvial silts			1.65	On tangent.
"	Snipes and Allen	"	Fine loam			1.65	On tangent.
"	Lower Richland	"	Fine sandy loam			1.00	On tangent.
"	Moxee Company	"	Volcanic ash			1.50	On tangent.
"	Old Reservation	"	Fine gravel			3.00	On tangent.
"	Yakima River	"	Coarse gravel			6.50	On tangent.
Kutter§	Mill Race	Switzerland	Side slopes, earth; bed, fine gravel			3.22	
"	Gurbeu Canal	"	Regular reach, detritus			4.79	

* *Water Supply Paper No. 43*, U. S. Geological Survey.

† *Bulletin 194*, U. S. Dept. of Agriculture.

‡ By letter.

§ Hering and Trautwine translation.

TABLE 5.—(*Continued.*)

Authority.	Canal.	Location.	Soil.	Flow, in second-feet.	Depth, in feet.	Mean velocity, in feet per second.	Remarks.
Kutter*	Smith Canal	Switzerland	Earth, no detritus	.....	10.8	5.53	
"	Simme Canal	"	Very coarse gravel and detritus	.....	.....	5.99	
"	Escher Canal	"	Very coarse gravel and detritus	.....	.....	8.36	
Fortier†	Hyrum Canal	Utah	.....	45.6	1.6	3.24	
"	Melville Canal (No. 1 Ditch)	"	.....	17.4	1.3	3.34	
"	Paradise I. and R. Co. Canal	"	.....	50.3	1.5	3.13	
"	Hooper Canal	"	.....	129.5	20+	3.90	
"	College and City Ditch	"	.....	7.72	0.5	3.45	
Cone-Trimble-Jones‡	Rio Grande	Colorado	In gravel	143.6	.....	3.86	
Cone-Trimble-Jones‡	" " Lateral 1	"	In graded gravel	38.0	.....	4.66	Bed scoured clean, sides silted, slick.
U. S. Reclamation Service§	.....	North Platte project, Nebraska	Sandy, fair condition	1 154	.....	2.89	Erodes, some brush rip-rap.
U. S. Reclamation Service§	.....	"	Sandy, good condition	1 096	.....	3.09	
U. S. Reclamation Service§	.....	"	Gravelly, fair condition	.....	.....	3.32	
U. S. Reclamation Service§	.....	Boise project	.....	.....	.....	.....	
Don Bark‖	South Side Twin Falls, Main Canal	Idaho	Clay and hard-pan	.....	.....	3.22	In good order, no weeds.
"	South Side Twin Falls, High Line Canal	Snake River, Idaho	Dark uniform clay loam	2 737	7.53	3.43	
" ¶	South Side Twin Falls, Farm Lateral	Snake River, Idaho	Dark uniform clay loam	877	5.31	2.72	
"	Randall Canal	Snake River, Idaho	Medium clay loam	222	.38	2.40	
		Snake River, Idaho	.....	135.7	2.49	3.10	

Hering and Trautwine translation.
Unpublished.
Bulletin 194, Colorado Agricultural College.
§ *Reclamation Record*, July, 1913.
‖ Ninth Biennial Rept., State Engr. of Idaho.
¶ Biennial Rept., State Engr. of Idaho, 1911-12.

predominating throughout their lengths. For the same canals in sinuous alignment a reduction of about 25% is recommended. Likewise, the figures are for depths of 3 ft. or less. For greater depths a mean velocity greater by 0.5 ft. per sec. may be allowed. The velocity in canals carrying water free from muddy silts, but bringing a powerful abrasive from the river, should be reduced 0.5 ft. per sec. Basalt ravelings are probably the sharpest scouring agency carried by any of the Western streams, and many of the streams from the Northwestern lava beds will contain this abrasive although the water may appear to be quite clear. Canals from streams more or less silt-laden through the year, like the Rio Grande or the Colorado River, may be designed for full-load mean velocities from 1 to 2 ft. per sec. greater than would be permissible for the same material or new excavation, served by clear water from a reservoir. Deep canals conveying water with clean sand will develop destructive sand boils which would be absent in the same canal with clear reservoir water. In short, the character of water conveyed must be considered as well as the materials through which the canal is first excavated. In theories of "loading", clear water is best fitted to take on a load of scoured material and a water carrying a partial load of sharp abrasive is better equipped to start erosion.

Explanation of Table 6.—The general basis of Table 6 is a tabulation of original soil materials in their order of ability to withstand erosion under usual conditions; that is, conveying relatively clear water, for the greater part of the season, either from a mountain stream or from a reservoir. In soil gradations finer than gravels, it is recognized that the resistance to erosion depends on the cohesion between individual particles. For the gravels—fine, coarse, and cobbles—the resistance is by virtue of weight, shape, and density of units, aided by the mechanical obstruction afforded one unit by its mixture with others. There is no element of cohesion due to viscosity, plasticity, or stickiness, such as is found in clay. In graded materials, loam to cobbles in size, such as are found on the Piedmont benches and alluvial fans of Colorado, Utah, and other mountain States, the canal bottoms become firmly bedded after a few years. At first, there being little plastic colloidal matter, the finer grained soils are washed free and carried farther down stream, to be deposited on the inside of curves or precipitated where velocities are reduced above check structures. At the same time, the gravel is moving slowly down stream, until finally a bed is developed in which voids are filled by smaller material, much on the order of proportioning concrete aggregates. This graded bed is quite stable, entirely because of mechanical intermixture and not because of plastic cohesion, although as indicated in Column (3) of Table 6, the fact is recognized that the admixture of effective colloids will make the bed all the more tough and tenacious, increasing its resistance to erosion. The shales are taken as smooth, soft rock, resistant to erosion by clear water, but not so resistant when a sharp abrasive, such as sand or gravel, is carried by this water. (Note comment by Mr. Bunger, page 949.)

TABLE 6.—Permissible Canal Velocities.

Original material excavated for canal. (1)	Velocity, in Feet per Second, after Aging, of Canals Carrying: Clear water, no detritus. (2)	Water transporting colloidal silts. (3)	Water transporting non-colloidal silts, sands, gravels, or rock fragments. (4)
Fine sand (non-colloidal)	1.50	2.50	1.50
Sandy loam (" ")	1.75	2.50	2.00
Silt loam (" ")	2.00	3.00	2.00
Alluvial silts when non-colloidal	2.00	3.50	2.00
Ordinary firm loam	2.50	3.50	2.25
Volcanic ash	2.50	3.50	2.00
Fine gravel	2.50	5.00	3.75
Stiff clay (very colloidal)	3.75	5.00	3.00
Graded, loam to cobbles, when non-colloidal	3.75	5.00	5.00
Alluvial silts when colloidal	3.75	5.00	3.00
Graded, silt to cobbles, when colloidal	4.00	5.50	5.00
Coarse gravel (non-colloidal)	4.00	6.00	6.50
Cobbles and shingles	5.00	5.50	6.50
Shales and hard-pans	6.00	6.00	5.00

For each grade of original material shown in Column (1) the fact is recognized in Column (2) that the final condition of a canal bed must be quite largely determined by the material through which it flows and likewise the fact that there is little or no mechanical abrasive in the water. The figures in Column (3) refer to water that contains effective colloidal silts, such as the waters from the Colorado River or Rio Grande. These silts will settle to greater or less extent under relatively high velocities. The difference between colloids and colloidal silts should be noted. The first will remain in suspension, even if all velocity be eliminated, for a long period of time, even years. The silts will precipitate under reduced velocities, and possess sufficient colloidal matter to form a plastic, highly cohesive mass. Obviously, such water will have a marked influence on the final character of the canal bed for all gradations up to the shales where the original material is more resistant than any deposit of silts. In Column (4), the figures indicate that waters conveying abrasive sand or gravel will make some materials more resistant by furnishing the constituents needed for a graded bedding. On the other hand, shales or slick tough clays are themselves resistant, and this resistance is reduced when a powerful abrasive is contained in the water.

Conclusions

1.—The laws of hydraulics governing the movement of loose silt and detritus in open channels are only distantly related to the laws governing the scouring of a canal bed and are not directly applicable.

2.—The material of seasoned canal beds is composed of particles of different sizes and when the interstices of the larger are filled by the smaller, the mass becomes more dense, stable, and less subject to the erosive action of water.

3.—The velocity required to ravel and scour a well bedded canal in any material is much greater than the velocity required to maintain movement of particles of that same material before becoming bedded or that have been raveled off by higher velocities than the bed would stand.

4.—Colloids in either the material of the canal bed or the water conveyed by it, or in both, tend to cement particles of clay, silt, sand, and gravel in such a way as to resist erosive effects.

5.—The grading of material running from fine to coarse coupled with the adhesion between particles brought about by colloids make possible high mean velocities without any appreciable scouring effect.

6.—Irrigation canals may be designed for the velocity that is permissible when seasoned by age, as the demand for water grows with the age of the canal and the maximum mean velocity grows with the supply necessary to satisfy this demand.

7.—Power canals are likely to be placed under peak load as soon as feasible after completion. For this reason a more conservative velocity should be chosen, otherwise scour may take place before seasoning.

8.—Canals, when new, may be operated with velocities less than the maximum permissible, by the use of check structures.

9.—A slight excess of velocity is preferable to insufficient velocity. Check-drop structures will correct a slope causing erosion, but there is no method within reasonable cost by which velocities may be increased.

10.—In the past, engineers have been led astray by too close an adherence to the results of experimental data on the transporting action of water on such materials as clay, silt, sand, and gravel, considered separately rather than on cohesive or mechanical combinations, and in consequence many canals have been designed and built on too flat slopes with correspondingly low velocities.

11.—The growth of aquatic plants is only partly connected with velocities. Canals designed for the highest permissible velocities, from the standpoint of erosion, will be as free from plant growth as design alone can effect.

DISCUSSION

H. B. Muckleston,* M. Am. Soc. C. E. (by letter).—This paper attacks a problem which is in great need of serious consideration. Its principal difficulty lies in the definition of the various soils. Such designations as "sandy with admixture of clay", "clayey loam", "dark uniform clay loam", and others in Tables 5 and 6 may convey varying ideas to different engineers, depending on the soils with which they have had experience. The man familiar with the loams around Winnipeg, Man., Canada, would consider the loams of Alberta as light to the point of being floury, whereas the Alberta man would consider the Winnipeg soil extraordinarily tough. Yet the designation as "dark uniform clay loam" might very well fit either material. This is not in disparagement of the authors' work, but to show the difficulties in the problem they have undertaken to solve.

The engineer who is faced with the necessity of deciding on the upper limit of permissible canal velocity for a project is confronted at the outset with the fact that he knows very little about the soils he will encounter in the excavation. No matter how numerous his test pits or borings, they can reveal little about the real character of the soils in which the canal will be dug. When the canal is entirely in an alluvial river valley there may be some assurance that the soils will be reasonably constant over an extended area, but for a canal in bench lands, particularly glacier-made lands, the character may change materially in a few hundred feet. In the writer's experience, a canal designed for a mean velocity of 2.5 ft. per sec. was badly eroded in some stretches, while in others, possibly in the next quarter-section, silting took place to such a degree as to become a nuisance. The difference in the soils was not especially noticeable on examination, but it must have been considerable in fact.

The authors are quite right in insisting that silting and scouring are phenomena of entirely different natures and following radically different laws. They involve fundamentally different processes. In the one case the force opposing is gravity; in the other, it is cohesion or friction. In well-packed, well-graded gravel, the resisting force is almost entirely friction assisted in some cases by the weight of the larger particles helping to hold down the smaller ones. In clays, or in materials containing a good proportion of clay, the resisting force is mainly cohesion. Before erosion can take place the particle must first be detached from its matrix and this can happen only as a result of unequal pressure on opposing sides. This inequality of pressures may be due to an upward current tending to lift the particle, or to a horizontal current tending to roll it along the bottom. If the surface is smooth and impervious, the opportunity for the disturbing currents to act on small particles is limited and a high mean velocity is permissible without scour. This explains the high resistance offered to scour by nicely slicked bed and banks. It also explains why alkali soils erode so easily.

* Cons. Engr., Vancouver, B. C., Canada.

Almost any material will erode if the water carries in suspension particles of considerable size. These particles by their momentum can detach from the bottom other good-sized particles which would not be detached by the water alone.

In discussing the quotation from the late P. J. Flynn, M. Am. Soc. C. E., the authors have mistaken his meaning in the reference to the formula of Dubuat. Mr. Flynn was referring to Dubuat's formula for the mean velocity of water in open channels and not to his formula for permissible velocity. The former was a "horrible" expression used in India until it was superseded by Kutter's formula.

The velocities recommended in Table 6 will be accepted by most irrigation engineers, subject to their own interpretation of the terminologies.

ELBERT M. CHANDLER,* M. AM. SOC. C. E. (by letter).—The collection of records of experience and observation in this paper will be very useful to canal designers. The paper will undoubtedly serve to call attention to and emphasize the desirability of the use of maximum velocities consistent with the physical conditions to be met in any given case, and particularly that, under normal circumstances, the limiting factor in fixing a canal velocity should be the elimination of erosion of a seasoned canal and not the minimum velocity required to prevent the deposit of silt, as has been the case in many instances.

Of course, each particular situation must be analyzed and the conditions met as they arise. For example, in the case of canals diverting water from the Snake River and carrying a moderate but not harmful quantity of silt, circumstances could easily require, not the maximum or limiting permissible velocity, but some velocity less than the maximum that will permit of a continuous slight deposit of silt, such silt deposits being useful and necessary to reduce seepage losses.

There are certain kinds of aquatic growth that thrive only in water having a very high velocity. The writer has noted the odd situation of no aquatic growth in the canal, but long strings of such growth in an adjacent regulating spillway having a continuous velocity well over 10 ft. per sec.

Another factor to bear in mind is that many circumstances will justify in initial construction a concrete lined canal, not only because of the reduction of seepage losses, but of the decrease in the cost of excavation owing to the use of a smaller sectional area. This possibility should always be studied when considering grades and maximum velocities.

It may not be amiss to call attention to the life work of one of the authors, Dr. Fortier. For more than forty years, he has studied, investigated, and worked to solve the many difficult problems incident to irrigation development. He was the first to emphasize that the work of construction and operation of irrigation works was purely incidental and secondary to the ultimate result—the success of the man on the land. Twenty years ago he was a pioneer in working to improve the methods of irrigation and the opportunities of the

* Civ. Engr., Olympia, Wash.

irrigation farmer. His rôle has not been spectacular, but the actual worth of his services to the irrigated West has been exceeded by none.

ROBERT S. STOCKTON,* M. AM. SOC. C. E. (by letter).—This paper emphasizes certain matters in connection with permissible canal velocities that have come under the observation of experienced operating engineers in this field, but that have not been fully discussed in textbooks. This subject is one into which many factors enter and for which the collection and tabulation of observed field data at a large number of locations as given in the paper are particularly valuable.

It has been noted that a canal carrying water at a velocity near the eroding point on a straight course will actually erode on the curves. It may be desirable to use such a velocity and protect the curves by rip-rap or by superelevation of the bottom to correspond with the increased velocity on the outside of the curve. This latter method is limited in its application but has not been used as much as it should have been.

As a practical matter of economical operation an engineer should be sure that the velocities in small canals and distributing ditches are high enough; on the other hand, in designing large canals he should perhaps lean the other way and be sure the velocity is not too high, if they carry mostly clear water through material where seepage may be an important matter. For very muddy waters, for a low construction cost, and for preventing the growth of water plants, the highest permissible velocity or the highest feasible velocity would be indicated. At many points the soil is in layers and the softest must, more or less, govern the velocity. As pointed out by Fred D. Pyle, M. Am. Soc. C. E. (see page 948), it is usually much easier to remedy a high velocity than to increase a flat grade in a canal. The lighter the soil, the flatter the side slopes must be for stability.

The canals on the Western Section Irrigation Project of the Canadian Pacific Railway, Department of Natural Resources, have been in use from 15 to 20 years. The soil varies from sand to heavy clay, but in most places contains no gravel and might be described as varying between a light sandy loam and a clay loam, all characteristic of the prairie soils east of the Rocky Mountains.

The Main "A" Canal of this System is through an average clay loam soil and is a good example of a large canal in good condition after twenty years' use. The constructed slopes were 1 on 2 and for long distances are in almost perfect condition, except for occasional gullies where storm water has entered from the side. Of course, the section is modified slightly and approaches the parabolic form so pronounced on the canals and ditches in the softer loam soils. The calculated velocity for a depth of 10 ft. and a discharge of 2 048 sec-ft., is 3.2 ft. per sec., with $n = 0.025$. The discharge of the canal has not exceeded about 1 500 sec-ft. and for most of each season runs at half capacity or less, so that conditions are favorable for maintaining the canal section. The Bow River water turned into the canal carries silt for about a month at the high-water period. The grade of the canal is

* Supt. of Operation and Maintenance, Western Section C. P. Ry. Irrig. Block, Strathmore, Alberta, Canada.

0.02% and the typical meter measurements at Chestermere Station are as given in Table 7.

TABLE 7.—Typical Meter Measurements of Main "A" Canal at Chestermere Station.

Width, in feet.	Area of cross-section, in square feet.	Mean velocity, in feet per second.	Discharge, in second-feet.	Depth of water, in feet.	Calculated roughness coefficient.
52.0	116.0	1.25	145	2.70	$n = 0.028$
63.0	254.0	1.90	436	5.10	$n = 0.028$
72.0	459.0	2.74	1 257	7.90	$n = 0.026$

The material would come under the heading of "ordinary firm loam" and the canal would be classified as "carrying water bearing colloidal silts during a part of the season." The condition of the canal substantiates the recommended velocity as shown in Table 6.

The Secondary "A" Canal runs for most of its length through a rather sandy loam and follows the contour closely so that it has a large curvature. Considerable cutting requiring rip-rap work has occurred on this canal, mostly on the curves. The designed velocity varied in the sandy section from 2.5 to 2.7 ft. per sec., but experience indicates that it should have a maximum mean velocity of about 2.0 ft. per sec., for stable banks, and the material should be classed as a fine or sandy loam in Table 6. This canal takes practically clear water from Chestermere Lake but at high stages is turbid from the erosion of its own banks. The meter measurements as given in Table 8, are typical.

In general, it will be admitted that a large number of complex and variable conditions affect the design of canals and ditches as expressed by the permissible velocities of the water and are difficult to place in their true perspective without considerable practical experience. It is of great benefit, however, to try and describe fully all the factors and discuss conditions in order to capitalize the united experience of engineers and express as clearly as possible all the principles involved.

TABLE 8.—Typical Meter Measurements at Bridge 16A, Cairnhill. (Grade, 0.03%; constructed side slopes, 1 on 2.)

Width, in feet.	Area of cross-section, in square feet.	Mean velocity, in feet per second.	Discharge, in second-feet.	Depth of water, in feet.	Calculated roughness coefficient.
27.0	34.3	1.47	50	1.6	$n = 0.020$
31.0	67.0	1.90	127	2.7	$n = 0.022$
41.5	158.8	2.56	408	4.8	$n = 0.024$

It very frequently happens that the cost of construction, plans for enlargement or future canal lining, the requirement for a high velocity to prevent weed growth, or other factors, lead to the selection of different velocities from those most favorable to a condition of no erosion or silting. It is fre-

quently the case that the material through which canals are built changes so often from one class of soil to another, that the only practical course is to meet a probable average condition.

On the Western Section Irrigation Project, it has proved wise to leave many small ditches with excess grade and to utilize the natural channels. The Operating Department has thus been able to take up the excess grade after use has developed its exact amount and determined the location of controlling structures.

The writer has had particular occasion to observe the difference between new and old canals and wishes to commend the emphasis the authors have laid on designing for the more or less permanent condition of the old canal with its compacted banks and growth of vegetation.

R. A. Hart,* M. Am. Soc. C. E. (by letter).—This paper should prove of great value not only in that the various factors relating to the question have been brought together but especially in that the difference between eroding and transporting velocities is so clearly emphasized.

An important consideration is the position of the ground-water table with respect to the water surface of the canal. As long as the latter is higher, seepage is out of the canal, and there is a tendency for the finer materials to be carried into the interstices between coarser particles, thereby permitting a silting-up process. On the other hand, if the ground-water table is higher, as frequently happens, seepage is into the canal and the whole process is reversed.

In localities having a high water-table there is likely to be an accumulation of soluble materials in the soil, in the ground-water, and even in the canal water. The nature of the soluble matter has a bearing on the case. Calcium salts aid in the flocculation of soil, forming a structure that is easily eroded, whereas sodium carbonate, for example, has a deflocculating effect.

With respect to the presence of sodium carbonate the effect of temperature is of possible concern. At 0° cent., sodium carbonate is soluble at the rate of 7.1 parts per 100 parts of water, while at 35° cent., the solubility is 51.0 parts per 100 parts of water. The warmer the canal water, therefore, the greater the extraction of the deflocculating salt.

Small irregularities in grade are of moment as is evidenced by the self-cleaning of clogged tile lines that have a grade too slight to provide an eroding velocity. The cleaning results from local erosion at the toe of the clogged part, due to the higher velocity of the water flowing down the short, steep section. Once eroded, the material is kept moving by virtue of the velocity of normal flow.

R. A. Hill,† M. Am. Soc. C. E. (by letter).—The very careful distinction which the authors make between canals carrying colloidal matter and those carrying only clear water or materials not colloidal in nature is one of especial value and in itself warrants all the efforts spent in gathering and interpreting the data.

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* Secy.-Mgr., Western Clay Products Assoc., Salt Lake City, Utah.

† Associate, Quinton, Code & Hill, Los Angeles, Calif.

Consideration of the effect of infiltration of colloidal matter into the sides and bottoms of canals has been disregarded too many times in the development of irrigation projects in the Southwest. Equally unfortunate has been the experience of those familiar only with canals carrying colloidal material when they are called on to design irrigation works for other conditions. As the writer's experience has been very largely with irrigation canals served from silty rivers, he will limit his discussion to this particular phase of canal velocities.

While the conditions found by Mr. R. G. Kennedy in Northern India probably are considerably different from those in America, it would seem that in dismissing his conclusions as fallacious, the authors have overlooked the fact that any relation found reasonably applicable there should likewise, when properly modified, be reasonably applicable to canals in those parts of the United States that have climatic conditions generally similar to Northern India; furthermore, that the so-called critical velocity is really not a single velocity but rather a range of velocities between which neither silting nor scouring will occur.

Where the material being transported by the water in canals is not colloidal in nature the range of velocity between which silting will cease and scouring begin will become smaller as the particles comprising the bed of the channel approximate the size of the particles carried in suspension. In practically all cases, however, the bed of the channel is composed either of colloidal material or of particles materially larger than those carried in suspension. Consequently, it should be obvious that a velocity which will not cause silting need not cause scour or erosion.

While the relation proposed by Mr. Kennedy, that the velocity should be proportional to the 0.64 power of the depth, would be applicable only in rare instances, nevertheless there must exist in any channel a relation between the depth and the permissible velocity. It is generally recognized on irrigation projects where silty water must be handled that a deep channel is far more apt to silt than a channel having an equal mean velocity but a less depth. This recognition on one project took the form of an empirical rule that the depth should be one-thirteenth of the bottom width. Analysis of this "rule-of-thumb" will show that when the depth is increased the hydraulic characteristics will be such, under ordinary circumstances, that the velocity will also increase. On other projects, attempts have been made to follow Kennedy's rule or some modification thereof.

If, for the sake of illustration, it is assumed that water is diverted from a silt-carrying river into two large canals similar in capacity and of equal area, one of which is, however, twice the depth of the other, it will be seen that consideration must be given to this factor of depth. At the head of each channel the same quantity of silt is present at all times. The sides of most artificial channels carrying silty water are rather steep, consequently the bottom width of these canals must be approximately in the ratio of 1 to 2; furthermore, the bottom width will be a large percentage of the wetted perimeter.

In order to prevent this silt from falling by the force of gravity to the bed of either channel, eddies must be created to raise the silt toward the sur-

face. About the same number of eddies of equal intensity would be required to keep the silt in the deeper channel from settling to the bottom as would be required in the broader and shallower channel. As the deeper channel is only one-half the width of the shallower channel, there would have to be about twice as many eddies per unit of wetted area, or the eddies would have to be of double intensity in order to maintain the silt in suspension. These eddies can only be created by the energy of the water in contact with the bed of the channel, consequently, in order to develop twice the number of eddies or eddies of double intensity over an equal area, it must be necessary to increase the velocity in the channel if the depth of the channel is increased.

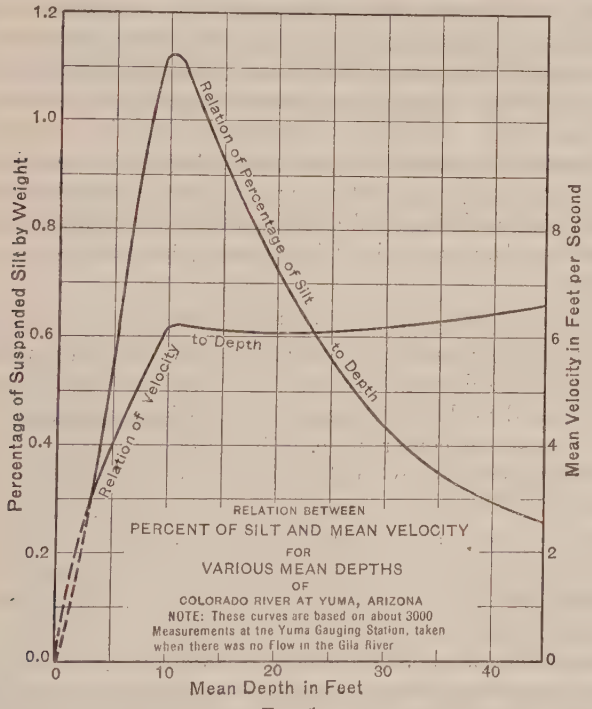

FIG. 1.

Although the most efficient hydraulic section is that in which the bottom width is roughly equal to the depth, no river naturally forms such a channel, the tendency being quite opposite in that the bottom width is usually from 10 to even 100 times the depth. Rivers carrying sediment and flowing between banks of unstable material naturally adapt themselves to a cross-section in which only local silting or scouring will occur as long as the velocity is fairly constant. On such streams bottom scour will commence as soon as the river rises and will continue until a depth has been reached which is sufficient to maintain equilibrium between the tendency to deposit suspended matter and the tendency to erode material from the bed of the channel. As the river recedes the channel gradually refills until equilibrium is again reached at some low stage.

Such fluctuations in the depth of the Colorado River are a most fortunate illustration of the effect of depth on the silt-carrying capacity of a stream. On Fig. 1 is shown the actual relations between depth and mean velocity, and depth and percentage of silt in the Colorado River at Yuma, Ariz. It will be noted that as the velocity increases rapidly the percentage of silt held in suspension also increases rapidly, but when the velocity becomes approximately constant, the percentage of silt steadily decreases as the depth increases. When the mean depth is about 10 ft. the silt content is normally about 1.1%, but when the depth is 30 ft. the silt content is only about 0.4%, although the mean velocity is practically constant throughout this range in depth.

Conversely, if silting of artificial channels is to be avoided, the percentage of suspended matter carried by the water must be held constant, hence the mean velocity must be in accord with some increasing function of the depth of the channel. Since bottom scour cannot take place except at velocities higher than non-silting velocities, consideration must be given to the depth at which the canal will normally be operated, otherwise the maximum velocity ordinarily considered safe against scouring might be insufficient to prevent silting of the deep canals required to carry very large quantities. As pointed out by the authors, bank scour is of little consequence due to the berm which naturally forms.

Jerome H. Fertig,* M. Am. Soc. C. E. (by letter).—The authors are to be commended on the able manner in which they have presented the subject and their conclusions, and desires to express his accord with their views.

It appears that the column of permissible velocities submitted by the writer in Table 3, is somewhat conservative. This is due, possibly, to the fact that his experience has been largely in the construction of irrigation works and in the operation of such works only during the first year or two. His observation has been that new canals and laterals in earth will sustain without scour only a comparatively low velocity; that pure sand banks will wash quite readily; and that shales containing an excessive quantity of alkaline salts will erode much more easily than those having little or none.

In seasoning new canals it is a good plan to allow the first water turned in to run only a short time. The wetting of the canal will cause the banks to settle and become more compact and by not allowing the water to run too long at the start, the finer particles of the soil will remain and tend to cement the material as it dries. Thus, the banks will become set, as it were. It is also desirable, as stated by the authors, to "check back" muddy water in the canal allowing the silt to be deposited, thus aiding in cementing the banks and making them more resistant against scour and seepage.

It is known, as stated in the paper, "that old, well-seasoned canals will stand much higher velocities than new ones". Therefore, where sufficient grade is available, canals should be designed for the highest velocities the materials will stand after several years of operation, taking into account the probability of self-riprapping and also of silting either by natural silt carried

* Los Angeles, Calif.

into the canal by the water supply or, where the latter is deficient, by sluicing fine material into the water, or by other means. If the velocities are too high, particularly during the first few years, they can be controlled by checks placed at suitable locations, as at drops or turnouts in the canal.

It is often essential to locate canals so that as much land as possible is supplied with water, in which case the engineer is confronted with the question of the minimum allowable velocity rather than the maximum. This, of course, presumes a thorough study of costs of larger canals and the position and elevation of the intake, the latter often being rather definitely fixed because of controlling topography.

E. S. Lindley,* M. Am. Soc. C. E. (by letter).—The authors quote the Kennedy formula for critical velocities, but what is said gives the impression that the original paper has not been fully understood; and it shows an inclination to generalize even outside the field that Kennedy only glanced at tentatively.

Twenty years elapsed before any concrete attempt was made to advance from Kennedy's pioneer work. In a few Panjab technical papers† various aspects of the problem have been attacked, and matters pertinent to the present discussion are given. In the course of these papers and the ensuing discussions, reference is made to most of the work known in the East.

Andrew Weiss,‡ M. Am. Soc. C. E. (by letter).—The excellent report on the subject of permissible canal velocities prompts the following supplementary comment. The determination of the most suitable canal velocities is associated with numerous difficulties, chief of which are:

1.—The abrupt and often frequent variation of the material through which the canal section must be built;

2.—The uncertainties attending the selection of the proper coefficients or friction factors in the design of the canals; and

3.—The fluctuations in demand, progressive and seasonal, for which the system should adequately provide.

The first of these difficulties is perhaps best met by attempting to obtain some suitable velocity best adapted to the general average conditions of the locality, and depending on the use of the proper corrective measures as unforeseen difficulties develop later in the use of the system. Sometimes Nature will assist in shaping the canal bed to meet the needs where such an adjustment is made possible. Thus, in the case of a part of the Interstate Canal of the North Platte Project extending through about four miles of shifting dune sands, the section of the channel changed itself from a bottom width of 22 ft. and a water depth of 10 ft. to a width of approximately 75 ft., with a corresponding reduction in depth and slackening of the velocity until the section became stable without further widening and erosion.

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* Executive Engr., Indian Public Works Dept., Panjab Irrig. Branch, London, England.

† "The Design of Head Regulators," and two other papers, Panjab Eng. Congress, 1916; "Régime Channels," Panjab Eng. Congress, 1919; and "Economic Sections of Kennedy Channels", *Panjab Irrigation Review*, 1925.

‡ Asst. Director of Reclamation Economics, U. S. Bureau of Reclamation, Denver, Colo.

As to the second difficulty, the growth of grasses and willows along the water edge of the channel may be so abundant as to interfere materially with the velocity of flow unless this vegetation is frequently removed with a corresponding increase in the cost of maintenance; on the other hand, the velocity may be greater than expected by reason of a friction factor less than that assumed in the design of the canal. Measurements of velocity on the Interstate Canal have indicated a value of n in Kutter's formula generally less than 0.020 and, in some instances, as low as 0.017, whereas in the design of the canal the coefficient, 0.025, was used. The result of this was that erosion started immediately when the canal was put into operation, at times with serious consequences. In addition, its discharge was only about two-thirds its rated capacity. The immediate effect of the excess velocity was the development of sand-boils in the canal bed, on the outside of curves, which were followed by the rapid undermining and destruction of the adjacent canal bank. This difficulty is more serious on the larger canals, because of the relatively greater rapidity of the development of these boils and the disproportionately great damages which may result from a breach of the canal.

This difficulty was successfully overcome by the use of check structures in the canal bed, which served as a means to control velocities and proved almost indispensable in regulating the diversions from the canal.

The third difficulty, namely, the fluctuation in the demand for which the system may be constructed, is due to seasonal causes as well as to the conditions which may prevail during the period of development of any system. If such development proceeds slowly on account of agricultural depression or other causes, the system may be only partly used for several years. There are also seasonal variations in the use of any canal system which are especially notable in the Northern latitudes, and which bring about a sharp rise in the demand on the system during June, or more often in July, according to local weather conditions and the variety of crops predominating.

As a result of the observations made on the North Platte Project the following conclusions have been reached:

(*a*) With the fine silt coating resulting from the use of the muddy waters from the direct flow of the North Platte the value of n in Kutter's formula should not exceed 0.020, except on the smaller canals or distributaries on which the growth of long grasses and willows along the water edge may obstruct the velocity of the water to a considerable extent, thereby necessitating more frequent cleanings and consequent larger expense for the proper maintenance of the system. Similar and more serious obstructions may result from the growth of aquatic plants in clear water as noted in the paper. For these reasons the use of the larger friction factor, 0.025 (Kutter's formula), is favored in the design of these smaller canals.

(*b*) Owing to the frequent changes of the structure of the soil through which these canals had to be built, it was best to attempt to obtain some suitable average friction coefficient and resultant velocity. In the case of the larger canals velocities ranging from 2.50 to 2.75 ft. per sec. were sought and, on the distributaries, velocities of about 0.5 ft. per sec. greater.

(*c*) In all cases the best ultimate results were obtained by the use of check structures in the canals and distributaries, these serving the dual purpose of controlling velocities and facilitating the regulation of diversions. It was found to be practically impossible to perform these functions without check structures. By this means provision was made for these uncertain factors and conditions which could not be foreseen in advance of the construction and operation of these canals.

(*d*) In the design of the check structures it is desirable to use a simple method of closure, preferably check planks, all or any part of which may be removed to correspond to the fluctuations in use and the demands incident thereto. By this means the removal of the planks will restore the original canal velocities if such is found desirable during certain periods, as when it is necessary to sluice accumulated material, or after the canal has become seasoned, or when for other reasons it is not desired to maintain these structures in active use.

(*e*) It is usually desirable to adopt a velocity somewhat in excess of the ultimate rate required because after some period of use the growth of obstructing grasses and willows may cause a considerable retardation of the flow, embarrassing shortages in capacity and additional cost of maintenance of the system. This suggests that a reasonable surplus capacity in the design of such channels may be desirable and economical if the cost of operation and maintenance is given due consideration.

The range of velocities indicated in Table 6 appears to be well applicable in all cases with which the writer has come in contact. There are occasional instances where it may be desirable to avoid the higher velocities in order to induce the deposition of silt for the purpose of reducing percolation losses.

J. C. Stevens,* M. Am. Soc. C. E. (by letter).—The authors deserve the thanks of the Engineering Profession for their excellent paper. It summarizes briefly existing knowledge on the subject and offers its final conclusions in Table 6, which the writer believes will become the guide for earth canal design for years to come.

However, a word of caution should be sounded against using such data blindly. A thorough knowledge of the conditions to be met in design and of the materials encountered is essential to the proper interpretation of such guiding data. The authors have presented the results of their intimate knowledge and study of this subject in as nearly a "fool-proof" form as appears possible. Nevertheless, the terms "non-colloidal fine sands", "sandy loam and fine loam", "colloidal silts", etc., must give many engineers great concern. The skill of the chemist will be required in many instances.

It is obviously impossible to standardize soil classifications so that the various designations will have the same meaning for all. Broad experience will yet be a prime essential in order properly to interpret and successfully to utilize the data presented by the authors.

* Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

CARL ROHWER,* ASSOC. M. AM. SOC. C. E. (by letter).—A knowledge of permissible canal velocities is an important factor in the design of irrigation systems, particularly where the soil is easily eroded, or the canals carry a large quantity of suspended material. Where both conditions obtain the velocity must be carefully determined to eliminate the possibility of erosion on one hand and sedimentation on the other. Fortunately, as pointed out by the authors, non-silting velocities are not necessarily scouring velocities.

An interesting example of a canal constructed under the conditions mentioned is shown in the view (Fig. 2), of the Fort Lyon Supply Canal in the Arkansas Valley near Rocky Ford, Colo. This canal is about 90 ft. wide, has a maximum capacity of 1 500 sec-ft., is excavated in Fresno fine sandy loam, and carries a large quantity of sand and colloidal silt. The silt remains in suspension in the canal but is deposited on the lands irrigated, amounting to as much as ¼ in. in a single irrigation. The type of silt carried is shown by the view (Fig. 3) of the material deposited in a depression alongside the canal. Under some conditions the sand or bottom load is deposited and under others is scoured out. The characteristic appearance of the sand deposit is shown in Fig. 2. The depressions shown are about 15 ft. in diameter and 3 ft. deep.

TABLE 9.—DISCHARGE MEASUREMENTS ON FORT LYON SUPPLY CANAL.

Date.	Gauge height, in feet.	Area, in square feet.	Discharge, in cubic feet per second.	Mean velocity, in feet per second.	Critical velocity, V_0*, in feet per second.
June 8, 1923	3.44	304.5	681.5	2.24	1.85
" 14, "	1.32	101.5	159.4	1.57	1.00
" 19, "	4.96	452.7	1 142.6	2.52	2.35

* Computed from the formula, $V_0 = 0.84\, d^{0.64}$.

Tables 9 and 10, submitted by Mr. H. D. Amsley, of the Colorado State Engineer's Office, give the flow characteristics of the canal based on current meter measurements and the gauge height record at the rating flume near the head. The quantities given are for the conditions in the rating flume, but they may be assumed to apply to the canal proper because the rating section does not differ materially from the canal section.

TABLE 10.—MAXIMUM GAUGE HEIGHTS, FORT LYON SUPPLY CANAL.

Date.	Hour.	Gauge height, in feet.
June 8, 1923	4:00 A.M.	4.75
" 16, "	6:00 P.M.	5.15
" 19, "	9:00 A.M.	5.00

On June 8, 1923, the depth in the flume for the gauge height, 3.44 ft., varied from 2.02 to 3.44 ft., and on June 19, for a gauge height of 4.96 ft., from 4.90 to 5.00 ft. This shows that more than 1.25 ft. were scoured from some parts of

* Associate Irrig. Engr., Div. of Agri. Eng., U. S. Dept. of Agriculture, Colorado Experiment Station, Fort Collins, Colo.

FIG. 2.—FORT LYON SUPPLY CANAL, NEAR ROCKY FORD, COLO., AUGUST 13, 1924, SHOWING ERODED AREA.

FIG. 3.—SILT DEPOSIT IN DEPRESSION ALONGSIDE FORT LYON SUPPLY CANAL, NEAR ROCKY FORD, COLO., AUGUST 13, 1924.

the flume in that interval, and as the conditions in the canal are similar it is reasonable to assume that scouring occurred there also. During this time, as shown in Table 10, the gauge height did not materially exceed the values for which the discharge is given, therefore, the velocities were probably never materially higher than the maximum indicated in Table 9.

The values of the critical velocity, V_0 (Table 9), computed from Kennedy's formula, $V_0 = 0.84\, d^{0.64}$, in which, V_0 is the critical velocity, in feet per second, and d, the depth, in feet, of the canal, show that the mean velocity of the canal was always greater than the critical velocity. This indicates, as was found, that the tendency was toward scouring rather than silting at the time. However, the accumulation of sand shown in Fig. 2 indicates that at other times considerable sand is deposited, but the established regimen of the canal indicates that from season to season as much sand is scoured out as is deposited.

Ivan E. Houk,* M. Am. Soc. C. E. (by letter).—The paper presents several good ideas and many valuable data, and the authors are to be congratulated on the clear and comprehensive manner in which they have handled the subject. Their discussion of the effect of colloidal silts in increasing scouring velocities is both interesting and valuable; and the practical ideas and data,† submitted by engineers in charge of irrigation projects, should prove of great value in future designing work. The velocities to be permitted in canals are primarily dependent on the nature of the material through which the canals are located, a condition which is susceptible of almost infinite variation. It is believed that, with the present knowledge and available data, detailed descriptions of the materials through which certain canals have been built, together with the actual velocities found feasible therein, are more valuable than generalized recommendations, such as are given in Table 6.

It is not clear to the writer where there has been any confusion, fallacy, or fundamental error in the work and discussions of other engineers, particularly the British irrigation engineers who have had long and valuable experience in such matters. They have frequently pointed out a wide difference between non-silting and scouring velocities. That they have discussed deposition and erosion in a more or less combined manner, is true; but it seems perfectly logical to do this. In designing a canal to be located through extensive silt deposits it is important to adopt velocities that will neither silt nor scour; and it is perfectly proper to consider such phenomena at the same time even if they are fundamentally different operations. However, experimental determinations of non-silting and eroding velocities are matters which undoubtedly should be considered separately.

It is believed that the experiments, experience, researches, and discussions of many Far Eastern irrigation engineers, including Kennedy, Willcocks, Buckley, Parker, Bellasis, and others, are well worthy of careful study by any one interested in the problem. Although their experience may not agree exactly

* Engr., U. S. Reclamation Service, Denver, Colo.

† See pp. 946–950.

with that of Western engineers, it should not be so briefly dismissed. It is believed that the difference between their experience and that of irrigation engineers in the United States is due to the differences in the materials through which the canals are located, rather than to any fallacy or fundamental error in their conclusions. Their studies and experiments, in some cases at least, may have been made on canals where no colloidal silt was present. There probably are many locations in the United States where Kennedy's laws are more applicable than the recommendations of the authors. The writer knows of several power canals in the East where the velocities recommended by the authors could not be permitted.

It might also be pointed out that the effect of the channel depth is not considered in the final recommendations of the authors. Although the writer has never attempted to express his observations by formulas, he has observed, and always considered, that higher velocities can be permitted in deep channels than in shallow channels, as has been maintained by many writers. On the Miami Valley flood prevention work at Dayton, Ohio, velocities as high as 10 ft. per sec. were considered allowable as extreme maximum values in straight channels having gravel or cobble beds, where the depths were 20 ft., or more. Of course, such high velocities could not be permitted in irrigation canals through similar materials. They would not have been considered on the Miami Valley work except as ultimate possible limits to occur only for a few hours' duration, but even under such conditions the writer would not consider them safe for channels 5 ft. deep. In curved channels on the Miami Valley work concrete slab paving was placed on the outer banks, and flexible concrete block mattresses on the bottom, adjoining the outer banks, wherever the mean velocity exceeded 6 ft. per sec.*

In the high-water surveys of the 1913 flood in the Miami Valley,† made during the summer after the flood, it was noticed that unusually high velocities can occur over alluvial soils when the depths are comparatively great and the flow comparatively uniform, without causing appreciable scour. Several instances were observed where mean velocities as high as 6 ft. per sec. occurred over alluvial fields along the main channels, with mean depths of about 15 ft., without causing any noticeable erosion. Cornstalks from the preceding year's crop were still in place after the flood had passed. In one instance a velocity estimated at 11.5 ft. per sec. occurred over fields flooded to a mean depth of 24.6 ft., where the flow conditions were unusually uniform, without causing apparent scour. Immediately below, where the velocities were practically the same, but the flow conditions greatly disturbed, a new channel about 100 to 150 ft. wide and about 10 ft. deep, was eroded. The material in both places consisted of an alluvial surface soil underlaid by a yellow clay containing some sand and gravel.

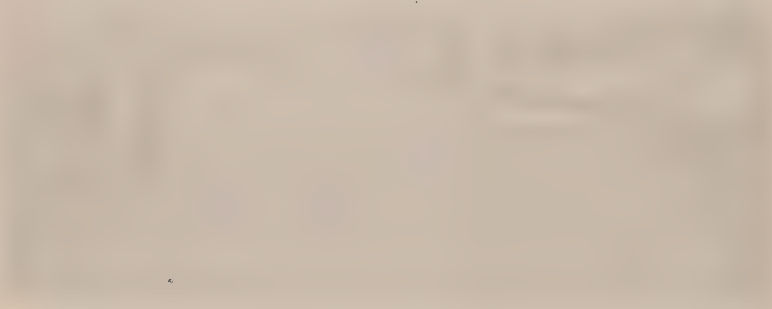
Although the velocities just mentioned were calculated from surveys made after the flood, they are not believed to be appreciably in error. The surveys were carefully made, and the calculations, using the Kutter formula, were

* "Hydraulics of the Miami Flood Control Project," by Sherman M. Woodward, Technical Reports, Pt. VII, The Miami Conservancy District, Dayton, Ohio, 1920.

† "Calculation of Flow in Open Channels," by Ivan E. Houk, Technical Reports, Pt. IV, The Miami Conservancy District, Dayton, Ohio, 1918.

FIG. 4.—VIEW OF MIAMI RIVER CUT-OFF, LOOKING UP STREAM ALONG THE CHANNEL.

FIG. 5.—GRAVEL FORMING BED AND BANKS OF THE MIAMI RIVER CUT-OFF.



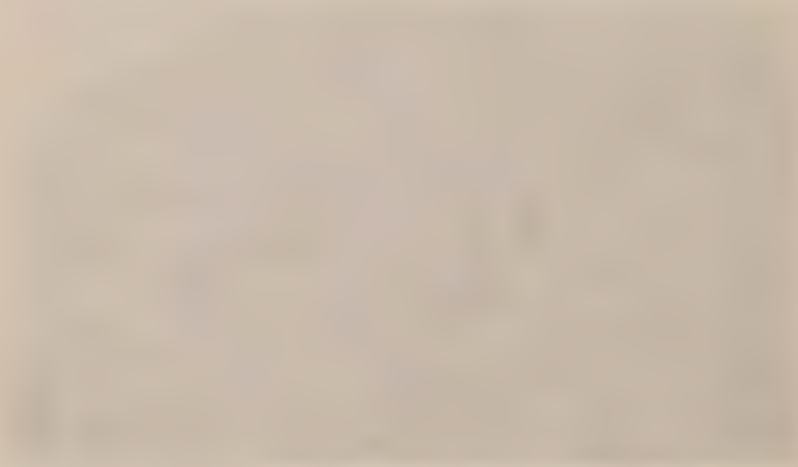


TABLE 11.—Summary of Scour Observations in the Great Miami Valley.

Location.	Maximum average velocity, in feet per second.	Mean depth, in feet.	Material.	Results.
Overflow area of Miami River, Miamitown, Ohio	11.5	24.6	Alluvial soil.	No evidence of scour.
"Big Four" Railroad bridge over Buck Creek, Springfield, Ohio	13.7	6.7	Clay, sand, and gravel, with some loose rock around abutments.	No evidence of scour.
Limestone Street Bridge over Buck Creek, Springfield	10.1	11.5	Clay, sand, and gravel, with some loose rock around abutments.	No evidence of scour.
"Big Four" Railroad bridge over the Miami River, Sidney, Ohio	12.4	15.7	Earth and gravel with probably rock in river bed.	Scour around east end of bridge and south side of embankment. None in river bed.
Miami River above the Cincinnati Hamilton and Dayton Railroad bridge, Troy, Ohio	6.11	18.6	Clay, sand, and gravel with some broken rock.	No evidence of scour.
Miami River at the Cincinnati Hamilton and Dayton Railroad bridge, Troy	11.1	16.3	Clay, sand and gravel with some broken rock.	Considerable scouring around piers.
Overflow areas of Miami River, below Miamisburg, Ohio	6.67	16.7	Alluvial soil.	No evidence of scour.
Miami River Bridge, Tadmor, Ohio	7.8	About 23	Sand and gravel with some broken rock.	No evidence of scour.
Miami River channel below Miamisburg	9.23	31.1	Clay and gravel.	No evidence of scour.
"Big Four" Railroad bridge over Miami River below Miamisburg	15.2	27.7	Clay, gravel, and rock.	Extensive scour.
Stillwater River above highway bridge, West Milton, Ohio	12.0	27.7	Clay and gravel.	No evidence of scour.
Stillwater River at highway bridge, West Milton	14.6	26.3	Clay and gravel with broken limestone around abutments.	Extensive scour.
Mad River at first "Big Four" Railroad bridge west of Springfield	12.2	17.2	Clay and gravel with some bed-rock in middle of channel.	No evidence of scour.
Mad River at second "Big Four" Railroad bridge west of Springfield	15.0	18.0	Clay and gravel with some bed-rock in middle of channel.	Extensive scour.
Miami River at Main Street Bridge, Dayton, Ohio	11.0	*24.6	Gravel.	Considerable scouring around piers.
Miami River overflow above Tadmor	5.87	13.3	Alluvial soil.	No evidence of scour.
Miami River channel above Tadmor	8.17	23.4	Gravel.	No evidence of scour.
Miami and Erie Canal opening at Miami River below Tadmor.	16.6	30.6	Gravel and rock.	Extensive scour.
Miami and Erie Canal stone arch opening at Turtle Creek, west of Sidney, Ohio	13.7	10.1	Clay and gravel.	Some scour.
"Big Four" Railroad concrete arch opening at Tawawa Creek, east of Sidney	23.1	8.2	Clay and gravel.	Scour.
Seven-Mile Creek, Seven Mile, Ohio	5.44†	2.58	Limestone.	No evidence of scour.
Miami River, Sidney	3.90†	5.06	Gravel and rock.	No evidence of scour.
Miami River, Tadmor	4.63†	9.85	Gravel.	No evidence of scour.
Miami River at Main Street, Dayton	4.54†	10.1	Gravel.	No evidence of scour.
Miami River just above Wolf Creek, Dayton	5.0 †	5.2	Gravel.	Slight shifting of material.
Stillwater River, West Milton	6.36†	8.70	Gravel.	No evidence of scour.
Miami River, Dayton	2.68†	.....	Gravel.	No scour.
Miami River cut-off channel, Dayton	5.5 †	1.8	Gravel.	No scour.
Miami River cut-off channel, Dayton	5.5 †	1.5	Clay.	Continuous erosion.
Miami River cut-off channel, Dayton	6.1 †	3.2	Gravel.	Extensive scour.

* Average depth under bridge.

† Velocity measured with a current meter.

checked by independent computations based on measured head at contracted openings, as described in full in the report previously noted. The more valuable observations, together with some later data obtained where the velocities were measured with a current meter, are given in Table 11. Velocities measured with a current meter are indicated by asterisks.

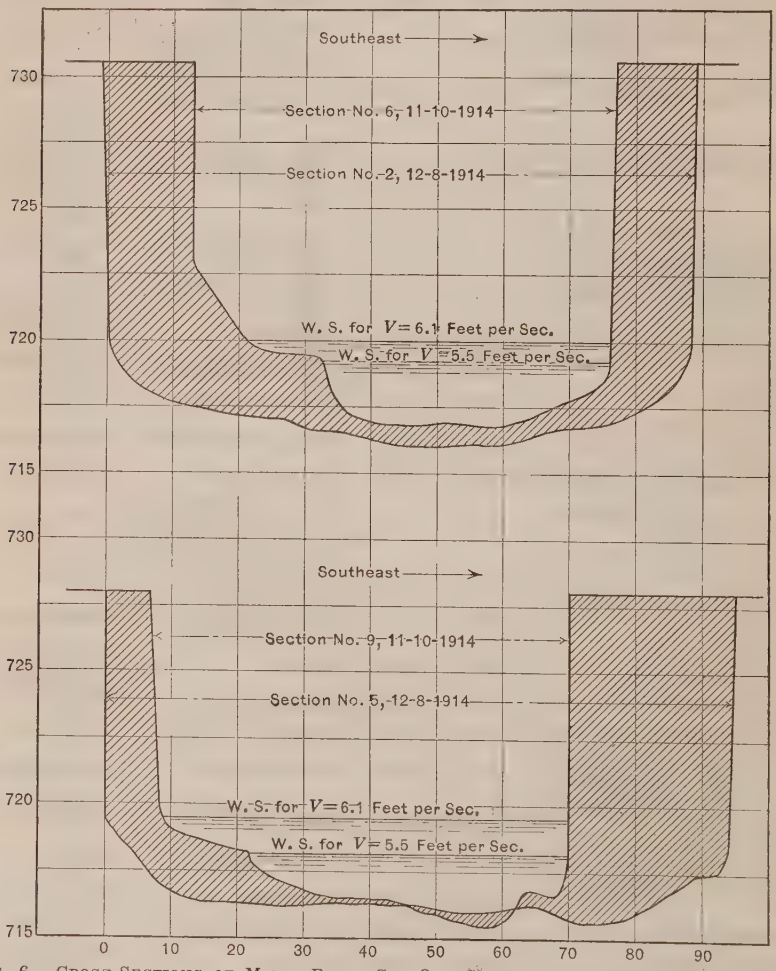


FIG. 6.—CROSS-SECTIONS OF MIAMI RIVER CUT-OFF CHANNEL, DAYTON, OHIO, SHOWING SCOUR CAUSED BY INCREASE IN VELOCITY FROM 5.5 TO 6.1 FEET PER SECOND.

The observations on the Miami River cut-off channel, which make up the last three items in Table 11, are probably worth some discussion. Fig. 4 is a view of the cut-off looking up stream along the channel, from a point near the lower end. The material forming the bed and sides of the cross-section above the place where the man is standing was clean sand and gravel; below this place it was a tenacious yellow clay. Fig. 5 is a close view of the gravel, taken on the berm which appears at the left of Fig. 4. The folding rule and

engineer's scale indicate the sizes of the particles. Although 2 to 3-in. stones constitute the greater part of the surface material, finer particles, fairly well graded, were beneath the surface. This material seemed to be fairly typical of the river-bed gravel found in other parts of the Miami Valley.

When the first set of current-meter measurements were taken, showing a mean velocity of 5.5 ft. per sec., the clay section was eroding continuously, whereas the gravel section was stable. The gravel seemed to be just on the point of moving, but no sand or pebbles were being transported in suspension and no movment along the bottom could be detected. Fortunately, the clay section was down stream from the gravel section so that the water above the clay was perfectly clear. When the second set of meter measurements was taken, four days after the stable conditions had been changed by a slight rise in the river, the mean velocity was 6.1 ft. per sec., only 0.6 ft. per sec. greater; but the gravel was being rapidly eroded. Pebbles of considerable size were being carried in suspension near the surface of the water, as could be determined by placing the sounding rod only a short distance into the water. Thus, it was definitely proved that this material began to erode at some velocity between 5.5 and 6.1 ft. per sec. probably nearer the lower value, when the depths were between 2 and 3 ft. The shaded portions of the cross-sections in Fig. 6 indicate the total cross-sectional area eroded during the four days. Taking into account the length of the cut-off, about 600 ft., rough calculations show that approximately 100 cu. yd. per hour were being transported.

On the basis of the cut-off experiments the writer believes that the maximum permissible velocities of 4.00 and 5.00 ft. per sec., given in Table 6 for limiting values to be used in straight channels where the beds are "coarse gravel" and "cobbles and shingles", respectively, and where the water is clear, could be raised to 5.0 and 5.5 ft. per sec., respectively, especially since very few canals designed for such velocities will be less than 2 or 3 ft. deep. For large canals, 10 or 15 ft. deep, the writer believes that still higher velocities can sometimes be permitted, possibly 7 or 7.5 ft. per sec. in emergencies. What really happens in a canal located through gravelly materials, subjected to relatively high velocities, is that the finer particles are washed out until the bottom is literally paved with the larger stones; so that the larger stones protect the finer particles below from further scour.

Samuel Fortier* and Fred C. Scobey,† Members, Am. Soc. C. E. (by letter).—The writers wish to express their appreciation of the additional constructive information contained in the discussion. From comments in the discussion there appear to be two main points that may be enlarged upon, namely,

1.—Kennedy's theory and formula and its relationship to "permissible canal velocities"; and

2.—Soil classification particularly as influenced by colloidal material in a dispersed state, contributing to plasticity.

* Senior Irrig. Engr., Div. of Agricultural Eng., Bureau of Public Roads, U. S. Dept. of Agriculture, Berkeley, Calif.

† Senior Irrig. Engr., U. S. Dept. of Agriculture, Berkeley, Calif.

The flow of water in canals may be said to consist of three phases of velocity in its relationship to the movement of soil particles:

(*a*) Non-silting velocities, or in the words of Kennedy* himself, "critical non-silting mean velocities", further explained by the statement:

"Thirty channels were selected, all known to be more or less in permanent régime, and all having self-silted and similar sand beds. Evidently each of them was fully charged with silt, since each had the option of picking up more sand from the bed or dropping any overcharge."

(*b*) Transporting velocities—those which will move down stream particles already detached.

(*c*) Scouring velocities—those which will erode a well-aged canal bed.

Phase (*a*) implies a condition of equilibrium in the flow of a silt-laden canal—a water completely loaded with silt for that particular velocity and depth of channel, and neither taking on more silt nor depositing what it has. Likewise, the implication exists in the Kennedy quotation that the bed silt which might be taken on as a load is quite free to join the silt being transported—that it is not firmly bedded down, as defined on page 944. It is readily seen that any particular soil unit from the finest of clay particles to cobble-stones, could be more easily picked up for further transportation within a short time after it last came to rest than after it had either adhered to adjoining bedded particles or had become interkeyed with adjacent units.

Phase (*b*) differs from Phase (*a*) in that it deals with a water that may not be already loaded. In other words soil particles that would not be moved under Kennedy's specifications would be transported under the same velocity conditions by a water not already loaded to capacity. This phase applies only to soil particles that are perfectly free to move—either particles journeying down stream that have never become part of the settled bed of the canal or particles that have been raveled off from the bed due to velocities under Phase (*c*). Gilbert differentiates between material moved by "stream suspension" and that moved by "stream traction".

Phase (*c*) covers velocities that will scour a canal bed which has become well-seasoned under lesser velocities. These are the velocities that "permissible canal velocities" may be allowed to approach, but not to equal. For other given conditions they are higher than the velocities in Phases (*a*) and (*b*).

The writers have the utmost respect for the work of Mr. Kennedy and feel that his theory and formula, adjusted by the proper coefficients for the waters and silts under consideration, should be the basis for all canal design where silted waters are encountered. Their objections are directed at various writers who misapply his formula in discussing scouring velocities.

There is the implication that below a certain velocity for a given channel and given soil conditions silting takes place and above it scour occurs. This may sometimes be true for the traction of loose particles. The writers' contention is that the well-seasoned bed will not scour under a velocity well above the silting velocity limit.

* "Hydraulic Diagrams for Channels in Earth, Giving Discharge, Mean Velocities, and Silting Data," Second Edition, 1907, p. 1.

In a similar manner transporting velocities have been used as criteria for scouring velocities. One of the most quoted authorities on hydraulics (Merriman), in discussing velocities to be used in the design of irrigation canals, states:*

"The following statements show approximately the velocities which are required to move different materials:

0.25	ft.	per	sec.	moves	fine clay.
0.5	"	"	"	"	loam and earth.
1.0	"	"	"	"	sand.
2.0	"	"	"	"	gravel.
3.0	"	"	"	"	pebbles, 1 in. in size.
4.0	"	"	"	"	spalls and stones.
6.0	"	"	"	"	large stones.

"The mean velocity in a channel may be somewhat larger than these values before the materials will move, because the velocities along the wetted perimeter are smaller than the mean velocity."

It is doubtful whether "somewhat larger" could be construed to mean from fifteen to twenty times as great for clay as is indicated in Table 6. Likewise, Merriman's list would indicate that a greater velocity is permissible in a canal with a sand bed than in one with either clay or loam or "earth", whereas sand alone will be eroded by lesser velocities than any other type of soil particles. Even pure sand will bed and pack down until it requires a higher velocity to move it than the "transporting velocity" which could move the same sand down a flume of such material that the sand could not start to "bed down". The amount of the "bedding down" is indicated in the difference between 1.00 ft. per sec., in Merriman's list, and 1.5 ft. per sec. given in Table 6.

Considering velocities that will scour a well-set canal bed, the following statements regarding soils must be taken as general only. The soil types are so varied and blend from one gradation to another and from "light" to "heavy", and through so many mineral and chemical modifications, that there are cases where general statements do not hold. For instance, ordinarily sand is not regarded as having plastic properties such as are usually associated with clay and probably it seldom occurs in a cohesive state in a canal bed. However, in a previous condition, such sand may have been mixed with colloidal material that would cause more or less cohesion between the sand particles. In its progress down a river or canal such a sand would probably have much of the colloidal material removed by attrition. It is suggested by Macy Lapham, Inspector for the Western Division, U. S. Bureau of Soils, that the colloidal matter so prevalent in the waters of the Colorado River may have come in part from the sand particles which are comparatively free from plastic material by the time they reach Yuma, Ariz., and the Imperial Valley, California.

In a paper of this nature, discussion of soils must be confined as nearly as possible to the phases as they appear to engineers, and its lack of detail will allow any soil technician to point out exceptions to the statements made. From the viewpoint of the engineer—particularly the irrigation engineer—it is unfortunate that the soil technicians have chosen the word, "silt", for their

* "Treatise on Hydraulics," Eighth Edition, N. Y., 1908, p. 288.

grade in size of material next above that of clay. When engineers speak of a reservoir "silting up" they mean the reduction of storage by everything brought in by the river. The engineer and the layman nearly always mentally associate silt and water, and this is borne out by Webster, yet some of the great areas of soil of high silt content as classified by soil technologists were not water-laid but derived from loessial deposits, probably wind-laid. On the other hand, "clay" is not immediately associated in thought with water, yet the predominating areas of arable clay soils of the Western United States are water-laid.

To enlarge on the soil materials listed in Table 6, Column (1): Fine sand, sandy loam, and silt loam are listed as non-colloidal. There seems to be little question as to sand. It is composed of hard distinct units that ordinarily have no cohesive attraction one for another. There are exceptions, as mentioned before. Any resistance to scour exerted by a canal bed of sand, other than the weight of the particles, lies in whatever interkeying has taken place since the last disturbing condition.

As to the various modifications of "loam" there may be more uncertainty. The word, "loam", whatever the modification, with the possible exception of "clay loam", conveys the conception of friability. It must be of a low plastic property or it would not be a loam. A loam does not "clod up" when plowed, and when a lump is moistened the tendency is for it to break down into a rather non-sticky mud. A dry lump of loam crushes freely under a boot-heel. Clods of silts or clays give back a resounding resistance when struck with a shovel. However, such clods may rapidly break down in water if the colloids are in a flocculated state. The fact is to be accented that in soil-particle sizes there is no such gradation as "loam". It must exist in a mixture. Sandy loam is rather more sandy than clayey. Clay loam indicates the predominance of clay rather than sand, and so on.

The next category is "alluvial silts when non-colloidal". Alluvial silts may or may not yield a slick sticky mud when wetted by canal water flowing in an excavation in such silts. The criterion of identity is based largely on the state of the colloidal matter in the silts. If the silts were laid down or filtered out in seepage while containing "effective colloids" as defined on page 944, the particles are in a dispersed—extremely fine—status; while if they were deposited as aggregates or flocculated units, they do not have the plastic attributes described by Hilgard, as quoted by the writers. In an unpublished paper on the silts of the Colorado River, J. F. Breazeale, Associate Bio-Chemist of the U. S. Department of Agriculture, points out that a silt may change back and forth many times in its progress from the upper benchlands of Colorado, Wyoming, or Utah, and from the "bad lands" of New Mexico and Arizona. This change is a physico-chemical reaction between the silt particles and the particular waters that are encountered as the silt travels down stream. This reaction is from a dispersed (effective) to a flocculated state, and *vice versa*.

"Volcanic ash" or "lava ash" is popularly applied to much of the finer grained soils of agricultural value east of the Cascade Range, in the States of Washington, Oregon, and Idaho. The term, "volcanic ash", is a misnomer

as this soil contains little or no volcanic ash material associated as to its origin with volcanic activity. It is mainly a wind-laid accumulation and derived from desiccated sediments of old lake beds and from the finely textured weathered particles of the basaltic lava flows. When wetted it forms a slick mud, but without the plastic characteristics of the clays. It is friable when the moisture is dried out of the mud. However, when wetted, as in a canal bed, this material—intermediate in size between the loamy soils and the plastic clays—will withstand velocities slightly greater than soils that are distinctly loamy and, therefore, friable in character.

The foregoing discussion on the characteristics of colloidal and non-colloidal materials will suffice to explain the terms, "very colloidal", as applied to stiff clays and, "when colloidal", or "when non-colloidal", as applied to graded material that may have fine silts or even particles of "clay" size or less that are effectively plastic if in a dispersed state, but are not plastic if in a flocculated state.

Taking up the individual discussions in the order in which they appeared: Mr. Muckleston senses the difficulty in soil determination as already described. He also mentions the same problem later found in the discussion of Mr. Weiss, that is, the fact of rapid changes in soil types within such a short distance that no adjustment in velocities would be feasible. He suggests that alkali soils erode easily. This is true for most alkalis found in the West with the exception of black alkali (sodium carbonate). (See discussion by Mr. Hart.) Other alkalis, being powerful electrolytes, flocculate colloidal matter so that it no longer has the property of forming sticky, extremely cohesive, canal beds.

Mr. Chandler brings out a point that perhaps should have been emphasized in the paper. This paper deals only with maximum permissible velocities—velocities just short of those that will erode a canal bed. Of course, there may be other factors, such as the need for silting of porous channels, which will entirely change the problem so that the velocities desired are just less than Kennedy's critical velocity or may involve transporting velocities, as already mentioned under Phase (*b*).

As to aquatic growth that seemed to flourish in high velocities, the writers have noted moss in chute-drops where the velocities approached 40 ft. per sec., and, furthermore, this moss was literally alive with small worms, some sort of larval growth. It is suggested that the "swell" of water of such high velocity, carrying almost 30% by volume of entrained air, gives the additional oxygen that makes this flourishing growth possible.

The writers appreciate the additional data contributed by Mr. Stockton. Few engineers have had the operating experience under many conditions that has been his lot. For about twenty years he has operated canals in the Mountain States and the Province of Alberta. His comments are authoritative. Mr. Hart makes an excellent point that has previously been overlooked although it involves most obvious hydraulics. If a canal loses by seepage there is a current available to carry silt and sand down into the original bed material, making it tough and well graded by filtration alone. If the water-

table is such that the flow of ground-water is inward no such current is available and there is less tendency for a canal bed to become well set by the grading of material, especially by the introduction of the fine plastic clays.

Mr. Hill misunderstands the writers when he states that they dismiss Kennedy's formula and theory as fallacious. Kennedy's theory is accepted. When misused the results are fallacious. Kennedy's first edition of "Hydraulic Diagrams for Channels in Earth, Giving Discharges, Mean Velocities and Silting Data, etc." appeared in 1895. Presumably the second edition, written in 1907, contains his retrospect after twelve years of applied use of his formula. In his title he mentions "silting data". A careful perusal of this second edition discloses that in no place does Kennedy confuse his critical velocity with a velocity that will scour a well set canal bed. The writers do dismiss as fallacious the findings of later writers who have misinterpreted Kennedy's silting data, applied it to scouring data, and offered limiting velocities for earthen canals based on such silting data. They regret exceedingly that any statements in the paper could be construed as disagreement with Kennedy's theory in general.

Thus far in this paper and discussion no mention has been made of canal depth as a function of the permissible velocity. As pointed out by Mr. Hill, it is certain that depth would enter the problem in any case where sand bottom exists. The scouring velocity and transporting velocity might approach the same figure, and fluctuating pressures would cause disruption of the canal bed by developing "boils".

It is rather difficult to reply to Mr. Lindley. It is regretted that he was not more specific regarding the Kennedy theory. Evidently, he, like Mr. Hill, misinterpreted the paper as disparaging to Mr. Kennedy and his work. The citations given by Mr. Lindley have not been located. It is to be regretted that the valuable reports and professional papers of the irrigation engineers of India, Egypt, and South Africa, are not more generally available in the United States.

The data in the contribution of Mr. Rohwer is similar to that of Mr. Hill in that it applies to silting velocities or transporting velocities rather than to limiting canal velocities. His photograph (Fig. 3) of cracked silt deposits along the Arkansas River, is typical of other rivers as well. Exactly the same condition exists in the Palo Verde and Imperial Valleys of California, the silt coming from the Colorado River,* and in the valley of the Rio Grande.

Mr. Rohwer's Fig. 2 does not appear typical of an erosion condition. Nearly all true erosion, or more properly "corrosion," shows ragged, nearly vertical slopes, at least on one side of the excavation. The tables given to show the movement of sand as the velocity increases apply to the transportation of a sand prism typical of many canals served by water from the eastern side of the Colorado Rockies. The true beds of these earthen canals are seldom exposed to view. This condition also exists in many of the concrete lined canals of the citrus region of Southern California. Here the concrete bottom

* See discussion and views of Colorado River silt, by Harry T. Cory, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), pp. 1223 *et seq.*

is often covered with a traveling load of sand ranging from isolated patches to dunes more than 1 ft. thick.

Mr. Houk adds a great deal of pertinent data, particularly as to the fact that higher velocities may be permitted in deep channels, for the same type of bed material. It is a question whether some of his cited figures may be applied directly to irrigation and power canals. One of the primary assumptions in the paper was that the canal bed should be well aged. For newly made canals an entirely different set of velocities would apply. Such velocities would not enter into the calculations of the designer unless the canal were to be used immediately at or near peak load and there was no possibility of checking the water by proper grade correction. Some of the velocities cited by Mr. Houk as scouring the channel in alluvial material recently inundated by a severe flood would undoubtedly not have been so effective in a well bedded canal gradually seasoned. On the other hand some of his velocities cited as without scouring effect, may have had too little time. It is probable that inundating floods, in their first stages, pick up a tremendous quantity of trash, too heavy for suspension but not too heavy for bottom traction. Such a condition would cause a great lag at the bottoms of the vertical velocity curves. It is conceivable that mean velocities of 10 to 15 ft. per sec. might have bottom velocities of only 4 or 5 ft. This would be particularly true over the usual inundated area, grown over with more or less brush and weeds. Given sufficient time, this protecting cover would be worn off and effective scour would develop.

The reason for not including the element of depth is the lack of data as to an increase in permissible velocities for an increase in depth. Mr. Houk mentions the work of several of the best known Far Eastern irrigation engineers. It is true that their writings yield much light on the subject of canal velocities, but all of them do not differentiate between these velocities as has been done in this paper. Specifically, Kennedy, as already mentioned, is dealing with non-silting velocities over a bed of shifting material. He may have written on other phases of velocity. Willcocks mentions scouring and silting velocities in several places; not under a specific heading dealing with either of these subjects. Regarding scour, he says:

"The laws of scour are not quite the same as the laws of transport, but are analogous. Here the scour depends on the pressure which the water is able to effect on the particles, on the size and weight of the particles, on the surface exposed to the water's action, and on the tenacity with which one particle clings to the next. Smoothness or roundness in the particles will diminish the pressure, but will decrease the tenacity. A mixture of various sizes of silt will probably diminish the surface on which the water can bear, and will decrease scour."

Again, in writing of silts, he says:

"In November and December the water is comparatively clear, and if advantage is taken of this clear water it can be utilised to scour out the deposits of fine mud in the canals before the mud has had time to harden and consolidate."

Evidently, Willcocks was under no misapprehension as to the various types of velocities to be encountered. It is to be regretted that he did not devote

a chapter exclusively to the subject of canal velocities, bringing together his scattered material on this general subject. It may be noted in his second quotation that he senses the writers' premise of aging in a canal bed.

Buckley does not confuse the various phases of velocity and he has a table of permissible canal velocities that agrees quite closely with a similar one developed years before by Clibborn and that also agrees within reason with Table 4 (Etcheverry), and Table 6, Column (2). Bellasis writes of silting velocities and transporting velocities only. Parker* in a large volume that goes exhaustively into some phases of irrigation and other hydraulics, treats of silting velocities and transporting velocities but makes no mention of those classed under the title of this paper. He must have had the writings of Clibborn, Willcocks, and Buckley as well as those of Kennedy and Deacon (on transporting velocities). He may have been convinced that permissible velocities were synonymous with those of one of the other phases mentioned at the beginning of this discussion.

* "Control of Water," Second Edition, Lond., 1925.

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Paper No. 1589

FLOOD FLOW CHARACTERISTICS*

By C. S. Jarvis,† M. Am. Soc. C. E.

With Discussion by Messrs. Robert Follansbee, O. W. Hartwell, E. C. La Rue, N. C. Grover, B. Okazaki, L. K. Sherman, George G. Honness, L. M. Winsor, H. V. Hinckley, Hardy Cross, E. W. Lane, Gerard H. Matthes, Ford Kurtz, Charles W. Comstock, Thaddeus Merriman, Weston E. Fuller, H. de B. Parsons, Robert E. Horton, Merrill M. Bernard, C. D. Purdon, and C. S. Jarvis.

Synopsis

The characteristics of flood flow that may have either general or special application and therefore must be considered in any preliminary studies for protecting a given locality are briefly stated, not by way of defining probable flood limits for individual cases, but of indicating methods of deduction that must be followed.

The available data that seemed reliable and valuable have been listed and platted to give a visual representation of the range of discharge per square mile for maximum recorded floods. Where possible, the portion of the maximum flow which represents the average annual maximum flood on each stream, has been indicated.

A method of attack is explained by which problems in several Central and Western Districts have been solved satisfactorily, both as to prevention of floods and protection of property.

In addition, there has been assembled in convenient form for reference a variety of representative basic and auxiliary data that are best portrayed diagrammatically.

* Presented at the meeting of May 6, 1925.

† Care, U. S. Bureau of Public Roads, Washington, D. C.

The important public service which the Engineering Profession owes society in analyzing each local situation and in inaugurating or even restricting improvements as the future safety of the community may demand, is especially emphasized.

Design of Structures to Cross Drainage Systems

An engineer endeavoring to design economical and safe bridges or similar structures is confronted with a maze of factors influencing the problem—scores of formulas and tables of data, hundreds of examples that serve as warnings, and thousands that appear satisfactory. Where records concerning the stream flow are lacking, he must rely on a study of the water-shed and channel, the alluvial bars and embankments, the drift lodged along canyon walls and in crevices, and the indefinite memories of observers. In general, he will be unable to reconcile the data thus gathered; and then remains the question of assigning proper weights to the various fragments of evidence.

Following such deductions, an important Western railroad company located nearly 100 miles of line along a valley floor, safely above the reach of the maximum assumed floods, only to experience its destruction within a few years. It was rebuilt on a higher line that would be safe against three or four times the maximum flow originally assumed. Similarly, various highway structures on which Federal aid was sought, had provided only one-half or one-fifth the proper capacity and during the torrential floods of 1923 some waterways that had been regarded as adequate proved able to carry only a small fraction of the maximum flow.

Proper design depends on knowledge of (1) the possible maximum flood; (2) the probable duration and frequency of its destructive stages; (3) the average yearly peak; and (4) the damage that will be likely to accrue from a channel overflow; as the fourth factor is a function of the first or the second, the importance of wisely forecasting the probable flood crests warrants patient labor and investigation.

Causes of Floods

Floods result from (*a*) intensive rainfall; (*b*) prolonged storms that produce a combination of swollen tributaries; (*c*) rapid melting of snow such as accompanies warm rains; or (*d*) sudden release of impounded water. Even the last named may operate without human agency, as, for instance, when an ice dam breaks, or when a landslide forms a temporary impounding dam, to be cut through after the main stream has gathered greater volume and power. Too often floods are traceable to errors of judgment, or perhaps to the demand for economy, as in the cases of restricted spillways, reduced free-board for dams of the non-overflow type, and infringement on the river channel by permanent improvements.

Destructive Stages

The highest flood stage of a stream ceases to be destructive when the endangered elements are either placed beyond its scope or protected from the

violence of the current. The beneficent habits of the Nile would undoubtedly have been known as "destructive floods" if the ancients had established their homes and perishable property in the area subject to inundation, if they had arbitrarily encroached with levees on the required effective channel, or if occasionally the high flood stage had occurred at harvest time instead of during the planting season. Thus, the elements of unpreparedness and helplessness of victims overpowered by a relentless force distinguish a calamity from an awe-inspiring display of latent power in proper control.

Within the past decade communities, after suffering property losses of several million dollars, have expended similarly large amounts for positive insurance against a recurrence. Just such calamities seem to be essential occasionally to crystallize public opinion in preventing channel encroachments that menace property and human lives. It is most regrettable that unscourged communities, exposed to the same peril, exhibit a hesitant attitude toward investigations based on new advances in the science of flood protection.

Rainfall Intensity and Resulting Run-Off

If 1 in. of rainfall runs off uniformly in 1 hour, then each acre will yield 1 sec-ft., or each square mile will yield 640 sec-ft. If the shape and slopes of the drainage basin, the lengths of the various tributaries, and the direction of movement of the storm unite to favor the worst combination, then the yield at some point might be a multiple of this rate. In most cases, due to absorption, percolation, and detention, the percentage that runs off is so low as to neutralize this cumulative effect of synchronous crests at junction points; yet there are apparently authentic records of yields from small areas mounting to 1 000 or even 4 000 sec-ft. per sq. mile, continuing at such stages for only a few minutes, but meanwhile causing incredible damage.

In precipitous localities, especially those denuded of timber, undergrowth, and sod, the first storms that fall after a period of drought seem to yield a greater percentage of run-off than subsequent ones; a sudden penetration of moisture is resisted by the parched and dusty surface. Perhaps the outer inch or two of soil may be saturated and partly eroded, leaving the greater depths entirely dry. The undercutting of such banks along the gullies adds to the weight and momentum of the moving mass, and even retards percolation into the gravel and sand beds along the channel. The first small rivulet, preceding the main body like a pilot, may lubricate the trough so as to increase the natural velocity of the rising torrent, and may finally be overtaken by the flood wave. The great resistance of a rock-strewn channel retards the lower parts of the moving mass, and the upper volume then flows over and descends in front of the retarded portion in a rotary motion; thus, the crest, laden with logs and drift, is comparable to a huge Juggernaut as it crushes obstructions along its path. The content of sediment, fine unctuous clay, gravel, stones, and boulders, picked up from the stream bed according to their relative mobilities, serves to increase the specific gravity and destructive battering power of this first wave, and acts like a moving dam holding this shifting force in reserve, and quickly mounting to greater heights if the progress of the front line is checked momentarily. If the barriers hold, the lateral banks

will be overtopped and the divided flood, disarmed of its heavy weapons, will seek out or carve for itself new channels. Examples of the effects of such mountain torrents are shown in Figs. 1 to 5. The value of such occurrences as warnings to communities similarly situated may even exceed the amount of damage caused if only steps are taken either to prevent a recurrence, or to control, guide, and dissipate the threatening crests.

Diverse Flood Flows

Between the combined avalanche and flood flow here described and the well ordered habits of other rivulets, especially those that have their sources largely in springs, or in snow deposits well protected against sudden thaws, there exists an endless variety of conditions. A north slope, a deep rocky gorge that entraps heavy drifts and provides security against direct sunlight, or topographic features such as mountain peaks on the windward side, serving alike to deflect the "chinooks" and the moisture-laden air currents, cooling them, and transforming the potential rain into snow, may modify run-off habits. As a result some main tributaries have flood problems of limited scope —rarely turbid and never departing far from mean flow—whereas others vary from a dry-channel condition to a peak exceeding that of the main river.

Such is the Gila River in Arizona; in its dry-channel stage it represents between 50 and 400 sec-ft. of underflow,* while as a maximum, its record has exceeded the Colorado River floods above the junction at Yuma, Ariz. The first severe storms in the summer of 1923 wrought havoc to railroad and highway traffic in various Western States. When a destructive flood crest tore out structures on up-stream tributaries of the Gila and approached the highway bridge at Florence, Ariz., a warning alarm was spread; but this flood never attained dangerous proportions nor even reached the first flood channel at this station, as the underground storage nearly absorbed it. A subsequent flood approaching with no higher crest was deemed of less importance and therefore was unheralded; yet it interrupted traffic at some of the lower crossings, because the underground storage space was already filled.

Rainfall Variants

Just as mountains lying across the path of moisture-laden atmospheric currents deflect them upward with a consequent chilling and condensation, so a sharp canyon or rift in a mountain range often causes an induced upward draft as through a chimney, elevating the warm moist air of the valleys to an altitude where the cold temperature results in copious rainfall.

If the convection currents moving upward have a velocity in excess of about 20 miles per hour, the condensation is blown back and scattered much like spray from a garden hose pointing into a violent wind; when the distribution is uneven some areas near the center of disturbance receive a double portion, while others are deprived of their normal supply. A notable example was the record-breaking rainfall in Central Texas, September 7 to 11, 1921, when 23.98 in. fell in 35 hours at Taylor, somewhat to leeward of the storm

* "The Underground Waters of Gila Valley, Arizona", *Water Supply Paper No. 104*, U. S. Geological Survey, p. 49.

FIG. 1.—JUNCTION OF CANYON DIABLO WITH LITTLE COLORADO RIVER, ARIZONA, SHOWING EFFECTS OF THE SEPTEMBER, 1923, FLOOD.

FIG. 2.—CANYON DIABLO, ARIZONA. FLOOD OF SEPTEMBER, 1923, REACHED ABOUT TO THE CROWN OF THE BRIDGE.

FIG. 3.—WILLARD, UTAH, FLOOD OF AUGUST, 1923. NOTE HEIGHT OF MUD DEPOSIT AGAINST HOUSE.

FIG. 4.—FARMINGTON CANYON, UTAH. THE LARGE ROCK, 25 FT. IN DIAMETER, WAS MOVED 300 YD. BY FLOOD.

FIG. 5.—STATE HIGHWAY NEAR WILLARD, UTAH, AFTER THE FLOOD OF AUGUST, 1923.

center, while Elgin, only 18 miles southward, and actually nearer the center of disturbance, recorded only 4 in. The area receiving more than 5 in. was nearly elliptical, the major axis extending along the pronounced Balcone fault zone for nearly 200 miles, from San Antonio to the Brazos River; Elgin in the midst of 15 and 20-in. rainfall areas had its portion depleted to increase Taylor's abundance, doubtless by the action here described.*

Run-Off Variants

The variations due to the presence or absence of vegetation; to the surface texture and condition; to the probability of sudden thaws, as from warm rains, or of intense precipitation on frozen ground; to the possibility of bank and flood-channel storage; to the direction of travel maintained by prevailing storms; to the altitude, temperature, annual rainfall, special topographical or meteorological features; and to the area, shape, and slopes of the drainage basin—all these variations must be considered in a given problem. The influence of the area appeals to the writer as the logical basis for a general formula, with the effect of the other variants grouped in the numerical coefficient corresponding to Kutter's or Chezy's C.

Determination of Basic Run-Off Formulas

A careful study of diagrams on which are platted data for the yield, q, in second-feet per square mile of drainage area, as ordinates against the drainage areas, M, in square miles, as abscissas, indicates the relation:

$$q = \frac{C}{M^{\frac{1}{n}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which C and n are constants.

If there is required, not the yield per square mile, but the total discharge from the basin, designated Q, it will be necessary to multiply each member of Equation (1) by M, that is,

$$q\,M = \frac{C\,M}{M^{\frac{1}{n}}}, \text{ or } Q = C\,M^{\frac{n-1}{n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

which is one of the most convenient forms for practical field use.

Another easily applied formula results from dividing each member of Equation (2) by the assumed mean velocity to be attained at the section considered; the left-hand member will be the prescribed area of waterway, a, in square feet. Assuming a velocity of 10 ft. per sec., for example, and reducing C correspondingly to C', there results:

$$\frac{Q}{10} = a = C'\,M^{\frac{n-1}{n}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

This fundamental form is followed, with slight variations, by many of the widely used and satisfactorily tested formulas for run-off or for area of

* "The Floods of Central Texas in September, 1921", *Water Supply Paper No. 488*, U. S. Geological Survey.

waterway, with n taking values between 1.5 and 6 generally, giving rise to expressions, such as $\sqrt[6]{M^5}$, and others much more complicated.

The bewildering number of factors, constants, and variants which have been so ingeniously devised to make the various formulas fit the needs of given conditions, apparently should be capable of reduction to a common denominator, so to speak, a means of translation without mastering each nomenclature or becoming an adept in the use of each of the various forms.

In devising a proper formula for run-off, the Myers type will be tentatively selected for modification or extension, and correlated with the recorded flood run-offs, as platted on Plate IX. In the Myers formula $a = C\sqrt{A}$, in which, A equals the area of the drainage basin, in acres, and C is a coefficient varying from 1 to 4. Transforming the area into square miles:

$$a = C \times 25.3 \sqrt{\frac{A}{640}} = C \times 25.3 \sqrt{M}$$

This agrees with Equation (3) if $C' = 25.3\ C$ and $n = 2$.

Trying a value of C equal to 40, giving the product of $C \times 25.3$ as approximately 1 000, and assuming the velocity to be 10 ft. per sec., the resulting equation becomes:

$$Q = a\ V = 10\ 000 \sqrt{M}$$

The resulting graph for Q (Plate IX) is found to lie above all the platted points representing flood run-off rates except two, and to conform closely with the enveloping curve, thus justifying the assumption of C as 40 for a maximum.

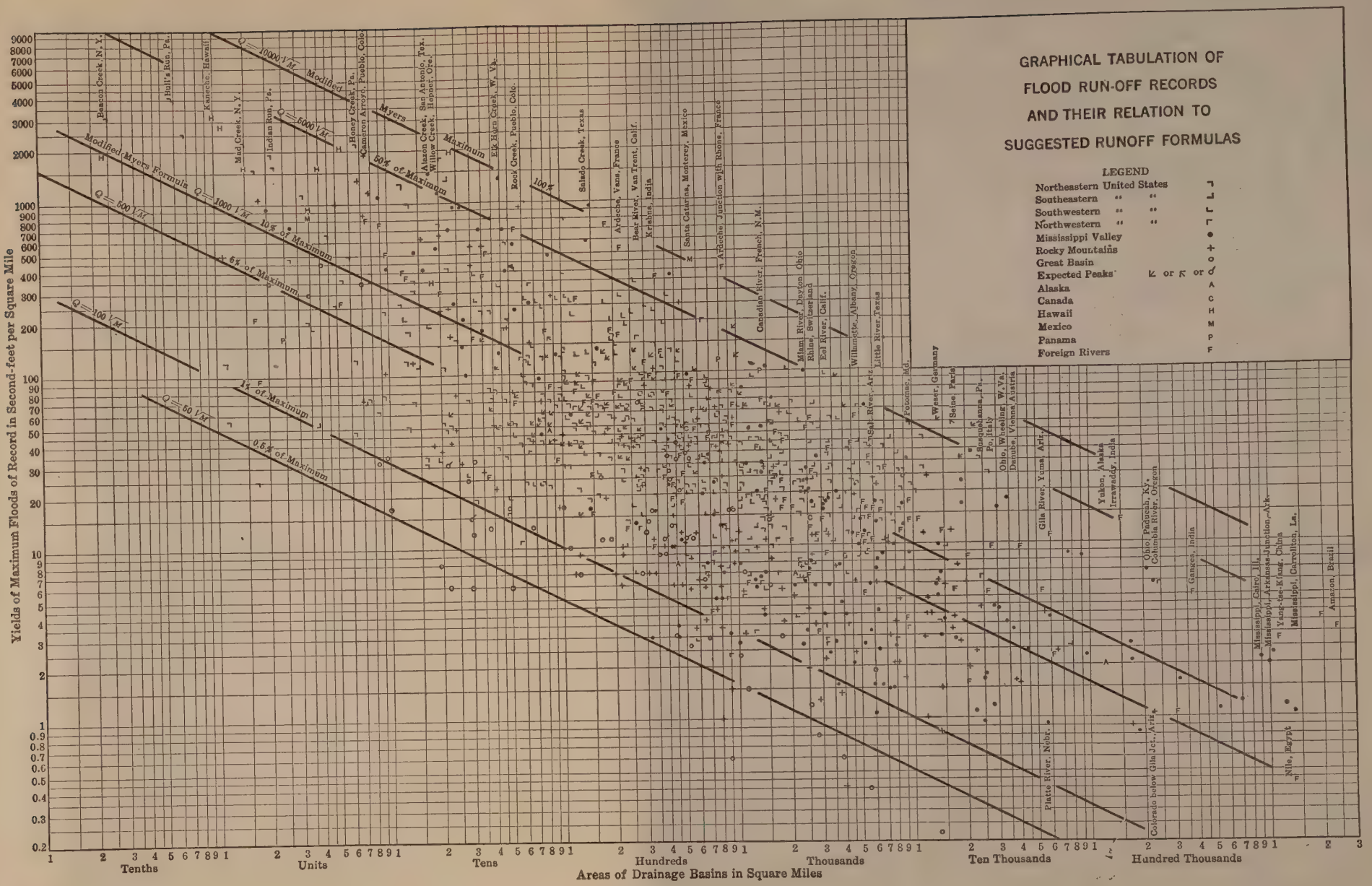
Similarly, for the lower limit, the path of the point, $Q = 100 \sqrt{M}$, conforms well with the general slope, and lies above only 5% of the more than 1 000 platted flood run-off rates. This treatment suggests the idea that the modified Myers formula might be considered as $Q = R \sqrt{M}$, in which, R is the expected rate of run-off, in second-feet, from 1 sq. mile, varying between 100 and 10 000.

To determine the required cross-sectional area of bridge opening, the allowable velocity would be used as a denominator, and,

$$a = \frac{R}{V} \sqrt{M} = S \sqrt{M}$$

in which, S is the coefficient for the sectional area of waterway.

Another method of comparing discharges would be according to the percentage, p, of the maximum for the corresponding area obtained from the modified Myers formula, $Q = 10\ 000 \sqrt{M}$, which percentage is readily scaled for any point platted, as shown on Plate IX; in that case the coefficient, R, in the previously given equation becomes $10\ 000\ p$; and $S = 1\ 000\ p$, if $V = 10$. Thus, the Colorado River below the Gila junction at Yuma would rate 5%; the Nile would be expressed as 4%; the Amazon, 50%; the Mississippi at Cairo, 21%; the Ohio at Paducah, Ky., 32%; the Gila both at Yuma and Florence, Ariz., 10%; the Salt River, prior to its control by storage reservoir, at the mouth and at Roosevelt, Ariz., 27%; Salado Creek, Texas, 117%, the highest rate platted; the Miami, Dayton, Ohio, 50%; Otay, Lower Otay Dam (causing failure), Calif., 40 per cent.


GRAPHICAL TABULATION OF
FLOOD RUN-OFF RECORDS
AND THEIR RELATION TO
SUGGESTED RUNOFF FORMULAS
LEGEND
Northeastern United States
Southeastern " "
Southwestern " "
Northwestern " "
Mississippi Valley
Rocky Mountains
Great Basin
Expected Peaks
Alaska
Canada
Hawaii
Mexico
Panama
Foreign Rivers
Yields of Maximum Floods of Record in Second-feet per Square Mile
Areas of Drainage Basins in Square Miles
Tenths
Units
Tens
Hundreds
Thousands
Ten Thousands
Hundred Thousands
Modified Myers Formula
Myers
Maximum
50% of Maximum
10% of Maximum
5% of Maximum
1% of Maximum
0.5% of Maximum
Beacon Creek, N. Y.
Bull's Run, Pa.
Kaneohe, Hawaii
Mad Creek, N. Y.
Indian Run, Pa.
Bonny Creek, Pa.
Cameron Arroyo, Pueblo, Colo.
Alazon Creek, San Antonio, Tex.
Willow Creek, Hooper, Ore.
Elk Horn Creek, W. Va.
Rock Creek, Pueblo, Colo.
Salado Creek, Texas
Aydeche, Vans, France
Bear River, Van Trent, Calif.
Krishna, India
Santa Catarina, Monterey, Mexico
Arfuche Junction with Rhone, France
Canadian River, French, N.M.
Miami River, Dayton, Ohio
Rhine, Switzerland
Eel River, Calif.
Willamette, Albany, Oregon
Salt River, Ariz.
Little River, Texas
Potomac, Md.
Weser, Germany
Seine, Paris
Susquehanna, Pa.
Po, Italy
Ohio, Wheeling, W. Va.
Danube, Vienna, Austria
Gila River, Yuma, Ariz.
Yukon, Alaska
Irrawaddy, India
Ohio, Pittsburgh, Ky.
Columbia River, Oregon
Ganges, India
Mississippi, Cairo, Ill.
Mississippi, Arkansas Junction, Ark.
Yang-tse-Kiang, China
Mississippi, Carrollton, La.
Nile, Egypt
Amazon, Brazil
Platte River, Nebr.
Colorado below Gila Jct., Ariz.

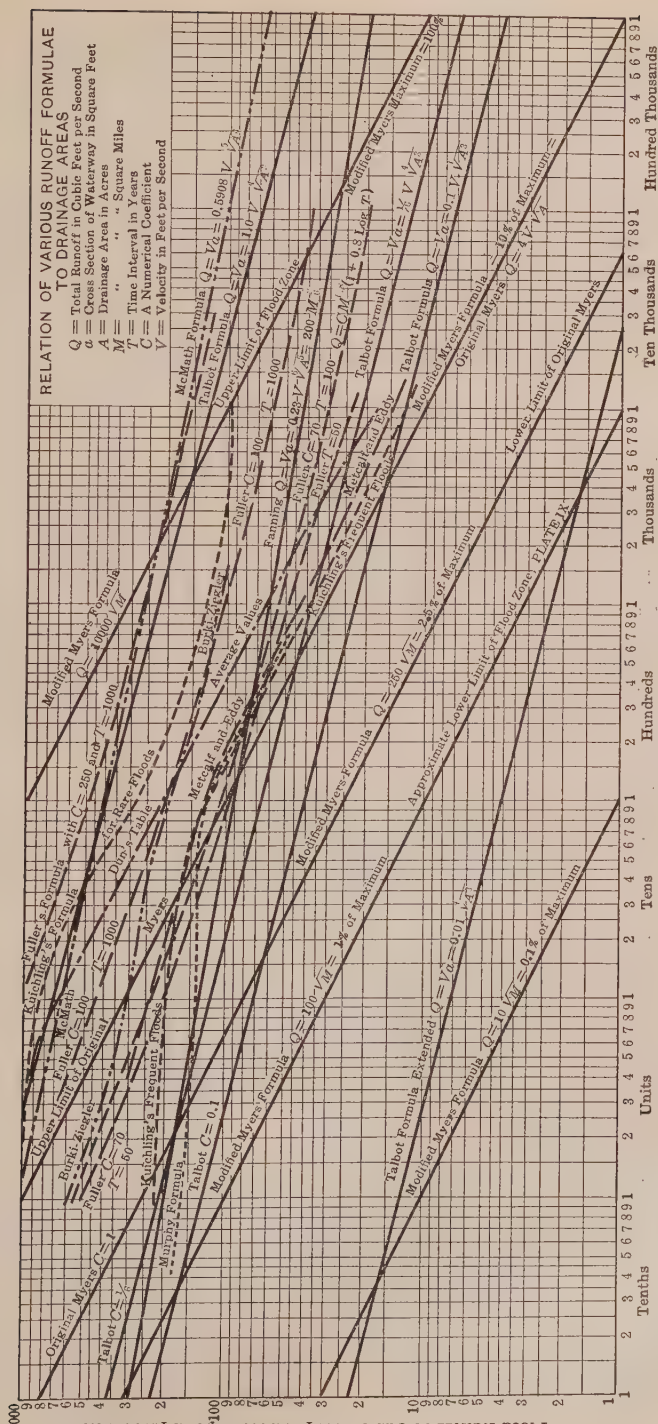

RELATION OF VARIOUS RUNOFF FORMULAE TO DRAINAGE AREAS
Q = Total Runoff in Cubic Feet per Second
a = Cross Section of Waterway in Square Feet
A = Drainage Area in Acres
M = " " " Square Miles
T = Time Interval in Years
C = A Numerical Coefficient
V = Velocity in Feet per Second
Flood Runoff in Cubic Feet per Second per Square Mile
Drainage Area in Square Miles (M)
Tenths
Units
Tens
Hundreds
Thousands
Ten Thousands
Hundred Thousands
McMath Formula Q = Va = 0.5908 V √A
Talbot Formula Q = Va = 1.10 V √A
Upper Limit of Flood Zone
Modified Myers Formula Q = 10000 √M
Fuller's Formula with C = 250 and T = 1000
Kuichling's Formula
McMath
Fuller C = 100
Upper Limit of Original
Burki-Ziegler
Fuller C = 70
T = 50
Original Myers C = 1
Talbot C = ½
Kuichling's Frequent Floods
Murphy Formula
Talbot C = 0.1
Modified Myers Formula Q = 100 √M = 1% of Maximum
Modified Myers Formula Q = 10 √M = 0.1% of Maximum
Talbot Formula Extended Q = Va = 0.01 V √A
Approximate Lower Limit of Flood Zone
Modified Myers Formula Q = 250 √M = 2.5% of Maximum
Kuichling's Frequent Floods
Metcalf and Eddy
Talbot Formula Q = Va = ½ V √A
Modified Myers Formula Maximum = 100%
Original Myers Q = 4 V √A
Modified Myers Formula = 10% of Maximum =
Original Myers Q = 10 V √A
Lower Limit of Original Myers
Fuller C = 100
Fanning Q = Va = 0.23 V √A = 200 M
Fuller C = 70
T = 100
Q = CM (1 + 0.8 Log T)
Fuller C = 50
Metcalf and Eddy
Average Values
Burki-Ziegler
Fuller C = 100
T = 1000
Dun's Table
for Rare Floods
Myers
Metcalf and Eddy
PLATE IX

FIG. 6.

There may be other formulas just as capable of extension and modification to cover the entire field, but it will be difficult to find one more readily applicable, and agreeing with the platted run-offs more consistently than the modified Myers formula. In Fig. 6 is graphically illustrated the relationship between the best-known expressions for run-off.

In some cases a first approximation of run-off for a given district may be obtained by the use of data from Plate IX, or from Table 2 (Appendix I), which is the same information in statistical form. As opportunity affords, further computations may be based on all the usual factors, representing expected rainfall intensities and duration, percolation, evaporation, detention, bank storage, slopes, forestation, tillage, elevation, shape of drainage basins, the time elements, geological formations, and other tangible influences.

Variable Factors

Among the considerations too often neglected, and so variable and elusive as to have escaped notice in standard formulas, are the following.

1.—Inherent Similarity.—Flood problems for small drainage areas may be essentially the same in various districts, even under widely dissimilar climatic conditions. If it be recognized that extreme rainfall and run-off rates for the arid and the humid regions of the United States, and, in fact, for nearly all known regions, are nearly identical, then there is a common basis for estimates. Whether in desert regions or on the coastal plains, however, the resulting run-off varies, being increased in violence by steep slopes and confluent crests from denuded areas, or reduced by detention, evaporation, percolation, and under-flow.

Vital differences do exist as regards the frequency, duration, and extent of such violent storms, thus justifying the classification of the resulting floods as "frequent" or "occasional" for one district, while they are regarded as of rare occurrence for another. The latter case encourages the appropriation and partial occupation of the channel for productive purposes.

2.—Periodic Maxima Expected.—The time element, so useful in connection with flood frequencies, is too often a basis of error. The popular notion is to place the probable occurrence of the 100-year maximum flow at a number of years hence, and, similarly, with the 20-year expected maximum. Either of these is as likely to occur during this year or next as in any other two years of the designated periods. This fact is well illustrated in Table 1, showing the time intervals of great floods.

3.—Factors of Safety.—On most American streams the records are altogether too meager to establish a fair approximation of the maximum flood to be expected in a century or even in a period of two decades. If there is any basis in fact for the theory of storm cycles, the platted curve representing the magnitude of expected floods would doubtless show harmonic variation, with nodes and slight inclinations as well as steep slopes. If the period of observation includes only one of the phases, there is a probability of a large error. Therein is the most potent argument for a physical examination of the watershed, channel, and modifying influences existing or to be provided.

TABLE 1.—TIME INTERVALS OF FLOODS

Stream.	Location.	Years of record.	Mean.	Intervals between Disastrous Floods, in Years.		Notable concentrations.
				Maximum.	Minimum.	
Jones Falls*.....	Baltimore, Md...	150	17	32	2	1837, '42, '58, '60
Los Angeles River†..	California.....	98	8	29	2	1884, '86, '89; 1911, '14, '16
Seine River‡.....	Paris, France ...	265	29	82	3	1879, '82, '85
Mississippi River §...	Cairo, Ill.....	80	3	7	0.25	April, 1912 January, 1913 April, 1913

* *Engineering News-Record,* April 20, 1922, p. 654.

† *Engineering News,* July 9, 1914, p. 95.

‡ *Engineering Record,* February 26, 1910, p. 240.

§ "Hydrologic Record, Mississippi River Floods," by E. N. Chisolm, Jr., Captain, Corps of Engrs., U. S. A., M. Am. Soc. C. E., *Engineering News-Record,* January 18, 1923, p. 112.

As an example, the City of Pueblo, Colo., and the Denver and Rio Grande Railroad Company previous to 1921 had made ample provision against maximum recorded or expected floods, based on the average annual crests, or on the possibility of local storms of the cloudburst type affecting small tributaries concurrently; but the demonstrations of June, 1921, placed this problem in another category. At that time (Fig. 7), there was a sustained violent storm over an area in excess of 1 000 sq. miles for at least 10 hours.

While making provision for increased channel capacity at Pueblo, engineers were astonished to find that the flood might have been of nearly double severity had the storm center been located a few miles farther westward. For this and other reasons, it was recommended that the old channel with a capacity of 40 000 sec-ft. (fully five times the average yearly maximum for the preceding 30 years) be replaced by one including a wide spillway, the combined capacity to be 175 000 sec-ft. The actual observed peak was 105 000 sec-ft.

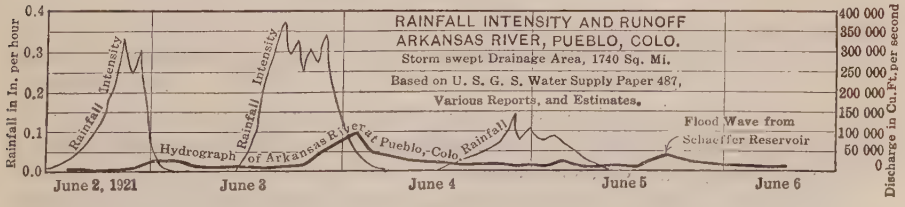


FIG. 7.

As another instance the spillway at the Lower Otay Dam, near San Diego, Calif., was overtaxed in January, 1916, the run-off being eight or ten times the quantity for which it was designed according to the known habits of the water-shed at that time. Such calamities emphasize the need for larger factors of safety to take care of these supposedly remote happenings.

4.—Flood Height vs. Discharge.—Where protective works are established or contemplated without storage the flood height is more important than the actual discharge passing the station. It is well established that a given stage

of a rising flood in a silt-laden stream represents a greater flow than the same height during recession. In Fig. 8 are graphically shown some of the reasons for this phenomenon. It has been claimed for the Colorado River channel at Yuma, that for each foot of rise on the gauge it scours an equal depth from the bottom. Evidently, after the peak has passed another cycle begins, the velocity being retarded, the silt burden depositing, the gradient and the hydraulic radius decreasing, thereby causing further retardation and deflection of currents laterally as if in quest of new material to transport. Although the tendency during the rising phase is to scour the channel bottom and induce central flow, the receding torrent is inclined toward lateral excursions. This often accounts for the fact that the main damage to alluvial farms occurs after the main crest has passed and the patrols have slackened their vigilance.

It remains an open question whether the tragedies too frequently recorded of passenger trains plunging from trestles into swollen streams are due to derailment or to inadequate inspection of the substructure. How is the track walker or inspector viewing the rising flood surface only 1 ft. or 2 ft. higher than the normal stream surface, to know that the channel may have scoured several feet deeper, possibly undermining the piers or piling?

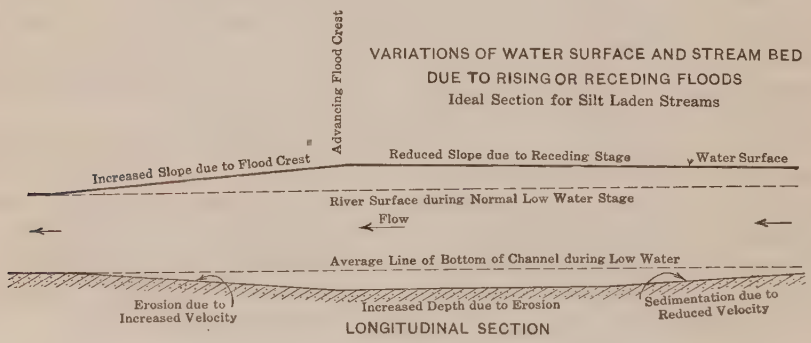


FIG. 8.

While Sir William Willcocks was on a tour through America after his notable success at the Assuan Dam on the Nile, he stated that the 15-ft. gauge height on a rising Tigris flood represents 180 000 sec-ft. of discharge; the 20-ft. stage if at the peak represents only 120 000 sec-ft.; and the 15-ft. stage during recession corresponds to 90 000 sec-ft.*

5.—Maximum Stage as Compared with Twenty-four Hour Average.—The importance of determining maximum flood rates instead of or in addition to the average for 24 hours, is well illustrated by the Pueblo flood (June 3, 1921), where the 24-hour rate was only 110% of the full channel capacity, while the maximum flow, lasting less than 1 hour but accountable for the main loss and damage, was more than twice as great.

In dealing with small areas it must be borne in mind that the maximum flood crest—sometimes coincident with the initial wave—and the return to normal flow may occur within a few hours. For this reason the collected data

* *Engineering Record*, June 27, 1914, p. 723.

for Table 2 (Appendix I), and Plate IX, deal with maximum rates wherever available.

6.—The Wooded Channel.—The regulating influence of great storage dams, such as those built by the U. S. Reclamation Service and enterprising irrigation districts, extends beyond the limits usually assigned. When they begin to function, the improvement of farms near the stream, where the alluvial soil is generally the most productive and at the same time most mellow and exposed to lateral erosion, increases the hazard of flood damage if maximum stage should recur. Even if the reservoir is full to the spillway floor at such a time it is effective, acting as a detention and regulating basin; and it has been conceded on some projects that the 6 to 10 ft. of storage depth above the spillway crest is as important as the entire capacity below that level. Additional depredations surely would have been made by the Colorado River in the Imperial Valley, California, had not the Roosevelt Dam been regulating the Salt River, the Gila's main tributary, when the Gila experienced a record flood in January, 1916, far exceeding any known discharge of the Colorado above Yuma.

If, however, a storm and flood period is protracted, the increased flow over the spillway or the necessity for the operation of emergency outlets to save the main structure, may overcome the natural equalizing effect and transmit the destructive flood stage to the lower river. In that case it would be discovered that the channel capacity has decreased as compared with the years when unrestrained floods limited the growth of vegetation. The islands and bars formerly denuded would have such a well established covering of quick-growing trees as to resist and retard the maximum flood—perhaps even collecting driftwood barriers—forcing the water to seek higher levels. Carefully conducted gaugings* on typical channels have established the fact that for a wooded course Kutter's n increases to nearly double that obtaining for cleared areas.

7.—Structural Design Limits and Flood Peaks.—Granting that the economics of design would not justify doubling the expense to prevent interruption of bridge traffic two or three times a century, it still remains for the Engineering Profession to define just where the excess flow shall be accommodated. The U. S. Bureau of Public Roads anticipates conditions by the following specification:†

"*Waterway.*—The waterway shall be adequate for the passage of ordinary flood water and drift at a velocity not exceeding the average velocity of the stream in the vicinity of the bridge and shall be adequate for the passage of extreme flood water unless other provisions are made for same. By ordinary flood water is meant the stage that is likely to occur once in ten years."

To cross the numerous intermittent flood channels that are in service only a few hours or days each year, the Bureau and several States have adopted standard dips or spillways, consisting of undulations in the highway grade with suitable cut-off walls to prevent erosion or undermining. Comparative unit costs of waterway area provided by various types of drainage structures, indi-

* *Engineering News-Record*, October 12, 1922, p. 599.

† Superstructures, General Clauses, Standard Specifications, U. S. Bureau of Public Roads, as revised July, 1921.

cate the great economy of the dip or spillway. Although it is conceded that this form of construction has its limitations, especially in connection with railroads and large drainage channels, it is believed that there is a broad field for its use in providing for the maximum flood stages that are beyond the economic range of other structures.

8.—The Definite Function of Spillways.—The moment a flood overtaxes its definite channel is the beginning of either a minor episode or a great tragedy. If the rising surface is confined by inadequate artificial embankments until relief is secured only by the formation of crevasses, then the benefit sought by the levee system is more than nullified by the destructive velocity of an impounded flood suddenly released. It would be far better to allow the gradual overflow and less violent inundation of the adjacent lands; perhaps the enrichment of the soil due to an occasional quiet sedimentation would go far toward repairing the loss and inconvenience, if only the land owner has planned his crops and plantings, his permanent improvements, and his pastures for live-stock so as to insure their safety or the minimum damage from such an expected overflow.

Sir William Willcocks, already quoted, evidently gave his own views of what should be done to control the Mississippi River more effectively, when he described the best practice of the ancients. He was altogether appreciative of the wonderfully effective work of the Corps of Engineers, U. S. Army, and of the civilian personnel, and reiterated the idea that an occasional relief through crevasses is inevitable unless some form of multiple spillway along one bank or both shall begin to function automatically whenever the flood stage passes the limit for which ample provision has been made.

The alarm following the moderate but protracted floods of 1922 that passed the Arkansas Junction and registered the greatest flow ever recorded at Carrollton, La., is really a commendation for effective construction and maintenance of a great levee system, and gives point to the argument of Sir William Willcocks for automatic spillways, regulated overflow channels, and valley storage. Examinations of the actual projects on the Mississippi will disclose that all the foregoing features are contemplated in some degree; but where a stage higher than 50 ft. continues for 50 days, as occurred even at Cairo during that year, the use of farm lands for storage almost amounts to their abandonment.

Applying the principles herein enumerated to projects under civilian control, it would seem like a measure of safety to compute the profile of an assumed or estimated flood crest, which should be some multiple of the present channel capacity, in many cases double or treble. If the resulting lateral flow would overtax structures under highways, railroad embankments, or other barriers, the question would be to determine the possibility of their sudden failure, and the changes required to provide a safe automatic discharge with minimum damage.

When destructive floods occur at other times than during the planting or growing season and consist of sharp crests carrying much sediment, there are strong arguments for not only allowing but insuring the action of wooded channel spillways, thus providing against scouring velocities. The recurrent

quiet sedimentation on the overflowed lands would add fertility and cause them to rise at least proportionately with the river delta.

Review of Flood Problems Desirable

As a result of floods during recent decades and especially during 1923, a re-appraisal of the attendant dangers should be made for many of the river channels.

After a flood the stricken community is confronted with the problem of abandoning the inundated area in whole or in part, or of insuring it against future damage. This latter solution may be accomplished either by positive control—that is, regulation of maximum crests—or by adequate protective works. Conditions may favor (1) regulation by detention basins, ordinary reservoirs, or those purposely located to favor outflow through underground channels; (2) diffusion of the water as by spreading it over detrital cones, gravel bars, or coarse sandy areas to replenish underground storage; (3) establishing timber cribs or other barriers as overfall dams to break the continuous gradient, to check a rolling initial wave and temporarily arrest the heaviest burden; (4) planting willow and timber copses to entrap sediment; or (5) diffusing the surplus by spilling it into pasture or woodlands. All these methods are based on retardation. The case may demand the widening, rectification, and clearing of the channel to promote accelerated flow, or it may warrant a combination of the two general methods—truly, a wide field for investigation. The number of possible solutions of such problems forms a maze through which the proper path is difficult to discover, requiring the best talent and the most mature judgment that experience and special training can develop.

Need for Unified Control

So complex and interlocked are the community interests to be served in the control of streams that there is scant opportunity for the rough and ready, independent, private initiative which pioneers had to develop in order to survive. According to present paternalistic policies, it seems imperative that this control become a State or municipal function for small streams, even as it is now administered under Federal jurisdiction for navigable rivers.

The shortcomings of the present methods are forcibly illustrated by a case that occurred during 1923. When a torrential flood had started to ravel the protective rip-rap and eat into city property, a mass meeting was called, at which the State Engineering Department, represented in an advisory capacity, recommended the anchoring of newly cut trees, with branches intact, as current deflectors and retards, and the placing of brush and rock revetment among hurriedly driven piles and stakes. In earlier days this very plan would have been carried out and reported as accomplished before the hour for mass meetings; but in the present case by the time each citizen had extolled his individual plan, the majority sentiment seemed to favor the use of rock because of its durable character. This disregard of sound engineering advice for wasteful emergency measures resulted in the continued progress of erosion, with the loss of all the new rip-rap, several city lots and comfortable homes, an

appeal to the Government for relief, and the acceptance of a plan furnished by a competent specialist requiring a timber pile jetty with brush and rock-filling to cover the scar and prevent further incision.

Opportunity for Public Service

The possibilities of the present situation must appeal to all members of the profession who are experienced and competent in flood protection, that is, the privilege of contributing to community welfare in determining whether great hazards do exist and what special protective measures are required. If thereby local officials are awakened to the reasonable needs, and relief projects are initiated, one step toward safety will have been accomplished; the danger zone of floods will be known and provision will be made for an emergency. If even a small percentage of the annual disasters should be averted, or the loss and damage reduced by wise preparation, the cost and labor involved in such studies will be fully justified.

Conclusion

In this paper, the writer does not assume to solve any particular problem, but merely lists certain data and considerations that are often left unrelated or neglected entirely. His purpose will be accomplished only when the essential suggestions herein recorded, or to be brought forth during the discussion, have resulted in a definite campaign to determine flood dangers and the advisable measures for control. The most positive step yet taken seems to be the studies involving the expected run-off for California streams made by the State Engineering Department.*

Specific acknowledgments have been made wherever practicable in the paper. In addition, the writer has obtained invaluable information from the published writings of the many eminent hydraulic engineers of the United States, most of them members of this Society. These data are included in Table 2 (Appendix I). For the convenience of members and others, the writer has included an alphabetical index, Table 2 (A) (Appendix I), in order that references to streams in Table 2 may readily be found.

Special appreciation is due those who have so generously assisted and encouraged this work, notably Government engineers and officials, State, municipal, and college officers, engineers in private practice, and associates.

* Reference No. (70), Table 2 (Appendix I).

APPENDIX I

TABLE 2.—Maximum Observed Flood Discharge Rates and Expected Run-Off from Various Drainage Basins Arranged According to Area.*

No.	River or tributary and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	Murchison Farm, Jackson, Tenn ...	0.17	1 900 , Apr., 1918	1 000	.....	116	M.V.	19.0
2	Beacon Creek, near Fishkill, N. Y...	0.25	3 200 , July, 1897	.....	.....	27	N.E.	32.0
3	Bull's Run, Long Level, Pa.....	0.58	4 170 , July, 1914	.....	.....	71	N.E.	41.7
4	Docker's Hollow, North Braddock, Pa.	0.60	4 000 , June, 1917	.....	.....	71	N.E.	40.0
5	Mann's Run, Creswell Station, Pa..	0.67	2 540 , July, 1914	.....	.....	71	N.E.	25.4
6	Manoa, West Branch, Oahu, Hawaii	1.00	3 250 , Jan., 1921	.....	.....	78	H.	32.5
6-A	Skyrocket Creek, Ouray, Colo.....	1.00	2 000 , July, 1923	.....	.....	112	R.M.	20.0
7	Manoa, East Branch, Oahu, Hawaii	1.10	2 810 , Jan., 1921	.....	.....	78	H.	29.5
8	China Wash, near Hurricane, Utah..	1.10	500 , Aug., 1916	300	500	74	R.M.	5.2
9	Budlong Creek, near Utica, N. Y....	1.13	120 , Mar., 1904	.....	.....	2	N.E.	1.3
10	Sylvan Glen Creek, New Hartford, N. Y.....	1.18	57 , Mar., 1904	.....	.....	71	N.E.	0.6
11	Mad Creek, Leroy, N. Y.....	1.5	2 300 , May, 1916	.....	.....	71	N.E.	28.2
12	Nuuanu Reservoir No. 4, Oahu, Hawaii.....	1.5	1 600 , Feb., 1907	.....	.....	78	H.	19.6
12-A	Magpie Gulch, Near Golden, Colo...	1.5	1 270 , July, 1923	.....	.....	112	R.M.	15.5
13	Pequest, Hunt's Pond, N. J.....	1.7	25 , ... 1904	.....	.....	71	N.E.	0.3
14	Green Branch, Bridgeville, Pa.....	1.7	1 595 , July, 1914	.....	.....	83	N.E.	20.8
15	South Arroyo, near Pueblo, Colo....	1.8	1 060 , June, 1921	.....	.....	68	R.M.	14.2
16	Provo (Head-waters), Wall Lake, Utah.....	2.0	350 , June, 1911	200	350	74	G.B.	5.0
17	Cherryvale Creek, Cherryvale, Kans.	2.0	930 , ...	.....	.....	31	M.V.	13.2
18	Indian Run, Letort, Pa.....	2.1	1 930 , ...	.....	.....	71	N.E.	29.0
19	Canodochly Branch, East Prospect, Pa.....	2.2	1 630 , July, 1914	.....	.....	71	N.E.	24.2
20	Rio Grande, near Culebra, Panama..	2.4	161 , ...	.....	.....	32	P.	2.5
20-A	Missouri Canyon, near Masonville, Colo.	2.4	1 820 , June, 1923	.....	.....	112	R.M.	28.2
21	Colvin Run, Grindstone, Pa.....	2.7	480 , July, 1912	.....	.....	91	N.E.	7.9
22	San Pedro Creek, San Antonio, Tex..	2.7	750 , Sept., 1921	.....	.....	67	M.V.	13.3
23-A	Arroyo, near Santa Fe, N. M.....	3.2	600 , Sept., 1924	200	500	74	R.M.	10.7
24	Starch Factory Creek, N. Y.	3.4	209 , Sept., 1905	130	.....	71	N.E.	3.9
25	Provo, Washington Lake, Utah.....	3.4	300 , June, 1911	200	300	74	G.B.	5.5
26	Estanguela, near Monterey, Mexico.	3.5	825 , Aug., 1909	300	1 700	35	M.	31.8
27	Switzer Canyon, San Diego, Calif...	3.5	188 , Jan., 1916	.....	....	66	S.W.	3.5
28	Nuuanu, Oahu, Hawaii.....	3.5	952 , Jan., 1921	.....	.....	78	H.	17.8
28-A	Honey Creek, West Fork, New Carlisle, Ohio.....	3.5	1 000 , July, 1918	.....	.....	109	M.V.	18.7
29	Venison Branch, near Mulga, Ala...	3.9	53 , ...	.....	.....	11	S.E.	1.0
30	North Canyon, near Centerville, Utah	4.0	450 , Aug., 1923	50	450	74	G.B.	9.0
31	Reel's Creek, near Dearfield, N. Y..	4.4	67.1 , June, 1903	55	.....	28	N.E.	1.4
32	Kaukonahua, Upper Dam, Oahu, Hawaii	4.5	1 605 , Jan., 1921	.....	.....	78	H.	34.0
33	Mad Brook, Sherbourne, N. Y.....	5.0	262 , Sept., 1905	.....	.....	91	N.E.	5.9
34	Hull's Gulch, Boise, Idaho.....	5.0	1 000 , July, 1913	.....	.....	91	R.M.	22.3

* Table 2 has been revised to include all corrections noted by discussors, by engineers of the U. S. Geological Survey and others, and information obtained subsequent to the original publication in *Proceedings*. Item numbers followed by letters indicate new data. Corrections, revisions, and the necessary re-arrangements have been made.

† Locations as given in Column (8) of Table 2, are abbreviated, as follows: Northeastern United States, N.E.; Southeastern, S.E.; Southwestern, S.W.; Northwestern, N.W.; Mississippi Valley, M.V.; Rocky Mountains, R.M.; Great Basin, G.B.; Alaska, A.; Canada, C.; Hawaii, H.; Mexico, M.; Panama, P.; and foreign, F.

TABLE 2.—(*Continued*).

No.	River or tributary and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
35	Breakbeck Run, Bullskin Tp., Pa...	5.2	310 , May, 1902	.....	.....	91	N.E.	7.1
36	Kaneohe, Oahu, Hawaii.....	5.3	2 070 , Jan., 1921	.....	.....	78	H.	47.6
37	Brush Creek, Jeanette, Pa.....	6.0	500 , July, 1903	.....	.....	91	N.E.	12.2
37-A	Hogan's Gulch, Eden, Colo.....	6.1	1 580 , Aug., 1904	.....	.....	112	R.M.	39.0
38	Skinner Creek, Mannsville, N. Y....	6.4	124 , July, 1891	.....	.....	71	N.E.	3.1
39	Cold Spring, Brook, Mass.....	6.4	48 , ... 1886	.....	.....	71	N.E.	1.2
40	Blue Ribbon Creek, Pueblo, Colo....	6.7	1 360 , June, 1921	.....	.....	68	R.M.	35.2
41	Honey Creek, East Fork, New Carlisle, Ohio.....	6.7	2 210 , July, 1918	.....	.....	71	M.V.	57.2
42	Farmington Canyon, Farmington, Utah.....	7.0	350 , Aug., 1923	70	350	74	G.B.	9.3
43	Templeton Gap, Colorado Springs, Colo.....	7.1	862 , May, 1922	.....	.....	76	R.M.	23.0
44	Cameron Arroyo, near Pueblo, Colo.	7.3	1 900 , June, 1921	.....	.....	68	R.M.	51.4
45	Camp Branch, Ensley, Ala.....	7.4	69 , ... 1909	.....	.....	11	S.E.	1.9
45-A	Sawpit Canyon, Los Angeles, Calif.	7.4	550 , ... 1889	.....	.....	107	S.W.	15.0
46	Osteen Arroyo, near Pueblo, Colo..	7.8	1 160 , June, 1921	.....	.....	68	R.M.	32.4
47	Croton, South Branch, N. Y.....	7.8	74 , ... 1869	.....	.....	71	N.E.	2.1
48	Burgoon's Run, Pennsylvania.....	8.0	400 , May, 1894	.....	.....	91	N.E.	11.3
49	Independence Creek, Calif.....	8.5	34 , ...	27	.....	69	G.B.	1.0
50	Israel, Jefferson Highlands, N. H...	8.7	63 , ...	41	.....	69	N.E.	1.9
51	Arroyo, Indiole, N. Mex.....	8.9	1 105 , July, 1915	.....	.....	82	M.V.	33.0
52	Fred Rohr Gulch, Pueblo, Colo.....	9.3	104 , June, 1921	.....	.....	68	R.M.	3.2
53	Woodhull Reservoir, Herkimer, N.Y.	9.4	78 , ... 1869	.....	.....	71	N.E.	2.4
54	Mill Brook, near Edmeston, N. Y...	9.4	241 , Sept., 1905	.....	.....	1	N.E.	7.4
55	Dry Run, Ohio.....	9.8	460 , ... 1912	.....	.....	69	M.V.	14.4
56	Baker Creek, Baker, Nev.....	10.0	17 , ... 1914	.....	.....	72	G.B.	0.5
57	Spring Creek, Harrisburg, Pa.....	11.6	259 , Feb., 1908	.....	.....	69	N.E.	8.8
58	Honey Creek, East Fork, New Carlisle, Ohio.....	11.8	1 285 , July, 1918	500	1 200	71	M.V.	44.2
59	Boulder Creek, near Julian, Calif..	12.0	217 , Jan., 1916	.....	.....	66	S.W.	7.5
60	Mill Creek, Erie, Pa.....	12.9	1 000 , Aug., 1915	.....	.....	91	N.E.	35.9
61	Goodyear Creek, Goodyear Bar, Calif.	12.9	120 , Dec., 1913	.....	.....	99	S.W.	4.3
62	Stony Brook, Boston, Mass.....	12.9	121 , ...	.....	.....	72	N.E.	4.3
63	Tomer, above Reservoir, Holyoke, Mass.....	13.0	52 , ...	33	.....	69	N.E.	1.9
64	Manhan River, Holyoke, Mass	13.0	182 , Feb., 1900	.....	.....	91	N.E.	6.6
65	Connoquenessing Creek, Oakland, Pa.....	13.6	315 , Aug., 1903	.....	.....	91	N.E.	11.6
66	Chandler Creek, near Pueblo, Colo..	13.6	118 , June, 1921	.....	.....	68	R.M.	4.3
67	Canodochly Creek near Mouth, Pa..	13.9	359 , July, 1914	.....	.....	80	N.E.	13.4
‡68	Great River, Westfield, Mass.....	14.0	71 , ...	.....	.....	72	N.E.	2.7
69	Panther Creek, Iowa.....	14.0	520 , June, 1905	.....	.....	91	M.V.	19.4
70	Grand Central, below Forks, Alaska.	14.6	100 , ... 1906	.....	.....	72	A.	3.8
71	Rocky Creek, near Ellisville, Miss..	15.0	1 110 , May, 1882	.....	.....	91	M.V.	43.1
72	North Arroyo, near Pueblo, Colo...	15.6	619 , June, 1921	.....	.....	68	R.M.	24.4
73	Rio des Perca, near St. Louis, Mo...	15.6	410 , Aug., 1915	.....	.....	81	M.V.	16.2
73-A	Le Noir Coulee, Malta, Mont.....	16.0	538 , June, 1906	.....	.....	120	R.M.	21.5
74	Broad Creek, Millgreen, Md.....	16.4	31 , ...	17	.....	69	N.E.	1.3
75	Arroyo Seco, near Pasadena, Calif.	16.4	343 , Feb., 1914	190	300	99	S.W.	13.9
76	Williamstown, Williamstown, N. Y.	16.5	34 , ...	.....	.....	72	N.E.	1.4
77	Alazon Creek, San Antonio, Tex....	17.1	1 515 , Sept., 1921	.....	.....	67	M.V.	62.6
78	Wahaiwa Waste, Oahu, Hawaii.....	17.5	343 , Jan., 1910	.....	.....	78	H.	14.3
79	Ford Creek, Augusta, Mont.....	18.0	40 , June, 1908	22	.....	99	R.M.	1.7
80	Little Devil's Creek, Iowa.....	19.0	560 , June, 1905	.....	.....	91	M.V.	24.4
81	Yuba, Bowman Dam, Calif.....	19.0	32 , ...	.....	.....	72	S.W.	1.4
82	City Creek, Salt Lake City, Utah...	19.2	8 , ...	6	50	69	G.B.	0.4
83	Rush Creek, near Pueblo, Colo.....	19.6	238 , June, 1921	.....	.....	68	R.M.	10.5

‡ Maximum daily flow.

TABLE 2.—(*Continued*).

No.	River or tributary and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
84	Martinez Creek, San Antonio, Tex..	19.6	1 223 , ...	.....	.....	71	M.V.	54.1
85	Ridley Creek, near Philadelphia, Pa.	20.0	750 , Aug., 1843	400	900	77	N.E.	33.5
86	Chase Creek of Gila River, Arizona.	20.0	647 , Dec., 1906	200	500	91	R.M.	28.9
87	Willow Creek, near Heppner, Ore..	20.0	1 800 , June, 1903	250	900	19	N.W.	80.5
88	Croton, West Branch, N. Y.....	20.5	54 , Jan., 1874	.....	.....	72	N.E.	2.5
89	Beaverdam Creek, Altmar, N. Y....	20.7	111 , ...	.....	.....	72	N.E.	5.0
90	Israel, below South Branch, N. H....	21.2	49 , ...	34	.....	69	N.E.	2.3
91	Mill Creek, Salt Lake City, Utah....	21.3	6 , May, 1912	4	30	90	G.B.	0.3
92	Brush Hollow Creek, Pueblo, Colo.	21.9	243 , June, 1921	.....	.....	68	R.M.	11.4
93	Crum Creek, near Philadelphia, Pa.	22.0	410 , Aug., 1843	.....	.....	77	N.E.	19.2
93-A	Little Tonawanda Creek, Linden, N. Y.....	22.0	114 , May, 1910	.....	.....	113	N.E.	5.3
94	Cane Creek, Bakersville, N. C.....	22.0	1 341 , May, 1901	400	900	34	S.E.	62.8
95	Penitencia Creek, San Jose, Calif...	22.0	.....	40	59	70	S.W.	2.8
96	Coal Creek near Pueblo, Colo.....	22.3	167 , June, 1921	.....	.....	68	R.M.	7.9
97	Dry Run, Decorah, Iowa.....	22.3	720 , Mar., 1915	.....	.....	91	M.V.	34.2
98	Ritchie Gulch, near Pueblo, Colo...	22.6	41 , June, 1921	.....	.....	68	R.M.	1.9
99	Trout Brook, Centerville, N. Y.....	23.0	51 , ...	.....	.....	72	N.E.	2.4
100	Wailua, near Lihue, Kauai, Hawaii..	23.0	1 955 , Jan., 1921	.....	.....	78	H.	93.7
101	Rio des Perca, St. Louis, Mo.....	23.8	256 , Aug., 1915	.....	.....	81	M.V.	12.5
102	Apache Creek, San Antonio, Tex....	23.8	948 , Sept., 1921	.....	.....	67	M.V.	46.3
103	Mill, North Fork, Pinkbed, N. C.....	24.0	49 , ...	32	250	69	S.E.	2.4
104	Six-Mile Creek, near Pueblo, Colo..	24.6	77 , June, 1921	.....	.....	68	R.M.	3.8
105	Trout Brook, Brooksport, N. Y.....	25.0	158 , ...	.....	.....	69	N.E.	7.9
106	Pequonnock, Bridgeport, Conn.....	25.0	157 , July, 1905	.....	.....	1	N.E.	7.9
107	Middle Creek, Bozeman, Mont.....	26.0	31 , ...	16	150	69	R.M.	1.6
108	Smith Creek, Augusta, Mont.....	26.0	31 , June, 1907	23	180	69	R.M.	1.6
109	Boggs Creek, near Pueblo, Colo.....	26.0	582 , June, 1921	.....	.....	68	R.M.	9.7
110	Olmos Creek, San Antonio, Tex.....	26.4	948 , Sept., 1921	.....	.....	67	M.V.	48.6
111	Spring Creek, above Dayton, Ohio..	27.0	210 , Mar., 1913	.....	.....	91	M.V.	10.9
112	Bear Grass Creek, Louisville, Ky...	27.5	100 , ... 1908	.....	.....	40	M.V.	5.2
113	Donnells Creek, above Dayton, Ohio.	27.5	147 , Mar., 1913	.....	.....	91	M.V.	7.7
114	Little Cottonwood, Salt Lake City, Utah.....	27.7	25 , June, 1912	15	60	87	G.B.	3.2
115	Cottonwood Creek, near Fresno, Calif.....	28.0	.....	20	26	70	S.W.	1.4
116	Wantuppa Lake, Fall River, Mass..	28.5	72 , ... 1875	.....	.....	72	N.E.	3.8
117	Emigration Creek, Salt Lake City, Utah.....	29.0	6 , ...	3	20	74	G.B.	1.1
118	American Fork, above Power Plant, Utah.....	30.0	12 , May, 1914	10	50	87	G.B.	2.7
119	Pinal Creek, Globe, Ariz.....	30.0	440 , Aug., 1904	.....	.....	91	R.M.	24.1
120	Arroyo Seco, near Pasadena, Calif..	30.5	374 , Feb., 1914	.....	.....	73	S.W.	20.6
121	Donner Creek, near Truckee, Calif..	30.5	32 , Mar., 1907	19	34	69	G.B.	1.9
122	Moosa Creek, near Bonsall, Calif....	31.0	269 , Jan., 1916	.....	.....	66	S.W.	15.0
123	Pequest, Huntsville, N. J.....	31.4	19 , ...	.....	.....	72	N.E.	1.1
124	Ink's Creek, near Red Bluff, Calif..	34.0	.....	120	153	70	S.W.	8.9
125	San Antonio, San Antonio, Tex.....	34.3	691 , Sept., 1921	.....	.....	67	M.V.	40.5
126	Peck's Creek, near Pueblo, Colo....	34.4	564 , June, 1921	.....	.....	68	R.M.	33.1
127	Sawkill, near mouth Kingston, N. Y.	35.0	229 , Apr., 1895	.....	.....	69	N.E.	13.5
128	Turtle Creek, above Dayton, Ohio..	35.0	175 , Mar., 1913	.....	.....	91	M.V.	10.4
129	Whippany River, Whippany, N. J..	38.0	94 , ... 1896	.....	.....	24	N.E.	5.2
	Chalk Creek, Fillmore, Utah..... ...	38.0	13 , ... 1914	10	25	74	G.B.	1.5
131	San Francisquito Creek, California.	38.0	51 , ...	51	62	70	S.W.	3.8
132	Lake Roland, Maryland.....	39.0	230 , ... 1868	.....	.....	91	N.E.	14.4
133	Cuyadutta, Johnstown, N. Y.....	40.0	72 , Spring 1896	.....	.....	71	N.E.	4.6
134	Mill, South Fork, Sitton, N. C.....	40.5	51 , Jan., 1916	39	.....	69	S.E	3.2
135	Red Creek, near Pueblo, Colo.....	40.6	22 , June, 1921	.....	.....	68	R.M.	1.4
136	San Pablo Creek, Berkeley, Calif....	41.0	.....	38	50	70	S.W.	3.2

TABLE 2.—(*Continued.*)

No.	River on tributary and location.	Drainage area, in square miles.	Run-Off in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
137	Little Johns Creek, near Stockton, Calif.	41.0	.....	70	107	70	S.W.	6.8
138	Basic Creek, Freehold, N. Y.	41.0	81 , ...	.....	.....	27	N.E.	5.2
139	Westfield Little, Blandford, Mass.	43.0	50 , ...	34	.....	69	N.E.	3.3
140	Little Gunpowder, Bel Air, Md.	43.0	37 , ...	20	.....	69	N.E.	2.4
141	Sweetwater River, Descanso, Calif.	43.7	226 , Jan., 1916	.....	.....	66	S.W.	14.9
142	Elkhorn Creek, Keystone, W. Va.	44.0	1 363 , June, 1901	.....	.....	34	M.V.	90.3
143	Cameron Creek, Hurley, N. Mex.	44.0	135 , Aug., 1913	40	135	91	R.M.	9.0
144	San Leandro Creek, Oakland, Calif.	44.0	.....	38	50	70	S.W.	3.3
145	Six Mile Creek, Ithaca, N. Y.	46.0	195 , June, 1905	.....	.....	1	N.E.	13.2
146	San Pedro, below Apache Creek, Texas	46.5	698 , Sept., 1921	.....	.....	67	M.V.	47.6
147	Wallowa, Joseph, Ore.	47.0	16 , July, 1904	14	.....	69	N.W.	1.1
148	Dog River, Northfield, Vt.	47.0	72 , ... 1913	.....	.....	72	N.E.	4.9
149-A	Fish Creek, West Camden, N. Y.	47.5	34 , Spring, 1884	.....	.....	120	N.E.	2.3
150	Dry Creek, near Fresno, Calif.	48.0	.....	42	51	70	S.W.	3.5
151	Pine Creek, Paris, Tex.	48.0	410 , May, 1920	.....	.....	91	M.V.	28.4
152	Prosser Creek, Hobart Mills, Calif.	48.0	19 , ...	11	.....	69	S.W.	1.3
153	Darby Creek, near Philadelphia, Pa.	48.0	580 , Aug., 1843	.....	.....	77	N.E.	40.2
154	Turkey Creek, near Pueblo, Colo.	48.0	188 , June, 1921	.....	.....	68	R.M.	13.0
155	Big Cottonwood, Salt Lake City, Utah	48.5	17 , June, 1909	9	30	69	G.B.	2.1
156	Conemaugh, causing Johnstown flood, Pa.	48.6	206 , May, 1889	75	200	69	M.V.	14.4
157	Parley's Creek, Salt Lake City, Utah.	50.1	6 , June, 1909	3	20	69	G.B.	1.4
158	Sauquoit Creek, New York Mills, N. Y.	51.5	53 , ...	.....	.....	71	N.E.	3.8
159-A	Palomas, near Hermosa, New Mexico	52.0	167 , July, 1925	10	150	74	R.M.	12.0
160	Cottonwood Creek, North Fork, Ono, Calif.	52.0	80 , Dec., 1913	.....	.....	99	S.W.	5.7
161	Lost Creek, above Dayton, Ohio.	52.0	571 , Mar., 1913	.....	.....	91	M.V.	41.2
162	Guadalupe River, California.	52.0	.....	55	75	70	S.W.	5.4
163	Rockaway, Dover, N. J.	52.5	43 , ...	.....	.....	71	N.E.	3.1
164	Santa Ysabel Creek, Mesa Grande, Calif.	53.4	395 , Jan., 1916	.....	.....	66	S.W.	28.8
165	Tawawa Creek, above Dayton, Ohio.	54.0	239 , Mar., 1913	.....	.....	91	M.V.	17.6
166	Yakima, Martin, Wash.	55.0	112 , Nov., 1906	.....	.....	99	N.W.	8.3
166-A	Kosk Creek, near Henderson, Calif.	55.1	53 , May, 1915	.....	.....	99	S.W.	3.9
167	Santa Maria Creek, Ramona, Calif.	57.3	125 , Jan., 1916	.....	.....	66	S.W.	9.5
168	Oneida Creek, Kenwood, N. Y.	59.0	41 , ... 1890	.....	.....	71	N.E.	3.1
169	Rock Creek, near Pueblo, Colo.	59.0	913 , June, 1921	.....	.....	68	R.M.	70.0
170	Wildcat Creek Group, Stockton, Calif.	59.0	.....	70	97	70	S.W.	7.4
171	Lytle Creek, San Bernardino, Calif.	60.0	267 , Jan., 1916	.....	.....	66	S.W.	20.7
172	Flat River, Rhode Island.	61.0	120 , Mar., 1843	.....	.....	72	N.E.	9.4
173	West Branch Fish Creek, Camden, N. Y.	61.4	24 , June, 1889	.....	.....	72	N.E.	1.9
174	Pequannock, Macopin, N. J.	62.0	97 , Oct., 1903	.....	.....	99	N.E.	7.6
175	Chester Creek, near Philadelphia, Pa.	62.0	1 000 , Aug., 1843	.....	.....	77	N.E.	78.7
176	Los Angeles, near Los Angeles, Calif.	62.0	117 , Jan., 1916	.....	.....	66	S.W.	9.2
177	Nine-Mile Creek, Stittville, N. Y.	62.6	125 , Aug., 1898	.....	.....	72	N.E.	9.9
178	Kachess, near Easton, Wash.	63.0	28 , Nov., 1906	24	.....	99	N.W.	2.2
179-A	Otter Creek, Casler's Mill, N. Y.	63.0	31 , Apr., 1869	.....	.....	120	N.E.	2.5
180	Wissahickon Creek, Philadelphia, Pa.	64.6	43 , ... 1898	.....	.....	72	N.E.	3.5
181	Eight-Mile Creek, near Pueblo, Colo.	65.0	154 , June, 1921	.....	.....	68	R.M.	12.4
182	Ludlow Creek, above Dayton, Ohio	65.0	266 , Mar., 1913	.....	.....	91	M.V.	21.4
183	San Jacinto, at Hemet Reservoir, Calif.	65.8	145 , Jan., 1916	.....	.....	66	S.W.	11.8

TABLE 2.—(*Continued*).

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
184	American Fork, American Fork, Utah	66.0	13 , ...	7	30	69	G.B.	2.4
185	Owens Creek Group, near Merced, Calif.	66.0	.....	33	44	70	S.W.	3.6
186	Daulton Creek Group, California...	66.0	.....	40	51	70	S.W.	4.1
‡187	Carson, West Fork, Woodfords Calif.	67.0	23 , May, 1906	.....	.....	70	G.B.	1.9
188	Oak Creek, near Pueblo, Colo	68.0	41 , June, 1921	.....	.....	68	R.M.	3.4
189	Sandy Creek, Allendale, N. Y.....	68.4	89 , ... 1891	.....	.....	72	N.E.	7.4
190	Jamul Creek, near Otay, Calif.....	69.8	259 , Jan., 1916	.....	.....	66	S.W.	21.6
191	Bear Creek, near Madera, Calif.....	71.0	.....	36	50	70	S.W.	4.2
192	Dutchman Creek Group, Madera, Calif.	72.0	.....	40	54	70	S.W.	4.6
193	Laramie Reservoir Outlet, Laramie, Wyo.	72.0	97 , Mar., 1913	.....	.....	38	R.M.	8.2
194	Butte Creek, Butte Valley, Calif...	73.0	23 , Jan , 1909	16	.....	8	S.W.	2.0
195	Brushy Creek, Round Rock, Tex..	74.7	462 , Sept., 1921	.....	.....	67	M.V.	39.9
196	San Vincente Creek, Foster, Calif..	74.9	248 , Jan., 1916	.....	.....	72	S.W.	21.5
196-A	Birch Creek, Swift Dam, Mont.....	75.0	70 , June, 1916	.....	.....	98	R.M.	6.1
197	Mission Creek Group, San Jose, Calif.	77.0	.....	43	59	70	S.W.	5.2
198	Rock Creek, Washington, D. C.....	77.5	126 , ...	.....	.....	72	N.E.	11.1
199	Sonoma Creek Group, Sonoma, Calif.	78.0	.....	52	66	70	S.W.	5.8
200	Sudbury, Framington, Mass.....	78.0	41 , ... 1897	.....	.....	72	N.E.	3.6
201	Hockanum River, Conn.....	79.0	78 , ...	.....	.....	72	N.E.	6.9
202	Chittenango Creek, Chittenango, N. Y.	79.0	24 , ...	17	.....	69	N.E.	2.2
203	Dry Creek, near Auburn, Calif.....	79.0	.....	100	124	70	S.W.	11.0
204	Smith River, North Fork, Crescent, Calif.	81.0	348 , Nov., 1915	.....	.....	99	S.W.	31.3
205	Bear Creek, near Cape Mendocino, Calif.	82.0	.....	130	162	70	N.W.	14.6
206	Claremont Creek Group, Claremont, Calif.	83.0	.....	40	50	70	S.W.	4.6
207	Lagunitas Creek, near Sausalito, Calif.	84.0	.....	32	42	70	S.W.	3.8
208	San Mateo Creek Group, California	84.0	.....	44	58	70	S.W.	5.3
209	Kraiz Gamepa Outlet, Salmon, Alaska	84.0	51 , ... 1902	.....	.....	72	A.	4.7
210	Nashua River, Massachusetts.....	84.5	71 , ... 1850	.....	.....	72	N.E.	6.5
211	San Antonio Creek, Texas.....	85.0	499 , Sept., 1921	.....	.....	67	M.V.	46.0
212	Roach Creek, Roach, Me.....	85.0	24 , May, 1906	19	.....	99	N.E.	2.2
212-A	Pequannock, Pompton, N. J.....	85.0	65 , Feb., 1896	.....	.....	99	N.E.	6.0
213	Dry Creek, near Pueblo, Colo.....	86.0	283 , June, 1921	.....	.....	68	R.M.	26.2
214	Willow Creek, near Augusta, Mont.	90.0	42 , May, 1917	.....	.....	68	R.M.	4.0
215	Gallinas, Las Vegas, N. Mex.....	90.0	129 , Sept., 1904	.....	.....	120	M.V.	12.2
216	Putah Creek, near Guenoc, Calif...	91.0	270 , Mar., 1904	.....	.....	91	S.W.	25.8
217	Independence Creek, Crandall, N.Y.	93.2	66 , Apr., 1869	.....	.....	71	N.E.	6.4
218	Boulder, Bruffeys, Mont.....	94.0	17 , ...	16	.....	69	R.M.	1.6
219	Asay Creek, Hatch, Utah.....	96.0	17 , ... 1913	12	17	72	G.B.	1.7
220	North Fork Creek, Brookville, Pa...	97.0	124 , July, 1912	.....	.....	91	N.E.	12.2
221	Malabu Creek, Cabasas, Calif.....	97.0	70 , ...	27	.....	69	S.W.	6.9
222	Yokohl Creek Group, near Tulare, Calif.	98.0	.....	30	38	70	S.W.	3.8
223	Otay, Lower Otay Dam, Calif.....	98.6	379 , Jan., 1916	40	300	70	S.W.	37.6
224	Churn Creek Group, California....	100.0	100 , ...	100	133	70	S.W.	10.0
224-A	Rondout Creek, Lackawack, N. Y..	100.0	140 , Nov., 1913	.....	.....	104	N.E.	14.0

‡ Maximum daily flow.

TABLE 2.—(*Continued*).

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
225	Passaic, Chatham, N. J.	101.0	30 , Jan., 1905	20	.....	69	N.E.	3.0
226	Wanaque, Pompton, N. J.	101.0	84 , Oct., 1903	24	.....	24	N.E.	8.4
227	Deer River, Deer River, N. Y.	101.0	78 , Apr., 1869	.....	.....	72	N.E.	7.8
228	Swift Current, near Babb, Mont.	101.0	45 , ...	22	.....	69	R.M.	4.5
229	Tohickon Creek, Mt. Pleasant, Pa.	102.0	138 , May, 1894	41	.....	72	N.E.	13.9
230	Little Stony Creek, near Ladoga Calif.	102.0	69 , Feb., 1909	.....	.....	8	S.W.	7.0
‡231	Mariposa Creek, near Merced, Calif.	103	.....	43	56	70	S.W.	5.7
232	Fish Creek, East Branch, Point Rock, N.Y.	104	80.5, Fall, 1897	.....	.....	72	N.E.	8.2
233	Valley, Tomotla, N. C.	106	97 , ...	47	.....	69	N.E.	10.0
234	San Jacinto, near San Jacinto, Calif	108	278 , Jan., 1916	.....	.....	66	S.W.	28.9
235	Onondago Creek, Syracuse, N. Y.	108	56 , ... 1920	30	56	100	N.E.	5.8
236	Nashua River, Massachusetts.	109	104 , ... 1848	.....	.....	72	N.E.	10.8
‡237	Red Bank Creek Group, near Tulare. Calif.	109	.....	67	89	70	S.W.	9.3
238	Whiterocks River, Whiterocks. Utah.	110	10 , ...	6	10	69	R.M.	1.0
‡239	Deer Creek, near Red Bluff, Calif.	110	.....	32	38	70	S.W.	4.0
240	Sandy Creek, North Branch, Adams, N. Y.	110	67 , Winter, 1897	.....	.....	72	N.E.	7.0
241	Santa Ysabel Creek, near Ramona, Calif.	110	258 , Jan., 1916	.....	.....	66	S.W.	27.5
24	Lewiston Reservoir Outlet, Ohio.	111	58 , ...	.....	.....	38	M.V.	6.1
243	Sweetwater River, near Dehesa. Calif.	112	217 , Jan., 1916	.....	.....	66	S.W.	23.0
244	Ohanapecosh, near Lewis, Wash.	116	65 , Nov., 1909	.....	.....	72	N.W.	7.0
244-A	Ausable, W. Branch, near Newman, N. Y.	116	54 , Apr., 1922	.....	.....	113	N.E.	5.8
245	Rockaway, Boonton, N. J.	118	49 , Oct., 1903	.....	.....	24	N.E.	5.3
246	Laurel Hill Creek, Confluence, Pa.	118	42 , ...	25	.....	69	N.E.	4.6
247	Scantic, North Branch, Connecticut	118	52 , ...	.....	.....	72	N.E.	5.6
248	Ramapo, Mahwah, N. J.	118	106 , Oct., 1903	.....	.....	120	N.E.	11.5
249	Clear Creek, Buffalo, Wyo.	118	7 , ...	6	.....	69	R.M.	0.8
250	Antelope Valley Group, Los Angeles, Calif.	119	.....	54	87	70	S.W.	9.5
251	Los Gatos Creek, near Hanford. Calif.	119	.....	27	38	70	S.W.	4.1
252	Cottonwood Creek, Morena, Calif.	119	128 , Jan., 1916	.....	.....	66	S.W.	13.9
253	Hobble Creek, Springville, Utah.	120	7 , ...	3	.....	69	G.B.	0.8
254	Los Gatos Group, near Redwood, Calif.	121	.....	46	62	70	S.W.	6.8
255	Martells Creek Group, near Stockton, Calif.	122	.....	70	97	70	S.W.	10.7
256	Suisun Creek Group, near Suisun, Calif.	125	.....	43	52	70	S.W.	5.8
257	South Boulder, Marshall, Colo.	125	9 , ...	5	.....	69	R.M.	1.0
258-A	Paulinskill, New Jersey.	126	54 , ...	.....	.....	120	N.E.	6.1
259	Santa Ysabel, near San Diego, Calif.	126	.....	80	154	70	S.W.	17.3
260	Patuxent, Laurel, Md.	127	40 , ... 1915	.....	.....	72	N.E.	4.5
261	Walnut Canyon, near Flagstaff. Ariz	128	82 , Sept., 1923	15	82	74	R.M.	9.3
262-A	Walla Walla River, Milton, Ore.	130	16 , Apr., 1904	.....	.....	120	N.W.	1.8
262-B	Payette, North Fork, Lardo, Idaho.	131	32 , June, 1909	.....	.....	103	R.M.	3.7
263	Fresno River, near Knowles, Calif.	134	34 , Feb., 1917	.....	.....	99	S.W.	3.9
264	Bear Creek Group, California.	137	.....	100	131	70	S.W.	15.3
265	San Antonio, near Santa Barbara, Calif.	138	.....	53	73	70	S.W.	8.6

‡ Maximum daily flow.

TABLE 2.—(*Continued*).

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
266	Petaluma Creek Group, Petaluma, Calif.	139	.....	46	57	70	S.W.	6.7
267	Union River (West Branch), Maine..	140	50 , Apr., 1923	15	.....	88	N.E.	5.9
268	Neshaminy Creek, below Forks, Pa.	140	98 , ... 1891	33	.....	72	N.E.	11.6
269	Cedar River, Ravensdale, Wash....	141	81 , Nov., 1906	35	80	99	N.W.	9.6
270	Dead River, Forestville, Mich.....	142	17 , ...	16	.....	69	M.V.	2.0
271	Devil's Creek, near Viele, Iowa.....	143	600 , June, 1905	.....	.....	91	M.V.	71.7
272	Musconetcong, Bloomsbury, N. J...	143	19 , ...	13	.....	69	N.E.	2.3
273	Oriskany Creek, Oriskany, N. Y....	144	53 , Spring, 1896	.....	.....	120	N.E.	6.3
274	Turtle Creek, East Pittsburgh, Pa..	146	64 , ...	.....	.....	26	N.E.	7.7
275	Salado Creek, Salado, Tex.....	148	966 , Sept., 1921	.....	.....	67	M.V.	117.5
276-A	Clyde, West Derby, Vt.....	150	42 , Mar., 1913	.....	.....	113	N.E.	5.1
277	Perkiomen Creek, Frederick, Pa....	152	116 , May, 1894	35	.....	99	N.E.	14.3
278	Mohawk, Ridge Mills, N. Y.....	153	46 , Spring, 1888	.....	.....	72	N.E.	5.7
279	Bolinas Creek Group, Bolinas, Calif.	158	.....	20	26	70	S.W.	3.3
280	Soquee, Demorest, Ga,.....	158	56 , ...	36	.....	69	S.E.	7.0
281	Mora River, below Mora, N. Mex....	159	140 , Sept., 1904	.....	.....	2	M.V.	17.7
282	Gunpowder Falls, Glencoe, Ind.....	160	35 , ...	34	.....	69	M.V.	4.4
283	Ramapo, Pompton, N. J.....	160	66 , Mar., 1832	.....	.....	24	N.E.	8.3
284	Kinzua Creek, Dew Drop, Pa.....	162	20 , ...	.....	.....	22	N.E.	2.5
285	Weber River, Oakley, Utah.....	163	25 , ...	17	25	69	G.B.	3.2
286	Salmon Creek, Malott, Wash.....	164	4 , Apr., 1904	3	.....	69	N.W.	0.5
287	Mono Lake Group, California.....	166	.....	20	25	70	G.B.	3.2
288	Little Truckee River, Starr, Calif..	166	11 , ...	9	.....	69	G.B.	1.4
289	Los Angeles River Tributary, California.....	167	.....	70	130	70	S.W.	16.8
290	Fish Creek, East Branch, Taberg, N. Y.....	169	65 , Mar., 1913	.....	.....	37	N.E.	8.4
291	Burns Creek Group, Merced, Calif.	171	.....	50	66	70	S.W.	8.6
292	Sweetwater River, Jamacha, Calif.	172	250 , Jan., 1916	.....	.....	66	S.W.	32.8
293	Hardscrabble Creek, Pueblo, Colo..	173	19 , June, 1921	.....	.....	68	R.M.	2.5
294	St. Mary River, near Babb, Mont...	177	45 , June, 1908	19	.....	69	R.M.	6.0
295	Backbone Creek Group, Red Bluff, Calif.....	178	.....	80	120	70	S.W.	16.0
296	Bear Creek, Morrison, Colo.....	180	48 , July, 1896	.....	.....	68	R.M.	6.4
297	Canyon Padre, Diablo Jct., Ariz..	180	60 , Sept. 1923	20	60	74	R.M.	8.0
297-A	Baker River, near Anderson Creek, Washington.....	184	200 , Dec. 1917	.....	.....	119	N.W.	27.1
298	Williams Fork, Parshall, Colo.....	185	14 , June, 1918	5	.....	99	R.M.	1.9
299	Fish Creek, West Branch, McConnellsville, N. Y.....	187	33 , ... 1884	.....	.....	72	N.E.	4.5
300	Smoke Creek Group, Honey Lake, Calif.....	188	.....	10	11	70	G.B.	1.5
301	Santa Ana River, Mentone, Calif....	189	154 , Jan., 1916	.....	.....	66	S.W.	21.2
302	San Diego River, Lakeside, Calif....	189	201 , Jan., 1916	.....	.....	66	S.W.	27.6
303	Seekonk, Providence, R. I.....	190	57 , ... 1867	.....	.....	72	N.E.	7.9
304	Tallulah River, Tallulah Falls, Ga..	191	47 , Aug., 1906	.....	.....	120	S.E.	6.5
305	Esopus Creek, Coldbrook, N. Y.....	192	171 , Sept., 1924	....	150	101	N.E.	23.7
306	Coyote River, Madrone, Calif.....	193	130 , Mar., 1911	.....	.....	99	S.W.	18.1
307	Santa Ana Tributaries, Mentone, Calif.....	199	146 , Jan , 1916	.....	.....	70	S.W.	20.6
308	Fishkill Creek, Groveville Dam, N. Y.	200	69 , Mar., 1902	.....	.....	120	N.E.	9.7
309	Mt. Diablo Creek, Martinez, Calif...	200	.....	47	63	70	S.W.	8.9
310	Cave Creek, near Phoenix, Ariz...	200	125 , Aug., 1921	.....	.....	91	S.W.	17.7
311	Cle Elum Lake, Roslyn, Wash.....	202	93 , Nov., 1906	36	.....	99	N.W.	13.2
312	Unadilla River, New Berlin, N. Y...	204	40 , Sept., 1905	.....	.....	67	N.E.	5.7
313	Cantua Creek, Hanford, Calif.....	208	.....	20	32	70	S.W.	4.6
314	San Luis Rey, Mesa Grande, Calif..	209	280 , Jan., 1916	.....	.....	66	S.W.	40.5
315	St. Vrains Creek, Lyons, Colo.....	209	6 , June, 1903	5	.....	69	R.M.	0.9

TABLE 2.—(*Continued*).

No.	River or tributary and location.	Drainage area, in square miles.	Run-off, in second-feet per square mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
316	Coon Creek, Sacramento, Calif.	210	.....	70	89	70	S.W.	12.9
317	Catskill Creek, South Cairo, N. Y.	210	100 , Spring, 1901	.....	.....	27	N.E.	14.5
318	Mojave River, Victorville, Calif.	211	64 , Mar., 1903	65	109	70	G.B.	15.8
319	French Creek, Kimmelton, Pa.	212	43 , ...	.....	.....	22	N.E.	6.2
320	Beaver Creek, Pueblo, Colo.	213	45 , June, 1921	.....	.....	68	R.M.	6.6
321-A	Greenville Creek, near Bradford, Ohio	213	85 , Mar., 1908	.....	.....	107	M.V.	12.4
322	Arroyo Seco, Soledad, Calif.	215	102 , Feb, 1917	19	100	99	S.W.	15.0
323	Owens Lake Group, California.	216	.....	21	24	70	G.B.	3.5
324	Sespe Creek, Sespe, Calif.	216	86 , Jan., 1916	.....	.....	66	S.W.	12.6
325-A	Octoraro River, Rolandsville, N. Y.	217	28 , ... 1897	.....	.....	120	N.E.	4.1
326	Uinta River, White Rocks, Utah.	218	10 , ...	5	20	74	R.M.	3.0
327	Logan River, Logan, Utah.	218	11 , June, 1907	7	30	74	G.B.	4.4
327-A	Santa Ynez, Santa Barbara, Calif.	218	60 , Jan., 1914	.....	.....	99	S.W.	8.9
328	Frazier River, Granby, Colo.	220	9 , ...	8	.....	69	R.M.	1.3
329	Sapella River, Los Alamos, N. Mex.	221	37 , Sept., 1904	.....	.....	120	R.M.	5.5
330	Salmon River, Altmar, N. Y.	221	28 , ...	.....	.....	72	N.E.	4.2
331	San Gabriel River, Azusa, Calif.	222	212 , Feb., 1884	50	250	70	S.W.	37.2
332	Pescadero Group, near San Mateo, Calif.	222	.....	35	48	70	S.W.	7.1
333	Napa River Tributaries, Napa, Calif.	226	.....	45	61	70	S.W.	9.2
334	Ventura River, Ventura, Calif.	226	.....	70	87	70	S.W.	13.1
335	Smith at Junction, Crescent City, Calif.	227	187 , Nov., 1915	.....	.....	70	S.W.	28.2
336	Salmon Creek Group, Sonoma, Calif.	230	.....	25	32	70	S.W.	4.8
337	Tocca, near Blueridge, Ga.	231	53 , ... 1901	150	300	16	S.E.	45.5
338-A	San Lorenzo River, California.	235	42 , ... 1900	.....	.....	120	S.W.	6.4
339	Schoharie Creek, Prattsville, N. Y.	236	123 , Sept., 1924	55	144	101	N.E.	22.1
340	Chowchilla River, Merced, Calif.	238	.....	35	44	70	S.W.	6.8
341	Salinas Tributaries, Soledad, Calif.	238	80 , Jan., 1916	.....	.....	70	S.W.	12.3
‡342	Cobbosseecontee Stream, Gardiner, Me.	240	19 , May, 1922	8	.....	99	N.E.	2.9
343	Jalama Creek, Santa Barbara, Calif.	242	.....	.....	60	70	S.W.	9.3
‡344	West Walker River, Coleville, Calif.	245	17 , July, 1907	.....	.....	70	S.W.	2.7
345	South River, Port Republic, Va.	246	37 , ... 1897	.....	.....	17	S.E.	5.8
346	Mattole River, New Petrolia, Calif.	249	228 , ...	.....	.....	70	S.W.	35.2
347	Patapsco River, Woodstock, Md.	251	44 , Feb., 1908	28	.....	69	N.E.	7.0
348	Crow Creek, Cheyenne, Wyo.	251	37 , May, 1904	.....	.....	68	R.M.	5.9
349	Clear Creek, Red Bluff, Calif.	251	.....	70	105	70	S.W.	16.6
350	Butte Creek Group, Oroville, Calif.	251	.....	100	134	70	S.W.	21.2
351	East Canada Creek, Dolgeville, N. Y.	256	54 , Mar. 1913	22	.....	37	N.E.	8.6
352	Cahokia Creek, Poag, Ill.	259	14 , ...	.....	.....	4	M.V.	2.3
353	Salmon River, Pulaski, N. Y.	260	42 , Mar., 1913	38	.....	37	N.E.	6.8
354	Bear River, Van Trent, Calif.	262	336 , Feb., 1907	100	400	70	S.W.	64.7
355	Mattole River, Cape Mendocino, Calif.	264	.....	130	164	70	N.W.	26.6
356	Tule River, Portersville, Calif.	266	25 , Jan., 1916	12	.....	99	S.W.	4.1
357	Black River, Forestport, N. Y.	268	39 , ...	.....	.....	72	N.E.	6.4
358	Whitewater River, Whitewater, Calif.	269	.....	40	67	70	G.B.	11.1
359	Miller's Creek, Lovella, Ore.	270	25 , ... 1907	.....	.....	6	N.W.	4.0
360	Fresno, near Fresno, Calif.	270	.....	36	46	70	S.W.	7.6
361	Cottonwood Creek, Jamul, Calif.	270	22 , ...	7	.....	69	S.W.	3.6
362-A	Twin Creek, Germantown, Ohio.	270	244 , Mar., 1913	.....	.....	107	M.V.	40.0
363	Notteley, Ranger, N. C.	272	21 , ...	21	210	16	S.E.	34.6
364	Choccolocco Creek, Jenifer, Ala.	272	43 , ...	19	.....	69	S.E.	7.1
365	Chewaucan, Paisley, Ore.	272	13 , ...	6	.....	69	N.W.	2.1
366	Navarro River, Ukiah, Calif.	273	.....	54	67	70	S.W.	11.1

‡ Maximum daily flow.

TABLE 2.—(*Continued*).

No.	River or tributary and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main Authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
367	Big Thompson Creek, near Drake, Colo.	274	29 , July, 1919	4	.....	99	R.M.	4.8
368	Redwood Creek, Eureka, Calif.	275	57 , ...	.....	.....	70	S.W.	9.4
369	Goose Lake, Goose Lake, Calif.	275	.....	12	13	70	G.B.	2.0
370	St. Regis River, St. Regis, Mont.	278	28 , May, 1917	.....	.....	98	R.M.	4.6
371	Crooked Creek, Hileman's Farm, Pa.	279	43 , ...	.....	.....	22	N.E.	7.2
372	Lamoille River, Cady's Falls, Vt.	280	31 , Oct., 1920	.....	.....	113	N.E.	5.2
373	Conejos, Mogote, Calif.	282	15 , ...	10	.....	69	S.W.	2.5
374	Sutter Creek Group, Lodi, Calif.	285	.....	55	75	70	S.W.	12.7
375	Piscataguis, Foxcroft, Me.	286	78 , ... 1909	43	.....	10	N.E.	13.2
376	Black Fork, Hyrum, Utah.	286	7 , ...	3	.....	69	G.B.	1.2
376-A	Genessee River, Scio, N. Y.	288	37 , May, 1919	.....	.....	113	N.E.	6.3
377	Yamhill River, Sheridan, Ore.	290	62 , ...	53	.....	69	N.W.	10.5
378	Brokenstraw Creek, Youngsville, Pa.	290	25 , ...	.....	.....	22	N.E.	4.3
379	Youghiogheny, Friendsville, Md.	294	28 , ...	28	86	22	N.E.	14.8
380	Mora River, Weber, N. Mex.	294	94 , Sept., 1904	.....	.....	68	R.M.	16.0
381	Panoche Creek, Mendota, Calif.	295	.....	30	43	70	S.W.	7.4
382	Antietam Creek, Sharpsburg, Ind.	295	23 , ... 1902	11	.....	17	M.V.	3.9
‡383	Carson, East Fork, Stateline, Nev.	298	12 , June, 1911	.....	.....	70	G.B.	2.1
384	San Dieguito, Bernardo, Calif.	299	241 , Jan., 1916	.....	.....	66	S.W.	41.6
384-A	Sandusky, near Upper Sandusky, Ohio.	299	14 , Apr., 1922	.....	.....	113	N.E.	2.4
385	Gunpowder River, Maryland.	302	83 , ... 1889	.....	.....	69	N.E.	14.4
386	Oil Creek, Rouseville, Pa.	302	28 , ...	.....	.....	22	N.E.	4.9
387-A	Black River, near Boonville, N. Y.	303	33 , Mar., 1913	.....	.....	113	N.E.	5.7
388	Chittenango Creek, Lake Oneida, N. Y.	307	13 , ...	.....	.....	120	N.E.	2.2
389	Honcut Creek, Oroville, Calif.	314	.....	90	124	70	S.W.	21.9
390	Rapid Creek, Rapid, S. Dak.	320	3 , June, 1904	2	.....	72	M.V.	0.5
391-A	Beaver River, Beaver Falls, N. Y.	322	17 , Apr., 1869	.....	.....	120	N.E.	3.0
392-A	San Antonio River, Jolon, Calif.	322	48 , ... 1900	.....	.....	120	S.W.	8.6
393	Soquel Creek Group, Santa Cruz, Calif.	324	.....	50	73	70	S.W.	13.1
394	San Luis Rey, near Pala, Calif.	325	231 , Jan., 1916	.....	.....	70	S.W.	41.6
395	San Jacinto Tributary, California.	330	.....	50	89	70	S.W.	16.2
396	Zion Creek, near Springdale, Utah.	330	11 , Aug., 1916	7	20	74	R.M.	3.6
397	Deerfield River, Charlemont, Mass.	332	126 , July, 1915	40	.....	99	N.E.	22.9
398	Silver Creek, near Lebanon, Ill.	335	16 , ... 1908	.....	.....	4	M.V.	2.9
399	Croton River, Croton Dam, N. Y.	339	75 , ... 1867	.....	.....	72	N.E.	13.8
400	Canyon Diablo, Arch Bridge, Ariz.	340	104 , Sept., 1923	.....	.....	74	R.M.	19.2
401	Carrabassett River, North Anson, Me.	340	40 , ...	27	.....	15	N.E.	7.4
402	Great River, Westfield, Mass.	350	151 , ... 1878	.....	.....	72	N.E.	28.2
403	Umatilla River, Gibbon, Ore.	353	28 , Mar., 1908	10	.....	69	N.W.	5.2
404	Ogden River, Ogden, Utah.	360	9 , ...	5	15	69	G.B.	2.8
405-A	Rondout Creek, Rosendale, N. Y.	365	38 , Mar., 1902	38	105	120	N.E.	20.0
406	Battle Creek, Red Bluff, Calif.	366	.....	90	121	70	S.W.	23.2
407	Moose River, Moose River, N. Y.	370	45 , Mar., 1913	17	.....	90	N.E.	8.6
408	West Canada Creek, Hinckley, N. Y.	372	105 , Apr., 1869	35	.....	37	N.E.	20.3
409	Ocoee, McCays, Copperhill, Tenn.	374	48 , ...	19	.....	69	M.V.	9.3
410	San Diego River, Santee, Calif.	375	187 , Jan., 1916	.....	.....	66	S.W.	36.2
411	Esopus Creek, Mt. Marion, N. Y.	378	65 , Apr., 1910	50	130	9	N.E.	25.3
412	Surprise Valley Group, Cedarville, Calif.	379	.....	9	11	70	G.B.	2.1
413	Malibu River Group, Chatsworth, Calif.	379	.....	60	77	70	S.W.	15.0
414	Rondout Creek, near Rosendale, N.Y.	380	51 , Apr., 1910	35	100	9	N.E.	19.5

‡ Maximum daily flow.

TABLE 2.—(*Continued.*)

No.	River or tributary and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
415	Pompton, Two Bridges, N. J.	380	62 , ... 1903	.....	.....	1	N.E.	12.1
416	Carson, East Fork, Gardnerville, Nev.	381	9 , ... 1904	6	.....	6	G.B.	1.8
417	Saco, Center Conway, N. H.	385	37.1 , Nov., 1907	27	.....	118	N.E.	7.3
418	Roanoke River, Roanoke, Va.	388	43.5 , Aug., 1901	23	.....	69	S.E.	8.6
419	Tule River, Tulare, Calif.	390	.....	20	28	70	S.W.	5.5
420	Fall River, near Squirrel, Idaho.	390	13.8 , June, 1918	8	.....	69	R.M.	2.7
421	Calaveras, Jenny Lind, Calif.	394	176 , Jan., 1911	170	229	70	S.W.	45.5
422	Willow Creek Group, Colusa, Calif.	394	.....	60	79	70	S.W.	15.7
423	Middle Oconee, near Athens, Ga.	395	49 , ... 1902	40	106	16	S.E.	21.1
423-A	Embarrass River, near Embarrass, Wis.	395	17 , Apr., 1922	.....	.....	113	M.V.	3.4
424	Pacolet River, Spartansburg, S. C.	400	89 , June, 1903	60	120	19	S.E.	24.0
425	Mojave River, below Victorville, Calif.	400	33 , Mar., 1903	20	40	6	G.B.	8.0
426	Blacklick Creek, Blacklick, Pa.	403	38 , ...	24	.....	69	N.E.	7.6
427	Tygerts Valley, Belington, W. Va.	403	41 , ... 1902	.....	.....	22	M.V.	8.2
427-A	Sheboygan River, near Sheboygan, Wis.	403	18 , Mar., 1920	.....	.....	113	M.V.	3.6
428	West Walker River, California.	405	.....	20	27	70	G.B.	5.4
429	Moose River, Ager's Mill, N. Y.	407	31 , ...	.....	.....	72	N.E.	6.2
430	Watauga River, Elizabethton, Tenn.	408	23 , ...	14	.....	69	M.V.	4.6
431	Potomac, North Branch, Piedmont, W. Va.	410	33 , ...	20	100	17	N.E.	20.2
432	Hiwassee River, Murphy, N. C.	410	54 , Mar., 1899	30	163	16	S.E.	33.0
433	East Walker River, California.	411	.....	19	24	70	G.B.	4.9
434	Mahoning Creek, Furnace Bridge, Pa.	412	30 , ...	.....	.....	22	N.E.	6.1
435	Carson, East Fork, Rodenbohs, Nev.	414	13 , ...	9	.....	69	G.B.	2.6
436	Elder Creek Group, Hamilton, Calif.	414	.....	47	63	70	S.W.	12.8
437	Esopus Creek, at Saugerties, N. Y.	417	132 , Dec., 1878	40	132	69	N.E.	27.0
437-A	St. Maries River, Lotus, Idaho.	420	21 , Mar., 1921	.....	.....	103	R.M.	4.2
438	Buffalo River, New York.	420	55 , Mar., 1902	.....	.....	69	N.E.	11.3
438-A	Winooski River, Richmond, Vt.	420	48 , Apr., 1912	.....	.....	99	N.E.	9.8
439	Oil Creek, near Pueblo, Colo.	423	6 , June, 1921	.....	.....	68	R.M.	1.2
440	South Sangammon, Taylorsville, Iowa.	427	10 , ...	.....	.....	4	M.V.	2.1
441	Stony Creek, Johnstown, Pa.	428	70 , May, 1889	.....	.....	72	N.E.	14.5
442	San Gabriel River, Georgetown, Tex.	431	371 , Sept., 1921	.....	.....	67	M.V.	77.0
442-A	San Diego River, San Diego, Calif.	434	173 , Jan., 1916	50	173	99	S.W.	36.0
443	Youghiogheny River, Confluence, Pa.	435	55 , Aug., 1888	27	100	69	N.E.	20.8
444	Laramie River, Woods Landing, Wyo.	435	10 , ...	7	.....	69	R.M.	2.1
445	Owens River, Round Valley, Calif.	439	3 , June, 1907	.....	.....	70	G.B.	0.6
446	Appalachie River, Brickhead, Ga.	440	17 , ...	11	100	69	S.E.	21.0
447	Whetstone, Big Stone City, S. Dak.	441	3 , ...	.....	.....	25	M.V.	0.6
448	Battenkill, Greenwich, N. Y.	444	22 , Mar., 1913	.....	.....	37	N.E.	4.6
449	Cow Creek, Red Bluff, Calif.	444	.....	90	114	70	S.W.	24.0
449-A	Ausable River, Ausable Forks, N. Y.	444	56 , Mar., 1913	.....	.....	113	N.E.	11.8
450	Missiquoi River, below Richford, Vt.	445	23 , Mar., 1913	.....	.....	72	N.E.	4.8
451	Cherry Creek, Denver, Colo.	445	25 , July, 1912	.....	.....	68	R.M.	5.3
452	Bishops Creek Group, Owens Valley, Calif.	446	.....	25	32	70	G.B.	6.7
453	Whiteface, below Meadowlands, Minn.	446	13 , ... 1916	.....	.....	72	M.V.	2.7
454	Truckee River, Stateline, Calif.	447	33 , Mar., 1907	.....	.....	70	G.B.	7.0
455	Casselman River, Confluence, Pa.	448	46 , ...	23	100	22	N.E.	21.1
455-A	Stillwater, Sugar Grove, Ohio.	448	115 , Mar., 1913	.....	.....	107	M.V.	24.3
456	Tionesta Creek, Nebraska, Pa.	451	20 , ...	.....	.....	22	N.E.	.2

TABLE 2.—(*Continued*).

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location,†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
457	St. Mary River, near Cardston, Canada	452	40 , June, 1908	.....	.....	69	C.	8.5
458	Pine River, below Pine Reservoir, Minn.	452	4 , ...	3	15	69	M.V.	3.2
459	Union River, near Junction, Me.	452	50 , Apr., 1923	.....	.....	88	N.E.	10.6
460	Adobe Meadows, Owens Valley, Calif.	453	.....	18	21	70	G.B.	4.5
461	Chatanika River, below Poker, Alaska.	456	8 , ... 1911	.....	.....	72	A.	1.7
‡462	Mad River, Arcata, Calif.	457	42 , ... 1912	100	142	70	S.W.	30.3
463	Santa Ana Tributaries, Calif.	460	.....	50	128	70	S.W.	27.4
464	Machias River, Whitneyville, Me.	465	24 , ...	12	.....	69	N.E.	5.2
465	Cattaraugus Creek, Versailles, N. Y.	467	64 , Mar., 1918	.....	.....	104	N.E.	13.8
466	Lake Fork River, Myton, Utah.	470	6 , ...	4	10	69	R.M.	2.2
467	Caliente Creek, near Tehachapi, Calif.	471	.....	30	48	70	S.W.	10.4
468	St. Charles River, Pueblo, Colo.	482	149 , June, 1921	.....	.....	68	R.M.	32.7
469	Little Wolf River, Royalton, Wis.	485	11 , ... 1914	.....	.....	72	M.V.	2.4
470	Kalamazoo River, Ceresco, Mich.	489	1 , May, 1897	3	12	120	M.V.	2.7
471	Coquitlam, above Coquitlam Dam, Canada	490	49 , ... 1921	25	50	94	C.	11.1
472	Eagle Lake Group, California.	498	.....	10	11	70	G.B.	2.5
473	Lake Tahoe Basin, California	499	.....	20	34	70	G.B.	7.6
474	Yadkin River, North Wilkesboro, N. C.	500	36 , Mar., 1899	25	90	69	S.E.	20.1
475	Cache Creek, Lower Lake, Calif.	500	30.5 , Mar., 1904	.....	.....	120	S.W.	6.8
476	Grande, North Branch, Haley, N. Dak.	500	11.6 , ... 1913	.....	.....	72	M.V.	2.6
477	Yakima River, Cle Elum, Wash.	500	51.2 , Nov., 1906	31	50	99	N.W.	11.4
478	Shoshone, South Fork, Marquette, Wyo	500	10.6 , ...	7	15	69	R.M.	3.4
479-A	Mohawk River, Utica, N. Y.	500	34.6 , Feb., 1891	30	75	120	N.E.	16.8
480	Feather, North Fork, Prattville, Calif.	506	19.8 , Mar., 1907	20	100	8	S.W.	22.8
481	Kaweah, Three Rivers, Calif.	514	28.6 , Jan., 1916	.....	.....	70	S.W.	6.5
482	Olentangy River, Columbus, Ohio.	514	98 , Mar., 1913	28	100	71	M.V.	22.7
483	West Canada Creek, Middleville, N. Y.	518	24.9 , Aug., 1898	.....	.....	72	N.E.	5.7
484	Truckee River, Tahoe, Calif.	519	2.6 , July, 1907	2	5	6	G.B.	1.1
484-A	Clear Fork of Trinity, Fort Worth, Tex.	522	142 , April, 1922	50	140	117	M.V.	32.4
485	Dolores River, Dolores, Colo.	524	5.6 , ...	4	.....	69	R.M.	1.3
486	Owens River (Upper), California.	524	.....	25	32	70	G.B.	7.3
487	Cosumnes River, Michigan Bar, Calif.	525	43 , Feb., 1917	40	55	99	S.W.	9.9
488	Price River, Helper, Utah.	530	8.5 , ... 1913	7	15	72	R.M.	3.5
489	Coosawattee River, Carters, Ga.	531	33.4 , Nov., 1906	25	80	120	S.E.	18.4
490	Cache la Poudre, Livermore, Colo.	532	38.0 , May, 1904	15	38	68	R.M.	8.8
491	Los Angeles Tributaries, California	534	71.2 , Dec., 1889	50	100	70	S.W.	23.1
492	Canyon Diablo, Leupp, Ariz.	544	82 , Sept., 1923	30	90	74	R.M.	21.0
493	Santa Catarina River, Monterey Mexico.	544	432± , Aug., 1909	100	400	91	M.	100.8
494	Occoquan Creek, Occoquan, Va.	546	38.3 , ... 1915	.....	.....	72	S.E.	8.9
495	Madeline Plains Group, North East, Calif.	548	.....	9	11	70	G.B.	2.6
496	Conewango River, Frewsburg, N.Y.	550	20.9 , Jan., 1911	.....	.....	22	N.E.	4.9
497	Uncompahgre River, Montrose, Colo.	565	3.7 , ...	3	25	69	R.M.	5.9
498	San Luis Rey, Oceanside, Calif.	565	169.0 , Jan., 1916	.....	.....	66	S.W.	40.2
499	Sacramento, above Pitt River, Calif.	568	.....	60	101	70	S.W.	24.1
500	Poso Creek Group, Tulare, Calif.	576	.....	20	26	70	S.W.	6.2

‡ Maximum daily flow.

TABLE 2.—(*Continued*).

No.	River or tributary, and location.	Drainage area, in square miles.	Run-off, in second-feet per square mile. Observed. Maximum and date.	Expected peaks. Frequent.	Expected peaks. Rare.	Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
501	Hoosic, River, Buskirk, N. Y.	579	23.6, ... 1913	18	45	69	N.E.	9.8
502	Farmington River, Connecticut	584	41.7, ...	.....	.....	72	N.E.	10.1
503-A	Payette, North Fork, Van Wyck, Idaho	586	15, May, 1921	.....	.....	103	R.M.	3.6
504	Tugaloo River, near Madison, S. C.	593	36.9, July, 1905	26	80	16	S.E.	19.5
505	Sun, North Fork, Augusta, Mont.	600	54.0, June, 1916	20	50	68	R.M.	13.2
506	Fort Pierce, Utah-Arizona Boundary	600	6.5, Aug., 1909	4	10	74	R.M.	2.4
506-A	Stony Creek, Fruto, Calif.	601	60.0, Feb., 1909	28	.....	99	S.W.	14.7
507	Etowah River, Canton, Ga.	604	31.5, Jan., 1895	23	55	71	S.E.	13.5
508-A	Bad River, near Odanah, Wis.	607	20, Apr., 1916	.....	.....	113	M.V.	3.9
509	Pennigewasset, Plymouth, N. H.	615	49.8, July, 1907	27	.....	69	N.E.	12.3
509-A	St. Regis, Brasher Center, N. Y.	621	26, Mar., 1914	.....	.....	113	N.E.	6.5
510	Gualala Group, Santa Rosa, Calif.	623	.....	45	59	70	S.W.	14.9
511	Smith River, N.W., California	627	.....	160	204	70	N.W.	51.1
512	Des Plaines, Riverside, Ill.	630	20.8, ... 1889	15	30	72	M.V.	7.5
513	Mokelumne, near Clements, Calif.	631	26.0, Jan., 1911	13	.....	99	S.W.	6.5
514	Saranac River, Plattsburg, N. Y.	634	11, Apr., 1914	7	15	113	N.E.	3.8
515	White River, Meeker, Colo.	634	5.8, ...	5	12	69	R.M.	3.0
‡516	Alameda Creek, Sunol Glenn, Calif.	639	23.0, Mar., 1911	.....	.....	70	S.W.	5.8
516-A	Naches River, Nile, Wash.	640	34.0, Nov., 1906	20	40	99	N.W.	10.1
517	Provo River, Provo, Utah	640	6.4, May, 1907	5	10	74	G.B.	2.5
517-A	Mad River, Osborn, Ohio	649	117, Mar., 1913	.....	.....	107	M.V.	29.8
518	Sandy River, Madison, Me.	650	21.2, ...	15	60	69	N.E.	15.3
518-A	Eagle River, Eagle, Colo.	650	10, June, 1914	6	.....	99	R.M.	2.5
519	Pigeon River, Newport, Tenn.	655	30.9, ...	15	.....	69	M.V.	7.9
520	Putah Creek, Winters, Calif.	655	91.6, Dec., 1913	28	.....	70	S.W.	23.5
521	Monocacy River, near Frederick, Md.	660	31.0, Mar., 1902	22	75	17	N.E.	19.3
522	Grande Ronde, Hilgard, Ore.	660	6.8, Apr., 1904	5	40	69	N.W.	10.3
523	Milwaukee River, Milwaukee, Wis.	661	18.3, Mar., 1918	8	18	113	M.V.	4.7
‡524	Russian River, Geyserville, Calif.	662	24.9, Mar., 1911	.....	.....	70	S.W.	6.4
525	Tuckasegee River, Bryson, N. C.	662	58.2, Mar., 1889	34	.....	72	S.E.	15.0
526	McCloud River, Baird, Calif.	669	82.2, Feb., 1904	33	.....	70	S.W.	21.2
527	Spanish Fork River, Spanish Fork, Utah	670	2.9, ...	2	5	69	G.B.	1.3
528	Uinta River, Fort Duchesne, Utah.	672	6.7, ...	4	10	69	R.M.	2.6
529	Black River, Neillsville, Wis.	675	34.2, ...	15	34	69	M.V.	8.9
530	Little Tennessee River, Jackson, N. C.	675	85.3, Dec., 1901	40	85	21	S.E.	22.2
531	Delaware, West Branch, Hancock, N. Y.	680	49.6, ...	28	100	69	N.E.	26.1
532	Santa Margarita, near San Diego, Calif.	690	.....	40	67	70	S.W.	17.6
533	Willamette, Coast Fork, Goshen, Ore.	690	45.4, ...	30	75	69	N.W.	19.7
534	Blue River, Kremmling, Colo.	700	8.6, ...	8	.....	69	R.M.	2.3
535	Mooyie River, Snyder, Idaho	717	15.1, June, 1916	.....	.....	103	R.M.	4.0
‡536	San Jacinto, Elsinore, Calif.	718	19.5, Jan., 1916	.....	.....	66	S.W.	5.2
537	Trinity River, Lewiston, Calif.	718	37.4, Jan., 1914	.....	.....	70	N.W.	10.0
538	Racquette River, Piercefield, N. Y.	723	8.1, May, 1900	8	15	5	N.E.	4.0
538-A	Huron River, Barton, Mich.	723	8, Mar., 1918	6	.....	102	M.V.	2.2
539	Salmon River, Somesbar, Calif.	734	44.9, Feb., 1912	.....	.....	70	N.W.	12.2
540	Big Muddy, near Cambon, Ill.	735	15.0, ...	.....	.....	4	M.V.	4.1
540-A	Chipola River, Altha, Fla.	740	17.0, Mar., 1918	7	17	119	S.E.	4.6
541	Indian Creek, Crescent Mills, Calif.	740	15.4, ...	9	.....	69	S.W.	4.2
541-A	Cuyahoga, Cleveland, Ohio	740	45, Mar., 1913	20	45	115	N.E.	12.2
542	Purgatoire River, Trinidad, Colo.	742	61.0, Sept., 1904	.....	.....	68	R.M.	16.6
543	West Fork, Enterprise, W. Va.	744	23.7, ... 1911	.....	.....	22	M.V.	6.5
544	Appomatox River, Mottox, Va.	745	16.2, May, 1901	11	55	120	S.E.	15.0

‡ Maximum daily flow.

TABLE 2.—(*Continued.*)

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
545	Santa Ynez River, near Lompoc, Calif.	750	55.3 , Feb., 1915	.....	.....	70	S.W.	15.1
546	Chico Creek, near Pueblo, Colo.	750	38.0 , June, 1920	.....	.....	68	R.M.	10.4
547	Huron River, Geddes, Mich.	757	4.9 , ...	5	.....	69	M.V.	1.4
548	Catawba, Morgantown, N. C.	758	42.4 , May, 1901	40	100	69	S.E.	27.5
549	Broad River, near Carlton, Ga.	762	62 , Aug., 1908	30	62	90	S.E.	17.1
550	Passaic River, Little Falls, N. J.	773	41 , Oct., 1903	15	.....	99	N.E.	11.4
551	Chagres River, Bohio, Panama	779	115.5 , ...	.....	.....	32	P.	32.2
551-A	Payette, South Fork, Garden Valley, Idaho	779	12 , June, 1921	10	30	103	R.M.	8.4
552	Noyo River Group, North West California	780	.....	60	78	70	N.W.	21.8
553	Youghiogheny River, Pennsylvania.	782	58.9 , Aug., 1888	40	100	69	N.E.	28.0
554-A	Salt Creek, below Reservoir, Wyo.	794	61 , Sept., 1923	.....	.....	112	R.M.	17.3
555	Pryor Creek, Huntley, Mont.	800	2.0 , July, 1912	1	2	99	R.M.	0.6
556	Escanaba River, Escanaba, Mich.	800	13.4 , ...	9	14	69	M.V.	4.0
557	Little Muddy, Williston, N. Dak.	800	5.4 , ...	2	5	69	M.V.	1.5
558	Shasta River, California	803	.....	12	13	70	N.W.	3.7
559	Hudson River, North Creek, N. Y.	804	37.3 , Mar., 1913	.....	.....	37	N.E.	10.6
560	North River, Port Republic, Va.	804	29.8 , ... 1896	.....	.....	120	S.E.	8.4
‡561	Raritan River, Bound Brook, N. J.	806	64.5 , Sept., 1882	25	60	1	N.E.	18.3
562	Animas River, Durango, Colo.	812	9.6 , ...	6	.....	69	R.M.	2.7
563	Scott River, Scott Bar, Calif.	813	.....	30	39	70	N.W.	11.1
564	Snake River, near Moran, Wyo.	820	18.4 , June, 1918	11	.....	103	R.M.	5.4
565	Kettle River, near Sandstone, Minn.	825	7.1 , ... 1912	.....	.....	25	M.V.	2.0
566	Holston, South Fork, Bluff City, Tenn.	828	39.8 , ... 1902	24	.....	72	M.V.	11.4
567-A	Boise River, near Twin Springs, Idaho	830	11 , May, 1917	.....	.....	103	R.M.	3.2
568	James, North Fork, Glasgow, Va.	831	44.8 , ... 1896	21	.....	71	S.E.	12.9
569	West Gallatin, Salesville, Mont.	860	12.5 , June, 1896	7	.....	98	R.M.	3.7
570	Silvies River, Burns, Ore.	865	5.5 , ...	3	.....	69	N.W.	1.5
571	Dead River, near Forks, Me.	870	20.7 , ...	.....	.....	15	N.E.	6.1
572	Deschutes, Allen Ranch, Lava, Ore.	880	2.4 , ...	1	.....	69	N.W.	0.7
573	San Pitch River, Gunnison, Utah.	886	0.6 , Mar., 1915	0.6	2	87	G.B.	0.6
574	Fish River, Wallagrass, Me.	890	10.0 , ...	8	.....	69	N.E.	3.0
575	Potomac, North Branch, Cumberland, Md.	891	22.8 , ... 1897	.....	.....	72	N.E.	6.8
576	Flint River, Molena, Ga.	892	7.4 , ...	7	40	71	S.E.	12.0
577	Black River, Lyons Falls, N. Y.	897	46.0 , Apr., 1869	.....	.....	72	N.E.	13.8
578-A	Flathead, North Fork, Belton, Mont.	900	54 , June, 1916	.....	.....	98	R.M.	16.2
579	Manistee River, Sherman, Mich.	900	3.2 , ...	3	15	69	M.V.	4.5
579-A	Schuylkill River, Reading, Pa.	900	89 , ... 1850	.....	.....	107	N.E.	26.7
580	Two Butte Creek, at Mouth, Colo.	900	39.0 , Oct., 1908	.....	.....	68	R.M.	11.7
581	Schoharie Creek, Fort Hunter, N. Y.	900	55.1 , Mar., 1901	.....	.....	37	N.E.	16.5
582	Tule Lake Group, near Mt. Shasta, Calif.	901	.....	14	16	70	G.B.	4.8
583	Clarion River, Clarion, Pa.	910	43.2 , ...	26	.....	22	N.E.	13.0
584	Santa Clara Tributaries, Ventura, Calif.	911	.....	60	82	70	S.W.	24.7
585	Delaware, East Branch, Hancock, N. Y.	920	78.8 , ... 1904	36	75	69	N.E.	23.9
586	Fountain Creek, Pueblo, Colo.	932	54.0 , June, 1921	5	50	91	R.M.	16.5
587	Cottonwood Creek, near Red Bluff, Calif.	937	....	60	75	70	S.W.	22.9
588	Clearwater, South Fork, Grangeville, Idaho	940	10.5 , May, 1912	.....	.....	103	R.M.	3.2

‡ Maximum daily flow.

TABLE 2.—(*Continued.*)

No.	River or tributary, and location.	Drainage area, in square miles.	Run-off, in second-feet per square mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
588-A	Arkansas, Florence to Pueblo, Colo.	940	80 , June, 1921	10	80	106	R.M.	24.5
589	French Broad River, Asheville, N. C.	949	94.8 , July, 1916	30	90	99	S.E.	29.2
590	Chelan River, Chelan, Wash.	950	12.2 , June, 1921	9	.....	99	N.W.	3.8
591	Mahoning River, Youngstown, Ohio.	958	20.2 , ...	15	81	69	M.V.	25.0
592	Teton River, near St. Anthony, Idaho	960	7.9 , June, 1909	5	.....	69	R.M.	2.4
593	Mill Creek Group, near Red Bluff, Calif.	971	.....	90	122	70	S.W.	38.0
594	Cut Bank Creek, Cut Bank, Mont.	971	9.1 , June, 1908	.....	.....	72	R.M.	2.8
595-A	Rock Creek, Hinsdale, Mont.	976	18.0 , June, 1906	.....	.....	120	R.M.	5.6
596-A	Passaic River, Dundee, N. J.	981	20.2 , Dec., 1878	15	40	120	N.E.	12.5
597	Stanislaus, Knight's Ferry, Calif.	983	61.0 , Jan., 1911	27	.....	70	S.W.	19.1
598-A	Genessee River, St. Helena, N. Y.	992	44.0 , May, 1916	.....	.....	113	N.E.	13.8
599	Huron River, Flat Rock, Mich.	1 000	2.8 , ...	2	.....	69	M.V.	0.9
600	Redwater River, Belle Fourche, S Dak.	1 006	8.0 , ... 1904	.....	....	72	M.V.	2.5
601	Virgin River, Virgin City, Utah.	1 010	11.9 , ... 1912	6	13	72	R.M.	4.1
602	San Luis Obispo Group, Monterey, Calif.	1 019	.....	55	73	70	S.W.	23.3
603	Hoosatonic, Gaylordsville, Conn.	1 020	31.0 , ...	16	.....	69	N.E.	9.9
604	Blackfoot River, Preston, Idaho	1 020	2.3 , April, 1907	2	.....	69	G.B.	0.7
605	Shenandoah, North Fork, Riverton, Va.	1 037	20.9 , ... 1901	.....	.....	17	S.E.	6.7
606	Cahaba River, Centerville, Ala.	1 040	16.4 , ...	15	45	69	S.E.	14.5
607	Snake River, North Fork, Ora, Idaho	1 040	5.2 , May, 1904	4	16	69	R.M.	5.2
608	Scioto River, Columbus, Ohio.	1 047	80.8 , Mar., 1913	14	80	39	M.V.	26.2
‡609	Merced River, near Merced Falls, Calif.	1 054	36.0 , Jan., 1911	.....	.....	70	S.W.	11.7
610	Saluda River, Waterloo, S. C.	1 056	18.0 , ... 1903	.....	.....	72	S.E.	5.8
611	Sacandaga, Hadley, N. Y.	1 060	33.4 , Mar., 1913	... .	.....	104	N.E.	10.9
612	Cache la Poudre, Fort Collins, Colo.	1 060	5.3 , ...	3	24	69	R.M.	7.8
613	Genesee River, Mount Morris, N. Y.	1 070	39.2 , May, 1894	.....	.....	72	N.E.	12.8
614	Truckee River, Reno, Nev.	1 070	7.0 , ... 1913	.....	.....	72	G.B.	2.3
615	French Creek, Carlton, Pa.	1 070	22.9 , ...	.....	.....	22	N.E.	7.5
616	Pajaro Tributaries, near Salinas, Calif.	1 070	.....	44	62	70	S.W.	20.3
617	Penobscot, East Branch, Grindstone, Me.	1 070	32.8 , Apr., 1923	14	.....	99	N.E.	10.7
617-A	St. Joe River, Calder, Idaho.	1 080	16.0 , May, 1922	.....	.....	103	R.M.	5.4
617-B	Flint River, near Woodbury, Ga.	1 090	32.4 , Mar., 1913	.....	.....	118	S.E.	10.7
618	Oconee River, Greensboro, Ga.	1 100	62.0 , Aug., 1908	16	60	119	S.E.	20.6
619	Walker River, Yerington, Nev.	1 100	1.5 , ...	1	.....	69	G.B.	0.5
620	Nolichucky River, Greenville, Tenn.	1 100	29.4 , ...	14	.....	69	M.V.	9.7
620-A	Milk River, Milk River, Canada.	1 104	3.1 , Feb., 1916	.....	.....	98	C.	1.0
621	Naches River, North Yakima, Wash.	1 120	19.6 , Nov., 1906	10	.. ..	69	N.W.	6.6
621-A	Miami River, Tadmor, Ohio.	1 130	113.0 , Mar., 1913	.....	.....	107	M.V.	38.0
622	Cowlitz River, Mossy Rock, Wash.	1 170	43.5 , Nov., 1906	.....	.....	72	N.W.	14.9
623	Racquette, Massena Springs, N. Y.	1 170	9.4 , ...	7	.....	5	N.E.	3.2
624	Chattahoochee, Norcross, Ga.	1 170	30.9 , Dec., 1915	15	30	118	S.E.	10.5
625	Neuse River, Selma, N. C.	1 175	6.7 , ...	16	45	119	S.E.	15.4
626	Pea River, Pera, Ala.	1 180	13.2 , Mar., 1913	8	15	119	S.E.	5.1
627	Hiwassee, Reliance, Tenn.	1 180	46.7 , Nov., 1906	24	.....	69	M.V.	16.0
628	Cache Creek, Yolo, Calif.	1 195	17.4 , Feb., 1909	12	.....	70	S.W.	6.0
629	Umatilla, River Yoakum, Ore.	1 200	19.9 , May, 1906	8	.....	120	N.W.	6.9
‡630	Yuba River, near Smartville, Calif.	1 200	92.5 , Jan., 1909	47	.....	70	S.W.	32.0

‡ Maximum daily flow.

TABLE 2.—(*Continued.*)

No.	River or tributary and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
631	Wenatchee River, Dryden, Wash....	1 200	22.6, Dec., 1917	14	.....	99	N.W.	7.8
631-A	Coeur 'D Alene, near Cataldo, Idaho.....	1 220	18.0, Mar., 1921	.....	.....	103	R.M.	6.3
632	Deschutes, West Ranch, Lava, Ore.	1 240	3.2, ...	2	.....	69	N.W.	1.1
632-A	Lehigh River, Bethlehem, Pa.....	1 240	69.8, ... 1902	.....	.....	107	N.E.	24.6
633	Heart River, Richardton, N. Dak...	1 250	6.4, ...	3	.....	72	M.V.	2.3
634	Thunder Bay River, Alpena, Mich..	1 260	5.8, ...	4	.....	69	M.V.	2.1
635	Dock River, Columbia, Tenn.....	1 260	20.3, ...	15	.....	69	M.V.	7.2
636	Conecuts, Beck, Ala.....	1 290	12.7, ...	8	.....	69	S.E.	4.6
637	Animas River, Aztec, N. Mex.....	1 300	9.6, ...	5	.....	69	R.M.	3.5
638	Mohawk, Little Falls, N. Y.....	1 306	26.6, Mar., 1913	15	.....	37	N.E.	9.6
639	Oneida, Euclid, N. Y.....	1 310	19.5, ...	8	.....	69	N.E.	7.1
640	Wapsipinicon, Stone City, Iowa.....	1 310	6.6, ...	5	.....	69	M.V.	2.4
641	Chagres River, near Gatun, Panama.....	1 320	93.9, ...	.....	.....	32	P.	34.1
642	Youghiogheny, Connellsville, Pa....	1 320	41.2, ... 1907	.....	.....	72	N.E.	15.0
643	Tygart Valley, Felterman, W. Va...	1 327	26.4, ...	.....	.....	22	M.V.	9.6
644	Orestimba Creek Group, Merced, Calif.....	1 340	.....	30	40	70	S.W.	14.6
645	Tejon Creek Group, near Hanford, Calif.....	1 341	.....	21	35	70	S.W.	12.8
646	Greenbriar River, Alderson, W. Va.	1 344	46.5, Mar., 1913	27	.....	69	M.V.	17.0
647	Grande Ronde, Elgin, Ore.....	1 350	6.2, ...	4	.....	69	N.W.	2.3
648	Cheat River, Morgantown, W. Va..	1 380	30.3, ...	20	.....	22	M.V.	11.3
649	Hondo Reservoir, N. Mex.....	1 387	7.4, July, 1905	.....	.....	120	R.M.	2.8
650	San Juan, Arboles, Colo	1 390	28.8, ... 1911	.....	.....	72	R.M.	10.7
651	Ocmulgee River, Jackson, Ga.....	1 400	18.2, ...	13	50	119	S.E.	18.7
652	Rio Grande, Del Norte, Colo.....	1 400	10 , Oct., 1911	3	10	90	R.M.	3.7
653	St. Croix, Woodland, Me.....	1 420	14.3, ...	7	.....	69	N.E.	5.4
654	Au Sable, Bamfield, Mich.....	1 420	3.0, ...	2	.....	69	M.V.	1.1
655	Potomac, Springfield, W. Va.....	1 440	17.8, ...	20	60	119	N.E.	22.8
656	Willamette, Middle Fork, Jasper, Ore.....	1 450	64.2, ...	33	....	69	N.W.	24.4
657	Merrimac, Franklin Junction, N. H..	1 460	28.1, Apr., 1923	13	.....	99	N.E.	10.7
658	Kalamazoo, Allegan, Mich.....	1 470	7.0, ...	3	.....	69	M.V.	2.7
659-A	Sun River, Fort Shaw, Mont.....	1 475	11.9, June, 1916	.....	.....	98	R.M.	4.6
660	Shoshone River, Cody, Wyo.....	1 480	10.7, ...	8	.....	69	R.M.	4.1
661	Canadian River, French, N. Mex....	1 480	105.0, Oct., 1909	.....	.....	68	M.V.	40.4
662	Mattawamkeag, Mattawamkeag, Me.....	1 500	29.3, May, 1923	11	.....	99	N.E.	11.3
663	Tuolume, La Grange, Calif.....	1 500	40 , Jan., 1911	.....	.....	99	S.W.	15.4
664	Honey Lake Group, California.....	1 507	.....	11	14	70	G.B.	5.4
665	Russian River, N. W. California....	1 508	.....	40	51	70	N.W.	19.8
666	Chenango, Binghamton, N. Y.....	1 530	23.5, Mar., 1913	18	36	119	N.E.	14.1
667	Catawba River, Catawba, S. C.....	1 535	71.7, July, 1916	19	70	118	S.E.	28.1
668	Bitterroot, Grantsdale, Mont.....	1 550	8.3, ...	6	.....	69	R.M.	3.3
669	Chattahoochee, Oakdale, Ga.....	1 560	31.3, ...1901	22	50	16	S.E.	19.8
670	Kennebec River, Forks, Me....	1 570	13.2, Apr., 1923	9	50	88	N.E.	19.8
671	Lower Scioto, Columbus, Ohio.....	1 570	70.5, Mar., 1913	20	70	71	M.V.	27.9
672	Shenandoah, South Fork, Front Royal, Va.....	1 570	48.9, ... 1902	16	48	17	S.E.	19.4
673	Osgewatchie, Ogdensburg, N. Y.....	1 580	10.0, ...	8	15	69	N.E.	6.0
674-A	Yadkin, Donaha, N. C.....	1 600	50 , July, 1916	20	50	99	S.E.	20.0
675-A	Suwannee River, White Springs, Fla.	1 600	9.0, Jan., 1908	.....	.....	119	S.E.	3.6
676-A	White River, Indianapolis, Ind.....	1 606	...., Mar., 1913	25	75	110	M.V.	30.0
677	Oostanaula, Branch, Mobile, Ga....	1 610	24.3, Mar, 1909	15	25	69	S.E.	10.0
678	Yakima River, Umtanum, Wash....	1 620	25.3, Nov., 1906	16	25	99	N.W.	10.2
‡679	San Joaquin, Friant, Calif.....	1 631	36.6, Jan., 1881	11	28	70	S.W.	14.8
680	Santa Maria River, Santa Maria, Calif.....	1 634	.....	40	61	70	S.W.	24.6

‡ Maximum daily flow.

TABLE 2.—(*Continued*).

No.	River or tributary and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
681	Allegheny River. Red House, N. Y.	1 640	24.4, Mar., 1913	15	25	22	N.E.	9.9
682	Weiser, Weiser, Idaho	1 670	10.7, May, 1896	6	12	99	R.M.	4.9
683	Kings, near Sanger, Calif.	1 694	35.3, Jan., 1914	12	.....	70	S.W.	14.5
684	Elk River, Elkmont, Ala.	1 700	28.8, ...	24	.....	69	S.E.	11.9
685	Yampa River, Craig, Colo.	1 730	5.6, ...	5	20	69	R.M.	8.3
686	Arkansas Tributaries, Pueblo, Colo.	1 740	57.0, June, 1921	5	55	91	R.M.	23.8
687	Truckee River, Clark, Nev.	1 740	3.0, ... 1911	.....	.....	72	G.B.	1.3
688	King River, Slate Point, Calif.	1 742	25.2, Jan., 1901	.....	.....	72	S.W.	10.9
689	Kiskiminatis, Avonmore, Pa.	1 750	44.3, ...	26	.....	69	N.E.	18.5
689-A	Guadalupe, New Braunfels, Tex.	1 760	32.2, Sept., 1921	.....	.....	99	M.V.	13.6
690	Umpqua, South Fork, Brockway, Ore.	1 800	39.2, ...	28	.....	69	N.W.	16.6
691	Etowah River, Rome, Ga.	1 800	33.0, Mar., 1906	19	33	69	S.E.	14.0
692	Gallatin River, Logan, Mont.	1 805	3.6, ...	3	10	69	R.M.	4.2
693	Black River, Carthage, N. Y.	1 812	21.2, Apr., 1869	10	21	72	N.E.	9.0
693-A	Methow River, Pateros, Wash.	1 850	8.1, May, 1910	7	.....	99	N.W.	3.5
694	Penobscot, West Branch, Millinocket, Me.	1 880	12.9, Apr., 1903	8	.....	10	N.E.	5.6
695	Black Warrior, Cordova, Ala.	1 900	30.0, Mar., 1902	21	30	120	S.E.	13.1
696	American River, near Folsom, Calif.	1 900	99.5, Jan., 1862	.....	.....	70	S.W.	43.3
697	Schuylkill River, Fairmont, Pa.	1 920	42.8, ...	.....	.....	69	N.E.	18.7
698	Feather, North Fork, Big Bend, Calif.	1 940	56.3, Mar., 1907	.....	.....	8	S.W.	24.8
698-A	Flint River, Culloden, Ga.	2 000	42.5, July, 1916	20	40	99	S.E.	19.0
699	Rogue River, Tolo, Ore.	2 020	23.9, ... 1909	17	.....	72	N.W.	10.7
700	Chemung River, Elmira, N. Y.	2 055	67.1. June, 1889	20	60	72	N.E.	30.4
700-A	James River, Buchanan, Va.	2 060	60.6, Mar., 1913	16	60	118	S.E.	27.5
701	Madison, Red Bluffs, Mont.	2 085	3.1, May, 1890	3	10	99	R.M.	4.6
702	Flambeau River, Ladysmith, Wis.	2 120	6.0, ...	5	14	69	M.V.	6.4
702-A	Maumee River, Antwerp, Ohio.	2 130	6.9, Apr., 1922	5	12	113	N.E.	5.5
703	Meadow Valley Wash, near Moapa, Nev.	2 150	3.8, Jan., 1910	1	5	74	S.W.	2.3
704	Birch Creek, Fourteen Mile House, Alaska	2 150	6.9, ... 1911	.....	.....	72	A.	3 2
705	Palouse River, Hooper, Wash.	2 210	12.6, Mar., 1910	5	.....	99	N.W.	5.9
706	San Francisco Bay Drainage, Calif.	2 219	.....	40	51	70	S.W.	24.0
707	Aroostook River, Fort Fairfield, Me.	2 230	15.4, ...	14	.....	69	N.E.	7.3
708	Payette River, Horseshoe Bend, Idaho	2 230	9.9, June, 1921	6	10	99	R.M.	4.7
708-A	Boise River, near Arrowrock, Idaho.	2 230	7.2, June, 1921	5	10	103	R.M.	4.7
708-B	Wolf River, New London, Wis.	2 240	6.9, Apr., 1922	5	10	113	M.V.	4.7
709	Tar River, Tarboro, N. C.	2 290	6.4, ...	10	30	71	S.E.	14.4
710	Hiwassee River, Charleston, Tenn.	2 297	20.0, ...	16	.....	69	M.V.	9.6
711-A	Blackfoot River, Bonner, Mont.	2 310	7.5, June, 1899	.....	.....	98	R.M.	3.6
‡712	Androscoggin, Rumford Falls, Me.	2 320	23.8, Apr., 1895	11	.....	120	N.E.	11.5
713	Klamath (River), (N. W.) California.	2 320	.....	15	20	70	N.W.	9.6
713-A	Auglaize River, near Defiance, Ohio.	2 320	15.5, Mar., 1919	.....	.....	113	N.E.	7.5
714	Merrimac, Garvins Falls, N. H.	2 340	18.3, ...	14	.....	69	N.E.	8.8
715	Susquehanna, Binghamton, N. Y.	2 350	26.6, Mar., 1902	16	.....	72	N.E.	12.9
716	Muskegon, Newaygo, Mich.	2 350	2.7, ...	2	.....	69	M.V.	1.3
717	Colorado (Grand), Kremmling, Colo.	2 360	9 , June, 1912	4	.....	99	R.M.	4.5
‡718	Genessee River, Rochester, N. Y.	2 365	21.2, Mar., 1865	10	.....	71	N.E.	10.3
719	Kern River, Bakersfield, Calif.	2 410	7.6, Jan., 1914	2	.....	70	S.W.	3.7
720	Menominee, near Iron Mountain, Mich.	2 415	6.2, ...	5	.....	69	M.V.	3.0

‡ Maximum daily flow.

TABLE 2.—(*Continued*).

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
721	West Fork, Trinity River, Fort Worth, Tex.	2 420	35.1, Apr., 1922	13	.....	99	M.V.	17.2
722	Ocmulgee, Macon, Ga.	2 425	37.1, Jan., 1925	14	.....	119	S.E.	18.3
723	Miami River, Dayton, Ohio	2 450	100. , Mar., 1913	19	.....	71	M.V.	49.5
723-A	Tittabawassee, Freeland, Mich.	2 461	20.1, Mar., 1919	10	20	102	N.E.	10.0
724	Little Tennessee, McGhee, Tenn.	2 470	28.3, ...	15	.....	69	M.V.	14.0
725	Elkhorn, near Norfolk, Nebr.	2 470	3.2, ... 1903	.....	.....	14	M.V.	1.6
726	Tallapoosa, Sturdevant, Ala.	2 500	23.6, Mar., 1906	15	.....	69	S.E.	11.8
727	Clark Fork, Fromberg, Mont.	2 500	4.3, ...	3	.....	69	R.M.	2.1
728	Sangamon, Riverton, Ill.	2 560	7.5, ... 1911	.....	.....	4	M.V.	3.8
729	Sevier River, Marysvale, Utah	2 560	1.2, ...	1	.....	69	G.B.	0.6
730	Rhine, Macon, Ga.	2 574	37.4, ...	.....	.....	69	S.E.	19.0
731	Grand River, Ontario, Canada	2 600	14.3, ...	.....	.....	79	C.	7.3
732	Marias River, Shelby, Mont.	2 610	11.3, ...	5	.....	69	R.M.	5.8
733	Yellowstone, Corwin Springs, Mont.	2 630	10.1, June, 1918	5	10	99	R.M.	5.2
734	Wisconsin River, near Merrill, Wis.	2 630	8.0, ...	6	.....	18	M.V.	4.1
735	Pecos River, Santa Rosa, N. Mex.	2 649	17.6, Sept., 1904	5	17	72	R.M.	9.0
736	Kennebec, near Waterville, Me.	2 700	48.6, ... 1901	.....	.....	15	N.E.	25.2
737	Flint River, Montezuma, Ga.	2 700	27.0, Jan., 1925	10	25	119	S.E.	14.0
738	Savannah, near Calhoun Falls, S. C.	2 712	27.8, Apr., 1897	15	40	119	S.E.	20.8
739-A	Miami River, Miamisburg, Ohio.	2 722	94.0, Mar., 1913	.....	.....	107	M.V.	49.0
740	New River, Radford, Va.	2 725	63.8, ... 1900	30	60	21	S.E.	33.4
741	Saline River, Beverly, Kans.	2 730	5.9, ... 1896	.....	.....	19	M.V.	3.1
742	Duchesne River, Myton, Utah	2 750	3.5, ...	2	.....	69	R.M.	1.8
743	Dan River, South Boston, Va.	2 750	18.6, Dec., 1901	8	.....	120	S.E.	9.7
744	Hudson, Glens Falls, N. Y.	2 760	25.4, Mar., 1913	.....	.....	37	N.E.	13.3
745	Canadian River, Taylor, N. Mex.	2 832	32.1, Oct., 1904	.....	.....	72	M.V.	17.1
746	Oconee, Milledgeville, Ga.	2 840	40.5, Jan., 1925	10	40	119	S.E.	21.6
747	Humboldt, Elko, Nev.	2 840	0.8, ...	0.5	.....	69	G.B.	0.4
748	Trinity, near Dunsmuir, Calif.	2 965	.....	35	44	70	S.W.	24.0
749	Catawba, near Rockhill, S. C.	2 987	50.5, May, 1901	22	.....	34	S.E.	27.6
750	Shenandoah, Millville, W. Va	2 995	46.6, ... 1896	15	.....	17	S.E.	25.5
751	Kennebec, Waterville, Me.	3 030	49.5, Dec., 1901	.....	.....	99	N.E.	27.2
752	Salt Creek, at mouth, New Mexico.	3 052	4.1, ... 1904	.....	.....	72	M.V.	2.3
753	Holston River, Rogersville, Tenn.	3 060	16.8, ...	12	.....	69	M.V.	9.3
754	Verdigris River, Liberty, Kans.	3 067	16.4, July, 1904	13	.....	2	M.V.	9.1
755	Eel River, Scotia, Calif.	3 071	94.4, ...	.....	.....	70	N.W.	52.3
755-A	Staunton River, Randolph, Va.	3 080	24.4, Dec., 1901	.....	.....	120	S.E.	13.5
756	Arkansas, Canon City, Colo.	3 090	6 , Aug., 1921	2	.....	99	R.M.	3.3
757	Seneca River, Baldwinsville, N. Y.	3 100	6.5, Mar., 1865	3	.....	120	N.E.	3.6
757-A	Connecticut, Orford, N. H.	3 100	18.5, Mar., 1913	9	.....	99	N.E.	10.3
758	Link River, Klamath Falls, Ore.	3 110	2.9, ...	.....	.....	6	N.W.	1.6
759	Pearl River, Jackson, Miss.	3 120	12.4, Mar., 1902	7	.....	69	M.V.	6.9
760	Iowa River, Iowa City, Iowa.	3 140	11.5, June, 1918	8	.....	99	M.V.	6.4
761	Wabash River, Logansport, Ind.	3 163	18.0, Mar., 1904	.....	.....	2	M.V.	10.1
762	Delaware, Port Jervis, N. Y.	3 250	28.5, Mar., 1914	18	.....	108	N.E.	16.2
763	Belle Fourche, Belle Fourche, S. D.	3 250	1.8, June, 1904	.....	.....	120	M.V.	1.0
764	Bitterroot, Missoula, Mont.	3 260	11.5, June, 1899	7	.....	69	R.M.	6.6
765	Chattahoochee, West Point, Ga.	3 300	42.1, Dec., 1919	14	40	99	S.E.	24.2
766	Saline River, Salina, Kans.	3 311	3.4, June, 1903	.....	.....	120	M.V.	2.0
767	Mohawk, Rexford Flats, N. Y.	3 384	23.1, ... 1892	.....	.....	72	N.E.	13.5
768	Yadkin, Salisbury, N. C.	3 399	38.3, ... 1889	19	.....	72	S.E.	22.3
769	Mohawk, Cohoes, N. Y.	3 472	28.5, Mar., 1913	14	28	37	N.E.	16.8
770	Juniata, Newport, Pa.	3 480	84.0, Mar., 1902	18	80	90	N.E.	49.6

TABLE 2.—(*Continued.*)

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
771-A	Meramec River, Eureka, Mo	3 497	14.6 , Sept., 1905	.....	.....	120	M.V.	8.6
772	Yakima, Union Gap, Wash	3 550	18.0 , Nov., 1906	9	.....	99	N.W.	10.7
773	Crow Wing, near Mouth, Minnesota	3 580	2.8 , ...	.....	.....	25	M.V.	1.7
774	Yellowstone, Livingston, Mont	3 580	7.5 , ...	5	.....	69	R.M.	4.5
775-A	Salmon River, Salmon, Idaho	3 600	4.5 , June, 1921	.....	.....	103	R.M.	2.7
776	Feather River, Oroville, Calif	3 627	51.6 , Mar., 1907	26	.....	70	S.W.	31.1
777	Cannon Ball, Stevenson, N. Dak	3 650	1.6 , ...	1	.....	69	M.V.	1.0
778	Neosho, Iola, Kansas	3 670	20.3 , July, 1904	14	.....	2	M.V.	12.2
779	Dan River, Clarksville, Va	3 749	8.8 , July, 1896	20	50	119	S.E.	30.6
780-A	Menominee River, below Koss, Mich.	3 790	6.1 , Apr., 1916	4	6	113	M.V.	3.7
781	South Platte, South Fork, Denver, Colo	3 840	2.3 , June, 1921	1	2	99	R.M.	1.4
782	Tallapoosa, Milstead, Ala	3 840	18.2 , Dec., 1901	.....	.....	72	S.E.	11.1
783-A	Miami River, Miami, Ohio	3 937	98 , Mar., 1913	.....	.....	107	M.V.	61.7
784	St. Joseph, Buchanan, Mich	3 940	4.7 , ...	3	.....	69	M.V.	3.0
785	Sevier River, Gunnison, Utah	3 990	0.6 , ...	0.3	0.6	69	G.B.	0.4
786	Coosa River, Rome, Ga	4 006	16.0 , Dec., 1901	14	25	16	S.E.	15.8
787-A	North Loup, St. Paul, Neb	4 020	19.0 , June, 1899	10	19	120	M.V.	12.0
788	Pit River, Bieber, Calif	4 040	9.8 , Mar., 1907	3	.....	99	S.W.	5.9
789	Salinas Tributaries, Salinas, Calif	4 042	.....	60	112	70	S.W.	71.1
790	Merrimac River, Lowell, Mass	4 085	19.8 , ...	.....	.....	72	N.E.	12.7
791	White River, near Interior, S. Dak	4 090	4.0 , ... 1905	.....	.....	72	M.V.	2.6
792	Oconee River, Dublin, Ga	4 180	10.5 , Mar., 1912	6	25	119	S.E.	16.2
793	Malheur, Vale, Ore	4 190	3.5 , Feb., 1904	2	.....	69	N.W.	2.3
794	Spokane River, Spokane, Wash	4 350	11.3 , May, 1894	6	11	99	N.W.	7.4
795	Tombigbee, Columbus, Miss	4 440	11.4 , Mar., 1902	8	.....	69	S.E.	7.6
796	Cape Fear River, Fayetteville, N. C.	4 493	25.6 , Aug., 1908	12	25	99	S.E.	17.2
797	Wateree River, Camden, S. C	4 500	8.1 , ...	7	24	119	S.E.	16.1
798	Hudson, Mechanicsville, N. Y	4 500	26.7 , Mar., 1913	11	25	37	N.E.	17.9
799	Mississippi, above Sandy River, Minn	4 510	2.1 , Sept., 1900	1.4	.....	25	M.V.	1.4
800	Merrimac River, Lawrence, Mass	4 553	20.3 , Apr., 1852	12	20	119	N.E.	13.7
801	Colorado (Grand), Glenwood Springs, Colo	4 560	7 , June, 1918	4.5	7	99	R.M.	4.7
802-A	Flathead River, Columbia Falls, Mont	4 560	19.3 , June, 1922	.....	.....	98	R.M.	13.1
803	Arkansas, Pueblo, Colo., before 1914	4 600	2.4 , May, 1901	1.2	.....	69	R.M.	1.6
804	Arkansas, Pueblo, Colo., after 1920	4 600	22.3 , June 1921	3	22	68	R.M.	15.1
805	Broad River, Alston, S. C	4 609	28.4 , May, 1901	17	28	72	S.E.	19.3
806	Yadkin River, Norwood, N. C	4 614	13.7 , ...	10	20	119	S.E.	13.6
807	Potomac River, Dam No. 5, Maryland	4 640	22.2 , ...	.....	.....	72	N.E	15.1
808	Clearwater, Kamiah, Idaho	4 850	15.8 , May, 1913	.....	.....	72	R.M.	11.0
809	Willamette River, Albany, Ore	4 860	62.2 , ... 1861	25	.....	72	N.W.	43.3
810	Grand River, Grand Rapids, Mich	4 900	10.1 , June, 1905	5	10	72	M.V.	7.1
811	Black Warrior, Tuscaloosa, Ala	4 900	38.8 , ... 1895	28	38	72	S.E.	27.2
812	Flint River, Albany, Ga	5 000	19.0 , Jan., 1925	6	19	119	S.E.	13.4
813	Guadalupe River, near Cuero, Tex	5 020	14.2 , ... 1903	.....	.....	72	M.V.	10.1
813-A	Milk River, Havre, Mont	5 050	1.9 , June, 1908	.....	.....	90	R.M.	1.3
814	Saint John River, Fort Kent, Me	5 280	14.3 , ...	12	.....	69	N.E.	10.4
815	Little River, near Leon, Tex	5 300	62.5 , Sept., 1921	.....	.....	67	M.V.	45.5
816	Chippewa River, Chippewa Falls, Wis	5 300	12.1 , June, 1905	7	12	69	M.V.	8.8
817	Red Lake, Crookston, Minn	5 320	2.7 , ...	1.6	.....	25	M.V.	2.0
818	Pitt River, Ydalpom, Calif	5 346	8.8 , Dec., 1913	5	.....	70	S.W.	6.4
819	Monongahela, Lock No. 4, Pa	5 430	38.1 , ... 1888	.....	.....	1	M.V.	28.1
820	Snake River, South Fork, Lyon, Idaho	5 480	9.4 , May, 1904	6	.....	69	N.W.	7.0

TABLE 2.—(*Continued.*)

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
821	Yakima River, Kiona, Wash	5 520	11.5 , Nov., 1906	.....	.....	72	N.W.	8.5
822	Sevier River, Leamington, Utah	5 595	0.4 , ...	0.3	.....	69	G.B.	0.3
823	Susquehanna, Williamsport, Pa.	5 670	61.7 , ...1889	19	60	107	N.E.	46.5
824	Salt River, Roosevelt, Ariz.	5 756	36.0 , Mar., 1893	.....	.....	72	R.M.	27.3
825	Little Missouri, Medora, N. Dak.	5 780	3.3 , ...	2	.....	69	M.V.	2.5
825-A	Wisconsin River, Necedah, Wis.	5 800	16.1 , June, 1905	.....	.....	120	M.V.	12.3
826	St. Croix River, St. Croix Falls, Minn	5 950	6.0 , ...	.....	.....	72	M.V.	4.6
827	Missoula River, Missoula, Mont.	5 960	6.0 , June, 1899	3.5	.....	69	R.M.	4.6
828	Elkhorn River, Arlington, Nebr.	5 980	1.6 , ...	1	.....	69	M.V.	1.2
829	Bear River, Collinston, Utah	6 000	1.9 , Jan., 1909	1	.....	69	G.B.	1.5
830	Verde River, McDowell, Ariz	6 000	24.0 , ... 1893	.....	.....	72	R.M.	18.6
830-A	Powder River, Arvada, Wyo.	6 050	15.7 , Sept., 1923	.....	.....	112	R.M.	12.2
831	Niobara, near Valentine, Nebr.	6 070	1.1 , July, 1903	0.6	.....	14	M.V.	0.9
832	Fox River, Rapid Croche Dam, Wis.	6 150	3.3 , ... 1895	.....	.....	113	M.V.	2.6
833	Pecos River, Fort Sumner, N. Mex.	6 191	7.3 , Oct., 1904	.....	.....	72	M.V.	5.7
834	New River, Fayette, W. Va.	6 200	17.8 , ... 1899	13	.....	72	M.V.	14.0
835	James River, Cartersville, Va.	6 230	17.0 , Dec., 1901	9	17	90	S.E.	13.4
836	Salt River, McDowell, Ariz.	6 260	22.0 , Mar., 1893	7	.....	69	R.M.	17.4
837	Rock River, below Rockton, Ill.	6 290	4.3 , ...	4	7	4	M.V.	5.6
837-A	Maumee River, Waterville, Ohio.	6 310	6.6 , Apr., 1922	4	6	113	N.E.	5.2
838	Cedar River, Cedar Rapids, Iowa.	6 320	9.0 , Apr., 1917	5	9	90	M.V.	7.1
839	Delaware, Riegelsville, N. J.	6 430	44.3 , Oct., 1903	15	40	99	N.E.	35.5
840	Penobscot, West Enfield, Me.	6 600	23.2 , May, 1923	9	23	99	N.E.	18.8
841	Chippewa River, Eau Claire, Wis.	6 740	9.0 , June, 1905	6	9	1	M.V.	7.4
842	Susquehanna, Northumberland, Pa.	6 800	17.5 , June, 1889	13	25	120	N.E.	20.6
843	Solomon River, Niles, Kans.	6 815	1.8 , June, 1903	1	.....	69	M.V.	1.5
844	Coosa, Riverside, Ala.	6 850	10.5 , Oct., 1898	8	20	119	S.E.	16.5
845	Delaware, Lambertsville, N. J.	6 855	37.1 , Jan., 1841	15	37	120	N.E.	30.7
846	Little River, Cameron, Tex.	7 010	92.3 , Sept., 1921	30	90	67	M.V.	77.4
847-A	Flathead, near Polson, Mont.	7 010	10.7 , June, 1913	.....	.....	98	R.M.	9.0
848	Mississippi River, Minnesota	7 283	1.5 , ...	.....	.....	71	M.V.	1.3
849	Cheyenne River, Edgemont, S. Dak.	7 350	1.5 , July, 1905	1	.....	69	M.V.	1.3
849-A	North Platte, above Pathfinder, Wyo.	7 410	2.5 , June, 1917	.....	.....	99	R.M.	2.2
850	Green River, Green River, Wyoming.	7 450	3.0 , ...	2	.....	69	R.M.	2.6
851	Savannah River, Augusta, Ga.	7 500	40.0 , Sept., 1888	15	40	120	S.E.	34.6
852	Penobscot, Bangor, Me.	7 700	14.9 , ...	7	14	10	N.E.	13.1
853	Connecticut, Sunderland, Mass.	7 700	13.4 , May, 1854	10	.....	69	N.E.	11.7
854	John Day River, McDonald, Ore.	7 800	2.9 , ...	2	.....	69	N.W.	2.6
854-A	Muskingum, Marietta, Ohio	7 850	32.0 , Mar., 1913	15	32	90	M.V.	28.4
855	Gunnison River, Whitewater, Colo.	7 863	3.6 , June, 1905	3	7	1	R.M.	6.2
‡856	Smoky Hill, Ellsworth, Kans.	7 980	2.6 , July, 1895	.....	.....	19	M.V.	2.3
857	Wisconsin River, Kilburn, Wis.	8 000	10.0 , ...	.....	.....	90	M.V.	8.9
857-A	Rio Grande, below Dam, El Paso, Tex.	8 000	1.05, Aug., 1925	1	2	111	R.M.	0.9
858	Bighorn, Thermopolis, Wyo.	8 080	3.7 , Aug., 1923	1.6	3	99	R.M.	3.3
859	St. John, Van Buren, Me.	8 270	16.2 , Apr., 1923	8	16	88	N.E.	14.7
860	Roanoke River, Old Gaston, N. C.	8 350	32.9 , ... 1877	12	32	72	S.E.	30.1
860-A	Coosa River, Childersburg, Ala.	8 390	17.9 , July, 1916	8	17	99	S.E.	16.4
861	Connecticut River, Holyoke, Mass.	8 660	21.1 , May, 1854	10	20	71	N.E.	19.6
862-A	Cheyenne River, Hot Springs, S. Dak.	8 720	17.2 , May, 1920	5	17	112	M.V.	16.1
863	Colorado (Grand), Palisade, Colo.	8 790	6. , June, 1921	3	6	99	R.M.	5.6
864	Tombigbee River, Epes, Ala.	8 830	6.9 , ...	5	15	119	M.V.	14.1
865	Kanawha, Charleston, W. Va.	8 900	13.5 , ... 1875	10	.....	71	M.V.	12.7

‡ Maximum daily flow.

TABLE 2.—(*Continued*).

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
866	Jefferson River, Sappington, Mont.	8 984	1.7 , ...	1	5	119	R.M.	4.7
867	Tennessee River, Knoxville, Tenn...	8 990	17.5 , Mar., 1902	10	30	119	M.V.	28.4
1868	Allegheny River, Kittanning, Pa....	9 010	29.9 , Mar., 1913	9	27	90	M.V.	28.4
868-A	Peedee River, Cheraw, S. C.....	9 100	30.0 , Sept., 1908	.....	.....	74	S.E.	28.6
869	Deschutes River, Biggs, Ore	9 180	3.3 , ...	2	.....	69	N.W.	3.2
870	Sacramento, Red Bluff, Calif.....	9 300	30.0 , Feb., 1907	15	30	99	S.W.	28.9
871-A	Altamaha River, Charlotteville, Ga.	9 400	16.5 , Jan., 1925	6	16	119	S.E.	16.0
872	Blue River, near Manhattan, Kans..	9 490	9.13, May, 1903	5	9	72	M.V.	8.9
873	Potomac, Point of Rocks, Md.....	9 654	33.7 , June, 1889	12	33	99	N.E.	33.1
874	Susquehanna, Wilkes-Barre, Pa....	9 810	22.1 , Mar., 1902	13	13	26	N.E.	25.8
874-A	Owyhee River, Owyhee, Ore.....	9 875	2.1 , Feb., 1904	2	.....	120	N.W.	2.1
875	Connecticut River, Hartford, Conn.	10 234	20.3 , May, 1854	10	20	11	N.E.	20.5
875-A	Kootenai, Libby, Mont.....	11 000	11.8 , June, 1916	.....	.....	98	R.M.	12.4
876	Susquehanna, Danville, Pa.....	11 100	27.4 , Mar., 1902	13	27	69	N.E.	28.8
877	Rio Grande, Rio Grande, N. Mex....	11 250	2.75, Oct., 1904	2	4	119	R.M.	4.2
878	Potomac, Great Falls, Md.....	11 427	28.4 , June, 1889	10	28	99	N.E.	30.2
879	Canadian River, Logan, N. Mex....	11 440	12.3 , Oct., 1904	6	12	120	M.V.	13.2
880-A	Yellowstone, Huntley, Mont.....	12 000	4.8 , June, 1916	2	4	98	R.M.	5.3
881	Salt River, below Phœnix, Ariz.....	12 000	24.7 , Feb., 1891	.....	.....	72	R.M.	27.0
882	Mississippi, Sauk Rapids, Minn.....	12 400	4.1 , ...	3	4	69	M.V.	4.6
883	Illinois, Peoria, Ill.....	13 480	5.94, Mar., 1904	4	6	72	M.V.	7.0
884	Loup River, Columbus, Nebr.....	13 540	5.17, June, 1896	3	5	1	M.V.	6.0
885	Salmon River, Whitebird, Idaho....	13 600	8.84, June, 1894	4	8	99	R.M.	10.3
886	Humboldt, Oreana, Nev.....	13 800	0.22, May, 1897	.....	.....	72	G.B.	0.3
887	Yazoo River, Mississippi.....	13 850	10.04, ...	5	10	72	M.V.	11.8
888	Des Moines River, Keosauqua, Iowa.	13 900	7.0 , ... 1903	4	7	99	M.V.	8.3
889-A	Milk River, Malta, Mont.....	14 040	0.82, Mar., 1918	.....	.....	98	R.M.	1.0
890	Minnesota River, Mankato, Minn....	14 600	3.0 , ...	1.6	.....	25	M.V.	3.6
891	North Platte, Orin Junction, Wyo..	14 828	1.5 , ...	1.1	.....	69	R.M.‡	1.8
892	Pecos River, Roswell, N. Mex.....	14 840	3.75, Oct., 1904	2	4	72	M.V.	4.9
893	Alabama River, Selma, Ala.....	15 400	9.5 , Jan., 1892	5	9	1	S.E.	11.8
894	North Platte, Guernsey, Wyo.....	16 200	1.9 , ...	.....	.....	69	R.M.	2.4
895	Missouri River, Townsend, Mont...	16 400	3.2 , June, 1899	2	.....	98	R.M.	4.1
896	North Pacific Drainage, Calif.....	16 543	.....	50	72	70	N.W.	92.6
897	Colorado (Grand), Fruita, Colo.....	17 100	7 , July, 1884	3	7	99	R.M.	9.1
898	Mississippi, Anoka, Minn.....	17 100	2.87, ...	2	.....	25	M.V.	3.7
898-A	Appalachicola River, Junction, Ga.	17 300	22 , ...	.....	.....	74	S.E.	28.8
899	Gila River, Florence, Ariz.....	17 750	7.5 , Feb., 1891	.....	.....	72	R.M.	10.0
900-A	Snake River, South Fork, Idaho....	17 900	2.12, ...	.....	.....	69	R.M.	2.8
901	San Joaquin Drainage, Calif.....	18 178	.....	30	39	70	S.W.	52.6
902	Missouri River, Cascade, Mont.....	18 300	2.70, June, 1908	2	.....	72	R.M.	3.6
903	White River, Clarendon, Ark.....	19 000	16.84, ... 1916	8	16	89	M.V.	23.2
904	Ohio River, Pittsburgh, Pa.....	19 100	22.98, ... 1907	10	22	33	M.V.	31.7
904-A	Clark Fork, near Plains, Mont.....	19 900	5.78, June, 1913	3	6	98	R.M.	8.5
905	Big Horn, Hardin, Wyo.....	20 700	1.97, ... 1908	1.4	3	72	R.M.‡	4.3
906	Tennessee River, Chattanooga, Tenn.	21 382	34.37, Mar., 1867	11	34	1	M.V.	50.2
907	Republican River, Bostwick, Nebr..	22 300	1.10, July, 1905	.....	.....	14	M.V.	1.6
908	Snake, South Fork, Minidoka Idaho.	22 600	2.40, June, 1896	1.9	.....	69	R.M.	3.6
909	Ohio River, Wheeling, W. Va.....	23 800	21.30, Feb., 1884	11	21	99	M.V.	32.9
910	Susquehanna, Harrisburg, Pa.....	24 030	30.60, June, 1889	12	30	1	N.E.	47.4
911	Clark Fork, Newport, Wash.....	24 200	8.98, June, 1894	4.3	9	69	R.M.	14.0
912	North Platte, Camp Clarke, Nebr...	24 800	0.95, ...	0.7	.....	14	M.V.	1.5
913	Red River, Grand Forks, N. Dak....	25 000	1.70, ... 1897	1.2	.....	25	M.V.	2.7
914	Republican River, Junction, Kans.	25 837	2.30, May, 1903	1	2	120	M.V.	3.7
915	Susquehanna, McCall's Ferry, Pa...	26 766	25.1 , Mar., 1904	.....	.....	69	N.E.	41.0
916	Illinois, at Mouth, Ill.....	27 914	4.48, Apr., 1904	.....	.....	90	M.V.	7.5
917	Rio Grande, San Marcial, N. Mex...	30 000	1.10, Oct., 1904	0.6	1.1	111	R.M.	1.9

‡ Maximum daily flow.

TABLE 2.—(*Continued.*)

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
918	Tennessee River, Florence, Ala.....	30 800	16.2 , Mar., 1897	.....	.....	72	M.V.	28.4
919	Brazos River, Waco, Tex.....	30 800	4.3 , May, 1908	4	10	119	M.V.	17.5
920	Colorado River, Austin, Tex.....	34 200	3.57, Apr., 1900	4	10	119	M.V.	18.5
921	Mississippi, St. Paul, Minn.....	36 085	3.32, Apr., 1881	1.7	3.3	25	M.V.	6.3
922	Green River, Blake, Utah.....	38 200	1.76, May, 1897	1	2	120	R.M.	3.9
923-A	Snake River, near Murphy, Idaho..	41 900	1.13, June, 1918	.....	.....	103	R.M.	2.3
924	Mississippi, Prescott, Wis.....	44 070	2.50, ...	.....	.....	30	M.V.	5.2
925	Gila River, Yuma, Ariz.....	56 000	3.93, Jan., 1916	2	4	75	R.M.	9.5
926	Platte River, near Columbus, Nebr..	56 900	0.83, May, 1905	0.4	1.8	14	M.V.	4.3
927	Kansas River, Lawrence, Kans.....	59 841	3.80, ... 1903	1	3	1	M.V.	9.3
928-A	Yellowstone, Intake, Mont.....	66 800	2.38, June, 1921	1	2	98	R.M.	6.2
929	Ohio River, Cincinnati, Ohio.....	75 800	8.70, Mar., 1913	3	8	90	M.V.	23.9
930	Mississippi, Clayton, Iowa.....	79 040	2.66, ... 1880	.....	.....	30	M.V.	7.5
931	Ohio, Louisville, Ky.....	90 600	8.5 , Mar. 1913	3	8	72	M.V.	25.6
932	Red River, Arkansas.....	97 000	2.32, ...	.....	.....	72	M.V.	7.2
933	Yukon River, Eagle, Alaska.....	122 000	2.08, ... 1911	.....	.....	72	A.	7.3
934	Colorado, above Gila Junction, Ariz.	169 000	0.89, ... 1909	0.6	0.9	6	R.M.	3.7
935	Arkansas, Little Rock, Ark.....	170 000	2.7 , ... 1916	1.4	2.9	89	M.V.	12.0
936	Mississippi, Grafton, Ill.....	171 570	2.10, ... 1883	.....	.....	30	M.V.	8.7
937-A	Niagara (Land Area Only), N. Y....	175 000	1.71, ...	0.5	1.7	114	N.E.	7.2
938-A	Rio Grande, near Rio Grande, Tex..	220 000	0.91, ... 1872	0.3	0.9	119	M.V.	4.3
939	Colorado, below Gila Junction, Ariz.	225 000	1.05, Jan., 1916	0.5	1.5	119	R.M.	7.1
940	Ohio River, Cairo, Ill.....	233 000	6.0 , Mar., 1913	3.6	6.0	90	M.V.	29.0
941	Columbia River, Dalles, Ore.....	237 000	5.87, June, 1894	3	5.8	72	N.W.	28.6
941-A	Niagara, Niagara, N. Y.....	263 440	1.13, ...	0.3	1.1	114	N.E.	5.8
941-B	St. Lawrence, near Ogdensburg, N. Y.....	298 080	1.07, ...	0.3	1.07	105	N.E.	5.8
942	Missouri, Sioux City, Iowa.....	323 462	1.64, ... 1881	.....	.....	30	M.V.	9.3
943	Missouri, St. Charles, Mo.....	530 810	1.13, ...	.....	.....	30	M.V.	8.2
944	Mississippi, St. Louis, Mo.....	702 380	1.28, ... 1883	.....	.....	30	M.V.	10.7
945	Mississippi, Cairo, Ill.....	902 900	2.23, ... 1912	1.3	.....	89	M.V.	21.2
946	Mississippi, Columbus, Ky.....	930 540	1.59, June, 1858	.....	.....	72	M.V.	15.3
947	Mississippi, Helena, Ark.....	1 000 000	2.04, ... 1912	1.1	2.5	119	M.V.	25.0
948	Mississippi, above Arkansas Jct.....	1 050 000	2.31, ... 1912	1.3	2.3	89	M.V.	23.7
949	Mississippi River, Mississippi.....	1 244 000	1.19, ...	.....	.....	72	M.V.	13.3
950	Mississippi, Carrollton, La.....	1 400 000	1.07, May, 1922	0.7	.....	89	M.V.	12.7
	FOREIGN RIVERS.							
1-A	Eyack River, Wurtemberg, Germany.....	7.34	788.8 , June, 1895	.....	.....	53	F.	21.4
1-B	Bargaglino Creek, above Genoa, Italy.....	8.8	732 , Oct., 1892	.....	.....	52	F.	21.7
1-C	Eyack River, Balingen, Germany..	34.75	356 , June, 1895	.....	.....	53	F.	21.0
1-D	Bargaglino Creek, Genoa, Italy.....	35.6	485 , Oct., 1892	.....	.....	52	F.	29.0
1	Tansa River, India.....	52.5	666.7 , ...	.....	.....	93	F	48.3
1-E	Ardeche River, Aubenas, France...	178.0	694.4 , ... 1890	.....	.....	63	F.	92.5
2	Queis River, Lauban, Germany.....	187.4	161.2 , July, 1897	.....	.....	46	F.	22.0
3	Moselle River, Epinal, France	313	90.45, ...	.....	.....	47	F.	16.0
3-A	Irrity River, India.....	336	446 , ...	.....	.....	107	F.	81.6
4	Krishna River, India.....	345.0	342.6 , ...	.....	.....	93	F.	63.6
4-A	Queis River, at mouth, Germany...	389	109 , ...1897	.....	.....	107	F.	21.5
4-B	Towbrapoorny River, India.....	587	324 , ...	.....	.....	107	F.	78.5
5	Kiewa River, Kiewa, Australia.....	700	3.8 , Feb., 1900	.....	.....	85	F.	1.0
5-A	Ardeche at junction with the Rhone.	831	382.48, ...1827	.....	.....	63	F.	110.2
5-B	Musi River, Hyderabad, India.....	862	493 , ...1908	.....	.....	107	F.	144.6

TABLE 2.—(*Continued.*)

No.	River or tributary, and location.	Drainage area, in square miles.	Run-Off, in Second-Feet per Square Mile.			Main authority, see Appendix II.	Location.†	Percentage of maximum rating on the Myers scale.
			Observed.	Expected peaks.				
			Maximum and date.	Frequent.	Rare.			
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	FOREIGN RIVERS.—(*Continued.*)							
6	Rhone, St. Maurice, Switzerland	1 812	17.40, ... 1856	.....	.....	44	F.	7.4
7	Mitta River, Tallaugatta, Australia.	2 400	6.4 , Jan., 1903	.....	.....	85	F.	3.1
8	Rhine, Lake Constance, Switzerland	2 555	48.4 , ...	.....	.....	55	F.	24.5
9	Ovens River, Waugaretta, Australia	3 000	3.3 , ...	.....	.....	85	F.	1.8
9-A	Vistula River, Cracow, Galicia	3 180	34.7 , ...1813	.....	.....	55	F.	19.6
10	Brisbane River, Brisbane, Australia	5 300	10.6 , Feb., 1893	.....	.....	85	F.	7.7
11	Glommen River, Elverum, Norway	5 650	15.63, ...	.....	.....	60	F.	11.7
11-A	Tiber River, Rome, Italy	6 455	16.4 , ...1598	.....	.....	107	F.	13.2
12	Goulburn, Murchison, Australia	9 000	3.6 , Mar., 1902	.....	.....	85	F.	3.4
12-A	Moselle River, Trier, Germany	9 200	17.28, ...	.....	.....	47	F.	16.6
12-B	Vaal River at Barrage, South Africa	10 000	18.7 , Nov., 1917	10	26	117	F.	26.0
12-C	Moldau Branch of Elbe, Prague, Czecho-Slovakia	10 400	15.3 , Mar., 1845	.....	.....	107	F.	15.6
13	Weser River, Baden, Germany	14 640	11.1 , Jan., 1841	.....	.....	41	F.	13.4
14	Seine River, Paris, France	16 859	5.24, Jan., 1910	3	.....	51	F.	6.8
14-A	Elbe River, Tetschen, Germany	19 730	10 , Mar., 1845	6	10	107	F.	14.0
15	Lachlan River, Forbes, Australia	20 159	1.5 , ... 1902	.....	.....	85	F.	2.1
16	Hun River, China	21 000	5.0 , ... 1917	.....	.....	92	F.	7.2
17	Po River, Ponte Lagosiuro, Italy	27 027	9.10, ...	.....	.....	52	F.	15.0
18	Danube, Vienna, Austria	39 212	8.86, Aug., 1897	6	10	50	F.	19.8
		39 400	9.5 , ...1899	.....	.....	107	F.	18.8
		39 400	12.6 , ...1501	.....	.....	107	F.	25.0
19	Huai River, China	52 000	5.0 , ...	.....	.....	92	F.	11.4
20	Fitzroy, Australia	58 000	10.56, Feb., 1896	.....	.....	85	F.	25.4
21	Elbe River, Altengamm, Germany	60 600	2.15, ...	.....	.....	55	F.	5.3
22	Irrawaddy River, India	149 800	12.9 , ...	.....	.....	93	F.	49.9
23	Yellow River, China	305 000	1.0 , ...	.....	.....	92	F.	5.5
24	Ganges River, India	367 970	4.9 , ...	.....	.....	93	F.	29.7
25	Yang-tze-Kiang, China	1 100 000	2.73, ...	.....	.....	92	F.	28.6
26	Nile River, Assuan, Egypt	1 300 000	0.35, ...	.....	.....	93	F.	4.0
27	Amazon, Obidos, Brazil	1 945 000	3.5 , ...	.....	.....	96	F.	48.8
28	Amazon at mouth, Brazil	2 368 000	3.0 , ...	.....	.....	96	F.	46.2

NOTES.—In addition to nearly 1 000 maximum recorded or expected flood rates from Table 2, Plate IX includes other points as follows:

From the discussion by the late Emil Kuichling, M. Am. Soc. C. E., of the paper entitled, "Flood Flows", by Weston E. Fuller, M. Am. Soc. C. E. (Reference No. 69), there are platted fifty flood rates designated (F) for European streams not otherwise listed.

In several instances, points with different designations were found to coincide, requiring either a slight displacement or the omission of one or more.

Reference No. 70, *Bulletin No. 5*, "Water Resources of California", includes several streams for which records are incomplete, but the expected maximum flood rates have been carefully estimated; these platted points are distinguished from those regularly listed.

Initials (Table 2), or characters (Plate IX), to designate each country or section have been selected with regard to their suggestive value.

TABLE 2 (A).—INDEX OF TABLE 2.*

Name of stream.	No.	Name of stream.	No.	Name of stream.	No.
Adobe	460	Black Warrior	695, 811	Chemung	700
Alabama	893	Blue Ribbon	40	Chenango	666
Alameda	516	Blue River	534, 872	Cherry	451
Alazon	77	Boggs	109	Cherryvale	17
Allegheny	681, 868	Boise	567-A, 708-A	Chester	175
Altamaha	871-A	Bolinas	279	Chewaucan	365
American Fork	118, 184	Boulder	59, 218	Cheyenne	849, 862-A
American	696	Brazos	919	Chico	546
Androscoggin	712	Breackbeck Run	35	China Wash	8
Animas	562, 637	Broad Creek	74	Chipola	540-A
Antelope	250	Broad River	549, 805	Chippewa	816, 841
Antietam	382	Brokenstraw	378	Chittenango	20, 388
Apache	102	Brush	37, 92	Choccolocco	364
Appalachicola	898-A	Brushy	195	Chowchilla	340
Appalachie	446	Budlong	9	Churn Creek	224
Appomattox	544	Buffalo	438	City Creek	82
Arkansas	588-A, 686, 756, 803, 804, 935	Bull's Run	3	Claremont	206
Aroostook	707	Burgoon's Run	48	Clarion	583
Arroyo	23, 51, 72, 75	Burns	291	Clark Fork	727, 904-A, 911
Arroyo Seco	120, 322	Butte	194, 350	Clear Creek	249, 349
Asay Creek	219	Cache Creek	475, 628	Clear Fork	484
Auglaize	713-A	Cache la Poudre	490, 612	Clearwater	588, 808
Au Sable	654	Cahaba	606	Cle Elum	311
Ausable	244-A, 449-A	Cahokia	352	Clyde	276-A
Backbone	295	Calaveras	421	Coal Creek	96
Bad River	508	Caliente	467	Cobbosecontee	342
Baker Creek	56	Cameron	44, 143	Coeur D' Alene	631-A
Baker River	297-A	Camp	45	Cold Springs	39
Bassic	138	Canadian	661, 745, 879	Colorado	717, 801, 863, 897, 934, 939
Battenkill	448	Cane Creek	94	Colorado (Texas)	920
Battle Creek	406	Cannon Ball	777	Columbia	941
Beacon	2	Canodochly	19, 67	Colvin Run	21
Bear Grass Creek	112	Cantua	313	Conecuts	636
Bear Creek	191, 205, 264, 296	Canyon Diablo	400, 492	Conejos	373
Bear River	354, 829	Canyon Padre	297	Conemaugh	156
Beaver	320, 391	Cape Fear	796	Conewanger	496
Beaverdam	89	Carrabassett	401	Connecticut	757-A, 853, 861, 875
Belle Fourche	763	Carson	187, 383, 416, 435	Connoquenessing	65
Big Cottonwood	155	Casselman	455	Coquitlam	471
Bighorn	858, 905	Catawba	548, 667, 749	Coon Creek	316
Big Muddy	540	Catskill	317	Coosa	786, 844, 860-A
Big Thompson	367	Cattaraugus	465	Coosawattee	489
Birch	196-A, 704	Cave Creek	310	Cosumnes	487
Bishops Creek	452	Cedar	269, 838	Cottonwood	115, 160, 252, 361, 587
Bitterroot	668, 764	Chagres	551, 641	Cow Creek	449
Blackfoot	604, 711-A	Chalk Creek	130	Cowlitz	622
Black Fork	376	Chandler	66	Coyote	306
Blacklick	426	Chase	86	Crooked Creek	371
Black River	357, 387, 529, 577, 693	Chatanika	461	Croton River	47, 88
		Chattahoochee	624, 669, 765	Crow Creek	348
		Cheat River	648	Crow Wing	773
		Chelan	590	Crum Creek	93
				Cut Bank	594
				Cuyadutta	133

* Does not include foreign streams.

TABLE 2 (A).—(*Continued.*)

Name of stream.	No.
Cuyahoga	541-A
Dan River	743
	779
Darby Creek	153
Daulton	186
Dead River	270
	571
Deer Creek	239
Deerfield	397
Deer River	227
Delaware	531
	585
	762
	839
	845
Deschutes	572
	632
	869
Des Moines	888
Des Plaines	512
Devil's Creek	271
Docker's Hollow	4
Dock River	635
Dog River	148
Dolores	485
Donnels Creek	113
Donner	121
Dry Creek	150
	203
	213
Dry Run	55
	97
Duchesne	742
Dutchman	192
Eagle	472
	518
East Canada	351
East Walker	433
Eel River	755
Eight-Mile	181
Elder	436
Elk River	684
Elkhorn	142
	725
	828
Embarrass River	423
Emigration Creek	117
Escanaba River	556
Esopus Creek	305
	411
	437
Estanguela	26
Etowah	507
	691
Fall River	420
Farmington	42
	502
Feather	480
	698
	776
Fish Creek	149-A
	232
	290
	299
Fish Creek (West Branch)	173
Fish River	574
Fishkill	308
Flambeau	698-A
Flathead	578
	302-A
	847-A
Flat River	172
Flint	576
	617-B
	698-A
	737
	812

Name of stream.	No.
Ford Creek	79
Fort Pierce Wash	506
Fountain Creek	586
Fox River	832
Frazier River	328
Fred Rohr Gulch	52
French Broad River	589
French Creek	319
	615
Fresno River	263
	360
Gallatin River	692
Gallinas	215
Genessee	376
	598-A
	613
	718
Gila River	825
	899
Goodyear Creek	61
Goose Lake	369
Grand Central	70
Grand River	731
	810
Grande River	476
Grande Ronde	522
	647
Great River	68
	402
Green Branch	14
Greenbriar	646
Green River	850
	922
Greenville	321-A
Guadalupe	162
	689-A
	813
Gualala	510
Gunnison	855
Gunpowder Falls	282
Gunpowder River	385
Hardscrabble	293
Heart River	633
Hiwassee	432
	627
	710
Hobble Creek	253
Hockanum	201
Hogan's Gulch	37-A
Holston River	566
	753
Honcut Creek	389
Hondo	649
Honey Creek	28
	41
	58
Honey Lake	664
Hoosatonic	603
Hoosic	501
Hudson	559
	744
	798
Hull's Gulch	34
Humboldt	747
	886
Huron River	538-A
	547
	599
Illinois River	883
	916
Independence	49
	217
Indian	18
	541
Ink's Creek	124
Iowa River	760

Name of stream.	No.
Israel Creek	50
	90
Jalama Creek	343
James River	568
	700-A
	835
Jamul Creek	190
Jefferson	866
John Day River	854
Juniata River	770
Kachess River	178
Kalamazoo	470
	658
Kanawha	865
Kaneohe	36
Kansas River	927
Kaukonahua	32
Kaweah	481
Klamath	713
Kennebec	670
	736
	751
Kern River	719
Kettle	565
King River	683
	688
Kinzua	284
Kiskiminatis	689
Kootenai	875-A
Kosk Creek	166-A
Kraiz Gamepa	209
Lagunitas	207
Lake Fork	466
Lake Roland	132
Lake Tahoe	473
Lamoille	372
Laramie	193
	444
Laurel Hill	246
Lehigh	632-A
Le Noir Coulee	73-A
Lewiston	242
Link River	758
Little Cottonwood	114
Little Devil's Creek	80
Little Gunpowder	140
Little John's Creek	137
Little Missouri	825
Little Muddy	567
Little River	815
	846
Little Stony	230
Little Tennessee	580
	724
Little Tonawanda	93-A
Little Truckee	288
Little Wolf River	469
Logan River	327
Los Angeles	176
	289
	491
Los Gatos	251
	254
Lost Creek	161
Loup River	884
Lower Scioto	671
Ludlow Creek	182
Lytle Creek	171
Machias	464
Mad Brook	33
Mad Creek	11
Madeline Plains	495
Madison	701
Mad River	462
	547-A
Magpie Gulch	12-A

TABLE 2 (A).—(*Continued.*)

Name of stream.	No.
Mahoning River	434
	591
Malabu	221
	413
Malheur	793
Manhan	64
Manistee	579
Mann's Run	5
Manoa	6
	7
Marias	732
Mariposa	231
Martell's Creek	255
Martinez Creek	84
Mattawamkeag	662
Mattole	346
	355
Maumee River	702-A
	837-A
McCloud River	526
Meadow Valley	703
Menominee	720
	780-A
Meramec River	771-A
Merced River	609
Merrimac	657
	714
	790
	800
Methow River	693-A
Miami	621-A
	723
	739-A
	783-A
Middle Creek	107
Middle Oconee	423
Milk River	620-A
	813-A
	889-A
Mill Brook	54
Mill Creek	60
	91
	103
	134
	593
Miller's Creek	359
Milwaukee River	523
Minnesota River	890
Mission Creek	197
Missiquoi	450
Mississippi	799
	848
	882
	898
	921
	924
	930
	936
	944
	to
	950
Missoula River	827
Missouri Canyon	20-A
Missouri	701
	895
	902
	942
	943
Mohawk	278
	479
	638
	767
	769
Mojave River	318
	425
Mokelumne	513
Mono	287

Name of stream.	No.
Monocacy	521
Monongahela	819
Moosa Creek	122
Moose River	407
	429
Mora River	281
	380
Mount Diablo Creek	309
Moye River	535
Murchison Farm	1
Musconetcong	272
Muskegon	716
Muskingum	854-A
Naches River	516-A
	621
Napa River	333
Nashua	210
	236
Navarro	366
Neosho	778
Neshaminy	268
Neuse River	625
New River	740
	834
Niagara	937-A
	941-A
Nine-Mile	177
Niobara	831
Nolichucky	620
North Arroyo	72
North Canyon	30
North Fork	220
North Loup	787-A
	849-A
North Platte	891
	894
	912
North River	560
Notteley	363
Noyo	552
Nuuanu	12
	28
Oak Creek	188
Occoquan	494
Ocmulgee River	651
	722
Ocoee River	409
	618
Oconee River	746
	792
Octoraro	325-A
Ogden River	404
	904
	909
Ohio	929
	931
	940
Oil Creek	386
	439
Olmos Creek	110
Ohanapecosh	244
Olentangy	482
Oneida	168
	639
Onondaga	235
Oostanaula	677
Orestimba	644
Oriskany	273
Osgewatchie	673
Osteen Arroyo	46
Otay	223
Otter	179-A
Owens Creek	195
	323
Owens	445
	486

Name of stream.	No.
Owyhee	874-A
Pacolet	424
Pajaro	616
Palomas	159-A
Palouse	706
Panoche	381
Panther	69
Parley's Creek	157
Passaic	225
	550
	596-A
Patapsco	347
Patuxent	260
Paulinskill	258-A
	262-B
Payette	503-A
	551-A
	708
Pea River	626
Pearl River	759
Peck's Creek	126
Pecos River	735
	833
	892
Peedee River	868-A
Penitencia	95
Pennigewasset	509
Penobscot	617
	694
	840
	852
Pequest	13
	123
Pequonnock	106
	174-A
	212
Perkiomen	277
Pescadero	332
Petaluma	266
Pigeon	519
Pinal Creek	119
Pine Creek	151
Pine River	458
Piscataquis	375
Pit River	788
	818
Platte	926
Pompton	415
Potomac	431
	575
	655
	807
	873
	878
Price River	488
Prosser	152
Poso	500
Powder River	830-A
Provo River	16
	25
	517
Pryor Creek	555
Purgatoire	542
Putah Creek	216
	520
Ramapo	248
	283
Rapid Creek	390
Racquette	538
	623
Raritan	561
Red Bank	237
Red Creek	135
Red Lake	617
Red River	913
	932

TABLE 2 (A).—(*Continued.*)

Name of stream.	No.	Name of stream.	No.	Name of stream.	No.
Redwater	600	San Dieguito	384		204
Redwood	386	Sandusky	384-A	Smith River	335
Reel's Creek	31		189		511
	907	Sandy	240	Smoke Creek	300
Republican River	914		518	Smoky Hill	856
Rhine	730	San Francisco	706		564
Ridley Creek	85	San Francisquito	131		607
Ritchie Gulch	98	San Gabriel	331		820
	73		442	Snake River	900-A
Rio des Perca	101	Sangamon	728		908
	652	Sangquoit	158		923-A
	857-A		183	Solomon	843
Rio Grande	877	San Jacinto	234	Sonoma	199
	917		395	Soquee	280
	938-A		536	Soquel	393
Rio Grande, Panama	20	San Joaquin	679	South Arroyo	15
Roach Creek	212		901	South Boulder	257
	418	San Juan	650	South Platte	781
Roanoke	860	San Leandro	144	South River	345
Rockaway	163	San Lorenzo	338	South Sangammon	440
	245	San Luis Obispo	602	Spanish Fork	527
	169		314	Spokane River	794
Rock Creek	198	San Luis Rey	394	Spring Creek	57
	595-A		498		111
Rock River	837	San Mateo	208	Stanislaus	597
Rocky Creek	71	San Pablo	136	Starch Factory	24
Rogue River	699	San Pedro	22	Staunton	755-A
	224-A		146	Stillwater	455-A
Rondout Creek	405	San Pitch	573		62
	414		301	Stony	441
Rush Creek	83	Santa Ana	307		506-A
	524		463	Sudbury	200
Russian River	665	Santa Catarina	493	Suisun	256
		Santa Clara	584		505
Sacandaga	611	Santa Margarita	532	Sun River	659-A
Saco	417	Santa Maria	167	Surprise Valley	412
	499		680		715
Sacramento	870	Santa Ynez	328		823
St. Charles	468		545		842
	653		164	Susquehanna	874
St. Croix	826	Santa Ysabel	241		876
	814		259		910
St. John	859	San Vincent	196		915
St. Joe	617-A	Sapella	329	Sutter	374
St. Joseph	784	Saranac	514	Suwannee	675-A
St. Lawrence	941-B	Sawkill	127		141
St. Maries	437-A	Sawpit Canyon	45-A	Sweetwater	243
	294	Savannah	738		292
St. Mary	457		851	Swift Current	228
	370	Scantic	247	Switzer Canyon	27
St. Regis	509		339	Sylvan Glen	10
St. Vrains	315	Schoharie	581		
Salado Creek	275		579-A	Tallapoosa	726
	341	Schuylkill	697		782
Salinas	789		608	Tallulah	304
	741	Scioto	671	Tar River	709
Saline	756	Scott River	563	Tawawa Creek	165
	286	Seekonk	303	Tejon	645
Salmon Creek	336	Seneca	757	Templeton Gap	43
	330	Sespe	324		867
	353		729	Tennessee	906
Salmon River	539	Sevier	785		918
	775-A		822	Teton	592
	885	Shasta	558	Thunder Bay	684
	554-A	Sheboygan	427-A	Tionesta	456
Salt Creek	752		605	Tittabawassee	723-A
	824	Shenandoah	672	Tocca	337
Salt River	636		750	Tohickon	228
	881		478		795
Saluda	610	Shoshone	660	Tombigbee	864
	125	Silver Creek	398	Tomer	63
San Antonio	211	Silvies	570		537
	265	Six-Mile	104	Trinity	721
	392		145		748
	302	Skinner Creek	38		99
San Diego	410	Smith Creek	108	Trout	105
	442-A				

TABLE 2 (A).—(*Continued.*)

Name of stream.	No.	Name of stream.	No.	Name of stream.	No.
Truckee	454, 484, 614, 687	Wailua	100	Willow Creek	87, 214, 422
Tuckaseegee	525	Walker River	344, 433, 619	Winooski	438-A
Tugaloo	504	Walla Walla	262-A	Wisconsin	734, 825-A, 857
Tule	356, 419, 582	Wallowa	147	Wissahickon	180
Tuolumne	663	Walnut	261	Wolf River	708-B
Turkey Creek	154	Wanaque	226	Woodhall	53
Turtle Creek	128, 274	Wantuppa	116	Yadkin	474, 674, 768, 806
Twin Creek	362	Wapsipinicon	640	Yakima	166, 477, 678, 772, 821
Two Butte Creek	580	Watauga	430	Yamhill	377
Tygart Valley	643	Wateree	797	Yampa	685
Tygerts	427	Weber River	285	Yazoo	887
Uinta River	326, 528	Weiser River	682	Yellowstone	733, 774, 880-A, 928-A
Umatilla	403, 629	Wenatchee	631	Yokohl	222
Umpqua	690	West Canada	408, 483	Youghiogheny	379, 443, 553, 642
Unadilla	312	Westfield	139	Yuba	81, 630
Uncompahgre	497	West Fork	543	Yukon	933
Union River	267, 459	West Gallatin	569	Zion Creek	396
Valley	233	West Walker	344, 428		
Venison	29	Whetstone	447		
Ventura	334	Whippany	129		
Verde River	830	Whiteface	453		
Verdigris	754	White River	515, 676-A, 791, 903		
Virgin River	601	Whiterocks	238		
Wabash River	761	Whitewater	358		
Wahiawa	78	Wildcat	170		
		Willamette	533, 656, 809		
		Williams	398		
		Williamstown	76		

APPENDIX II

List of Authorities Referred to in Table 2

Nos. (1) to (65) inclusive, *Transactions,* Am. Soc. C. E., Vol. LXXVII (1914), p. 662, Authorities as listed and cited by the late Emil Kuichling, M. Am. Soc. C. E.

No. (66) U. S. Geological Survey, *Water Supply Paper No. 426,* "Southern California Floods of January, 1916".

No. (67) U. S. Geological Survey, *Water Supply Paper No. 488,* "The Floods in Central Texas, September, 1921".

No. (68) U. S. Geological Survey, *Water Supply Paper No. 487,* "Arkansas River Flood of June 3-5, 1921, Pueblo, Colo.".

No. (69) *Transactions,* Am. Soc. C. E., Vol. LXXVII (1914), p. 564, "Flood Flows", by Weston E. Fuller, M. Am. Soc. C. E.

No. (70) *Bulletin No. 5,* "Water Resources of California; Report to the Legislature of 1923 by State Engineering Department".

No. (71) *Transactions,* Am. Soc. C. E., Vol. LXXXVII (1924), p. 135, "The Design of Earth Dams", by Joel D. Justin, M. Am. Soc. C. E.

No. (72) American Civil Engineers' Handbook, 1920 Edition, p. 1170 *et seq.,* Mansfield Merriman, M. Am. Soc. C. E.

No. (73) *Engineering News-Record,* Vol. 88, 1922, p. 771.

No. (74) "Observations and Estimates", by C. S. Jarvis, M. Am. Soc. C. E.

No. (75) *Engineering Record,* Vol. 74, 1916, p. 346.

No. (76) *Engineering News-Record,* Vol. 89, 1922, p. 921.

No. (77) " " " Vol. 89, 1922, p. 402.

No. (78) " " " Vol. 89, 1922, p. 239.

No. (79) *Transactions,* Am. Soc. C. E., Vol. LXXXV (1922), p. 1534 *et seq.,* "Flood Problems: A Symposium"; Data on Canadian Rivers by J. B. Challies and W. H. Breithaupt, Members, Am. Soc. C. E.

No. (80) *Transactions,* Am. Soc. C. E., Vol. LXXXV (1922), p. 1383 *et seq.,* "Flood Problems: A Symposium".

No. (81) *Engineering News,* Vol. 74, 1915, p. 742.

No. (82) *Bulletin,* American Railway Engineering Association, August, 1915.

No. (83) Water Supply Commission of Pennsylvania.

No. (84) *Engineering Record,* Vol. 53, 1906, p. 170.

No. (85) *Transactions,* Am. Soc. C. E., Vol. LXXVII (1914), p. 638, Discussion by Herbert E. Bellamy, Assoc. M. Am. Soc. C. E.

No. (86) *Engineering News-Record,* Vol. 89, 1922, p. 345.

No. (87) Tenth Biennial Report (Hydrography), State Engineer of Utah, 1916.

No. (88) *Engineering News-Record,* Vol. 90, 1923, p. 891.

No. (89) *Engineering News-Record,* Vol. 90, 1923, p. 112, "Hydrologic Record of Mississippi River Floods" by E. N. Chisolm, Jr., M. Am. Soc. C. E.

No. (90) "Relief from Floods", by Alvord and Burdick.
No. (91) *Transactions,* Am. Soc. C. E., Vol. LXXXV (1922), p. 1528, Discussion of "Flood Problems", by H. P. Eddy, M. Am. Soc. C. E.
No. (92) *Transactions,* Am. Soc. C. E., Vol. LXXXV (1922), p. 1405, Discussion on "Flood Problems" by John R. Freeman, Past-President, Am. Soc. C. E.
No. (93) *Engineering News,* Vol. 62, 1909, p. 315, "The Recent Floods at Monterey, Mexico", by G. R. G. Conway, M. Am. Soc. C. E.
No. (94) *Transactions,* Am. Soc. C. E., Vol. LXXXV (1922), p. 1383 *et seq.;* various minor references to Symposium on "Flood Problems".
No. (95) Senate Document 142, Report by A. P. Davis, Past-President, Am. Soc. C. E., as Director, U. S. Reclamation Service, on "Development of the Imperial Valley", to 67th Congress.
No. (96) Encyclopedias: Britannica, International, *et al.*
No. (97) *Engineering News-Record,* Vol. 83, 1919, p. 28, "Spillway Capacities" by J. T. Whistler, M. Am. Soc. C. E.
No. (98) *Engineering News-Record,* Vol. 91, 1923, p. 1016, "Flood Flows of Montana Streams", by G. H. Ellis.
No. (99) Corrective and Supplemental Data from the U. S. Geological Survey.
No. (100) *Engineering News-Record,* Vol. 95, 1925, p. 153, "Flood Protection Studies for Syracuse, N. Y."
No. (101) Discussion on "Flood Flow Characteristics", by George G. Honness, M. Am. Soc. C. E., see p. 1041.
No. (102) Hydrographic Records of Gardner S. Williams, M. Am. Soc. C. E., Ann Arbor, Mich.
No. (103) Third Biennial Report, Department of Reclamation, State of Idaho, 1923-24.
No. (104) Annual Report, New York State Engineer, 1923.
No. (105) Discussion by H. de B. Parsons, M. Am. Soc. C. E., on "The St. Lawrence Waterway to the Sea", see p. 517.
No. (106) *Engineering News-Record,* Vol. 88, p. 1073, "Hydraulic Design of Bridge Waterways", by Ivan E. Houk, M. Am. Soc. C. E.
No. (107) Technical Reports, Miami Conservancy District, Pts. IV and VI.
No. (108) Report of the Water Supply Commission, Pennsylvania, 1919-20.
No. (109) *Engineering News-Record,* Vol. 87, p. 480, "High Rainfall and Run-off from Small Areas", by Ivan E. Houk, M. Am. Soc. C. E.
No. (110) *Engineering News-Record,* Vol. 95, August 6, 1925, p. 213.
No. (111) *Engineering News-Record,* Vol. 95, September 3, 1925, p. 372, "Effect of Rio Grande Storage on River Erosion and Deposit", by L. M. Lawson, M. Am. Soc. C. E.
No. (112) Discussion on "Flood Flow Characteristics", by Robert Follansbee, M. Am. Soc. C. E., see p. 1033.
No. (113) U. S. Geological Survey *Water Supply Paper No. 544,* Pt. IV, "St. Lawrence River Basin."

No. (114) Report of Board of Review, Chicago Sanitary District, Pt. II, pp. 72 and 78.

No. (115) *Engineering News,* Vol. 69, p. 221, by Edgar B. Thomas, M. Am. Soc. C. E.

No. (116) Agricultural Engineering Investigations, 1925, U. S. Bureau of Public Roads.

No. (117) Sundry News Notes of the Current Engineering Press.

No. (118) U. S. Geological Survey *Water Supply Paper No. 472.*

No. (119) Co-ordinated Records of U. S. Weather Bureau and U. S. Geological Survey.

No. (120) *Proceedings,* Am. Ry. Eng. Assoc., Vol. 12, Pt. 3, 1911, p. 504.

NOTE.—Column (8) of Table 2 (Appendix I) devoted to references is entitled "Main Authority." Several conflicting sources have often been consulted; perhaps the date was supplied from one and the most reliable observed data from another. In such a case, the latter authority is the only one quoted.

The quantities in Columns (6) and (7) of Table 2, "Expected Peaks," were in general gleaned from sources other than the ones quoted, except where the reference is No. (70); this California *Bulletin* was found to be better adapted than any other publication examined for giving visual presentation of such hydrographic data. It has been endorsed and adopted as the most desirable form for displaying such information by the Society's Special Committee on Flood Protection Data.

Wherever the U. S. Geological Survey publications contained the data for a given stream they were given preference, as being both the most reliable and the most nearly standardized. The majority of observations tabulated were either made or collected and authenticated by the Hydrographic Branch of that Department.

DISCUSSION

ROBERT FOLLANSBEE,* M. AM. SOC. C. E. (by letter).—This paper contains the most complete tabulation of flood flows that has come to the writer's attention, and the Engineering Profession is indebted to the author for his able treatment of the subject.

The list of maximum observed discharge rates should be brought down to date where possible, especially in the case of records for stream-gauging stations still being maintained, where floods greater than those presented by the author have been recorded.†

During 1922 and 1923, the States of Colorado and Wyoming experienced a number of severe floods, some of which were the greatest ever recorded. These are described in a report of the U. S. Geological Survey.‡

In addition, two other severe floods of previous years which have been recorded, are not included in the author's list. Table 3 shows the maximum discharge of these floods, arranged according to the size of the drainage areas.

TABLE 3.—MAXIMUM OBSERVED FLOOD DISCHARGE RATES.

Stream.	Drainage area, in square miles.	Crest discharge, in second-feet per square mile.	Date.
Skyrocket Creek, at Ouray, Colo.	1.0	2 000	July 20, 1923
Magpie Gulch, near Golden, Colo.	1.5	1 270	July 26, 1923
Missouri Canyon, near Masonville, Colo.	2.4	1 820	June 16, 1923
Hogan's Gulch, at Eden, Colo.	6.1	1 580	Aug. 7, 1904
Bayou Gulch, near Franktown, Colo.	19*	460	July 28, 1922
Redstone Creek, at Masonville, Colo.	21*	325	June 16, 1923
Tough Creek in Sec. 28, T. 39 N., R. 94 W., Wyo.	24	62.5	July 24, 1923
Buckhorn Creek, near Masonville, Colo.	40*	262	June 16, 1923
Cherry Creek, near Parker, Colo.	87*	195	July 28, 1922
Paintrock Creek, near Hyattville, Wyo.	164	30.2	July 24, 1923
Salt Creek, below Salt Creek Reservoir, Wyo.	520	61.5	Sept. 27, 1923
Badwater Creek, at Bonneville, Wyo.	794	23.4	July 24, 1923
North Fork, Shoshone, near Wapiti, Wyo.	800	10.1	July 22, 1923
Powder River at Arvada, Wyo.	6 050	15.7	Sept. 29, 1923
Cheyenne River, at Hot Springs, S. Dak.	8 720	17.2	May 12, 1920

* Drainage area affected by storm.

O. W. HARTWELL,§ ASSOC. M. AM. SOC. C. E. (by letter).—The author has made a valuable contribution to the long list of papers on this subject. In the past much has been written about flood flows, and many formulas have been evolved in attempts to predict the size of floods. This problem will be studied again and again in the future, with results increasing in value as the fund of actual flood data increases.

The author has overlooked one source of information which will give him many valuable additions to Table 2. In all the recent *Water Supply Papers*

* Dist. Engr., U. S. Geological Survey, Denver, Colo.

† This suggestion has been carried out by a revision of Table 2 (Appendix I) included in the paper as published.

‡ "Some Floods in the Rocky Mountain Region", *Water Supply Paper No. 520-G*, U. S. Geological Survey.

§ District Engr., U. S. Geological Survey, Trenton, N. J.

of the U. S. Geological Survey the maximum and minimum discharge during the total period of record is published for each gauging station. In the report for any given year may also be found the extremes of discharge for that year.

The author dismisses consideration of the time element too hastily. There are many projects where human life is not endangered and where the cost of various degrees of protection to property must be compared with corresponding losses. Such problems can best be solved by the theory of probability. In this connection, Allen Hazen, M. Am. Soc. C. E., wrote,* in 1914:

"As time goes on, data covering longer periods and more streams may change some of the numerical values; but the underlying idea of treating the recurrence of floods as a matter of probabilities, to be determined by an examination of the records of many streams, will stand."

Ten years have already been added to the continuous records for a large number of the stream-gauging stations which existed in 1914, many new stations having been installed since that time. Furthermore, the accuracy of flood data has been greatly increased through the development of the automatic water-stage recorder and through other improvements in methods of conducting stream-gauging work. It will take 1 000 years to obtain an actual 1 000-year record, but long before the end of this period engineers will be able to determine the 1 000-year probability within a reasonable degree of accuracy.

E. C. La Rue,† M. Am. Soc. C. E. (by letter).—Since the automobile became a dependable means for long-distance travel the demand for good roads has been almost universal. During recent years hundreds of millions of dollars have been expended for the construction of automobile highways in the United States. The cost per mile of many of the hard-surface highways is comparable with that of railroad construction. It is generally conceded that some of this road work has been done without providing adequate structures to take care of transverse drainage. How many engineers make a serious study to determine the magnitude of the floods which may be expected in each running stream or dry wash?

There is much to be said in defense of the engineer. He is not always responsible for the poor location of a railroad or highway, or for drainage structures of inadequate capacity. The author states that "an important Western railroad company located nearly 100 miles of line along a valley floor, safely above the reach of the maximum assumed floods, only to experience its destruction within a few years". Some years ago, the writer talked with the engineer who had charge of the location of this railroad. He had strongly recommended another location to avoid the floods which he, at that time, knew to be dangerous, but was overruled by those who were financing the construction. The estimated initial cost for the location in the valley was less than that recommended by the chief engineer.

The author has presented a simplified formula for computing maximum floods which may be expected from a given drainage basin. Although this is an important step in the right direction, he has left plenty of work for the

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 626.

† Hydr. Engr., U. S. Geological Survey, Pasadena, Calif.

engineer who must design the structures, and who must make certain that they will carry the maximum load with safety, that they will not be damaged by the maximum expected floods, and that their estimated costs are reasonable. It will be noted that the author's formula $\left(Q = R\sqrt{M}\right)$, carries a factor which may vary from 100 to 10 000. This formula will unquestionably prove useful in actual practice. However, the engineer must make extensive studies to determine the flood flow characteristics of the streams in the territory through which he must build a railroad or highway. Where the construction of a dam is being considered the studies of flood flows may be confined to its particular drainage basin.

The writer, being familiar with the flood flow characteristics of the Colorado River and many of its tributaries, has applied the author's formulas for the purpose of comparing the calculated values for flood flows with those obtained from recorded data or traditional evidence. It was found that the formulas give erroneous results when used to determine flood flow maxima on the main stream of the Colorado River. According to the formulas the greater the area of drainage basin, the greater the maximum flood. This is not true of the Colorado River. The maximum flood flow at Lees Ferry (drainage area, 108 000 sq. miles) may be 250 000 sec-ft., while the maximum flood at the Laguna Dam (drainage area, 186 000 sq. miles) may be only 200 000 sec-ft. The records show that on this stream the maximum flood flows occur at a point 800 to 850 miles above its mouth (except floods from the Gila River).

The writer believes that the maximum flood that may be expected in a stream with a drainage basin of 1 000 sq. miles, or more, can be estimated with greater accuracy than in streams having drainage basins of a few square miles. The greatest run-off per square mile of drainage area occurs in streams having small drainage basins; however, the frequency of these floods is much less than those which occur in streams having large drainage basins. The engineer has to decide as to how much money may reasonably be spent on structures where the maximum flood may not occur once in 100 or 200 years. It is true that the 100-year flood may occur next year; it is also true that if the structures over every dry wash were built to withstand the 100 or 200-year maximum flood, the mileage of highways to be constructed would be seriously reduced. Where the problem does not involve possible loss of human life, it seems reasonable for the engineer to design flood by-pass structures on a less conservative basis, taking into account the cost of replacing structures which may be destroyed once in 50 years.

The author's graphical tabulation of run-off records (Plate IX) is a valuable contribution to the subject, and he is to be commended for his exhaustive search for the data which form the basis for this diagram. Its accuracy and value are only limited by the flood-flow data which were available. The diagram, perhaps, fails to show the maximum 50-year or 100-year flood for many of the streams due to the lack of available records, but the author has plotted sufficient data to show in a general way the limits of maximum floods that may be expected in streams with drainage basins varying

in size from less than 1 sq. mile to more than 1 000 000 sq. miles. The diagram would obviously be more valuable if sufficient data were available to show the maximum flood that has occurred during the past 100 years on each of the respective streams considered.

The author is unfair to himself, when in his conclusion he says:

"In this paper, the writer does not assume to have solved any particular problem, but merely lists certain data and considerations that are often left unrelated or neglected entirely."

He has suggested a simple formula which may prove to be of considerable value to engineers who must calculate maximum flood flows that may be expected in a given region. The paper as a whole may prove an important factor in furthering the work that should be done to determine flood dangers along streams throughout the country.

N. C. Grover,* M. Am. Soc. C. E. (by letter).—This paper emphasizes the importance of a thorough study of floods, the need for which was anticipated by the Board of Direction of the Society when, in 1922, it created a Special Committee on Flood Protection Data. That Committee, after compiling the records of many floods, has recommended that the Society undertake an elaborate study of flood data and the publication of the results in such form as to afford engineers a satisfactory working basis for handling the many problems of design and construction related to floods.

In the absence of sufficient data to serve as a basis for studying floods by river basins, or even by regions, the author has followed the common practice of showing on a single diagram (Plate IX), the available flood data for a wide area (in this instance, the whole world) and of plotting thereon limiting, average, and regional curves of various kinds that may serve to aid him and others in establishing elevations of highways and railroads and capacities of culverts and other bridge openings and of spillways and flood channels. The diagram has served to generalize the available data and to show how the regional curves are related to all the data and to the limiting curves of the world's floods. He has shown also, by sixteen symbols, the flood data for different regions, but the plotting of so many points on a single diagram has made difficult the separate utilization of the information pertaining to any particular region. He has also presented a diagram (Fig. 6) showing the various flood formulas and, by modifying one of these formulas, has suggested a possible method of approach to the problems that arise in certain fields of engineering.

The results are evidently not wholly satisfactory to the author, as he "does not assume to solve any particular problem but merely lists certain data and considerations that are often left unrelated or neglected entirely." His inability to reach satisfactory conclusions should perhaps have been anticipated, as the only data available are unsystematic and even incomparable, because they include without discrimination both peak discharges and averages for days of maximum discharge. As a net result of reading

* Chf. Hydr. Engr., U. S. Geological Survey, Washington, D. C.

the paper one is left with the feeling that organized engineers, and specifically the Society, should find a better way to serve the profession by providing a rational and adequate basis for treating the many important problems related to floods. Evidently, the author is similarly impressed, as he states in his "Conclusion" that his purpose in preparing this paper will have been accomplished when "a definite campaign to determine flood dangers and the advisable measures for control" has been inaugurated.

Floods are essentially local phenomena in which precipitation is the major factor, even if temperature, in its effect in storing water as snow and ice and, later, in releasing it by melting, may at times be controlling. Other factors also serve to modify the effects of precipitation in varying degrees. Precipitation and temperature vary widely within relatively short distances, so that each of the factors modifying the effect of precipitation in producing floods is strictly local in its application. Floods, therefore, must be studied locally by river systems and sometimes even by tributaries, as well as broadly by continents, in order to reach reliable conclusions. A composite picture of the world's floods will disclose, of course, the limits of floods that are due to the worst conditions of rainfall and run-off, but will not show what may reasonably be expected on any particular stream. Obviously, engineers should not design structures for all rivers on the basis of floods that occur in only a few. The cost would be unwarranted because unnecessary, even if such structures would have the saving grace that they would surely be ample and, therefore, never the causes of physical disaster. Moreover, much legitimate development would be prevented because of prohibitive costs.

California has made a study of the floods on the rivers of that State and has published the results in graphic form.* Similar studies should be made for many rivers in other States by using as the principal source of information the systematic records of discharge collected by the U. S. Geological Survey and supplementing them by all other pertinent data obtainable. This is the first step in the program of the Special Committee on Flood Protection Data, for which it is seeking sufficient funds to warrant the employment of a mature and competent engineer who can work continuously under the direction of the Committee. Following this step the Committee expects, if available funds permit, so to correlate floods with climate, topography, geology, vegetation, and other modifying factors that reasonably accurate predictions of floods may be made not only for the rivers in the United States that have been systematically gauged, but also for the many others for which actual records of discharge are not available.

The writer feels, therefore, that although this paper has direct value in meeting the present needs of engineers for aid in applying the meager available flood data to immediate problems, its greatest value is indirect, for it accentuates the range and importance of flood problems and the unsatisfactory data and methods now available to engineers for solving them. He hopes that its presentation and discussion will assist in promoting a rational study of the magnitude and control of floods in the rivers of the United States

* "Flow in California Streams," *Bulletin No. 5*, Department of Public Works, California.

and the publication of the results in form for ready use, thereby making available to engineers not only the mass of flood data that is awaiting compilation and analysis, but also methods for its application to particular problems. This will change the present unsatisfactory, expensive, and even dangerous procedure into one that will serve as a basis for economic and safe development and that will bring credit to the profession.

B. Okazaki,* M. Am. Soc. C. E. (by letter).—It is well known that where an improved river, in its primitive state, overflows the natural river banks and adjoining lands, the actual maximum discharge at a point down stream from the valley subject to overflow is far less than if the river were diked or "short-cut" to prevent inundation. The natural controlling action caused by the temporary reserve or detention of flood water over the submerged area makes it sometimes necessary to fix the criterion for maximum flood discharge in determining the stability or size of a structure contemplated and the extent of waterway to be provided for probable future floods. The known maximum flood discharge may be greatly exceeded if the river is improved by short-cuts or embankments that do away with the natural flood reserve.

In dealing with a river in its natural state, the writer suggests the need for considering this controlling action due to overflow. As the volume of water temporarily impounded over a large area changes continually, it is difficult to determine the overflow in reserve and the time of maximum intensity, and, likewise, the quantity to be added to the ordinary maximum discharge in order to forecast the probable future discharge after the up-stream portion has been improved.

The writer made a careful investigation of the Ishikari River, which is one of the two largest rivers in Japan and which had long been unregulated. It was found that the known maximum discharge, in July, 1904 (150 000 cu. ft. per sec.) at a place called Tsuishikari, which is about 25 miles up stream from the river mouth and which has 5 600 sq. miles of tributary drainage area, should be doubled, namely, 300 000 cu. ft. per sec., and that the observed run-off of 26.8 cu. ft. per sec. per sq. mile might be increased to 53.6 cu. ft. Therefore, this information should be considered in plans for improvement.

It should also be noted that on the same river for a point called Kamuikotan about 132 miles up stream from the mouth, where the drainage area is 1 400 sq. miles and the impounding action is negligible, the same flood gave a maximum discharge of 129 000 cu. ft. per sec., or 92.2 cu. ft. per sec. per sq. mile.

L. K. Sherman,† M. Am. Soc. C. E. (by letter).—The data which the author has compiled and interpreted, are valuable contributions to engineering progress in the determination of rates of flood run-off.

In forecasting the run-off from relatively small areas the considerations of rainfall, time of flow, and coefficient of imperviousness (such as is used in storm-sewer design) will suffice. In forecasting flood run-off from larger areas, however, the results of actual observation must govern. The usual

* Engr. in Chf., Upper Liao River Conservancy, Newchwang, China.
† Pres., Randolph-Perkins Co., Chicago, Ill.

problem presented is the forecasting of floods from some particular drainage area which has few or no recorded run-offs. Such forecasting must be predicated on analogy with recorded data, such as Mr. Jarvis has presented. The question is how to interpret the existing data so as to apply them to another water-shed.

Referring to observed run-off data, the four principal variants in the rate of flood run-off are:

1.—The size of the drainage area;
2.—The topography and surface characteristics;
3.—The frequency period for which any particular recorded run-off applies; and
4.—The rainfall characteristic.

If the recorded run-off is known for a water-shed identical in all four of the foregoing features with the new location, the problem is solved; but this is seldom the case. In order to apply the recorded data to a special case, it is important to know the relative values of these four principal run-off factors.

Size of Drainage Area.—The effect of this factor on the rate of run-off is quite well established; in larger drainage areas it is relatively small. The run-off rate increases as the area decreases.

Topography and Surface Characteristics.—The effect of these factors has been established only in a general way. In the light of present information, reliable results can be had only by gleaning out and utilizing data based on drainage areas of similar topography and surface characteristics.

Flood Frequency.—In any list of recorded run-off, this factor is the least certain, hence the apparent wide discrepancy in observed run-off from drainage areas almost identical in size, physical character, and rainfall intensities. The author is to be commended in that, where possible, the part of the maximum flow which represents the average annual flood has been indicated in his table of flood records (Table 2).

While it is true that the flood of 100-year frequency may occur within a year or two, there are many cases where it is poor engineering economy, if not impracticable, to build structures for an ultimate maximum possible flood. The extent of the investment in a structure for flood capacity is a matter of a comparative balance between the cost of insurance and that of flood damage.

It is important, therefore, to know the law of variation of flood frequency, not only for co-ordinating flood data, but also for determining the proper frequency to adopt in the design of any particular structure.

Weston E. Fuller, M. Am. Soc. C. E., has developed the law of flood frequencies.* Mr. S. L. Moyer, of Montevideo, Minn., has presented a similar investigation of flood frequency.† Mr. Moyer has developed the expression, $Q = (c - d)e$, for the relation between the rate of run-off and the period of

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 564.

† "An Approach to Run-Off Expectancy", *Monthly Weather Review*, November, 1924.

expectancy for such a rate. In the foregoing expression:

Q = the magnitude of flow, in cubic feet per second.

$c\,e$ = the limit to which Q approaches.

$$d = \left(\frac{1}{2\,f} + \frac{4.5}{f + 8}\right)$$

in which,

f = the frequency or interval, in years, between peak flows of a given or greater magnitude.

In Fig. 9 has been platted the curve showing the relationship between run-off and frequency according to the Moyer formula when c and e were each unity. The law of frequency as expressed by Fuller reduced to the Moyer form, which is approximately, $Q = (0.99 - d)\ 0.8$, is also platted on Fig. 9 for comparison.

The dashed curve represents the recorded relation between the amount of rainfall and its frequency at Chicago, Ill. The point for the 22-year rainfall frequency has been platted arbitrarily to coincide with the 22-year point on the Moyer curve.

Both the Fuller and Moyer curves show the relatively rapid increase in the expected rate of run-off with slight increase in the frequency periods up to 10 or 20 years, the lesser increases from 20 to 40 years, and the slowest, almost straight-line, rate of increase after 40 years.

The dashed curve indicates what might be expected—the similarity between run-off intensity frequency and rainfall intensity frequency.

Rainfall Characteristic.—Rainfall is, of course, the prime factor in flood run-off. This factor, however, has the least variant (as between different water-sheds) of any of the four principal factors, namely, area, topography, frequency, and rainfall.

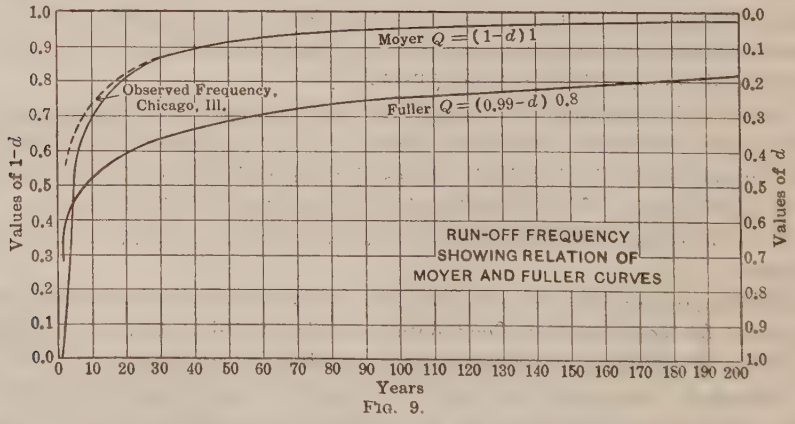


Fig. 9.

For a 20-year frequency, observations at twenty-two stations scattered throughout the United States show a general range within 2 to 3 in. of intensity of rainfall for 60 min. and a range within 7 to 9 in. of intensity of rainfall per hour for a period of 5 min. Meyers* has pointed out that more

* "Elements of Hydrology."

reliable results can be obtained by consideration of the rainfall data from a group of stations within a certain large area than from one particular station.

In the writer's opinion the chief value of a rainfall record in relation to any observed flood discharge, is its aid in determining the probable frequency period applicable to the flood in question.

Flood Height vs. Discharge.—Under this heading, Mr. Jarvis has described interesting phenomena regarding flood waves and the channel regimen of certain streams. The writer has investigated the time of travel of flood waves in open channels.* In many situations a study of the height and velocity of flood waves on existing channels is of value in forecasting flood stages. Such investigation can determine how flood waves and velocities may be modified by changes in the channel section.

George G. Honness,† M. Am. Soc. C. E. (by letter).—The author has performed a valuable service by so forcefully calling attention to conditions only too often neglected in the design of many engineering works and by supplying formulas which may be applied to all localities. These formulas, however, should be used with discretion so as to avoid uneconomical conclusions.

Little can be added by the writer except to furnish additional data which may be considered reliable for Northeastern United States.

On September 29 to October 1, 1924, inclusive, the Catskill Mountain region in New York State was visited by an unprecedented rainfall which led to flood discharges on the Schoharie and Esopus Creeks larger than any theretofore measured. A flood occurred in October, 1869, on Schoharie Creek, which is known to have exceeded the flood of September 30, 1924. At the time of the 1869 flood, it was said to have been the largest in the knowledge of the oldest inhabitants, and, therefore, it seems safe to assert that the flood of September 30, 1924, is the second largest in more than 100 years.

The storm began on September 29 and ended on October 1. For this period, 6.47 in. represented the average precipitation on the Schoharie water-shed above Prattsville (varying from 3.63 in. at Prattsville to 13.32 in. at Elka Park). On the Esopus water-shed above Cold Brook the average precipitation for the same period was 8.63 in., and varied from 5.35 in. at Highmount to 11.51 in. at Edgewood. The bulk of the rain fell on September 29 and 30, during a period of 36 hours.

In Fig. 10 is shown the region under discussion, together with the location of the rain gauges and the lines of equal rainfall for the period. Table 4 gives essential data in reference to the locality and the storm of September 29 to October 1, 1924.

By referring to the isohyetal map (Fig. 10) it will be noted that the storm centered in the extreme southeastern part of the Schoharie water-shed and the northeastern part of the Esopus water-shed. On the Esopus, however, the rainfall was considerably greater, as the storm came from the southeast and the high mountains intervening between the Esopus and the Schoharie

* *Journal*, West. Soc. of Engrs., August, 1923.

† Dept. Engr., Board of Water Supply, New York City, Grand Gorge, N. Y.

Valleys intercepted the moisture-laden winds and caused the higher precipitation on the southern and eastern slopes.

FIG. 10.

TABLE 4.—DETAILS OF WATER-SHEDS AND RECORD STORM.

Location of gauge.	Drainage area, in square miles.	Average yearly rainfall, 18 years, in inches.	Rainfall, September 29-October 1, 1924, in inches.	Maximum discharge, in cubic feet per second.	Maximum discharge, in cubic feet per second per square mile.	Time of maximum discharge.
Prattsville, N. Y., Schoharie Creek.	236	41.3	6.47	29 000	123	9:00 P. M., September 30
Cold Brook, N. Y., Esopus Creek...	192	47.6+	8.63	32 800	171	" "

A masonry dam on a stream draining an area on the eastern side of the mountains, where the maximum precipitation occurred, afforded an opportunity

to estimate closely the maximum discharge of an area of 1.82 sq. miles located between Palenville and West Saugerties, known locally as Whittaker's. This discharge was determined to be at the rate of 508 cu. ft. per sec. per sq. mile.

A determination has been made of the probable effect had the storm been geographically centered over either of the water-sheds. On a conservative basis, it may be asserted that if this condition had obtained, the maximum discharge at Prattsville on Schoharie Creek would have been at the rate of about 177 cu. ft. per sec. per sq. mile, and if at Cold Brook on Esopus Creek, at the rate of about 184 cu. ft. per sec. per sq. mile.

L. M. WINSOR,* ASSOC. M. AM. SOC. C. E. (by letter).—The author has made a clear and comprehensive statement of this extremely important subject and has presented facts and figures which will be helpful under certain restricted conditions. In the discussion which follows one phase of the question will be amplified and presented according to a somewhat different viewpoint from that of the author.

A distinction must be made between the two types of flood which appear in arid sections, in order to avoid confusion and to obtain a real appreciation of the actual conditions which confront the engineer in designing structures to handle this run-off.

There is a decided difference between the flood that comes as a result of melting snow, or excessive spring rain, and the raging torrent that follows a cyclonic storm in summer—what is generally known and perhaps erroneously termed a "cloudburst." Furthermore, there is as much difference in floods within humid and arid sections as there is in the appearance of the landscape. A flood in a humid country merely means the swelling of a stream beyond its normal volume. In a desert land a flood often means the appearance of a torrent where water had never been seen before.

Spring high water in arid climates is characterized by the swelling of a natural or an intermittent stream over a protracted period. If the flow is augmented by melting snow there is a constant fluctuation between night and day, with a constant rise and fall in total volume from day to day, depending on the temperature and humidity. Practically every mountain stream carries large quantities of drifting sand and gravel at high water, which cause an ever shifting stream bed within the limits of canyon walls, or over the entire surface of gravel cones and alluvial fans built by stream action at the mouth of every canyon. This condition often makes designing for waterways a most intricate problem and one to which run-off formulas have not yet been successfully applied for calculating probable requirements.

In streams of this type drifting gravel becomes the chief factor in designing waterways across highways, railways, and canal systems, or through towns and farming lands. Where attempts have been made to confine such a stream to a narrow channel, the result has invariably been the building up of this channel until it occupies the highest ground in the district, which makes the stream a real menace to adjoining property at flood periods. Under such a condition it is impossible to design bridges or culverts of sufficient capacity. As the stream bed is built up the opening across a roadway or under a canal

* Irrig. Engr., Utah Experiment Station, U. S. Dept. of Agriculture, Logan, Utah.

is filled until finally there is no head-room left. Thus, a stream of a few second-feet in size, if it carries drifting sand and gravel, may cause greater trouble than one of 1 000 sec-ft. that carries only muddy water during flood periods.

The summer flood resulting from a heavy thunder-storm is different in every detail from the spring freshet or high-water flow. Usually, the summer flood reaches its maximum flow immediately. In other words, the very first wave is generally the highest. In extreme cases the breast of such a flood is composed of mud so thick that it cannot flow. It is forced along by the weight of water behind it, and rolls, rather than flows, forward. This is the type of storm referred to by the author, which swept the west slope of the Wasatch Range on August 13, 1923, from Nephi, Utah, on the south, to Brigham City on the north. It struck Nephi at 5:00 P. M. and at 8:00 P. M. had reached its northern limit 200 miles away.

It was not the quantity of water that caused the terrible destruction, but the manner in which it fell. It was such a storm as is frequent in tropical countries where probably 1 in. or 1½ in. of water falls in 10 min. In this case, however, it fell on soil and mountain steeps unprotected by tropical vegetation and unaccustomed to carrying the run-off from such a sudden and severe downpour. The result was inevitable. The entire upper foot in depth of soil in many areas was washed down into ravines and finally into the main canyon leading to the country below. The conglomeration of soil, rubbish, and trees, mixed with a little water made a thick mortar which formed a dam in front of the flood, in places reaching a height of 50 or 60 ft. This dam was rolled forward when the accumulation of water behind it created sufficient pressure. The sliding and rolling mass of earth and débris literally swept the canyon clean of rock and vegetation, and, when released from the canyon walls, spread over the valley below, carrying houses, barns, orchards, livestock, and people to destruction.

The author shows (Fig. 4) a huge boulder, 25 ft. high, which was swept along for 300 yd. In another instance, a large, modern dairy barn filled with hay was carried more than ¼ mile and landed in the main street of Willard, Utah.

In these floods, following the advance wave of mud, rubbish, and huge boulders, there came an after-flow of water which spread the débris to a certain extent; but the actual volume of water was negligible, except where it ponded behind the advance wall of mud.

Needless to say such floods have never been measured, nor will any theory of applied hydraulics as evolved in the laboratory aid in making an estimate as to the type, kind, and size of structure required to handle them. They are calamities which come once or twice in a lifetime under arid conditions, in mountainous regions.

In locating highways through desert regions this problem is ever before the engineer who is acquainted with the real flood conditions. There is no such thing as determining beforehand where or when the next flood will come. In many instances there is no such thing as a defined channel. In certain cases what appears to be a ridge may be the next flood channel. Close observa-

tion in desert regions will reveal the fact, usually, that the ridges at the foot of mountain slopes have been made by water.

Designing for such emergencies is a difficult problem and one which can hardly be accomplished by formulas. It is recommended, therefore, that application of the author's deductions be confined to established waterways and living streams, rather than to desert areas which are subject to heavy summer floods.

It is also important that the engineer who is unacquainted with the character of floods in desert lands, give proper weight to the information available from natives who are familiar with the territory through which roads or canals are to be built.

H. V. Hinckley,* M. Am. Soc. C. E. (by letter).—For some time the writer has had in mind advancing a formula similar to Equation (2), and of the form, $Q = 5\,000\sqrt{M}$. He realizes that the constant, 5 000, might have to be enlarged to apply to possibly forty localities. The use of the symbol, Q, is somewhat objectionable, as no quantity is to be expressed, only a factor in the quantity.

Since the writer has studied the paper (which is both timely and interesting) and, in connection with it, the previous contributions on flood flow by Weston E. Fuller, M. Am. Soc. C. E., and the late Emil Kuichling, M. Am. Soc. C. E., it seems to him unwise to propose definitely the constant, 5 000, at this time because the author's Table 2, covering more than 1 000 records, shows 57 cases where the factor, 5 000, is materially exceeded, of which 23 cases exceeded even 10 000. In studying these data, to offset the inadequate length of observations, a factor was applied to the readings multiplying them by quantities between 1 and 3.40, the member, 3.40, applying to records of one year only or of indefinite and unknown periods. This is in accordance with the recommendation in Fuller's paper.†

For example, Table 5 shows 6 of the 23 cases, in which the author's value of C (10 000) would be replaced by numbers from 21 400 to 37 400. The multiplier, 3.40, was used in each case. It may yet be possible to eliminate discrepancies and make use of the formula, $Q = 10\,000\sqrt{M}$.

The writer believes that the first essential is to have the constant, C, large enough to cover all the absolutely maximum cases, no matter how large it may need to be. Then, as suggested by the author, definite percentages might be adopted to represent certain drainage basins—if it is possible to tell the relation of future floods in different parts of the continent or of the globe.

It seems hardly necessary to mention that an average of the flood flows recorded would be far less than the average actual maximum—if there were such a thing—as fully one-half the values given as maximum floods are not maximum at all except possibly for a day.

On Fig. 11, Line C, by the writer, from studies made for the State of Oklahoma, compared with the lines, M and K (all three lines being for maxi-

* Res. Bridge Engr., Oklahoma City, Okla. Mr. Hinckley died on April 21, 1926.

† "Flood Flows", *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), Table 2, p. 568.

mum second-feet per square mile), shows how the maximum recorded floods have become greater as the records have increased the working data.

TABLE 5.—COMPUTATION OF COEFFICIENT, C.

Area, in square miles.	Years of record.	Run-off, in second-feet per square mile.	Multiplier.†	Maximum run-off expected, in second-feet per square mile.	$\sqrt{M}$	C.
20*	Indefinite	1 800	3.40	122 000	4.47	27 400
22*	"	1 341	"	100 000	4.69	21 400
44*	"	1 363	"	204 000	6.63	30 800
143†	"	600±	"	292 000	11.96	24 400
544†	"	432	"	818 000	23.32	35 000
831*	"	382	"	1 079 000	28.83	37 400

* Data taken from Table 37 of the paper on "Flood Flows", by Weston E. Fuller, M. Am. Soc. C. E., *Transactions*, Vol. LXXVII (1914), p. 650.

† Discussion on "Flood Problems", by H. P. Eddy, M. Am. Soc. C. E., *Transactions*, Vol. LXXXV (1922), Fig. 45, p. 1526, and Table 5, p. 1528.

The author seems to have his flood values under good control. Nos. 275 and 493 (Table 2), however, apparently give values for the constant, C, of slightly more than 10 000, being 11 683 and 10 029, respectively.

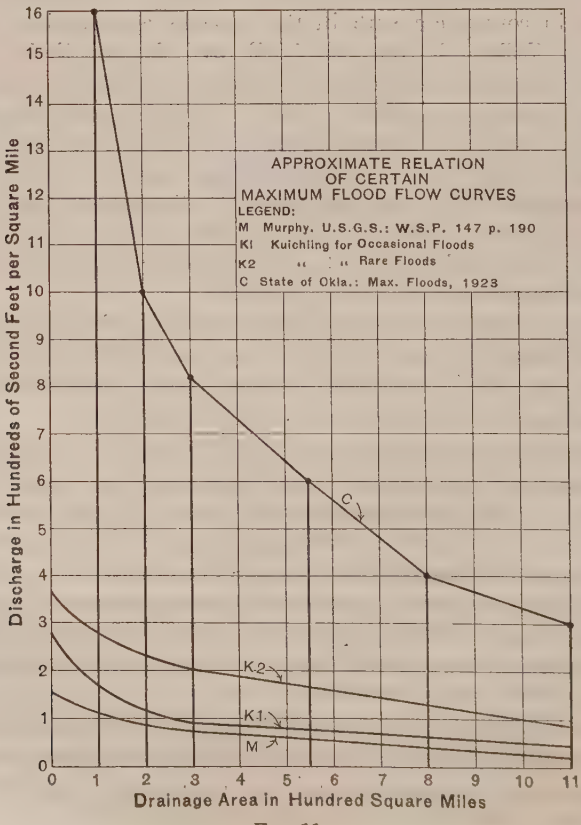


FIG. 11.

It would be interesting to know whether the author finds that any of the drainage basins named in Plate IX are found to have definite values or percentages of the constant, $C = 10\,000$; for example, whether the flood values for any one drainage district are within the limits of the 10% class and have maximum values less than $Q = 1\,000\,\sqrt{M}$.

HARDY CROSS,* M. AM. SOC. C. E. (by letter).—By listing "certain data and considerations that are often left unrelated", the author has prepared a most interesting paper. The whole subject is in a chaotic state, and discussion of it necessarily depends much on individual opinion. Any effort to clarify, systematize, or in any way make definite the problems involved, is a professional and public service. The next fifty years will see much development along the line of placing river hydraulics on a scientific basis.

The special problem of provision for flood flow at river crossings is commonly one of erosion phenomena and only occasionally or secondarily one of back-flow and consequent damages. It seems important to emphasize a distinction between streams with very mobile beds, in which, owing to bed erosion, the flood velocity is constant, and streams in which the bed is so stable that increased flood velocities encourage lateral erosion. The former undercut bridge piers and cause them to overturn, due partly to the unbalanced pressure of silt not eroded; or they may allow the abutments to slide into the bed of the stream, often together with the foundations. Streams of the second class are more likely to back-cut their abutments. Any bridge engineer familiar with the rivers of the Middle West can cite numerous examples of both types of phenomena.

Scour and fill in any one cross-section may be due to:

(*a*) Changes of location of the thalweg in the section considered.

(*b*) Changes of location of the thalweg in up-stream sections resulting in a change of burden of water threads where they cross the given section. (Note that increased erosion in the up-stream thalweg does not necessarily mean increased burden in the thalweg of the given section, for, owing to its greater inertia, the sediment does not move exactly in the line of the water threads.)

(*c*) Changes in erosive activity due to causes other than velocity at any point in the section. (Differences in the eddy-forming power of obstructions with changes of stage and, if Kennedy's law is applicable, changes in the relations of depth to velocity at various points are here indicated.)

Of these causes, the shifting of the hydraulic axis with the stage is important at bridge crossings. Many bridges carefully located square to the thalweg of mean flow are skew to the destructive currents of the falling flood stage; but this migration of the thalweg with the change of stage is only one of the causes of increased erosion on a falling stage. Water-soaked banks are another important element. Most serious in many cases is the tendency to flank the abutment by a back-cutting eddy below the approach fill.

* Prof., Structural Eng., Univ. of Illinois, Urbana, Ill.

It is important also to distinguish between the action of protracted and that of "flashy" floods. The latter, of course, have not time to improve the hydraulic efficiency of the bridge opening and, therefore, back-water may produce considerable hydrostatic heads against the approaches.

As bearing out and supplementing certain statements of the author with reference to erosion in silt-bearing streams, some selected conclusions of the twenty general rules formulated in the case of the Indus by the Indus River Commission* are worth quoting:

"The progress of erosion at any point is usually determined by the condition of the river for some miles up stream.

"Erosion is most severe when a bank, which consists of clay or heavy soil at the top, is underlain by sandy strata which can be readily undermined."

* * * * * * * * * *

"Erosion is generally more active during a falling than during a rising stage of the river, as in the former case the unsubmerged and overhanging bank is charged with moisture which weights it and diminishes its cohesion."

* * * * * * * * * *

"An embayment is, almost invariably, formed first below this point [a resisting projection of the bank] owing to the swirl of the water produced by the construction.

"If this embayment becomes sufficiently deep, the projecting bank will be outflanked and washed away."

Although it is of quite as much economic importance as any other phase of bridge design, the problem of the design of river crossings, as distinguished from the structural design of bridges, is one on which little accurate information is available. It is encouraging to note how fully the author and his coworkers realize the intricacy of the problem. It is to be hoped that such papers as this may bring out the huge mass of experience on this subject possessed by drainage engineers of large railways. The writer believes that the subject can be put on a basis approximately as scientific as has been reached in recent years in the design of spillways.

E. W. Lane,† M. Am. Soc. C. E. (by letter).—This paper is valuable in that it brings together a large amount of data on flood flows. This information may serve as a rough guide in the design of hydraulic structures, but the writer is of the opinion that a much more reliable indication of probable floods may be obtained, particularly for those in streams for which a few years' discharge records are available, by a different method from that used by the author.

The discharge rate of any flood is the product of innumerable factors, and the combinations of these factors which cause floods on any stream give rise to discharges which have wide ranges of magnitude. One of the important factors is the area of the water-shed, and this is the only one which the author includes in the formula which he adopts. After its effect is eliminated as far as possible, the data given still show a range of discharge represented by numerical coefficients of from 50 to 10 000. When the selection

* Indus River Commission Record, Sind, P. W. D., Pt. IV, Conclusions.

† Lafayette, Ind.

of the proper coefficient is left to the judgment of the individual engineer, it is obvious that the formula itself is of comparatively little assistance.

Although many factors combine in causing floods, they may be divided into two groups: (1) Those depending on the physiography of the basin in question; and (2) those depending on the weather. In the first group, the most important factors are the area, topography, geology, shape, and permeability of the water-shed, and the availability of storage in lakes and swamps. Those of most importance in the second group are the precipitation, temperature, and the snow and ice conditions on the water-shed. Vegetable cover and the condition of storage of water in soil, lakes, and swamps are really products of factors from each of the two groups.

For a given point on a stream, the factors in the first of these groups are constant; that is, these conditions remain the same. Weather conditions are proverbially variable, but, although they cover a wide range, they follow some probability law. This is particularly true of the weather phenomena that give rise to floods.

As a rational flood formula must take into account both these classes of influences, it should be made up of two parts. One would be a factor expressing the physical characteristics of the stream; this would be a constant for a given point, but would vary somewhat at different points on the same stream, still more for the streams of a given region, and widely for the streams of a continent. The other part would express a probability law which would be practically the same over a comparatively large area, but not necessarily the same for all water-sheds.

The writer believes that any formula which does not include the effect of both these groups of influences would be only a rough approximation and of little usefulness. Most formulas have a factor for the water-shed area, some consider to a limited extent the physical characteristics of the drainage basin, and a few have some indication of the time element; but the Fuller formula* seems to be the only one that gives numerical values to the probability factor and also considers the basin characteristics. As an example of the necessity of considering the water-shed characteristics may be cited the comparison of the Deerfield River at Charlemont, Mass., with Miller's River at Erving, Mass.; these streams have nearly the same drainage area, but the former gives flood discharges five or six times as great as the latter. The Souhegan River at Merrimack, N. H., gives discharges about three times as large as the Swift River, at West Ware, Mass., although their drainage areas are nearly equal. If as great variations as these occur in as small a region as New England, where weather conditions are so nearly uniform, how much larger variations can be expected when the whole United States or the entire world is considered?

The author has objected to the time element in flood formulas as "too often the basis of error". It is true that it may sometimes be misunderstood by those who blindly apply formulas without much thought as to their mean-

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 567.

ing; but must formulas be made so simple that the least intelligent or informed may readily understand them, thus denying a useful tool to those who are capable of using it? Progress is not made by that method. As a matter of fact, progress in engineering science is so rapid that the products of the best minds in one decade are used by all the fairly well informed in the next, so that even if a development seems complicated at the time it is made, in a few years it will be considered relatively simple.

The writer believes that the time element in a rational flood formula is a simple thing, well within the comprehension of an ordinary mind. Unfortunately, its meaning has been needlessly obscured in the development of the Fuller formula. Suppose there was a long record (say, 1 000 years) of the flood discharges of a stream at a given point, arranged in order of their magnitudes, beginning with the largest. Suppose the one-hundredth in the list had a discharge of 5 000 sec-ft. There have been 100 floods in the 1 000 years with a discharge of 5 000 sec-ft., or more. Therefore, the average interval between floods of 5 000 sec-ft., or more, is $\frac{1\,000}{100} = 10$ years. If a spillway on this stream at this point had a capacity of 4 999 sec-ft., its capacity would be exceeded once in 10 years on the average. The probability of its capacity being exceeded in any one year would be 0.10. For convenience, call the average interval, in years, I. Then the probability of occurrence in any one year would be $\frac{1}{I}$. The interval used by Mr. Fuller, in which the chances of a flood of that magnitude occurring are equal to the chances that it will not occur, is, therefore, $\frac{I}{2}$. To use the example of the spillway again, the interval used by Mr. Fuller, which he computes from his average maximum flood or median maximum flood, is the interval for which the chance of the spillway capacity being exceeded is equal to the chance that it will not be exceeded—one chance out of two, or 0.5. Since the chance in any one year is 0.10, the period for which the chance would be 0.5 would be five years, or $\frac{I}{2}$. The writer believes that the use of the interval, I, is more easily understood than the interval used by Mr. Fuller.

This discussion may seem elementary, but it indicates how simple the element of time in a rational flood formula really is. Some doubt as to the reliability of the probability formula is apt to arise in the mind of the engineer when he considers a case where several unusually large floods come within a short interval, or where severe flood conditions occur simultaneously over a wide region, as happened in March, 1913. He should remember that the theory on which life insurance is based is not disproved because several members of the same family may die in the same year, or because an epidemic of influenza sweeps over the country, claiming the lives of thousands. These cases in the field of insurance are paralleled in the field of flood probability.

To try out the writer's ideas on the development of a logical flood formula, a study was made of the floods of New England rivers from the data compiled by C. H. Pierce, M. Am. Soc. C. E.* The probability or average interval curve was computed for each station having sufficiently complete records. The flood discharges at the station in question were arranged in order of magnitude, beginning with the largest, and numbered consecutively, the largest being No. 1. Disregarding the error due to the fact that the record covers only a short period, the average interval, I, between floods as large or larger than any one of those listed, can be found by dividing the length of the record at that station, in years, by the serial number of the flood. Thus, with a 10-year record, the value of I for the magnitude of Flood No. 4 would be $\frac{10}{4} = 2.5$ years. Average interval curves were then drawn showing the relation between the flood magnitude, in second-feet per square mile, and the average interval, I. Logarithmic, semi-logarithmic, and probability paper were used, but the best results were obtained from the semi-logarithmic paper.

In Fig. 12 are shown the resulting curves. Many of the data were in the form of peak flows, but other records consisted of one reading per day, the higher of two readings per day, and the average flow, the various kinds being indicated by different symbols. On account of the wide difference in unit flows, it was necessary to divide the data into two parts, and use different scales, in order to plot them on a diagram of reasonable size. Table 6 gives the principal data regarding the stations the records of which were used. On account of the fact that the length of record was insufficient to determine accurately the average interval for floods of greater magnitude, the values indicated by the highest points are made less prominent. It will be seen that all the curves, as indicated by the data, approach reasonably close to straight lines, and converge to approximately the same point.

Fig. 12 shows that the average interval between floods of various magnitudes for the New England rivers can be expressed by an equation of the form:

$$Q = K (\log I + B) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which, Q is the discharge, in second-feet per square mile, which would be equalled or exceeded on the average once in I years; K is a constant for the station in question; and B is a constant for New England streams and probably nearly constant for a much larger area.

It will be seen that this formula is made up of the two parts previously mentioned as being what one would expect in a rational flood formula, namely, a constant, K, representing the physical characteristics of the water-shed, and a probability factor, $\log I + B$, representing the weather variability.

Although the writer had available only the data for New England streams, from the results of Mr. Fuller's studies, he believes that an expression of the form of Equation (4) would apply to nearly all the rivers in the United States east of the 95th Meridian, which is roughly the western boundary of

* "Flood Flows of New England Rivers", *Journal*, Boston Soc. of Civ. Engrs., October, 1924, Vol. XI, No. 8.

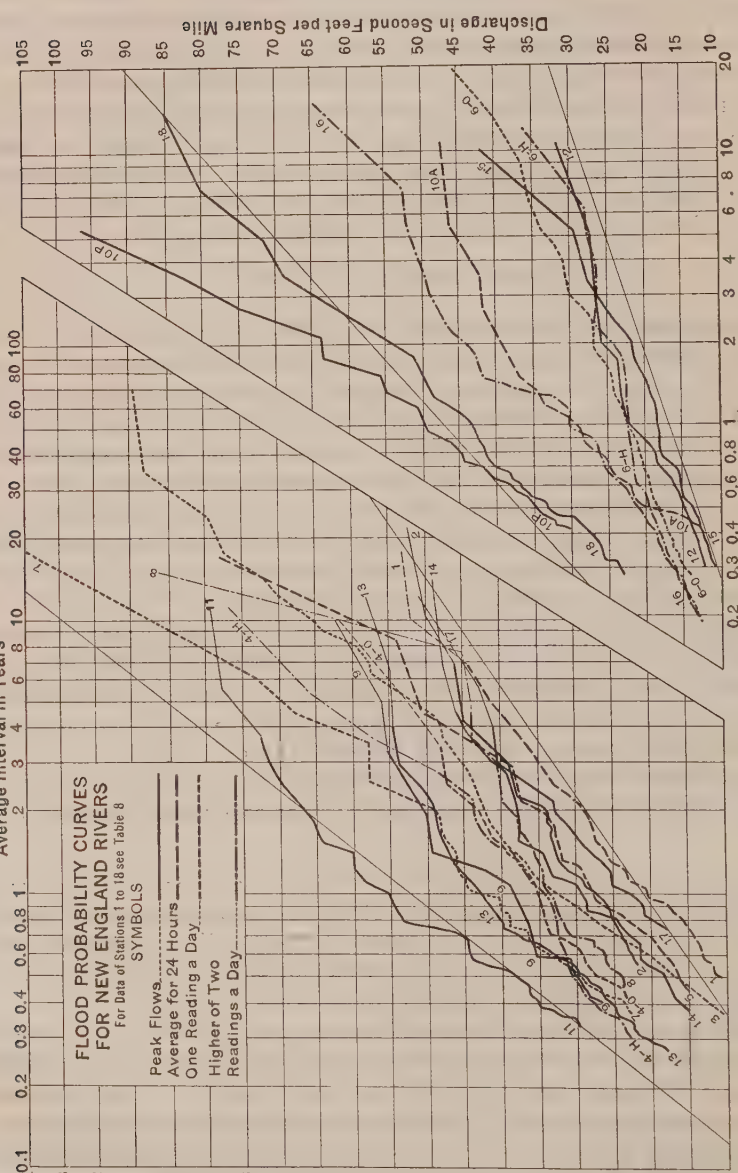

FLOOD PROBABILITY CURVES
FOR NEW ENGLAND RIVERS
For Data of Stations 1 to 18 see Table 8
SYMBOLS
Peak Flows
Average for 24 Hours
One Reading a Day
Higher of Two
Readings a Day
Average Interval in Years
Discharge in Second Feet per Square Mile

FIG. 12.

the States immediately west of the Mississippi River. He is inclined to doubt its applicability to the "plains" rivers, but believes that it would apply to many of the mountain and Pacific Coast rivers.

TABLE 6.—RUN-OFF STUDIES OF NEW ENGLAND RIVERS.

No.	River.	Station.	Area, in square miles.	Length of record, in years.	Type of record.
1....	Connecticut	Holyoke, Mass	8 390	20	Mean for 24 hours.
2....	Connecticut	Sunderland, Mass	8 000	21	Nearly all peak flows.
3....	Merrimack	Lowell, Mass	4 097	72	Nearly all one reading per day.
4-*O*.	Connecticut	Orford, N. H	3 100	10	One reading per day.
4-*H*.	Connecticut	Orford, N. H	3 100	11	Higher of two or more readings per day.
5....	Saco	West Buxton, Me	1 500	17	Average for 24 hours.
6-*O*.	Pemigewasset	Plymouth, N. H	615	26½	One reading per day.
6-*H*.	Pemigewasset	Plymouth, N. H	615	12	Higher of two readings per day.
7....	Machias River	Whitneyville, Me	465	18	One reading per day.
8....	Ashuelot	Hinsdale, N. H	440	15	Higher of two readings per day.
9....	Miller's	Erving, Mass	372	10	Peak flow.
10-*A*.	Deerfield	Charlemont, Mass	332	10½	Mean for 24 hours.
10-*P*.	Deerfield	Charlemont, Mass	332	10½	Peak flow.
11....	Housatonic	Great Barrington, Mass	280	11	Peak flow.
12....	Lamoille	Cady's Falls, Vt	280	10	Nearly all peak flow.
13....	Ware	Gibb's Crossing, Mass	201	12	Peak flow.
14....	Swift	West Ware, Mass	186	14	Nearly all peak flow.
15....	Souhegan	Merrimack, N. H	168	10	Peak flow.
16....	Westfield	Knightville, Mass	162	15	Higher of two readings per day.
17....	Quaboag	West Bumfield, Mass	150	12	Peak flow.
18....	Middle Branch, Westfield	Goss Heights, Mass	53	14	Peak flow.

Representing the magnitude of a flood with an average interval of one year as Q_1, which, for convenience, may be called the "one-year flood", from Equation (4):

$$Q_1 = K B, \quad \text{or} \quad K = \frac{Q_1}{B}$$

Substituting this in Equation (4):

$$Q = \frac{Q_1}{B} (\log I + B) = Q_1 \left(1 + \frac{1}{B} \log I\right) \ldots\ldots\ldots\ldots (5)$$

This is of the same form as the Fuller formula:

$$Q = C A^{0.8} (1 + 0.8 \log T)$$

and shows that the magnitude of floods of various intervals is a logarithmic function of the "one-year flood" discharge. With the data on discharge expressed in terms of their ratios to the "one-year flood" all the "discharge-average interval" curves should fall on the same line. This permits of combining all the records, as Mr. Fuller did, into a single record, with the result shown in Fig. 13.

In determining this curve, all the different kinds of records were used, as there seems to be no great variation between the shapes of the curves from different kinds of observations. It is possible, however, that if enough records were available, a difference might be shown.

The value of $\frac{1}{B}$ was found to be 0.690, which differs somewhat from the 0.80 found by Mr. Fuller. It is possible that if only the "average for 24 hours" and "one reading a day" records were used, a better agreement with the Fuller formula would have resulted. Mr. Fuller's data were based on records supposed to represent the average for 24 hours. In a great many cases, however, the observation was made once daily, which reading was assumed to give the mean flow for the day.

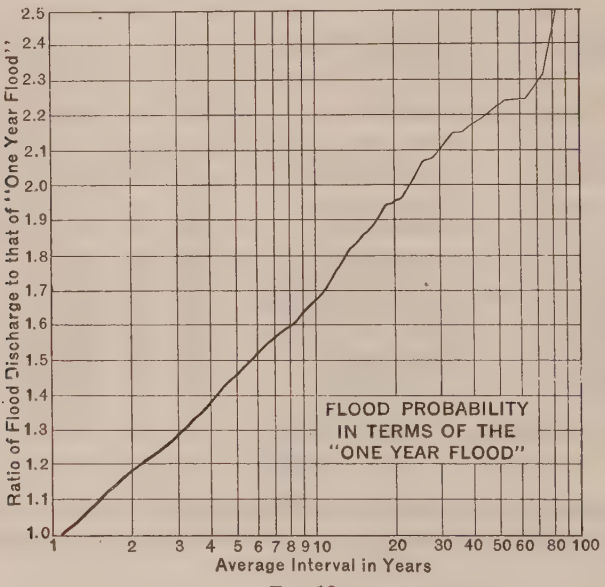


FIG. 13.

The writer does not claim originality for his method of determining flood probabilities, or for the formula, although they were independently developed to find later that some points were previously used by others. He has not hesitated to borrow from others when their methods seemed sound. He does believe, however, that the method he has outlined leads to a flood formula which combines rationality, simplicity, and accuracy to as great an extent as is possible for so complex a phenomenon as floods.

The method here given for working out probability curves and deducing the flood probability formula has certain advantages over the methods heretofore proposed. Although the form of the result is very similar to that reached by Mr. Fuller, the method is less laborious, and has the great advantage that the meaning of the result is definite and readily understandable, as compared with Mr. Fuller's "average maximum flood" and "median maximum flood", which are very difficult to understand, "if not indeed practically meaningless", to use the words of a well-known hydraulic engineer. By the writer's method, the larger floods, the average intervals of which are less reliably determined, are automatically eliminated in the determination of the

one-year flood, while in the Fuller method, a single flood considerably larger than the other members in the record exercises too much influence on the results.

The writer's method is almost identical with that outlined by R. E. Horton, M. Am. Soc. C. E., in his discussion* of Mr. Fuller's paper, but is an improvement on the Horton method in that it takes into consideration all the floods of the stream, and not merely the largest one in each calendar year. In using only the largest flood in any one year, Mr. Horton's method is subject to the criticism he makes of Mr. Fuller's method, namely, the failure to use all the available data.

If time and data were available to carry out the work for other sections of the United States as the writer has done for New England, and a study was made of the conditions which give rise to different index values, in order to determine with reasonable accuracy, from the character and location of the water-shed, what the flood index would be, a tool of great value would be in the hands of hydraulic engineers.

The work necessary for collecting these data and analyzing them is so great as to be prohibitive to a man of ordinary resources, but its value to a single piece of construction work could easily exceed the cost of the entire investigation. The logical body to make such a study is the Society's Special Committee on Flood-Protection Data, and the request of this Committee that the Society appropriate funds for an investigation of flood flows should be granted. The results would add many times the cost of the work to the wealth of the nation.

For the last ten years, the writer's work has been on flood prevention; before starting the studies outlined in this discussion, he was quite skeptical of flood flow formulas. Working entirely independently, he has arrived at the same general form of equation that was reached by Mr. Fuller, although the meaning of the terms of the two formulas, and the method of deriving them, are somewhat different. He is convinced, therefore, of the correctness of the general formula and believes that further study should be along the line of determining the constants for various streams, and the limit of applicability of the formula.

Some objection has been made to the logarithmic form because it fails to give a maximum value which can never be exceeded. The writer does not consider this a serious objection. It is based on the assumption that there is a maximum flood, but this has never been proved. With the logarithmic form discharges only a few times as great as the 10-year magnitude become so infrequent as to be almost geological periods apart, in which time the river valley may become a sea bottom. It is much better to have an empirical formula agreeing with the data within practical limits than one which is logical according to an improved theory, but may not agree so well. With equally good agreement with the flood data, the writer would prefer a simple logarithmic formula to one having a maximum limit and being thereby complicated.

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 665.

In this discussion it is not the writer's intention to develop a satisfactory flood discharge formula, for much study is necessary before such a result can be reached, but he desires to point out some of the basic requirements of a satisfactory formula, the lines along which study should be made, and some of the methods that he believes most likely to lead to the desired result.

One of the points requiring further study is the influence of area on discharge. Mr. Jarvis has adopted the area factor as the 0.50 power, Mr. Fuller derives the 0.80 power from nearly equally good data. Talbot used the 0.75 power, and other results have been found by other investigators. It is doubtful whether any single figure will cover all cases, and where an attempt is made to predict flood discharges at one point on a river from the data at another point having a considerably different drainage area, or different water-shed conditions, an exercise of judgment would be necessary.

For example, at Dayton, Ohio, two rivers of roughly the same drainage area as the Miami River join it at nearly the same point. As all the areas are small enough to be covered in many cases by the same storms, the flood discharges of the Miami below the junction, per unit of area drained, is only slightly less than that of the individual tributaries, although the area drained is about three times as large. As an example of another condition, the case of the Arkansas River might be cited. Its source is in the mountains, but the lower portion flows through the plains. The flood characteristics of the mountainous parts of the drainage area are entirely different from those below. It is evident that any area factor would have to be applied with judgment, but the writer believes that when flood probabilities are worked out for many stations on the same rivers, it will be possible to make quite close estimates of the area factor for ordinary cases.

The most important feature of a flood is usually the peak flow, but the published records are generally in the form of the average flow for the day, and give peak flows only when they are very large. For this reason, in computing flood probabilities, it is usually necessary to work out probabilities of average daily flows, and from this to determine the probability of peak discharges. A great deal of further study is necessary before this relation can be satisfactorily established. Mr. Fuller has derived an expression for the relation between average daily flow and peak flow, but this expression is one of the greatest objections to the Fuller formula. With great care he finds the probability of average daily flows for all parts of the United States, and to obtain the result nearly always desired—the peak flow—he multiplies this by a factor based on only thirty-three observations, many of which differ 100% from the values computed from the formula adopted. Data on the ratio of peak to average daily flow are scarce, and probably Mr. Fuller used all that were available.

His assumption that this ratio is only a function of the drainage area leaves out an important consideration and probably accounts for much of the variation between his formula and the actual results. For instance, the Hudson River, at Mechanicsville, N. Y., and the Oswego River, at Battle Island,

N. Y., have nearly the same drainage area, but a glance at their hydrographs* will show that the ratio of the peak flow to the average 24-hour flow for these rivers would differ widely. For the region east of the 95th Meridian, the writer believes that an expression combining the drainage area and the value of Q_1, which is really an index of the variability of the flow, would give much more satisfactory results.

As an example of the error resulting in some cases from the Fuller formula might be cited the Arkansas River at Pueblo, Colo., with a drainage area of 4 600 sq. miles. The average ratio for twenty-two peaks was 2.5, which agrees well with that cited by the author for the great flood of 1921, while the ratio of the Fuller formula for a stream having this area would be 1.159. In one case the discharge rose to 12 500 sec-ft., dropped to 2 100 sec-ft., and rose to 10 800 sec-ft. in a single day. All these floods were due to cloudbursts on a small part of the drainage area, while there may have been no evidence of a storm at other points.

If a great many records were available, by plotting a curve like that on Fig. 13, using peak-flow data only, and another using the averages for the 24-hour data only, some clue could be obtained regarding the ratio, but much more data like those shown on Fig. 12 for the Deerfield River, giving both the peak flow and the average daily flow for a long period of record, will be necessary before a satisfactory result will be obtained.

Although the application of the rational type of formula is not as easy a matter as that of the type suggested by the author, it is really a rather simple process. In general, there are three conditions under which predictions of future floods are desired. The point may be (1) on a stream near a station having a long discharge record; (2) on a stream near a station having a short-period record; or, (3) on a stream remote from a discharge station or having no measuring station.

The most accurate predictions can be made from the conditions in Class (1), but even here the general formula is of material assistance. For example, an attempt was made to develop a flood probability curve for the Miami River, at Dayton, from a long-period record there. It was found that the data on the larger floods were so limited that formulas giving widely different probabilities for large floods fitted the data equally well. By means of the general formula, a much more accurate flood probability could have been determined.

The number of flood records is rapidly increasing and will continue to increase. This is not only beneficial to a determination of an accurate formula, but also to the application of it. At present, some form of discharge records exist at one or more points on most of the large and medium sized streams of the United States, and reasonably reliable predictions will be possible for them when a thorough study has been made to determine the best prediction methods. Most of these fall in Class (2). The flood probability for the point in question can be determined by examination and comparison of the probabilities as determined for adjacent measuring stations.

* "The Economics of Hydro-Electric Development," by Daniel W. Mead, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 168 (Fig. 3).

This can best be done by drawing the probability curves as was done for Fig. 12, and determining the "one-year flood" from the straight line most closely fitting the points, but an approximate method may be used without recourse to plotting. In this approximate method, the floods are arranged in order of magnitude, beginning with the largest, and numbered. The one having the number equal to the length of the record may be considered as the "one-year flood". Substituting this in the formula,

$$Q = Q_1 \left(1 + \frac{1}{B} \log I\right)$$

gives the probability of floods of other magnitudes and intervals.

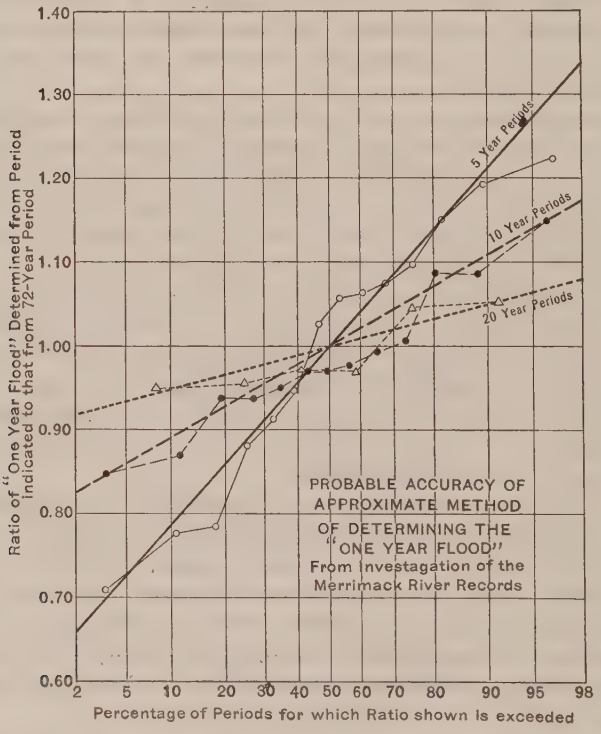


FIG. 14.

To investigate the extent of the error which might occur in the probability curves thus determined, a study was made by dividing the 72-year record of the Merrimack River, at Lawrence, Mass., into periods of various lengths, and comparing the "one-year floods" determined from these periods with that determined from the 72-year period. The result, as shown in Fig. 14, is highly satisfactory. For example, from 5-year periods, more than 75% of the determinations were within 20% of the proper value, and with 10-year periods about 98% were within 20% and about 75% were within 10%, while with 20-year periods about 98% were within 10 per cent.

Fig. 15 was prepared to show how closely the probability curves, as determined from different parts of the record, agreed with each other. For this purpose, the 72-year record of the Merrimack was divided into two 36-year periods and four 18-year periods and the curves for all the periods plotted together. The result shows a very satisfactory agreement.

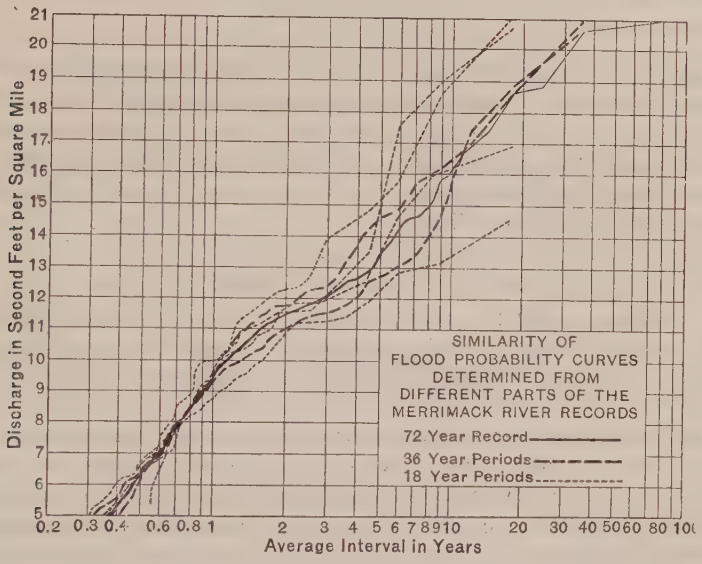


FIG. 15.

Class (3), the determination of the flood probability of a stream which has no flow records or only records at remote points, is the case most frequently met by bridge and highway engineers. As there are comparatively few records for small streams, most of them come in this class. In this field, formulas of the type adopted by the author have their greatest usefulness. Nevertheless, the writer believes that, with the exercise of care and a reasonable amount of judgment, better results can be obtained by the method outlined by him than from formulas of the other type.

To determine flood probabilities from cases in this class, it is necessary to compare the water-shed characteristics with those of adjacent water-sheds for which records are available. The results thus determined are not so reliable as those for which records at closer points are available, as streams with apparently the same water-shed conditions give different results. The Mad and Stillwater Rivers, tributaries of the Miami River in Ohio, have basins which in outward appearances are as nearly similar as could be found. The Stillwater River gives considerably higher flood flows than the Mad River, which is entirely contrary to what one would expect from their names and from the opinions generally held by local residents.

Summarizing the writer's ideas on flood formulas, it is his belief that a satisfactory flood formula must take into consideration both the basin characteristics and the time factor, and that, although considerable progress

already has been made in the development of such a formula by Mr. Fuller, great improvements are still possible, but, as these require time and expense beyond the resources of a single man, this study should be carried on by the Special Committee on Flood-Protection Data. The writer heartily agrees with the author as to the need for this type of study and believes that the Society should provide the Committee with the necessary funds to make an adequate investigation possible.

GERARD H. MATTHES,* M. AM. SOC. C. E.—This noteworthy paper clearly represents the fruits of considerable labor on the part of the author in compiling data for use in proportioning waterway areas for bridges and culverts. The treatment of the subject appears to be principally from that angle, which accounts for the fact that the author is concerned mainly with maximum discharge rates, as shown in Table 2.

Of practical interest is the following specification of the U. S. Bureau of Public Roads mentioned by the author on page 999:

"The waterway shall be adequate for the passage of ordinary flood water * * * and be adequate for the passage of extreme flood water unless other provisions are made for same. By ordinary flood water is meant the stage that is likely to occur once in ten years."

This appears to be a sensible basis for designing such structures, and should operate to provide bridges and culverts adapted to local conditions. Thus, in a region like Colorado where cloudbursts are common, the 10-year criterion would provide structures capable of passing extreme flood flows except where they could be by-passed, while in a State like Maine where cloudbursts are of rare occurrence, much smaller waterways will suffice. It would be interesting to know what considerations led to the adoption by the Bureau of the 10-year flood as the criterion for ordinary flood flow, as this might be of value also in the design of other structures built to resist flood waters.

Mr. Jarvis could have made Table 2 of more direct value if in Column (5) he had given figures representing the "ordinary flood water" as here defined.

Two aspects relating to flood studies have suggested themselves to the speaker in reading this paper: First, the imperative need for better means of interpreting the enormous dissimilarity in the flood characteristics of streams; and, second, the desirability of establishing uniform methods of compiling flood data for publication.

As regards the first, Plate IX which is the most complete tabulation of its kind that the speaker has seen, brings out strikingly the wide variation that exists in the maximum flood rates of North American streams, and, by inference, the magnitude of the task that confronts any engineer, not a specialist in flood data, who is called upon to decide what rate of flood run-off to assume in the design of a structure on a given stream. Whether he uses a formula or a diagram, he must exercise a certain amount of judgment in order that the rates of discharge selected shall be commensurate with the character of the stream and the requirements of the particular problem in hand. Methods or devices that make possible the efficient exercise of such judgment is what

* Cons. Engr., New York, N. Y.

the Engineering Profession has been seeking for a long time, and this want is, even to-day, far from being filled. The speaker's experience indicates that formulas can be used with safety only by experts who are familiar with the basic data from which they have been derived. This would seem especially true of the modified Myers formula in which the coefficient ranges all the way from 100 to 10 000. This formula fits the data in Plate IX admirably, but it is not clear how a set of coefficients might be prepared for practical use. Possibly, the author has undertaken such a task, and, if so, it would add materially to the value of his paper if he would supply this information in workable form.

Suitably prepared diagrams are, on the whole, more intelligible than formulas when judgment must be exercised, because they enable the expert and the non-expert alike to visualize many things which a formula will not disclose. For example, a glance will tell what part of a curve is supported by actual observations, and what part has been produced beyond the limits of such data. Again, in dealing with a drainage area for which few or no recorded data are available, the use of a diagram is superior to that of a formula because it permits of more intelligent exercise of judgment. Furthermore, diagrams of the right kind may be kept up to date by entering on them new data as they become available; this cannot be done conveniently in the case of formulas.

The advantages of the diagrammatic method are well illustrated by reference to curves showing the probable frequency of flood discharges of various magnitudes for 173 California streams.* A separate curve is drawn for each gauging station, or section of river, showing the observed data on which it is based, the entire range of floods that may be expected, and the probable frequencies of floods of any given magnitude. For streams on which few or no observations have been made curves are prepared based on other comparable drainage areas for which observed data are available. With such diagrams it is practicable to solve problems relating to the maximum discharge that is likely to occur at a given point in a stated period of years, and direct comparison may be effected between different streams or points on the same stream, while new data may be readily added from time to time or the curves altered in the light of later information. Such diagrams make available basic information the substance of which is open to inspection, and the derivation of each curve is readily seen. Other commendable features are the compactness of the material thus presented, and the uniformity of scale of the diagrams, which permits of showing with reasonable clearness four diagrams on a 6 by 9-in. page.

In giving consideration to the use of formulas and diagrams for publishing flood data the Society's Special Committee on Flood-Protection Data has expressed unanimous preference for diagrams of the general type used in the California report previously mentioned.

The author has well said that "there exists an endless variety of conditions that affect flood run-off". Many of these conditions have been made the subject

* *Bulletin No. 5*, Report on California Water Resources Investigations.

of study, but in the case of large streams two factors do not appear to have gained the recognition that is due them, one being the effect of the geographic arrangement of the tributaries on the building up of the flood wave in the parent stream, and the other the general direction of flow with respect to the movement of great storms. With regard to the first factor, high peak discharges in large rivers are usually traceable to synchronism in the arrival of flood waves from certain major tributaries. Other streams, exposed to storms in all respects similar, may lack this synchronism because of the different configuration of their basins, and their flood crests will be less in height but the duration of flood flow will be correspondingly prolonged.

An interesting illustration of the importance of these factors is afforded by Table 7, showing the greatest flood discharge in a century or more from each of three drainage areas of nearly equal extent. The river stage records, in each case, are of approximately the same length, and the relations between stage and discharge have been so well established as to be subject only to minor corrections.

TABLE 7.—Flood Data for Three Comparable American Rivers.

Stream.	Locality.	Drainage area, in square miles.	Date of maximum flood.	Maximum Rate of Flood Flow:		Coefficient in modified Myers formula.
				Second-feet.	Second-feet per square mile.	
Susquehanna..	Harrisburg, Pa.....	24 100	June 2, 1889...	700 000	29.1	4 520
Ohio.....	Wheeling, W. Va.....	23 800	Feb. 7, 1884...	494 000	20.8	3 210
Tennessee.....	Chattanooga, Tenn...	21 400	Mar. 11, 1867...	393 000	18.4	2 700

It will be noted that the Susquehanna River at Harrisburg discharges per square mile of drainage area, 58%, and the Ohio River at Wheeling, 13%, more than the Tennessee River at Chattanooga, although the latter drains a basin noted for its heavy storms and having a mean annual discharge greater than those of the others. All three streams are situated in the Eastern United States, two of them, the Susquehanna and Ohio, having their head-waters partly adjacent to each other; all of them drain country in many respects comparable, with mountains at the head-waters partly covered with heavy timber stands; and the climatic conditions do not differ materially except that the Tennessee River Basin lacks the cold winter seasons. Ordinarily, there would be reason to look for a fair degree of similarity in the flood behavior of these three streams. Radical differences exist, however, in the geography of their drainage basins. In the case of the Susquehanna River, two large tributaries, the Juniata and the West Branch, are responsible for the great floods in the main stream. Their basins adjoin and are usually covered by the same storms; their courses are from west to east in the same direction as the movement of storms, and their flood waves crest at about the same time, the flood peak of one joining that of the other in the channel of the Susquehanna.

On the Ohio River, the conditions are nearly reversed, there being no marked synchronism in the arrival of the main flood crests of the Allegheny and Monongahela Rivers at Pittsburgh, Pa., owing to the peculiar geographical arrangement of their basins and tributaries. In consequence, the maximum flood rates at Pittsburgh and Wheeling are not very high from the standpoint of flood run-off. The flood crests in the Ohio are caused mainly by synchronism in the flood waves of the Kiskiminetas and Youghiogheny Rivers, the water-sheds of which are contiguous and often subject to the same storms. Both streams drain from east to west, contrary to the storm movement, and, before their waters can join, one of them must first empty into the Allegheny and the other into the Monongahela. With an arrangement of this sort maximum flood rates such as occur on the Susquehanna River are physically impossible on the Ohio.

As for the Tennessee, this river and all its main upper tributaries flow in directions opposite to that of storm travel. In addition, there is little synchronism in the discharge of the larger tributaries. Thus, the flood crest of the Hiwassee River passes out first, and that of the French Broad River last.

Obviously, in the case of large streams the synchronism, or the lack of it, is an important factor. On smaller streams, the time of collection and concentration of storm run-off is usually so brief that the geographic arrangement of the tributaries cannot sensibly affect the net result. Thus, in a small basin like that of the Miami River (4 000 sq. miles), the maximum rates of flood discharge caused by the three-day flood of March, 1913, would have been no different if the tributaries, instead of converging in the existing crow-foot arrangement, could have been made to empty into the Miami many miles apart.

To what extent the coefficient of a formula can be made to take account of such divergent geographic conditions is not at all clear. It will be seen from Table 7 that the coefficient in the modified Myers formula is 4 500 for the Susquehanna, 3 200 for the Ohio, and 2 700 for the Tennessee. In the last analysis the coefficient in a flood formula is nothing more than a classification index intended to make possible a comparison between things not strictly comparable. In contrast, the California diagrams treat each stream as a law unto itself and comparisons are avoided. This, to the speaker, appears to be the rational method of dealing with flood phenomena.

The speaker feels that this is an opportune time for emphasizing the desirability of establishing uniform methods of compiling flood data. It is an undeniable fact that, of the existing mass of valuable flood statistics collected to date, only a comparatively small portion is in form for general use. Much of it has appeared in printed form, and some of it remains inaccessible to the profession at large, lying as it does in private files. No uniform system of collating any of it appears to have been advocated.

The demand of the Engineering Profession to-day is for more intensive and extensive compilations than have been made heretofore—more intensive, in the sense that data should receive closer scrutiny and be made to include

something more than mere dates and maximum flood rates; more extensive, in the sense that more streams should be studied and existing records be extended back to include information pertaining to early floods. Reviewing the long lists of flood data that have been published from time to time by the Society one is impressed with the wealth of information of this kind that may well serve as a foundation on which to build. The same may be said of the stream flow records of the U. S. Geological Survey. Its early publications give little information regarding floods, but the later ones are quite complete in that respect. Good work is being done also by State bureaus, and here and there much valuable material is available in private offices.

Flood data, to be of maximum serviceable value, must be in such form as to lend themselves to any of the many uses that engineers may wish to make of them. Engineers having dams to build and spillways to design are interested not only in maximum rates, but in the duration of floods and the total volumes of water discharged day by day. The same considerations affect the proportioning of reservoirs for storing flood waters and of detention basins. On the other hand, to the designers of bridges, culverts, flood channels, and levees, maximum rates are of chief interest and knowledge of flood duration or volumes of discharge is not essential. In all these operations, however, one factor stands out in importance above all others, controlling as it does the major aspects of any design, and that is, the probability of the recurrence of floods of various magnitudes. This may well be called the economic factor in any flood problem.

For some time the Society's Special Committee on Flood Protection Data has been at work devising a simple but comprehensive system to meet these various requirements. This has been referred to in the discussion by Mr. Grover, Chairman of that Committee. Attention is here invited to some of the features introduced, in the belief that they go far toward solving the problem of a uniform system for compiling flood data. To facilitate matters and insure uniformity, special blank forms on sheets 8½ by 11 in., have been printed by the Society, which may be had on application. The headings for these forms are shown on Fig. 16. The complete forms have space for 80 records in Form No. 1; 100 additional records in Form No. 2; 26 records (*a* to *z*) in Form No. 3, Table 2; 11 records in Form No. 3, Table 3; and 100 records in Form No. 4. Each form has space at the foot for the reporter's name and the date.

Forms Nos. 1 and 2 show in chronological order all floods that have occurred on a stream during the period covered by continuous records. By "all floods" is meant every rise of which there is record down to and including the least maximum annual flood. The latter, commonly referred to as the "one-year flood", serves as a starting point in the scale of floods, and its adoption places the flood records of widely differing streams on a comparative basis. Special pains are taken to indicate by means of a symbol whether the figures refer to peak flow or average 24-hour conditions (Form No. 1, Table 1, Column 1) (Fig. 16). For convenience of reference the three greatest known floods are entered at the top of Form No. 1 (Fig. 16), irrespective of whether or not they

Form No. 1. Am. Soc. C. E. Research Committee on Flood Protection Data.

FLOOD DATA Sheet No.....of.....Sheets.

STREAM.....
Location.....
Drainage Area.....Sq. Miles
Source of Data.....
Grading of Data..... Total Length of Period of Daily Records .....Years.
(for period of daily records) From.....to.....and from..... to.....

GREATEST FLOODS

Greatest known flood in.....years.....c.f.s.=.....c.f.s. per sq. mile on.....
Second largest known flood.....c.f.s.=.....c.f.s. per sq. mile on.....
Third largest known flood.....c.f.s.=.....c.f.s. per sq. mile on.....
The greatest flood is known to be the maximum in at least.....years.

TABLE 1. FLOODS DURING PERIOD OF DAILY RECORDS

Including all flows equalling or exceeding the "one-year flood" arranged chronologically

Ref. No.	Date	Gage Height (feet)	Flood Disch. c.f.s. P=Peak rate A=Aver. rate (24 hrs.).	Ref. No.	Date	Gage Height (feet)	Flood Disch. c.f.s. P=Peak rate. A=Aver. rate (24 hrs.).
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
1				41			
2				42			
3				43			

Form No. 3. Am. Soc. C. E. Research Committee on Flood Protection Data.

FLOOD DATA Sheet No..... of .....Sheets.

STREAM.....
Location.....
Drainage Area.....Sq. Miles

TABLE 2. FLOODS NOT INCLUDED IN PERIOD OF DAILY RECORDS

Ref. Letter	Date	Gage Height (feet)	Maximum Discharge		Grading	Source of Information and Remarks
			c f s	c f s per sq mile		
(1)	(2)	(3)	(4)	(5)	(6)	(7)
a						
b						
c						
d						

TABLE 3. CHARACTERISTICS OF CERTAIN HIGH FLOODS

(in order of magnitude as far as practicable)

Ref. Mark *	Date	Gage Height (feet)	Maximum Discharge		Aver. Rate of Discharge (c.f.s.) for Maximum Day	Duration of Flood (days)	Total Discharge during Flood	
			c.f.s	c f s per sq. mile			Acre Feet	Watershed Inches
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)

*Same as in Table 1 or Table 2,—i. e., use the same number or letter for a particular flood as in preceding tabulations

Form No. 4. Am. Soc. C. E. Research Committee on Flood Protection Data.

FLOOD DATA Sheet No.....of.....Sheets.

STREAM.....
Location.....
Drainage Area.....Sq. Miles.

TABLE 4. DATA FOR PLATTING FLOOD HYDROGRAPHS

Date.	Hour.	Gage Height.	Discharge.	Date.	Hour	Gage Height.	Discharge
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)

FIG. 16.—SAMPLE FORMS FOR COMPILING FLOOD DATA.

occurred within the period covered by daily observations. The data so listed are fundamental for many studies. In order that its degree of reliability may be quickly judged, each record is graded with a number, the scale of which runs from 5 for data of the best character to 0 for information of questionable accuracy.

Form No. 3, Table 2 (Fig. 16) is designed for listing floods not included in the periods covered by daily records, but concerning which definite facts are known. It is not amiss to state here that early flood data of considerable value have been obtained from a variety of sources, and have proved of much value in determining the frequencies of recurrence of great floods. On Form No. 3, Table 3 (Fig. 16) may be tabulated in greater detail several of the floods listed in Table 2.

Finally, on Form No. 4, Table 4 (Fig. 16), may be entered data for platting flood hydrographs, consisting of both gauge heights and discharges. This form was adopted by the Committee in preference to graphic representations, because no one form of hydrograph could be devised that would answer the many different requirements of engineers, and it was deemed best to leave the plotting to individual tastes. All the forms were designed to be capable of indefinite addition and expansion as future data are obtained.

These forms and the data to be entered on them represent the first step in the program of the Committee, the next step being the condensation of them into diagrammatic or other form for general use and reference. To date (1925), 139 streams have been compiled on the forms, thanks to the co-operation received from various sources. Engineers possessing flood data will confer a favor on the Society and on the profession as a whole if they will co-operate with the Special Committee on Flood-Protection Data in this important work.

Ford Kurtz,* M. Am. Soc. C. E.—Flood problems can be divided into two major groups, namely, those requiring for their solution knowledge of flood heights under existing physical conditions and those requiring the estimation or prediction of flood heights under other than existing conditions. The solution of all problems coming under the latter classification depends on the rate as well as the height of flood discharge.

Nearly all flood investigations required in highway and railroad design belong to the first group, since, in general, only slight channel encroachments are involved. Information as to the maximum stage of streams is often obtainable when data on the rate of discharge are completely lacking, so that highway and railroad engineers can usually secure the records they need, whereas the designer confronted with problems in the second group finds in many cases only very meager data to aid in their solution. Thus it happens that irrigation, public water supply, and hydro-electric engineers, who nearly always deal with design problems of the second group, are much more deeply involved in the study of stream flow characteristics than either the highway or the railroad engineer. However, the final difference in the difficulties encountered by the two groups of engineers in their flood-control problems is not quite so great as indicated, by reason of the fact that highways and rail-

* Care, J. G. White Eng. Corporation, New York, N. Y.

roads involve many streams in a single project and often pass through regions where irrigation, public water supply, and hydro-electric developments could not be financed, since bankers are loath to sponsor projects of which water supply is the basic factor unless stream flow records are available for a period of years, or the discharge can safely be estimated from records on adjacent streams. Thus, some observations of flood discharge as well as of flood heights are usually available for solving problems of the second group.

This paper presents in both tabular and graphic form a great amount of data on maximum recorded flood discharges and in a general way interprets them and indicates their application to specific problems. The latter feature is the chief measure of the value of the paper, since it determines the practical use. Plate IX, which summarizes the data graphically, shows such great variations in the maximum recorded flood discharges for a given area of drainage basin as to create at once the impression that the author has consolidated data too diverse (both geographically and in the period of observation) to permit of drawing conclusions of value. Furthermore, the method adopted in preparing Plate IX takes no account of the frequency of floods of different magnitudes, which is often an important factor in design.

Geographical diversity, as representative of different hydrological conditions, can be taken care of by preparing more diagrams from a given number of observations, but the correction for difference in length of period of observation, which is at least equally important, is not so simple. For this latter purpose, the speaker proposes the analysis of flood flows by the probability method. He considers the practical application of the mathematical theory of probabilities to the problem of frequency and magnitude of floods the most noteworthy single contribution of the last twenty-five years to the science of hydrology. Its uses are many, and it is the best method for correlating flood records having different lengths and made during different periods of years, as well as for determining probable frequencies of various sizes of floods, where such data are needed for purposes of design.

If the records presented by the author were first segregated into at least as many geographical divisions as are listed on Plate IX and if certain representative streams of each division, beginning with those having the longest records, were then studied intensively by the probability method, the plotting of the resulting values for the probable 100-year or 1 000-year flood would undoubtedly show a tremendous decrease in the width of the "belt" of points as compared with Plate IX. This method of studying an individual stream often results in the discovery and rejection of erroneous or doubtful data, which might otherwise go unnoticed, and will always smooth out in a consistent manner the irregularities of records on the stream. Thus, it may happen that the maximum flood observed in a 10-year period may actually be one which occurs only once in every 100 years on the average. It is truly remarkable how the probability method will show such a flood to be "out of line". By that it is not meant that the flood could not have occurred, but rather that it will appear quite plainly as a 100-year flood which chanced to occur in the 10-year period of observation. No other method will bring out

these features so clearly. The speaker, therefore, believes that intensive and careful study of a fewer number of streams in the manner just described would be much more valuable than the compilation of a mass of data on maximum recorded flood flows so great as to preclude detailed study of them.

There still remains for evaluation the important factor of shape of drainage basin, which causes wide variation in the rate of flood flow from basins having the same area and subject to the same hydrological conditions. This is a matter of individual examination for each stream. Obviously, the effect of this factor can be studied much better if geographical diversity and difference in period of observation have already been eliminated in the manner suggested.

The speaker by no means wishes to disparage the use of general curves based on the records of a number of streams, but he does feel that such curves must be used with great care. In general, if as many as ten years of flow records for any stream are available, intensive study of the higher recorded flows of the stream itself is often more valuable in predicting flood frequencies than curves based on longer records of adjacent streams. Some engineers might feel that a 10-year period of observations is too brief to serve as a basis of probability studies. However, examination of the hydrograph of any stream will show a great many peaks projecting above the general level, rather than one only. In the speaker's opinion these should all be classified and studied as floods, the problem being to fix the "basic stage" for a flood, that is, the line between those flows governed by a few well-defined factors and those governed by the laws of probability and chance. If this opinion is correct, ten years of records may yield, not ten, but twenty, thirty, or even forty, values for probability determination. These values will give a very good idea of the trend of floods—not the true trend, to be sure, but an indication of it that represents a great advance over the results of any other method thus far presented for the study of flood problems.

In order to fix this dividing line, or "basic stage", for floods on any given stream, it is only necessary to select various values, assume every peak above that value to be a flood, and from the points thus obtained derive a probability curve. That value of basic stage the points of which lie nearest to a straight line on probability paper is chosen as the correct value, and the resultant straight line as the flood probability curve.

As already mentioned, many design problems require a solution based on the frequency of floods of different magnitudes, because it is not always necessary or even desirable to design structures in such a manner that there will be no damage caused by a large, infrequent flood. One of the great advantages of the probability method is that, more than any other, it presents a quite definite idea of frequencies as a guide in the two major features of flood-control design, that is, the economic and the social. The economic feature is embodied in the question, "Will it pay from the standpoint of probable property loss over a long period of years to protect against certain floods?" Here the frequency studies are of utmost value. The social feature involves

the safeguarding of human life and the frequency studies point the way to the determination of the flood having such a small percentage of probability of occurrence that there is practical assurance it will never happen.

The speaker, therefore, wishes to urge the general adoption of the probability method as the best presented thus far for the study of compiled flood data, such as those submitted by the author, and for the study of the individual flood problems confronting all hydraulic engineers.

Charles W. Comstock,* M. Am. Soc. C. E.—The author has rendered a valuable service in collecting and arranging the records of about 1 000 flood discharges from drainage areas of all sizes in different parts of the world.

The speaker believes, however, that it is a mistake to relate the rate of run-off to the area of the drainage basin. This is, to use a crude analogy, as if one sought an empirical formula for stress in a beam in terms of the length as the only argument, leaving the load, the cross-sectional dimensions, and the method of support to be covered by an all-inclusive coefficient to be guessed at subsequently.

The author enumerates a considerable list of variables, both meteorological and topographic, which influence flood discharges, and then selects one (probably not the most important), "as the logical basis for a general formula", ignoring the others except as they may be included in a coefficient the values of which range from 100 to 10 000. Such procedure is certainly not scientific, nor is it likely to throw any light into the dark corners of this complicated subject.

It may well be doubted whether a formula is worth while in view of the large number of variables, most of which it is impracticable, if not impossible, to evaluate for any given drainage area. Careful study of Table 2 will give quite as much information and a far better understanding than can be had from any formula that attempts to summarize the data. Moreover, the very existence of a formula is likely to lull engineers into a comfortable, but unjustified, belief in the precision of results, instead of stimulating that healthy skepticism and critical examination of every step which so often keep them from serious blunders.

As already noted, the speaker believes that the area of the drainage basin is probably not the most important factor in determining flood run-off. The author states that he considers it the "logical" variable to which run-off may be related. May he not have been drawn to it, perhaps subconsciously, because it is the only factor which can ordinarily be ascertained?

The area covered by a violent storm is relatively small. A limited drainage basin may get the full effect of such a storm over its entire area at one time. A large basin may be crossed by the storm over a short line or a long line. In the latter case the total precipitation on the basin may be large, but it does not fall on all parts simultaneously. If the path of the storm is down stream the cumulative effect may build up an enormous flood from a moderate rate of precipitation. If the path is up stream a violent storm may produce

* Engr., Dwight P. Robinson & Co., Inc., New York, N. Y.

only ordinary high water. Again, a persistent and prolonged rain covering the entire basin at one time may cause a greater flood than either of the storms mentioned, although the total volume of water falling may be no greater. In these three assumed cases, all of which are not only possible but of frequent occurrence, the area and all other features of the water-shed are identical, and the total precipitation in a given time may be the same, yet the discharges at the gauging station will differ greatly.

An element of great importance (perhaps more than the area itself) is the permeability of the underlying rocks. The author mentions this only casually. This point was first emphasized by Belgrand in his studies on the basin of the Seine. He pointed out that 75% of this basin is underlaid by permeable rocks, the annual discharge of the river being only 28% of the precipitation; for the Garonne, of which about one-half the basin is impermeable, the run-off is 52%; and for the Po, the basin of which is almost wholly impermeable, the run-off is 64 per cent. These basins are of nearly equal size, that of the Seine being 30 000 sq. miles; that of the Garonne, 33 000; and that of the Po, 27 000.

A peculiar feature of the Po floods is that the discharge is nearly the same at all points along the lower 150 miles of the river. This seems to be due in part to the lack of synchronism of flood discharges on the numerous tributaries, and in part to the enormous storage capacity of the endiked channel.

The upper part of the Arkansas River in Colorado lies between the Continental Divide on the west and the Mosquito Range to the east. At the time of the spring run-off and consequent flood stage of the Arkansas, almost the entire supply comes from the tributaries which drain the granite massif of the main range, while the limestones, sandstones, and other permeable rocks of the Mosquito Range contribute only a trifle to the gulches which feed the river from the east.

It seems impossible that the various elements here mentioned, as well as many others, should receive suitable quantitative designation and be incorporated in any formula for general use. The engineer will necessarily be driven back on his judgment of the significance and importance of each factor in a given case, and he might better apply that judgment directly to the tabulated data.

It is easy to be critical. As Byron said:

> "One must serve his time to every trade
> "Save censure. Critics all are ready made."

It is another and far more difficult thing to contribute a new fact or a new principle. It is important, however, that in the desire to render knowledge more useful by classifying and formulating it engineers should not jump to hasty generalizations which may mislead.

As the author points out, the gauge height at flood stage is more important than the volume of discharge. It matters little whether the quantity of water flowing is large or small, as long as the surface does not rise high enough to

endanger lives or destroy property. It is well known that on the front of a flood wave the maximum velocity, maximum discharge, and maximum gauge height occur in succession in the order named. Thus, two flood waves of equal maximum discharge may have quite different gauge heights, depending on the steepness of the wave front.

There is a tendency to think that floods become increasingly destructive as time goes on. This is not so much because floods are greater in volume now than formerly as it is that there is more to destroy. For instance, the Seine flood of 1910 had a maximum gauge 0.39 m. less than that of 1658, but did incomparably more damage for an obvious reason.

The author's warning against the misuse of calculated flood frequencies is worthy of careful attention, especially in view of the prevalent, and in the speaker's opinion improper, tendency to base probability calculations on small numbers of observations.

In 1873, Belgrand published his monograph on "The Seine", one of the most elaborate and exhaustive treatises ever devoted to the study of a single river. In discussing the flood of 1658, he wrote:

"The meteorological phenomena which produced the flood of 1658 have manifested themselves again in our day. They have been renewed three times between 1732 and 1872 (140 years) in the floods of 1784, 1836, and 1866, or once every 47 years, and they were separated by intervals averaging 35 years. But, in order that there should result a flood similar to that of 1658, it is necessary that they should occur not more than five or six days apart. The probability of such an approximation is very small, and, by calculations which I think it useless to reproduce here, I have demonstrated that the recurrence of the flood of 1658, under the same conditions, *i. e.*, by floods in only two of the affluents, would require thousands of years."

Thirty-seven years later, in 1910, it happened.

A probability, however logically derived, is not a forecast. It is certain that what has occurred may recur, and even worse conditions may present themselves. There comes a point beyond which it is not financially practicable to go in providing protection against possible but improbable floods. At this point the engineer, like all good gamblers, must be willing to risk his money on a "long shot".

Thaddeus Merriman,* M. Am. Soc. C. E.—The speaker has been interested in floods for many years. He was fortunate in getting on to the ground in the Ohio Valley very shortly after the flood of 1913 and has kept up a lively interest in all matters pertaining to floods ever since. This problem is one that will always be with us because every time a new flood occurs it becomes necessary to revise a large part of our so-called fundamental data.

The speaker is heartily in accord with the point of view presented by Mr. Comstock. This matter of floods is not one which can be fairly treated by any method of probabilities. Maximum flood discharges are related to the duration of the storm and its intensity, to the direction of its path, to the characteristics of the topography and of the geology of the water-shed, far more than to any simple function of the area of the drainage basin.

* Chf. Engr., Board of Water Supply, City of New York, New York, N. Y.

On account of the ease with which these problems can be discussed with pencil and paper there is a tendency to neglect one source of information respecting flood heights that is worth more than all the direct observations which have been accumulated to date. This information is to be found in the records which the floods themselves have left on the walls of the valleys they traversed. This does not refer to the drifts of wood and leaves or other similar perishable substances, but to the sand and gravel bars which lie above the stream bed. They are to be found in every river valley and are better indices of past flood heights than any so-called records as to observed maxima. No analysis of possible flood heights can be considered complete, which does not include the records which the great floods of the past have engraved on the topography.

If, on the Catskill drainage area, there had been used any method of 10-year periods and had a probability curve been developed from them, the results would have been shattered beyond recognition by the flood of October, 1924; and that flood, moreover, was far from being a maximum.

There is a fundamental reason for the anomalous position in which we find ourselves with respect to this problem of floods. There are existent very few dependable records prior to 1900. Many floods occurred before that year and many have since occurred and passed without being recorded. Most of what are called flood records are not worthy of the name. Table 2 as presented is not a record of floods; it is merely a tabulation of observed stream flows. This is a fact which must be held clearly in mind, else we shall continue the vain task of studying and analyzing a mass of data that bears no known relation to real floods as such.

Weston E. Fuller,* M. Am. Soc. C. E. (by letter).—The paper calls attention again to the importance of the flood problem and sounds another warning of the inherent danger which lies in the encroachment of the channels that streams have used in the past for the purpose of carrying the great floods which come only at rare intervals. The great disasters that have come to many communities in the United States show this danger clearly. That similar disasters will occur at many other places can scarcely be doubted. The mere fact that for many years streams have not experienced floods that have caused any damage is, in itself, no assurance as to the future. The unusual conditions that produce great floods may occur on any stream, just as, in the past, they have come on other streams, and floods greatly in excess of the usual ones are possible at any time.

The author has discussed a number of characteristics of floods in a very helpful manner. He proposes a formula which, he concludes, represents the effect of the size of the drainage area on the maximum flood. Many investigations have been made for the purpose of determining this effect. As a result of these studies, a number of formulas have expressed the relation as an exponential function, in most of which the relation was found to be quite different from that now proposed by the author.

* Prof., Civ. Eng., Swarthmore Coll.; Cons. Engr., Swarthmore, Pa.

Grouping all the other factors affecting floods into a coefficient designated C in each case, the formulas resulting from these investigations may be expressed, as follows:

Talbot	$Q = C\,M^{0.75}$	
Fanning	$Q = C\,M^{0.83}$	
Burkli-Ziegler	$Q = C\,M^{0.75}$	
Dickens	$Q = C\,M^{0.75}$	
Cooley	$Q = C\,M^{0.67}$	
Ryves	$Q = C\,M^{0.67}$	
Dredge	$Q = C\,M^{0.67}$	
Metcalf and Eddy	$Q = C\,M^{0.73}$	
Fuller	$Q = C\,M^{0.67}$	for small streams, increasing to
	$Q = C\,M^{0.8}$	for very large rivers.
Cramer	$Q = C\,M^{0.7}$	to
	$Q = C\,M^{0.8}$	

The author's proposed formula is $Q = C\,M^{0.5}$, which is quite different from the previous formulas and which calls for an analysis of his data and methods in order to ascertain his reasons. He derives his formula by plotting the maximum rate of flow of many different streams, and draws a curve to envelop the highest of these maximum flows. The writer believes that this method of treating flood data has a fundamental error which leads to an erroneous conclusion. This opinion the writer has previously expressed in discussing other formulas derived in a more or less similar manner. In the closing discussion of his paper on "Flood Flows,* the writer made the following statement:

"* * * other formulas, derived by plotting the maximum floods which have occurred on streams of different sizes, give the greatest flood which has occurred on any of the large number of streams in the varied intervals covered by the several records.

"As there are many more small streams than large ones, it is obvious that there are more chances of obtaining an extraordinary flood on some one of the many small streams than there are of obtaining a similar flood on one of the few large ones."

This criticism was offered respecting data selected for streams which had similar characteristics so far as slope, rainfall conditions, and other elements affecting floods are concerned, and was simply directed toward the greater probability of securing relatively great floods on the large number of small streams than of securing a similar result from a few large rivers. In his paper, however, the author has not confined himself to a consideration of streams of a similar nature as to other characteristics affecting floods, but has plotted on his diagram (Plate IX), the extraordinary floods which have occurred on a few of the many small streams that are, in the main at least, mountain streams, and has compared these floods with those on the few large rivers on which no similar conditions exist.

In this case, therefore, the data are unbalanced in two different respects, and it is no wonder that the effect of the size of the catchment area is greatly exaggerated. An examination of Plate IX will show that the enveloping curve is controlled at the upper end by a few points, all of which are extraor-

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 681.

dinary floods which have occurred on a few streams having unusually favorable conditions for great floods so far as their drainage areas are concerned, and that these floods represent most extraordinary occurrences among thousands of similar streams. The other end of the curve is controlled by the floods which have occurred on the few large rivers in the United States. It will be noted that these rivers do not have similar characteristics so far as slope and other conditions are concerned. The author's formula, therefore, covers the extreme conditions for small streams, but only ordinary conditions for the large rivers, and does not properly indicate the effect of size of drainage area.

This is well illustrated by considering the two extreme points on Plate IX. Beacon Creek, the smallest stream plotted, has a catchment area on the side of a steep mountain, most of which is impervious; from it the run-off is very large. At the other end of the diagram is the Mississippi River, on which the conditions are nearly opposite. That the difference in run-off per square mile between these two extreme cases is largely effected by the character rather than the size of the water-shed, cannot be doubted.

Bearing in mind that the author has plotted on Plate IX floods which have occurred on nearly all the large rivers in the United States, it would be well to consider what the diagram would look like if he had followed the same course with small streams. In such a case he would have had a mass of points plotted on the left side of the diagram, representing the largest floods on the enormous number of small streams that have never had a flood as great as those plotted. A curve through the center of these points representing average conditions on small streams, and through the center of the points for the larger rivers, would indicate the effect of the drainage area.

Methods of treating data similar to that used by the author are quite common. For this reason the writer believes that it is well worth while to discuss the matter in considerable detail in order to show the reason why erroneous conclusions may be drawn. It may clarify the subject somewhat to indicate what would have resulted if a similar method had been followed in deriving other well-known formulas.

For instance, consider the formula to which the author likens his own, that is, the Chezy formula for the flow of water in conduits. Suppose that, in deriving the Chezy formula, data had been used which represented the maximum flows obtained in any conduits, irrespective of their slope or surface conditions, and that the formula had been based on the maximum points obtained by plotting these results. The result would have been a curve controlled at one end by the flow in small conduits with smooth surfaces, and, at the other, by the flow in large rivers, for which much less slope would have been found, and for which the surfaces would have been rough. It is needless to say that the formula thus obtained would have given the relation between the velocity and the hydraulic radius very different from that which was obtained by comparing conduits of similar slopes and similar surface conditions. Such a formula would have been misleading indeed if applied to similar conduits of different sizes. The author has used a similar method

in obtaining his formula. It is essential, in using data of this kind for the purpose of ascertaining the effect of any element, that the effect of all other elements be made at least similar, otherwise the results do not show the true relation.

The author tabulates the maximum floods for a large number of streams, the information having been collected from many different sources. These data are useful, and Mr. Jarvis has performed a real service in bringing them into a single table (Table 2). Any one who has traced data of this type to their sources, must have been impressed with the likelihood of errors, as the items are transferred from table to table. Flood data are particularly unreliable in this respect.

In the first place, the records of these floods are of varying degrees of accuracy. Some of them are undoubtedly close approximations to the actual peak flows of the floods in question; others are probably far from the actual run-off. This must be true for many of the floods, particularly for those which have caused great destruction. In the great majority of such cases the discharge has been estimated by the use of the Kutter formula. The data for determining this flow have been obtained subsequently by observation of the water marks of the maximum height attained by the flood at any time during the flood period. Whether or not the slope thus obtained represents the slope of the water flowing in the channel cannot be ascertained with any degree of accuracy where the channel has been materially changed by the floods. Moreover, the value of the coefficient, n, in the Kutter formula at the time the maximum height was attained at the point under consideration, is doubtful. It is impossible to say how much the flow in the channel is being retarded at that particular time by the mass of débris flowing down the stream. It is impossible to know whether or not the height which the flood reached at that point was caused by water flowing in a free channel or whether it was backed up by submerged obstructions. Little is known of the effect on the flow of great masses of soil, trees, brush, rocks, and other material which are being carried down the stream by these floods.

Mr. Winsor, in his discussion, vividly sets forth the condition that exists in some extreme cases where the water is practically pushing the large mass of matter ahead of it. Is it not true that for most destructive floods similar conditions, to a lesser degree, occurred at some time during the flood, thus establishing high-water marks which are afterward taken as the slope of the water which is assumed to have been flowing freely in the channel as left after the flood recedes? It must not be overlooked that, in estimating the discharge for many of the floods, values of n in the Kutter formula as low as 0.030 have been used, and seldom, so far as the writer can ascertain, have values of more than 0.040 been taken.

The writer is inclined to think that for most destructive floods the discharge has been over-estimated. In tables such as Mr. Jarvis has presented, as well as in many similar tables, there is no indication of the accuracy of the data, yet from the original source of the information, it is found—in many

cases at least—that the engineer who estimated the flood had a clear conception that his estimate may have been far from the actual flow.

As an instance of this, consider the floods on Cane Creek, in North Carolina, and on Elkhorn Creek, in West Virginia, two of the largest, relative to the drainage area, of which there are records. The quantity of water flowing during the peak of these floods was estimated by Mr. E. W. Myers, who described* the difficulties of ascertaining the discharge during these floods, and frankly stated that his estimates might have been far from the truth. It appears from his description that in estimating the flow he used the slope of the channel as it existed after the flood had receded.

To illustrate the nature of these data it may be well to quote a little of what Mr. Myers has said in regard to his measurements. As to Cane Creek, he says:

"For these reasons the writer has used all the means in his power to obtain such information and it is presented below, though with diffidence, for the possibility of considerable error is great, but the results are the best that can be obtained under the existing circumstances.

"* * *. Unfortunately, no gaugings of this stream at anything approaching flood heights has ever been made, so that a rating table giving the relation between height and discharge cannot be constructed in the usual manner. Failing in this, therefore, there are two approximate methods which can be used for obtaining this information, one based on the consideration of the amount of the rainfall during the period, its distribution, the condition of the earth as to saturation, etc., together with the shape of the basin, its average slope and the general topography; the second based on the cross-section of the channel at some point, its condition as to roughness and the slope for some distance above and below the point where the cross-section is taken.

"* * *. This method is open to objection on the ground that there is great difficulty in selecting the proper value of n, and also from the fact that the slope at flood stages is probably not the same as at low water, except at the crest of the flood, when the water is neither rising nor falling."

As to Elkhorn Creek, he says:

"As stated before, in this way no estimate of the maximum flood volume is possible and we are compelled to resort to estimates based on the slope and cross-section of the channel. Such estimates in this case present even greater difficulties than in the case of Cane Creek, since the waters of Elkhorn Creek were much more obstructed and the cross-section at different points is much more variable, making the coefficient of roughness greater than before, and introducing more uncertainty as to its proper value. As the method used was precisely the same as that used on Cane Creek, the calculations will not be given here in detail as on that stream."

Other floods, which rank high among the extraordinary recorded floods, are those which occurred in the vicinity of Philadelphia, Pa., in 1843. The record for the discharge during these floods is found by a recent re-computation based on data obtained at the time of the flood. The writer is not criticizing either the accuracy of the data taken at that time, or the method of computation, but simply noting that there is a large chance of error, and that too much dependence cannot be placed on such data. For many of these extraordinary floods, therefore, there is merely an estimate based on doubtful

* *Engineering News*, Vol. XLVIII, August 7, 1902, p. 102.

data, with no assurance that these estimates in different cases are properly comparable.

It appears further, in comparing the various tables of maximum flood flows, that there are considerable differences as to the magnitude of these floods in the different tables. In some cases these differences are accounted for by revisions of the original estimate; in others they probably are due to typographical errors and similar causes. Each time a new table is presented, some of these errors creep in. Evidently, such data are in much confusion, and there is real need of a careful study of the original records of all these great floods in order that their relative reliabilities may be determined.

The author's proposed method of determining the maximum flood on a stream is evidently based on consideration of the single flood on that river. For many of the rivers there is no method of telling whether or not this single flood is an extraordinary event. It is true that, for many streams, the author has listed what he indicates as frequent and rare peaks. So far as the writer can ascertain, these frequent and rare peaks are taken from two or three different sources. Most of them represent the data taken from the writer's paper on "Flood Flows"; others are taken from the study of floods in California, and a few others from observations and estimates by the author. So far as those taken from the writer's paper are concerned, the frequent peaks represent the yearly floods as obtained by averaging the largest flood for each year, over a period which was then available. The end of this period was about twelve years ago, and more reliable data must now be available for many of these streams. The rare peak merely represents the largest 24-hour average flow during that period.

The basing of any estimate on the single maximum flood, without consideration of any other floods which have occurred on the stream, leaves the time element entirely out of consideration.

Mr. Lane's excellent statement of the meaning of flood frequency and its importance, will serve to clarify this subject. His study, which is confined to New England streams, indicates a frequency relation for flood flow expressed by $Q = Q_1 (1 + 0.69 \log T)$. The writer, basing his study on all the streams of the Eastern Coast, which included, of course, Southern as well as Northern rivers, found a relation as to frequency for those rivers in accordance with the formula, Q maximum $= Q$ average $(1 + 0.75 \log T)$; for other parts of the United States, he found a larger variation in respect to T and adopted as a general average, $1 + 0.8 \log T$, believing that the data then available did not justify any attempt to use a different relation for different localities.

In making his study, Mr. Lane has used a method which gives the flood to be equalled or exceeded, corresponding closely to that described by the writer.* Using Mr. Lane's illustration of a series of 1 000 floods on a stream, he is correct of course when he states that the one-hundredth of the series is that which will be equalled or exceeded on an average of once in a 10-year period. But what about the largest flood? It is not the flood which will be equalled or exceeded in 1 000 years. It is the flood which will probably

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 580.

occur. It is the flood for which there is an equal chance that it will occur in another thousand years. This is what the writer's formula gives, and, in his opinion, it is what is desired. With Mr. Lane's method, this largest flood cannot properly be included in his series unless he includes it for a period of 2 000 years. From the method outlined in his discussion, it would appear that he has not done this, but has taken the largest flood as the one which is equalled or exceeded for the full period. If so, he is slightly in error for about one-half his records.

If it were desired to find the flood for which to design, so as to be quite sure that the structure would fail within a given period, then the flood that is equalled or exceeded in the period would be the proper one to use. The writer's view of the matter, however, is that for the design of the structure, it is desired to ascertain the flood for which there is an even chance that it will not fail during the period, and for that reason he used the average flood. It really matters little which method is used, if the computer understands what he is doing. Substantially the same results would be obtained if the value of T in the Lane method is made twice as great as in the writer's method. The relation between these methods has been discussed previously* at some length by the writer.

Mr. Lane apparently obtains his yearly flood by plotting the data rather than by averaging the floods. There is much to be said in favor of this method and something to be said against it. As he states, it eliminates any consideration of the larger floods and, in cases where these floods are unusual, this result would be better. It is not certain, however, that these larger floods are unusual. Is it not equally possible that, for the period in question, the stream has had an unusual number of smaller floods? The writer considered this question at the time he prepared his paper. He was much inclined to adopt a method similar to that proposed by Mr. Lane, but concluded that, while in some cases the average was unduly affected by including the larger floods, the error in neglecting them, while less apparent, was, in other cases, quite as great. In his own work, however, he takes into consideration both the actual average yearly flood and the yearly flood as indicated by plotting. For long records either method will produce satisfactory results, while for short records there is, of course, bound to be some error.

Mr. Lane has computed the probable error in obtaining the one-year flood by means of records of different lengths, based on the Merrimack record. His results seem to agree quite closely with those found by the writer from a study of many different rivers to ascertain the probable error in finding the average yearly flood.† In so far as the probable error is concerned, there seems little to indicate which method is the better.

A study of the floods on streams where large ground storage exists, or where there are considerable storage reservoirs which are partly emptied at times when floods occur, has convinced the writer that in some cases a method almost the opposite of that suggested by Mr. Lane is necessary. For such streams the floods are divided into two series, one representing the floods that come

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 564.

† *Loc. cit.*, Fig. 19, p. 685.

when storage is to be had in the ground or in the reservoir, and the other including the floods when no storage is available. The plotting then may become two parallel lines, one distinctly higher than the other. This is because the storage, when it is available, affects the total run-off from large and small storms by about the same amount, thus being a much greater proportion of the small storms than it is of the larger ones. In such conditions the flood to be used in the place of the yearly flood in the formula, should be determined largely by consideration of those floods which occur when little or no storage is available. It can be approximated by considering only the upper points representing the larger floods, drawing through these points a line representing the frequency relation and considering the "yearly flood" as that given by the intersection of this line with the one-year line. This, of course, may be higher than the average and much higher than that obtained by the Lane method. While such cases are not common in most sections of the United States, yet in a study of Southern California streams several were found, and the influence is undoubtedly felt for other rivers, although it is less marked. Here the method proposed by Mr. Lane would lead to a much larger error than the use of the average flood.

In arid regions these conditions are exaggerated, so that no floods of any moment occur during many years, yet large floods sometimes come. In his paper on "Flood Flows," the writer stated:*

"* * * in arid and semi-arid regions * * * conditions are such that no floods worthy of the name occur in some years, though occasionally there is a large one. For such cases the average yearly flood becomes low and the percentage of the maximum flood becomes unduly high. These conditions are quite different from those generally found, and it seems best to exclude such cases. It follows, of course, that the frequency relation here proposed does not hold for such rivers."

Mr. Lane suggests that the relation between the average 24-hour flood and the maximum rate of flood, as included in the writer's formula, is the weakest part of the formula. The writer agrees. At the time the study was made little data were available on the maximum rate of flow, only occasional records could be found for most streams, and there were few continuous records. What data were to be had consisted, in the main, of records based on the 24-hour average, so that it was necessary to make the study on that basis. The writer appreciated the fact that the peak flow for small streams would be a larger proportion of the 24-hour average flow than for large streams, and established the relation as best he could with the data at hand.

At present, there is a large amount of additional data available, and a far better relation can undoubtedly be established. This the writer hopes to do at an early date. Mr. Lane suggests that, instead of using this relation, his Q_1 may be used. With this conclusion the writer is agreed, providing that it is possible to obtain directly the flow at the peaks of the floods. At the time the writer proposed his formula, it was quite impossible to obtain these peak flows for any considerable number of rivers. Even to-day the bulk of the data is still expressed in terms of a 24-hour average, so that the relation between

* *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), pp. 581–582.

the peak and the 24-hour average is a necessary part of the formula. Where, however, the peak flows are available, the writer certainly would agree that they should be used in preference to any formula which could possibly be devised. In other words, where facts are available, they should always be used in preference to any indirect figures obtained by means of formulas.

There is one point in regard to this relation between maximum rate and 24-hour average rate to be made clear. Mr. Lane criticizes the relation because it does not agree with some individual cases. The relation as expressed is intended to be that between the maximum flow at the peak, during a period of years, and the maximum 24-hour average, during that same period. This relation cannot be expected to hold for individual floods, as the two maximums do not necessarily come during the same flood.

A few years ago the writer made a partial check on the frequency relation in his formula by again plotting the data for selected streams with long records, including therein the floods for five or six additional years. For most of these streams the indications were that the individual plottings approached more closely to the general relation than previously. With the large amount of additional data now available, it may be that frequency relations for different sections of the country are justified. It is the writer's intention to make a re-study of this phase of the problem.

The investigations which have been made have only scratched the surface of the flood problem. The maximum peak flow, although an important element in many problems, is in others less important than the maximum total flow during flood periods. In fact, the hourly flow for small streams, representing as it does the bulk of the run-off, is perhaps properly comparable to daily flows of larger streams, weekly flows of still greater rivers, and flows of several weeks for the Mississippi River in its lower stretches. Each represents the run-off for somewhat similar conditions. A study along such lines might tend to clarify some points not yet understood. There have been many studies of the effect of such elements as the average slope of the drainage area, the relation of the length to the width of drainage area, the average rainfall over the area, and some other elements. The effect of the percentage of the water area of lakes and ponds is another factor which could well be studied. Information as to the effect of these elements, as well as to the effect of the size of the drainage area and of the frequency relation, would place in the hands of the engineer tools with which to work in solving the problem for individual streams; but they can never eliminate the necessity for the careful study of the stream itself or the exercise of judgment.

Formulas which confuse the effect of any element are, however, not useful tools, as they tend to confuse the judgment. In the writer's opinion, the author's formula, purporting to show the effect of drainage area, while in fact including thereunder the effect of many other elements, comes in this class.

H. de B. Parsons,* M. Am. Soc. C. E. (by letter).—This paper calls attention to a most important subject, and the data of maximum observed flood discharge rates given in tabular form will be useful to those studying flood problems.

* Cons. Engr.; Prof. Emeritus, Rensselaer Polytechnic Inst., New York, N. Y.

The maximum run-off in any district must be strongly influenced by the characteristics of the district, such as the geological formation, surface contours, number and size of tributaries of the main stream, forestation, etc. When the area of a drainage basin is large, the physical characteristics of part of the area may (and often do) differ widely from those of other parts.

If the flood discharges given in Table 2 were segregated into groups according to physical characteristics and to geographical location, the variation in the value of constant, C, between upper and lower limits, as shown in Plate IX, would be greatly reduced for each group. This would make the practical application of the formula less liable to error.

It would seem that a formula to express the discharge in second-feet as a maximum to be expected, based on known flood records, should be of the form,

$$Q = C\,M^{\frac{n-1}{n}}\,K$$

This is the author's Equation (2), with the addition of K, a variable to cover the physical and geographical characteristics of the drainage area. It might be possible to determine values for K by classifying the data in Table 2, and platting the points in a manner similar to that used for Plate IX. Then, K would be constant for each classified group, and C would be constant for all groups, but sufficiently large to cover the maximum expected floods.

The writer suggests that this matter be referred to the Society's Special Committee on Flood-Protection Data and to the author, who deserves great credit for the work he has done. Perhaps a detailed study of classified drainage basins would develop the fact that values might be determined for K which would be characteristic for certain physical and geographical conditions and which would bear some simple relation to the discharge corresponding to known precipitation. This idea, expressed mathematically, might take the form of:

$$K = \text{Function of } \frac{\text{Discharge}}{\text{Corresponding precipitation}} = f\left(\frac{D}{P}\right).$$

Robert E. Horton,* M. Am. Soc. C. E. (by letter).—The author adopts tentatively the Myers formula in which flood discharge varies as the square root of the area. In most such formulas, larger exponents of A have been used, generally $\frac{3}{4}$, $\frac{4}{5}$ or $\frac{5}{6}$. The author does not give reasons for the use of so small an exponent of A; presumably he believes it better adapted to Western streams.

The average slope of the line of relation between Q and A could be determined by taking group means of all the data plotted on Plate IX within the limits of selected increments of A and plotting the mean values of Q in terms of A. This the author has not done. The data on Plate IX seem to show some justification for the reduction of the exponent of A below its customary value. A line through the center of gravity of the points having a slope corresponding, say, to $A^{0.8}$ would clearly fall below the majority of the points to the left and above a majority of those to the right of the central point. It is to be noted, however, that in nearly all cases where the exponent

* Cons. Hydr. Engr., Albany, N. Y.

of Q is taken in terms of the $\frac{3}{4}$ to $\frac{5}{6}$ power of A, slope is also included as a factor in the formula. Large areas are generally flatter than small ones. The steeper the slope the greater the flood discharge, other things being equal. The flood discharges recorded on Plate IX for small areas are, therefore, greater than they would be if the slope of these small areas was the same as that of the larger areas; hence, to fit the plotted points the line must be steeper or, in other words, a smaller exponent of A must be used, than if the slope of the drainage basins was considered separately. However, it seems certain that the data given by the author would serve to evaluate the law of slope in relation to flood discharge quite as readily as the law of area. If this was done, one less factor would be left to the uncertainties of human judgment. The author's formula apparently does not relate to basins of any particular slope, but is based, tacitly at least, on the presumption that each basin has the average slope of all basins of the same area, this average slope differing for each size of area and being steeper for small than for large areas. This assumption is hardly a safe guide in computing flood discharge in specific cases. Two basins of equal size in the same region may differ widely in slope. Drainage basins on the Great Plains are generally much flatter than basins of equal area either in the East, or in the Western mountain regions.

The author compares the Myers flood formula with the Kutter-Chezy formula for flow in channels and pipes. In the Kutter-Chezy formula two important variables, R and S, are directly included and only the effect of roughness is covered by the judgment coefficient, n. In the Myers formula only one of the two main variables is directly provided for, namely, area. Rainfall and all the factors which affect the rate of run-off from a given area are thrown into a judgment coefficient.

The writer believes it is better in general to reduce the factors depending on judgment to a minimum. That this can be done in some degree, at least, in flood discharge calculation, is evidenced by the general success attending the application of the so-called "rational method" to storm-sewer design. At present, the laws connecting surface slope, permeability of soil, and other factors with flood run-off rates are only imperfectly known. Methods by which data may be obtained for the correlation of some of these factors with flood discharge have been developed by the writer and are presented herewith, some of them for the first time.

In addition to area, the principal physiographic factors which are correlated with the flood discharge of a drainage basin are:

(1) Length of basin, measured along the main stream channel.
(2) Drainage density or length of streams per square mile of area.
(3) Average slope of the stream channels.
(4) Average slope of the ground surface.
(5) Slope ratio or ratio of stream channel slope to land surface slope.
(6) Average length of overland flow.

Quantitative values of these factors can be determined without great difficulty if suitable topographic maps are available.

Let,

A = area, in square miles.
Σl = total length of streams in the drainage basin, in miles.
h = contour interval used, in feet, measured vertically.
Σc = total length of contours of vertical interval, h, on a given area.
S_c = average slope of stream channel.
S_g = average slope of land surface.
d = drainage density = $\frac{\Sigma l}{A}$.
$\frac{S_c}{S_g}$ = slope ratio.
w = average distance between contributory streams.
z = average horizontal angle between the direction of stream flow and the direction of overland flow.
l_g = average length of overland flow.

The obvious method of determining drainage density is to measure with an opisometer the total length of the streams shown on a topographic map and divide this result by the area. For large areas this is laborious, and a sufficiently accurate value can generally be obtained by the "sample area method", which consists in determining the drainage density on a few selected representative rectangular areas and taking the average.

The slope of streams can be determined by noting the elevation of the source and mouth of each contributory and measuring its length. If ΣE_1 is the sum of the elevation of stream sources and ΣE_2 is the sum of the elevation of their mouths, then the mean slope of the streams, in feet per mile, is:

$$S_c = \frac{\Sigma E_1 - \Sigma E_2}{\Sigma l}$$

This determination can also be made by counting the number of contours intersecting all streams. Calling this number Σc:

$$S_c = \frac{h \Sigma c}{\Sigma l}.$$

The latter method has an advantage in that the small streams, which largely determine the average slope, are not so likely to be omitted. This determination should generally be made for the whole basin, although the "sample area method" can be applied after selecting representative sub-basins for measurement.

If the total length, Σc, of the contours of the vertical interval, h, in feet, crossing the basin is measured by an opisometer, then $\frac{A}{\Sigma c}$ is the average distance between contours measured horizontally, and the average slope of the land surface is:

$$S_g = \frac{h \Sigma c}{A}.$$

To determine by opisometer the length of contours on a large area, even at 100-ft. intervals, is very laborious. Although the "sample area method"

may be applied, it is better to use what may be called the "intersection method", which gives a fair determination of the slope for all parts of the area with moderate labor. To apply the "intersection method" the portions of topographic maps covering the area are divided by pencil base lines into, say, 4-in. squares. Choosing a suitable contour interval, the number of times, N, that the base lines are crossed by contours is counted and the total length, L, of base lines within the area is determined. If the contours crossed the base lines at right angles, the average slope would be $\frac{h\,N}{L}$. The contours, however, cross at varying angles from zero to 90 degrees. If x is the distance between the points where two adjacent contours cross a base line, and a is the angle between a normal to the contours and the base line, then the distance between contours, measured normally or at right angles to the contours, is $x \cos a$. Hence, the average normal distance between the contours is:

$$m = \frac{N \cos a}{L}.$$

The average slope of the ground surface is:

$$S_g = \frac{h}{m} = \frac{h\,L}{N \cos a} = \frac{h\,L}{N} \sec a$$

The average secant of angles from zero to 90° is 1.57; hence:

$$S_g = 1.57\,\frac{h\,L}{N}$$

The slope ratio, $\frac{S_c}{S_g}$, may now be determined. This is never greater than unity. In other words, the slope of the ground surface of a drainage basin is always greater than that of the streams. The average length of overland flow or the distance which the water must travel on the ground surface before reaching a definite stream channel is an important factor in relation to flood discharge, especially for flat, poorly drained areas. The larger the distance the greater is the time of rainfall concentration at any given point on the stream. If α_g and α_c are the slope angles for the ground surface and stream channels, respectively, then:

$$\tan \alpha_c = \frac{S_c}{5\,280}$$

$$\tan \alpha_g = \frac{S_g}{5\,280}$$

Given these values, the average direction of overland flow or the average angle, z, between the direction of overland flow and the channel direction, can be determined by the formula, readily derived by geometrical construction:

$$\cos z = \frac{\tan \alpha_c}{\tan \alpha_g} = \frac{S_c}{S_g}$$

In other words, the slope ratio is the same as the cosine of the angle between the direction of overland flow and the stream direction.

The reciprocal of the drainage density, or $\frac{l}{d}$, is the average distance between streams. As there is a divide between every two adjacent streams the average distance from the divide to a stream is one-half this, or:

$$\frac{w}{2} = \frac{l}{2\,d}$$

The average distance which water must travel overland from a point on any divide between streams or the average length of overland flow can be derived from the relation:

$$\frac{w}{2\,l_g} = \sin z$$

from which,

$$l_g = \frac{w}{2 \sin z}$$

The writer has applied these and certain additional methods to various drainage basins for the purpose of providing data for the study of the relation of run-off to slope and other physiographic factors. It has been deemed worth while to present the methods in some detail here in order to lay a foundation for attacking the problem of the laws of flood discharge for different areas on a quantitative basis.

When a sufficient body of flood discharge data has been accumulated, accompanied by physiographic and soil or infiltration data, a beginning can be made toward the derivation either of a "rational" flood discharge formula, or perhaps better, the use of the existing rational method can be extended to large natural areas. To accumulate such a mass of data may require a long time, but the Engineering Profession worked for fifty years with the Kutter and similar formulas before reaching the point where the data for their application could be considered reasonably complete.

Flood discharge data unaccompanied by adequate data for the correlation of physiographic and other factors with the discharge coefficients are little more useful than a heterogeneous table of values of Kutter's n unaccompanied by data on the kind and character of surface for which the given values of n were determined.

Viewing the subject of maximum flood discharges from another angle, the writer believes that there is for each drainage basin a certain finite rate of flood discharge which Nature is incapable of transcending, that is, the true maximum discharge. Reasons for this belief are founded on: (1) Experience; and (2) meteorological considerations. The Hudson River cannot produce a Mississippi River flood for very much the same reasons that an ordinary barnyard fowl cannot lay an egg a yard in diameter—the hen is not large enough.

Rain can only be produced as fast as moist ascending air currents can be brought over the area on which the rain falls. The larger the area covered by a storm the smaller in general are the barometric gradients around the storm center. With less wind velocity and less rate of ascent of moist air the

average rainfall rates over large areas must necessarily be less than may prevail over small areas.

On the other hand, in purely convective storms where the air ascends vertically, the moisture supply must in general be local. There is thus a natural inhibition of rain intensities exceeding certain limits, the limit depending on the size of the area. In view of these considerations a flood discharge formula such, for example, as the Fuller formula which implies that a flood of infinite magnitude may occur on any area, however small, if one only waits long enough, is, to say the least, irrational in form.

The question naturally arises: Is it possible to determine, even with approximate certainty, the true maximum or limiting flood discharge for any area? The writer believes it is possible provided a good record of flood discharges is available.

Flood discharge data when plotted in terms of frequency take the form of a skew frequency curve to which the ordinary or Gaussian law of error does not apply. Skew frequency curves, however, can be represented accurately by other forms of expression. For floods the frequency equation should be of such form that the flood magnitude approaches a finite limit as the "exceedance" interval approaches infinity.

The following expression was developed by the writer to meet this requirement and has been applied to a variety of phenomena dependent on rainfall, including floods:

$$R = R_g \left[1 - e^{-kt^n}\right]$$

in which,

R = ratio of the magnitude of the event to its average magnitude.
t = average "exceedance" interval of an event of magnitude, R.
e = base of Naperian logarithms.
R_g = true maximum or limiting value of the magnitude of the flood or other phenomenon.
k, n = constants.

Values of k, n, and R_g can all be determined from the lower definitely located portion of the "exceedance" interval curve.

Data of flood records for 70 years for the Connecticut River at Hartford, Conn. (1843-1917), give the equation:

$$R = 1.82 \left[1 - e^{-0.255\, t^{0.54}}\right]$$

The highest recorded value of R is 1.70, but there have been nine floods with R exceeding 1.40. In other words, floods having magnitudes of more than 75% of the limiting flood magnitude occur at fairly short intervals, in this case about once in eight years.*

Merrill M. Bernard,† Assoc. M. Am. Soc. C. E. (by letter).—A review of the many contributions to engineering literature on flood flow and related subjects, impresses one with the fact that eventually there will develop, out of

* Application of this form of frequency equation to the determination of maximum and minimum annual rainfall in New England will be found in the *Journal*, New England Water Works Assoc., Vol. 38, March, 1924.

† Civ. Engr., Crowley, La.

the maze of meager data and unsupported theory, a definite and clean-cut working knowledge of the relation between rainfall and the resulting run-off.

The engineer, having to do with drainage works, flood control, and the design of hydraulic structures and bridges, grasps eagerly at current articles pertaining to the subject, only to find, in many cases, that they fail to clear up many of the doubtful points.

The author has made available, in most satisfactory form, an exhaustive list of flood discharges, and has made some valuable suggestions. He presents a basic run-off formula, which should be followed up and utilized by those advocating this means of estimating flood flow.

In Table 8 are listed eighteen of the more important factors entering into an analysis of flood flow on any particular water-shed, many of them influencing run-off as much as the catchment area. Six of these eighteen factors

TABLE 8.

Constant factors.	Variable factors.
(1) Area of shed.	(1) Degree of efficiency desired.
(2) Shape of shed.	(2) Intensity of rainfall producing maximum flood to be considered.
(3) Arrangement of lateral drains.	(3) Frequency of rainfall producing maximum flood to be considered.
(4) Average slope of shed.	(4) Time of concentration.
(5) Average slope of drains.	(5) Effect of vegetable cover.
(6) Geographical location.	(6) Degree of improvement of lateral drains.
	(7) Losses: Evaporation, transpiration, and seepage.
	(8) Relation of storm center to water-shed.
	(9) Travel direction of storms.
	(10) Season in which storm occurs.
	(11) Relation of storm to storm period.
	(12) Available storage.

are constant and can be given definite values. The remaining twelve are variable, but in the study of a particular water-shed can be assigned maximum or average maximum values tending to give results that are on the side of safety.

The rational expression of the relation of rainfall to run-off, $q = C\,i$, in which, q is the run-off, in cubic feet per second, from a unit of catchment area, resulting from a rainfall of intensity, i, and C is a coefficient reflecting the physical characteristics of the water-shed, is becoming popular in the field of storm sewer design. It is based on a theory that maximum flow will result from a rain the duration of which is equal to the time of concentration.

The diagram, Fig. 17, illustrates the relation of the listed factors to the formula. In application, the value of C will range from 0.11 to 0.15, where the acre is the unit of area. The time of flow can be analyzed from head-water to outlet, which, in a co-ordinated system of drains, will take care of peculiarities in the shape of the area, the arrangement of lateral drains, etc. The writer recalls a water-shed made up in part of cut-over pine hills, having an average slope of about 12 ft. per mile, changing abruptly at the stream itself to a dense palmetto swamp, with a slope of about 0.4 ft. per mile. The outfall stream ran for 30 miles without adding 1% of area to the water-shed,

due to the fact that the adjacent lands sloped away from the stream. In this case, it would be manifestly impossible to arrive at an intelligent result by considering the flood discharge as a function of the area alone.

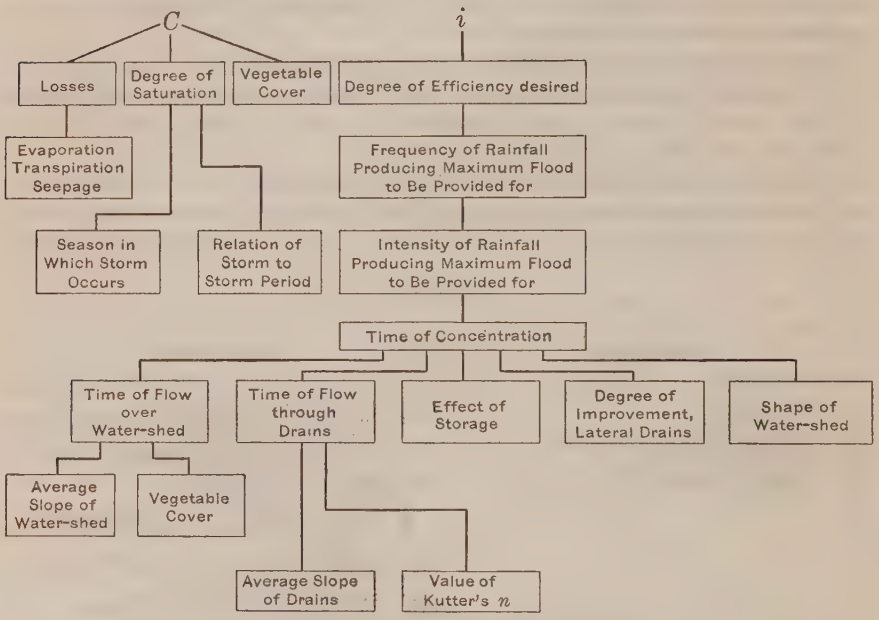


FIG. 17.—RELATION OF RUN-OFF FACTORS.

Assumptions must be made. If assumed values are applied to all factors, errors of judgment become compensative, and the designer has the satisfaction of knowing that he has dealt with all phases of his problem, even if he is in error as to some of them.

CONSTANT FACTORS

Area of Water-Shed.—The area can be actually measured or estimated.

Shape of Water-Shed.—The shape of the water-shed is reflected in the analysis of the time of flow. A long narrow water-shed will produce a shorter concentration time than a broad, fan-shaped one.

Arrangement of Lateral Drains.—The topography usually decides this arrangement, but should future improvement demand a re-arrangement of lateral drains the fact should be considered.

Average Slope of Water-Shed.—This factor is subject to measurement and is necessary in determining the time of flow over the ground surface. It also aids in selecting what must be considered as the most remote part of the water-shed.

Average Slope of Drains.—This factor, too, is measurable, and is necessary in arriving at the time in transit through the drains.

Geographical Location.—The geographical location will place the water-shed under consideration in the proper weather characteristic group, and will permit the selection of a satisfactory rainfall intensity curve.

Degree of Efficiency Desired.—An economic study of the problem will determine a value for this factor. The flood selected as the maximum to be provided for carries with it the selection of a rainfall intensity curve which will be reached or exceeded with a given estimated frequency.

Intensity of Rainfall Producing Maximum Flood to Be Provided For.—Having determined the time of concentration at any point under consideration, the intensity of rainfall producing the maximum flood can be taken directly from the rainfall intensity curve.

Frequency of Rainfall Producing Maximum Flood to Be Provided For.—Where no other data are available, such works as "Elements of Hydrology", by Meyer, will provide a source from which can be selected an intensity curve of known frequency for a given geographical location.

Time of Concentration.—Although considerable error may enter into the estimate of the time of flow over the surface, it can be rather closely estimated after the run-off has reached the lateral and main drains. Where a knowledge of the flood flow is necessary in sizing the drains themselves, the increased flow, due to the decreased time of concentration, becomes apparent.

Effect of Vegetable Cover.—An inspection and study of the water-shed will determine that part which will be improved and cleared as a result of drainage or flood protection and that part which will remain for an indefinite period as woodland. Lateral water-sheds subject to such change should be dealt with, keeping the improved condition in mind.

Degree of Improvement: Lateral Drains.—Where lateral drains are subject, within reasonable time, to extension or improvement, they should be so considered in estimating the time of flow through them.

Losses: Evaporation, Transpiration, and Seepage.—Evaporation, transpiration, and seepage are considered in giving a value to *C*, vegetable cover, type of soil, and ground slopes being the controlling factors.

Relation of Storm Center to Water-Shed.—The available rainfall record is the basis for the selection of the rainfall intensity curve. The record of a heavy downpour may or may not represent the maximum intensity of the storm, depending on the relative location of the gauging station to the storm center. The record, therefore, must be assumed to represent the average intensity to be expected. This factor, like rainfall intensity, is governed by the law of probability, and can be grouped with it.

Direction of Travel of Storm.—Where the design of structures such as dams, spillways, and bridges, is involved, the path of travel of the maximum storm should be assumed to take the direction of flow, unless long-time records disclose the fact that such a condition is not probable.

Season of Year in Which Storm Occurs.—In localities subject to freezing weather and snowfall, the combination of conditions resulting in maximum run-off should be considered.

Relation of Storm to Storm Period.—The maximum storm should be assumed to occur during or at the end of the storm period when saturation is complete.

Available Storage.—What may be a storage basin to-day may be reclaimed for other purposes on the completion of a contemplated improvement. No storage should be depended on, which will not be available for a reasonable period.

The writer feels justified, therefore, in attempting to analyze and give values to the many factors involved in the problem of run-off and flood flow. Surely the results will justify the added effort.

C. D. PURDON,* M. AM. SOC. C. E. (by letter).—Reference has been made by the author to a table of waterway areas devised by the late James Dun, M. Am. Soc. C. E., who was much interested in finding an approximate formula for the size of openings required for different drainage areas.

When Chief Engineer of the St. Louis and San Francisco Railway, he had brought this formula to the point of giving the drainage area that would be accommodated by a certain pipe, box culvert, etc. Generally, a rainfall of 2 in. per hour was assumed, but for small pipes, 4 in. was used; thus, an 18-in. pipe drained 3 acres, and a 24-in. pipe, 5.5 acres; a 3 by 4-ft. box culvert served 25 acres; and a 6-ft. arch with walls 4 ft. high, 100 acres. Open bridges or trestles were rated according to their span and depth of water—as a 12-ft. span and water 2 ft. deep for 65 acres, and 30 ft. span and 4 ft. of water for 700 acres.

Later, when Mr. Dun became Chief Engineer of the Atchison, Topeka and Santa Fé Railway, the work was continued, he being assisted by the writer. The table was intended to apply to the country adjacent to the Santa Fé, and at that time the Santa Fé Pacific (Lines West of the Rio Grande) was not included. Adjoining railways were asked for data on such openings as had proved to be about the right size, during a long term of years. Where data were not given, an assistant was sent to measure the opening and its drainage area and give a general description.

Those data were plotted on cross-section paper and a curve to envelop them was applied. This curve for areas of more than 16 sq. miles was then reduced to the formula:

$$W = \sqrt{A}\,(240 - 12\sqrt[6]{A})$$

in which, W is the area of the waterway, in square feet, and A the drainage area, in square miles. For small areas an arbitrary addition was made to allow for drift, etc.

Various adjustments were made from time to time, Missouri and Kansas being taken as a base. Illinois, Indian Territory (now Oklahoma), Texas, and New Mexico required a certain percentage to be applied. This was not intended as an absolute rule, but to be used with judgment, for instance, an absorbent soil would require a smaller opening than a rocky soil.†

* Cons. Engr., St. L. S. W. Ry., St. Louis, Mo.

† A lengthy discussion on waterways was held by the Western Society of Engineers, in which the Dun table appears. This will be found in its *Journal*, Vol. XI, No. 2, April, 1906. There also was a discussion in *The Area*, Vol. 12, Pt. 3, p. 481, and in the same volume a compilation of formulas (p. 490) and on p. 505 an application of the Dun formulas, converted into run-off by multiplying the waterway by 10 (his formulas assuming a speed of 10 ft. per sec. in a culvert or bridge), to all the streams in the United States of which run-off data were obtainable.

C. S. Jarvis,* M. Am. Soc. C. E. (by letter).—The generous response in discussions from members of the profession reflects the widespread interest in floods and their control. As aptly stated in a recent editorial:†

"In many respects the serious study of the subject may be taken as dating from the memorable Central States floods of March-April, 1913. Long before that time many individual flood problems had been investigated, and in addition engineers specially concerned with stream problems had recognized and given attention to the more general questions of floods. Nevertheless, the common tendency was to look upon floods of disastrous size as catastrophic happenings, like earthquakes or lightning strokes, and the attitude of engineers was greatly influenced thereby. Each community not yet visited by a large flood was inclined to believe itself immune. Only after 1913 did it come to be generally understood that flood danger exists along every water-course and that there is enough system in the attendant circumstances to warrant abandoning the fatalistic attitude of waiting for a flood to come, and substituting therefor scientific study."

Such a view is quite opposed to the theory that "every river is a law unto itself, and unrelated to other streams even within the same area."

Just as C. E. Grunsky, Past-President, Am. Soc. C. E., has defined‡ a simple relation between inches of rainfall and the percentage that will appear as run-off, it seems reasonable similarly to seek the relation between drainage areas and their discharges under assumed or known rainfall intensities and surface conditions within the water-sheds. Variable coefficients have long been used to account for those functions, as illustrated by Talbot's formula; and in Kuichling's and Fuller's formulas the time element is included.

Run-Off Expressed as Percentage on the Myers Scale.—From the fundamental equation of the paper:

$$Q = a\,V = 10\ 000\ \sqrt{M}$$

or,

$$q\,M = 10\ 000\ p\ \sqrt{M}$$

Dividing by $100\ \sqrt{M}$ there results,

$$100\ p = \frac{q}{100}\ \sqrt{M}$$

which gives the numerical value of the percentage; that is, the product of the run-off, in second feet per square mile, times the square root of the drainage area, in square miles, divided by 100, is the desired number. A single setting of the slide-rule affords a solution.

The ease with which such ratings may be ascertained is one point in their favor. For example, the Miami River, at Miami, Ohio, during the flood of March, 1913, had $q = 98$, and $M = 3\,957$. Then,

$$100\ p = \frac{q}{100}\ \sqrt{3\ 957} = 61.7\%$$

as the rating for the discharge of that stream in terms of the modified Myers formula (the "fundamental equation" just noted).

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* Care, U. S. Bureau of Public Roads, Washington, D. C.

† *Engineering News-Record*, June 4, 1925, p. 919.

‡ *Transactions*, Am. Soc. C. E., Vol. LXI (1908), p. 514, and Vol. LXXIX (1915), p. 1165.

At one stage of the writer's investigations the desired equation appeared to have the form, $x\ y = C\ k$, in which, C is a variable coefficient and k is a constant; but this equation was found to have only limited or local application. In small drainage areas of the Western States, affected by intense rainfall, it seems to have been demonstrated that up to certain limits the maximum flood is a constant irrespective of area. This may be explained by considering the usual storm, with its vortex traversing a path only a few miles in width at most. If from 30 sq. miles the run-off amounts to a depth of nearly 1 in. per hour, the maximum flood wave will register 18 000 sec-ft. Investigators in Wyoming have concluded that such a maximum seems to prevail for areas from 1 000 to 4 000 sq. miles. Reduced to the Myers scale these floods would rate 5.7% and 2.8% at the lower and upper limits, respectively. These values occur quite frequently throughout the Rocky Mountain District for both large and small areas.

The same tendency toward a constant maximum for varying areas may be observed by reference to Items Nos. 41 and 58 in Table 2. The flood in question resulted from a precipitation estimated at 2 in. in 3 hours, over an area of 15 sq. miles which included only minor parts of the Honey Creek watershed, yet yielded a run-off at a greater rate than that which obtained during March, 1913. The maximum intensity when reduced to the Myers scale is seen to be 57.2%, as compared with 61.7% at Miami during 1913. If properly interpreted, the observations during 1918 might be an index of possible floods in this region. The extensive investigations carried on by the Miami Conservancy District resulted in a design based on a run-off 40% greater than that recorded for 1913. This would rate at various critical points in the valley from 50 to 86% on the Myers scale.

As a practical application, the same intensity should be expected for the White River floods at Indianapolis, Ind., with such modifications as the dissimilar topographic, surface texture, and climatological factors may warrant. After diligent research the designers of the protective works at Indianapolis adopted as a basis a flood stage about 4 in. higher than that of 1913. The Pueblo, Colo., flood of June, 1921, rated 15.2% for the total drainage area, or 28.5% for the part affected by the intense storm; the basis of design for the new channel is 60% in excess of these values, even though they might be regarded as indicating a 100-year, or even a 1 000-year, flood on well-known formulas.

Design of Drainage Structures for Highways.—Due to the ability of highway structures to withstand occasional overflows for brief periods without serious damage, more latitude is usually accorded in designing them than in railroad or municipal practice; thus, the 10-year maximum is often prescribed as the basis of reckoning in highway work. The trend seems to be toward using the rare flood as the datum, not necessarily in providing full capacity under the main structure, but rather using by-pass spillways corresponding with flood channels. This method precludes the possibility of a high embankment failing suddenly and without warning, as at Monterey, Mexico (see Item No. 493, Table 2). In that instance, it was reported that the loss of

human life exceeded 5 000, or double the casualties in the Johnstown flood. The sudden failure of the high embankment forming the bridge approach and impounding dam literally engulfed the peon districts of the city. Rare flood heights should be traced out so that they would be known well in advance and warnings issued accordingly.

The present situation in practice is well illustrated by the highway bridge specifications of the State of Ohio, approved June 10, 1925, in which Talbot's formula is used for small areas, and Fanning's and Kuichling's for larger drainage basins. These formulas are shown in Fig. 18 by the broken curves. The modified Myers curves of 5, 10, and 50% are also platted, together with the observed maximum flood data from Table 2 and Item No. 107 for Ohio; also representative maxima from California, Colorado, New York, and North Carolina. This demonstrates that the modified Myers formula may be substituted for a group of unrelated formulas, or may serve as a common denominator and a scale of comparison for them. Also, it applies equally well to any district if the coefficient is properly chosen.

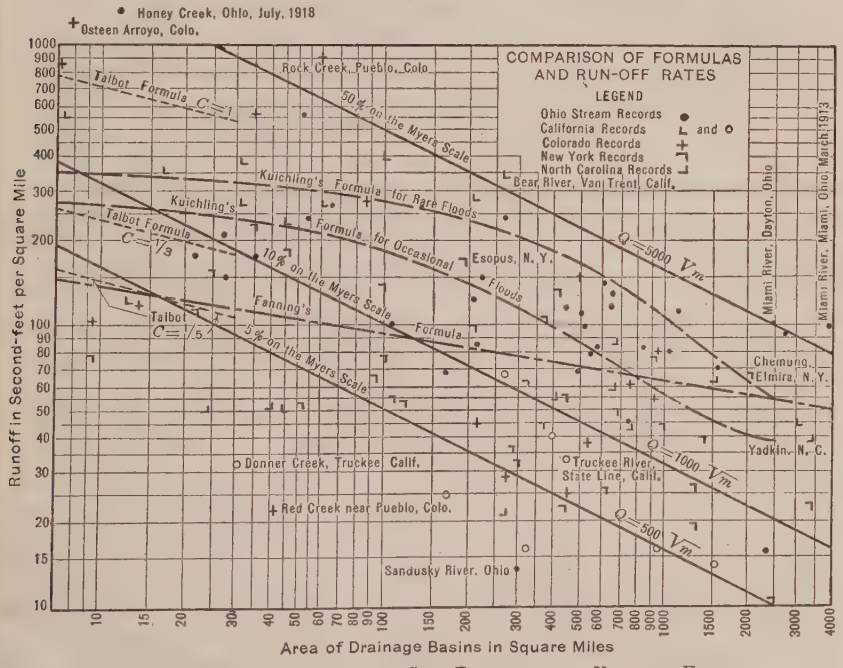

FIG. 18.—RELATION BETWEEN RUN-OFF RECORDS AND VARIOUS FORMULAS.

The North Carolina State Highway Department has used the same method for Talbot's formula,* with C ranging from 0.2 on the coastal plains to 1.0 in the mountains. Reduced to the Myers scale, these ratings would be from 10% to 35%, as shown on Fig. 19.

Need for a General Formula.—The need for a general formula was evidently recognized by the American Railway Engineering Association, for its

* *American Highways*, July, 1925, p. 8.

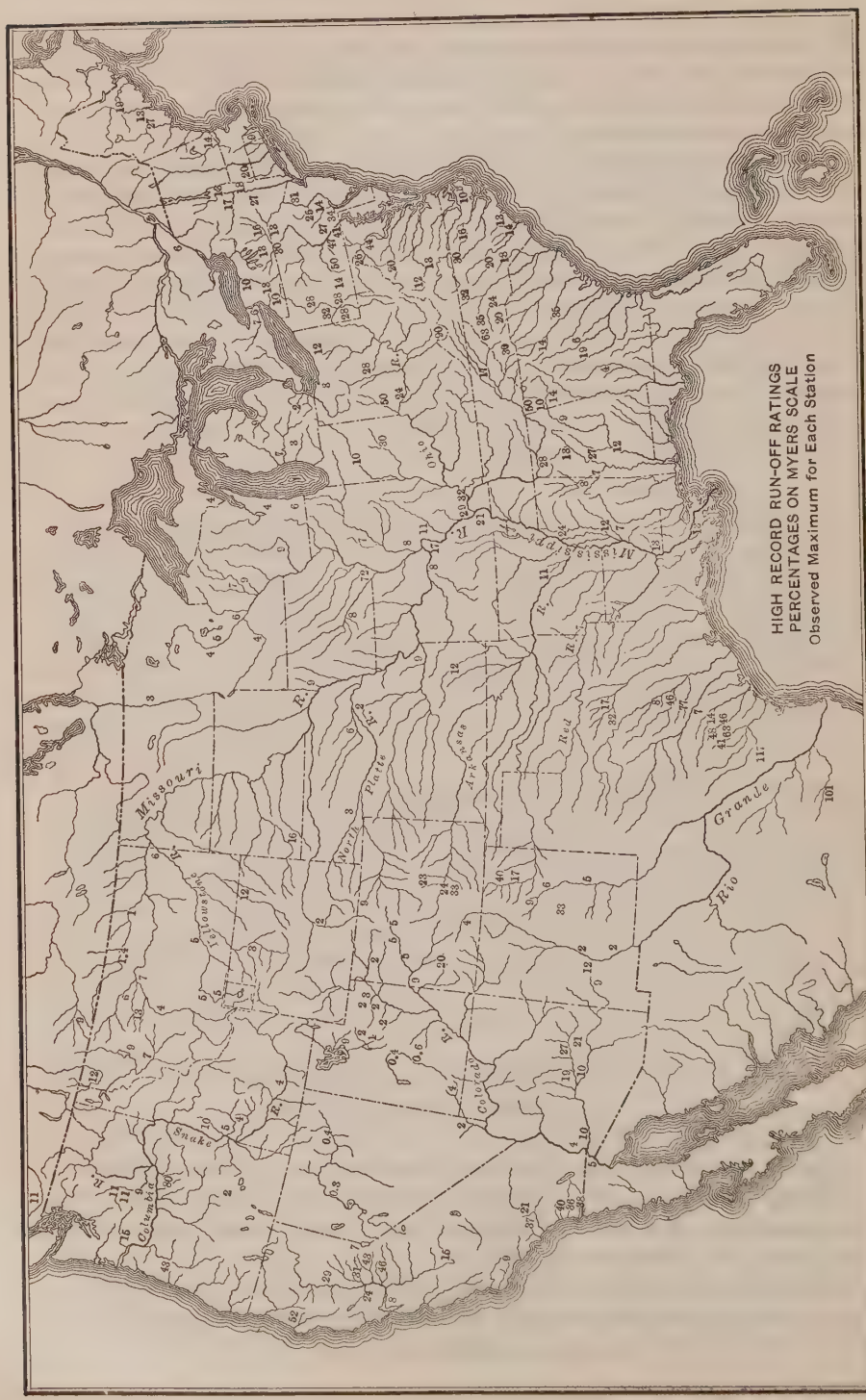

FIG. 19.—MAXIMUM OBSERVED RUN-OFFS (Q), IN TERMS OF PERCENTAGES OF MYERS' FORMULA ($10\,000\sqrt{M}$).

Special Committee seeking such an expression made progress reports in both 1909 and 1911.* Its final conclusion was that no existing formula could be recommended for general application, but that Dun's tables best represented the relation between drainage area and run-off. It is significant that the average values of these tables rate 20% on the Myers scale. Specifically, they are 12% at 1 sq. mile, 21% at 7 sq. miles, and 19% at the upper limit, 6 500 sq. miles.

Applying a General Formula.—As Chief Engineer of the Santa Fé Railroad System, the late James Dun, M. Am. Soc. C. E., found it necessary to increase or reduce his tabular values before applying them to certain districts through which he operated, so that they rate for practical purposes from 5 to 50% on the Myers scale.

Other curves in Fig. 6 which show close parallelism in their upper ranges are those of Kuichling and Murphy, while that of Metcalf and Eddy will rate 10% at 10 sq. miles and 13% at 1 000 sq. miles, and gradually increase to 14% at 10 000 sq. miles. In effect, therefore, all these formulas involve the square root of the drainage area as an important factor, although this function may not be explicitly stated. Those with greater or less slope may be excellent for a limited range, but have to be discarded when they digress from the trend of the populated area as shown on Plate IX, and others must be utilized to cover the remaining territory, as shown in Fig. 18.

Simple tests are afforded by the river diagrams in Figs. 20 and 21. If the slope of the Myers curve is correct, it should parallel the graphs of typical streams. Then the determination of the rating at any section would provide an index for all other points along its course. Thus, the Monongahela and the Ohio are fairly consistent in ratings from 25 to 35%, likewise the Sacramento and Columbia (Fig. 21). The abrupt changes of slope for the Sweetwater and the Pequannock (Fig. 20) herald the influence of the humid coastal regions in the one case, and of channel storage and flood-plains in the other. The Willamette River increases very definitely as it invades the moist valley near Portland, Ore., and the Truckee has paid toll to the desert (Fig. 21).

Discussions.—Mr. Hartwell deplores the apparent neglect of the time element. This factor is involved with others, additive or subtractive, in the value of the coefficient to be adopted in any given instance.

Mr. La Rue rendered valuable service both before and after the paper was written, drawing from his wide experience, subjecting the formula to practical tests, and lending untold encouragement to continue the tedious quest. He points out that the maximum flood of the Colorado River at the Laguna Dam is only 80% of that registered at Lees Ferry from only 58% of the drainage area. This is only natural, for the greater channel storage and percolation losses, together with the reduced slopes and desert conditions below the Grand Canyon, would smooth out the flood peak. These factors warrant the change in the coefficient, C, from 760 to 460, or the rating on the Myers scale, from 7.6 to 4.6 per cent.

* "Bridge Engineering", by J. A. L. Waddell, M. Am. Soc. C. E., Vol. II, p. 1111.

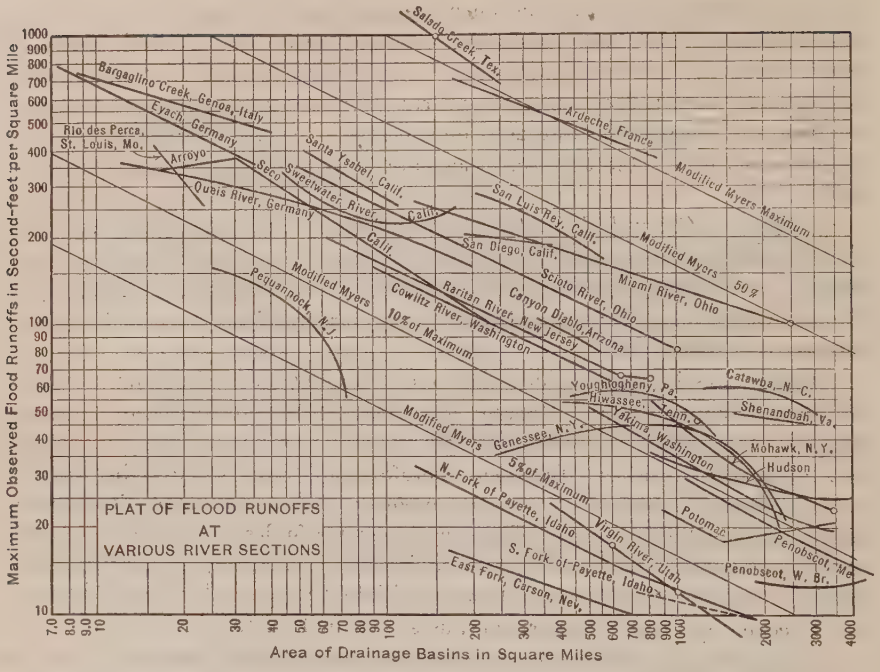

FIG. 20.—COMPARISON OF RIVER DISCHARGES.

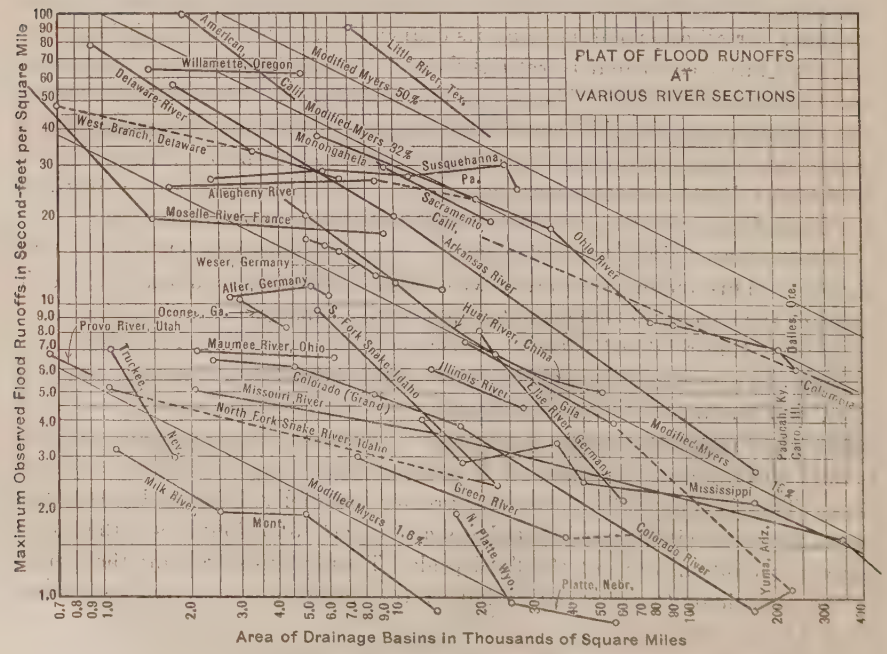

FIG. 21.—COMPARISON OF RIVER DISCHARGES.

Mr. Grover has pointed out the limitations that must be observed in the use of any general or special formula dealing with natural phenomena. In the course of his official duties he secured through the District offices a careful checking of all the flood data in the paper derived from the U. S. Geological Survey, and completed all records to date as incorporated in Table 2. Under his guidance the Society's Special Committee on Flood-Protection Data should evolve some systematized data from the chaotic mass that now exists. The writer's efforts have been co-ordinated wherever practicable with the plans of this Committee; he realizes that the task of digesting the available records must be accomplished, if at all, by some such agency.

Mr. Okazaki calls attention to the inevitable results of diking and channel rectification without due regard to by-pass functions and lateral storage. There are meadows and other lands that would derive positive benefit from inundation by a turbid crest, yet they are enclosed by great levees which insure the maximum severity of flood stage for the lower valleys.

Mr. Sherman gives valuable suggestions regarding the elements of flood frequency and rainfall intensity. The highway bridge specifications of the State of Ohio, approved June 10, 1925, have included such basic data as the following:

"Thirty-three weather bureau stations in Ohio selected at random, covering a period of observation averaging 24 years, show that a rainfall exceeding 2½ in. in 24 hours and 3 in. in 48 hours occurs more frequently than once in 5 years, also, that a 3½-in. rainfall in 24 hours and a 4-in. rainfall in 48 hours occurs more frequently than once in 20 years.

"Also six stations at which hourly rainfalls have been observed for an average period of 40 years show that a 1-in. rainfall occurs more frequently than once every 2 years, and a 2-in. rainfall (per hour) more frequently than once each 16 years."

The importance of such data as a basis for analysis and scientific design is being recognized as never before.

Mr. Winsor has doubts as to the applicability of any formula to the mud slides and rolling débris which occasionally precede a flashy flood. His careful study of this subject and his direct observations during the storm of August, 1923, along the Wasatch Range, Utah, lend additional weight to his opinions, yet, somehow, a bridge engineer must design a structure that may span such a grinding torrent. The photographs of flood effects from that storm (Figs. 3, 4, and 5), were kindly contributed by Mr. Winsor for publication with the paper.

As illustrating how a formula may apply to certain phases of desert runoff as well as to the wooded areas, consider the floods in the Rio Grande at El Paso, Tex., during the early part of August, 1925. On July 30 and 31, the U. S. Weather Bureau observed the basic data on which Fig. 22 is based. Precipitation as great as 6.3 in. occurred over a great area, centering at Hermosa, N. Mex., 30 miles west of the Elephant Butte Dam, and averaging a depth of 3 in. over 1 000 sq. miles, or more. Part of the drainage was toward desert sinks, and nearly one-half of that in the Rio Grande Valley discharged above the dam; yet about 10 800 acres were flooded when the smoothed-out

crest reached El Paso, with an estimated damage of $100 000, despite all efforts at control.

Table 9 is based on data* supplied by L. M. Lawson, M. Am. Soc. C. E., Project Superintendent, U. S. Reclamation Service, El Paso, and shows not only the desilting and flood protection produced by ample storage, but also the sixfold increase of minimum dependable yearly flow. The last three columns have been added by the writer.

The dangers outlined in the paper under the sub-title, "The Wooded Channel" (page 999), were exemplified in the havoc wrought near El Paso from a flood crest of 8 400 sec-ft., or nearly one-half the previous record.

Is it merely a coincidence that the maximum ratings on the Myers scale for San Marcial and El Paso are so nearly comparable, when the active drainage areas are considered? If this method of comparison means anything, the plane of a 20 000-sec-ft. stage should be traced out below El Paso, so that the limits of the danger zone may be known well in advance in case the recent storm intensity should be repeated, with the center at Hillsboro, N. Mex., instead of Hermosa, or else a storage reservoir should be established for the joint purpose of flood control and power development below Elephant Butte.

On July 24, 1924, the intensity of a local storm near Hondo, N. Mex., about 100 miles east of Elephant Butte, equalled or exceeded the one recorded in Fig. 22. The writer had occasion to observe the effect on a newly constructed Federal Aid road project, and reported in part as follows:

"The drainage structures on 70% of the 16-mile length were overtaxed by 10%, and on the remaining mileage they were not required to perform full duty. As the crest which caused overtopping of the road grade evidently lasted only a few moments, the damage was insignificant."

The culverts were proportioned by tables comparable with Talbot's $C = 0.2$, or from 2 to 6% on the Myers scale for the areas considered, with due regard to special local conditions. Spillways or dips in the grade proved very effective in safeguarding the major structures. At one of these locations traffic was halted for a few hours; but even desert habits of run-off have proved amenable to rational design.

Mr. Hinckley and the writer have evidently traversed parallel paths of investigation and their conclusions are in close agreement. The fundamental equation of the paper may be written as:

$$Q = \frac{10\,000\,M}{\sqrt{M}}$$

This shows that the run-off from each square mile which rates 100% on the Myers scale is 10 000 cu. ft. per sec. for 1 sq. mile; 3 162 cu. ft. per sec. for 10 sq. miles; 1 000 cu. ft. per sec. for 100 sq. miles; and 316.2 cu. ft. per sec. for 1 000 sq. miles of drainage area. Where $M < 1$, the modified Myers formula becomes $Q = 10\,000\,M$ for the upper limit, or 100%; and for any rating, $Q = 10\,000\,p\,M$, as illustrated by the first five items in Table 2.

The Musi River, India, is reported to have discharged at the rate of 493 sec-ft. from each of the 862 sq. miles above Hyderabad.† The corresponding

* *Engineering News-Record*, Vol. 95, September 3, 1925, p. 372.

† Technical Repts., Miami Conservancy Dist., Pt. IV, p. 66.

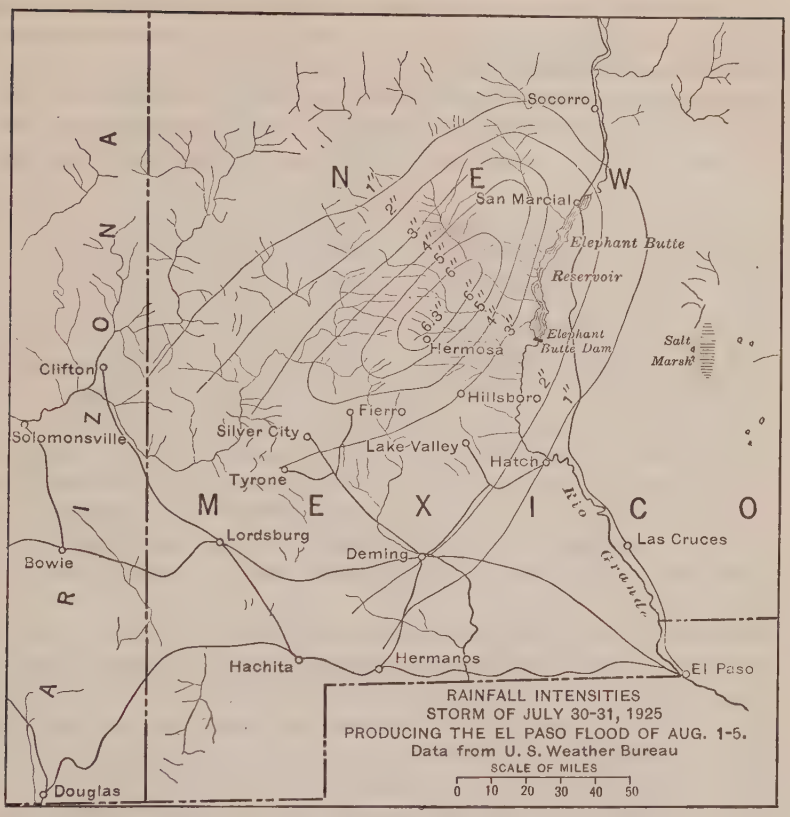

FIG. 22.

TABLE 9.—DISCHARGES AND SEDIMENT CONTENT OF RIO GRANDE AT SAN MARCIAL, N. MEX. (ABOVE RESERVOIR) AND AT EL PASO, TEX.

(Storage in Elephant Butte Reservoir began in 1915.)

Place and period.	Annual Run-Off, in Acre-Feet.			Maximum discharge, in second-feet.	Mean annual silt content, in acre-feet.	Run-off, in second feet per square mile (*q*).	Drainage area, in square miles (*M*).	Maximum rating on the Myers scale percentage.
	Maximum.	Minimum.	Mean annual.					
San Marcial, 1895-1924.....	2 422 008	201 000	1 225 727	33 000 (Oct. 11, 1904)	19 000	1.10	30 000	1.90
El Paso, 1889-1914.	2 010 000	50 700	925 400	17 000 (Oct. 15, 1904)	15 300 (1906–09)	0.45	38 000	0.89
El Paso, 1916-1925.	870 418	348 951	685 232	8 400 (Aug. 2, 1925)	480	1.05	8 000 (Below reservoir)	0.94

position on the Myers scale is 144 per cent. The four items thus far discovered that exceed the 100% limit adopted for convenience of reckoning ought not to detract from its applicability.

The discharge of 10 000 sec-ft. from 1 sq. mile is equivalent to 1 in. of rainfall run-off every 4 min., and requires special extreme conditions or coincidences, such as a deluge on melting snow, or exact synchronism of violent flood waves.

By far the majority of ratings fall below 50%; the 10% line is the axis of gravity, apparently, on Plate IX; and many streams stay consistently below 5% throughout their courses, as illustrated on Figs. 20 and 21.

Table 5 illustrates the fallacy resulting from the application of a "rule-of-thumb" method outside its proper bounds. It seems absurd to multiply the outstanding maximum flood rates by any factor such as 3.4 as there used, simply because the period of observation was unknown or indefinite in each case. The fact that each of the items listed in Table 5 has a rating so nearly approaching 100% on the Myers scale, and far exceeds any other observation in the same district, should indicate that these are maximum floods of a century or more. The recommendation in Mr. Fuller's paper evidently sought to achieve such a maximum rating by multiplying the frequent flood by a suitable factor, and did not contemplate increasing such run-offs as those listed in Table 5.

Mr. Lane has furnished an instructive elementary analysis of many factors which the writer recognized and enumerated, but could not develop without obscuring the central idea. The results derived from due consideration of the time, climatic, and physical elements will surely be expressed numerically; and if so, they will take a definite rating on the Myers scale. As an example, the rare floods at Pueblo, Colo., may rate 15% or even 25%, according to recent design data for protective works built under Mr. Lane's direction; but the frequent floods will probably rate less than 5 per cent. Between these values lies a wide range for refinements such as analytic or rational methods will produce. The Myers scale affords a direct method of first and, perhaps, second approximations, but does not take the place of the later extended research and the exercise of mature judgment. It is essentially a common denominator for diverse expressions.

Mr. Matthes has made an attractive presentation of governing principles for this subject and has furnished one of the best practical tests that could be devised for the Myers scale. The Susquehanna, Ohio, and Tennessee Rivers are shown in Table 7 to have ratings of 45.2, 32.1, and 27%, respectively, for comparable drainage areas with a century of record or observation on which to depend. It would be interesting to extend the comparison to some of the typical tributaries of these and other rivers.

As regards Mr. Matthes' suggestion for providing run-off coefficients, Fig. 19 and Column (9) of Table 2 express percentage ratings on the Myers scale. If these are multiplied by the factor, 10 000, in the fundamental equation (see page 994) there results a set of coefficients in workable form for practical use, if, indeed, the percentage ratings have not already filled this need pointed out by Mr. Matthes.

Whether statistical data are portrayed by graphs, numbers, or diagrams, the final step in comparing the similar or contrasting the divergent results will generally involve the reduction to a percentage basis. Whether the St. Lawrence River at Ogdensburg is rated at 5.8%, or has a coefficient of 580 in the modified Myers formula, in which the factor, 10 000, represents 100%, the modes of approach are identical.

Mention of the St. Lawrence flood rating of 5.8% as compared with the Mississippi's 21.2% at Cairo should call forth as definite a mental picture as could be conveyed by the charted lines of proportionate lengths on a diagram of flood intensities. Moreover, this ratio holds among their average or typical tributaries, irrespective of the area drained. Any great divergence from these ratings may be ascribed to variations in the precipitation, concentration, or detention factors peculiar to the given water-sheds, some of which are readily ascertained in advance of the actual test.

The flood problem resolves itself into a scientific and practical study of relative capabilities for discharging or detaining surface water, and also for estimating the precipitation that may occur.

The scope of the work and the arrangement of the data outlined by Mr. Matthes, as Secretary of the Society's Special Committee on Flood-Protection Data, lend assurance that its efforts are well directed. Although this promises to be one of the most fruitful fields for engineering endeavor the activities have been halted by lack of funds. From the standpoint of insurance the expenditure of one-millionth of the values at stake would be repaid many-fold during the next year of violent storms.

Mr. Kurtz has displayed the same conception of the best form for presenting the data as first appealed to the writer. This had to be deferred because of time limitations, and because the Society and the Government possess the proper agencies to evolve the elaborate analyses and comparisons. From a tangled mass of unrelated data the transformation must proceed step by step. The imperfect, incomplete, somewhat fragmentary assemblage which Table 2 represents may be most useful in showing that most of the work still remains to be done. The methods of interpolation and exterpolation used by Mr. Kurtz have proved basically sound in many cases observed by the writer.

Mr. Comstock opposes the practice, almost universally adopted, of relating the rate of run-off to the area of the drainage basin. Some standard or well-known formulas use few, and some involve many, factors of influence; but where is there an expression that omits consideration of the drainage area? This factor may be measured, and it remains nearly constant; but most of the others depend on probabilities and estimates at best.

Mr. Merriman develops the idea that Table 2 is not so much a record of floods as of maximum observed discharge. This thought was clearly impressed on the writer during the tedious process of tabulation, for many streams would rate on existing records far below the probable high flood intensity, and portrayal on the Myers scale gave a basis of comparison not otherwise furnished. The river diagrams shown in Figs. 20 and 21 result from merely connecting the points representing run-off intensities on the Myers scale for various sec-

tions. The Mohawk River is there shown to yield only about one-third of the Miami discharge from an equal area.

The Pueblo flood of 1921 taught engineers more about Arkansas River run-off capacities than did the data from previous decades, and multiplied the run-off given by previous records by the factor, 9. Likewise, the maximum observed on the Santa Ana River, in California (Item No. 301, Table 2) has increased more than fivefold during the past ten years and the San Gabriel (Item No. 331, Table 2) has quadrupled its rating. Others have doubled since the paper was first written. On the other hand, the great mass of records, probably 90%, has undergone only minor changes, if any, in twenty years.

If Esopus Creek might be used as an index to the Catskill water-shed the record run-off of September, 1924, ought not to be regarded as a radical upsetting of data. It rated only 23.9% on the Myers scale (Item No. 305, Table 2), as compared with 27.0% for the same stream at Saugerties, N. Y. (Item No. 437, Table 2). The Chemung, at Elmira, N. Y., reached 30.4% in 1889 (Item No. 700, Table 2), and the Mohawk rated 16.8% at Cohoes, N. Y., while the Hudson was 17.9% at Mechanicsville, N. Y., during 1913.

The Schoharie adjoining the Esopus water-shed reached only 18.9% at Prattsville, N. Y.; but with a similar storm centering differently there seems to be no insurance against its rating 25% or more. The same might apply to the Catskill Creek Basin or the Rondout as described and illustrated in the discussion by Mr. Honness.

After scanning the percentages recorded on the map in Fig. 19, the basic data in Table 2, and the information on Fig. 18, a generalization might be ventured regarding the St. Lawrence drainage from Wisconsin to New York: It seems to range below 15% on the Myers scale, averaging nearly 6%, the same as the Niagara River. This may be accepted as a first approximation, and further research may show that capacities of one-half or even one-tenth would suffice on several streams in this area. Furthermore, it may be deduced by the same process that the Mississippi drainage in the same States would rate double or, in some cases, five times those determined for the St. Lawrence water-shed. The disparity is evidently not in rainfall but is due in part to the presence of lakes among glacial débris in the one area, and the increased slopes in the other.

The writer must differ with Mr. Merriman's views regarding the reliability of flood flow data prior to 1900, believing that there is much to be gained from even the crude approximations which necessarily served on the frontier. No doubt, present methods will be regarded as inadequate by later investigators. However, there is thorough agreement as to the value of Nature's historical monuments along each water-course. The rings denoting annual growth of timber may by their varying widths indicate the trend of climatic changes and the extremely dry or humid years of past ages. The record afforded by the gravel bars, detrital cones, and laminated sediment should be interpreted by expert geologists. Otherwise, many signs may be misinterpreted; for example, the back-water effects from a temporary impounding dam at a junc-

tion, or a landslide such as occurred on the Gros Ventre River, Wyoming,* June 23, 1925, or the successive stages occupied during the process of erosion.

Mr. Fuller rendered his first service to this paper when he published his notable work on "Flood Flows"† in 1914. Mr. Hazen's discussion of that paper included a map‡ of the United States with Fuller's coefficients recorded on various streams; and when the writer observed that Central Texas was rated at less than one-tenth of what prevailed in the Pacific Northwest, he began his definite quest for a basic formula capable of more satisfactory comparison. The paper is without doubt a composite of the experience, observations, and analyses of many in the field of research; and among the prominent influences are the works of Messrs. Fuller, Hazen, Metcalf and Eddy, and the late Emil Kuichling, M. Am. Soc. C. E. Acknowledgments have been made wherever practicable, but there are numerous factors which cannot be defined.

The only reply that will be ventured to the objections outlined by Mr. Fuller regarding the slope of the Myers formula is the suggestion that his own curves be platted on Plate IX for any small-area floods worthy of the name, and that he observe how far above the large-area floods these lines will fall.

Mr. Parsons evidently has understood the writer's purpose more clearly than most readers, for his suggestion as to an additional factor, K, to represent the influence of physical and other characteristics of drainage areas coincides with the original conception of a percentage scale on which all streams may be rated. It would give a product, $CK = C'$; or in other words, a coefficient in which are combined and reflected all the factors and influences that claim consideration.

Mr. Purdon rightfully speaks with authority regarding the development of Dun's waterway table, because of his participation in that achievement. His statements confirm accounts hitherto published. The resulting tabulation, based on field observations along the Santa Fé Railroad System from Illinois to New Mexico, almost coincides with an element rating 20% on the Myers scale derived from a study of flood flow from all continents.

The application of the Dun formula referred to by Mr. Purdon indicates the hypothetical velocity which would have prevailed in each case if the maximum observed run-off had occupied the entire waterway prescribed by the table. The computed velocities range from 0.02 for Item No. 330§ to 26 ft. per sec. for Item No. 156, instead of the assumed rate of 10 ft. per sec. In view of the wide departure from the assumed rate of flow displayed by that test, it appears that the subtractive term in the original Dun formula may be neglected because of its slight influence; computations show that for drainage areas between 8 and 6 500 sq. miles, this effect is less than 3% on the modified Myers formula, 20% rating. This proves that the Myers and Dun expressions are fundamentally similar, and that they differ mainly in scope.

* *Engineering News-Record*, Vol. 95, September 17, 1925, p. 467.
† *Transactions*, Am. Soc. C. E., Vol. LXXVII (1914), p. 681.
‡ *Loc. cit.*, p. 629, Fig. 8.
§ *Proceedings*, Am. Railway Eng. Assoc., 1911, Vol. 12, Pt. 3, p. 505 *et seq.*

The discussions may be the source of the greatest benefit derived from this paper. The writer gratefully acknowledges the contributions of Messrs. Follansbee, Honness, Cross, Horton, and Bernard. Although these discussions do not call for special comment, the data contained therein have added greatly to the value of the paper. Thanks are also due to the many engineers—in the U. S. Geological Survey and elsewhere—who have so kindly checked and corrected the statistics regrading flood flow. Table 2 has been corrected accordingly and as here published represents the data in their most complete and up-to-date form.

The task of segregating and digesting the available data should be carried on, not by individuals who will duplicate each other's efforts, but by a well-equipped organization such as the Society's Special Committee on Flood-Protection Data, working hand in hand with the U. S. Geological Survey.

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Paper No. 1590

RELATION OF THE OHIO RIVER AND ITS TRIBUTARIES TO TRANSPORTATION IN THE UNITED STATES

A SYMPOSIUM*

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* Presented at the Annual Convention, Cincinnati, Ohio, April 22, 1925.

THE RELATION OF THE OHIO RIVER AND ITS TRIBUTARIES TO TRANSPORTATION IN THE UNITED STATES

By C. W. Kutz,* M. Am. Soc. C. E.

The Ohio River extends from the confluence of the Allegheny and the Monongahela Rivers at Pittsburgh, Pa., to the Mississippi River at Cairo, Ill., a distance of 968 miles. Its width varies from less than 900 ft. to nearly 6 000 ft. and its slope from 11½ in. per mile in the upper reach, to 3 in. per mile near the mouth.

The more or less uniformly changing slope has one break at the Falls of the Ohio (Louisville, Ky.), where there is a low-water drop of 26 ft. in a distance of 2 miles. The low-water discharge varies from 1 100 sec-ft. at Pittsburgh, where the water-shed is 18 970 sq. miles, to 27 500 sec-ft. at the mouth, where the water-shed is 203 900 sq. miles, and the maximum high-water discharge (estimated) varies from 440 000 sec-ft. at Pittsburgh to 1 500 000 sec-ft. at Cairo. The extreme fluctuations of stage at different points along the river due to floods, and the year in which they occurred, are indicated in Table 1.

TABLE 1.

Place.	Highest recorded stage above low water, in feet.	Year.	Place.	Highest recorded stage above low water, in feet.	Year.
Pittsburgh, Pa.....	35.5	1907	Portsmouth, Ohio....	67.9	1913
Beaver, Pa.....	47.1	1907	Cincinnati, Ohio....	71.1	1884
Steubenville, Ohio....	48.6	1884	Louisville, Ky. (below Falls).....	70.0	1884
Wheeling, W. Va.....	52.1	1884	Paducah, Ky.....	54.3	1884
Parkersburg, W. Va..	58.9	1913	Cairo, Ill.....	54.8	1913
Point Pleasant, W. Va.	68.8	1913			

In its original condition, the river, in addition to the Falls at Louisville, was obstructed by snags, rocks, and bars composed of sand and gravel. The minimum depth over bars at extreme low water was about 1 ft. in the upper, and about 2 ft. in the lower, section. In some of the intervening pools the depths were as great as 30 ft. In 1825, just 100 years ago, the Legislature of Kentucky empowered a stock company to construct a canal around the Falls of the Ohio with suitable locks, docks, and basin. It was completed in 1830. In addition to a canal 64 ft. wide, there was a flight of three locks of equal lifts at the western end, each chamber having a width of 50 ft. and a usable length of 185 ft. The Federal Government became a stockholder in this company in 1826. Tolls were charged and the project was a profitable one. In 1874, the Government acquired complete title and tolls were reduced from 50 cents to 5 cents per ton. A few years later it was made a free waterway.

* Col., Corps of Engrs., U. S. A., Cincinnati, Ohio.

Until 1879 work elsewhere on the river was limited to the removal of obstructions and the building of contracting dikes with a view to securing a low-water depth of 3 ft. In this year Congress authorized the construction of a lock and dam about 5 miles below Pittsburgh, which proved to be the beginning of a slack-water system for the entire river. As first proposed the canalization project contemplated a depth of only 6 ft. and covered a distance of only 90 miles. Even as late as 1896, it was limited to the reach above Marietta, Ohio, 174 miles in length.

As now formulated the project provides for a navigable depth of 9 ft. throughout by the construction of fifty low-lift dams, each with a lock chamber 110 ft. wide and 600 ft. long in the clear. To prevent any increase in flood height and to permit open river navigation when the natural depth is 9 ft. or more, all the dams are of the movable type except the one immediately below Pittsburgh. This is of recent construction and replaced Dams No. 1 and No. 2 as originally built.

The main part of each movable dam, through which navigation passes when the dam is lowered, is known as the navigable pass. It is from 600 to 1 000 ft. long and consists of a series of wickets or shutters, 3 ft. 9 in. wide and 16 to 20 ft. long, depending on the lift of the dam, each supported in an inclined position by a prop. The dam is lowered by means of a derrick-boat, known as a maneuver-boat, the props being released from their seats one by one and the wickets lowered so as to lie flush with the bottom of the river. In addition to the lock and navigable pass there are weirs of different types for the purpose of regulating the pools and facilitating the raising and lowering of the navigable pass.

Of the fifty dams, thirty-eight are completed and in operation, and the remaining twelve are under construction. Of these, four are nearly completed and on two work of a preliminary character only has been done. The reach from Pittsburgh to Cincinnati, Ohio, will be completed in 1925 and the last dams should be completed in the season of 1929. This relatively long construction period is due to the fact that the various parts of a lock and dam are built within coffer-dams, designed to hold out the river to a stage of 16 ft. When the river rises above this stage, work must be suspended until the water subsides. With this limitation, the working season rarely exceeds 5 months—July to November, inclusive—and sometimes is as short as 100 working days. Under normal conditions, 4 years are required for the construction of a lock and dam, and, in the lower river where the dams are longer, 5 years will be required.

Tributaries.—In Table 2 are listed the principal navigable tributaries of the Ohio with information that tends to show their relative magnitude and the nature of the improvements made to date. Navigable sections of those rivers not now connected with the Ohio are not included in Table 2. An exception to this rule is made in the case of the Cumberland River, the canalized part of which will not have a 12-months connection with the Ohio until Dam No. 52, Ohio River, is completed. On the Allegheny, Big Sandy, Cumberland,

and Tennessee Rivers, where authorized projects are incomplete, only the completed sections at the lower end are given.

The lack of uniformity in lock dimensions is partly accounted for by the fact that on five of these tributaries, the canalization projects were first undertaken by the States or by private capital and subsequently acquired or taken over by the Federal Government, but even where not thus hampered, there was an apparent failure on the part of the Government to standardize lock dimensions to the extent that now seems desirable.

TABLE 2.—PRINCIPAL NAVIGABLE TRIBUTARIES OF THE OHIO RIVER.

Mouth, in miles below Pittsburgh.	Name.	Drainage area, in square miles.	Low-water discharge, in second-feet.	Length canalized, in miles.	Number of dams.	Project depth, in feet, canalized section.	Dimension of locks, in feet.	Notes.
0	Allegheny	11 580	1 440	24	3	7	55 by 286	Locks Nos. 2 and 3 slightly larger.
0	Monongahela	7 390	160	101 30	8 6	8 7	56 by 360 56 by 182	This assumes completion of new Locks and Dams Nos. 7 and 8 now being built to replace old Nos. 7, 8, and 9. Locks Nos. 1 to 6 have double lock chambers. At Lock No. 3 one of the lock chambers is 720 ft. long.
172	Muskingum	7 797	672	91	11	5.5	36 by 160	Lock No 1 is 56 by 360 ft.
184	Little Kanawha	2 288	....	48	5	4	22.7 by 125	One lock is 26 ft. wide.
265	Kanawha	12 200	1 100	88	6 4	6 6	55 by 313 50 by 271	.....
317	Big Sandy	4 182	100	55	5	6	55 by 158	Lock No. 3 is only 52 ft. wide. Three locks on main river and one on each fork.
543	Kentucky	5 904	100	270	14	6	37 by 145 52 by 148	.....
779	Green	13 000	....	217	7	5.5	35.5 by 137.5	One of the locks is on Barren River, a tributary.
910	Cumberland	17 758	1 100	333	14	6	52 by 280	.....
924	Tennessee	40 569	10 000	256	*	6	80 by 287	Florence, Ala., to mouth only.

* One lock and lateral canal, 8 miles long.

Table 3 shows the first cost of the existing river improvement projects in the Ohio Basin and the cost of operation and maintenance. In the case of the Ohio River, the amount given is 80% actual cost and 20% estimated. On the Monongahela River, the figure includes the estimated cost of Locks and Dams Nos. 7 and 8, now under construction, to replace old Locks and Dams Nos. 7, 8, and 9. In other cases the cost is the historical cost as shown by the Annual Reports of the Chief of Engineers, U. S. Army. The costs of operation and maintenance are compiled in a similar manner. In Column (4) of Table 3 is shown the cost per mile and, in Column (5), interest on the first cost at 4 per cent. Column (7) shows the total fixed charges and Column (8) the fixed charges per mile.

Commerce.—Table 4 shows, for the Ohio and for the tributaries listed, the commerce for certain selected years as taken from the Annual Reports of the

Chief of Engineers, U. S. Army. The year 1900 was selected as marking the peak of water-borne commerce on the Ohio, and the year 1913 as typical of conditions just prior to the World War. Present conditions are shown by the tonnage reported for the calendar year 1924, and by the average for the period 1919-24, inclusive. The approximate ton-mileage in 1924, the fixed charges per ton-mile, and the cost of maintenance and operation per ton-mile are also shown.

TABLE 3.—OHIO RIVER AND THE CANALIZED PARTS OF ITS PRINCIPAL TRIBUTARIES.

River.	Length, in miles.	Cost.	Cost per mile.	Interest on first cost at 4%	Cost of operation and maintenance, average for 5 years.	Fixed charges, Column (5) + Column (6).	Fixed charges per mile.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Ohio	968	$103 000 000	$106 350	$4 120 000	$2 000 000	$6 120 000	$6 320
Allegheny	24	1 446 000	60 200	57 840	136 080	193 920	8 075
Monongahela	131	13 612 000	103 900	544 480	1 185 500	1 729 980	13 200
Muskingum	91	1 452 595	15 963	58 104	76 670	144 774	1 591
Little Kanawha	48	282 000	5 800	11 280	29 857	41 137	857
Kanawha	88	3 580 000	40 700	143 200	166 160	309 360	3 515
Big Sandy	55	1 512 000	26 500	60 480	34 707	95 187	1 731
Kentucky	260	4 266 000	16 400	170 640	234 541	405 181	1 558
Green	217	670 428	3 100	26 817	102 770	129 587	597
Cumberland, from mouth to Lock and Dam No. 9	333	7 174 000	21 544	286 960	105 036	391 996	1 177
Tennessee below Florence, Ala	256	5 166 000	20 180	206 640	45 000	251 640	983

TABLE 4.—COMMERCE ON THE OHIO RIVER AND ON PARTS OF ITS TRIBUTARIES DIRECTLY CONNECTED THEREWITH (EXCLUSIVE OF FLOATED TIMBER).

River.	Tonnage, 1900.	Tonnage, 1913.	Tonnage, 1924.	Average tonnage, 1919–24, inclusive.	Ton-mileage, 1924.	Fixed charges, in mills per ton-mile.	Cost of maintenance and operation, mills per ton-mile.
Ohio	14 054 000	9 814 000	8 446 322	7 416 119	978 421 930	6.25	2.04
Allegheny	1 611 103	2 159 530	4 339 511	3 940 686	35 370 300	5.5	3.85
Monongahela	7 000 000	12 039 175	21 878 815	19 558 108	919 564 942	1.88	1.29
Muskingum	78 595	61 755	152 012	120 667	912 000	16.1	8.4
Little Kanawha	119 439	97 099	45 951	40 907	1 378 530	29.7	21.6
Kanawha	1 475 930	1 416 804	901 069	1 146 395	54 064 140	5.7	3.18
Big Sandy	300 000	172 403	29 497	55 274	450 780	211.0	77.0
Kentucky	106 336	211 794	195 372	177 475	40 521 667	10.0	5.9
Green	1 190 756	1 057 068	436 916	313 206	*	.....	.....
Cumberland	696 306	626 832	390 700	358 846	15 949 200	24.5	6.6
Tennessee, below Florence, Ala	.....	584 191	1 255 436	558 000	59 202 500	4.25	0.76

* Not available.

Strictly speaking this commerce is not all applicable to the sections of the rivers listed in Table 4, as, in some cases where separation was impossible, it includes all the commerce reported for the entire river, but the error thus introduced is not large. The tonnage reported for recent years is exclusive of floated timber and timber products. For earlier years such tonnage was not

separately reported. The tonnage listed includes sand and gravel much of which is transported for very short distances. To show the nature of the traffic, the reported tonnage for 1924 has been divided into four groups: Coal, sand and gravel, other bulk freight, and package freight, as given in Table 5.

TABLE 5.—Reported Tonnage of Four Commodities on the Ohio River and on Parts of Its Tributaries in 1924.

River.	Coal.	Sand and gravel.	Other bulk freight.	Package freight.
Ohio	5 274 313	1 993 278	812 241	366 490
Allegheny	841 923	3 066 969	429 873	746
Monongahela	17 749 150	2 385 099	1 686 122	58 444
Muskingum	21 343	121 526	3 202	5 941
Little Kanawha	320	14 335	7 618	23 678
Kanawha	703 555	163 768	15 340	18 520
Big Sandy	555	0	15 459	13 483
Kentucky	33 655	15 375	139 691	6 651
Green	105 918	3 339	304 085	23 574
Cumberland	95	286 746	73 196	30 669
Tennessee	18 275	962 163	209 210	65 788
Total	24 749 102	9 012 598	3 696 037	613 984–38 071 721
Percentage	65	23.68	9.70	1.62

From Table 5 it will be seen that the Ohio River System is primarily a coal carrier and in the small percentage of other freight resembles some of the principal coal-carrying railroads.

Decrease in Tonnage.—The great decrease in tonnage on the Ohio notwithstanding the improved navigation facilities is due in large measure to a change in the place of marketing the coal mined in the river mines of the Monongahela Valley. In 1901, 3 200 000 tons of coal from such mines were shipped by water to points on the Lower Ohio River. None is shipped to-day either by water or rail.

The recorded decrease is also due in part to the greater care that is exercised to prevent duplication in the collection of statistics. Some years ago it was the practice on some rivers to arrive at the total tonnage by a summation of the tonnage passing each lock. However, even with such errors eliminated, there has been on some of the rivers in this Basin an actual reduction in the amount of water-borne commerce. A marked exception to this general rule is the Monongahela River, on which the commerce has increased from 7 000 000 tons in 1900, to 12 039 175 tons in 1913, and to an average of 19 558 000 tons during the past six years. The Allegheny River also shows an increase, largely in sand and gravel which made up 75% of its traffic in 1924.

The decline in water-borne commerce has not been limited to the Ohio River Basin, but is more or less general on inland rivers. This subject was brought to the attention of the Society in 1915 in a paper by W. W. Harts, M. Am. Soc. C. E., entitled "'Rivers and Railroads in the United States."* Many reasons for the decline are set forth in that paper and in the voluminous

* *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), p. 919.

discussion thereon. Among the discussors was H. Burgess, M. Am. Soc. C. E., who stated:*

"* * * the more favorable freight rate charged by railroads on any portion of their lines for which there is a competing water route. The result of the lower rates for towns connected by a water route is that 'inland' towns must pay a portion of the rail transportation charge on freight received by river towns. These latter, therefore, receive the superior facilities offered by the rail route over carriage by water at practically what such freight would cost if carried by boat, and these superior facilities are obtained at the expense of the 'inland' towns."

In the same discussion, J. H. Bernhard, Assoc. M. Am. Soc. C. E.,† after making a series of comparisons between rail rates on lines with and without a competing water route, states:‡

"Although rebates are forbidden by State or Federal statutes, railroads grant them quite as much to-day as at any time in the past, the only distinction being that formerly they were granted to individuals surreptitiously, and now they are granted to cities and towns openly; the former became illegal, the latter is legalized. These rebates arise from the unjust practice of railroads in basing rates to river points on water competition."

TABLE 6.—Railroad Rates on Coal, April, 1925.

Place.	Kanawha District.			Pittsburgh District.			Ohio No. 8 District.		
	Distance, in miles.	Rate, in cents per ton.	Mills per ton-mile.	Distance, in miles.	Rate, in cents per ton.	Mills per ton-mile.	Distance, in miles.	Rate, in cents per ton.	Mills per ton-mile.
Columbus, Ohio	200	189	9.45	200	189	9.45	145	139	9.58
Dayton, Ohio	288	214	7.43	270	214	7.92	216	183	8.47
Toledo, Ohio	323	239	7.4	260	239	9.19	230	189	8.21
Indianapolis, Ind.	338	252	7.45	388	252	6.49	333	227	6.81
Terre Haute, Ind.	409	287	7.01	460	287	6.26	405	262	6.47
Fort Wayne, Ind.	382	270	7.07	320	270	8.43	296	220	7.43
Columbus, Ind.	317	252	7.98	...	...	....	...	...	....
Bedford, Ind.	360	252	7.00	...	...	....	...	...	....
Mitchell, Ind.	354	252	7.11	...	...	....	...	...	....
Seymour, Ind.	315	252	8.00	...	...	....	...	...	....
Vincennes, Ind.	417	287	6.88	...	...	....	...	...	....
Greensburg, Ind.	291	252	6.88	...	...	....	...	...	....
South Bend, Ind.	465	304	6.53	408	304	7.45	384	254	6.61
Logansport, Ind.	398	304	7.64	398	304	7.64	343	254	7.40
Chicago, Ill.	512	309	6.03	468	309	6.60	444	274	6.17
Peoria, Ill.	549	334	6.08	476	334	7.01	515	309	6.00
Joliet, Ill.	524	309	5.89	...	...	....	...	...	....
Mattoon, Ill.	466	334	7.16	...	...	....	...	...	....
Danville, Ill.	423	309	7.30	...	...	....	...	...	....
Cincinnati, Ohio	228	189	8.29	320	229	7.15	257	214	8.32
Jeffersonville, Ind.	358	198	5.53	450	292	6.48	382	275	7.24
Louisville, Ky.	342	189	5.52	455	292	6.41	378	275	7.27
Evansville, Ind.	497	321	6.46	589	372	6.31	533	347	6.51
East St. Louis, Ill.	564	312	5.53	609	372	6.11	586	347	5.92

To determine to what extent this practice prevails in the Ohio River Basin, existing rail rate tables have been examined and the results which are summarized in Tables 6, 7, and 8, are platted on Figs. 1, 2, and 3. On Fig. 1 is

* *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), 941.

† Mr. Bernhard resigned from the Society on April 12, 1926.

‡ *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), p. 946.

shown a series of coal rates from bituminous coal fields to points with and without river competition. A curve representing a rate of 4.5 mills per ton-mile plus $1.00 per ton, may be taken as the mean. The short hauls are comparisons between rail rates in the valley of the Monongahela which is canalized and rates in the uncanalized section of the Allegheny. These rates show

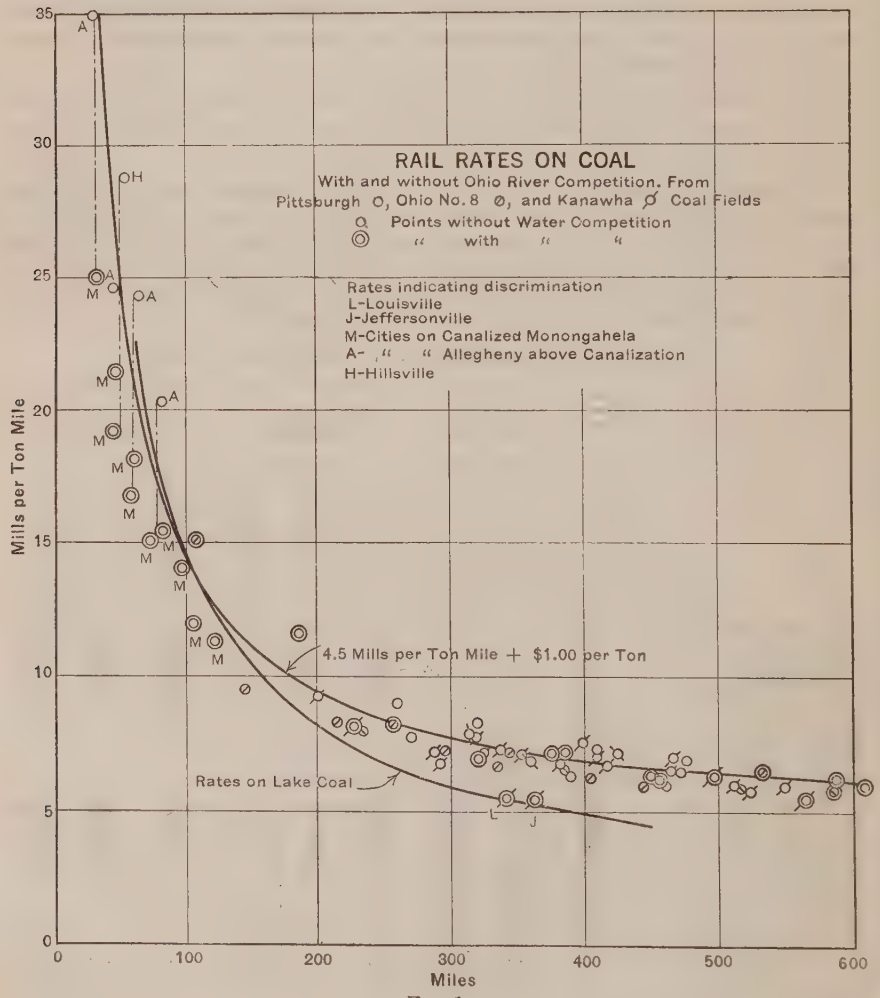

FIG. 1.

marked discrimination in favor of points with water competition, the average difference being 7 mills per ton-mile. In the longer hauls no such discrimination is apparent except on hauls from Pittsburgh to Louisville and to Jeffersonville, Ind. On the same diagram, as a matter of general interest, a line is drawn representing the rates on coal from a number of bituminous coal fields to lake ports for re-shipment on lake vessels.

TABLE 7.—Railroad Class Rates, April, 1925.

Cincinnati, Ohio, to:	Distance, in miles.	First-Class Rate.		Fifth-Class Rate.		Sixth-Class Rate.	
		Cents per ton.	Mills per ton-mile.	Cents per ton.	Mills per ton-mile.	Cents per ton.	Mills per ton-mile.
Columbus, Ohio	116	1 170	10.0	410	35.3	330	28.4
Cleveland, Ohio	244	1 550	64.0	540	22.1	430	16.8
Toledo, Ohio	208	1 460	72.0	510	25.1	410	20.2
Chicago, Ill.	285	1 610	56.4	560	19.3	450	15.7
Indianapolis, Ind.	110	1 130	10.3	400	36.3	320	29.1
Terre Haute, Ind.	182	1 420	78.0	500	27.4	400	22.0
Birmingham, Ala.	481	3 020	62.7	1 580	32.8	1 300	27.0
Atlanta, Ga.	488	3 020	61.8	1 580	32.3	1 300	26.6
Pittsburgh, Pa.	311	1 670	53.7	580	18.6	470	15.1
Parkersburg, W. Va.	195	1 450	74.3	510	26.1	410	21.0
Huntington, W. Va.	154	1 310	84.9	460	29.8	370	24.0
Portsmouth, Ohio	107	1 130	105.6	400	37.3	320	29.9
Louisville, Ky.	114	1 130	99.0	400	35.0	320	28.0
Evansville, Ind.	269	1 500	55.8	530	19.7	420	15.6
Cairo, Ill.	374	1 770	47.3	620	16.5	500	13.3
St. Louis, Mo.	339	1 700	50.0	600	17.7	480	14.1
Memphis, Tenn.	494	2 720	55.0	1 410	28.5	1 170	23.6
New Orleans, La.	836	3 740	44.7	1 940	23.2	1 610	19.2
Kansas City, Mo.	618	3 370	54.5	1 210	19.5	1 190	19.2
Nashville, Tenn.	297	2 070	69.7	1 080	36.3	890	29.9
Chattanooga, Tenn.	338	2 140	63.3	1 030	33.4	940	27.8
Knoxville, Ky.	282	2 140	75.9	1 030	40.0	940	33.3
Jackson, Miss.	674	3 420	50.7	1 780	26.4	1 470	21.8
Meridian, Miss.	634	3 420	54.0	1 780	28.1	1 470	23.1
Hattiesburg, Miss.	719	3 600	50.1	1 870	26.0	1 550	21.5
Montgomery, Ala.	578	3 040	52.5	1 610	27.8	1 330	23.0
Mobile, Ala.	751	3 740	49.8	1 940	25.8	1 610	21.4
Atlanta, Ga.	475	3 200	63.5	1 580	33.2	1 300	27.3
Pittsburgh, Pa., to:							
Cincinnati, Ohio	311	1 670	53.7	580	18.6	470	15.1
Louisville, Ky.	425	1 890	44.5	660	15.5	530	12.4
Evansville, Ind.	580	2 070	35.7	720	12.4	580	10.0
Cairo, Ill.	693	2 510	36.2	880	12.7	700	10.1
Memphis, Tenn.	805	3 240	40.0	1 680	20.8	1 400	17.3
Vicksburg, Miss.	1 026	3 940	37.4	2 050	20.0	1 690	16.4
New Orleans, La.	1 147	4 270	37.1	2 210	19.2	1 840	16.0
St. Louis, Mo.	612	2 450	40.0	860	14.0	690	11.2
Kansas City, Mo.	891	4 120	46.2	1 470	16.5	1 520	17.0
Davenport, Iowa	651	2 450	37.6	860	13.2	690	10.6
Dubuque, Iowa	653	2 450	37.5	860	13.2	690	10.6
St. Paul, Minn.	866	3 360	38.8	1 200	13.8	960	11.1
Cleveland, Ohio	132	1 230	93.1	430	32.5	340	25.7
Detroit, Mich.	316	1 670	52.8	580	18.3	470	14.8
Chicago, Ill.	468	1 950	41.7	680	14.5	550	11.7
Columbus, Ohio	191	1 450	76.0	510	26.7	410	21.4
Indianapolis, Ind.	371	1 770	47.7	620	16.7	500	13.4
Terre Haute, Ind.	443	1 950	44.0	680	15.3	550	12.4
Fort Wayne, Ind.	320	1 670	52.2	580	18.1	470	14.7
Logansport, Ind.	389	1 820	46.7	640	16.4	510	13.1
Peoria, Ill.	561	1 950	34.5	686	12.1	550	9.8
Springfield Ill.	568	2 330	41.0	820	14.4	650	11.4
Birmingham, Ala.	792	4 320	54.5	2 040	25.7	1 680	21.2
Nashville, Tenn.	608	3 140	50.0	1 630	26.7	1 350	22.2
Chattanooga, Tenn.	649	3 570	55.0	1 680	26.0	1 380	21.2
Knoxville, Tenn.	598	3 570	60.0	1 680	28.3	1 380	23.3
Jackson, Miss.	985	4 860	49.3	2 260	22.9	1 870	19.0
Meridian, Miss.	945	4 860	51.4	2 260	23.9	1 870	19.8
Hattiesburg, Miss.	1 030	4 960	45.2	2 310	22.4	1 940	18.8
Montgomery, Ala.	889	4 360	49.0	2 080	23.4	1 720	19.3
Mobile, Ala.	1 062	4 270	40.2	2 210	20.8	1 840	17.3
Atlanta, Ga.	786	4 380	55.7	2 160	27.4	1 770	22.5

On Fig. 2 a series of fifth-class rates (bulk products, carload lots) from Pittsburgh and from Cincinnati to points with and without water competition, have been platted. The results show that for points in Central Freight Association territory (that is, the territory bounded on the north by the Great Lakes, on the south by the Ohio River, on the west by the Mississippi River,

and on the east by a line drawn from Buffalo, N. Y., to Pittsburgh), the rates are based on mileage, and no difference is noted between rates to inland points and to points on the Ohio. The rates to many points south of the Ohio, however, are noticeably higher, and some distinction is made between inland points and points on the Mississippi River, such as Memphis, Tenn., Vicksburg, Miss., and New Orleans, La.

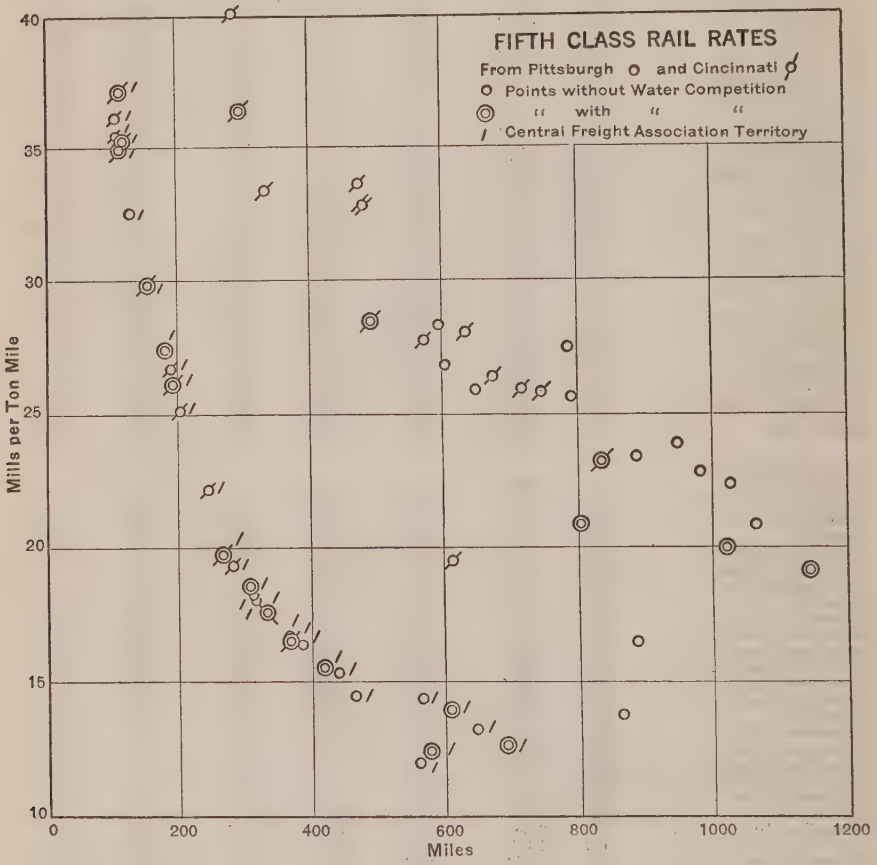

FIG. 2.

On Fig. 3 are platted a series of rail rates on finished steel products from Pittsburgh. In Central Freight Association territory such products move at fifth-class rates. In territory south of the Ohio, they move under a commodity rate which is not only much less than fifth-class rates in that territory, but is practically uniform with fifth-class rates in Central Freight Association territory. The rates indicated on Fig. 3 show no distinction between inland points and points having water competition.

There is a considerable movement of coal from the West Virginia coal fields through Cincinnati to points in Indiana. Some of this coal comes in by rail and is turned over to another railroad for delivery to destination.

Another part comes in by river, is transferred to cars, and delivered to the same railroad for delivery to the same destination, but the rates charged on the coal coming in by rail are lower than the rates charged on the coal coming in by river, other conditions being identical. This differential in favor of the all-rail movement to Indianapolis, Ind., is now 18½ cents per ton and several years ago was considerably higher.

TABLE 8.—RAILROAD RATES ON FINISHED STEEL PRODUCTS, APRIL, 1925.

Pittsburgh, Pa., to:	Distance, in miles.	Rate.		
		Cents per hundred weight.	Cents per ton.	Mills per ton-mile.
Cincinnati, Ohio	311	29	580	18.6
Louisville, Ky	425	33	660	15.5
Evansville, Ind	580	36	720	12.4
Cairo, Ill	693	44	880	12.7
Memphis, Tenn	805	56	1 120	13.9
Vicksburg, Miss	1 026	64	1 280	12.4
New Orleans, La	1 147	67	1 340	11.7
St. Louis, Mo	612	43	860	14.0
Kansas City, Mo	891	73.5	1 470	16.5
Davenport, Iowa	651	43	860	13.2
Dubuque, Iowa	653	43	860	13.2
St. Paul, Minn	866	60	1 200	13.8
Cleveland, Ohio	132	19	380	28.8
Detroit, Mich	316	29	580	18.3
Chicago, Ill	468	34	680	14.5
Columbus, Ohio	191	25.5	510	26.7
Indianapolis, Ind	371	31	620	16.7
Terre Haute, Ind	443	34	680	15.3
Fort Wayne, Ind	320	29	580	18.1
Logansport, Ind	389	32	640	16.4
Peoria, Ill	561	34	680	12.1
Springfield, Ill	568	41	820	14.4
Birmingham, Ala	792	58	1 160	14.6
Nashville, Tenn	608	50	1 000	16.4
Chattanooga, Tenn	649	50	1 000	15.4
Knoxville, Tenn	593	50	1 000	16.8
Jackson, Miss	985	64	1 280	13.0
Meridian, Miss	945	62	1 240	13.1
Hattiesburg, Miss	1 030	64	1 280	12.4
Montgomery, Ala	889	62	1 240	13.9
Mobile, Ala	1 062	67	1 340	12.5
Atlanta, Ga	786	58	1 160	14.7

With the exceptions noted, the results of the investigation seem to justify the general statement that in the movement of coal and structural steel in the Ohio River Basin, the existing rail-rate structure does not discriminate between points having water competition and points that are without water competition.

Cost of Transportation by Water, Ohio River System.—In determining the economic advisability of creating an inland waterway or in measuring its usefulness as a part of the country's general transportation system, one of the most important factors and one concerning which available information is limited is the cost of water haulage, that is, the cost of transportation by water exclusive of fixed charges. This cost is subject to the combined influence of many factors, among which may be listed:

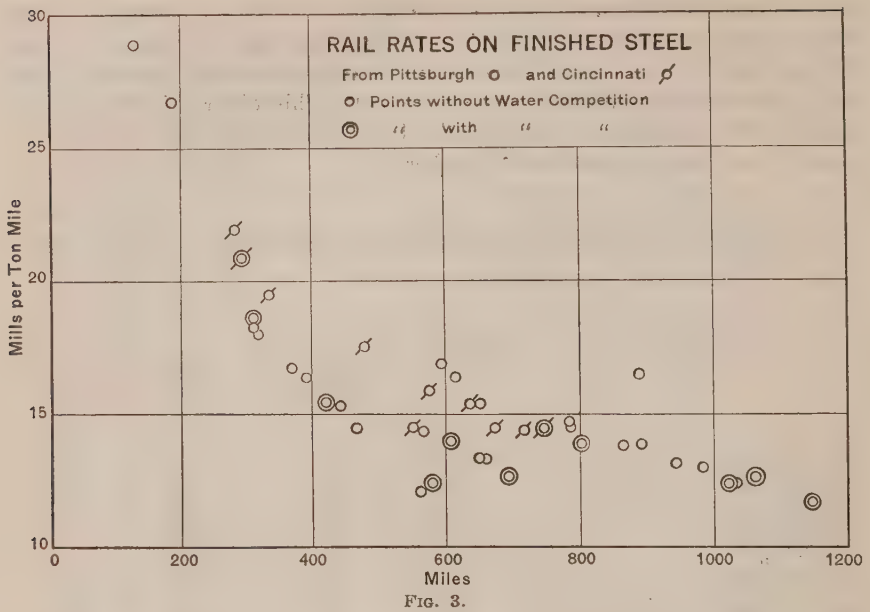

FIG. 3.

(*a*) The physical characteristics of the waterway, of which in canalized rivers the more important are the size of the locks and the depth and width of the intervening pools.

(*b*) Types of transportation service, which may be grouped as private industrial, contract industrial, and common carrier.

(*c*) The kind of freight: In the comparisons to follow only bulk freight will be considered and in some cases only coal.

(*d*) Type of floating plant: As more than 98% of the tonnage on the Ohio and its tributaries is carried on barges, open or decked, no consideration will be given to a movement of freight on packet-boats or other self-propelled carriers.

(*e*) Annual volume of tonnage: The volume of tonnage handled annually by a given operating agency affects the number of barges required per towboat, and the plant performance. On the Monongahela River, with its large volume of tonnage, a standard tow consists of six barges, each approximately 175 ft. long and 26 ft. wide, with a maximum capacity of 1 000 tons. These barges are lashed together, two wide, and pushed by a stern-wheel steamer, the overall dimensions of which are somewhat less than a standard barge. For passage through a standard Monongahela River lock such a tow must be broken into two parts. On the Ohio a fleet of eight such barges with towboat can pass through the locks. Tows with twelve to fourteen barges are not infrequent on the Ohio between the mouth of the Kanawha River and Cincinnati, and formerly even larger tows were not uncommon.

(*f*) Back haul: Very little freight is available as back haul that can be carried on barges used for the transportation of coal, sand, and gravel. Where

back-haul traffic is available, the costs of transportation are from 60 to 65% of those without back haul.

(*g*) Length of haul: This factor influences the investment per ton-mile and the plant performance.

(*h*) Co-ordination of terminal and transportation services: This factor gives private industrial operation an advantage over contract industrial or common carrier and results in lower costs.

In a discussion of the paper presented by William M. Black, M. Am. Soc. C. E., entitled "Waterways and Railway Equivalents",* C. I. Grimm, Assoc. M. Am. Soc. C. E., analyzed† waterway haulage costs on rivers similar to the Ohio, and reached the conclusion that 2.5 mills per ton-mile was a fair measure of the cost under the following conditions: 250-mile haul, no back haul, 6 000-ton cargoes in 1 000-ton barges, wide river, large tonnage, and steady movement.

The principal users of the Monongahela River have kept comprehensive cost records for a number of years. These records were furnished in confidence to the office of which the speaker is in charge and, together with the analysis made by Mr. Grimm, have been made the basis of the water haulage costs shown on Fig. 4. These costs are based primarily on tonnage moving with the current. Other conditions are shown on the diagram itself.

Rail vs. Water Rates.—Fig. 4 also shows existing rail rates on coal on roads coming into direct competition with the Ohio River or its coal-carrying tributaries. Ton-mile rates by rail and water are, however, not directly comparable, one important variation being the difference in the length of the haul.

Fig. 5 shows water and rail distances from Pittsburgh to points on the Ohio and the Mississippi Rivers. For a rail haul of 230 miles the water distance is approximately 50% greater than the rail distance and, for convenience in comparison with rail distance, a line representing this ratio has also been shown. On the Monongahela River the competing railroads parallel the river, and differences in lengths of haul are insignificant.

Another point that must be taken into consideration in any economic comparison is the difference in terminal costs. Experience on the Monongahela River indicates that it is somewhat cheaper to load into cars than into barges. On the Ohio the difference is greater, due to the greater range between normal and high-water stages. Coal cars can be unloaded from a trestle or elevated and tipped bodily, while coal barges must be unloaded by a grab-bucket. With modern unloading plants of large capacity this difference amounts to less than 5 cents per ton.

Every transfer of bituminous coal, whether loading or unloading, involves some loss due to breakage and, as this breakage is generally proportional to the drop, loading into barges has been considered as involving greater loss than loading into cars. To minimize this loss, three modern plants in Huntington, W. Va., for transferring coal from rail to barge are in the form of inclined belt conveyors. For joint hauls involving two railroads, cars can

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 538.

† *Loc. cit.*, p. 568.

be switched from one to the other at small cost, whereas in joint hauls—part river and part rail—both a transfer charge of about 10 cents per ton and a breakage loss must be absorbed.

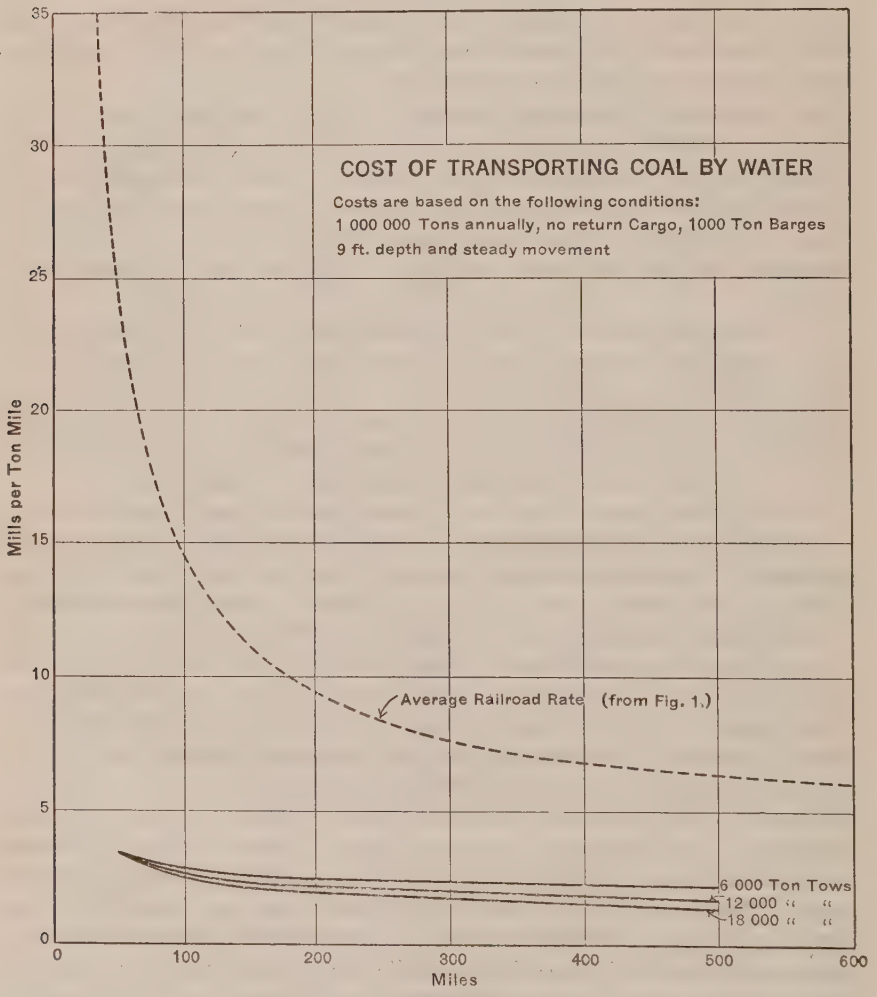

FIG. 4.

Economic Comparisons.—If the points of origin and destination and the ton-mileage of each commodity hauled on these rivers are known, as well as the water haulage costs, the direct benefits or savings resulting from their use can be readily ascertained by comparison with existing rail rates and their economic justification determined by comparing the savings or benefits with the fixed charges. Although neither the traffic data nor the water-haulage cost data are sufficiently comprehensive for a complete comparison, approximate results can be secured by limiting the comparison to coal and assuming

that the ton-mile savings on that commodity represent the average savings on the total ton-mileage.

The average cost of moving coal on the Monongahela River is quite well determined and is 3 mills per ton-mile and the rail rate on coal on parallel roads for the same average haul—42 miles—is 19 mills per ton-mile, the rail and water distances being practically equal in this case. This difference applied to the total ton-mileage—919 000 000—reported for 1924, shows a gross annual saving of $14 700 000, or allowing for difference in terminal costs, a net annual saving of $12 700 000, in comparison with fixed charges of $1 729 980.

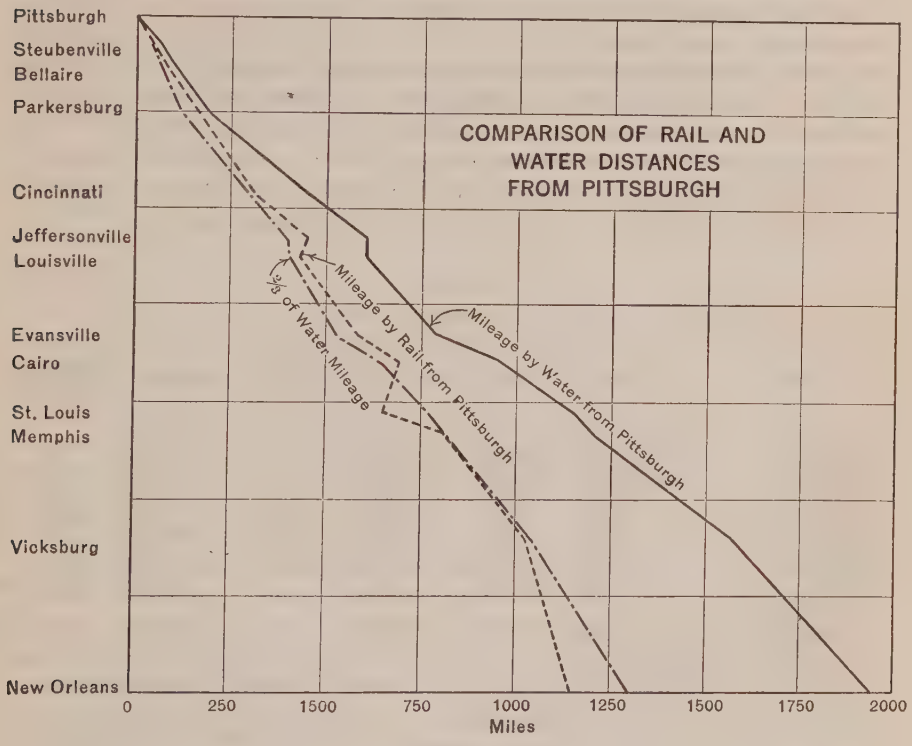

FIG. 5.

A similar comparison on the Ohio is limited to the reach between Pittsburgh and Cincinnati, 483 miles in length. There are only three uncompleted dams above Cincinnati and as they will be completed in 1925, the cost of that part of the project and the cost of operation and maintenance can be closely approximated. With a first cost of $54 500 000, interest at 4%, and operation and maintenance cost of $1 500 000, the fixed charges will be $3 680 000.

The coal moving on the Upper Ohio may be divided into three groups:

(*a*) That coming out of the Monongahela River and moving to points in the industrial district between Pittsburgh and Wheeling;

(*b*) That coming out of the Kanawha; and

(*c*) That coming into Huntington, W. Va., by rail and being transferred to barge.

Part of Movements (*b*) and (*c*) goes to the industrial district at Ashland and Ironton, Ohio, but the major part goes to Cincinnati. Some of the latter coal is re-shipped by rail, involving a rail-river-rail movement, the coal being hauled from Guyan Valley by rail, 70 miles, transferred to river barges, hauled by water to Cincinnati, 162 miles, and re-shipped by rail to points in Indiana.

The computation of direct benefits on the Upper Ohio is not as simple as on the Monongahela, but a study of the major movements and a comparison with rail rates and distances indicate a weighted average saving of 4.0 mills per ton-mile. If this is applied to the total ton-mileage—800 000 000—reported for this part of the Ohio in 1924, the gross savings amount to $3 200 000 or, allowing for difference in terminal costs, an estimated net saving of $2 200 000. This is approximately 60% of the fixed charges. As this part of the project is not yet completed, a characteristic of every large transportation enterprise, the showing is regarded as promising of ultimate economic success.

TABLE 9.

River.	Cost.	Tonnage.	Ton-miles.	Average haul, in miles.	Fixed charges.	Fixed charges, in mills per ton-mile.†
Ohio	$103 000 000	8 446 322	978 421 930	116.0	$6 120 000	6.25
Allegheny	1 446 000	4 339 511	35 370 300	8.15	193 920	5.5
Monongahela	13 612 000	21 878 815	919 564 942	42.0	1 729 980	1.88
Muskingum	1 452 595	152 012	9 120 720	60.0	144 774	16.1
Little Kanawha	282 000	45 951	1 378 530	30.0	41 137	29.7
Kanawha	3 580 000	901 183	54 064 140	60.0	309 360	5.7
Big Sandy	1 512 000	29 497	450 780	15.0	95 187	211.0
Kentucky	4 266 000	195 372	40 521 667	208.0	405 181	10.0
Green	670 428	436 916	.....	.....	129 587	.....
Cumberland	7 174 000	390 706	15 949 200	41.0	391 996	24.5
Tennessee, below Florence, Ala.	5 166 000	1 255 436	59 202 500	47.0	251 640	4.25
Total*	$141 490 595	37 634 805	2 114 044 709	56.2	$9 683 175	4.6

	Six Coal Roads.	Ohio River System.
Investment per annual ton (not including equipment)	$3.50	$3.75
" " " ton-mile	0.016	0.0675
Fixed charges per ton-mile	0.002	0.0046

* Not including Green River.

† The fixed charge per ton-mile for the entire system is 4.6 mills, 2.3 times the fixed charge per ton-mile on the principal coal-carrying railroads in this region. If upon completion of the Ohio canalization project the commerce is doubled, as seems probable, the fixed charge per ton-mile will be quite comparable with that on coal-carrying railroads.

Similar comparisons made on other tributaries show some as yielding benefits comparable with the fixed charges, others have not yet reached the stage of economic justification, while still others have apparently passed beyond it, as did the old canals constructed 100 years ago. Certain data concerning the Ohio River System as a whole for 1924 are shown in Table 9.

On the lower half of the Ohio, the tonnage for 1924 amounted to 1 400 000, of which, 420 000 tons was a seasonal movement of coal from the West Kentucky fields to Paducah, Ky., Cairo, and points on the Mississippi River. With the completion of the canalization project this tonnage bids fair to increase greatly. There is now a seasonal movement of steel products from Pittsburgh to Memphis and Baton Rouge, La., where two of the large steel corporations have established distributing points for supplying the Southwest; 66 000 tons were shipped in 1924 and with year-round navigation the tonnage will increase.

On the Mississippi River, in 1923, there was an up-stream tonnage (Memphis to Cairo), of 400 000, consisting principally of sugar, molasses, logs, bauxite ore, and miscellaneous items. Similar traffic will develop on the Ohio when the project is completed. For some of this tonnage, covered barges will be required which, in turn, will be suitable for the shipment of manufactured products from the Ohio Valley to points on the Mississippi River, resulting in a balanced haul.

The coal movement on the Ohio and its tributaries is nearly all down stream with practically no back haul. There are many indications pointing to an increase in this movement, but very little prospect of developing back-haul traffic that can be carried in coal barges. In this respect, however, the river does not differ from the principal coal-carrying railroads. As evidence of increase in the movement of coal by river, two additional coal companies are planning to ship, to Cincinnati by water, coal brought into Huntington, W. Va., by rail. The combined capacity of the three rail-river transfer plants at Huntington will then be 4 000 000 tons annually.

A modern unloading plant at Cincinnati is equipped with a bucket having a capacity of 5 tons of coal and the speed is such as will permit a round trip from barge to hopper in 40 sec. The cost of unloading is reported as less than 10 cents per ton. The new steam power plant under construction 20 miles below Cincinnati is planning to receive its coal by river. The initial installation will require from 250 000 to 300 000 tons annually and its ultimate requirements will be nearly three times that quantity. A proposed by-product plant at Cincinnati contemplates the shipment by water of 500 000 tons of coal annually.

The steel-making industries on the Ohio are responsible for the largest part of the present coal movement, 2.6 tons of coal being required directly or indirectly in the manufacture of each ton of steel, and if these districts continue to grow or other similar districts are established, the Ohio may eventually be as essential to their success as the Monongahela River is to the steel industries in that valley.

In connection with its Parkersburg, W. Va., refinery, the Standard Oil Company has established a water delivery service for its refined products. With a fleet of six steel barges, with a capacity of 100 000 gal. each, deliveries are being made to points on the Ohio between Wheeling and Huntington and to points on the Kanawha River. Another development is the movement

of 100 000 tons annually of crude oil from points on the Kentucky River to the Ohio, thence to Louisville.

Many of those interested in the completion of the Ohio River project look forward to such reduction in transportation costs as to make Baton Rouge and New Orleans the exporting points for a great part of the Ohio Valley instead of points on the Atlantic. Others foresee the time when iron ore will be brought by sea to Baton Rouge and barged up the Mississippi and Ohio Rivers to meet the coal; but even if some of these forecasts are not realized, it seems probable that the canalization of the Ohio will eventually yield benefits in excess of the fixed charges and thus justify the investment.

By some it is held that inland waterway improvements are justified even if the benefits are less than the carrying charges, but in its report to Congress, a number of years ago, the National Waterways Commission stated that:

"The cost of facilities for carrying freight, whether borne by the Federal Government or by private capital, is a burden upon the resources of the country. The fundamental criterion should be whether a railway or a waterway, when constructed or improved, will be a profitable investment of capital."

The same view was expressed recently by General Black in his paper on "Waterway and Railway Equivalents", previously mentioned, as follows:

"New waterway lines of transportation should not be established unless an analysis of the existing railway, highway, and waterway lines and of the need for transportation shows that the proposed new line is required and that if established it will produce an annual saving in the cost of transportation greater than the interest on construction plus maintenance and operating costs."

The writer would phrase a similar thought by saying that the people of the United States are entitled to have the products of the farm, mine, and factory move from origin to destination by the cheapest means, all elements of cost considered. Every inland waterway project should be gauged on that basis. Projects that are economically sound should be prosecuted vigorously and their use encouraged by wise regulation of rates and by the establishment of joint rail and water rates so as to make them an integral part of the country's transportation system.

THE RELATION OF THE OHIO-MISSISSIPPI WATERWAYS TO PRODUCTION AND INDUSTRIAL DEVELOPMENT

By H. B. Luther,* M. Am. Soc. C. E.

The proper utilization of the Ohio-Mississippi Waterways for transportation is rapidly becoming essential to the safeguarding of National prosperity and to the proper economic and industrial development of the United States. This is considered to be a fair statement because two distinct types of studies have indicated that these waterways afford a means of solving important industrial problems. This particular paper is confined to a description of these two fundamental studies and the relation of the Ohio-Mississippi route to them. The first was a war-time study of production which is equally applicable to the general problem of production, and it stresses a serious situation which is worthy of frequent re-statement from all angles. The second study is concerned with the industrial development of the Cincinnati District.

Prosperity is linked with production. Periods of prosperity make demands on the entire capacity of the nation for production, and in time of war the country's ultimate military effort is limited by production. There are seven limiting factors of production, namely, (1) demand; (2) fluid capital; (3) transportation; (4) labor; (5) raw material; (6) equipment; and (7) power.

It is impossible to place these factors in the order of their importance, because at different periods any one of the various items may become the limiting factor. At present (1925), demand is the limiting factor in production. The country is so equipped with labor, raw materials, and sources of power in its coal supply and water-power possibilities, that the proper handling and utilization of these commodities should make them meet any demands that can be anticipated. At critical periods, however, transportation is the real limiting factor of production. The great panic of 1907 was caused fundamentally by the inadequacy of transportation facilities, and the transportation facilities again failed to meet the demand after the entrance of the United States into the World War.

That Division under the U. S. Chief of Ordnance, which handled economic problems, emergency production, and labor matters, made a survey of transportation conditions in the fall of 1917. The flow of material was primarily eastward. Congestion appeared very early. Outline maps of the United States, such as are used by the U. S. Weather Bureau, were prepared with each railroad yard represented by a red dot. The congestion in each yard was indicated by an arrow pointing backward from the yard, westward. This arrow lengthened week by week and the rate at which this length increased was the basis for the prediction of the date of complete congestion. In December, 1917, it was clear that this would occur by February 1, 1918. Addi-

* Prof. of Civ. Eng., Univ. of Cincinnati, Cincinnati, Ohio.

tional factors—in particular, severe weather conditions—hastened the tie-up and by January 17, 1918, the Coal Commission issued its orders closing down all factories for one week and for one day per week thereafter, until the congestion was cleared. It was necessary to suggest many changes in orders for war material which various Departments had placed in plants admirably adapted by equipment and personnel, but situated where material could not be shipped in or out with needed regularity.

Herman Schneider, Assoc. M. Am. Soc. C. E., who was closely concerned with those studies, gives a simple illustration of the transportation situation in one of his professional papers. The transportation facilities correspond to the aisles of a factory along which the raw materials are carried to the machines and the finished products are carried away from them. There is an efficient width of aisle which will avoid congestion and consequent tie-up of various machines. Further, the proper layout of the equipment must be considered in its relation to a proper sequence of functions and a minimum transport of material from the entrance of the plant to its issue as finished product. Individual plant layout is a matter of careful analysis. The industrial layout of the entire country should be viewed in much the same manner, treating the country as one vast factory. At present, the aisles are not wide enough nor is the machinery in all cases properly placed.

In 1904, there was $1 in railroad property for each $9 of other property; in 1912, the ratio was 1 to 12, and, in 1920, there was nearly $25 worth of other property for each $1 of railroad plant. Although the basis of valuation has changed somewhat in that period, the comparison of these ratios is an important index. In case of a war, the nation's efforts would still be limited by transportation. It is stated that during the period of congestion on the Eastern seaboard, under World War conditions, it was easier to ship goods from Pittsburgh, Pa., to New York, N. Y., *via* New Orleans, La., than from Pittsburgh to New York direct by rail. On the completion of the work on these waters, a similar tie-up into the North Atlantic ports would make exporting by way of New Orleans feasible, that is, it would be easier to ship from Cincinnati, Ohio, to New Orleans by river and then by boat to France than to ship from Cincinnati to New York and export from there. The increased utilization of these waterways is necessary for general business demands and as a measure of National security.

Again the matter may be viewed from a study of proper industrial development. Such development for any particular locality should be based on sound searching studies of all the economic principles involved and on a proper consideration of the industrial relation of that particular section to the country at large. In the past, industries have been encouraged for the purpose of building up communities, without due care in the selection of the proper type of industry. There are exceptions, of course, primarily in the location near the source of supply of those industries using bulky raw materials.

In this problem of the selection of proper industries, the Ohio-Mississippi Waterways, is an important factor for those States bordering on it. Investigations of the proper industrial developments of the Cincinnati District are

under way. The work has not yet reached the stage where direct recommendations can be made, but a discussion of the fundamental principles involved, with stress on the question of transportation, together with an illustration of the specific type of problem which arises, will indicate the importance attaching to the waterways.

The question of the proper industry for a site involves many factors. First, the natural resources of the surrounding country should be studied; the demands of the community must be analyzed; the type of power and labor available must be known; and the contacts with neighboring industrial centers considered.

For example, assume that a plant to manufacture a given commodity is to be located in Cincinnati and that a request is made for capital. In the investigation of the wisdom of such investment it is necessary, in order to determine the transportation charges, to determine the sources of supply of its raw materials and to learn the dependency to be placed on a continual flow of this supply. Then, with the cost of the finished product known, there may be determined the distance this product can be sent in competition with similar factories which may be located in Chicago, Ill., St. Louis, Mo., and Pittsburgh. The demand for this product in the territory it may economically supply must be studied and the output of existing local plants determined. Obviously, if the demand in the local marketing area is already 90% filled by existing agencies, a 25% increase in production capacity would mean failure of some of the companies or severe financial stress for all. This leads toward the question of decentralization for certain types of industries.

In order to have a basis for solving these industrial problems, two surveys have been started in Cincinnati. The first is determining the natural resources available to the Cincinnati District. The area to be studied varies with the type of material, in some instances a radius of 50 to 100 miles is taken, in others, one of 500 to 1 000 miles, depending on the bulk of the commodity and the means of transportation. This work is being carried on by the University of Cincinnati, with aid from the Commercial Club. At the same time, a study of the existing manufactures in this District, the demands in raw materials made by these plants, and the output from this District, is being carried on by the Technical Advisory Corporation. This work is supported financially by the Chamber of Commerce.

To illustrate how Ohio River transportation has entered this study, the following illustration serves excellently. Among the studies being made in connection with these investigations has been a review of the uses of salt and the sources from which it is now obtained for local purposes. At present, salt is brought chiefly from Northern Ohio and Michigan. Rock salt is mined somewhat more cheaply in New York State, but transportation charges make the Northern Ohio and Michigan sources more available. However, rock salt can be obtained in large quantities in Louisiana, a source which bids fair to become of great importance in supplying the demands of the country. It may be brought from this supply in barges on the Ohio and Mississippi Rivers and delivered to factories established on the banks of the Ohio at favorable rates

in competition with the present sources. There it can be crushed and screened and the demands for various sizes of rock salt for all local needs in the chemical industry supplied. The excess of all sizes can then be used at these new factories on the river to manufacture certain chemicals. Alkali (sodium hydroxide), suggested as the lime needed in conjunction with it, is available in great plenty in this region, and the electrolytic manufacture of sodium hydroxide could use any excess electric power if this proves feasible. Salt cake for the glass industry has also been suggested.

Also, salt cake (Na_2SO_4) demands sulfur, and this demand carried the investigation to a study of the source of this raw material. The sulfur used at present comes chiefly either from the Eastern seaboard, from places such as Perth Amboy, N. J., where it is obtained from pyrites imported in bulk, or from pyrites found in Tennessee. Sulfur may be obtained practically pure from wells in Louisiana and Texas and transported on the inland waterway. With cheap sulfur for the manufacture of sulfuric acid, an ample salt supply, and adequate power, the materials are assembled for the maintenance of a heavy chemical industry, for the manufacturing of such products as the salt cake, already mentioned, soda ash, caustic soda, and hydrochloric acid, all of which are products that find places in the industrial demands of Cincinnati.

This is the type of problem now confronting those engaged in the investigations of accessible materials and their utilization. The feasibility of this particular manufacture is still to be determined. However, recent studies seem to indicate that the Ohio-Mississippi route will place rock salt and sulfur among those raw materials so situated geographically as to fall among the natural resources which determine the proper industry for this locality.

There is a local demand for pig iron, and the question has been asked, should blast furnaces be located in the more immediate vicinity of Cincinnati? The transportation factor involved is that of barging coal from the Pittsburgh region in West Virginia—a water haul—and the rail shipment of the ore from a lake port, such as Lorain, Ohio, a haul comparable to the haul from the Great Lakes to Pittsburgh. Local factories that use by-product coke and slag cements deserve consideration in this connection, as well as the practicability of smelting at Cincinnati, a mixture of Birmingham iron, Bessemer and Lake Superior ores. The use of sulfur and salt is stressed, however, to indicate the need of an investigation of the natural resources of the Ohio-Mississippi Basin as a means to intelligent industrial expansion, along proper lines, of those industrial centers adjacent to the waterway.

COMPARISON OF RAIL AND RIVER TRANSPORTATION COSTS IN RELATION TO THE OHIO RIVER AND ITS TRIBUTARIES

By Frank H. Alfred,* M. Am. Soc. C. E.

The relation of the Ohio River and its tributaries to transportation in the United States may mean either of two things: First, how the one compares with the other as to amount, importance, expedition of service, economy, or some other characteristic; and, second, how the two are connected with or related to each other—the nature and extent of their interdependence and interaction. The first is largely a matter of statistics. The salient facts in regard to transportation on the Ohio River† are contained in Tables 10 and 11.

TABLE 10.—Freight Traffic, 1923.

Commodities.	Net tons.	Percentage of total.	Value.	Percentage of total.
Coal and coke	6 041 708	73.0	$27 466 401	25.0
Sand, gravel, and stone	1 372 404	16.6	1 542 881	1.4
Metals and metal products	389 792	4.7	36 714 894	33.4
All other commodities	476 617	5.7	44 298 077	40.2
Total	8 280 520	100.0	110 022 253	100.0

The figures given in Tables 10 and 11 include freight handled in car ferries. Since 1918 this item has been kept separately, and figures exclusive of car-ferry traffic are, as follows:

Year.	Tons.	Value.
1919	5 004 378	$88 912 336
1920	9 382 464	223 433 492
1921	7 307 880	93 274 814
1922	6 291 828	144 796 250
1923	8 280 520	110 022 253

Characteristics.—The characteristics of the Ohio River are, as follows:

Length, in miles		968.5
Drainage area, in square miles		203 900
Discharge, in cubic feet per second (estimated):		
	Low water.	High water.
At head, Pittsburgh, Pa.	1 600	440 000
At mouth, Cairo, Ill.	27 000	1 500 000

* Pres. and Gen. Mgr., Pere Marquette Ry., Detroit, Mich.

† Annual Report, Chf. of Engrs., U. S. Army, 1924.

Average fall per mile at low-water stage:

	Miles.	Inches.
Pittsburgh, Pa., to Wheeling, W. Va.	90	11½
Wheeling, to Cincinnati, Ohio.....	378½	5½
Cincinnati, to Cairo, Ill.....	500	4

Construction of Locks and Dams.—The existing project for improvement was adopted by the River and Harbor Act of June 25, 1910, and contemplated a 9-ft. channel from Pittsburgh to Cairo. It includes fifty-two dams, with a lock, 110 ft. by 600 ft., at each dam. Of these, thirty between Pittsburgh and Portsmouth, Ohio, are in operation, being all that are contemplated between those points, a distance of 358 miles. Between Portsmouth and Uniontown, Ky. (480 miles), seven dams are in operation, nine are under construction, and one is not yet begun. Between Uniontown and Cairo (130 miles), the five dams contemplated are not yet begun. This makes a total of thirty-seven in operation, nine under construction, and six not begun, on June 30, 1924.

TABLE 11.—Freight Traffic, 1892-1923.

Year.	Tons.	Value.
1892	6 901 186	.....
1893	7 371 804	.....
1894	7 795 501	.....
1895	7 963 478	.....
1896	9 914 435	.....
1897	11 265 638	.....
1898	6 756 627	.....
1899	13 529 742	.....
1900	14 054 322	.....
1901	10 064 978	.....
1902	12 202 017	.....
1903	12 499 842	.....
1904	10 142 551	.....
1905	13 163 656	.....
1906	11 427 784	$79 994 488
1907	11 306 544	79 145 808
1908	8 498 754	59 491 278
1909	8 676 701	60 736 907
1910	11 112 216	358 899 889
1911	12 046 294	82 074 878
1912	8 618 369	71 064 229
1913	9 814 123	77 026 901
1914	9 530 309	93 294 479
1915	9 273 184	188 510 914
1916	7 917 112	165 130 487
1917	6 149 213	309 272 755
1918	6 662 941	145 654 917
1919	5 469 424	186 710 332
1920	9 869 325	368 607 567
1921	8 037 788	464 576 262
1922	7 039 138	968 327 994
1923	9 245 647	372 534 148

In 1910, prior to the adoption of the present project, $17 657 273 had been expended, with $58 974 278 since that time, making a total of $76 631 551. The estimated cost of completing the project is $34 084 625, which would make the total cost $110 716 177. The cost of maintenance and operation in the year ending June 30, 1924, was $1 420 574.

The classification of freight in 1923 by principal commodity groups and by up-stream and down-stream movement, with the ton-miles and average haul in each case, is given in Table 12.

TABLE 12.—Freight Classification, 1923.

Up Stream.

Commodities.	Tons.	Percentage of total.	Ton-miles.	Average haul, in miles.
Coal and coke	343 225	20.6	13 563 726	40
Sand, gravel and stone	972 645	58.2	10 946 066	11
Metal and metal products	167 055	10.0	22 353 055	134
All other commodities	187 361	11.2	75 233 212	401
Total	1 670 286	100.0	122 096 059	73

Down Stream

	Tons.	Percentage of total.	Ton-miles.	Average haul, in miles.
Coal and coke	5 698 483	86.2	489 752 210	86
Sand, gravel and stone	399 759	6.0	6 328 085	16
Metals and metal products	222 737	3.4	101 619 654	456
All other commodities	289 256	4.4	29 324 478	101
Total	6 610 235	100.0	627 024 427	95

Total.

	Tons.	Percentage of total.	Ton-miles.	Average haul, in miles.
Coal and coke	6 041 708	73.0	503 315 936	83
Sand, gravel and stone	1 372 404	16.6	17 274 151	13
Metals and metal products	389 792	4.7	123 972 709	318
All other commodities	476 617	5.7	104 557 690	219
Total	8 280 521	100.0	749 120 486	90

No attempt to compare Ohio River traffic with all classes of transportation in the United States will be made, as that would include not only other water-borne traffic, but also that carried on the highways. The comparison will be confined to the freight traffic carried by the railways. This is shown in the following statement (Table 13) for 1923, the last year in which statistics as to river traffic are available.

In this comparison (Table 13) it should be noted that the cost of railroads includes rolling stock, which amounts to about one-third of the total cost, whereas the cost for the river does not include boats and barges, nor does it include anything to correspond to shops, fuel and water stations, freight and passenger stations, or terminal facilities in general, which form so large a part of the cost of a railroad. It is really equivalent to the cost of the roadway only, which, in the case of the Pere Marquette Railway, amounts to only about one-third of the total.

TABLE 13.

	Railroads in United States.	Ohio River.	Percentage of railroad.
Length, in miles	250 183	968.5	0.39
Tons of freight	2 333 000 000	8 280 520	0.35
Tons of freight per mile of length	9 065	8 550	94.3
Ton-miles	416 211 000 000	749 120 486	0.18
Ton-miles per mile of length	1 663 228	773 485	46.5
Cost, including equipment	$22 681 000 000	.....	.....
Cost per mile, including equipment	$ 90 658	.....	.....
Estimated cost of improvement	.....	$110 716 177	.....
Estimated cost per mile of improvement	.....	$ 114 318	.....

In 1923, the average railway revenue per ton-mile was 1.116 cents. On the Pere Marquette Railway the average revenue per ton-mile was 1.118 cents, which was disbursed, as follows:

	Cents.
Maintenance of way and structures	0.141
Maintenance of equipment	0.233
Transportation	0.390
Traffic, general and miscellaneous	0.063
Taxes	0.044
Rentals	0.071
Interest	0.053
Surplus	0.123
Total	1.118

This distribution is on the assumption that the ratio of expense to revenue is the same for freight as for all traffic.

Data are not available for arriving with accuracy at corresponding figures for Ohio River traffic. However, the uncertain factors are not important, so that a total may be reached, which is fairly comparable with that for railroads.

The improvement project will probably be completed within the next three or four years at a cost now estimated at $110 000 000, on which interest at 4% would be $4 400 000. The maintenance in 1923, with thirty-seven dams in operation, cost $1 420 574, so it may be assumed that the cost of maintenance of the completed project will average $2 000 000 per year.

Depreciation on the railroads is included in their maintenance cost, but not in the figures given for the river. In a paper on "Traffic on the Ohio River"*, Mr. Charles Whiting Baker states that: "The first dams built on the Ohio River already require reconstruction. A depreciation charge of 1½% allows at least as long a life for these works as can safely be assumed." This rate gives a yearly depreciation charge of $1 650 000.

The Government, by making this expenditure, will have withdrawn $110 000 000 from the taxable wealth of the nation, on which otherwise it would be receiving taxes. A fair comparison with railroad costs, therefore, should

* *Engineering News-Record*, January 15, 1920.

include taxes on this amount. In 1923 the railroads paid taxes of $332 000 000 on an investment of $22 681 000 000, or at the rate of 1.46%, which applied to the cost of $110 000 000 is $1 606 000.

The total annual charge against the improvement is summed up as follows:

Interest at 4%	$4 400 000
Depreciation at 1½%	1 650 000
Taxes at 1.46%	1 606 000
Maintenance	2 000 000
Total	$9 656 000

This amounts to a cost of 1.29 cents per ton-mile, based on the volume of traffic for 1923. As the volume of traffic decreases, this cost will increase.

This charge does not include the amount paid the carriers for actually moving the freight. Such figures are not available, but H. G. Moulton, M. Am. Soc. C. E., has estimated* that a rate of 0.089 cent per ton-mile would be necessary to enable a carrier to haul coal from Pittsburgh to Cairo at a profit of 5% on its investment. Although coal constitutes by far the greater bulk of the tonnage, there is enough other material handled at a much higher rate to raise the average rate level. Further, this rate assumes a through movement from Pittsburgh to Cincinnati; but the average haul on all traffic is only 90 miles, with correspondingly greater terminal delays. These two factors would result in at least doubling the rate quoted, and the increase in cost of labor, fuel, etc., since this estimate was made, would add about 60% more to adjust it to present-day costs. These adjustments would raise Mr. Moulton's estimate to approximately 0.27 cent per ton-mile, or about one-fourth the rail rate. Of course, rates on most commodities are much higher, for there is little competition by water, and rates are based largely on railroad competition.

River carriers no longer make the rates; instead, they adjust their rates to a little below the scheduled rate of the railroads, which is based on the experience of all the railroads of the United States and must stand the test of study by the Interstate Commerce Commission, a fact-finding body. This new condition has come about through the shifting of traffic from water to railroad, and the attendant specialization of traffic on the river.

The average rate is kept down by the relatively large tonnage of coal; but making all due allowance for this, it is not probable that the average cost is less than 0.27 cent per ton-mile, if interest, taxes, depreciation, and maintenance on the floating equipment, in addition to the actual cost of operation, are properly taken into account.

Adding the cost of operation, 0.27 cent, to the ton-mile charge for improvements, gives a total ton-mile cost of 1.56 cents, which is 40% greater than the rail rate. Even this figure is not comparable to the railroad cost per ton-mile, as a ton-mile by river is not the equivalent of a ton-mile by rail. All towns

* "Waterways versus Railways", 1912.

of any considerable size on the river are also accessible by rail. Table 14 shows a comparison of distances by water and the shortest rail routes.

TABLE 14.

Place.	Rail.	River.	Ratio: River to rail.
Pittsburgh to Wheeling, W. Va.	68	90	1.32
" " Cincinnati, Ohio.	326	468	1.42
" " Louisville, Ky.	455	604	1.33
" " Evansville, Ind.	509	784	1.54
" " Cairo, Ill.	670	968	1.45
Total.	2028	2914	1.44

In order to obtain a fair comparison of rail and river cost it is necessary, therefore, to multiply the river cost by 1.44, resulting in a cost of 2.24 cents per ton-mile by river, as against 1.116 cents by rail, or just twice as much. There are two other features which, if taken into account, would further modify this comparison to the advantage of the railroads. Coal, sand, and gravel constitute 90% of the river traffic, which is more than twice the proportion on the railroads. Railroad rates on these commodities are considerably lower than the average, and if such rates were used in this comparison instead of the average, it would result in a still more unfavorable showing for the river. Further, the rail rate of 1.116 cents per ton-mile covers revenue freight only. The traffic figures on the Ohio, as compiled by U. S. Engineers, may be based only on revenue freight, but it is probable that they include all freight moved on the river, as well as the fuel for steamboats and the materials used in constructing the river improvements. If this is true, a cost per ton-mile for river traffic, based only on revenue tonnage, would be appreciably higher. To sum up, it may be said conservatively that river transportation costs at least twice as much as railway transportation. In addition to costing twice as much, river traffic is subject to other disadvantages. It is a slow movement down stream, and much slower up stream. It is subject to interruption from the periodic floods and even ice at certain times.

River transportation is not comparable with railroad transportation in regard to service, inasmuch as it is limited as to point of loading and unloading. In the case of the railroads, the service is not thus restricted; and to this is due no doubt the development of the railroads in the river territory to the disadvantage of water-borne transportation.

This comparison shows that as a transportation facility, the Ohio River may be likened to a very crooked railroad of similar length with less than one-half the average traffic density; with a very badly balanced traffic, but with grades favorable to the heavier movement; with a roadbed which cost more than three times that of the average railroad; and operating at a total cost per ton-mile at least twice that of the average railroad. Fortunately for the

shipper on this route, he does not have to pay interest, taxes, depreciation, or upkeep on its roadway, for this expense is borne by the whole United States.

This brings us to the other phase of this discussion: How Ohio River transportation and railway transportation affect each other. When the writer began this brief study, his impression was that, in this sense, there was no relation between the two. Ohio River traffic seemed so self-contained, so independent of rail transportation, and so restricted to local traffic and to certain commodities which it can handle to advantage, that he failed to see much connection between it and transportation at large. The history of river and railroads since the latter invaded the Ohio Valley, however, shows that the railroads have had a very unfavorable effect on river traffic. On the other hand, the influence of river traffic on the railroads has been negligible, although it has resulted in favoring certain communities at the expense of others, as will be shown later.

The steamboat was invented nearly a generation before the locomotive, and water transportation was in full swing on the Ohio before the railroads arrived. As the railroads developed, however, the cost of transportation was reduced, although it was fifty years before they could seriously compete with water traffic. As this reduction continued, the railroads made serious inroads on river traffic. This reduction in cost of rail transportation, if measured by the purchasing power of the dollar, is still going on, and there is every reason to believe that it will continue indefinitely. On the other hand, river transportation has made little or no progress in that direction, and, from its very nature, will not be able to do so. The result may be noted in the trend of traffic on the Ohio. The peak was reached in 1900, since which time it has grown less at a fairly uniform rate, if the fluctuations due to varying business conditions are eliminated. This is shown in Table 15, which also shows the increase of railroad freight traffic for the same periods.

TABLE 15.

Year.	Tons of Freight.		Percentage.
	Ohio River.	Railroads.	
1900	14 054 322	1 081 983 301	1.30
1910	11 111 216	1 849 900 101	0.60
1920	9 869 325	2 427 622 267	0.41
1923	8 280 520	2 411 239 000	0.34

An editorial in *Engineering News* of May 4, 1916, stated that "Commerce has practically deserted most of the inland waterways. The great and controlling cause is the development of railways." This is undoubtedly true, but there are other contributing causes. The mines that can ship by river to advantage, on account of being directly on the river bank, are becoming exhausted, and the new mines that are being opened, are inland and are served by railroads.

The production of coal, per capita, is declining, not only on the Ohio River, but throughout the United States. This decrease is shown by the following information taken from the reports of the U. S. Geological Survey:

Calendar years.	Production of coal in net tons.
1913	569 961 000
1914	513 523 000
1915	511 617 000
1916	590 101 000
1917	651 399 000
1918	678 212 000
1919	546 155 000
1920	658 264 932
1921	506 395 401
1922	476 951 121
1923	657 446 000
1924	573 494 000 (estimated)

This statement shows that the production in 1924 is practically the same as in 1913. In the meantime, however, the population of the United States had increased from approximately 96 000 000 in 1913 to 112 000 000 in 1924.

The reason for this decline per capita is not difficult to find. The use of electricity has been growing by leaps and bounds, as shown by the following figures obtained from the U. S. Census Bureau:

Year.	Electricity generated, in 1 000 kw-hrs.
1907	5 862 277
1912	11 569 110
1917	25 438 303
1922	40 291 536

Part of this electricity is derived from water power, but the major part is obtained from coal. Much of the coal, however, is consumed at central locations provided with ample transportation facilities. The widening use of electricity results in less transportation of coal whether the electricity is derived from water power or from coal.

Another reason for the decline per capita in the use of coal is the rapidly increasing production of petroleum, as shown in the following figures obtained from reports of the U. S. Geological Survey:

Year.	Production of petroleum, in thousands of barrels of 42 gal.	Year.	Production of petroleum, in thousands of barrels of 42 gal.
1910	172 769	1917	335 316
1911	220 449	1918	355 928
1912	222 935	1919	378 367
1913	248 446	1920	442 929
1914	265 763	1921	472 183
1915	281 104	1922	557 531
1916	300 767	1923	725 702

The automobile is responsible for a large part of this increase, but not for all of it. The majority of the locomotives in the Southwest are now equipped for burning oil, and its use for this purpose is rapidly extending. On January 1, 1924, there were in use on the railroads of the United States 6 763 oil-burning locomotives. Still more marked is the increase in the use of fuel oil for vessels, domestic consumption, etc.

In fact, it is evident that the railroads of the United States have reached their peak of tonnage per capita. The decrease in the quantity of coal transported has been made up to a certain extent by the large quantity of materials handled for highway construction. This will probably continue for many years, but it is doubtful whether at any time it will exceed the tonnage now handled.

The relative decline in coal production will affect both river and rail traffic, but will affect river transportation more seriously, as coal constitutes about three-fourths of the Ohio River traffic, and when it goes, there is nothing to take its place. On the other hand, the railroads will have more capacity to handle higher grades of freight; they now have excess capacity, and could probably handle 25% more business than they do, without taxing their present facilities. For this reason, the declining coal traffic may cause some hardship for a time, but as the country develops, this loss will be more than made up by greater shipment of higher grade commodities.

Railroad freight rates generally are so much higher in Europe than they are in the United States that there is a wider margin for successful operation on the rivers and canals. The average revenue per ton-mile for 1913 in a number of European countries and in the United States is shown in the following tabulation, pre-war figures being chosen in order to eliminate the effect of varying conditions and exchange rates since the war:

	Cents.
Great Britain	2.23
France	1.28
Germany	1.37
Austria-Hungary	1.51
Sweden (1912)*	1.52
Belgium (1912)*	1.13
Holland	1.36
United States	0.729

Waterways have always been favored as to railroad rates for the reason that the original rate structure was based on the existing traffic and competition. This has often resulted in lower rates to towns along the rivers. In his discussion of the admirable paper† by William M. Black, M. Am. Soc. C. E., on "Waterway and Railway Equivalents", Edward Burr, M. Am. Soc. C. E., states that:‡ "The waterways, however, have justified the public's confidence and willingness to spend its money on them, although they may carry little actual

* Figures for 1913 not available.

† *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 538.

‡ *Loc. cit.*, p. 564.

commerce. To use a naval phrase they have the value of a 'fleet in being' ". This may have been true at one time, but Colonel Burr has evidently forgotten the Transportation Act of 1920. This Act establishes a fair return for the railways, and authorizes the Interstate Commerce Commission to adjust rates so that they will earn such a return. This being the case, it is evident that if any community or set of communities enjoys rates lower than they would pay in the usual run of affairs, this deficiency must be made up by other communities which pay a higher rate. Colonel Burr's "fleet in being"—in this instance, the improvement of the Ohio River—serves only to protect those who dwell along the banks of the river in an advantage secured at the expense of other communities. The shipper and consumer in Columbus and Dayton, Ohio, and Indianapolis, Ind., not only help pay for these improvements, but if the improvements fulfill their purpose and stimulate river traffic, the shipper and consumer at inland points will also pay a higher rate on account of them.

It does not seem likely, however, that the completion of the improvement project will result in any considerable increase of river traffic. Coal which formerly went from Pittsburgh to New Orleans is now obtained from Tennessee and Alabama, partly by rail and partly by water. The sand and gravel business is purely local and would have existed in about its present volume if no improvements had been made. In the light of present experience, one is forced to the conclusion that the construction of these works was an economic waste. The Ohio is the one river in the United States on which there seemed to be a fair prospect of developing a large and important traffic. These great expectations have not been realized, and the writer feels it must be admitted that the experiment is a failure.

It is not probable that such an improvement would be undertaken at the present time (1925). The attitude of the people toward the railroads has changed since 1910. The thinking part of the population now realizes that the railroads are its friends and servants, and not its enemies and would-be masters. As a people, Americans think more in economic terms than they did a few years ago. Even so, it is difficult to predict what may result from an active propaganda directed by interested parties and acclaimed by politicians who are seeking a campaign slogan.

There is a lesson to be learned from the results of the Erie Canal. In his Annual Message to the New York Legislature in 1925, Governor Smith stated:

"The Barge Canal, including construction, terminals, grain elevators, repairs, maintenance, operation, and payment of claims for damages, has cost the people since 1905 up to date, $191 626 000; between 1905, when the first canal bonds were sold, up to 1924, inclusive, we have paid in interest $39 880 387, making the total cost of the barge canal to date, $230 881 014."

In the six years following the opening of the new Barge Canal in 1919, the freight averaged 1 640 481 tons. Interest on the cost of the Canal at 5% is $11 544 000, to which should be added $4 182 696 for maintenance and operation, making a total annual charge of $15 727 000, which equals $9.59 for every ton of freight handled.

In 1911, when the agitation for a deep waterway from the Great Lakes to New Orleans was at its height, Mr. H. G. Moulton, after describing the earlier waves of popular enthusiasm for the development of waterways, stated:

"Attention should be directed to the incessant suggestion in public discussion and in the press that transportation upon our rivers and canals should be revived. The remarkable conjunction of influences should be recalled, namely, the lingering glory of our waterways, the widespread movement for the conservation of all our natural resources, the relation of waterways to the reclamation of flooded lands, to the development of water power, and to sanitation, the supposed remarkable cheapness of water transportation, the opposition to the monopolistic tendencies of the railways, the inadequacy of the railway service to meet the country's needs, the alleged benefits of waterways even to the railways, the avowed advocacy of waterway development by railroad men, and the extensive use of waterways in foreign countries. Finally, it should be noted that, in addition to this incessant and broadside suggestion, the movement has had the active support of some economists, and of many prominent business men and politicians, who have served to give the necessary sanction of authority. Under such a cumulation of causes and combination of influences, it was utterly impossible for the average individual to weigh the various arguments which were presented or to subject them to the test of reason; and it is not surprising that a large portion of the populace should have grown excited over the matter and jumped to the conclusion that the industrial growth of the country is fundamentally dependent upon the development of water transportation."

The country is in the midst of a similar agitation for the "Lakes-to-Ocean" waterway. It is to be hoped that a decision in regard to this gigantic project will only be reached after it has been most carefully considered in all its engineering and economic aspects, and that such a decision shall be made on the basis of the findings of the most expert investigators and not in response to an ill-informed and more or less hysterical popular enthusiasm, as has too frequently been the case in the past.

RIVER-RAIL TERMINAL

By Julian A. Pollak,* Esq.

What is the meaning of a river-rail terminal? First, it means that the river will help the railroads; second, it means that the railroads will help the river; and, finally, that both will help the port as well as the so-called hinterland. Every city of prominence and importance has been made in two ways—either by water, such as lakes and seaports, or by railroads. The largest cities in the United States, and those of the greatest importance, have a combination of both water and rail. New York, N. Y., Chicago, Ill., Cleveland, Ohio, Baltimore, Md., Pittsburgh, Pa., Detroit, Mich., and San Francisco, Calif., are a few examples of the wonderful combination of water and rail.

Engineers are familiar with the improvement for navigation of the Ohio River and the great plan of development of inland waterways. When this entire program is complete in the near future, the question is, what next? The answer to this question is the problem of each locality so situated that it can and will use the facilities at its disposal. For this reason the writer will confine his remarks to a definite locality—the City of Cincinnati, Ohio.

Begin with a natural highway equivalent to a 10-track trunk line from Pittsburgh to Cincinnati, built at a total cost of $103 000 000 for engineering development to make it navigable for approximately twelve months in the year, barring ice and exceptional conditions. When this waterway is completed, it will connect with other waterways such as the Mississippi River which, in turn, connects with seaport, ocean, canal, and more ocean.

Locally, this means that a single city on the banks of the Ohio River begins to think in terms of New Orleans, La., Seattle, Wash., even Hong Kong, China, to say nothing of neighboring cities such as Pittsburgh, Pa., Louisville, Ky., and St. Louis, Mo. By visualizing such commercial connections, the inland neighbors in Springfield, Columbus, and Dayton, Ohio, will see that trade opportunities await them also.

It is obvious that with a highway passing the door, with money already spent on river improvements, and with trade developing in such a startling manner, that locally the citizens of Cincinnati should think about handling it. When millions of dollars are being spent on river craft, such craft should be invited to stop at this port.

One of the difficulties in the interchange of traffic between water and rail is the transfer, the cost of which in money and time must be reduced to a minimum. It must be done in the most expeditious and cheapest manner which modern science can provide. The key to securing this minimum of time and expense in transfer is the terminal—the rail-water terminal located on the banks of the navigable river, connected by belt line with all the railroads serving the community, and provided with cranes and other modern

* Vice-Pres., Pollak Steel Co., Cincinnati, Ohio.

facilities for transferring freight between the warehouse and the boat, or the boat and the car, cheaply and expeditiously; and it follows that this key must be provided. Who shall provide it, the United States, the States, or the political sub-divisions—the municipalities?

In order to handle this approaching situation, a river-rail transfer company has been formed in Cincinnati to connect the rail and water facilities. The first idea was to form a company which eventually would become a municipal enterprise, thus opening the Port of Cincinnati to all. Owing to the lack of funds in the City Treasury, as well as the political complications that might arise, it was decided to organize this company other than municipal and strictly on a commercial basis and for profit. The advantage of such a plan is that it stimulates the individual initiative which developed the railroads and their terminals.

The group interested plans to make this enterprise quasi-civic by encouraging the large and small shipper alike to share in the company. There are no large stock subscriptions, but it is planned to have a city-wide list of stockholders. Already the subscription list includes industrials, lawyers, department stores, advertising agencies, insurance companies, and banks—some are directly interested and others are interested purely for civic reasons. The company will be free from any monopolistic tendency, and such that any one can have any commodity handled at any time. The project, in a sense, is pioneer work, for the company has neither handled costs nor guaranteed shippers, but it has faith in water transportation, and feels that if the Congress of the United States has sufficient faith in the project to make the Ohio navigable, the citizens should have enough confidence to use such facilities.

In order to design a transfer terminal to accommodate a variety of shippers, many things must be considered before proceeding with the work. First, there is the usual incorporation to cover the business aims or business franchise; next, the location of property easily accessible to the railroads and automobile trucks. This is the first problem, because, in addition to being on the water, the facilities must be above high water for protection of machinery, storage, railroads, and streets. A high abrupt river bank will overcome the flood-water obstacle, but when the bank is abrupt, the cranes must be of ample capacity and have sufficient swing to reach the barge in pool stage as well as the top of the bank where the tracks and trucks are situated.

The next problem is a natural harbor, free from ice, and situated so that drift does not injure the barges moored for unloading and loading. In this respect valuable assistance was obtained from the Government engineers who aided by surveys and reviewed the plans. When the location and plans were submitted to the Government engineers the permits and harbor-line locations were granted. From this stage, it was possible to estimate the cost of dredging, excavation, piling, fill, and tracks, and thus work back to a first unit capacity for crane, storage, and handling.

Added to all this is the usual easements, property rights of approach, and arrangements with the railroads. The writer has only attempted to describe

briefly a few of the problems involved in locating a transfer terminal with the idea that some engineers may utilize the experience of the company in so far as it has gone.

It is felt that now is the time to go ahead so that when the 9-ft. stage is completed, facilities will be ready to handle the traffic and to point out to others that there is ample room for more companies of a similar character to do likewise.

DISCUSSION

R. N. Begien,* M. Am. Soc. C. E. (by letter).—Prior to the era of railway building, the Ohio River, then a navigable stream during certain parts of the year, was the principal means of transportation and communication for a large tributary territory. As the railway lines were extended across the mountains to Ohio River points, river transportation came to be used conjointly with railway transportation in serving this territory and in reaching communities which were without a railway. As the railway system of the United States developed, competition between the river and the railways grew apace and the joint rail and river transportation was utilized less and less, but it has never been entirely superseded and is still used to good advantage in some instances.

The construction of the Chesapeake and Ohio Railway to Huntington, W. Va., on the Ohio River, was completed January 29, 1873. From that time until August, 1889, when the extension of the road to Cincinnati, Ohio, was placed in operation, the Ohio River formed the connecting link for Chesapeake and Ohio traffic between Huntington and Cincinnati. The river was thus used as an adjunct to the railway in reaching this important city.

Coal from inland mines is shipped in some volume by rail to river points and then moved by barge and towage to industries which the originating railroad does not reach, either by its own rails or by satisfactory through rates with other lines. In this manner river transportation is still being used as a supplement to railroad transportation, and to the advantage not only of industries but of individual railroads.

When the Chesapeake and Ohio Railroad was extended to Cincinnati, there immediately arose a sharp competition between the railway and the steamboats on the river. This competition still prevails to some extent, for the influence of the river has been, and still is, a factor in the determination of rates on coal. On other business, however, the effect of this competition is negligible and is virtually ignored in present practice.

At one time quite low rates on local merchandise and miscellaneous freights along the Ohio River were made to meet the competition of the irregular steamboats, but it was found that reliable service was of greater importance than the lower rates. Accordingly, rates which were thought to be fair and reasonable were established and since then, notwithstanding the operation of some river steamboats, the local business along the river handled by the railway has continued to increase and the railway has retained its share of the competitive business through superior service.

During the past fifty years a number of steamship companies of apparently ample financial strength and other resources have engaged in transportation on the Ohio River, but they have dropped out one by one and now only a few of them are operating as common carriers. The territory is restricted, the cost

* Vice-Pres., in Chg. of Operation, C. & O. Ry., Richmond, Va.

of transferring freight from cars to boats and *vice versa* is high, and comparatively few commodities can be handled advantageously. These factors combined with the hazards and seasonal character of river transportation and the competition of efficient railroad transportation at low cost have reduced river transportation to a minimum.

It is well to emphasize that in the development of American industry reliable transportation has been found of such great value as to constitute an essential. Dependability is the important thing. The transportation afforded by the railways, at somewhat higher cost perhaps, has proved to be of greater economic value than that provided by the less dependable and more restricted inland waterways.

The traffic handled on the Ohio River, excluding that moved from shore to shore by the ferries, for the period 1918 to 1923, inclusive, is as follows:

Year.	Tons.
1918	6 171 413
1919	5 004 337
1920	9 382 463
1921	7 307 880
1922	6 291 825
1923	8 280 520
Total	42 438 478
Average	7 073 080

The tonnage of 1923 was made up as shown in Table 16. It is apparent that the volume of business is subject to quite wide fluctuations, this being brought about by variations in the total volume of available traffic; by the number and extent of interruptions to navigation by low water, ice, and floods; by railroad competition; and, moreover, by railroad congestion.

Of the total tonnage for 1923, minerals (coal) constituted 90%, ore, metals, etc., 5%, and chemicals 2½%, leaving 2½%, or about 200 000 tons of other freight. Assuming 20 tons per car this would amount to 10 000 car loads. Any one of the great railroad systems which serve the Ohio River territory will load this number of cars in from 1 to 3 days.

Under favorable conditions river transportation is well adapted to handling certain tonnage which moves in large volume to consuming industries located on the river banks. The Monongahela River, for example, is used by the steel mills and other industries in the Pittsburgh District for transporting coal from the mines to the plants. This coal is dumped by the mine tipples into barges and is unloaded from them by bucket conveyors or other mechanical devices, so that transportation is afforded at low cost to the users; but even in this case there are substantially no common carriers in operation, and the river bears the same relation to the users as an industrial railroad bears to the industry which it serves.

In the Annual Report of the Chief of Engineers, U. S. Army, 1924, the following items in connection with the Ohio River improvements will be found:

Net amount expended on all projects to June 30, 1924	$76 631 552
Total appropriations to date of this report	86 406 577
Amount estimated required to be appropriated for completion of existing projects	24 309 600
Total estimated to complete projects (appropriated and to be appropriated)	110 716 177

TABLE 16.

Commodity.	Tons.	Percentage.
Minerals (non-metallic)	7 483 126	90.4
Ores, metals, etc	389 792	4.7
Chemicals	208 069	2.5
Wood and paper	141 673	1.7
Vegetable food products	35 068	0.4
Animals and products	13 392	0.2
Machinery and vehicles	3 531	0.1
Textiles	2 041	
Miscellaneous	3 828	
Total	8 280 520	100.0

Interest on the total estimated cost of the completed projects at 4½%, amounts to $4 982 228 per annum. The cost of operation and maintenance of the Ohio River locks and dams for the fiscal year ending June 30, 1924, was $1 420 575. Assuming a proportionate increase in these costs as the project is continued, the amount per annum on completion will be $2 052 427; adding the interest to this it will be found that a total annual charge of $7 034 655 will be required by the Ohio River waterway.

It has not been practicable to make a reliable estimate of the cost of handling miscellaneous freight by river transportation because of the lack of accurate information and the quite wide variation in the methods followed by the different users of the river. Believing, however, that some comparison of the cost of river and rail transportation would be of interest, an effort has been made to determine the cost of handling coal, which amounts to 90% of the river traffic, from inland mines to a river point in the vicinity of Huntington, thence by river to another river point in the vicinity of Cincinnati, and also by river and rail to an inland point beyond, as an example of which Indianapolis, Ind., has been selected. Although admitting the probability of considerable variation in the cost of dumping, towing, and elevating coal, it is thought that the estimates given in Table 17 are sufficiently accurate and representative for the purpose.

Table 17 indicates that a probable saving by river transportation of 28 cents per ton to river points (rail delivery), and of 12 cents per ton to inland points can be made. For river-side industries the saving would be about 50 cents per ton. Any large increase in this tonnage would undoubtedly have to go principally to inland destinations. Assuming 20 cents as the average, the river traffic would have to reach a total volume of approximately 35 000 000

tons for the saving to equal the interest charges on the capital expenditure and the annual cost of operating and maintaining the river improvements. This is four times the tonnage of the peak year and about five times that of the average six-year period ending with 1923.

TABLE 17.

	To river point (Cincinnati) per ton.	To inland point (Indianapolis) per ton.
Railroad rate, mines to river	$0.50	$0.50
Dumping cost (estimated)	0.06	0.06
Towing cost "	0.55	0.55
Elevating cost "	0.25	0.25
Railroad switching rate	0.25	....
Railroad rate, river to destination	....	1.01
Total	$1.61	$2.40
Railroad rate, mines to destination	$1.89	$2.52

An effort has been made to ascertain what effect the lower cost of river transportation has on the prices of coal in the communities along the river having both rail and water transportation. No reason could be found for believing that the river has any appreciable influence. The price of coal, both retail and wholesale, seems to be fixed on the basis of the rail rates, so that any saving in the cost of transportation by river, as compared with that by rail, is not transmitted to the consumer.

The formation of ice in the Ohio River during severe winter weather renders it ineffective as a means of transportation for one to three months at a time. During such interruptions the burden of handling the tonnage normally moved by the river is thrown upon the railroads. It is during severe weather that the railroads are subjected to the greatest load, not only because the effectiveness of locomotives is from 20 to 50% less than during summer weather, due to increased resistance and the greater demands on the boiler, but also because the consumption of coal and other fuel for industrial and domestic uses is greatest during these seasons. If, therefore, the railroads are to handle the normal river traffic under such conditions, it will mean either one or two things:

1.—That they will have to provide facilities, consisting of locomotives, cars, engine-houses, shops, yards, etc., to take care of this business for a short time each year, or at intervals of two or three years, the cost of which in interest, taxes, and maintenance charges will have to be borne by the other traffic through a higher general average of rates.

2.—If such additional facilities are not provided, the normal railroad traffic increased by the tonnage from the river during severe winter seasons will so tax the railroad facilities that the entire railroad transportation system in the river territory will become congested, traffic will be seriously delayed, the cost of handling it will be unduly increased, and industry and business will suffer. At the same time the capital expenditures involved in the river

improvements and the steamboat and other shipping facilities will lie idle. This means, of course, that the rates for river transportation must be high enough to cover the fixed charges and operating costs of the river transportation companies during these periods of enforced idleness.

There is little doubt but that through the proper development of their facilities all the transportation required in the territory served by the Ohio River can be provided by the railroad alone at less cost than the same transportation can be provided jointly by the river and the railroads. It is true that by far the greater part of the capital required for river development is furnished by the United States Government, and that the comparatively few industries and individuals who make use of river transportation contribute a very small part toward the interest and carrying charges of such improvements. Nevertheless, these charges have to be borne by some one, and, in the last analysis, they constitute a burden in the form of taxes on industry in general, the railroad industry not escaping its share.

L. D. Cornish,* M. Am. Soc. C. E.—There is an important improvement that is under construction by the State of Illinois, namely, that of a 65-mile stretch of the Des Plaines and Illinois Rivers to connect the Great Lakes with the Mississippi River System. That improvement runs from Lockport, forty miles south of Chicago and the terminus of the Chicago Sanitary Canal, down to La Salle, which is the head of navigation on the Illinois River. The Illinois Waterway will have locks of the same dimensions as those on the Ohio River, consequently, the tonnage capacity will be 9 000 in one cargo; the capacity of the entire waterway per year is estimated at 60 000 000 tons.

The paper by Colonel Kutz is a valuable and timely contribution on the subject of inland waterway transportation. In the past much has been said about the engineering design or construction of waterways, but not enough about the economics of waterway transportation. Such inland waterways as this country possesses have been obtained against the persistent opposition of powerful interests that now maintain that the improvement and maintenance of waterways is unsound, that waterway tonnage is decreasing, and "that all the canals and other inland waterways planned would not make a dent in the American transportation problem", to quote from a recent statement† by Mr. Charles H. Markham.

A summary of Columns (3), (4), and (5) of Table 3 of Colonel Kutz's paper shows that the tonnage in 1924 was 31% greater than in 1913, and 10% greater than the average for the last five years. This indicates a very healthy condition in view of the fact that the main line, the Ohio River, is not only incompleted as to its total length, but has several widely separated incompleted sections which prevent the economical development of through transportation. If statistics should be gathered to show the amount of money that has been expended by private capital in the past three years, and is planned to be spent in the next five years, in the development of terminals and floating plant for the Ohio River System, the total would give a better indication of

* Asst. Chf. Engr., Div. of Waterways, State of Illinois, Chicago, Ill.

† *Chicago Daily News*, February 23, 1925.

the future of transportation on the Ohio than can be secured from tonnage statistics of the past and present.

In any question of national importance, much depends on the point of view, and this is true of waterways. Many advocates see only the waterway side; its opponents, only the railroad side, or the point of view of materialistic capitalists. The latter ask, Will it pay interest? If not, away with it. This is the view expressed by the National Waterways Commission in the paper from which extracts have been quoted by Colonel Kutz, who expresses a similar thought when he considers the right of the people to "have the products of the farm, mine, and factory move from origin to destination by the cheapest means". Note his use of that little word "move", then reflect on the periods of railroad congestion during the past twenty years when products could not be moved, or could only be moved so slowly that thirty or sixty days' credits were due before the goods were received. Also, reflect on the fact that tonnage doubles every decade, and then try to estimate what the transportation needs of this country will be thirty years hence.

Professor Luther forecasts the future national prosperity, and places transportation as the most cardinal requirement. It is the speaker's belief that thirty years hence the need will be so great that no method or artery of transportation will be considered uneconomical. If such be only partly true, then it is the duty of this generation to give proper consideration to the needs of the next, and to continue improving the principal waterways. If such standards had prevailed in the last century as prevail to-day, the United States would not be the leading nation of the world, because one-half of the present railroad mileage would probably never have been built, for fear of the railroads becoming bankrupt. In fact, about two-thirds of those railroads did become bankrupt. Yet who would dare assert now that the construction of those thousands of miles of railroads was not justified?

If such standards should govern waterway improvement, they should also govern highway construction. Fortunately, the people of the State of Illinois and its Chief Executive do not judge the needs of that State by those standards. If they did, Illinois would not be, as it is to-day, the leading State on hard road construction. The State has approximately 3 000 miles of hard roads completed and expects to build 5 000 more. Judged by economic considerations of interest, maintenance, and operating cost, it may be unsound, but judged by the need of the people of Illinois to move, not only their goods, but themselves, they feel that it is justified.

Charles S. Churchill,* M. Am. Soc. C. E.—The paper by Colonel Kutz, brings out many points of interest and value in connection with the great improvement work on the Ohio River; and the tabulated statements, tables, and diagrams contained therein have enabled Messrs. Alfred and Begien to make correct and careful comparisons between the true costs of such improved transportation and the corresponding costs of railroads. On this account, as well as many others, the paper by Colonel Kutz is to be commended. The discussions of Messrs. Alfred and Begien contain very reliable statements

* Vice-Pres., in Chg. of Purchases, Real Estate and Valuation, N. & W. Ry., Roanoke, Va.

of railroad statistics which have been compared with the statistics and statements of Colonel Kutz, and deserve the careful consideration, not only of men engaged in, and users of, river transportation, but also an equally careful consideration of men engaged in, and users of, railroad transportation.

When considered in connection with the discussions of Messrs. Begien and Alfred, the paper may become of great use to those officials of the Government who are engaged either in the improvement of waterways, or in the recommendations of further improvements, and especially to citizens who pay their individual share toward the cost of such improvements and who have a right to know that the expenditure is in all particulars a useful one.

That there were no unanswerable objections to these improvements of water transportation on the Ohio River itself seems to be proved by the near and successful completion of the work. The undertaking had a general public approval, and it was not made a political question. However, in the past, some of the improvements by the Government on the smaller rivers have been made on political solicitation and at times against the best judgment of the Government engineers. Some improvements of this kind were made on branches of the Ohio River, and are included in Tables 3 and 5 presented by Colonel Kutz. The speaker refers to some of this work as being of vital interest to the public, both as to the mistaken local or political reason for its undertaking; the consequent small present public use, and also the great relative fixed charges, plus maintenance and operation costs, and consequent loss of public funds; and the importance of preventing other such wastes of Government funds.

In the spring of 1911, the speaker had occasion to interview J. G. Warren, Lt.-Col., Corps of Engineers, U. S. A. (now Col., U. S. A. (*Retired*)), M. Am. Soc. C. E., who was then Chairman of a Board of three Army Engineers detailed to pass on the question of the improvement and enlargement of the bridge of the Norfolk and Western Railway crossing the Ohio River at Kenova, W. Va. After this business matter was finished, mention was made of the expenditure of moneys on rivers smaller than the Ohio. Colonel Warren became somewhat reminiscent and on this subject he said in substance that when he was a Lieutenant and afterward as Captain of Engineers, his recommendations against the advisability of an improvement on such a river amounted to something. Later, with his higher rank, it was often not worth a tinker's dam. In explanation, Colonel Warren stated that politics had often governed the appropriation of Government funds directly against the recommendation of the Government engineering official in charge of the District. One of these cases was the improvement of the Big Sandy River. Another case was a branch of the Ohio River in Kentucky.

Later, the writer met Colonel Warren's Assistant Engineer, at one time connected with the Norfolk and Western Railway, who stated that he had just completed the determination of the maintenance costs of the Erie Canal and had found that these costs were higher per ton-mile than the rates being actually received by the railroads of New York State for carrying the same kinds of freight. These data were regarded by him as proof of the lack of

wisdom in recommending the expenditure of large sums of money on canals, especially in territory where railroads exist.

Colonel Kutz presents Table 5 which shows that the fixed charges alone, in mills per ton-mile, on the Ohio River and tributary improvements averaged 4.6 mills. The large sections of the Ohio itself, together with the Monongahela and Allegheny, averaged 4.2 mills; the Big Sandy, however, was 211.0 mills per ton mile; the Little Kanawha River, 29.7 mills; the Cumberland River, 24.5 mills; and the Muskingum River, 16.1 mills. By making use also of the data in Table 3, which gives the cost of "operation and maintenance" in mills per ton-mile, the total of fixed charges, maintenance, and operation, for the Ohio, Allegheny, and Monongahela are found to have averaged 5.88 mills per ton-mile; whereas the Big Sandy was 288.0 mills; the Little Kanawha, 51.3 mills; the Cumberland, 31.1 mills; and the Muskingum, 24.5 mills.

As the economic value of any great engineering work or improvement can be determined only by a complete knowledge of the fixed charges and the maintenance and operation costs of the entire plant, it would have been better if Table 5 had been made complete by the inclusion of all these items.

Charles Wuest, Jr.,* Assoc. M. Am. Soc. C. E.—Having been engaged in railroad as well as river improvement work, the speaker feels that at present his is a neutral position in a discussion concerning the relative merits of railways or waterways as freight carriers. As engineer for a corporation producing about 200 000 tons of paving brick per year, he now has the viewpoint of a contributor to the shipping wants of the carriers.

Under the head of "Economic Comparisons" (page 1118) Colonel Kutz states:

"If the points of origin and destination and the ton-mileage of each commodity hauled on these rivers are known, as well as the water haulage costs, the direct benefits or savings resulting from their use can be readily ascertained by comparison with existing rail rates, and their economic justification determined by comparing the savings or benefits with the fixed charges."

The question that presents itself is, To whom are these direct benefits or savings to accrue? It is here conceded that waterways afford an easy, cheap, and most natural means of moving heavy, bulky freight, and that the development of the rivers should be encouraged to help bring about a general reduction of the prevailing high railroad freight rates. Let us see, however, how far the latter is true at present. On the basis of recent experiences, there seems to be little encouragement in the action of several operating companies on the Ohio River to produce the much hoped-for low freight rates, because of low transportation costs.

The transportation companies in quoting rates for the transportation of freight by river are not blind to opportunities afforded them by a beneficent Government in opening a way for the conduct of their business, but are totally indifferent about benefits which should rightfully, and, at least, in part, accrue to the public which provides for the way.

In April, 1925, the company which the speaker represents had a prospect of shipping 8 000 tons of paving brick from Portsmouth, Ohio, to Madison, Ind.

* Chf. Engr., The Peebles Paving Brick Co., Portsmouth, Ohio.

Having facilities for shipping by river, as well as by rail, the company requested several transportation companies operating on the Ohio River for rates by barge delivery, in expectation that a low and favorable rate by water would insure the movement. The quotations, with certain restrictions of time in loading and unloading attached, were the same in all cases. The rate quoted was $2 per ton.

Consider the fairness of such a rate in view of what has been said about the costs of transporting coal by water. As coal and paving brick differ little in the classification of bulky, heavy freight, there should be little, if any, difference in actual transportation costs.

Colonel Kutz has stated that, based on the following conditions, 250 miles haul, no back haul, 6 000-ton cargoes in 1 000-ton barges, and wide river and large tonnage, 2.5 mills per ton-mile was a fair measure of haulage cost by waterway. This is practically confirmed by Mr. Alfred, who has placed the cost at 2.7 mills per ton-mile.

By river, the distance from Portsmouth to Madison, is approximately 200 miles. To be conservative, by using the larger of the two figures, the cost of transporting heavy freight for this distance would be 54 cents per ton. The rate asked was $2. This carries with it 270% for profit and overhead. Why?

On other occasions in the past, rates were requested of the same companies for shipments under similar conditions from Portsmouth to Cincinnati, Ohio. As in the previous instance, a rate of $2 per ton was quoted, in one case, equal to, and, in another, in excess of, competing rail rates between the latter points. This distance by river is approximately 110 miles. From Colonel Kutz's Fig. 4, the cost per ton-mile for this distance is shown as approximately 3 mills, therefore, the actual cost of transporting 1 ton of heavy freight, based on Colonel Kutz's specifications, between these points would be 33 cents. The rate quoted was $2 per ton, which carries with it 506% for profit and overhead. Why?

This would seem to indicate that the river transportation companies are basing their rates to river towns on rail competition, or as near to it as they deem safe. It would indicate, too, that this is the reversal of a condition mentioned by Mr. J. H. Bernhard in referring* to the practice of rebating arising from "the unjust practice of railroads in basing rates to river points on water competition."

In conclusion, it may be said that a regulation of such conditions is in order, and that fair rates for river hauling must be established if the work of making the rivers into navigable waterways is to proceed unchallenged.

The speaker wishes to express his disapproval of the practices cited, which he believes tends to destroy the real purpose of the splendid work being done by the U. S. Corps of Engineers in the improvement of the national waterways.

C. E. Grunsky,† Past-President, Am. Soc. C. E.—This discussion indicates the attention which engineers are to-day giving to the economic aspects of their problems.

* *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), p. 946.

† Cons. Engr. (C. E. Grunsky Co.), San Francisco, Calif.

What would this section of the Middle West be like if there were no Ohio River and if this river had not fixed the transportation rates? The railroads have been compelled to take the water competition into account. In this connection a proposed canal comes to mind which was under consideration about thirty years ago but was never built. It was an easy matter to show that if this canal had been built, as proposed, for the purpose of moving grain crops out of two counties, it would have compelled a reduction in railroad freight rates which, from the standpoint of the two counties mainly involved, would have justified its construction even if it was never actually used for freight transportation. It might have been unfair to the railroads. Some one would have had to pay the cost.

It is only by a thorough study of the economic aspects of such projects that the question of undertaking them can be satisfactorily answered. The economic problems are equally as important as those of an engineering character.

Charles A. Wilson,* M. Am. Soc. C. E.—These papers and the relative discussion raise the question as to rates for transportation by water and rail and the basis on which such rates are now fixed in the United States. The Interstate Commerce Commission has always been inclined to base rates on the cost of the service and, in this, it has been abetted by Congress. It could depend perhaps on no other basis for support by the judiciary of the Government.

The complaint of Mr. Wuest against the water carrier was based on his desire to have the value of the service considered in fixing the water rate. Great efforts have been made before commissions in rate cases to have the value of the service considered. To-day, there is no rule or basis for fixing rates and, in the speaker's opinion, there is no scientific basis for fixing rates alike for all carriers. If the law of supply and demand is sound economics, then the value of the service should be considered.

The railways running along the valley of the chain of the Great Lakes are apparently successfully competing with the water carriers using the lakes. The tonnage of the carriers using the Great Lakes is tremendous, yet there is no serious competition between the lake carriers and the railways. This water transportation carries very little between producer and consumer except by additional services of the rail carrier.

Rail transportation and water transportation can never be considered as competitive. Highway and waterway transportation differ in their functions and capacities for transportation service and if any are to be governmentally regulated all should be regulated by the same agency and that service assigned which each is fitted by its nature to perform best and most economically.

The comparisons disclosed by the papers and the discussion show no cost for the use of waterways, and without such charges a comparison with the railways is not a fair one. If the railway, through the special characteristics of its type of transportation, can deliver a car at the warehouse of the shipper

* Cons. Engr., Cincinnati, Ohio.

(and that service is impossible for the waterway carrier), then before a fair comparison can be made between water and rail transportation the cost of delivering the water-borne freight from the waterway to the store door must also be added to the cost of water transportation. One of the principal functions of the Government is to deal justly between all its citizens, but to encourage some by charter to build railways from their private funds and then for the Government to build other ways, such as highways and waterways, out of public funds and give their free use to other citizens, is neither just nor fair and not to be upheld, at least by fair-minded men. The water carrier is forced to use the rail carrier in gathering and distributing the freight constituting the bulk of its tonnage.

The United States Government through its War Department is improving the rivers and seems to be fostering their use in transportation, but in this latter is ignoring the Interstate Commerce Commission. Until all commerce on these rivers comes under the same Government regulating agency, it cannot be expected that the railways will co-operate willingly with either shippers or river carriers in promoting river and rail interchange.

To place waterways, highways, and railways on a parity, the use of the waterway, the highway, and the railway should be paid for by the user, and their use and the charges therefor, placed under the same Government regulating agency. Without such regulation it is neither fair nor right to charge the railway carriers as seeking to discourage the development of waterways because the railways do not co-operate with river carriers.

Hunter McDonald,* Past-President, Am. Soc. C. E.—The Cumberland River Terminal, already mentioned, was completed in 1920 out of funds produced by a bond issue of the City of Nashville of $300 000. Its cost consumed practically all the bond issue. It was built on faith that transportation on the Cumberland River would greatly increase in the near future.

A study of the Cumberland River reveals the fact that above Nashville, and for some distance below it, the river flows through a deep gorge which it has eroded through limestone rock. It is bounded, therefore, on each side by rocky hills some distance apart. With the exception of the Tennessee Central Railroad for a short distance below Nashville, it is not, and never will be, paralleled by any railroad on account of the prohibitive cost of construction due largely to its crooked course. The country through which it flows would not sustain a railroad built at such cost. The coal which lies on the upper waters of the Cumberland is beyond the reach of present navigation, and it is a question whether if the river is made navigable as far up as this coal region it would move by water or over the Cincinnati-Southern Railroad, which crosses the Cumberland at Point Burnside, the present head of navigation. Such coal as might be moved out by water would necessarily be from mines adjacent to the river.

If it is incumbent on the Government to furnish transportation to people living along the Cumberland River who cannot hope for rail facilities, then the canalizing of that river for transportation purposes only might have been

* Chf. Engr., N. C. & St. L. Ry., Nashville, Tenn.

justified; but it is not the duty of the nation to furnish at public expense transportation to people living along rivers any more than it is to furnish transportation to communities living along the non-productive branches of railroads. On many such branches, the cost to the present operating companies is such as to require the expenditure of from $2 to $5 for every dollar that is earned. Nevertheless, solvent roads are required in most cases by public authority to continue the operation of these unproductive lines, the burden falling on the stockholders of the carrier as long as the carrier remains solvent. Furthermore, such solvency often depends on the profits derived from through business.

The traffic on the Cumberland River above Nashville is handled by two small steamboats. From Nashville to the mouth of the river, only one boat operates regularly and urgent appeals for support of this line have been made to the citizens of Nashville, such appeals being based upon sentiment and the desirability of controlling competitive railway rates by water competition.

The upper floor of the terminal warehouse at Nashville was rented shortly after its completion and is still rented to a wholesale grocery firm in an effort to meet a part, at least, of the interest on the bonds. It is entirely proper that Nashville should have provided a terminus to take care of the Upper Cumberland trade, but, in constructing the present facilities, the future has been largely and erroneously anticipated.

Mr. Alfred has referred to some extent to conditions in Europe with respect to water-borne freight as compared with those in the United States, but he did not go far enough into the reasons for the existing sentiment in America favorable to freight moving by water. American tourists who travel over Europe come back much impressed by the amount of water-borne commerce on the rivers and inland canals of Europe; but they are ignorant of the conditions which have brought this about.

The industries located along those inland waterways were established and in a flourishing condition many years before railroads were ever thought of. The political influence of these industries and of the people operating craft on the waterways has been so powerful as to throttle the development of the railroads in those countries and they have been prohibited from charging the rates which might have been remunerative but which would take the business away from the water carriers. The result of this policy has been to force the railroads, through foreclosure, into Government ownership, the railroads in Great Britain and a small proportion of those in France being the only privately operated railroads.

Shortly after the Federal Government took over the American railroads, instructions were issued by Walter B. Hines, Assistant Director General of Railroads, to the Regional Directors of the Western and Southern Districts, R. H. Aishton and C. H. Markham, respectively, to investigate and report on the advisability of the Government undertaking to establish and operate barge lines on the Mississippi and the Warrior Rivers. These Regional Directors appointed a committee of which the late John Howe Peyton, M. Am. Soc. C. E., at that time President of The Nashville, Chattanooga and St. Louis Railway,

was Chairman. This Committee held extensive hearings and in May, 1918, submitted its report, which was adverse to the project, such conclusion being based on data fully set out in the report. Having assisted to some extent in the collection of data, the speaker remembers that the five railroads operating between St. Louis, Mo., and Gulf ports had never been congested and were then in position to move speedily and effectively millions of tons more of freight; that they possessed ample ocean terminals; and that congestion on the Eastern seaboard could quickly be relieved if the Government would order the diversion of traffic originating in the West from the congested Eastern ports to Gulf ports and would provide the necessary shipping to take care of such traffic.

It was the conclusion of that Committee that the establishment of the proposed barge lines and their operation by the Government could not be justified under transportation conditions existing at that time.

Now, it is commonly admitted that the Eastern and Western railroad trunk lines were congested during the World War and that they were taken over by the Government largely as a result of this congestion, but, if the speaker remembers the situation correctly, the cause of that congestion was the fact that there was not sufficient shipping at the Atlantic ports to admit of prompt unloading and dispatch of the goods moved to such ports. Everybody knows that damming up the mouth of a river backs up the river. The condition on the railroads was similar. The congestion at the Atlantic terminals prevented further cars being delivered to the seaports and the traffic gradually backed up into the inland terminals, blocking them. This explains to some extent how the blocking of the terminals was progressive and how, by measuring the rate, it was possible to predict when a complete blockade would take place.

Personally, the speaker has never felt that the taking over of the railroads by the Government was a necessary step. If the Government had given the railroads a very short stay of action and if Great Britain had come to the rescue earlier and furnished enough ships to move the goods promptly from the seaports, the railroads would have worked their way out in an entirely satisfactory manner and without Government control.

Harry Taylor,* M. Am. Soc. C. E.—The speaker believes that this discussion and the interest which the Society has shown in this particular question will be of great benefit to the country in general.

There are two factions in the United States that look at river improvement from two different points of view: One thinks that if there were a little more water all their troubles would be over; the other that the rivers are no good anyway, that no matter what is done to them, they will be of no benefit.

The improvements to the Ohio will go quite a long way toward answering the question. It has been stated by Mr. Alfred that if the matter of cost were being considered at the present time, it is doubtful whether the work would be undertaken. Whether that is so, or not, the speaker does not know, but it is perfectly safe to say that if there is no increase in the present business on the river, no one would think of spending the amount of money required to

* Maj.-Gen., U. S. A.; Chf. of Engrs., U. S. A., Washington, D. C.

improve it; but those who have studied the problem do not believe that the present business represents by any means the ultimate amount that will be carried by the river. That may be shown pretty conclusively by looking at what some of the business institutions on the river are doing, such as the steel corporations at Pittsburgh, Pa. Jones and Laughlin and the Carnegie Steel Company are not firms that spend large sums of money in unprofitable business ventures as a rule; and they are spending large sums in the development of floating plants. During the season of the year when there is sufficient water in the Ohio, these companies carry large shipments down the river into the Mississippi, for delivery at St. Louis, Memphis, and New Orleans, and even for shipment abroad. The speaker has recently noted a statement in the press that one of the largest shipments of steel products that had ever left Pittsburgh was recently taken down the Ohio and delivered at Memphis in eleven days from the time of departure. That is not bad time for a railroad, so that the element of slow movement can be omitted.

In making recent allotments for the river improvements, the Ohio River has been given very liberal consideration by the Federal Government. The engineers engaged on this work would like to see the improvements finished, and, with ordinary luck, it is hoped to be able to finish them in the fall of 1929.

That there has not been a very large through movement of freight on the river to date is natural, when it is considered that during much of the year there are only 2 or 3 ft. of water at the lower end, making navigation of the river impracticable. With the improvement completed and with the 9 ft. depth throughout, that handicap will be overcome.

Something of the future of the Ohio River can be predicted from what is happening on the Mississippi. As is probably known, the Government is operating a barge line between St. Louis and New Orleans. The equipment was built during the World War, and a large part of it is not adapted to the service. The towboats are too large and heavy, with too deep draft for the river between Cairo and St. Louis; some of the barges were not exactly suited to the business, and the towboats were picked up where the management could find them; but notwithstanding these physical handicaps, that barge line is developing a large and profitable business.

In addition to the physical handicap that the line had to overcome, the lack of interchange of freight with the railroads had to be met. At present (1925), the barge line has traffic arrangements with 165 railroads, so that it covers practically the entire country. It has been of great benefit to the farms in Kansas. In a pamphlet recently published by the National Rivers and Harbors Congress for distribution among the agriculturists is quoted a statement from an address before the Twentieth Convention of the National Rivers and Harbors Congress held at Washington, D. C., on December 10, 1924, by the Hon. F. E. Frizell, of Larned, Kans., who is a farmer, a cattle raiser, and a member of the State Senate. He said:

"We have learned that the Federal Barge Line between St. Louis and the Gulf has made a substantial reduction in the freight rate on wheat shipped from Kansas for export. The rate from Kansas to New Orleans, all rail, is

30½ cents per 100 lb., while the rail-and-barge rate is 25 cents, a saving of 5½ cents per 100 lb.—something over 3 cents per bushel saved by shipping our wheat from Kansas to either St. Louis or Cairo to reach the northern terminus of the Federal Barge Line. I have it on good authority, in fact from experts, that with the Missouri River properly improved to Kansas City, Kansas, a saving of another 3 cents per bushel could be made to the farmers of Kansas.

"Since the farmer, in effect, pays the freight on all of his wheat to Liverpool (since that market fixes the price of all the wheat whether the wheat is actually exported or not), when we compute the value of Missouri River navigation to the Kansas wheat farmer, we cannot merely calculate the saving on the entire Kansas crop, because, if you raise the price of the export wheat 3 cents per bushel by reducing the cost of the freight to Liverpool by 3 cents, you automatically raise the price of all the wheat crop in Kansas.

"A reduction of 3 cents per bushel in the freight to New Orleans means a saving of $4 500 000 on the 1924 Kansas wheat crop of 150 000 000 bushels. With the Missouri River improved, a net saving of 6 cents per bushel could be made, which would mean a saving of $9 000 000 to Kansas wheat farmers for the year 1924.

"It is estimated that the State of Kansas has raised and sent to the markets of the world an average of 100 000 000 bushels of wheat each year since the plan to improve the Missouri River was first made fourteen years ago, in 1910. That would make the stupendous total of 1 400 000 000 bushels that has been shipped out by Kansas wheat farmers during the fourteen years we have been patiently waiting for Congress to furnish the money to improve this great water highway. It would have cost not to exceed $20 000 000 to improve the river to a permanent depth of 6 ft., but it has cost the wheat farmers of Kansas more than $40 000 000 not to improve the Missouri River as was recommended by the Engineers Corps."

In the fall of 1920 crops were good, the demand was urgent, and prices were high, but the railroads could not furnish cars and the crops remained unmarketed until the demand had been supplied from other sources and prices had been cut in half.

That was a case where, while the river probably could not carry all the wheat that was offered, it might have carried an appreciable portion of it. The small relative amount of commerce carried by the Ohio compared to that carried by the railroads was also referred to by Mr. Frizell. The Panama Canal carries a relatively small commerce as compared with the transcontinental railroads, but, as is well known, the Panama Canal has had a marked effect on transportation conditions in the United States.

The rivers, the speaker believes, should be made a part of a great transportation system; they must fit in as parts of the system. They cannot operate by themselves; they can only reach the cities on their banks. Operating in connection with the railroads, they can reach practically every part of the United States, as the barge line is doing. As to the barge line, it may be stated that, while it is a Government-owned transportation system at present, operated under the authority of Congress, the endeavor is to establish a successful line and then sell it to private transportation interests. The Government has not authorized that line to continue indefinitely. The speaker doubts whether any private corporation could have financed it through its struggle for existence to build up a business. It appears to be reaching a condition, however, where its success is assured, with more business offered to it than it

can possibly handle; and it has these interchange rates established throughout the country.

Reference has also been made by Mr. Alfred to a waterway connecting the Great Lakes and the ocean. The speaker does not know what particular waterway was referred to, whether it was the St. Lawrence Waterway, the one across the State of New York, or that across the State of Ohio. They have all been under consideration.

Several years ago, Congress authorized an investigation of the practicability and advisability of building a barge canal to connect the Ohio River and the Great Lakes. This subject is still under investigation. Congress has also authorized the investigation of the question of the construction of a ship canal across the State of New York. Little progress has been made on this investigation to date, but it may be said that before the existing Barge Canal across the State of New York was constructed the State authorities went into the question as to whether the canal to be constructed should be a barge canal or a ship canal. The ship canal project was rejected as too expensive. Since that time, the cost has increased very greatly.

The St. Lawrence River is being intensively studied unofficially at the present time by Herbert Hoover, Hon. M. Am. Soc. C. E. Probably that waterway will some day be improved. When or how cannot be stated, but on the St. Lawrence River to-day there is being wasted about 4 000 000 h.p., which waste cannot be allowed to go on indefinitely. The power will have to be developed. With the increasing price of coal and the greater need for power, the time will come when the demand for that development will be so great that it cannot be resisted. It is a complicated question, because it involves international problems; it also involves very difficult, but not unsolvable, engineering problems. When it is developed for water power, it will require a comparatively small additional expenditure for navigation. What kind of navigation will take place is a problem that only the future can answer. Something of the same condition exists on the Tennessee River, which is one of the tributaries of the Ohio. The U. S. Corps of Engineers is making a survey at present of the Tennessee River water-shed for navigation and power development. The indications are that between 2 000 000 and 4 000 000 h.p. can be profitably developed. That will involve a system of dams which will practically canalize the river and will make navigation possible with comparatively little additional expense. With the demand for power in the South, which is increasing very rapidly and which is outgrowing the capacity of the present companies, it is only a question of time when that development will be made. Whatever navigation there will be, therefore, on that river, will be added to that on the Ohio. Taking into consideration the development of the Tennessee and of the Ohio, the growth of the Mississippi traffic, and the rapidly increasing interchange of rail and water freight, undoubtedly in time the improvement of the Ohio River will justify itself.

Upon what takes place on the Ohio and the Tennessee Rivers will depend very largely the development on other rivers. Three or four years ago, the streams which the U. S. Engineer Corps had under consideration were divided

into rivers of first importance and rivers of secondary importance. The Ohio, of course, is included under rivers of first importance. Of an appropriation of about $50 000 000, $80 000 was allotted to the rivers of secondary importance, showing a practical standstill with respect to the smaller rivers. Actually, there was a sum larger than $80 000—it amounted to about $700 000 or $800 000 for all the rivers in the country for maintenance, that is, for keeping them *in statu quo,* so that if they were subsequently improved there would be no great difficulties. If all work should stop, there would be an accumulation of sand-bars, which would eventually cost a great deal of money to remove.

In connection with the smaller rivers there was formerly a great deal of politics which controlled largely the appropriations and improvements. In 1902, conditions reached such a stage that Congress directed that a board known as the River and Harbor Board, be authorized to pass on all improvements. Since then there has been practically no improvement authorized by Congress that has not been recommended by that Board. About 70% of all the projects reported on by the Board have been reported unfavorably. Since 1920, Congress has adopted the method of making lump sum appropriations instead of itemized appropriations. Formerly, Congress directed where the money should be spent; it is now appropriated as a lump sum available for allotment by the Chief of Engineers, so that if anything is spent on worthless engineering projects, the Chief of Engineers is responsible.

The Muskingum River was improved many years ago by the State of Ohio, and turned over to the United States to carry. These improvements cost a considerable sum of money. The Big Sandy River was improved by the Government, as was also the Cumberland River, the first appropriation for the latter stream having been made in 1832. While the ton-miles of commerce on those streams is small, the only money being spent on them at present is that necessary to keep up the improvements already made. Those improvements consist of locks and dams, at each of which there is a small crew. The total expenditures on the Big Sandy amount to a few thousand dollars only per annum. The same is true of the Muskingum. On the Cumberland River System the last remaining dam below Nashville was completed in 1924; the expenditures on that stream will also be very small hereafter unless future developments justify further work above Nashville. So that, to say that what is now being spent or that the improvements now being made are for political reasons, is misleading.

W. M. Black,* M. Am. Soc. C. E. (by letter).—Colonel Kutz's valuable paper should be considered together with the papers by H. B. Luther and Frank H. Alfred, Members, Am. Soc. C. E., relating to the same subject, and the discussion on the writer's paper "Waterway and Railway Equivalents".† The diversity in the opinions expressed by the railway officials on the one hand and by the advocates of water transportation on the other is marked and, to the mind of the writer, unfortunate.

* Maj.-Gen., U. S. A. (*Retired*); Cons. Engr. (Black, McKenney & Stewart), Washington, D. C.

† *Transactions,* Am. Soc. C. E., Vol. 88 (1925), pp. 553-577.

The question of transportation is vital to the prosperity of the people. The solution of the many intricate problems involved in its development should be based on the general welfare rather than on the interests of any industry, locality, or section.

The railway is an essential factor of modern economic life. No other agency of transportation as yet devised can to-day do its work. Yet when it is considered that in times past the transportation agency of primary importance has been, successively, the trail, the waterway, and the railway, and that, to-day, the motor truck and the airplane are successfully displacing the railway car for certain transportation uses, it seems possible that some day, the railway in turn may be relegated to a secondary place.

It is a platitude to state that each transportation agency has its own truly economic place and work, and that the nation is entitled to have its transportation bills reduced to a minimum by having each agency perform its economic share of the work. Such is not the case to-day; and is it not because a destructive competition exists between the owners and managers of the various agencies?

Colonel Kutz's Table 9, giving the fixed overhead charges per ton-mile for transportation on the rivers of the Ohio Valley, shows that with the limited use made of these waterways to-day, not only are some of them uneconomical agencies now, but, also, that their time of usefulness seems past and that the propriety of the discontinuance of operations on them should be considered, in conformity with the policy already followed in the cases of some other waterways, as well as of certain branch railways. The other tables in the papers, together with data given in the discussion, show some of the reasons why the tonnage of the waterways is so limited.

Carriage by water, mile for mile, is inherently less costly than carriage by rail. There is less likelihood of breakage, and, within limits, actually quicker delivery. The advent of the motor truck, which has curtailed short-haul railway service, has only increased the value of the waterways by broadening the territory which they can serve.

Why, then, are not the waterways in greater use in their own sphere of usefulness? The writer believes it is because of the improper conception of the use of railways which has produced in the course of years that cumbersome and unstable structure, the railway freight rate tariff.

This tariff is full of inconsistencies and full of discriminations. It is so complicated that only an expert can use it to discover a rate. In 1921, the writer found it necessary to ascertain the rates on certain ordinary package freight in five cases of movement between large cities of the United States. It required 9¾ hours for an expert of the Interstate Commerce Commission to pick out the rates from the mass of matter he had to go over!

The railway freight rate tariff is a subject concerning which railway officials are very sensitive, both as to freight classifications and freight rates. It is defended by many strong arguments, and under existing conditions some of these arguments are unanswerable. It is, however, an inheritance from the days when railways had to struggle for existence as radical and imprac-

ticable innovations, and from the later days of destructive competition among the rival roads. In those days public service in return for privileges granted was a secondary thought. The policy of the railway managements was based instead on the desire of gain for themselves. "The public be damned" slogan reflects this thought.

The railway freight tariff is an anomaly in the scheme of government of the United States. It is not in accord with the spirit of the Constitution that any body of men, other than duly elected legislative representatives, should be empowered to tax one community of the United States for the benefit of another, to enforce what is really a protective tariff in favor of one city or locality as against another, or to foster one class of products at the expense of another. This, however, is done by the existing railway freight rate and freight classification system. Proposals for any radical changes are strongly resisted by those now enjoying the favors conferred.

The entire structure is artificial and is almost as unstable as a house of cards. Change it anywhere and there is danger of a complete collapse. Is this a healthy condition for the principal agency of the basis of the nation's welfare—transportation?

In order to obtain its transportation at the lowest cost, the nation requires a co-ordinated transportation system, in which each agency—rail, water, highway, and air—shall perform its economic share of the work. Referring to the claim that "the development of inland waterways will relieve the railways of a large part of their freight traffic that is carried at low rates", Mr. C. H. Markham states:* "The truth about the latter argument is that almost any American railroad would starve to death if its heavy freight business were taken away from it, because that is the freight that pays best."

If this freight can be carried more cheaply by another agency, this is not an entirely fair argument from the standpoint of the general welfare. No sane person can wish the railroads to be other than prosperous. If the full use of waterways would, in truth, kill the railways, or even seriously injure their prosperity, some means of providing an adequate income for them must be furnished.

Since transportation is a fundamental element of commerce, it should be organized and managed on the most economic business basis. Should not a transportation company be prepared and able to utilize the most economical and advantageous class or classes of carrier for any specific cargo offered for a given destination? If each auxiliary transportation agency were used for the work it can do best, and the returns from all placed in a common fund, not only would the cost of transportation be reduced, but the destructive competition, with its resulting artificial tariffs, would also disappear. A transportation company should control and utilize the operation of carriers by railway, waterway, highway, and airway along a given line of traffic movement.

It is not fully apprehended how really artificial are the present-day tariff rates, and how little the relative rates are based on the relative costs of trans-

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 555.

portation. In other branches of industry the prices asked for products or for service rendered are based on the cost of production or of the service. For the railways, the cost of the service is known in the aggregate only. The relative cost of the transportation, including the necessary care, of the various classifications of freight is unknown. Similarly, the relative costs of the line haul and of the terminal services have not been determined, although the terminal service charges are "absorbed" in the freight rate charges.

Consequently, when railway revenues become insufficient there is no recourse other than that of an arbitrary increase of one or more of the specific rates. An example of the peculiar attitude taken toward freight rates is afforded by the recent action of the Interstate Commerce Commission in ordering an arbitrary decrease in the rate on a certain class of coal between the West Virginia mines and New England territory. Is it safe or sane to keep so vital a thing as the national transportation on such an unstable basis?

At one time, competition between railways was deemed necessary in order to secure an economic service. To-day, it is recognized that each railway line enjoys a natural monopoly for certain territory, and, further, that a railway affords a public utility service and that it, like other analogous utilities, serves the public best as a regulated monopoly. Is there any valid reason why the service should not be extended to cover other classes of common carriers, and thus to serve the public to greater advantage?

Water competitive railway rates are allowed by the Interstate Commerce Commission on the grounds that such lessened rates are necessary in order to permit the railways to meet the competition of the waterways. Those water competitive rates necessarily imply higher rates for interior points, and, therefore, unequal treatment for similar service. If it be the case that freight can be carried by water more cheaply than by rail (which is denied by some railway experts), and if transportation companies were authorized to operate inland waterway lines, the reasons for the water competitive railway rates would disappear, with the resulting discriminations against interior points. If, as is contended, water transportation is not cheaper than rail, why the water competitive rates?

It is impossible to consider the subject of interior transportation without being confronted with the basic subject of railway freight rates and freight classifications. Without a radical change in these, a harmonious economic system of interior transportation cannot be devised. The subject is so complex and so little is known as to actual costs, that a rational solution of the question cannot be had without an investigation and study which will be long and costly. Is not the question worthy of the time and expense?

Even with a solution presented, changes would have to be made slowly, step by step. In the meantime, cannot a start toward reform be made by permitting the railway companies, under due regulation, to take over and operate interior waterways lines within their respective zones, and the abolition of water competitive railway rates?

M. G. Barnes,* M. Am. Soc. C. E. (by letter).—The writer has been very much interested in Mr. Alfred's paper. Through a series of mathematical

* Chf. Engr., Div. of Waterways, State of Illinois, Chicago, Ill.

exercises the author has arrived at the conclusion that the ultimate cost of haul *via* the Ohio River is 40% in excess of the haul by rail, and that "even this figure is not comparable to the railroad cost per ton-mile, as a ton-mile by river is not the equivalent of a ton-mile by rail."

The conclusion is so opposed to the writer's conclusions as to the relative cost of rail and water haul, that he was impelled to make a more detailed analysis of the paper. He did not think that railroads could compete with transportation by water over such an important river as the Ohio. Mr. Alfred's train of thought reminds one of the argument pursued by the Bureau of Railway Economics in its consecutive *Bulletin No. 293,* in which the Bureau attempted to prove that the cost of transportation *via* the New York State Barge Canal was greater than the cost of transportation by rail. The argument made very interesting, if true, reading, but having reached such astounding conclusions the writer was in that case also impelled to investigate more closely. In checking the Bureau's arithmetic he found that it had quite conveniently misplaced the decimal point in computing the cost of haul by water, which resulted in finding "cost of operation, maintenance, and repair" ten times too great. When this correction was made, and using the Bureau's own argument, it was shown that the cost of haul by water was less than the cost of haul by rail, as one must assume would be the result when the figures are properly applied. Has not Mr. Alfred fallen into a similar error in his train of thought?

The author is challenging the economy of transportation by the Ohio River from Pittsburgh to Cairo as a unit, whereas he is using shipping statistics that apply almost entirely to local freight shipping over a reach of 358 miles of river, which he states was all that was completed in 1923, the year in which his shipping statistics were compiled. Through shipment on the Ohio River over an improved waterway such as the Government is now in the process of constructing, has not yet been possible. Several of the locks and dams are yet incompleted. Manifestly, it is not fair to use the present local shipments as an argument to show the wisdom or fallacy of improving the entire length of the Ohio River. Moreover, the Ohio River is only one unit in a system of waterways, as is one line of a railroad a unit in a railroad system, such as the Pere Marquette, for example. Obviously, it would be unfair to select some part of the Pere Marquette Railroad which might be operated at a serious loss, and argue from that that the entire railroad system is a failure.

The writer has compiled Table 18 as a modification of Mr. Alfred's Table 13, to show comparison between railroad shipments and water shipments over the Ohio River System. The data in the second column of Table 13 are derived from Tables 3 and 4 of the paper by Colonel Kutz.

A similar comparison with the completed portion of the Ohio River tonnage shows that the tons of freight per mile of length is twice as much as on the railroads of the United States.

It will be seen from Table 18 that the length of the improved waterways of the Ohio River System is practically 1% of the length of the railroads of the United States, whereas the freight shipments by the Ohio River System were 1.63% of rail shipments in the United States. Also, that the shipment per

mile of length on the Ohio River System was 70.0% greater than the average shipment by rail over the railroads of the United States. This is not such a bad showing for a transportation system yet incomplete and unusable in many places except for local business. The average haul is short because of the fact that through shipments cannot be made over the improved waterways.

TABLE 18.—Water and Rail Shipments Along the Ohio River.

	Railroads in United States.	Ohio River System, 1924.	Percentage of railroad.
Length, in miles	250 183	2 471	0.99
Tons of freight	2 333 000 000	38 071 601	1.63
Tons of freight per mile of length	9 065	15 400	170.00
Ton-miles	416 211 000 000	2 105 836 000	0.51
Ton-miles per mile of length	1 663 228	852 220	51.25
Cost, including equipment	$22 681 000 000	.....	.....
Cost per mile, including equipment	$90 658	.....	.....
Estimated cost of improvement	.....	142 161 023	.....
Estimated cost per mile of improvement	.....	57 532	.....
Balance for floating equipment and terminals to make cost equal to cost of railroads per mile	.....	33 126	.....
Total for Ohio System equipment and terminals	.....	81 854 346	.....

The estimated cost per mile of improvement on this river system is $57 532, whereas the cost per mile, including equipment, on the railroads of the United States, is shown by Mr. Alfred to be $90 658. This leaves a balance of $33 126 per mile of waterways in the Ohio River System to be used for floating equipment or terminals, or for the total length of the system of 2 471 miles, the handsome sum of $81 854 346 as available for floating equipment and terminals for the Ohio River System before the total sum has reached the average total cost, including equipment, of the railroads of the United States. It will be seen that the amount available for the floating equipment and real estate on the Ohio River System is surely adequate to take care of a very substantial business.

Mr. Alfred has computed the items of interest, depreciation, taxes, and maintenance on the Ohio River as $9 656 000 annually, and that "this amounts to a cost of 1.29 cents per ton-mile based on the volume of traffic for 1923." Here, again, the writer wishes to invite attention to the fact that at this time only 358 miles of the river had been completed, and that no through traffic over the improved waterway was possible.

The author states further "as the volume of traffic decreases, this cost will increase." He fails to state the converse of this—that as the traffic increases, the cost will decrease. Although the writer does not endorse the findings of Mr. Alfred in these four items he will take them at their face value and compute the value of the Ohio River improvement on this basis. By using the argument of H. G. Moulton, M. Am. Soc. C. E., Mr. Alfred arrives at a present cost of haul for coal over the Ohio River from Pittsburgh to Cairo of 2.7 mills per ton-mile. Adding about 10% to this for the shorter haul will give a shipping cost by barge of 3 mills per ton-mile, which is equal to the transportation cost on the Monongahela River over a distance of 42 miles,

as reported by Colonel Kutz, who also states that the rail charge for shipping coal over the same distance is 19 mills per ton-mile, or a difference of 16 mills per ton-mile in favor of transportation by water. Further, Colonel Kutz submits a chart (Fig. 1) showing the cost of haul of coal by rail for distances up to about 600 miles. In Table 12 Mr. Alfred finds that the average water haul for coal and coke in 1923 on the Ohio River was 83 miles. From Colonel Kutz's table for this distance is found the rail charge of about 16½ mills, or, say, 16 mills. This would show a saving of 13 mills per ton-mile in favor of water haul.

In the case of sand, gravel, and stone shown in Mr. Alfred's Table 12, the haul averages only 13 miles, and the difference between the water and the rail haul over such a short distance is even greater than for the coal haul of 83 miles, but for the purposes of this discussion it will be assumed that the saving is the same. In the case of the metal and metal products, Mr. Alfred finds the average haul is 318 miles.

Colonel Kutz shows that for a distance of 311 miles the rail charge for fifth-class freight, which is the same as for steel products, is 18.6 mills per ton-mile. Mr. Alfred feels that the cost by water for these higher class commodities should be greater than the cost for transporting coal. The writer agrees with Mr. Alfred on this subject and suggests an increase of 20% over the coal charge. This would give a cost of haul by water of 3.6 mills per ton-mile for fifth-class freight, such as steel products. It is interesting to note that Theodore Brent, Federal Manager for the Mississippi-Warrior Service, states that the average revenue to that barge line is about 3½ mills per ton-mile. This checks very closely with the writer's figure.

Mr. Alfred argues that if these commodities were forwarded by rail the distance would be less than 318 miles. That is probably true, but, on the other hand, for distances less than 400 miles the cost of haul by rail per ton-mile increases at about the same rate as the distance decreases, and hence this difference in distance is negligible. (See Table 8 and Fig. 3 of the paper by Colonel Kutz.)

TABLE 19.—Economy of Ohio River Transportation.

Commodities.	Tons.	Percentage of total.	Ton-miles.	Average haul, in miles.	Savings per ton-mile, in mills.	Total savings.
Coal and coke	6 014 708	73.0	503 315 936	83	13	$6 543 107
Sand, gravel, and stone	1 372 404	16.6	17 274 151	13	13	224 564
Metals and metal products	389 792	4.7	123 972 709	318	13.5	1 673 685
All other commodities	476 617	5.7	104 557 690	219	15.5	1 620 644
Total	8 280 521	100.0	749 120 486	90	....	$10 062 000

For example, in Table 14, Mr. Alfred finds the average length of water haul to be 44% greater than the rail haul. This relation applied to the 318-mile water haul gives an equivalent rail haul of 221 miles. By interpolation

from Fig. 3 of the paper by Colonel Kutz, the rail rate for 221 miles is 25 mills. The total rail charge then becomes $5.52, or 17 mills per ton-mile of water haul. In the same manner it may be shown that the rail rate per ton-mile of water haul for "all other commodities" is 19 mills. The saving to shippers for these two items is 13.5 and 15.5 mills per ton-mile. Inserting these figures in the totals given by Mr. Alfred in Table 12 gives Table 19.

Table 19 shows that the total saving to shippers on the Ohio River alone for the year 1923 was $10 062 000, which exceeds the $9 656 000 overhead charges found by Mr. Alfred.

Be it remembered that this is on a river 968 miles in length only 358 miles of which had been completed and opened. This is not such a bad showing for a stream in the condition of the Ohio River in 1923. When the Ohio River improvement has been completed steel products can be forwarded on regular schedules from Pittsburgh to St. Louis, Memphis, New Orleans, and other points, and heavy commodities, such as lumber, sulfur, bauxite, sugar, coffee, canned goods, and similar articles, can be brought to Pittsburgh from down-stream points. A very material through commerce will be created, which will show even greater benefits than now exist over a waterway admittedly incomplete. Surely with this increased through traffic the shipping interests will realize the converse of Mr. Alfred's statement and may well expect that as the volume of traffic increases the fixed charges per ton will decrease.

C. I. Grimm,* Assoc. M. Am. Soc. C. E. (by letter).—The paper by Colonel Kutz presents in condensed form much valuable data that have not been readily available to engineers and permits making an intelligent analysis of a subject that has frequently been discussed from an inaccurate or biased standpoint. Both Colonel Kutz and Mr. Alfred have as a result of an analysis reached the same general conclusion, namely, that present savings in transportation costs on the Ohio River are less than fixed charges plus the cost of maintenance and operation. While Mr. Alfred infers that the former never will be as great as the latter, Colonel Kutz estimates that the savings are approximately 60% of the fixed and operating charges on the completed reach of the river, extending from Pittsburgh to Cincinnati. The writer whose duties include the making of detailed studies of this subject accepts this figure as reasonably correct, realizing that the available data do not permit of mathematical accuracy in such an estimate.

Mr. Alfred has based fixed and operating costs on a completed project, assuming that the traffic will never be greater, but probably less, than at present. If his view of the future is accepted success cannot be predicted for the project, but there are many reasons for assuming a far more optimistic attitude. A railroad in an incompleted state and working at the low ratio of capacity that now prevails on the Ohio River, could not make much of a showing in competition with completed and interconnected lines.

The traffic density on the Ohio River at present is less than 1 000 000 ton-miles per mile of river, while its ultimate capacity is easily ten times this

* Asst. Engr., U. S. Engr. Office, Cincinnati, Ohio.

amount. The principal coal roads in the Ohio Valley had a traffic density of 4 000 000 ton-miles per mile of main track in 1922, which was not a year of heavy traffic, and with this traffic density, the fixed charges plus maintenance of way, was 1.9 mills per ton-mile, or about 40% of the corresponding charges on that part of the Ohio extending from Pittsburgh to Cincinnati.

The Monongahela River has a traffic density of 20 000 000 ton-miles per mile on its lower 60 miles, while for the entire 125 miles of improved river, the traffic density is about one-half as great, the traffic on the upper half being very light. It seems obvious that the capacity of the Ohio River exceeds by far that of a double-track railroad, but as its future use will be governed by its ability to get traffic rather than by its capacity to carry this traffic, the latter is not an important economic consideration.

Some further statistics, more or less general, that may be of assistance toward forming a general view of the traffic possibilities of the river are, as follows.

Population.—The population of the Ohio Valley in 1920 was 15 500 000, or 76 per sq. mile as compared with 116 for Illinois, 142 for Ohio, and 195 for Pennsylvania. In these three States, there is 1 mile of railroad for every 650 population. The population of this basin amounts to 10 000 per mile of Ohio River and canalized tributaries. In 1920 the population of cities and towns on the banks of the Ohio was 2 150 000, or 2 200 per mile. The population of the Ohio Basin increased 10.7% from 1910 to 1920 while the cities and towns on the river showed an increase of 13.7%, as compared with 14.9% for the entire United States.

Potential Tonnage.—The tonnage per capita handled by the railroads of the United States has been increasing rapidly as indicated by the following statistics:

1880	6.5 tons
1890	11.1 "
1900	13.2 "
1910	19.6 "
1920	24.0 "

These figures include originating tonnage and that received from other lines. Statistics show that originating tonnage is about 55% of the total, or that it was about 13 tons per capita in 1920, 5 tons of which was coal. The originating tonnage other than coal on the Upper Ohio is particularly heavy on account of the great steel industry practically all the furnaces and mills of which are situated on the river. The annual capacity of the blast furnaces on the Ohio and its tributaries is approximately 20 000 000 tons, with an average production of 15 000 000 tons.

The basin of the Ohio River produces 55% of the 600 000 000 tons of bituminous coal that is mined annually in the United States and nearly 200 000 000 tons of this coal is shipped from mines situated in counties that border on the river or its canalized tributaries.

The originating tonnage in the Ohio Basin in 1920 may be roughly estimated as follows:

Coal	300 000 000 tons
Other tonnage (at 10 tons per capita)	155 000 000 "
Total	455 000 000 "

The ton-mileage based on the average haul for the United States of 310 miles per originating ton is 141 050 000 000, less than 1.5 of which was handled by the rivers.

Growth in Tonnage.—The tonnage handled by the railroads of the United States has increased since 1880 as shown in Table 20.

TABLE 20.—Growth in Railroad Tonnage in the United States.

Year.	Tonnage.	Ratio of 1880.	Percentage of increase.
1880	320 000 000	1.	...
1890	700 000 000	2.18	118
1900	1 000 000 000	3.	43
1910	1 800 000 000	5.63	80
1920	2 500 000 000	7.8	39

The total increase has been equivalent to a steady rate of 66.7% each decade, while the rate since 1900 corresponds to 58% each decade. Since these figures do not include tonnage handled by waterways and highways, it is conservative to predict total future tonnage increases as 60% in each decade.

From the foregoing it appears that the tonnage in the Ohio Basin other than coal may be expected to increase about 100 000 000 tons from 1920 to 1930, while the tonnage of coal as correctly shown by Mr. Alfred will not increase greatly. There seems to be no doubt that the growth in this valley will demand extensive expansion of transportation facilities and that the Ohio River System if it provides economical transportation may easily absorb during the next twenty-five years a sufficient tonnage to justify the investment in locks, dams, terminals, and fleets, without depriving the railroads of the business they require for profitable operation, providing there is proper co-ordination between the railroads and this waterway system.

Coal has always been an important item in Ohio River tonnage as it is with the railroads in the Ohio Valley. The Ohio River tonnage from 1890 to 1925 was 60% coal. The 1924 tonnage was 53% coal and coke and 34% sand and gravel, with a similar percentage indicated by the 1925 statistics. The tonnage of the Chesapeake and Ohio, Norfolk and Western, and Hocking Valley Railroads was 80% products of mines (mostly coal) in 1922. The ton-mileage of coal, sand, and gravel handled on the Ohio is at present about 75% of the total, due to the fact that the average haul of coal, sand, and gravel is short. The average haul of coal on the railroads mentioned is long and their ton-mileage of coal will be at least 75% of the total.

The accompanying diagram (Fig. 6) shows, as far as records are available, the total Ohio River coal tonnage as well as the shipments from Pittsburgh to the lower river and shipments from Huntington, W. Va., and the Kanawha River. The former has disappeared, but the latter is now greater than at the date of opening the competing Chesapeake and Ohio Railroad in 1889, while as a result of developments in the steel industry other movements of coal have developed to replace the loss of the large tonnage of Pittsburgh coal moving to the lower river. The steel industry is extending down the river and its further development promises continued growth in tonnage, of river coal.

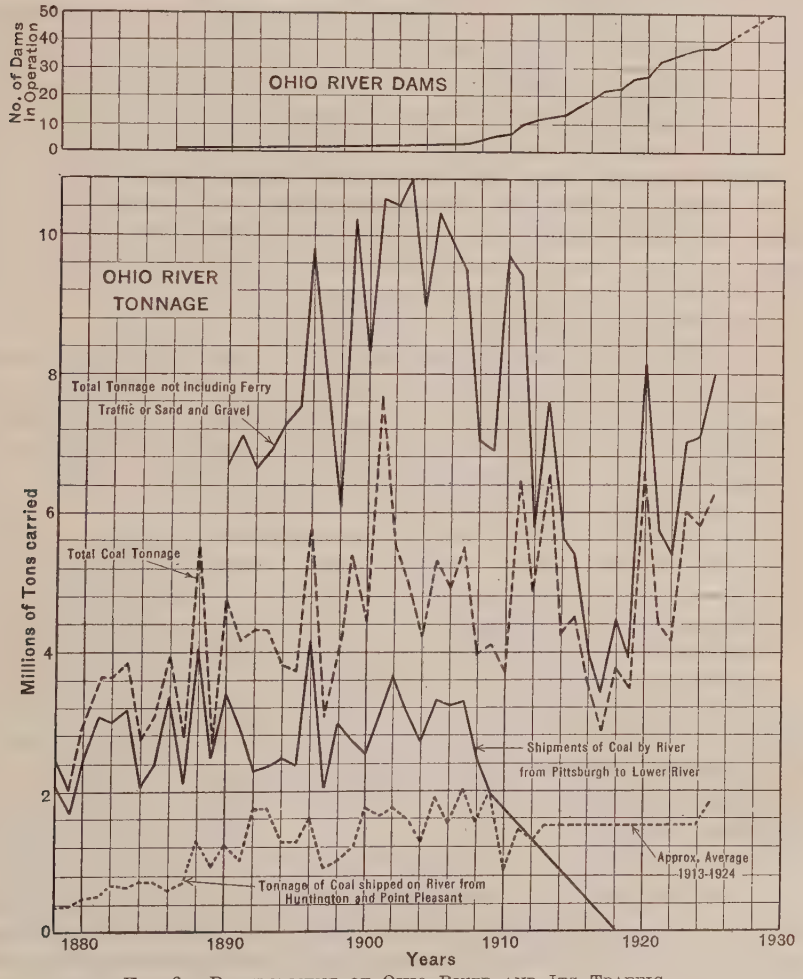

FIG. 6.—DEVELOPMENT OF OHIO RIVER AND ITS TRAFFIC.

The rapid growth in the demand for electrical energy will result in a large expansion of steam power plants in this great coal region and these plants by reason of their water requirements must locate on large streams. As the cost of transporting coal from the mines to the load center of a power system is

generally equal to, or less than, the cost of transmitting the power from a power plant at the mines to this load center, it is evident that steam plants having their load center near the Ohio River, or to the north and west of it, and their coal supply in West Virginia or Kentucky, will ship this coal to power plants on the Ohio. This is actually indicated by the action of two large power plants on the Ohio that receive their coal by river.

The increased use of oil as fuel, while it tends to decrease the rate of growth in coal production increases the field of water transportation since this commodity is particularly adapted to river transportation by reason of safety, low haulage costs, and low terminal costs. The oil traffic on the Ohio River has grown steadily since 1917, until it is now an important factor. The same may be said of the iron and steel products, and it is estimated that from 20 to 25% of the present Ohio River ton-mileage is in these two commodities, traffic in which has developed almost entirely during this period.

It will be noted from Fig. 6 that there was a decline in Ohio River tonnage from 1905 to 1918 coincident with the decline in shipments of Pittsburgh coal to the lower river, while from 1918 to 1925 there was a general increase in tonnage coincident with the increase in tonnage of iron, steel, and oil, and partly composed of these commodities.

Colonel Kutz estimates (Table 4) fixed charges on the completed project at 8.29 mills per ton-mile with 1924 traffic, while Mr. Alfred (page 1131), based on the 1923 traffic, estimates 12.9 mills. It is evident that traffic must increase so as to reduce the fixed charges well below these figures in order that the project may be justified.

From a former discussion by the writer* are quoted data (Table 21) relative to coal-carrying railroads.

TABLE 21.—Statistics of Ohio River Valley Railroads.

	Pittsburgh and Lake Erie Railroad.	Buffalo and Lake Erie Railroad.	Hocking Valley Railway.	Norfolk and Western Railroad.	Chesapeake and Ohio Railroad.
Products of mines, percentage..	64	90	83	80	79
Average haul, in miles.....	60	108	117	279	263
Average receipts, in mills per ton-mile.....	13.3	7.76	7.84	7.45	6.86
Average receipts per ton.....	$0.86	$0.84	$0.92	$2.08	$1.88
Interest on road plus maintenance of way in mills.....	2.31	1.58	1.96	1.82	1.60

To reduce fixed charges, plus cost of maintenance and operation on the river, to a figure as low as the corresponding charge on these roads, the river traffic must increase fivefold. This is, however, not a true criterion since the sum of other river transportation costs is less than those of railroads, as shown in the discussion referred to.

The average haul on the Ohio River is about 80 miles and the traffic as a whole is comparable with that of the short-haul (first three) railroads listed,

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 571.

but is practically all one-way, whereas these railroads have about 65% of their car-mileage in loaded cars. Taking these factors into consideration, it is estimated that the cost of handling all Ohio River traffic by rail would be $1.00 per ton, or 12.5 mills per ton-mile of river distance. The probable water cost is 5 mills per ton-mile, or $0.40 per ton, and the terminal differential is estimated at $0.20 per ton, leaving a saving of $0.40 per ton as compared with rail shipments. On this basis, the tonnage on the completed project must increase to 20 000 000 or 25 000 000 to effect annual savings equal to annual costs carried by the Federal Government.

The foregoing estimate involving the use of averages is subject to many inaccuracies and a detailed estimate based on an analysis of the savings on each specific movement is the only method of obtaining more accurate results. As shown by Mr. Begien, one of the important river hauls is a part of a longer through haul and this is true of many other river hauls. For such cases the estimate of savings is not simple, and results for specific movements may be widely different from those obtained by the methods of the preceding estimate. Data are not available for an analysis of all river movements, but a consideration of the major movements indicates that the foregoing estimate based on average conditions is reasonably correct.

As pointed out by Messrs. Alfred and Begien, dependability of service is a recognized advantage in a transportation facility and in this respect the railroads that compete with the Ohio have been far superior to the river in its uncanalized or partly canalized state. However, with the completion of the improvement, the railroads will possess no marked advantage in this respect. Navigation on the river will then be interrupted only by ice and extreme high water. Interruptions by the former will be greatest in the upper part of the river. At Lock No. 11, 76 miles from Pittsburgh, navigation has been suspended on account of ice an average of 7 days per year during the past 7 years, while high water has added 3 days more. On the Monongahela River during the past 25 years, ice and high water have interrupted navigation an average of 9 days per year. Velocities exceeding 3 miles per hour occur on the Ohio about 10% of the time in the steep upper end and about 5% of the time in the lower river, while velocities exceeding 2½ miles per hour prevail about 25% of the time. For an operation that is loaded down stream, towing conditions when the dams are lowered are no less favorable than those prevailing while the dams are up, until the river reaches a stage where the velocity is about 3½ miles per hour, but become less favorable as the velocity increases further and make towing impracticable when a velocity in excess of 4 miles per hour prevails.

The early growth of commerce on the Ohio River resulted primarily from its use in connection with marketing, which use gradually declined because of its inability in an uncanalized or partly canalized condition to meet railroad competition in this field. The recent increase in river tonnage has resulted primarily from its use as an agency of production. The former use resulted in long hauls of various products, while the latter involves shorter hauls of raw materials. It is recognized that extensive foreign and domestic markets to the south and southwest are open to the industries of the Ohio Valley and

it is expected that the improved facilities offered by the river when it is completed will again build up its marketing commerce. An industrial survey report made by the Technical Advisory Corporation of New York, N. Y., for the City of Cincinnati, refers particularly to these markets for Cincinnati products and states as follows regarding the Ohio River: "The Ohio River improvement is to be completed in 1928. This simply gives Cincinnati an opportunity—nothing more".

The situation as viewed by the writer is summarized, as follows:

1.—The Ohio Valley with its great mineral and other resources is one of the richest in the world.

2.—Population and industry in this valley will keep pace in growth with that of the nation.

3.—Industry and population of the valley will center in the future, as they have in the past, on the Ohio River.

4.—The Ohio River where canalized offers an economical and dependable means of transportation.

5.—With equitable relations between it and the railroads which are recognized as the major transportation agency of the valley, this river will carry in the future a commerce that will justify the expenditures made on it by the Federal Government.

Frank H. Alfred,* M. Am. Soc. C. E.—One of the most useful functions of the Society is the discussion of subjects like this. When discussion precedes the inception of the project, it is even more valuable than when it follows it, as in the present instance. Difference of opinion is to be expected and is desirable. Each party considers the subject from his own standpoint and it is thus illuminated from many angles. When all the facts have been assembled, there should not be much difficulty in agreeing on a conclusion. Twenty-eight engineers recently rendered a unanimous report on the causes and remedies of low water on the Great Lakes, in connection with the diversion of water for the Chicago Drainage Canal; thus affording a remarkable example of the openness of mind with which the engineer should approach his problems.

When internal waterways is the topic, however, especial care should be taken to distinguish facts from fancies; for this subject has been so obscured in the popular mind by a sort of sentimental haze that it is easy to mistake one for the other and to accept bald assertion as Gospel truth. It is astonishing to find this lack of discernment in quarters where it would be least expected. For example, General Taylor quotes from a Kansas enthusiast, presumably with approval. This quotation is taken from a pamphlet "published by the National Rivers and Harbors Congress for distribution among the agriculturists." It is a fair sample of propaganda on this subject and is also an excellent illustration of appeal to a special interest without considering the cost to others. This quotation in part is as follows:

"Since the farmer, in effect, pays the freight on all of his wheat to Liverpool (since that market fixes the price of all the wheat whether the wheat is actu-

* Pres. and Gen. Mgr., Pere Marquette Ry., Detroit, Mich.

ally exported or not), when we compute the value of Missouri River navigation to the Kansas wheat farmer, we cannot merely calculate the saving on the entire Kansas crop, because, if you raise the price of the export wheat 3 cents per bushel by reducing the cost of the freight to Liverpool by 3 cents, you automatically raise the price of all the wheat crop in Kansas."

This is followed by a computation which purports to show that the farmers of Kansas have lost $40 000 000 in 14 years on account of the lack of improvement on the Missouri River. This is based on an estimated saving in cost of transportation of 3 cents per bushel, a figure obtained "on good authority, in fact from experts." Assuming the existence of an expert wise enough to know what reduction in the freight rate would accrue, who pays the extra 3 cents to the Kansas farmer? Evidently, the person who buys his wheat. If 10% of this wheat is exported, 90% of the fortunate Kansan's increased emolument is contributed by his fellow citizens. If only 1% goes to Liverpool, they contribute 99% to his added profit. Statistics on this point are lacking. It would be of interest to know just what portion of the Kansas wheat crop finds its way to European markets, and also to know how the domestic consumer is to be convinced that he should pay Federal taxes for the privilege of contributing to the Kansas farmer's welfare.

In the case of river improvement, however, there is another difficulty which the twenty-eight engineers previously referred to did not encounter. They were dealing with facts, but any present conclusion as to the ultimate value of river improvements must depend on a forecast of their future use. It is easy to agree as to conclusions drawn from established facts; but where they must be based on probable future happenings, perhaps it is too much to ask even an engineer to lay aside the rose-tinted spectacles of hope.

The speaker has no prejudice against inland water transportation, and the railroads have no fear of it as a competitor. Many railroads are now engaged in it in some form or other and the speaker himself is "Rear Admiral" of a fleet of seven large car-ferries that operate across Lake Michigan, and are a very important part of the Pere Marquette's transportation system. Incidentally, he knows that the cost of this lake transportation per ton-mile of freight is as high as by rail, in spite of the fact that no maintenance-of-way expense is incurred. He maintains, however, that a broad distinction should be made between transportation on a great natural channel like the Great Lakes, or a great artificial channel like the Panama Canal where tolls are collected, and transportation on rivers, where immense sums are necessary to construct and maintain channels required even for boats of light draft.

Based on an experience of many years in the transportation field, his conclusion is that the future holds little of promise for river transportation, for the simple reason that except for the limited amount of traffic which originates and terminates on or near the river banks, it costs more to move freight from shipper to consignee by river than it does by rail. Transportation by river also requires more time and is less dependable. It is for these reasons that the rivers lost their traffic to the railroads and have never regained it and, in the speaker's opinion, never will regain it, no matter how much is spent on improving their channels. Patrick Henry's dictum as to how to judge the

future still holds, and there is certainly nothing in the past history of traffic on the Ohio River to justify optimism. The expected increase in traffic has always been just around the corner, where it bids fair to remain.

The railroad takes freight at the shipper's mine or factory and delivers it at the consignee's warehouse. This the vessel cannot do, unless both shipper and consignee are located at the water's edge. No comparison of transportation cost by rail and river is possible unless the terminal cost of delivery is included in the latter. The railroad rates include this cost on all but a negligible part of goods handled, but neither this cost nor any profit to the carrier is included in the figures which have been given in this discussion as representing the cost of river transportation. To give an idea what terminal costs amount to, the Pere Marquette Railway Company in 1923 had nearly one-half as many switching miles with locomotives at terminals as it had road miles in hauling the goods.

As to the alleged necessity for improved waterways to care for the greatly increased traffic of the future, why not let future generations attend to this if and when the need develops? The Ohio River improvement when complete will have cost $114 000 per mile, enough either to build and equip a double-track railroad, or to add a second track to each of the three existing single-track lines, which would give each railroad a capacity equal to at least ten times the present tonnage carried on the river. At this rate it will be a long time before extensive river improvements are called for on this account. Give the railroads a fair chance—which means simply enough return to attract the capital necessary for additions and betterments—and they will render cheaper and better service even to river communities than can be given by the river, and there will be no more of the congestion to which reference has been made.

It is hardly fair to refer to congestion that occurred under war-time conditions, or in 1920 when the railroads had not recovered from Federal control. The railroads are progressing, just like everything else. The total tractive power of locomotives used in the United States increased from 1 932 000 000 lb. in 1914 to 2 673 000 000 lb. in 1924, an increase of 38% in 10 years. In the same period the number of freight cars increased 16% and the average capacity of the freight car increased 12%, resulting in a total increase of freight car capacity of 30 per cent. In addition, there has been a large increase in the average distance which each freight car moves per day. Traffic in 1923 was 414 000 000 000 ton-miles as against 288 000 000 000 in 1914, an increase of 44 per cent. Taking all factors into consideration the railroads are capable of giving at least 60% more service now than they were ten years ago. In the second week of January, 1925, they handled the largest tonnage of coal ever handled in the history of coal mining or railroad operation. They actually moved 12 500 000 tons of coal in this one week. They never felt it; in fact, they did not know until afterward that they had moved nearly twice as much coal as in the previous week.

Let waterways be improved where the economic value of such improvement can be demonstrated, but let not assumption be mistaken for proof. Further,

let it be assured that the resulting benefit will be fairly well distributed among the general public, which pays the cost, and not accrue only to certain favored interests or localities.

C. W. Kutz,* M. Am. Soc. C. E. (by letter).—A review of the papers presented in this Symposium and the discussion thereon shows widely varying conclusions as to the value of the Ohio River as a transportation agency notwithstanding the fact that nearly all the economic comparisons are based on a consideration of the same basic data.

The principal points of disagreement may be grouped as follows:

1.—The annual charges on the waterway when the present project is completed;
2.—The benefits as measured by the difference between rail rates and water haulage costs when applied to existing tonnage; and
3.—Prospective tonnage.

Annual Charges.—The annual charges depend on first cost, cost of maintenance and operation, and depreciation. For purposes of comparison with a privately owned transportation agency, an item for lost taxes should also receive consideration. The writer estimated the cost of this project to be $103 000 000; Mr. Alfred used $110 000 000. The present project was adopted in 1910. There was expended under previous projects, including the amount for the purchase of the Louisville and Portland Canal, approximately $17 500 000. This amount also includes the cost of Dams Nos. 1 and 2—since replaced. If all this amount, as well as the cost of Emsworth Dam which replaced Dams Nos. 1 and 2, is considered as part of the first cost, the total will aggregate approximately $110 000 000. This total also includes the entire cost of the proposed power navigation dam at Louisville, but, as the Power Company will pay annually interest and depreciation on the difference between the cost of the navigation dam and the cost of the power-navigation dam, this difference, amounting to approximately $1 500 000, should be deducted from the total, leaving the net cost of the Ohio River Canalization Project as $108 500 000. If the cost is limited to that of work now used and useful in the interest of navigation, the total of $103 000 000 would be more nearly correct.

The writer used a 4% rate in determining interest charges, as did Mr. Alfred. Mr. Begien used 4½ per cent. The 30-year Treasury bonds, bearing 4% interest, sold at a premium, and the trend is toward lower, rather than higher, rates.

Mr. Alfred assumes an item of $2 000 000 for maintenance, plus 1½% for depreciation, the latter being based primarily on the age of Dams Nos. 1 and 2 when replaced. The replacement of these dams in 1921 by a fixed dam is not a fair index of the length of useful life of locks and dams of the Ohio River type, as their replacement was primarily a betterment, for the purpose of improving conditions in Pittsburgh Harbor. The replacement of Dam No. 3 and Dam No. 4 by a single fixed dam is also under consideration and for a similar reason.

* Col., Corps of Engrs., U. S. A., Cincinnati, Ohio.

The movable parts of these structures such as wickets, gates, valves, etc., have only a limited life and require replacement from time to time, but this is not equally true of the concrete walls and foundations. Based on experience, the cost of maintaining and operating Ohio River locks and dams, including the average annual cost of replacing depreciable parts, was estimated at $40 000 per lock and dam, or $2 000 000 for the entire river. A reconsideration of this item leads to the belief that it is too small and that the average annual cost during a long period will be approximately $2 250 000.

If this type of waterway is to be superseded eventually by some superior form of transportation agency, as were the old canals of 100 years ago, an economic comparison should properly include in the annual charges an item whereby the entire first cost will be amortized during the useful life of the project. Although this supposition might be applied with almost equal force to the privately owned railroads, to provide against such a contingency for the waterway only it is proposed to assume a life of 75 years and set up as part of the annual charges an item for depreciation of 0.22% of the first cost, an amount which is sufficient when compounded at 4% to amortize the entire first cost during the assumed life.

Mr. Alfred includes in his estimate of annual charges an item of taxes of 1.46%, the average paid by the railroads of the United States in 1923. As to the propriety of including such an item, the following is quoted from a report made by J. M. Clark, Associate Professor of Economics, Chicago University, a number of years ago, in connection with preliminary investigation of canal routes connecting Lake Erie with the Ohio River:

"Another question is whether taxes should be charged on the public investment, on the ground that if that capital had been left in the market to be used in private investments, the Government would have received taxes on it. The principle is sound: a public investment which yields the Government only what it has to pay out again on its bonds, leaves the Government poorer than if the same funds had gone into a private investment on which the Government receives taxes. The fact that the bulk of the taxes would not go to the Federal Government does not affect this principle: it merely makes the loss fall on the States.

"There is, however, one real question. Does an investment of this sort always result in reducing private investments by just the same amount? 'Orthodox' economics would say yes: but, in cases where the financing can be done partly through credit expansion, it seems probable that the correct answer is no; $100 000 000 of Government funds invested in this way would displace some private investments, but less than $100 000 000. It would depend on whether the credit mechanism had some reserve lending power at the time the loans were floated, and also on the existence, at the time, of a certain reserve of labor and plant capacity which could be drawn upon without diverting labor from other things. These conditions might or might not prevail. * * * If taxes are counted at the usual rate on one-half of the investment rather than on the whole, all reasonable allowance will have been made for the elasticity of the industrial system."

Although there is room for a difference of opinion as to the inclusion of such an item, the writer believes that full justice will be done the railroads by the adoption of Professor Clark's views. Accordingly, in the revised summary of annual charges, taxes will be included at 0.73%—one-half the average

rate paid by the railroads in 1923, which is equivalent to the full rate on one-half the investment. The total annual charges on completion of the project will then become:

Interest at 4% on $108 500 000	$4 340 000
Taxes at 0.73%	792 050
Depreciation (obsolescence) at 0.22%	233 400
Maintenance and operation (including replacement of depreciable parts)	2 250 000
Total	$7 615 450

This revised total is $1 500 000 greater than the annual charges given by the writer in Table 3; $600 000 greater than the estimate of Mr. Begien, but still $2 040 000 less than that of Mr. Alfred. Measured by the expenditures the project is 80% completed, so that in an economic comparison based on 1925 tonnage it will be fair to use 80% of $7 615 450, or $6 092 000, as the measure of the annual charges in that year.

Tonnage in 1925.—Some of the papers in the Symposium use mileage and ton-mileage data for the calendar year 1923 and others for the calendar year 1924. As a study of these data and of the reports from which they were compiled disclosed certain discordant features, the method of collecting tonnage statistics on the Ohio for 1925 was changed, a higher standard of accuracy being established. This fact will undoubtedly account for the large increase in the quantities of sand and gravel, as reported for 1925, and may account for part of the other increases. In Table 22 comparison is made between the tonnage of 1923, 1924, and 1925, as reported, no attempt being made to correct any of the errors, plus or minus, that are believed to exist in the reports for 1923 and 1924.

TABLE 22.—Variation of Ohio River Tonnage.

Commodities.	Traffic for Year:		
	1923.	1924.	1925.
Coal and coke, in tons	6 041 708	5 811 552	6 881 354
Iron and steel, in tons	377 156	550 363	539 464
Oil and gasoline, in tons	50 187	82 200	275 532
Packet, in tons	277 073	220 029	262 393
Unclassified, in tons	332 471	455 657	998 521
Sand and gravel, in tons	1 201 925	3 746 882	6 768 585
Total, in tons	8 280 520	10 866 683	15 725 849
Total, in ton-miles	708 302 795	670 486 423	845 000 000
Total, in ton-miles, exclusive of sand and gravel	693 865 289	621 391 897	757 000 000

Mr. Alfred deduced annual charges of 12.9 mills per ton-mile, but by using the revised annual charges and the tonnage of 1925 this rate becomes 7.25 mills per ton-mile.

Benefits of Savings Based on 1925 Tonnage.—While Mr. Alfred's annual charges on the waterway are regarded as too high, his water haulage cost, 2.7

mills per ton-mile, is too low. This rate does not differ greatly from the 2.5-mill rate set up by the writer for hauls of 250 miles; but it is not applicable to hauls of 86 miles (the average on the Ohio, exclusive of sand and gravel). Furthermore, his comparison of the sum of the two (annual charges per ton-mile plus water haulage cost per ton-mile) with the average railway revenue is unfair, as it fails to take into consideration the difference in the average length of haul, that on the railways being more than 200 miles whereas that on the Ohio is less than 90 miles.

The most direct method of showing benefits is by analyzing specific movements as was done by Mr. Begien in Table 17, and that method will be used in connection with the analysis of the 1925 tonnage (Table 23). Some elements in Table 17, however, call for comment. The canalization project, Huntington, W. Va., to Cincinnati, was not completed until late in 1925, but even under this handicap cost data covering operation in 1925 place the water haulage cost at 50 cents per ton as against 55 cents. It should be less hereafter. An elevating cost of 25 cents is fair for unloading plants of the old type. With modern plants on the Monongahela the cost is less than 10 cents. Making allowance for the smaller movement at Cincinnati, a cost of 15 cents is deemed to be fair. With these changes there is a saving of 68 cents per ton to river-side plants, and to householders within hauling radius of the river terminal 43 cents per ton on coal which requires a switching haul and 27 cents per ton on coal re-shipped by rail to Indianapolis, Ind. An average saving on this movement is nearer 40 cents than 20 cents, as assumed by Mr. Begien.

In connection with this rail-river-rail movement of coal to Indianapolis, the river is not regarded by the railroads as an integral part of the country's transportation system and is not accorded the same treatment as would be given a railroad; for coal brought to Cincinnati by river for shipment to Indianapolis pays a higher rate (Cincinnati to Indianapolis) than that brought in by rail, the differential being 18½ cents per ton.

Mr. Barnes' estimate of savings is subject to a correction due to the fact that many of the Ohio River hauls which go to make up the average are parts of through hauls involving tributary streams and in some cases parts of through rail and river hauls. In such cases it is necessary to compare the through water rate or through rail-river rate with the through rail rate and apportion the savings between rivers. This may be illustrated by reference to the largest coal movement—Monongahela River points to the Upper Ohio. The total difference or saving averages 94 cents per ton on a total haul of 88 miles (50 miles on the Monongahela and 38 miles on the Ohio). The share accruing to the Ohio is 44 cents, or 11.5 mills per ton-mile.

It has been alleged that the economic comparisons are faulty in that they fail to include in the cost of transportation by river either the terminal cost of delivery or any profit to the carrier. In Table 23 based on the tonnage of 1925, as well as in all other comparisons made by the writer, a terminal differential has been included. As to whether profit was included depends on the type of service, that is, whether by private carrier, contract carrier, or com-

mon carrier. In dealing with private carrier movements cost only was considered, the cost including a 6% return on the investment in floating plant and on the privately owned facilities. Although such a return may not be regarded as embodying a profit, it is equal to or in excess of what the railroads are permitted to earn on their investment under the Transportation Act. In dealing with contract carrier or common carrier movements, rates have been used which do include an item of profit.

In preparing Table 23, cost data covering a large number of movements were studied, but in order not to disclose costs of individual movements received confidentially, they have been consolidated into groups and the rates and savings listed are the average for the group.

The saving on sand and gravel is more or less indeterminate. While a series of comparisons indicate an average saving of $0.50 per ton, only part of this amount can be fairly credited as a benefit resulting from the canalization project as the industry would exist without the dams. On the other hand the dams in many cases double the length of the season and give access to deposits that could not otherwise be profitably mined. The saving of $0.20 per ton is the estimated reduction in the cost of producing sand and gravel.

It will be noted from Table 23 that the accumulated savings for 1925 amount to 87% of the annual charges for that year and to 70% of the estimated annual charges on completion.

Prospective Tonnage.—If Mr. Alfred is right in his assumption that the commerce of the Ohio will not increase but rather decrease, his conclusion is justified although his reasons are not convincing. It is true that the per capita production of coal is decreasing—and that the actual production, with minor fluctuations, has remained stationary since 1913, but the quantity moving on the Ohio has been increasing since 1920 when the movement from Pittsburgh to points on the Mississippi River ceased.

It is true that coal will move increasingly to central power stations as a substitute for coal formerly moved to local stations, but as such stations require 500 tons of condensing water for each ton of coal consumed they must locate on large rivers. Furthermore, if they can secure their coal by water, it is more economical to build such stations near the center of distribution than at the mouths of coal mines 200 miles distant.

It is true that petroleum is taking the place of coal, but the prospect that the Ohio River will receive a large share of this substitute tonnage is shown by the remarkable increase in this item in 1924-25.

While in all probability coal will always remain the largest single item of tonnage, the prospective movement of steel and iron to points on the Mississippi River bids fair to yield the largest returns eventually. Although a seasonal movement is all that is possible at present, the large steel companies in the Ohio Valley foreseeing the approaching completion of the Ohio improvement, already have taken steps to move their products by water on a large scale. In addition to seasonal movements to the Lower Mississippi, a common carrier service has been established whereby such products are being shipped by

TABLE 23.—SAVINGS ON OHIO RIVER COMMERCE FOR YEAR 1925.

Item No.	Commodity. (1)	Origin. (2)	Point of transfer to Ohio River. (3)	Destination. (4)	Average haul on tributary, in miles. (5)	Average haul on Ohio, in miles. (6)
1	Coal	Monongahela River	Pittsburgh, Pa.	Upper Ohio	50	38
2	"	Logan Field	Huntington, W. Va.	Cincinnati, Ohio, Louisville, Ky. and intermediate points	Rail, 70	199
3	"	" "	"	Portsmouth, Ohio	Rail, 70	48
4	"	" "	"	Indianapolis, Ind., etc.	Rail, 180	163
5	"	Kanawha River	Point Pleasant, W. Va.	Cincinnati, Ohio	60	204
6	"	West Kentucky Field	Caseyville, Ky.	Paducah, Ky., and Cairo, Ill.	Rail, 10	96
7	"	Monongahela River	Pittsburgh, Pa	Pittsburgh, Pa.	.....	2
8	"	Miscellaneous Movements			.....	100
9	Iron and steel	Monongahela and Ohio	Pittsburgh, Pa.	Ohio and Monongahela	8	59
10	" "	Ohio River points	.....	Ohio River points	.....	178
11	" "	Upper Ohio River	Cairo, Ill.	Mississippi River	Mississippi River, 457	926
12	Oil and gasoline	Parkersburg, W. Va.	Point Pleasant, W. Va.	Charleston	58	81
13	" "	Ravenna, Ky.	Carrollton, Ky.	Louisville, Ky.	222	63
14	" "	Ohio River points	.....	Ohio River points	.....	150
15	Stone	" " "	.....	" " "	.....	22
16	Cement	" " "	.....	" " "	.....	63
17	Packet	Ohio River and tributaries	.....	Ohio River and tributaries	.....	....
18	Logs and lumber	" " "	.....	" " "	70	35
19	Unclassified	Ohio River points	.....	Ohio River points	.....	87
20	Sand and gravel	" " "	.....	" " "	.....	13
	Total or average	.....	.....	.....	.....	54

TABLE 23.—(*Continued*).

Item No.	Average rail rate, per ton. (7)	Average cost, per ton, *via* water or rail and water. (8)	Terminal differential, per ton. (9)	Total savings, per ton. (10)	Tonnage. (11)	Savings accruing to tributaries. (12)	Savings accruing to Ohio River. (13)
1	$1.34	$0.30	$0.10*	$0.94	3 929 997	$1 914 300	$1 738 700
2	1.89	Rail, 0.50 Water, 0.60	Tipple, 0.06 Unloading, 0.15 Delivery, 0.125†	0.455	491 620	.....	223 000
3	1.39	Rail, 0.50 Water, 0.25	Tipple, 0.06 Unloading, 0.15	0.43	374 002	.....	161 000
4	2.52	Rail 0.50 Water, 0.50 Rail, 1.04	Tipple, 0.06 Unloadiug, 0.15	0.27	310 802	.....	84 000
5	1.89	Rail, 0 125‡ Water, 0.90	Tipple, 0.06 Unloading, 0.375 Delivery, 0.125†	0.305	646 755	45 000	152 000
6	1.36	Rail, 0.20 Water, 0.35	Tipple, 0.06 Unloading, 0.375 Delivery, 0.125†	0.25	293 251	.....	73 300
7	.....	....	....	....	480 110	Not estimated.	.....
8	1.45	0.40	Tipple, 0.15 Unloading, 0.50	0.40§	354 817	.....	142 000
9	1.18	0.54	Loading, 0.15‖ Unloading, 0.15‖	0.34	273 816	11 000	82 000
10	3.84	1.09	Loading, 0.25 Unloading, 0.75	1.75	165 675	.....	290 000
11	11.50	5.65	Loading, 0.25 Unloading, 0.75	4.85	99 973	160 000	325 000
12 13 14	3.87	1.47	0.40	2.00	275 532	133 000	418 000
15	0.60	0.20	Loading, 0.05 Unloading, 0.10	0.25	542 281	.....	135 570
16	1.75	0.60	Loading, 0.15 Unloading, 0.50	0.50	36 935	.....	18 467
17	.....	80% of rail rates.	20% of rail rates.	....	262 393	Not estimated.	.....
18	.....	....	....	1.00§	186 413	124 280	62 140
19	Sixth class.	....	....	0.30§	232 892	.....	70 000
20	3.00 0.70	....	.....	0.20¶	6 768 585	.....	1 353 717
	Total or average.....	.....	.....	.....	15 725 849	.....	$5 328 894

* River mines, river delivery, large movements, and efficient terminals. † Delivery cost of $0.25 on one-half of tonnage. ‡ Rail cost of $0.25 on one-half of tonnage. § Rough estimate without details of specific movements. ¶ Cost of dredging, transporting, and elevating sand and gravel is reduced $0.20 per ton by canalization of the river. ‖ Traffic between steel mills having large tonnages and efficient terminals.

river to Louisville, thence by rail to St. Louis, and Memphis, pending completion of the canalization project.

Mr. Begien makes the point that "the comparatively few industries and individuals who make use of river transportation contribute a very small part toward the interest and carrying charges of such improvements" and that in connection with the movement of coal by river none of the saving in transportation costs is passed on to the consumer.

The theory underlying the improvement of rivers at public expense assumes that the savings in transportation costs will be transmitted to the taxpayers who pay the annual charges, or to a large and representative group thereof. Not all the tax-paying public is benefited by a given improvement, but it may benefit to an equal extent from some other similar public improvement. If, however, the saving is not passed on as alleged then some means should be devised for placing the maintenance burden on the users. This could be done either through the collection of tolls or by a licensing system similar to that which governs the use of highways created at public expense, as was suggested a number of years ago by a prominent waterways advocate.

This question, however, is separate and distinct from the one at issue, namely, whether a canalized Ohio River will serve as an economic link in the country's transportation system. The writer believes that the use now being made of the river, the savings now being effected, and the prospective increase in use on completion of the project in 1928 justify an affirmative answer.

The Ohio River's full usefulness, however, will not be realized until the Interstate Commerce Commission is given the same control over rates by river as it has over rail rates, and until the railroads recognize the value of the river by granting joint rail and river rates as freely as they grant joint rail rates.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1591

THE ORADELL DAM OF THE HACKENSACK WATER COMPANY*

By Nicholas S. Hill, Jr.,† M. Am. Soc. C. E.

With Discussion by Messrs. Stuart K. Knox, Howard T. Critchlow, A. T. Ricketts, Thaddeus Merriman, Lazarus White, Albert C. Arend, Paul E. Gisiger, and Nicholas S. Hill, Jr.

Synopsis

This paper describes a unique type of dam of considerable size designed by the writer in 1911 and built by the Hackensack Water Company in 1920-22. The unusual design was the outgrowth of an attempt to plan a dam to meet special conditions found at a particular site. It was not intended to develop a type of dam which would be applicable to conditions likely to be found frequently, nor is it suggested that the type described offers a more economical substitute for the usual types of dam under ordinary conditions.

The special conditions for which, in combination, this design was intended to provide were:

1.—Spillway crest with maximum height about 20 ft. above the stream bed.
2.—Maximum possible length of dam between abutments, 402½ ft.
3.—Flood flows which would produce a head of 3 ft. over any practicable length of weir (402½ ft.).
4.—Firm foundation of hardpan and shale 15 to 45 ft. below the stream bed.
5.—Porous sand and gravel overlying the deep firm foundation, with intercalated and lenticular masses of stiff dense clay over part of the section.
6.—Impracticability, because of cost, of adopting a masonry section of the gravity type extending to the deep underlying hardpan and shale.

* Presented at the meeting of November 4, 1925.
† Cons. Engr., New York, N. Y.

7.—Impracticability of allowing a head of more than 1 ft. over the spillway crest, because of flooding of railroad and other unowned lands.
8.—Necessity of providing a dam section which might be overtopped without damage in times of flood.
9.—Necessity of holding foundation and toe pressures within lowest possible limits.
10.—Necessity of providing solid cut-off all the way down to the underlying hardpan and shale.

Introduction

The Hackensack Water Company, chartered in 1869, is one of the largest private water companies in the United States. At the time the Oradell Dam was constructed it supplied water to forty-eight municipalities or civil divisions in Hudson and Bergen Counties, New Jersey, all lying within the Metropolitan Area of New York, N. Y. The most populous of these communities were Hoboken, West Hoboken, West New York, North Bergen, Weehawken, and the Town of Union, in Hudson County, and Hackensack, Englewood, Rutherford, and Ridgefield Park, in Bergen County.

According to the 1920 Census, the total population in the territory supplied was approximately 340 000, of which about 92% was connected to the mains of the Company. It is estimated that this population will have increased to 530 000 by 1940.

The daily volume of water delivered for distribution increased from about 26 000 000 gal. in 1912 to from 35 000 000 to 36 000 000 gal. in 1918, 1919, and 1920. It is estimated that the water delivered for distribution will have increased to more than 80 000 000 gal. daily by 1940. From 40 to 45% of the consumption is, and probably will remain, industrial in character.

The supply is taken from the Hackensack River at New Milford, N. J., just above tide-water, at which point the stream has a drainage area of 115 sq. miles. At New Milford the entire supply is filtered through mechanical gravity filters and pumped for distribution at heads of 300 and 600 ft.

The Water Company has two impounding reservoirs on the water-shed above New Milford to regulate the flow and augment the supply during dry weather. The necessity for impounding storage is indicated by the fact that the natural flow of the Hackensack River at the New Milford intake decreases to 10 000 000 gal. per day or less in extremely dry seasons. The safe yield of the stream with the present storage (about 3 900 000 000 gal.) is estimated to be 40 000 000 gal. per day. Prior to the construction of the new Oradell Dam, the safe yield was only 32 000 000 gal. per day, and the impounding storage 2 190 000 000 gal.

The new Oradell Dam creates a storage reservoir of 3 023 000 000 gal. capacity, which in conjunction with the smaller Woodcliff Reservoir of 889 000 000 gal. capacity, constructed in 1906, comprises the total impounding storage of the Water Company on the Hackensack River. The new dam is situated on the main stream just above Oradell, N. J., from which it takes its name. The site is only about ¾ mile up stream from the New Milford pumping station and intake.

Aside from the much smaller Saddle River, the basin of which adjoins that of the Hackensack on the west, the Hackensack River is the only readily available source of water supply for the large and rapidly growing population served by the Hackensack Water Company. With proper storage it is estimated that the Hackensack River may be made to supply the needs of this territory until 1940, when the population supplied will exceed 500 000.

Since 1903, the Company has maintained a dam on the present site at Oradell, which originally was an old mill dam of low elevation (spillway crest, 8.60 ft.*) and negligible storage. The basin above the dam was a swamp quite heavily timbered. Beginning in the spring of 1911, the work of creating a storage reservoir at Oradell, which had been started in 1903 and then abandoned in favor of the Woodcliff site, was resumed. The clearing of the basin was again undertaken and a temporary timber crib dam, with wooden sheet-pile cut-off, was constructed on the site of the old mill dam. The crest of this crib dam which was about 11 ft. above the stream bed (or Elevation 11), created a storage basin of sufficient depth to float a dredge. Two suction dredges, one 12 in. and the other 20 in., were constructed on the bank of this small basin, and launched. Between 1911 and 1920, the basin, up stream from the dam, and below the 11-ft. flow line, was dredged to Elevation 0, the excavated material being used to build up the banks and fill in depressions, to which it was pumped through pipe lines supported on floats.

Coincidentally with the starting of this work, borings were made across the valley of the Hackensack River below the temporary crib dam, to determine the best site for a permanent dam to create a reservoir having a flow line 20 ft. above the stream bed. As a result of these borings the location for the permanent dam was chosen on a line at right angles to the thread of the stream and 50 to 90 ft. down stream from the curved crib dam. The permanent dam was designed in 1911 and its construction was begun in 1920, at which time the dredging was practically complete. By the close of 1921 the spillway section had been built to Elevation 16. The dam was completed to Elevation 20 in 1922.

Considerations Controlling the Selection of the Oradell Reservoir Site

The primary consideration controlling the selection of the Oradell Reservoir site was the desire to obtain a considerable volume of storage in the lower reaches of the drainage basin, near the intake, at a point so located that all branches of considerable size would be tributary to it, thus insuring the highest possible degree of stream regulation and control over flood flows.

The selection of any site other than that at Oradell would have excluded from the drainage basin tributary to the reservoir, an area of at least 25 sq. miles, or 22% of the total drainage area. The run-off from this area would have been ineffective in replenishing any other reservoir, following depletion, and the flood flows from its 25 sq. miles entirely uncontrolled

* All elevations given in this paper refer to Atlantic mean high tide.

except in so far as regulation could be effected by the method of operating the suction basin and pumping station at New Milford.

Studies indicated that the cost per 1 000 000 gal. of developing storage in Oradell Reservoir would be no higher than at any other desirable site.

Considerations Controlling the Selection of the Flow-Line Elevation of Oradell Reservoir

The divide between the Hackensack drainage and that of the Sparkill, a tributary of the Hudson, has an elevation only about 30 ft. above mean sea level, or 27.5 ft. above the Company's datum which is mean high tide. The two-track New Jersey and New York Railroad at Oradell is about at Elevation 23. Surveys of the reservoir basin below Contour 25, which were started in 1910, indicated that the placing of the spillway crest above Elevation 20 would necessitate the relocation of railroads and highways and the purchase of land areas the cost of which would have been greater than warranted by the advantages obtained. It was decided, therefore, after due study and consideration, to fix the spillway crest at Elevation 20 and to provide in the dam a sluice-gate section of a capacity sufficient to pass all extremes of flood with a head not exceeding 1 ft. over the spillway crest. In this way it is expected that troubles due to back-water and flooding above the dam will be avoided.

Flood Flows and Proportions of Sluice-Gate and Spillway Sections

The severity of the floods which occur on the Hackensack River is not extreme when measured in second-feet per square mile. Considerable trouble from flooding, however, has been experienced at times of extremely high run-off because of channel obstructions below the dam at Oradell. There are islands in the stream between Oradell and the pumping station at New Milford. The New Milford pumping station is located on an island, the surface of which is at a general elevation of 10 ft.

The suction basin has been created by damming the various arms of the stream between the islands and the mainland in the vicinity of the pumping station. The crests of these dams stand at Elevation 4.93, so that under normal conditions of stream flow the flow line in the suction basin and stream between Oradell and New Milford stands at about 5 ft.

On one occasion, during a period of very high run-off, the water surrounding the New Milford pumping station rose for a short time to a height above the engine-room floor, which stands at Elevation 10. Following this experience the suction basin and stream channels between Oradell and New Milford were considerably enlarged and improved by dredging. This work was done about 1911 or 1912.

As a result of the removal of obstructions and other improvements accomplished at this time, it is believed that the water level below the New Milford pumping station cannot now rise to an elevation exceeding 8 ft., and that the maximum elevation of back-water at the toe of the Oradell Dam, ¾ mile above the pumping station, cannot in the future exceed Elevation 12.25. The

minimum net head on the sluice-gates in the Oradell Dam, with 1 ft. over the spillway crest, will, therefore, not be less than 21.00 — 12.25, or 8.75 ft., at times of most extreme flood.

It is believed that the most extreme flood likely to occur on the Hackensack River will not exceed 46 000 000 gal. daily per sq. mile of tributary drainage area. Unfortunately, the run-off data on the Hackensack, which are meager, do not afford a basis for determining the extent of the floods which have occurred in the past. This figure, therefore, is an estimate. As a basis for the estimate, a study was made of the recorded flood flows on many streams of similar size in New Jersey and New York, some of which are given in Table 1.

TABLE 1.

Name of stream.	Area of water-shed, in square miles.	Maximum recorded flood flow, in millions of gallons per day, per square mile.	Date of occurrence.
Pequannock, N. J.	85.0	41.8	Feb. 6, 1896
Ramapo, N. J.	86.0	36.2	"
Wanaque, N. J.	101.0	46.1	"
Rockaway, N. J.	118.0	29.8	"
Paulins Kill, N. J.	126.0	34.9	"
Musconetcong, N. J.	130.0	29.1	"
Ramapo, N. J.	160.0	42.7	Sept. 22, 1882
Esopus Creek, N. Y.	234.0	43.6	Feb. 15, 1908

With these figures, a comparative study was made of the Hackensack River and the other streams mentioned, and in almost every case it was found that the drainage basins of the other streams were steeper and had a more rapid run-off than that of the Hackensack.

The length of spillway and the number of sluice-gates were proportioned, therefore, to pass a flood of 46 000 000 gal. daily per sq. mile, equivalent to 71 sec-ft. per sq. mile, or a total of 8 094 sec-ft., with 1-ft. head on the spillway crest, this flood to be passed when the back-water against the down-stream face of the dam stands only 7.75 ft. below the spillway crest elevation of 20 ft.

In connection with these assumptions as to the maximum probable flood flows, it may be noted that a flood in excess of the assumed maximum will not endanger the stability of the dam, which has a sufficient margin of safety to permit a head of 3 ft. on the spillway crest. The top of the sluice-gate section and the abutment and wing-walls were also placed 3 ft. above the spillway crest. With this head on the crest, and back-water on the down-stream face at 7.75 ft. below the spillway crest, the dam will pass a flood of 75 000 000 gal. per day per sq. mile, which is equivalent to 4⅓ in. of depth on the drainage area in 24 hours. It is not believed that such a flood can occur, but if it should, it will result in nothing more serious than the temporary flooding of certain railroad and unowned lands above the dam.

Stratigraphy at the Dam Site

In June and July, 1911, thirty-one test borings, ranging in depth from 20 to 45 ft., were made in the vicinity of the proposed dam.

The contract for the boring called for thirty-one 3-in. borings, by the auger, hydraulic, and drive-well process, at points designated by the writer. Casings, 3 in. in diameter, were driven to impenetrable hard bottom, of hardpan, boulder, or rock, and samples furnished for every 2 ft. of depth and at changes in the stratification. The price for this work was $3 per ft. In addition, diamond drill core borings were made, as directed, in test wells which had been driven to refusal, and core samples furnished. The diamond drill core borings were made with a $\frac{1}{8}$-in. diamond core bit, at a cost of $8 per ft.

The contract also called for auger borings to be made wherever samples could be obtained. When material could not be brought up with the auger a sand bucket was used. Wash borings were made only when the auger and sand-bucket methods failed.

The location of the borings, and the stratigraphy disclosed by part of the borings are shown on Fig. 1. The outlines of the dam shown in this view are those of the original mill dam previously mentioned, the crest elevation of which was 8.6 ft. Two lines of borings, 15 ft. apart, with staggered holes, were made entirely across the valley just below the old dam. This was regarded as the most desirable location for the new dam because of the surface topography. A third line of borings was made across the valley about 375 ft. down stream from the first two lines, and these parallel lines were connected by a series of borings which approximately paralleled the thread of the stream.

The nature of the foundation as disclosed by the borings was disturbing. The hardpan or shale which lay at a depth of only 10 to 15 ft. below mean high tide at the extreme westerly end of the sections, fell off to a depth of 45 ft. below mean high tide at the easterly end.

A masonry dam of the gravity type might have been extended to this foundation, but its cost would have been prohibitive. The overlying materials were pervious and of unsatisfactory bearing value. An earth dam with a sheet-pile cut-off, and a large sluice-gate section for passing flood flows, would have served, but it was considered unwise to rely solely upon sluice-gates for the discharge of floods, and an earth dam if overtopped would have been destroyed.

The purpose of the design adopted was to retain the advantages of an earth dam with sluice-gate section and sheet-pile cut-off, and, at the same time, insure it against damage from overtopping the crest.

Design of Sluice-Gate Section

No difficulty was presented by the sluice-gate section, which was located at the extreme westerly end of the dam, opposite the main channel, where the underlying hardpan or shale lay reasonably near the surface. The sluice-gate section, therefore, was carried down to a secure underlying foundation at all points.

The sluice-gate section, from the west abutment to the wall which separates it from the spillway section, was divided into seven equal bays by concrete piers (as shown by the general plan, Fig. 2, and the horizontal and vertical

section, Fig. 3). Each pier was founded on the underlying hardpan. The space between the piers was floored over at Elevation 0 with a reinforced

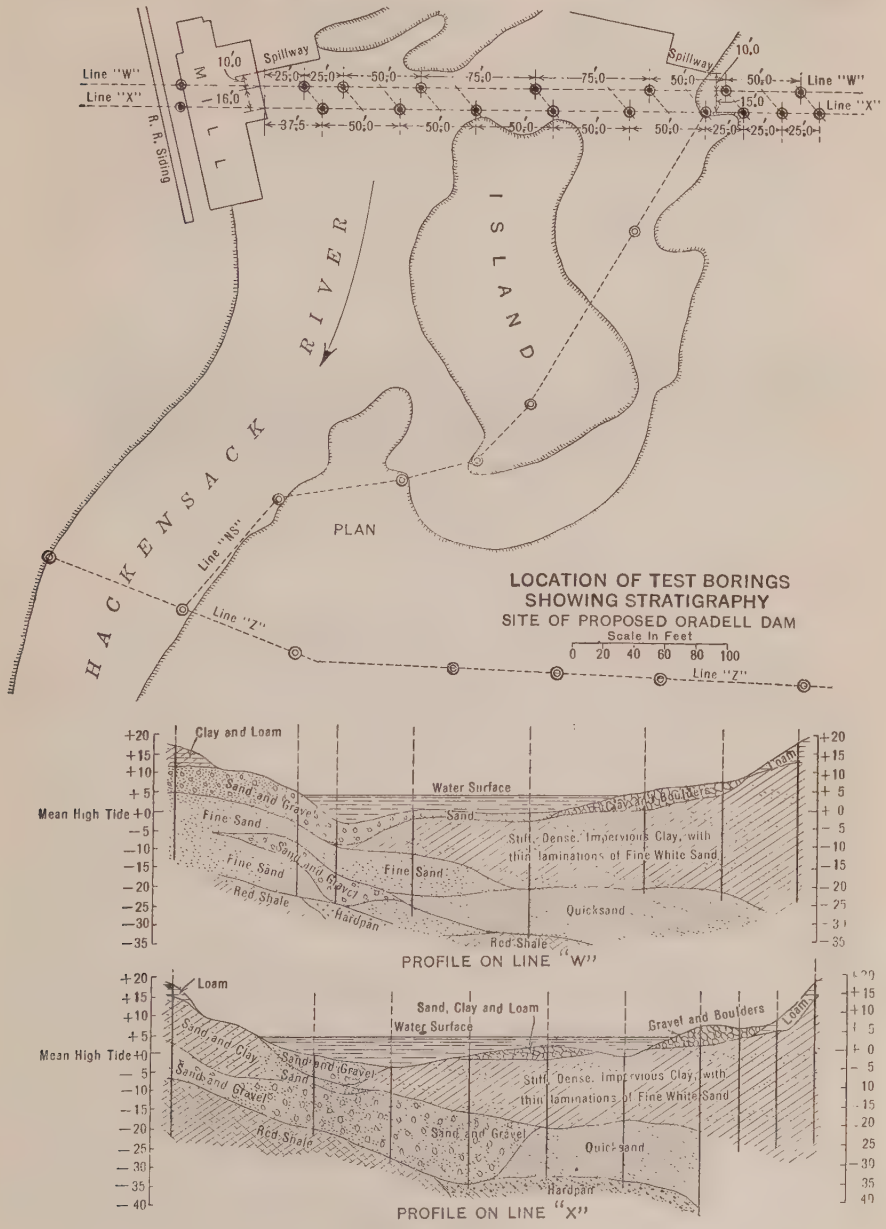


FIG. 1.

concrete slab, and a concrete cut-off wall reaching from end to end of the sluice-gate section was extended from the under side of the floor to the hard-

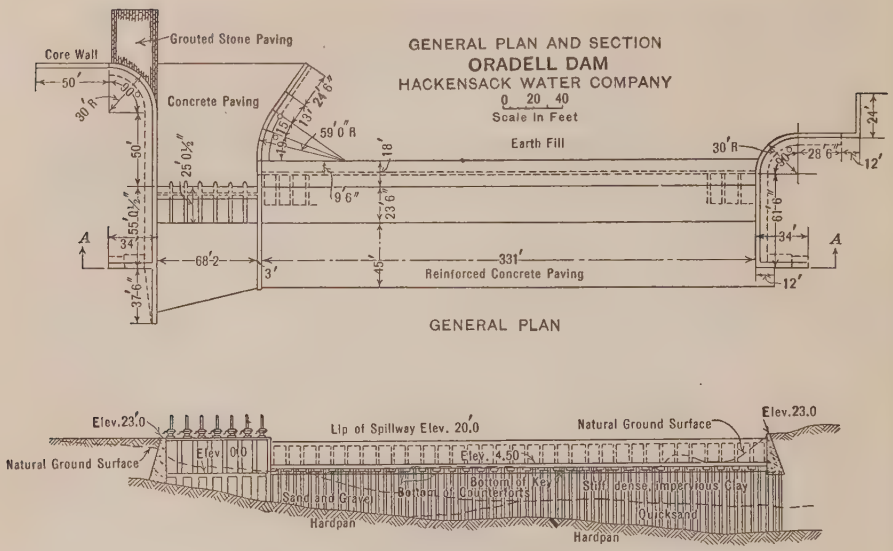

GENERAL PLAN AND SECTION
ORADELL DAM
HACKENSACK WATER COMPANY
0 20 40
Scale in Feet
Core Wall
Grouted Stone Paving
Concrete Paving
Earth Fill
Reinforced Concrete Paving
GENERAL PLAN
Elev. 23.0
Lip of Spillway Elev. 20.0
Natural Ground Surface
Elev. 0.0
Elev. 4.50
Bottom of Key
Bottom of Counterforts
Sand and Gravel
Stiff dense impervious Clay
Quicksand
Hardpan
LONGITUDINAL SECTION
AT DOWNSTREAM FACE OF DAM

FIG. 2.

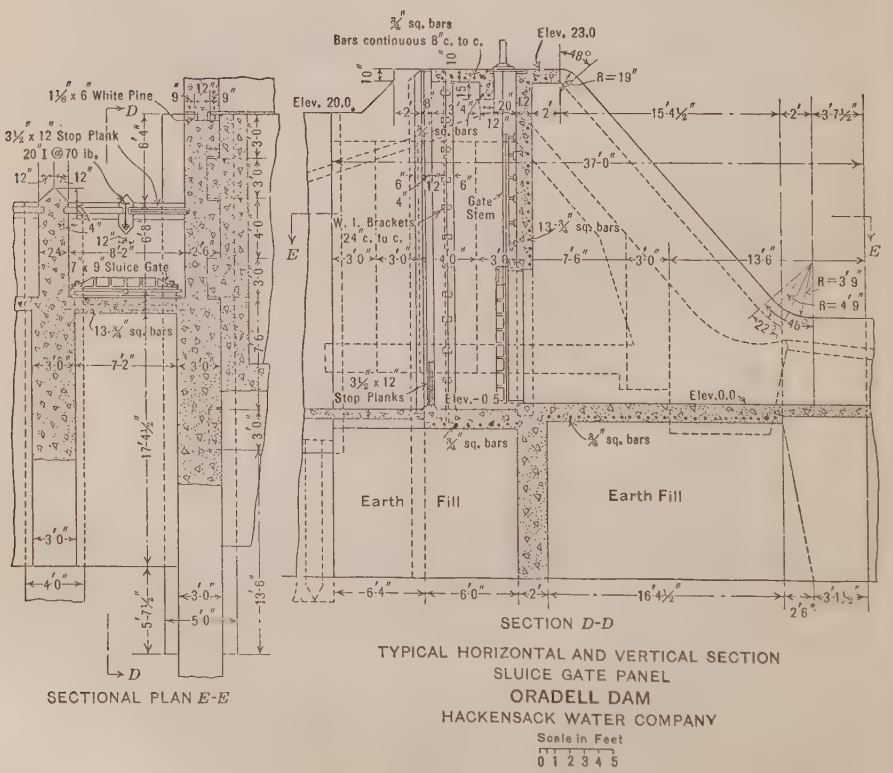

1⅛" x 6" White Pine
3½" x 12" Stop Plank
20" I @ 70 lb.
7" x 9" Sluice Gate
13-¾" sq. bars
SECTIONAL PLAN E-E
¾" sq. bars
Bars continuous 8" c. to c.
Elev. 23.0
Elev. 20.0
R=19"
W. I. Brackets
24" c. to c.
Gate Stem
13-¾" sq. bars
3½" x 12" Stop Planks
Elev. -0.5
Elev. 0.0
Earth Fill
Earth Fill
SECTION D-D
TYPICAL HORIZONTAL AND VERTICAL SECTION
SLUICE GATE PANEL
ORADELL DAM
HACKENSACK WATER COMPANY
Scale in Feet
0 1 2 3 4 5

FIG. 3.

pan, thus affording a solid masonry cut-off the full length of the sluice-gate section. Above the floor on the line of the cut-off wall were placed the seven rectangular sluice-gates, each 7 by 9 ft. in size, one in each bay. The sills of the gates were placed at Elevation 0. Each gate is operated manually from a stand seated on the concrete platform or walkway which extends from end to end of the sluice-gate section spanning the tops of the piers which separate the bays.

Under normal conditions all water passing the dam will be passed through the sluice-gates. No water will be allowed to flow over the spillway except during the most extreme flood. Views of the sluice-gate section are shown in Figs. 4 and 5.

Design of Spillway Section

The spillway section, as may be seen from the general plan, Fig. 2, and the horizontal and vertical section, Fig. 6, is essentially an earth dam 50 to 90 ft. thick at the base. The up-stream face of the embankment was formed by the existing earth and timber crib dam with its wooden sheet-pile cut-off extending about 20 ft. into the earth. The down-stream face and toe are encased in a reinforced concrete envelope, supported on round piles, each capable of sustaining a load of 20 tons. (See Fig. 6.)

At the up-stream edge of the floor of the concrete envelope there is a continuous line of interlocking steel sheet-piling which is also carried far into the easterly bank, following along the face of the easterly concrete abutment. These sheet-piles extend to the hardpan. Their tops are embedded in the floor of the concrete envelope and in the concrete of the easterly abutment wall. Together with the concrete cut-off wall of the sluice-gate section, they form a continuous cut-off entirely across the new dam, approximately on the line of the up-stream edge of the concrete envelope. (See Fig. 6.)

The fill between the existing earth and timber crib dam and the concrete envelope, which forms the body of the new dam, is hydraulically placed and of stiff clay. This fill forms a stable and impervious dam in itself, except that it is not proof against the inroads of burrowing animals, nor against scouring if it should be overtopped.

The reinforced concrete envelope, supported on round piles, performs the function of a masonry core-wall by resisting the inroads of burrowing animals in the fill above the steel sheet-pile cut-off, and also would protect the top and down-stream face of the fill against scouring if this section of the dam should be overtopped, or because of failure promptly to control the sluice-gates.

The heads of all round piles are embedded from 1 to 2 ft. in the base of the concrete envelope, as shown on Fig. 6. This ties the piles and the whole concrete envelope together, giving resistance against sliding and upward thrust as well as support against downward thrust.

The interlocking steel sheet-piles of the cut-off wall were driven to refusal and their tops cut off evenly at Elevation +3.00 (see Fig. 6), by an oxy-acetylene torch. Their upper ends, therefore, are embedded in the concrete

envelope to a depth of 1 to 4 ft. Reinforcing rods, ¾ in. square, 4 in., center to center, were placed in the bottom slab of the concrete envelope and bent over the tops of the steel sheet-piles. The rods extend back into the floor slab for a distance of 65 diameters, thus effecting a strong bond between the steel sheeting and the concrete envelope and insuring a strong resistance against sliding.

It will be noted from Fig. 6 that the spillway section has a substructure of piling and concrete, and that the part of this substructure between Elevations +2.00 and +4.00 forms a solid, level, reinforced concrete floor 2 ft. thick across the full length of the spillway section. On this floor, structural steel frames, triangular in shape, were set up at 10-ft. intervals and secured to the floor by anchor-bolts. The reinforcing rods for the concrete overfall or facing of the spillway section were threaded through the front members of the steel frames, and when all steel was set the inclined facing slabs were poured, the structural steel frames being encased at the same time in concrete counterforts 2 ft. thick. The horizontal top slab forming the spillway crest was not poured until after all the earth fill had been placed and given ample opportunity to settle.

Earth fill, deposited by the hydraulic process and thoroughly puddled, occupies the triangular space between the floor and face slabs, between counterforts, and extends back to the top of the existing timber crib dam. The sloping top surface of this fill is paved for a distance of 15 ft. back from the up-stream edge of the spillway crest.

In each panel between counterforts of the spillway section a pair of 2-in. weep-holes were placed through the face slab of the concrete envelope at the floor level. The upper ends of these 2-in. pipes were screened with fine copper mesh, and the acute angle between the floor and face slabs of the envelope was then filled with a triangular fillet of graded gravel and sand, which was intended to facilitate the drainage and consolidation of the fill and, at the same time, prevent the washing out of the filling material. The weep-holes should also relieve the face slab from any direct water pressure which might otherwise occur if the fill should settle away slightly from the concrete.

It will be noted that a considerable volume of concrete was saved by the adoption of the type of construction shown, thus lessening the dead weight of the dam, and also that under the extreme conditions of loading described later the downward pressure is considerably reduced (as compared with the pressures which would have obtained with a solid masonry section) by the action of the upward water pressure on the under side of the face slab.

Other Features of Design

The westerly abutment of solid gravity section was carried down to hardpan. Its up-stream end was curved on a 30-ft. radius and extended into the bank for a distance of 80 ft.

The easterly abutment, also of solid masonry and similar in form to the westerly abutment, was extended down to Elevation —3.00 and founded on round piles driven to hardpan. As previously stated, the easterly abutment

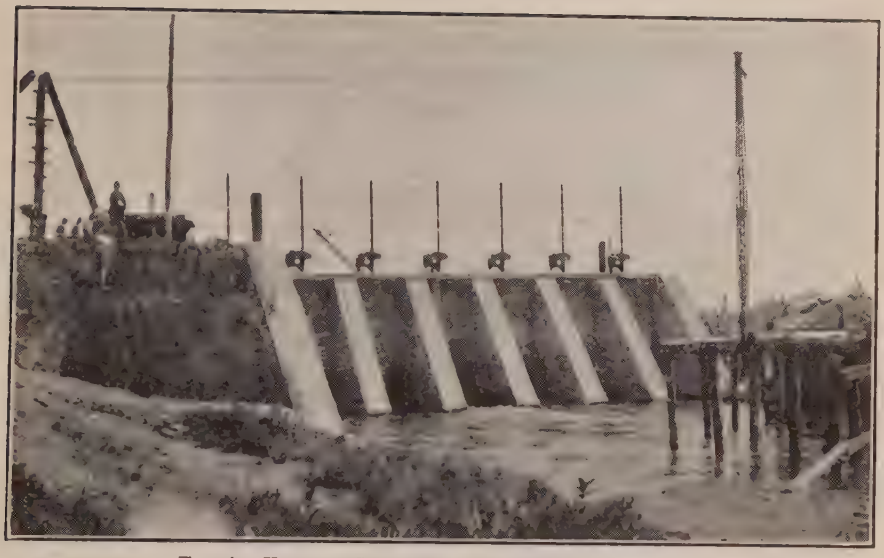


FIG. 4.—VIEW OF SLUICE-GATE SECTION, ORADELL DAM.

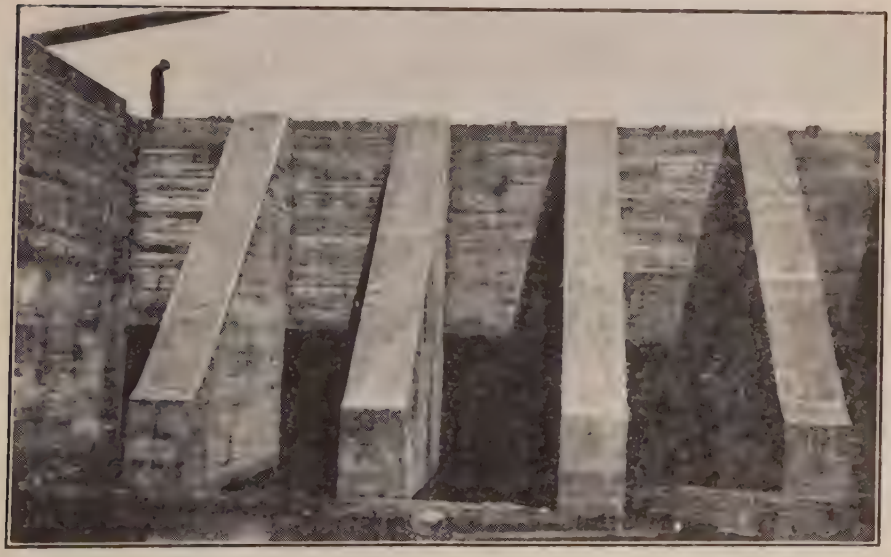


FIG. 5.—VIEW OF SLUICE-GATE SECTION, ORADELL DAM.

was also provided with a steel sheet-pile cut-off where necessary to intercept possible under flow.

The approach to the sluice-gates, which has a uniform elevation of —0.5 ft., was paved with a reinforced concrete slab for about 90 ft. up stream from the gates and across the full width of the sluice-gate section.

In order to retain the earth filling back of the concrete envelope of the spillway section and to separate this fill from the channel approach to the sluice-gates, a dividing wall was constructed, on a slightly curved sweep, as shown in Fig. 2. This wall extends from its junction with the dividing wall between the concrete envelope and the sluice-gate section for about 80 ft. up stream. It is of solid masonry to a depth of 3 ft. below the floor of the approach to the sluice-gates, and is supported on round piles which extend to the hardpan. It is also provided with a sheet-pile cut-off which is carried to hardpan.

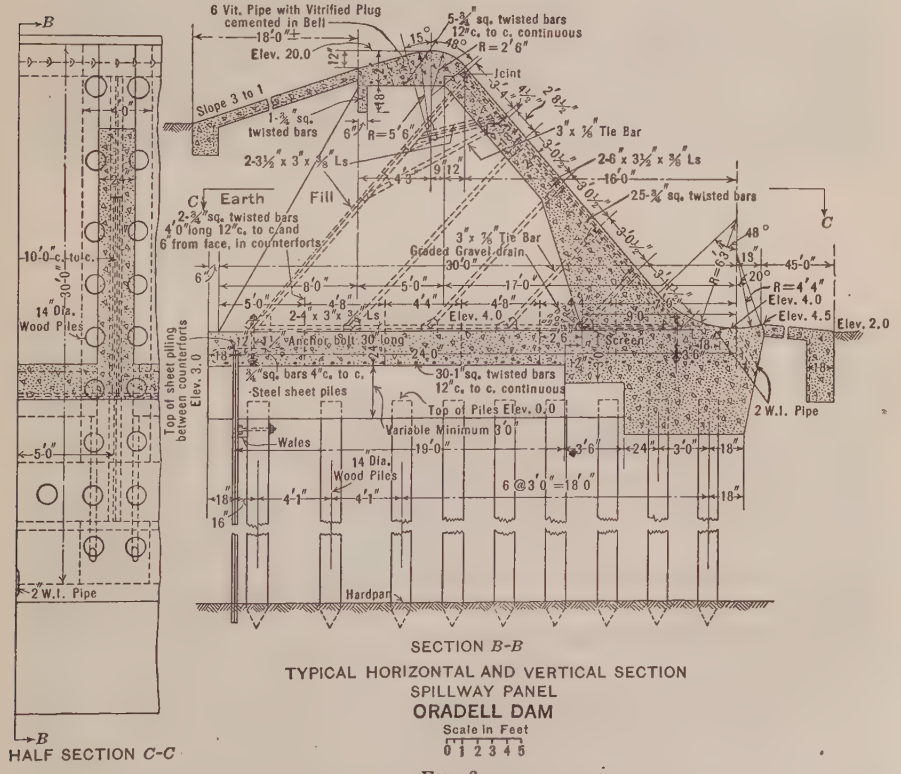


FIG. 6.

Below the toe of the concrete envelope of the spillway section, and the down-stream edge of the sluice-gate section, the stream channel is paved for a distance of 45 ft. across the full width of the dam between abutments. This pavement or apron is of reinforced concrete 8 in. thick and is turned down into the earth for a distance of several feet at its down-stream edge.

The concrete envelope was keyed into the face of the easterly abutment wall and into the face of the dividing wall between the sluice-gate and spillway sections, thus forming expansion joints at these points. There are no expansion joints in the 330 ft. of concrete envelope except at the ends, as noted. The concrete envelope has been subjected to a wide range of temperature with full water pressure behind it. The end joints have opened as much as $\frac{1}{8}$ in. No visible cracks have occurred at intermediate points in the envelope and no visible leakage has appeared at any point. There are practically no damp spots visible on the face of the envelope under full water pressure.

The abutment walls, which are unreinforced, were cast in sections with tongue and grooved joints at approximately 30-ft. intervals. Under extreme range of temperature each of these joints has opened about $\frac{1}{8}$ in.

Unit Stresses

Each round pile under the spillway section was driven to sustain safely a load of 20 tons as determined by the so-called *Engineering News* formula for drop-hammer driving. As previously stated, all steel sheet-piles were driven to refusal.

All concrete was mixed in proportions of 1 : 2.5 : 5, and all reinforced concrete was designed on the basis of the following units:

$K = 0.65$	$C = 450$
$p = 0.0052$	$r = 15$
$S = 14\,000$	$E_c = 2\,000\,000$

Construction Methods

No cableway was used in the construction of the dam. Excavated material was handled with derricks. All concrete was mixed in a single mechanical batch mixer situated on the bank nearest which work was being done, and handled to place by derricks and the ordinary steel double-side dump-cars running on portable narrow-gauge track.

A Blaw-Knox steel form and form carrier was used in casting the face slab of the concrete envelope which was poured in 30-ft. sections. The joints were made at the center of the counterforts in order to secure water-tightness. The heavy face reinforcement was carried through these joints. The steel form and form carrier are shown in Fig. 7.

Two types of pile-driver were used on the work. The round and steel sheet-piles across the full length of the spillway section were driven by a drop-hammer weighing 2 950 lb. The other piles were driven with a double-acting steam hammer suspended from a derrick boom. The total weight of this hammer was 3 500 lb., and the weight of the ram was 575 lb. The steam cylinder was 6 by 10 in., and at a steam pressure of 60 lb. per sq. in., the strokes averaged about 170 per min.

Experiments were made on test piles to determine the ratio of penetration per stroke to give the same bearing value, the double-acting steam hammer

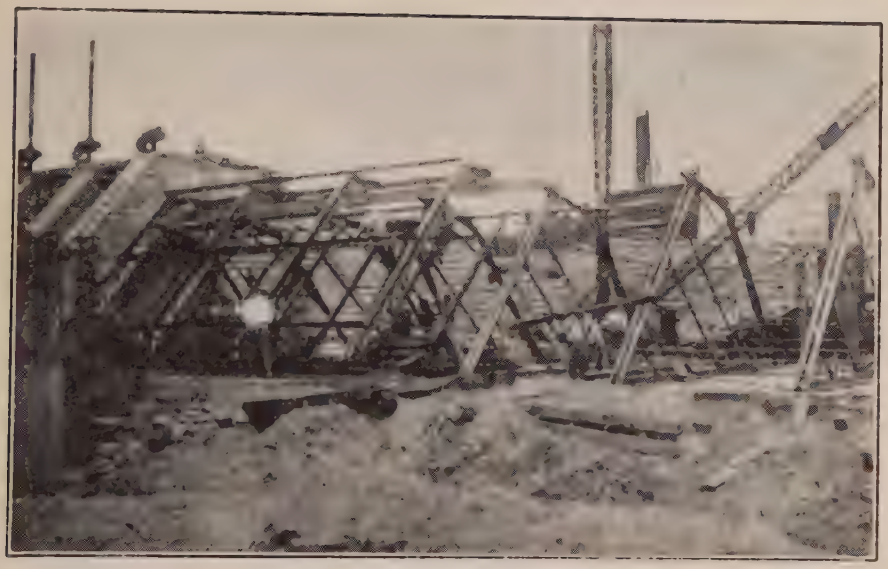


FIG. 7.—STEEL FORM AND FORM CARRIER USED IN CASTING FACE SLAB OF CONCRETE ENVELOPE, ORADELL DAM.

FIG. 8.—WEDGE-PILE USED ON ORADELL DAM.

FIG. 9.—CUTTING OFF HEADS OF STEEL SHEET-PILES, AFTER DRIVING, BY OXY-ACETYLENE TORCH, ORADELL DAM.

FIG. 10.—METHOD OF DISTRIBUTION OF HYDRAULIC FILL BACK OF SPILLWAY SECTION, ORADELL DAM.

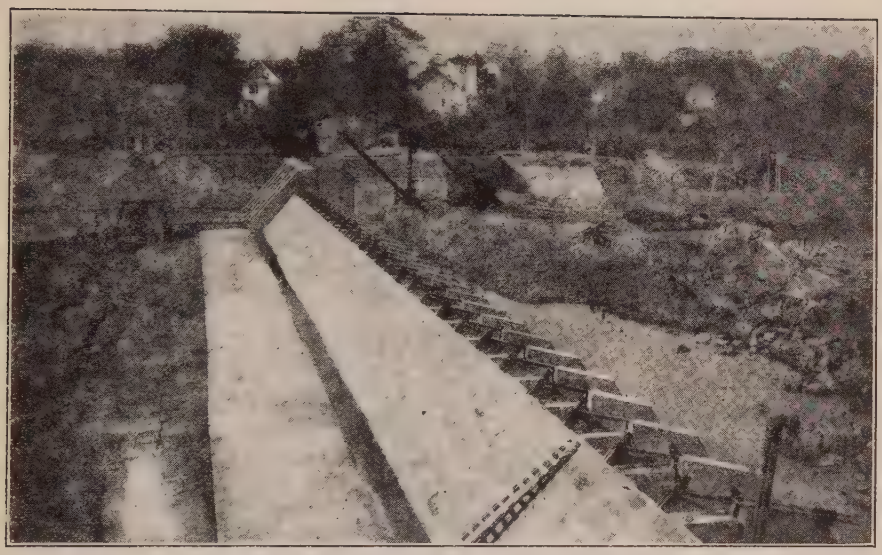


FIG. 11.—VIEW FROM EASTERLY ABUTMENT OF SPILLWAY AND SLUICE-GATE SECTIONS, ORADELL DAM, BEFORE COMPLETION.

FIG. 12.—VIEW OF COMPLETED ORADELL DAM FROM DOWN-STREAM SIDE.

being compared with the drop-hammer under the particular conditions which obtained on this work. The experiments suggested the following as a desirable general formula applicable to all methods of pile-driving, including drop-hammer, single-acting steam hammer, and double-acting steam hammer:

$$L = \frac{2\ W \times h}{S + \frac{\text{Constant}}{N}}$$

in which

L = the safe bearing value, in pounds per pile;
W = the weight of the ram, in pounds, for drop-hammers and single-acting steam hammers, and the weight of the ram plus the mean effective steam pressure for double-acting steam hammers;
h = the length of stroke, in feet;
S = the penetration, in inches, under the last blows of the hammer;
N = the number of strokes per minute.

The constant in the second term of the denominator should be determined by a comprehensive series of experiments.

The average number of piles driven per day on the spillway section was 33 for round piles, and 15 for steel sheet-piles. The driving of these piles occupied 2 months with a crew of 8 men.

As the driving of the steel sheet-piles progressed, some difficulty was experienced from the tendency of the joints between the sheet-piles to spread at the top while held tightly together at the bottom, thus causing the piles to get out of plumb. This was corrected by the occasional driving of wedge-piles as required. Fig. 8 shows one of the wedge-piles which was made on the ground by cutting and riveting together two of the ordinary piles. Fig. 9 shows the operation of cutting off the heads of the steel sheet-piles after driving by the oxy-acetylene torch.

The hydraulic fill back of the spillway section was distributed from an elevated hopper constructed on the easterly abutment of the dam adjacent to a large spoil pile. The filling material was elevated to this hopper by derricks. A hose stream was then played upon it, by which the filling material was washed down through a steel pipe having flexible joints, to the desired point of discharge. This operation is shown on Fig. 10. By the construction of proper dikes most of the filling material was deposited under water, and all of it was thoroughly puddled.

Fig. 11 is a general view, from the easterly abutment, of the spillway and sluice-gate sections as the dam was approaching completion, but before the completion of the hydraulic fill or the placing of the top slab. The hydraulic fill was completed late in the fall of 1921. The top slab was not cast until 1922. Fig. 12 shows a general view of the completed dam from the down-stream side.

Stability—Spillway Section

Forces.—Two conditions of loading, subsequently referred to as *A* and *B*, were selected as representing the most extreme conditions which might conceivably occur. The spillway section (concrete envelope) was made stable under both these extreme assumed conditions of loading.

Condition A.—The concrete envelope was assumed to have been constructed to its ultimate height (crest elevation, 20 ft.), and then subjected to a maximum flood overtopping its spillway crest by 1 ft. before any of the earth fill had been made. This condition could have occurred only during the construction period and only in event the dam had been built to its full height at the first operation. As actually constructed (concrete envelope and earth fill carried to Elevation 16 and raised later to Elevation 20), the nearest approach to Condition *A* which can be imagined would be that in which, after construction to the full height, the earth fill should settle away slightly from the inner face of the concrete slab, at a time when water was flowing over the spillway, thus causing full water pressure against the slab; but as constructed this could not occur without, at the same time, having the benefit of the weight of a considerable portion of the earth fill on the floor of the envelope in resisting overturning.

Condition B.—The same conditions as to loading as Condition *A* were assumed except that the resisting moment and downward pressure due to the weight of earth on the floor of the envelope was also taken into account.

With one exception which represented a most extreme condition not possible to be found save under very exceptional circumstances during the construction period, the four resultants tested cut the base practically at the mid-point.

DISCUSSION

STUART K. KNOX,* M. AM. SOC. C. E.—The most interesting part of the Oradell Dam is the spillway, which extends throughout the greater part of its length. This is an earthen section protected on its down-stream side by a concrete envelope, so that it may be overtopped without injury. It is this concrete envelope with its support of round piles and its steel sheet-pile cut-off, which constitutes the novel feature of the design.

When the plans for the dam were prepared, about 1911, the ideas were supplied by Mr. Hill. The speaker's function was merely that of developing the author's notions and in working out details. He recalls that when this particular idea was suggested, it did not, at first, appeal to him. His first impression of the proposed construction was that of a reinforced concrete dam perched on stilts, in a somewhat precarious position; of a buttressed dam turned hindside before. He feared the difficulty of insuring the structure against undermining, and that its purpose would thereby be defeated.

However, these unfavorable first impressions were quickly removed after a detailed analysis had demonstrated the practicability of the construction proposed. After careful examination of the stresses and of the resultant forces to which the dam would be subjected under the most extreme conditions of loading that could be imagined, he became convinced that the design represented an intelligent and effective solution of a really difficult problem, and that is still his opinion.

The actual conditions of loading to which the concrete envelope would be subjected were unusually difficult to predetermine. Three or four conditions were assumed, therefore, which it was thought represented the greatest possible extremes. The stresses and resultant forces were computed on the assumption that the full water pressure had been applied to the concrete envelope itself, without any earth fill, a condition which might conceivably have arisen after the concrete envelope had been completed and before the back-fill had been placed. They were also computed on the basis of the back-fill having been placed before the water pressure was applied. Sets of calculations were made on the assumptions that there was, and that there was not, upward water pressure on the bottom of the concrete envelope.

The stresses were first computed graphically and finally were carefully checked analytically. The resultant of the forces under each of the various assumed conditions of loading cut the base of the envelope close to its center.

HOWARD T. CRITCHLOW,† ASSOC. M. AM. SOC. C. E. (by letter).—In 1912 the State of New Jersey passed a law (Chapter 243, P. L. 1912) authorizing the State Water Supply Commission to have supervision over the erection and maintenance of dams on certain rivers and streams or reservoirs within the State or between it and any other State, in order to protect life and property which might be endangered by the improper construction of any dam and

* (Nicholas S. Hill, Jr.), New York, N. Y.

† Chf., Div. of Waters, Dept. of Conservation and Development, Trenton, N. J.

reservoir. In 1916, the Board of Conservation and Development became the successor of the Commission. As Hydraulic Engineer of the Department the writer was called on to examine the plans for the Oradell Dam on the Hackensack River.

On account of the unique design and the importance of the structure, the Board appointed a committee of experts, consisting of William H. Burr and Arthur H. Pratt, Members Am. Soc. C. E., and the writer, to examine the plans and specifications and determine whether they were in accord with good engineering practice. It is to the credit of the author and his associates that the Committee of Experts found so little to criticize in the plans for the structure. After careful consideration of the foundation materials immediately beneath the concrete envelope, the Committee recommended that protecting paving below the spillway and sluice-gate sections be extended farther down stream in order to protect the structure against scouring of the unconsolidated materials on which it rested. It was also pointed out that, inasmuch as the sluice-gate section was designed to handle a large part of the flood waters, the gates must have adequate attendance to insure that they may be opened promptly and be kept clear of ice and floating débris which might cause plugging of the openings or fouling of the gate mechanism.

In checking the capacity of the sluice-gates and spillway to pass extreme floods, a rate of 100 sec-ft. per sq. mile was considered. With this assumption the head on the spillway with all sluice-gates open would be about 2.2 ft. Although this head is 1.2 ft. higher than that assumed by the author the dam can safely sustain such a flow with all the gates open. The author bases his assumption of maximum flood flow of 71 sec-ft. per sq. mile largely on data in Table 1 which is given in millions of gallons per day instead of the usual second-feet. Some of these records do not represent the maximum floods. The discharge of the Pequannock River at Macopin Dam (drainage area, 62.5 sq. miles) is recorded* as 201 350 000 cu. ft. in 10 hours from 10:00 P. M., October 9, to 8:00 A. M., October 10, 1903, which represents an average discharge of 88 sec-ft. per sq. mile for 10 hours. What may have been the maximum rate is not recorded, but it must have been considerably more than 100 sec-ft. per sq. mile.

In examining the stability and safety of the Pompton Lakes Dam on Ramapo River, New Jersey, which was built in 1908 prior to State supervision, the writer had occasion to study the flood records on this stream. The following is quoted from his official report:

"The drainage area of the Ramapo at the confluence with the Pompton River is 160.7 sq. miles, while at the Pompton Lakes Dam 1.5 miles above the mouth, the drainage area is 159.5 sq. miles. In Volume III of the New Jersey Geological Survey (1894), page 155, it is stated that during the flood of September, 1882, the Ramapo had a maximum discharge of 10 540 cu. ft. per sec., which is equivalent to a water level in the lake of about 4.5 ft. above the elevation of the crest of the dam. In the U. S. Geological Survey *Water Supply Paper No. 88,* page 35, it is stated that during the flood of February-March, 1902, 'on the Ramapo River at Pompton Furnace the highest water was observed at 6:00 P. M. on March 1, when it reached 4.1 ft. above the dam crest'.

* *Water Supply Paper No. 92,* U. S. Geological Survey, p. 16.

In the Annual Report of the New Jersey Geological Survey for 1903, pages 27 and 28, it is stated that during the flood of October, 1903, 'the Ramapo at about 5:40 A. M. of October 9, when the Pompton Steel Works dam broke, had reached a discharge of 7 123 cu. ft. per sec. * * * Undoubtedly the maximum discharge was much in excess of this.' This same report in Table V, on page 27, states that the maximum discharge of the Ramapo during the flood of 1896 was 8 731 cu. ft. per sec. Thus it is seen that during a period of less than 25 years there were 4 floods that caused a lake level of over 4 ft. above the crest of the dam and a possibility that during the flood of 1903 the level would have been 5 ft. above the crest if the dam had not broken. [The crest of the old dam was of the same elevation as the existing structure.] Therefore, in analyzing the strength of the dam and wing-wall a flood level 5 ft. above the crest of the dam has been considered."

A 5-ft. head on the Pompton Lakes Dam (300 ft. long) is equivalent to a discharge of 85 sec-ft. per sq. mile, as compared with the record of 66 sec-ft. per sq. mile given in Table 1 for the flood of September, 1882. The flood of October, 1903, appears to have been a more severe flood than that of September, 1882, on North Jersey streams. Unfortunately, few permanently established stream-gauging stations were being operated during that flood. Thirty-six permanent gauging stations are maintained in New Jersey, twenty-six of which are equipped with continuous water-stage recorders. No extreme flood heights have yet been recorded. The heights given in Table 2 are, however, of record.

TABLE 2.—Recorded Run-Offs.

Name of stream and location.	Drainage area, in square miles.	Maximum flood flow, in second-feet per square mile.	Date.
Swimming River, near Red Bank, N. J...	47.7	80	July 23, 1919
Lawrence Brook, at Weston's Mills, N. J.	45.0	63	July 20, 1921
Shabakunk Creek, near Trenton, N. J....	13.1	107	April 7, 1924

In this connection the spillway capacity of some of the more important dams in New Jersey, as given in Table 3, may be of interest.

TABLE 3.—Spillway Capacities of New Jersey Dams.

Name and location of dam.	Drainage area, in square miles.	Spillway capacity, in second-feet per square mile.	Remarks.
Pompton Lakes Dam, on Ramapo River...	160	70	Entire dam masonry
Boonton Dam, on Rockaway River.....	122	92	Entire dam masonry
Oradell Dam, Hackensack River.....	114	110	With sluice-gates
Wanaque Dam, Wanaque River.....	91	192	Earth dikes
Tintern Manor Reservoir, on Swimming River.....	48	180	Earth dikes
Oak Ridge Dam, on Pequannock River.....	27.3	190	Earth dikes

The Oradell Dam is the most important structure in New Jersey equipped with sluice-gates for passing flood waters. As the author points out, however, this was necessary in order to hold down the water level in the reservoir and

prevent the flooding of railroad and other unowned land. There are many dams in streams in the Coastal Plain of New Jersey that depend entirely on manual control of sluice-gates to pass flood waters. Most of these are structures built prior to State supervision. It is very desirable to eliminate so far as possible the necessity for operating gates to pass floods and to rely on overflow or other automatic spillways. In the Oradell Dam, under present conditions of operation as a water-supply reservoir and with attendants constantly on duty at the pumping station immediately below the dam, it would seem that adequate provision has been made for gate control.

In some structures where inadequate spillway capacity is provided and it becomes the writer's duty to require enlargement and often added construction cost, he is met with the argument that the builder is not justified in making the increased expenditure to take care of a condition which may occur only once in a lifetime. Engineers use factors of safety of as much as six or more in many of their designs and yet the hydraulic engineer is often limited to a factor of two in many parts of his design. The writer is very strongly of the opinion that an endeavor to save money by providing a spillway of other than liberal capacity is poor economy for the owner as well as for the general public.

A. T. Ricketts,* M. Am. Soc. C. E.—The wedge-piles one of which is shown in Fig. 8 were made to meet a difficulty experienced in driving the sheet-piling due to a tendency of the piles to drive out of plumb. The tops, which were held in perfect alignment by waling strips, traveled ahead of the lower ends. After 50 or 75 ft. of piling had been driven the inclination of the piles was sufficient to cause lateral vibrations under the hammer. An attempt was made to correct the difficulty by beveling the points so as to spread the joints at the bottom, but this was not successful.

No difficulty was experienced in driving the wedge-piles with their broad ends down. The angle of the wedge was made slightly greater than the inclination of the last straight pile driven so that the top of the pile following the wedge-pile inclined slightly toward the piles in place. Two or three of the wedge-piles were used under the spillway section.

Thaddeus Merriman,† M. Am. Soc. C. E.—Mr. Critchlow touched on a very important point when he spoke of the spillway capacity. The maximum recorded floods as reported by the author are about 70 cu. ft. per sec. per sq. mile. This figure, however, does not represent the possibilities. Since the design of the Oradell Dam was made there has been recorded a flood discharge of 171 cu. ft. per sec. per sq. mile from a drainage area of 192 sq. miles on Esopus Creek in the Southern Catskills, New York State. Had the storm which produced this flood been centered over that water-shed, the resulting discharge would have been substantially greater.

Among the great storms on record is that of October, 1869,‡ which was central near Canton, Conn., where the total rainfall was 12.35 in. At Middle-

* Associate Engr., Nicholas S. Hill, Jr., New York, N. Y.

† Chf. Engr., Board of Water Supply, City of New York, New York, N. Y.

‡ *Transactions*, Am. Soc. C. E., Vol. VII (1878), p. 224.

town, Conn., out of a total precipitation of 9.37 in., 7.15 in. fell in 24 hours. This quantity of rainfall in 24 hours compares with 10.42 in. as measured* at Newark, N. J., on October 8 and 9, 1903.

The design of the Oradell Dam is most interesting and instructive, because, at first glance, it seems to be turned 180° from a normal position. Close study, however, reveals a most carefully executed design, admirably adapted to meet local conditions.

The design of any structure constitutes about one-half the entire story; the other half is in the kind and quality of the materials of which the structure is put together. This dam was built of concrete and it would be of value if, for the purpose of record, the author would describe the sands and the aggregates that were used. A statement as to the source of these materials, as to the kind of cement used, and as to the methods of mixing and proportioning, with particular reference to the water-cement ratio, would be most helpful to future students of concrete.

With regard to most structures the history of the materials entering into them is unknown and it is therefore practically impossible to trace back and determine the probable causes of disintegration or failure if it occurs. Records of this kind regarding many structures of the past twenty years would, to-day, be of great value.

Lazarus White,† M. Am. Soc. C. E.—The writer wishes to call attention to the dangers of a dam founded on wooden piles. In case of settlement of the material below the dam a space will be created through which circulating water may wash away such material with the danger of "piping". Of course the steel sheet-piling cut-off is designed to prevent this, but the danger is there in case of any defect of the cut-off.

The wooden piles were driven to sustain safe loads of 20 tons as determined by the *Engineering News* formula. The speaker's experience has demonstrated to his satisfaction the utter unreliability of the so-called *Engineering News* formula when checked with tests made by reliable hydraulic apparatus. In one piece of work not very far from the Oradell Dam, piles were driven to "refusal" in a stiff clay; a few weeks afterward they could sustain loads of only a few tons each from a hydraulic jack. He does not believe in the variation of the *Engineering News* formula proposed as "a formula applicable to all methods of pile-driving, including drop-hammer, single-acting steam hammer and double-acting steam hammer." In his opinion, the only safe way to determine the value of piles is to test a group of them against an actual load. Where piles are closely spaced the test of a single one may be deceptive because of overlapping effects.

As to the rusting of steel sheeting when buried below the ground, the speaker has often noted that it lasts very well. Steel sheeting dug up after being buried for about ten years was surrounded by a coating of rust which, when removed, disclosed that the sheeting itself retained nearly its full cross-section. This is also true of steel pipe buried in the ground. The coating of iron rust which forms in contact with the steel preserves it from the attack of oxygen in the ground or in circulating water.

* *Water Supply and Irrigation Paper No. 92*, U. S. Geological Survey.

† Pres., Spencer, White & Prentis, Inc., New York, N. Y.

ALBERT C. AREND,* M. AM. SOC. C. E. (by letter).—The writer believes that the conditions found to exist at the Oradell Dam are very similar to those found at points on the Big Blue and Republican Rivers, in Nebraska, but where no funds have been available to provide for such costly construction. These structures were evidently safe during all stages of construction work, and consequently they must have been unnecessarily safe after the earth fill was properly completed.

The author's list of ten special conditions does not include any specific cost limitation although Provision No. 6 indicates that there was a limit.

The life of the completed work will evidently be measured by that of the steel sheet-piling. The end of the east abutment, where the steel sheet-pile cut-off stops, is evidently the weakest point of the structure, and the next weakest point would probably be where this solid masonry wall ends in the clay and white sand.

This criticism merely calls attention to the comparative inconsistency of a very safe structural design having ends or connections of only one-half or one-fourth the resistance of the connecting or adjacent parts. "A chain is still as strong as its weakest link."

It would be interesting to know the cost of this dam, and what the cost studies showed in comparison with a masonry section of gravity type extending to hard foundation. Then, if not already investigated, comparison could be made of the cost of a Taintor gate section of equal capacity and of an earth dam with sheet-pile cut-off.

Where the density of population to be served (or menaced) and the volume of business warrant the cost of an abnormally safe type of design, that part of the cost which might be termed security insurance is undoubtedly justified when it does not become a financial hindrance to the project; but how much less security, because of cost, would be justified where the population requirements, the volume of use, and the returns on the investment would not provide for any more than a necessary minimum cost? That is often the controlling and most important business feature connected with such work, and the writer would appreciate some discussion on this particular phase of the design.

PAUL E. GISIGER,† ASSOC. M. AM. SOC. C. E. (by letter).—The dam described is of a very unusual and interesting type and its designers and builders are to be congratulated for discarding precedent and originating something new.

Studying the cross-section of the Oradell Dam, the question arises as to whether it would not be of advantage to go one step further, discarding the earth back-fill on the base slab and letting the water itself be part of the weight of the structure. Such a condition has in fact been assumed in the design, since it could have occurred while the dam was under construction.

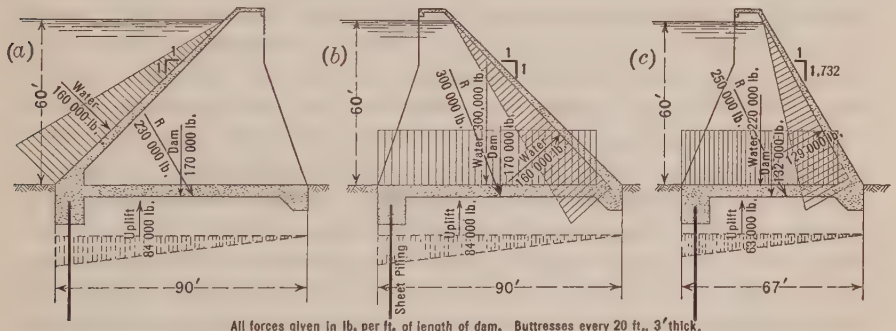
It is easy to show that a dam of this type, the reverse of the conventional reinforced concrete structure, can be designed for the same degree of safety with a considerably smaller cross-section; and it is hard to understand why

* Cons. Engr., Omaha, Nebr.

† Care, Pennsylvania Water & Power Co., Baltimore, Md.

this never has been done before. An aversion to putting tension in the buttresses and in their connection to the face slab may partly account for this reluctance, but it did not stop engineers from building reinforced concrete retaining walls of such a type. A very good solution of this detail problem has been found in the Oradell Dam by the use of structural steel.

The writer has analyzed three skeleton designs of reinforced concrete dams, assuming similar height and foundation conditions. Fig. 13 (*a*) shows a dam of conventional design, Fig. 13 (*b*) the same cross-section with water acting from the other side, and Fig. 13 (*c*) a reduced cross-section, which is even safer than Fig. 13 (*a*). The accompanying computations show that for two dams of the same cross-sectional area, Types (*a*) and (*b*), the one with water standing on the base slab, Type (*b*), is considerably safer against sliding and overturning; but, for the same safety as with the conventional type, Fig. 13 (*a*), the cross-section of the other type, Fig. 13 (*c*), can be reduced by at least 25 per cent.



All forces given in lb. per ft. of length of dam. Buttresses every 20 ft., 3′ thick.

Volume of Dam = 42 cu. yds. per lin. Foot

$$\frac{\text{Stabilizing Moment}}{\text{Overturning Moment}} = 2.2$$

$$\frac{\text{Sum of Vertical Forces}}{\text{Sum of Horizontal Forces}} = 1.75$$

Volume of Dam = 42 cu. yds. per lin. Foot

$$\frac{\text{Stabilizing Moment}}{\text{Overturning Moment}} = 2.4$$

$$\frac{\text{Sum of Vertical Forces}}{\text{Sum of Horizontal Forces}} = 2.35$$

Volume of Dam = 32 cu. yds. per lin. Foot

$$\frac{\text{Stabilizing Moment}}{\text{Overturning Moment}} = 2.2$$

$$\frac{\text{Sum of Vertical Forces}}{\text{Sum of Horizontal Forces}} = 2.10$$

FIG. 13.—ANALYSES OF SKELETON DESIGNS OF REINFORCED CONCRETE DAMS.

A further advantage that should not be under-estimated is the easy adaptation of this cross-section to a spillway, whereas with the usual type the transition from dam to spillway as a rule presents many difficulties.

The objection might be raised that, due to the weight of water on the base slab, the foundation pressures become higher. This would be erroneous, however, since the load, instead of being concentrated under the buttresses as in the conventional type, will be almost evenly distributed over the whole base.

NICHOLAS S. HILL, JR.,* M. AM. SOC. C. E. (by letter).—In several of the discussions comment is made upon the flood flow assumed in proportioning the sluice-gate and spillway sections. These sections have a discharging capacity of 71 cu. ft. per sec. per sq. mile when the water in the reservoir stands 1 ft. above the spillway crest. It was desired not to exceed this elevation because of back-flooding but, as stated, a flood in excess of this quantity

* Cons. Engr., New York, N. Y.

will not endanger the stability of the dam which has a sufficient margin of safety to permit a head of 3 ft. on the spillway crest. With this head the discharging capacity of the sluice-gate and spillway sections is 115 cu. ft. per sec. per sq. mile.

This discharge capacity is considerably greater than that of the Boonton Dam on the Rockaway River, mentioned by Mr. Critchlow. This dam has been in service for twenty years and, as far as the writer knows, its spillway capacity has never been overtaxed nor its sufficiency questioned. The discharge capacity of the Oradell Dam is also greater than that of the Pompton Lakes Dam on the Ramapo River, although less than that of the Wanaque Dam now building on the Wanaque River. The drainage areas tributary to these dams are roughly comparable in size and they are not far distant from each other.

The question of relative size of drainage area is, of course, important in comparing rates of flood discharge to be anticipated, as are also the topography of the water-shed and other factors. The writer himself has investigated and found well authenticated a flood discharge of 557 cu. ft. per sec. per sq. mile from a small area of 1.7 sq. miles. In the closing discussion on his paper, "The Design of Earth Dams", Joel D. Justin, M. Am. Soc. C. E., has compiled a table (Table 9*) which indicates recorded floods on Pennsylvania streams in excess of 4 000 cu. ft. per sec. per sq. mile from areas less than 1 sq. mile in extent. This table, which is entitled "Maximum Recorded Floods in American Rivers", shows a discharge rate of 3 200 cu. ft. per sec. per sq. mile from a drainage area of 0.2 sq. miles on Beacon Creek near Fishkill, N. Y., and a discharge rate of 2 300 cu. ft. per sec. per sq. mile from a drainage area of 1.5 sq. miles on Mad Creek, Le Roy, N. Y. With only a few exceptions, however, the flood discharges from areas exceeding 100 sq. miles in extent are less than 100 cu. ft. per sec. per sq. mile.

Mr. Merriman mentions the fact that in the storm of October 8 and 9, 1903, the quantity of rainfall in 24 hours as measured at Newark, N. J., was 10.42 in. While this is true, it is believed to have little bearing on the maximum intensity of flood flow to be anticipated on the Hackensack River. In this same storm the rainfall was 11.45 in. as measured at Paterson, N. J., which is only a few miles from the Oradell Dam. There are published figures which indicate that on July 26, 1819, 18 in. of rain fell at Catskill, N. Y., in a period of 7½ hours, and there are records which indicate that in the storm of August 5, 1843, 16 in. of rain fell in 3 hours at Concord, Pa., and 10 in. in 2 hours at Brandywine Hundred, Pa. Storms of extreme intensity are likely to be local and the average over a water-shed comparable with that of the Hackensack, to be smaller than those indicated by these maxima.

Mr. Merriman states that since the Oradell Dam was designed there has been recorded a flood discharge of 171 cu. ft. per sec. per sq. mile from a drainage area of 192 sq. miles on Esopus Creek in the Southern Catskills, New York State. Such data have a more direct bearing than the references to rainfall. However, the Esopus is a mountain stream. The Ramapo, Rockaway, and

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), pp. 135 to 139, inclusive.

Wanaque Rivers mentioned by Mr. Critchlow are highland streams. These considerations are important in estimating the maximum intensity of flood discharge to be anticipated.

Although the drainage area of the Hackensack River is not far distant from those of the rivers just mentioned, and does not differ greatly from them in extent, there are important factors of a geographic, topographic, and geologic nature which operate to make the period of accumulation of storm waters on the Hackensack relatively long. Among these may be mentioned the shape of the water-shed, the porosity of its soil, the large extent of its flats and swamp areas, the evenness of its topography, and its low elevation. The maximum elevation on the Hackensack water-shed is only 830 ft., above mean tide, and the greater part of its area does not exceed 300 to 400 ft. in elevation. It is the opposite of flashy. The slopes are flat. There is practically no outcropping rock. The dry-weather flow is much better sustained than on such streams as the Croton River, due to the porosity of the soil. Conversely, the flood flows are less. The temporary retention of the flood flows on the flats, and in the swamp areas, contributes to a leveling of the peaks of flood as do the other factors mentioned. All these matters were studied and taken into account in fixing the discharge capacity required. In the light of these studies the writer does not feel apprehensive concerning the adequacy of the flood-flow assumptions made in proportioning the Oradell Dam.

In reply to Mr. Merriman's questions regarding materials and methods used for the concrete, the ingredients of the concrete were trap rock from near-by sources, sand of excellent quality from a local pit at New Milford, N. J., and Vulcanite Portland cement. These were mixed in the proportions 1 : 2.5 : 5 for mass concrete and 1 : 2 : 4 for reinforced work. The ingredients were measured to insure proper proportioning. The consistency may be described as "medium". As stated by the writer a batch mixer was used, and the concrete handled to place by derricks and the ordinary double-side dump-cars running on portable narrow-gauge track, so that a wet mix was unnecessary.

Mr. Arend speaks of the construction as "costly". This was not the case. The most important object of the designer, the one that led him to the adoption of this unusual design, was to secure a structure adapted to the conditions at a minimum of cost. An earth dam with sheet-pile cut-off was not adapted to the conditions. The use of Taintor gates was considered. A masonry section of gravity type extending to hard foundation would have been much more costly than the design adopted. Having determined that the selected type was more economical than the others available, the only remaining opportunity for economy lay in lightening the section. In view of its novelty the writer would not have felt justified in assuming less rigid conditions in the design of the dam. The total cost of the construction was approximately $390 000.

Mr. Arend seems to the writer to misapprehend some features of the design. He states that the end of the east abutment is the weakest point, that "a chain is still as strong as its weakest link", and that a safe structural design having ends or connections of only one-half or one-fourth the resistance of the con-

necting or adjacent parts is "inconsistent", which is granted. The premises, however, are believed not to accord with the facts. The misunderstanding perhaps arises from a belief that the sheet-pile cut-off terminates at the face of the east abutment, whereas it is stated in the paper that the cut-off is "carried far into the easterly bank, following along the face of the easterly concrete abutment."

Mr. Gisiger has presented some interesting cost comparisons which constitute a valuable supplement to the paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1592

UTILIZABLE CAPACITY OF STEEL MEMBERS OF STRUCTURES*

By Henry S. Prichard,† M. Am. Soc. C. E.

With Discussion by Messrs. C. A. P. Turner, H. F. Moore, G. B. Woodruff, R. von Fabrice, F. E. Schmitt, and Henry S. Prichard.

Synopsis

This paper gives the results of a study to determine the fundamental facts on which the utilizable capacity of steel members of structures depends, and to devise methods whereby the utilizable capacity of any member can be determined, as nearly as practicable.

The utilizable capacity of any steel member is measured, as the case may be, by the greatest load which statically or repeatedly, or by the greatest opposite loads which, alternately, can be applied during the lifetime of the structure (as determined by other considerations) without causing failure, ruin, or seriously objectionable deformation (plastic, elastic, or combined plastic and elastic).

The subject is sub-divided and developed as follows:

Part 1.—Constitution, Elasticity, Deformation, Elastic Fatigue, and Permanent Fatigue of Steel and Iron. (Pages 1216 to 1217.)

Part 2.—Originally Imperfect Elastic State of Specimens of Steel and Iron; the Development, by Repeated Cycles of Stress, of Fields of Seemingly Perfect Elasticity of Chosen Range and Position; and the Limitations to Such Development Imposed in Practice by Critical Deformation. (Pages 1217 to 1218.)

Part 3.—Explanation of the Perfecting of Elasticity within Limits by the Modification of Initial "Auto-Stresses". (Pages 1218 to 1220.)

* Presented at the meeting of December 9, 1925.

† Analytical Engr., Am. Bridge Co., Pittsburgh, Pa.

Part 4.—Endurance Tests and Their Significance; and the Possibility of Remote Fatigue. (Pages 1220 to 1228.)

Part 5.—Interpretation of Tests as Criteria of Utilizable Capacity of Members in Direct Tension or Compression. (Pages 1228 to 1236.)

Part 6.—Utilizable Capacity of Compression Members. (Pages 1237 to 1249.)

Part 7.—Utilizable Capacity of Tension Members and of Members Subjected to Alternate Stresses; and the Results of Loading Tension and Compression Members Beyond Their Useful Limit Points. (Pages 1249 to 1254.)

Part 8.—Beams and Plate Girders under Transverse Loads. (Pages 1254 to 1262.)

Part 9.—Combined Compression and Flexure. (Pages 1262 to 1265.)

Part 10.—Combined Tension and Flexure. (Pages 1265 to 1267.)

Part 11.—Variation of the Characteristic Useful Limit Point Between Different Members of Nominally the Same Grade of Steel. (Pages 1267 to 1269.)

Part 12.—Limitation of the Useful Limit Point and Maximum Tensile Strength, as Criteria of Utilizable Capacity, to Steel of Good and Suitable Quality and Workmanship. (Page 1269.)

Part 13.—Factor of Safety. (Page 1270.)

In the systematic development of the subject under the headings noted, pertinent conclusions are successively reached which are briefly stated, as follows:

(*a*) Owing to the presence in plates, shapes, and bars, as they come from the mill yard, of a balanced system of internal forces (herein termed "autostresses"), the elastic limit, as determined in a long length with very delicate micrometers and measured by the intensity of the external forces which produce it, is in nearly all cases very low.

(*b*) The elastic limit, strictly defined, would be of no value as a criterion if determined, and should not be specified.

(*c*) In commercial practice a specified "elastic limit" is understood by both the seller and the buyer to be the point in a commercial test, as ordinarily conducted, properly known as the "yield point."

(*d*) The elasticity of a piece of steel, over a wide range of stress, can be perfected, or seemingly perfected, and then remain perfect or seemingly perfect, as nearly as may be gauged by very delicate micrometers, during long continued static or fluctuating stress. The limitations to this range are the criteria for gauging utilizable capacity in cases of alternate tension and compression, but are not critical, at least for untreated steel,* in cases where the stress is always direct tension or direct compression.

(*e*) The point which is the best criterion of utilizable capacity in a short column or tension member not subject to alternating stress, is the point named by the Society's Special Committee on Steel Columns and Struts

* Steel which has been treated by cold-rolling, cold-drawing, or quenching, is outside the scope of this paper.

the "Useful Limit Point", or "U. L. P.",* beyond which plastic deformation is a dangerous factor and soon becomes either ruinous or seriously objectionable *per se*. As to just what degree of plastic deformation is seriously objectionable *per se* there is room for some difference of opinion, but in most cases the yield, when well started, develops so rapidly that the zone of room for reasonable difference of opinion is limited. The Committee chose and the writer has adopted as the U. L. P. a point where the rate of combined plastic and elastic deformation reaches twice the rate of elastic deformation.

(*f*) The U. L. P. in a short column or a tension member can be fairly well approximated from the U. L. P.'s and tensile strength of representative specimens by a method described near the end of Part 5.

(*g*) As determined in ordinary commercial tests, the U. L. P.'s cannot be inferred from the yield point, but can be directly determined with fair precision in scientifically conducted tests.

(*h*) When used with judgment, the U. L. P.'s of short, centrally loaded columns and tension members are the best criteria of utilizable capacity of such columns and members, but tests of centrally loaded medium and long columns are usually made under conditions less severe than in actual service, and indicate the U. L. P.'s which are not the best criteria of utilizable capacity of members in service.

(*i*) The best criterion of the utilizable capacity of a medium column is sound analysis, based (1) on the U. L. P. of a corresponding short column; (2) on reasonable assumptions as to unintentional eccentricity; and (3) on the closely ascertained and well established modulus of elasticity. The best criterion of the utilizable capacity of a long column is sound analysis based on the maximum deflection consistent with good practice as well as on Conditions (2) and (3). Reasonable assumptions and simple methods are described and explained in Parts 5 and 6.

(*j*) For either direct compression or direct tension combined with intentional flexure, reliance must be placed on sound analysis as there are only a few tests. These few tests, however, corroborate sound analysis.

(*k*) The ordinary theory of flexure is faulty, so that very short beams and girders should have considerably more section than such theory indicates, making greater provision for spreading the load and for ample web section and reinforcement at points of reactions and heavy concentrations of load. This is fully explained in Part 8.

(*l*) In beams and girders supported against lateral deflection and with flanges and webs of sufficient thickness to be amply stiff, the possibilities in the way of shifting the elastic zone and widening its limits under constant and fluctuating, but not alternating, loads, without undue deformation, are greater than for columns or tension members under direct stress.

(*m*) The U. L. P.'s of different pieces of steel of nominally the same grade, purchased under the same specifications, may differ greatly one from another. For instance, the U. L. P. of a short, 8-in., light H-column (web, 0.31 in., and flange, 0.40 in. nominal thickness) was 35 000 lb. per sq. in.,

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1618.

whereas the U. L. P. of a short, 8-in., extra heavy H-column (web, 0.78 in., and flange, 1.212 in. nominal thickness) was only 19 000 lb. per sq. in.

(*n*) While great differences are possible in the U. L. P. of plates, shapes, and bars, and of members built from them, furnished under the same specifications, ordinarily they are all used with the same unit stress.

The paper concludes with the query whether it would not be well for manufacturers, fabricators, physical metallurgists, and structural engineers to investigate the practicability and desirability of so grouping different sizes of plates, shapes, and bars of the same nominal grade that the variation in the U. L. P. within each group would be comparatively small, and of proportioning structural members accordingly.

Part 1.—Constitution, Elasticity, Deformation, Elastic Fatigue, and Permanent Fatigue of Steel and Iron

In general, metals are composed of irregular crystals, termed grains. Both steel and wrought iron are composed of sets or groups of crystalline grains. For either steel or wrought iron, separately considered, these sets have nearly the same modulus of elasticity but different elastic limits.

The deformation of metals is of two kinds: Elastic deformation, from which recovery is complete on release of stress; and plastic deformation, which is permanent. Elastic deformation, within the limitations of delicate measurements, is proportional to stress. Ewing, Rosenhain, and other eminent physical metallurgists have shown that plastic deformation of metals takes place within their grains by the sliding, one over another, of crystalline layers similarly as the cards within a pack slide while its form is being altered. Beilby has shown that thin films, between the sliding crystalline layers of the grains, acquire, by this mechanical movement, the mobility of the liquid state. While this mobility continues, the grains in which it occurs are in a state of elastic fatigue. This state is temporary and is followed either by recovery or permanent fatigue. After the stress that imparts mobility to thin films between crystalline layers of the grains is removed, the films resolidify, and either cement the adjacent layers, thus restoring to the grains their integrity and elasticity, or, if the over-straining has been too severe or long continued, leave the layers separated by cracks. The development of cracks constitutes permanent fatigue.

Short specimens of steel and iron, when tested in the usual way by increasing tension or compression, do not, as a rule, exhibit any plastic deformation discernible by ordinary inspection until a certain zone of comparatively small range is reached within which plastic deformation develops in such a large number of grains that a pronounced yield becomes plainly evident without the aid of a micrometer. The point at which the yield becomes plainly evident is known as the "yield point". In commercial practice, this point generally was, and still frequently is, erroneously termed the "elastic limit".

There are great differences in the recorded observations of the yield point of specimens from the same piece of steel in tests made under different condi-

tions by different observers, as shown in Table 9; but for scientific purposes, a certain point in the yield (hereinafter designated) can be closely defined, and so well determined that it constitutes a reliable criterion.

Part 2.—Originally Imperfect Elastic State of Specimens of Steel and Iron; the Development, by Repeated Cycles of Stress, of Fields of Seemingly Perfect Elasticity of Chosen Range and Position; and the Limitations to Such Development Imposed in Practice by Critical Deformation

Measurements with delicate micrometers on long gauged lengths show that some slight plastic deformation occurs below, and even far below, the yield point in pieces of steel and iron when tested in the condition in which they come from the rolls, forge, or mould. This indicates either that the elastic limits of pieces in such condition are very low or that there are no limits within which they are elastic. Experience and numerous tests show, however, that elastic limits bounding wide ranges of stress can be developed in pieces of steel and iron by repeatedly straining them through cycles of stress. Tests, especially those of Bairstow, also show that the elastic limits which can be developed in a piece of steel or iron are not constant quantities, characteristic of the piece, but are variable, depending on the conditions of straining.

Bairstow's experiments established what Bauschinger's tended strongly to show, that failure under cycles of stress repeated within certain limits is evidence that elastic limits have not been developed, and *vice versa*. This fact renders available, as a means of determining developed elastic limits and the laws governing their development, many published experiments in which failure or non-failure under cycles of stress from external forces has been recorded without stating the deformations.

The center of range of any cycle of stress from external forces is the mean between its extreme intensities. For instance, if the range is between a compression of 10 000 lb. per sq. in. and a tension of 20 000 lb. per sq. in., the center of range is a tension of 5 000 lb. per sq. in.

The maximum range of elastic deformation, with its opposite elastic limits, that can be developed by cycles of stress from external forces, varies with the position of the center of range, as shown in Fig. 1, in which, first, tension is measured upward; second, compression is measured downward; third, the loci of the elastic limits that bound the elastic range are shown for a typical piece of steel and indicate these limits for each elevation of the center of range above zero and each depression of the center of range below zero; and, fourth, each elevation and each depression of the center of range is indicated by an ordinate and an abscissa which are equal. For comparison, the critical deformation points and the maximum tensile strength, as determined in scientifically conducted tensile tests, are also shown in Fig. 1.

The writer has adopted as critical that point in the deformation below which it is predominantly elastic and above which the additional deformation is predominantly plastic. Plastic deformation is useful in adjusting structural members to the service to which they are subjected but, otherwise, the plastic

state is objectionable *per se* while it lasts and the deformation remains an objectionable permanent set. Additional deformation predominantly plastic is seriously detrimental and in many cases is either dangerous or induces destruction under loads only slightly above those causing critical deformation, as discussed more in detail in Part 7. The point adopted as critical is the one designated by the Society's Special Committee on Steel Columns and Struts as the "Useful Limit Point", or "U. L. P." If a member (including its details) is strained beyond its U. L. P. and survives the consequences of such temporary fatigue, its loss or impairment in usefulness is not, ordinarily, due to changes which took place in the structure of the steel, but to the lengthening or shortening, sometimes combined with bending, of the member and its details.

The elevation, even to very high limits, and the moderate depression of the elastic limits shown by the full lines in Fig. 1 are in direct accordance with experiments. The great depression indicated by the dotted lines is inferred, as search has failed to discover any direct experimental evidence.

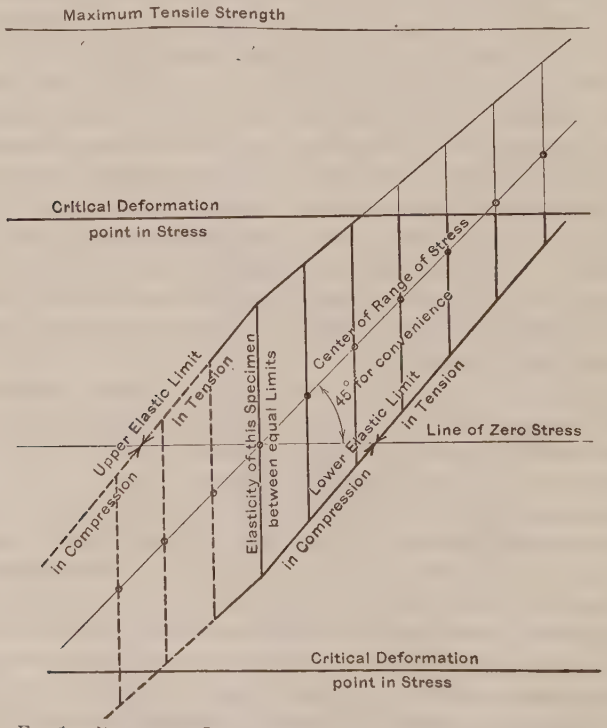


FIG. 1.—RANGE AND LIMITS TO SEEMINGLY PERFECT ELASTICITY DEVELOPED IN DIFFERENT SPECIMENS OF SAME PIECE OF STEEL BY REPEATEDLY STRAINING THEM BETWEEN LIMITS IN DIRECT TENSION, OR COMPRESSION, OR BOTH.

PART 3.—EXPLANATION OF THE PERFECTING OF ELASTICITY WITHIN LIMITS BY THE MODIFICATION OF INITIAL AUTO-STRESSES

As a basis for a satisfactory analysis and explanation of the development of elastic limits in a piece of steel or iron, it is necessary to consider the condi-

tion of the piece as regards auto-stresses and the changes that take place in these auto-stresses when the piece is over-strained and subsequently recovers from the resulting elastic fatigue.

Auto-stresses are the stresses within a body which, considered by themselves, without regard to those from external forces, form a balanced system of internal stresses. They may be present in a piece of steel or iron whether or not, at the same time, it is subjected to stresses from external forces. Auto-stresses are developed in a body while it is cooling from a hot state, and during recovery, or partial recovery, from the elastic fatigue of plastic deformation. They are also developed in structures during and by reason of such mechanical operations as tightening counter rods.

Auto-stresses which originate during the cooling of a body are caused by unequal rates or different periods of cooling of different parts. Every piece of steel or iron as it comes from the mill, forge, or foundry is liable to and probably does contain, in some of the grains of its weakest group, auto-stresses of such great intensity that the addition of even small stresses due to external forces will strain the metal beyond its elastic limit. A grain which has been over-strained may happen to recover from its elastic fatigue at such a point in the cycle of stress that the intensity of the total stress will be just the same as that of the stress from external forces, but the chances are greatly against such a phenomenon.

When the intensity of the total stress in a rehabilitated grain is different from the intensity of the stress from external forces, the difference is a new auto-stress. These new auto-stresses in rehabilitated grains are liable to be of any intensity between zero and the greatest possible, according to the point in the cycle at which recovery from elastic fatigue happens to take place. If this new auto-stress is less than a certain critical limit, and the intensities of the stresses from external forces under succedent cycles do not increase, the grain will not again be over-strained but will remain elastic; but if the new auto-stress is more than the certain critical limit the grain will again be over-strained during the next cycle without, however, destroying the possibility that during the following or some later cycle it will recover from elastic fatigue and thereafter remain elastic. Under continued cycles of stress the fatigued grain will eventually either crack or recover with an auto-stress so low that the combined auto-stress and extreme stress from external forces during subsequent cycles will be within the elastic limits.

By reason of the changes in auto-stresses effected by elastic fatigue and recovery, if the range of stress in a given continuously repeated cycle is within certain limits successive adjustments of the auto-stresses in the various grains of a piece of steel or iron gradually take place through the operation of the law of chance, although in extreme cases a million or more cycles occur before the auto-stress in the last grain is adjusted, the plastic deformation in the meantime growing less and less until no plastic deformation or growth in permanent set under continued cycles of stress can be detected, even with delicate micrometers.

While the adjustment of auto-stresses and the cracking of some of the grains are taking place an improvement occurs in the elasticity of those grains which do not become cracked during recovery from elastic fatigue. This improvement is due to the fact that with the solidification of the thin mobile films new junctures, stronger than the original ones, are formed between the parts of the grains adjacent to these films.

From the foregoing analysis it appears that the results of repeated cycles of stress on pieces of steel and iron are the net sum of various and, to some extent, opposite effects. Analysis by process of reasoning is useful in explaining why and how changes take place, but the phenomena involved are far too complicated to admit of determining by mathematical analysis the limits to the range within which elasticity will be perfected by repeated cycles of stress. Quantitative results of repeated cycles of stress can only be determined by experience and experiments.

Part 4.—Endurance Tests and Their Significance; and the Possibility of Remote Fatigue

Experiments on the endurance of specimens of steel and iron subjected to repeated cycles of direct stress have been made by Wöhler, Bauschinger, Reynolds and Smith, Bairstow, and the Sub-Committee of a Committee of the British Association for the Advancement of Science. The tests by the Sub-Committee were made by repeated tensions of specimens of two lots of steel with tensile strengths of 59 100 and 52 400 lb. per sq. in., respectively. These tests have not been published, but Mr. A. R. Fulton reported* "it was found that:

"1.—After ordinary yield is exceeded and permanent elongation has taken place, the limits of the proportionality of stress and strain are raised. The extent of this elongation for any maximum stress depends on whether the stress has been applied in one stage or with a sufficient number of repetitions at each of several stages.

"2.—The range of stress over which the proportionality is maintained is limited; once established it is independent of the number of cyclical repetitions to which it is subjected.

"3.—The range of stress may be again varied by further elongation, but to this there is a limit.

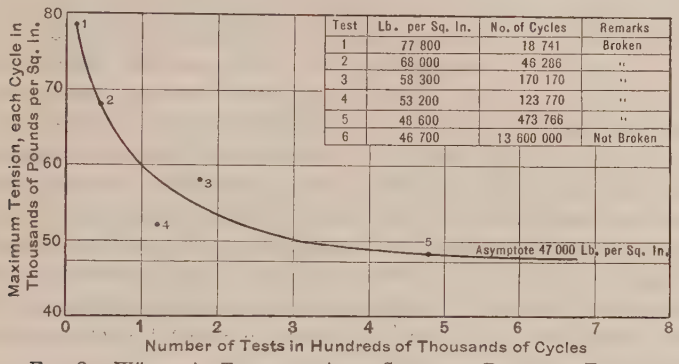
"4.—Failure under repetitions of tensile stress occurs by the movement of the crystals relatively to one another, consequent on the existing range of stress being exceeded, or during the transition period when one range is being changed to another.

"5.—Failure frequently originates by the range of stress, or even the maximum static stress of the material, being exceeded in parts of a section, and is due to mechanical flaws in the material or non-axial loading."

The fourth finding is open to the criticism that the movement occurs within the crystals. In addition to the tests mentioned, Thurston made endurance tests of steel and iron wires by subjecting them to long-continued constant tension. From these various experiments Tables 1, 2, 3, and 4 have been compiled. The data recorded, together with the supplementary data accom-

* Report, British Assoc. for the Advancement of Science, 1919, p. 484.

panying Figs. 2 and 3, and a few additional scattered citations, constitute an epitome of the important information to be gleaned from the published tests under direct stress.



Test	Lb. per Sq. In.	No. of Cycles	Remarks
1	77 800	18 741	Broken
2	68 000	46 286	"
3	58 300	170 170	"
4	53 200	123 770	"
5	48 600	473 766	"
6	46 700	13 600 000	Not Broken

FIG. 2.—WÖHLER'S TESTS OF AXLE STEEL BY REPEATED TENSIONS FROM ZERO, SHOWING THE GRAPHIC DETERMINATION OF LIMITS TO THE CYCLE OF STRESS WHICH SEEMINGLY CAN BE ENDLESSLY REPEATED WITHOUT CAUSING FRACTURE. BY INFERENCE ELASTICITY IS PERFECTED WITHIN THESE LIMITS.

Endurance tests by repeated cycles of transverse loading are comparatively numerous and embrace those of a recent and extensive investigation by the Engineering Experiment Station of the University of Illinois in co-operation with the National Research Council and others.* One set from

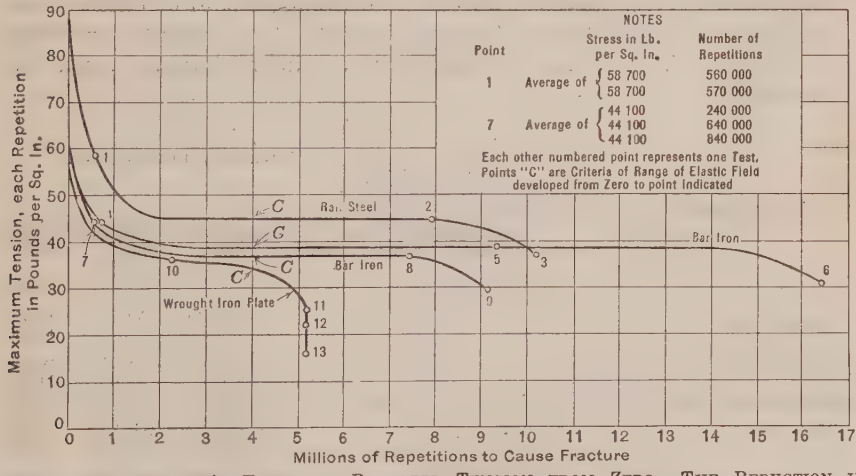


FIG. 3.—BAUSCHINGER'S TESTS BY REPEATED TENSION FROM ZERO. THE REDUCTION IN RESISTANCE UNDER THE EXTREME NUMBER OF REPETITIONS MAY HAVE BEEN CAUSED BY A HYPOTHETICAL "REMOTE FATIGUE", OR BY SOME UNRECORDED VARIATION IN CONDITIONS.

the Watertown Arsenal Reports of ordinary hot rolled steel bars has been chosen for illustration (Table 5). In this set the only important variable in the fundamental conditions was a wide range of carbon in the steel. Auxiliary pulling tests were made. The experiments were well planned and

* *Bulletin No. 124*, Eng. Experiment Station, Univ. of Illinois.

scientifically conducted; and the number of cycles in each case was great and, in some cases, extreme.

Bairstow made the tests recorded in Table 1 for Lots 1, 2, and 3, using the ordinary form of direct tension and compression testing machine with special attachments. His method was to test the specimen between the limits of stress selected for the experiment and to observe the deformation. In those cases in which the range between the limits was too great, the deformation progressively increased until the specimen broke. On the other hand, when the range was not too great, the increments of plastic deformation contributed by successive cycles became smaller and smaller so that eventually the deformation during each cycle seemed to be entirely elastic and no increase in permanent set under thousands of additional cycles could be detected with a measuring device sensitive to 0.00001 in. As long as the metal was to any appreciable extent plastic, the mobile films within the specimen acted as a drag on the elastic metal, delaying its recovery of shape, a phenomenon known as hysteresis.

TABLE 1.—Elastic Limits Developed by Cycles of Direct Stress with Different Centers of Range in Various Kinds of Steel and Wrought Iron.

(Stresses are in terms of tension from external forces (+) and compression from external forces (—). Lots 1, 2, and 3 were tested by Bairstow to determine the development of elastic limits under cycles of stress. The elastic limits given were scaled from his diagrams on p. 51 of *Philosophical Transactions,* Royal Soc. of London, Vol. 210. Lots 8 and 9 were tested by Wöhler. The developed elastic limits are inferred from stresses and endurance.)

Kind of materials.	Lot.	Tensile strength, in pounds per square inch.	Yield point, in pounds per square inch.	Developed Elastic Limits.			
				Cycles per minute.	Upper elastic limit.	Lower elastic limit.	Elastic range.
Unannealed charcoal iron.....	1	43 900	32 500	2	+22 000	−20 600	42 600
					+40 000	0	40 000
Unannealed axle steel.....	2	85 600	55 800	2	+30 000	−28 000	58 000
					+39 000	−15 000	54 000
					+51 000	+ 4 000	47 000
					+63 000	+18 000	45 000
					+74 000	+34 000	40 000
					+83 000	+45 000	38 000
Unannealed Bessemer steel.....	3	99 900	71 700	2	+38 200	−35 700	73 900
					+58 000	−11 000	69 000
					+65 000	− 2 000	67 000
					+83 000	+18 000	65 000
					+94 000	+34 000	60 000
Iron axle, Phœnix Company....	8	46 700	.....	60 to 80	+31 000	0	31 000
					+42 800	+19 400	*23 400
Axle steel, Krupp's.....	9	94 500	.....	60 to 80	+47 000	0	47 000
					+77 800	+34 000	*43 800

* Wöhler's experiments show that these ranges were developed, but do not indicate that they are maxima.

The tests of Lots 1, 2, and 3 recorded in Table 1 were made at the rate of 2 cycles per min. For purposes of comparison some of these tests are again cited in Table 2. The elastic limits and elastic ranges developed in the other tests cited are not directly indicated in the published records but were inferred from the resistance to fracture under the long continued repetitions of stress.

TABLE 2.—Ranges of Stress Between Elastic Limits Developed by Cycles of Alternate Direct Stresses.

(Lots 1, 2, 3, 7, 10, 11, 12, 13, 14, 15, 16, and 17 were tested by Bairstow (See *Philosophical Transactions,* Royal Soc. of London, Vol. 210, and *Minutes of Proceedings,* Inst. C. E., Vol. CLXVI, 1906, Pt. IV). Lots 4, 5, and 6 were tested by Reynolds and Smith (See *Philosophical Transactions,* Royal Soc. of London, Vol. 199, pp. 265-297.)

Material.		Ordinary Specimen Tensile Tests.		Elastic Range Developed Between Tension, *T*, and Compression, *C*.		
Kind.	Lot.	Maximum strength, in pounds per square inch.	Yield point, in pounds per square inch.	Cycles per minute.	Ratio of *T* to *C*.	Range, in pounds per square inch.
Charcoal iron (Unannealed)	1	43 900	32 500	1 200	1.07	40 800
" " "	1	43 900	32 500	800	1.4	42 600
" " "	1	43 900	32 500	2	1.07	*42 600
Lowmoor iron (Annealed)	5	51 700	.....	1 900	1.14	26 500
" " "	5	51 700	.....	1 600	1.17	39 200
" " (Unannealed)	5	52 800	36 800	....	....	.....
Wrought iron bar (Unannealed)	16	53 200	36 900	800	1.4	46 000
" " " "	17	57 300	32 900	800	1.4	43 000
Mild steel (Annealed)	4	57 400	41 800	1 930	1.13	27 700
" " "	4	57 400	41 800	1 750	1.14	34 000
" " "	4	57 400	41 800	1 670	1.12	40 800
" " "	4	57 400	41 800	1 500	1.15	43 200
" " "	4	57 400	41 800	1 400	1.16	45 000
" " "	4	57 400	41 800	1 325	1.17	46 600
" " "	4	57 400	41 800	....	....	.....
" " (Unannealed)	4	55 000	38 300	2 030	1.13	29 100
" " "	4	55 000	38 300	1 305	1.18	47 300
" " "	4	55 000	38 300	....	....	.....
Cast steel (Annealed)	6	107 500	.....	1 900	1.13	27 100
" " "	6	107 500	.....	1 750	1.15	37 400
" " "	6	107 500	.....	1 660	1.16	41 000
" " "	6	107 500	.....	1 300	1.18	45 000
Bessemer steel (Unannealed)	3	99 900	71 700	1 200	1.07	59 400
" " "	3	99 900	71 700	2	1.07	*73 900
" " "	10	106 600	65 800	800	1.40	70 000
" " "	11	98 000	62 800	800	1.40	66 000
Steel bar, ⅞ in. in diameter (Unannealed)	12	98 200	50 000	800	1.40	63 400
Axle steel (Unannealed)	2	85 600	55 800	2	1.07	*58 000
Bessemer steel (Unannealed)	13	63 900	53 300	800	1.40	58 800
Steel bar, 2½ in. in diameter (Unannealed)	14	63 400	35 400	800	1.40	57 000
Large steel forging (Unannealed)	15	66 000	32 500	800	1 40	46 100
Steel bar, ⅞ in. in diameter (Unannealed)	7	49 100	30 100	800	1.40	42 400

* Scaled from Bairstow's diagrams, see Table 1.

TABLE 3.—Resistance to Fracture Under Repeated Cycles of Direct Stress.

(Compiled from experiments by Wohler, Bauschinger, Reynolds and Smith, and Bairstow. All stresses are given in units of 1 000 lb. per sq. in.)

	Tests made by	Specimens from	Carbon, percentage.	Maximum tensile strength.	Primitive elastic limit.	Resistance to Fracture, 4 000 000 Cycles of Stress.			Extreme Tests.					
						From tension.	To tension.	Range.	From tension.	To tension.	Range.	Number of cycles, in millions.	Frequency per minute.	Results.
1	Wohler.	Iron axle	.....	46.7	.....	0	31.0	31.0	0	31.1	31.1	10.1	70 about.	Broken.
2		" "	.....	46.7	.....	23.3	42.8	19.5	19.4	42.8	23.4	2.4	"	Not broken.
3		Axle steel	.....	94.5	.....	0	47.0	47.0	0	46.7	46.7	13.6	"	Not broken.
4		" "	.....	94.5	.....	34.0	77.8	43.8	34.0	77.8	43.8	12.1	"	Not broken.
5		Cast iron	.....	.....	.....	0	9.7	9.7	0	9.7	9.7	7.6	"	Broken.
6		" "	.....	.....	.....	.....	.....	.....	0	9.7	9.7	0.2	"	Broken.
7	Bauschinger.	Mild steel plates	.....	63.8	34.9	0	35.8	35.8	0	35.8	35.8	11.0	.....	Not broken.
8		" " "	.....	63.8	34.9	.....	.....	.....	0	35.8	35.8	3.5	.....	Broken.
9		Mild steel boiler-plates	.....	59.6	39.4	0	39.0	39.0	0	36.7	36.7	6.5	.....	Not broken.
10		" " " "	.....	59.6	39.4	.....	.....	.....	0	41.9	41.9	4.9	.....	Not broken.
11		" " " "	.....	59.6	39.4	.....	.....	.....	0	36.7	36.7	0.3	.....	Broken.
12		Steel axle	.....	89.8	39.4	0	44.1	44.1	0	44.1	44.1	9.0	.....	Broken.
13		Rail steel	.....	87.4	42.6	0	44.1	44.1	0	36.7	36.7	10.2	.....	Broken.
14		Bar iron	.....	59.8	33.2	0	38.5	38.5	0	30.9	30.9	16.5	.....	Broken.
15		" "	.....	59.6	26.4	0	36.7	36.7	0	29.6	29.6	9.1	.....	Broken.
16		Wrought-iron plate	.....	56.4	15.2	0	34.0	34.0	0	15.9	15.9	5.2	.....	Broken.

TABLE 3.—(*Continued.*)

	Tests made by	Specimens from	Carbon, percentage.	Maximum tensile strength.	Primitive elastic limit.	Resistance to Fracture, 1 000 000 Cycles of Stress.				Extreme Tests.					
						From compression.	To tension.	Range.	Ratio, tension to compression.	From compression.	To tension.	Range.	Number of cycles, in millions.	Frequency per minute.	Results.
		Mild Steel Bars:													
17	Reynolds and Smith.	Unannealed	.....	55.0	.....	22.2	25.1	47.3	1.13	13.3	15.1	28.4	2.0	2 015	Not broken.
18		"	.....	55.0	.....	13.3	15.8	29.1	1.18	21.6	25.6	47.2	1.1	1 305	Broken.
19		Annealed	.....	57.8	.....	13.0	14.7	27.7	1.13	12 3	14.0	26.3	1.8	1 888	Not broken.
20		"	.....	57.8	.....	15.9	18.1	34.0	1.14	15.0	17.1	32.1	5.1	1 696	Not broken.
21		"	.....	57.8	.....	19.4	21.7	40.8	1.12	19.4	22.1	40.5	0.7	1 682	Broken.
22		"	.....	57.8	.....	20.1	23.1	43.2	1.16	20.7	23.9	44.6	0.5	1 544	Broken.
23		"	.....	57.8	.....	20.8	24.2	45.0	1.16	21.2	24.7	45.9	0.7	1 441	Broken.
24		"	.....	57.8	.....	21.5	25.1	46.6	1·17	21.8	25.6	47.4	0.7	1 345	Broken.
		Cast Steel:													
25		Annealed	.....	107.5	.....	12.7	14.4	27.1	1.13	12.4	14.1	26.5	1.3	1 892	Broken.
26		"	.....	107.5	.....	17.4	20.0	37.4	1.15	17.9	20.5	38.4	0.7	1 838	Broken.
27		"	.....	107.5	.....	19.0	22.0	41.0	1.16	18.8	21.8	40.6	2.3	1 650	Broken.
28		"	.....	107.5	.....	20.7	24.5	45.2	1.18	20.2	23.8	44.0	0.9	1 303	Broken.
		Lowmoor Iron:													
		Annealed	.....	51.7	.....	12.4	14.1	26.5	1.14	12.3	13.9	26.2	1.2	1 890	Broken.
30		"	.....	51.7	.....	18.1	21.1	39.2	1.17	18.2	21.3	39.5	0.8	1 630	Broken.
31		Cast iron	.....	.....	.....	.....	.....	.....	.....	5.9	6.9	12.8	0.6	1 305	Broken.
		Steel:													
32	Bairstow.	Bessemer	0.645	106.6	62.0	29.4	41.2	70.6	1.4	29.2	41.0	70.2	1.6	795	Not broken.
33		"	0.446	98.0	56.0	28.4	39.7	68.1	1.4	27.2	38.0	65.2	1.3	796	Not broken.
34		"	0.170	63.9	48.0	25.0	35.0	60.0	1.4	24.0	33.5	57.5	2.1	795	Not broken.
35		Bar, ⅞ in. in diameter.	0.446	98.2	43.9	26.4	37.0	63.4	1.4	26.0	36.5	62.5	3.4	796	Not broken.
36		" 2½ in. in diameter.	0.331	63.4	32.0	24.1	33.7	57.8	1.4	21.7	30.5	52.2	2.0	794	Not broken.
37		" ⅞ in. in diameter.	0.065	49.1	24.0	17.7	24.7	42.4	1.4	17.7	24.7	42.4	1.1	791	Broken.
38		Large forging	0.336	66.0	29.0	19.2	26.9	46.1	1.4	17.8	25.0	42.8	1.8	794	Not broken.
39		Charcoal iron bar	0.039	43.9	29.0	17.8	24.8	42.6	1.4	17.3	24.1	41.4	1.4	792	Not broken.
40		" " "	0.039	48.9	29.0	20.5	22.3	42.8	1.09	20.2	22.1	42.3	1.3	818	Not broken.
41		" " "	0.039	43.9	29.0	21.9	20.2	42.1	0.92	20.6	18.9	39.5	1.0	800	Not broken.
42		" " "	0.039	48.9	29.0	23.8	17.2	41.0	0.72	24.1	17.3	41.4	1.5	800	Not broken.
43		" " "	0.039	43.9	29.0	18.6	19.9	38.5	1.07	18.6	19.9	38.5	1.6	1 200	Not broken.
44		Wrought-iron bar	0.029	53.2	32.0	20.2	28.2	48.4	1.4	18.9	26.4	45.3	1.1	797	Not broken.
45		" " "	0.195	57.3	29.9	17.9	25.1	43.0	1.4	18.0	25.2	43.2	0.9	798	Broken.
46		" " "	0.195	57.3	29.9	20.5	22.7	43.5	1.09	20.7	22.6	43.3	1.0	818	Not broken.
47		" " "	0.195	57.3	29.9	22.7	20.8	43.5	0.92	25.2	18.1	43.3	1.0	800	Not broken.

TABLE 4.—Thurston's Tests of Endurance of Iron Under Dead Loads.*

Percentage of maximum static load.	Time Under Load Before Fracture.	
	Hard unannealed wire.	Soft unannealed wire.
95	8 days	3 min.
90	35 days	5 min.
85	Unnoted, but 1 or 2 years	261 days
80	91 days	266 days
75	Unbroken after several years	17 days
70	Same results	455 days
65	Same results	455 days (probable jar)

* Thurston's Table No. 11, *Transactions*, Am. Soc. C. E., Vol. XLI (1899), p. 516. Thurston states that "some of these wires were still unbroken in 1898, after 15 years loading." His records do not mention deformation, but, from the known relation between endurance and elasticity, it can be inferred that each specimen which did not break had developed an elastic limit at and under the load endured.

TABLE 5.—Endurance Tests of Rotating Shafts. Material, Hot-Rolled Bars of Open-Hearth Steel Turned to 1.009 Inches in Diameter, from 1.25 Inches.

(Length between supports, 33 in.; loaded over 4-in. length at middle; speed of rotation, 500 rev. per min. Compiled from Watertown Arsenal Reports for 1903-08.)

Percentages of Chemical Constituents.			Pulling Tests Auxiliary to Endurance Tests. Gauged Length, 4 In.		Endurance Tests of Rotating Shafts.			
					Developed elastic limit or one-half range, in pounds per square inch.	Extreme Tests.		
Carbon.	Manganese.	Silicon.	Maximum strength, in pounds per square inch.	U. L. P.* in pounds per square inch.		Extreme fiber stress, in pounds per square inch.	Number of rotations.	Result of test on shaft.
0.17	0.57	0.04	68 000	†51 000	35 000	30 000	100 000 000	Not broken.
0.34	0.65	0.34	84 600	54 300	40 000	40 000	63 667 320	Broken.‡
0.55	0.75	0.14	106 100	57 000	35 000	35 000	75 006 000	Not broken.
0.73	0.64	0.04	140 700	64 500	45 000	40 000	58 400 000	Not broken.
0.82	0.36	0.10	142 250	63 000	44 000	40 000	202 000 000	Not broken.
1.09	0.39	0.11	132 800	76 200	38 000	35 000	175 280 000	Not broken.

* In each case, the U. L. P. was obtained from deformation given in the reports of the Watertown Arsenal, but the term does not appear therein.

† Load fell to 42 000 lb. per sq. in. which is 61.8% of the maximum strength.

‡ Bar ruptured midway between bearings, at a score mark made by the head of the screw which holds up the middle bearing fixture.

The usual method of inferring the greatest resistance of specimens of steel or iron of a given quality, size, and shape to endlessly repeated cycles of stress is to subject different specimens from the same lot or, preferably, from the same long piece, to different ranges of stress in rational series, and to repeat each test until failure occurs or circumstances indicate that the test might be repeated indefinitely without breaking the specimen. The plotted results of a series of such tests indicate by an asymptote the maximum

range within which seemingly the stress can be endlessly repeated without fracture; and within which, by inference, seemingly perfect elasticity has been developed. This is illustrated in Fig. 2. Such tests are commonly called tests of fatigue.

Besides elastic fatigue and permanent fatigue, steel and iron may be liable to what may be called, for want of a better name, remote fatigue. Every change in the temperature or stress to which a specimen of steel or iron is subjected affects it, perhaps permanently, as regards electricity and magnetism. Moreover, some permanent fatigue may be developed at such a slow rate, and be so localized and, for a long time, confined within such narrow limits, that it gives no appreciable indication of its presence in endurance tests carried to what seems to be a sufficient number of cycles. Slow changes are continually taking place in all material things. Even atoms disintegrate, electrons are radiated into space, and planets, suns, and universes come into being, exist for a while, and then pass away. It is not reasonable to expect metals to endure forever no matter how moderate the strain nor how great the protection against corrosion and abrasion.

Bauschinger's experiments on rail steel, bar iron, and wrought-iron plate, cited in Table 3, and in Fig. 3, seem to show that failure from remote fatigue is a reasonable possibility even after seemingly perfect elasticity has been developed within certain limits. The results tabulated may be due, however, to differences in quality of the specimens or, possibly, to unrecorded occurrences during the tests; or it may be that there are some other unexplained circumstances by reason of which these tests are not proper criteria. Reciprocating parts of locomotives, motor vehicles, and stationary engines are sometimes subjected in service to repetitions of stress much greater in number than those noted in Table 3.

Endurance tests by bending stresses are much more numerous than by direct stresses; and the extreme tests are usually carried to a much larger number of cycles, the largest of which the writer has knowledge being 202 000 000, as shown in Table 5. Among the bending tests there are very few failures after elasticity has been seemingly perfected and these few can generally be accounted for by scratches, set screws marks, or other causes incident to testing; in other words, bending tests do not strongly indicate "remote fatigue". Endurance tests by direct stress carried to 50 000 000 cycles or more are much to be desired to disprove or confirm the indications of the extreme tests made by Bauschinger (Fig. 3).

In dealing with machinery, the possibility should be seriously considered of failure from "remote fatigue" after elasticity has been seemingly perfected and no appreciable increase in permanent set has been developed thereafter under millions of cycles of stress; but few if any structures, or parts thereof, will be strained often enough to fail from "remote fatigue" before they fail from other causes or are put out of service. The possibility, if there is a possibility, of the failure of a structure from "remote fatigue" is not an element to be considered in its design, but rather in deciding whether or not an old structure should be condemned.

The development of upper and lower elastic limits, both in tension, is, perhaps, the most astonishing result achieved by straining pieces of ordinary iron and steel by cycles of stress. The occurrence of this phenomenon indicates that a large proportion of the grains either had at the start, or acquired during the test, elastic limits in tension considerably greater than the range of stress between the upper and lower elastic limits of the piece. The stress in the grains that determines the lower elastic limit of the piece is the net result of an "auto-compression," developed in them during the test, and tension from external forces. This auto-compression is so great that when the minimum tension from external forces is reached at the lower elastic limit of the piece the auto-compression in the critical grains exceeds the tension from external forces by an amount equal to the elastic limit of these grains in compression.

It may be inferred that a piece of ordinary iron or steel can have its elastic field lowered by cycles of direct loading to such an extent that both the upper and lower elastic limits will be in compression; but tests to substantiate this are lacking. As shown in Table 2 experiments on pieces of iron and steel under cycles of direct stress indicate that a variation in the rate of from 2 to 800 cycles per min. makes little difference in the range of the elastic field that can be developed. They also indicate that the range is reduced with an increase in the rate above 800 cycles per min.

Some experiments with intervals of hours, days, or even years between successive cycles indicate the development of much greater ranges of elastic stress than with rates of from 2 to 800 cycles per min.; but because of the small number of cycles in these experiments the accumulation of defects and permanent sets may be so minute as to be undetectable with the micrometers used.

Experiments indicate that the elastic limit can be increased by allowing a specimen to remain at rest for a considerable period under strain, provided the strain is not too great. Thurston's experiments in this regard are shown in Table 4. His records do not mention deformation; but from the known relations between endurance and elasticity it can be inferred that each specimen which did not break had developed an elastic limit corresponding to the load.

Part 5.—Interpretation of Tests as Criteria of Utilizable Capacity of Members in Direct Tension or Compression

For members subjected to direct tension only, or if sufficiently stiff, to direct compression only, the criterion of utilizable capacity is either the elastic limit that can be developed or the point of critical deformation, whichever is the lower; and the ratio of this criterion to the computed direct stress in service, is the factor of safety. Unfortunately, this basic criterion can only be determined directly and accurately by endurance tests.

In practice, the utilizable capacity has to be inferred from the results of simple tensile or compressive tests under gradually increasing loads. Some of these results, such as the elongation and reduction in area, are indications of quality. The ultimate tensile strength and the development of the yield, if the

steel is of suitable quality, are fairly reliable criteria of utilizable capacity when jointly considered as hereinafter described.

When, in a tension or fairly rigid compression member under a constant load, any part of the metal is strained beyond its elastic limit by the stress from the load combined with auto-stress, plastic deformation will occur and will continue for a time; but, unless the load is too near the maximum, it will eventually cease and become a permanent set. Slight permanent sets in structural members are unavoidable; large permanent sets are ruinous. Between these conditions are sets which are seriously detrimental without being immediately ruinous. Plastic deformation (which is the original form in which the extension or linear compression constituting a set exists) after it becomes serious, because of the alterations it causes in the lengths of structural members, generally increases with the stress much more rapidly in soft, than in medium, and in medium, than in hard, steel; hence after serious plastic deformation has begun, there is much greater liability to the development of ruinous deformation in soft, than in hard, steel without, in some cases, any increase in load.

It is desirable for purposes of analysis and of definiteness in stating requirements to define precisely a readily ascertainable point in tests which will be regarded as the beginning of serious plastic deformation. In formulating such a rule close approximation to accuracy is the primary consideration and ease of ascertainment secondary.

In the Final Report of the Society's Special Committee on Steel Columns and Struts, it is stated*:

"In the Progress Report of 1917, the Committee mentioned the discussion held by the American Society for Testing Materials at its Annual Meeting, June, 1916, on the relation between proportional limit, elastic limit, and yield point. For the purpose of studying the column tests, the Committee gave careful consideration to this discussion to find whether it was possible to determine some point which, for practical purposes, might be easily located, clearly defined, and at the same time represent the limit where the metal ceases to have structural value. None of the terms defined by the discussions of the American Society for Testing Materials has appealed to the Committee as having these qualities. In searching for a more satisfactory definition, the Committee considered a modification of the suggestion made some years ago by the late J. B. Johnson, M. Am. Soc. C. E. The Committee has defined the critical point as the point which is determined graphically by drawing a line tangent to the envelope of the stress-strain curve, having a slope of one-half that of the last run-up line for its straight, or nearly straight, portion. Fig. 7 [of the Committee's report] illustrates this method of determination. So as not to confuse this with former definitions of yield point or elastic limit, the Committee has adopted a new term, and calls this the Useful Limit Point, or U. L. P.

"* * * It will be noted that the method is applicable to both tension and compression tests."

The normal U. L. P. is the same in compression as in tension for all steel likely to be used in structural members. This is shown in Table 6. It was also shown exhaustively and conclusively by the late Charles A. Marshall,

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1618.

M. Am. Soc. C. E., in his paper entitled "Compressive Strength of Steel and Iron".*

The Special Committee on Steel Columns and Struts precisely defined the most useful point that has yet been selected as the demarcation between characteristically elastic and characteristically plastic deformation. This was in terms of graphics; defined without recourse to graphics, the U. L. P. is that point at which under gradually and slowly increasing stress the rate of combined plastic and elastic deformation becomes twice the rate of elastic deformation. For instance, if the increase in deformation during the yield is measured for each additional 1 000 lb. per sq. in. and the modulus of elasticity, E, is 30 000 000 lb. per sq. in., the U. L. P. will be reached during the first increase of 1 000 lb. per sq. in. in which the increase in deformation reaches $\frac{\text{gauged length}}{15\ 000}$.

TABLE 6.—Comparative Tests in Tension and Compression of Open-Hearth Steel Bars.

(All specimens turned to 1.0092 in. in diameter from 1.25 in. in diameter; tension specimens, gauged length, 30 in.; compression specimens, 12 in. long, with flat ends and a ratio, $\frac{l}{r}$, of 47.6. Tests made at the Watertown Arsenal and given in reports for 1886, pp. 1635-1653; and for 1887, pp. 802-822.)

Marks.	Composition.			Tension, in Pounds per Square Inch.			Compression, in Pounds per Square Inch.		
	Carbon, percentage.	Manganese, percentage.	Silicon, percentage.	Maximum strength.	U. L. P.*	Modulus of elasticity.	Maximum strength.	U. L. P.*	Modulus of elasticity.
833	0.09	0.11	....	52 475	30 000	30 151 000	32 125	30 500	30 120 000
123	0.20	0.45	....	68 375	39 500	30 151 000	39 190	37 000	30 303 000
782	0.31	0.57	....	80 600	46 500	30 000 000	45 500	44 500	30 612 200
795	0.37	0.70	....	85 160	50 000	30 151 000	50 875	47 000	31 250 000
803	0.51	0.58	0.02	98 700	58 000	30 000 000	58 000	57 000	30 075 000
797	0.57	0.93	0.07	117 440	55 000	30 104 000	65 500	55 500	30 201 000
823	0.71	0.58	0.08	116 750	55 000	30 088 000	65 440	55 500	31 034 000
750	0.81	0.56	0.17	149 600	70 000	29 923 000	87 750	74 500	30 000 000
756	0.89	0.57	0.19	141 290	78 000	29 864 000	84 125	77 000	30 612 000
334	0.97	0.80	0.28	152 550	80 000	29 817 000	91 500	83 000	30 822 000

Note.—The modulus of elasticity of a nickel steel, flat-ended column with a ratio, $\frac{l}{r}$, of 100, a resistance to failure of 48 910 lb. per sq. in., and a U. L. P. of 47 000 lb. per sq. in., computed from compression sets measured at four points, in a gauged length of 50 in., was 29 800 000 lb. per sq. in. The test is reported in Watertown Arsenal Report, 1910, pp. 176-177. The moduli of elasticity, similarly computed except that the gauged length was 80 in., from column tests reported in *Technologic Paper 101*, U. S. Bureau of Standards, are as follows: (p. 48) chrome steel, 29 100 000; (p. 36) silicon steel, 30 400 000; (p. 46) silicon steel, 29 500 000; (p. 32) Mayari steel, 29 000 000; and (p. 42) Mayari steel, 30 300 000. For the chemical analyses, see Table 18.

* In each case, the U. L. P. was obtained from deformation given in the reports of the Watertown Arsenal, but the term does not appear therein.

Precise values of E cannot be observed in a piece of steel after it has been strained to its U. L. P. until the films of metal made mobile by the strain have

* *Transactions*, Am. Soc. C. E., Vol. XVII (1887), p. 53.

resolidified. The value of E for steel of all carbon contents up to and including 0.97% is approximately 30 000 000 as shown in Table 6 and, therefore, it is hardly worth while obtaining values of E (which may be only approximate) for each piece by "run-ups". The value indicated by the last run-up in the columns for which the Special Committee on Steel Columns and Struts submitted graphs varies from 28 000 000 to 30 000 000, with an average of 29 139 000.

The acceptance of the U. L. P. as a criterion of utilizable capacity without serious plastic deformation is qualified by four considerations: First, the rate at which the test is conducted must not be excessive; second, when the U. L. P is greater than the limit to which elasticity can be seemingly perfected by cycles of stress, a suitable deduction should be made in determining the utilizable capacity; third, allowance should be made for the fact that, in testing specimens by gradually increasing the load, the load is liable to drop to a point below the U. L. P. immediately after a yield point is reached; and, fourth, all the U. L. P.'s of the full-sized columns tested by the Column Committee were lower, as far as data were available for comparison, than the weighted average of the U. L. P.'s of the specimen tests.

TABLE 7.—Tests of Wrought Iron to Show the Effect of Slow and Rapid Fractures on Specimens of Varying Length.

(Specimens were taken from a bar of iron $1\frac{3}{8}$ in. square, reduced to a diameter of 1.008 in. (area, 0.08 sq. in.). From Watertown Arsenal Report for 1887, p. 924.)

Length of reduced section, in inches.	Tensile strength, in pounds per square inch.	Duration of test.	Gauged length, in inches.	Percentage of elongation in gauged length.	Percentage of contraction.	Area at rupture, in square inches.
0.08	49 380	6 min.	....	....	49.1	0.407
0.08	50 750	8 sec.	....	....	37.1	0.503
1.60	47 730	10 min.	1	56.0	47.6	0.419
1.60	49 130	13 sec.	1	50.0	46.2	0.430
2.40	47 980	8 min.	2	39.0	43.2	0.454
2.40	49 000	14 sec.	2	41.5	49.1	0.407
3.20	47 070	10 min.	3	39.0	49.1	0.407
3.20	48 120	15 sec.	3	36.0	47.6	0.419
4.80	46 860	10 min.	4	32.0	43.2	0.454
4.80	48 000	18 sec.	4	32.8	47.6	0.419
6.40	47 450	9 min.	6	25.0	49.1	0.407
6.40	48 250	20 sec.	6	32.0	47.6	0.419
8.00	45 650	9 min.	8	25.9	41.7	0.466
8.00	46 000	30 sec.	8	25.9	41.7	0.466
Mean of slow tests..	47 446	8.9 min.	....	36.1	46.1	0.431
Mean of rapid tests..	48 464	16.9 sec.	....	36.4	45.3	0.438
Grooved.....	57 200	6 min.	....	....	32.4	0.541
Grooved.....	59 250	6 sec.	....	....	32.4	0.541

No attempt is here made to define what constitutes an excessive rate of speed in testing for the purpose of determining the maximum strength and

the U. L. P. Although published information on the subject is scarce, a search has disclosed the material in Tables 7, 8, and 9. It is reasonably certain that none of the Watertown Arsenal's or of the U. S. Bureau of Standard's tests cited, was made at an excessive rate of speed.

The Special Committee on Steel Columns and Struts states*:

"The ultimate strengths shown on the supplementary specimen tests are very close to the ultimate strengths given for the specimen mill tests, and neither the ultimate strengths from the supplementary tests nor from the mill tests indicate the falling off in strength of the thicker material. It is also evident that the yield point, as recorded by the ordinary commercial tensile specimen tests, even when the machine is run at comparatively slow speeds, as was done in the Pittsburgh Laboratory of the Bureau of Standards, does not give the correct index of the strength of the material."

TABLE 8.—Effect of Variations in Speed of Testing on the Physical Properties of Steel Rivet Rods.

(*a*)* Tests Made by the Pennsylvania Steel Company on Ten Rivet Rods Taken from an Acid Open-Hearth Heat. From Each Rod Five Bars Were Cut and Each Bar Was Broken at a Different Speed.

Physical properties.		Speed of Testing Machine, 1 Inch in:				
		13.8 sec.	20 sec.	1.5 min.	2.6 min.	14.3 min.
Tensile strength, in pounds per square inch	Maximum	61 870	61 580	60 640	60 240	59 760
	Minimum	60 200	59 580	58 760	58 240	58 200
	Average of 10	61 075	60 672	59 523	59 387	59 027
Yield point by drop of beam, in pounds per square inch	Maximum	46 920	46 300	44 810	43 430	40 480
	Minimum	44 360	43 400	41 690	40 810	38 950
	Average of 10	45 708	44 410	42 904	41 763	39 647
Elongation in 8 in., percentage	Maximum	32.00	32.00	31.00	31.00	34.00
	Minimum	27.75	27.00	27.50	28.00	30 25
	Average of 10	29.65	30.18	29.45	29.33	31.93
Reduction of area, percentage	Maximum	67.1	66.3	68.0	67.1	68.4
	Minimum	62.3	62.4	63.9	63.1	63.4
	Average of 10	65.0	64.8	65.8	65.7	66.5

(*b*)† Tests Made by Mr. E. A. Custer for The Baldwin Locomotive Works.

Number of tests.	Pulling speed, 1 in. in:	Tensile strength, in pounds per square inch.	Yield point by drop of beam, in pounds per square inch.
6	3 min.	56 820	37 510
3	15 sec.	58 870	40 530

* These data were taken from "The Manufacture and Properties of Iron and Steel", by Harry Huse Campbell, Gen. Mgr., The Pennsylvania Steel Co., 1903, p. 453.

† *Loc. cit.*, p. 457.

In this regard a report of the Committee on Iron and Steel Structures of the American Railway Engineering Association, of tests on columns by the U. S. Bureau of Standards, states:† "The specimen tensile tests on which material is ordered and accepted afford no proper criterion for the strength of a column."

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1612.

† *Proceedings*, Am. Ry. Eng. Assoc., 1918, p. 793.

From Tables 1 and 2 it appears that under alternate direct tension and compression the limits to the range within which elasticity both in tension and compression can be perfected are lower than the yield point and probably lower in all cases than the direct tension and compression U. L. P.'s. It is probably the uncombined soft alpha iron in steel that determines the limits to the range of developed elastic fields. This fact tends to limit the range in steel that has not been cold-drawn or cold-rolled, or heated and quenched,* to what it is in wrought iron or in soft steel (sometimes called ingot iron). The opportunity under direct stress for increasing the elastic range of iron by resolidification of films made mobile by strain is, however, greater in the iron of high carbon steel than in that of medium carbon steel, likewise, greater in the iron of medium carbon steel than in that of soft low carbon steel or wrought iron. On the whole, the advantage is with soft steel and wrought iron over medium and high carbon steel as regards the ratio of the elastic range to the U. L. P. that can be developed by alternate direct tension and compression both of which are within the U. L. P. There is no reasonable chance of developing in service resistance to repeated direct stresses much above the U. L. P. in compression or tension members of structures (except possibly in adjustable rods of Howe trusses), on account of the serious, and in some cases, ruinous deformation that would occur under such stresses. After Wöhler had shown that endurance limits in test pieces can be developed much above the yield point and, therefore, much above the U. L. P., Launhardt published a formula designed to show this endurance limit for different ratios of maximum to minimum stresses, both in tension or both in compression; but, with the possible exception of adjustable rods in Howe trusses, Launhardt's formula has no rational applicability to tension and compression members of structures and machines. The occasion for referring to it is the fact that William Cain, M. Am. Soc. C. E., modified Launhardt's formula empirically "to allow for all the influences of impact", introduced a factor of safety, and recommended the formula thus modified for the design of railroad bridge members. This was forty-eight years ago,† but the modified formula is still extensively used.

As a criterion of utilizable capacity, the U. L. P.'s from specimen tests are too high rather than too low even for direct stress that is static or fluctuating but never reversed. In an extreme case (Test 7934, Table 10), soft steel with a maximum tensile strength of 48 240 lb. per sq. in. seemed in a careful pulling test to be perfectly elastic up to 40 000 lb. per sq. in. and had a U. L. P. of 40 600 lb. per sq. in., or 84.2% of the tensile strength. This U. L. P. is far above any valuation of utilizable capacity justified by endurance tests. Valuable information on this subject was obtained in Bairstow's experiments on an axle steel, with a yield point of 24.9 tons per sq. in., which in repeated loadings from 0 to 23.2 tons per sq. in. showed no lack of elasticity until after 6 000 loadings, when a permanent extension of the order of the yield took place.‡

* Steel which has been cold-drawn or cold-rolled or heated and quenched, is outside the scope of this paper.

† *Van Nostrand's Magazine*, November, 1887, p. 459.

‡ *Philosophical Transactions*, Royal Soc. of London, Vol. 210, p. 35.

Marshall* gives a number of test diagrams in which the elastic limit, as he calls it, is quite close to the U. L. P., and states:

"*First.*—When a load equal to the elastic limit of any of these materials is imposed and allowed to remain for some time, it causes permanent change of length, amounting, it is believed, to much more than has been generally understood, and which is apparently a very definite quantity for each case. *Second.*—This change of length, having begun under a load equal to elastic limit, will continue under a less load.

TABLE 9.—Differences in Recorded Results of Tests Made at Different Laboratories of Specimens of Carbon Steel from the Same Plates.*

(*a*) Yield Point and Drop of Beam.

Laboratory at which tested.	Point observed.	Speed of testing machine †	Load per Square Inch at Point Observed.					
			⅝-in. plate edge.	½-in. plate edge.	¾-in. plate edge.	1-in. plate edge.	1-in. plate interior.	Average of all tests.
"A"	Yield point‡	1 in. in 6 min.	38 800	36 880	33 420	.....	.....	32 400
		1 in. in 20 min.	.....	.....	.....	27 380	25 700	
"A"	Drop of beam	1 in. in 6 min.	40 800	37 320	34 050	.....	.....	33 670
		1 in. in 20 min.	.....	.....	.....	29 230	26 950	
"B"	Drop of beam	1 in. in 3 min.	Interior 38 000	Interior 33 900	Interior 30 900	.....	28 500	32 800
"C"	Drop of beam	1 in. in 15 sec.	46 700	40 800	36 400	.....	38 500	40 600

(*b*) Tensile Strength.

Material.	⅝-inch plate.			½-inch plate.			¾-inch plate.			1-inch plate.		
Speed, 1 in. in:	6 min.	3-2 min.	15-10 sec.	6 min.	3 min.	15-10 sec.	6 min.	3 min.	15-10 sec.	6 min.	3-2 min.	15-10 sec.
Tensile strength at edge, in pounds per square inch	.....	63 700	.....	63 800	.....	.....	62 200	.....	.....	.....	61 900	.....
Tensile strength in interior, in pounds per square inch	.....	62 400	66 100	.....	62 000	65 300	.....	61 100	62 700	59 000	58 300	61 600

* As given by J. A. L. Waddell, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXIII (1909), p. 222.

† At yield point and drop of beam.

‡ A permanent set of 0.01 in. was assumed by Waddell to indicate the yield point.

"The plotted tests in tension were all so made as to develop the amount of this stretch; and it is demonstrated in the cases of the 100 000-lb. steel, the 67 000-lb. steel, and the hard iron made from scrap, that the total is obtained equally as well, though requiring greater length of time, with a load 1 000 or 500 lb. less than primitive elastic limit as with a load equal to that limit. This becomes clear from the diagrams, when it is understood that the loads just above the limit were each allowed a minute or more in which to develop stretch."

* * * * * * * *

* "Compressive Strength of Steel and Iron," *Transactions*, Am. Soc. C. E., Vol. XVII (1887), pp. 56–57.

"The amount of stretch at elastic limit as taken from the tension diagrams:

"For the 100 000-lb. steel	0.3	per	cent.
" " 80 000 " "	1.4	"	"
" " 67 000 " "	1.85	"	"
For hard iron	1.3	"	"
" soft iron	2.3	"	" "

The reduction in the load in the extreme case was about 7 per cent.

From tests conducted at the Watertown Arsenal and published in its reports for the period, 1881-1910, results given in Table 10 have been selected as illustrative of extreme cases of drop in load and as suggestive of its significance. In every one of the tests constituting the first set given in Table 10 the load dropped to either 41 000 or 42 000 lb. per sq. in.; and it is reasonable to infer that the points to which the loads dropped in these ten tests are for them the real criteria of utilizable capacity.

TABLE 10.—Comparison of Tensile Strength, Elastic Limit, U. L. P., and Point to Which Load Dropped.

(Tests 7737-7745, steel bars for comparison of testing machines in Germany; gauged length, 8 in.; comp. from Watertown Arsenal Report, 1903, pp. 455-467. Tests 7934-7936, steel specimens for comparison of testing machine at works of Vicker's Sons and Maxim (Limited), London, England; gauged length, 8 in.; compiled from Watertown Arsenal Report, 1904, pp. 197-199. Up to the elastic limit the elasticity was as nearly perfect as could be gauged by micrometer readings to 0.0001 in. The load dropped immediately after reaching the U. L. P.)

Test No.	Tensile strength, in pounds per square inch.	Elastic limit, in pounds per square inch.	U. L. P.*		Load Dropped to:		
			In pounds per square inch.	Percentage of tensile strength.	Pounds per square inch.	Percentage of tensile strength.	Percentage of U. L. P.
7737	65 360	46 000	47 000	71.9	41 000	62.7	87.2
7738	64 680	50 000	51 000	78.9	41 000	63.4	80.4
7739	65 200	47 000	47 640	73.1	41 000	62.9	86.1
7740	64 740	49 000	49 000	75.7	42 000	64.9	85.7
7741	65 000	49 000	49 360	75.4	42 000	64.6	85.1
7742	65 080	49 000	50 200	77.1	41 000	63.0	81.7
7743	64 560	47 000	47 600	73.7	42 000	65.1	88.2
7744	64 780	51 000	51 400	79.3	41 000	63.3	79.8
7745	64 840	48 000	48 940	75.5	42 000	64.8	85.8
7746	64 260	47 000	47 560	74.0	41 000	63.8	86.2
7934	48 240	40 000	40 600	84.2	30 000	62.2	73.9
7935	69 080	51 000	51 000	73.8	46 000	66.6	90.2
7936	107 200	†55 000	†58 600	54.7	55 000	51.3	93.9
From Table 5	68 000	.....	51 000	75.0	42 000	61.8	82.4

* In each case, the U. L. P. was obtained from deformation given in the reports of the Watertown Arsenal, but the term does not appear therein.

† The elastic limit may have been higher and U. L. P. lower, as no data are given between 55 000 and 58 600 lb. per sq. in.

As a full-sized structural member usually lacks the homogeneity of a specimen, there is little chance, if any, that in a test the load on such a mem-

ber, after reaching its U. L. P., will fall much below it, except during subsequent failure by bending or local crippling. If it ever occurs, it would be most likely in the case of eye-bars. The report of the Watertown Arsenal for 1883 shows tests of six eye-bars; that for 1886, tests of twelve eye-bars; and, that for 1901, the test of one eye-bar. Some of these eye-bars showed pronounced yield points but the load in no case dropped below the U. L. P. previously indicated. In its report the Special Committee on Steel Columns and Struts states*:

"In straining a column, there is a point beyond which its structural value is uncertain, and consequently unsafe to rely upon. This point lies somewhere above the region of perfect elasticity and well below the place where manifest yielding occurs. For the study of column tests, the U. L. P. as above defined seems to fulfill these conditions satisfactorily. Careful observations and plotting of the stress-strain curve locate it without chance for controversy. It will be noted that the method is applicable to both tension and compression tests."

As far as was possible the Committee determined the U. L. P.'s of specimens of the shapes and flats composing the columns, and compared them with the tests of the full-sized members. From these tests, and all the foregoing data (in particular those recorded in Table 10), the writer has concluded that tensile tests of specimens of ordinary size, representative of the steel in a given column, conducted with the care and precision characteristic of the engineers of the U. S. Bureau of Standards and the Watertown Arsenal, are very valuable criteria of the strength of the column; and that they are better criteria than tests by compression of individual shapes and plates representative of those of which the column is composed; and he has devised the following:

Rule for Determining Critical Value.—The critical value of each specimen test in tension, of steel of good quality, as a criterion of the U. L. P. of a full-sized tension or short compression member of a structure, shall be considered as either 87% of its own U. L. P., or 60% of its tensile strength, whichever is the smaller.

As to the use of the specimen tests as criteria, the following is suggested:

Rule for Finding the U. L. P. of Built-Up Members from Specimen Tests.—To gauge the U. L. P. of a full-sized tension or short compression member, from tensile specimen tests, divide the area of its cross-section into representative parts; make specimen tests of each part and find the critical value of each specimen from its U. L. P. or tensile strength according to the foregoing rule; multiply the average of the tests for each part by its area, add the results, and divide their sum by the total area. The final result is the weighted average of the critical values of the specimens and indicates the U. L. P. under simple compression or tension.

This latter rule applies to properly designed, properly fabricated, centrally loaded members, made of steel of good quality, as nearly free from flexure as is practicable in tests of full-sized members. In practice, flexure is rarely confined to these limits.

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1618.

Part 6.—Utilizable Capacity of Compression Members

For convenience, all compression members of structures are herein termed "columns".

Each tension or compression member of a structure has some undesigned eccentricity of its axis with reference to the line of force herein termed "unintentional eccentricity". In addition, there may be intentional eccentricity; but Part 6 deals exclusively with columns to which the load is applied longitudinally and without intentional eccentricity. (For combined compression and flexure, see Part 9.)

In practice even short columns are liable to considerable unintentional eccentricity. The U. L. P. of a column depends partly on the character of the steel of which it is composed and partly on the unintentional eccentricity. The latter in suitable tests of centrally loaded short columns is so small that the U. L. P. may be considered as due to the character of the steel for resistance to force.

In this paper the term, "characteristic U. L. P.", is used to designate—in the steel composing a column—the property of resisting force, measured by the U. L. P., the column, if short, would have in a suitable compression test. In the absence of such tests the characteristic U. L. P. may be inferred from suitable tensile specimen tests.

A comparison of the observed U. L. P.'s of the short full-sized columns tested for the Special Committee on Steel Columns and Struts, with the U. L. P.'s inferred from tensile tests of specimens, as described in Part 5, is given in Table 11.

For the purposes of analysis, columns are classified as short, medium, and long. In the analysis hereinafter given, the utilizable capacities of short, medium, and long columns are determined by different equations corresponding to different conditions which for different ranges of length are critical. The points of intersection of the loci of the different equations define the limits to the ranges of length to which they apply. The divisions between these classes, and the influence of length on the utilizable capacity of medium columns, vary with each different U. L. P.

For the purpose of determining the utilizable capacities of columns both tests and analyses are desirable. When they harmonize the analysis explains the results of the tests and the tests confirm the results of the analysis. Other things being equal, full-sized tests are better criteria than analyses, but where sound analyses based on reasonable assumptions as to the unintentional eccentricity and the end conditions indicate smaller utilizable capacities than tests, analyses are the better criteria for the reason that the conditions which obtain in tests are generally less severe than those to which columns are liable in practice.

Columns are never absolutely straight. It may happen that the crucial eccentricity between the axis and the line of force is due to an initial curve in the axis. If the largest reasonable initial curve in the axis is assumed, it will lead to smaller computed utilizable capacities for medium and long col-

TABLE 11.—USEFUL LIMIT POINTS OF SHORT COLUMNS TESTED BY THE U. S. BUREAU OF STANDARDS FOR THE SPECIAL COMMITTEE ON STEEL COLUMNS AND STRUTS. COMPARED WITH MAXIMUM TENSION AND DROP OF BEAM IN SPECIMEN TESTS* AND THE U. L. P.'s DERIVED FOR COLUMNS FROM THE CRITICAL VALUES OF SPECIMEN TESTS BY THE RULE AT THE END OF PART 5.

(All U. L. P.'s and stresses in thousands of pounds per square inch.)

TYPE.			SPECIMEN TESTS.						FULL-SIZED TESTS OF SHORT COLUMNS.					
			Maximum Tension.		Drop of Beam.		Derived from Critical Values.		Average.					
										Percentages.				
No.	Section.	Weight.	No. of tests.	Stress.	No. of tests.	Stress.	No. of tests.	U. L. P.	U. L. P.	Of derived U. L. P.	Of drop of beam.	Of maximum tension.	Minimum U. L. P.	Maximum U. L. P.
1		Light.....	7	58.9	7	34.0	8	29.3†	28.3	96.6	83.2	48.0	**26.0**	**30.0**
		Heavy.....	7	59.5	7	38.3	2	25.9	25.0	96.5	65.3	42.0	24.0	25.7
		Extra heavy.	4	59.6	4	36.4	1	27.1	26.2	96.7	72.0	44.0	**26.0**	**26.5**
2		Light.....	5	57.7	5	36.5	8	28.0†	28.1	100.4	77.0	48.7	26.9	**29.2**
		Heavy.....	4	59.7	4	36.9	6	26.8†	26.8	100.0	72.6	44.9	26.0	27.5
3		Light.....	5	55.6	5	39.8	4	28.6	28.2	98.6	70.9	50.7	**26.9**	**29.0**
		Heavy.....	2	59.2	2	37.9	No	data	25.5	.....	67.3	43.1	25.0	**26.0**
4		Light.....	5	58.1	5	36.5	13	29.8†	31.6	106.0	86.6	54.4	29.9	33.0
		Heavy.....	4	59.4	4	37.7	13	22.9†	22.9	100.0	60.8	38.6	22.0	**24.0**
5		Light.....	2	61.5	No	data	3	29.2†	33.8	‡	....	55.0	**32.0**	35.0
		Heavy.....	3	57.9	"	"	3	29.0†	30.9	106.6	....	53.4	**29.3**	32.5
		Extra heavy.	12	56.4	"	"	12	20.0†	20.0	100.0	....	35.5	19.0	21.1
6		Light.....	9	57.6	9	34.6	No	data	24.5	.....	70.8	42.5	**23.7**	**26.2**
		Heavy.....	3	60.4	3	38.3	"	"	22.6	.....	59.0	37.4	21.0	**23.5**
7		Light.....	5	59.4	5	34.6	3	29.1	30.0	103.1	86.7	50.5	**28.5**	31.0
8		Light.....	6	56.3	6	39.2	No	data	31.2	.....	79.6	55.4	**30.0**	32.5
		Heavy.....	3	58.7	3	37.6	"	"	27.8	.....	§	‖	**20.0**	**31.0**
10		Light.....	16	58.1	16	37.6	"	"	26.8	.....	71.2	46.1	26.0	27.7
		Heavy.....	3	61.7	3	37.6	"	"	25.2	.....	67.0	40.8	**24.5**	26.5

NOTES.—For each type of column, the maximum, minimum, and average U.L.P. of six columns are given; three with $\frac{l}{r} = 50$, and three with $\frac{l}{r} = 85$, except for Type 1 Extra Heavy for which only three were tested $\left(\text{with } \frac{l}{r} = 85\right)$. Omitting these three from consideration, the U.L.P. of columns with $\frac{l}{r} = 85$ is smaller than for columns with $\frac{l}{r} = 50$ by less than 1 per cent. Where the maximum or minimum U.L.P. is for a column with $\frac{l}{r} = 85$, it is given in bold-face type.

*Comp. from Final Report of Special Committee on Steel Columns and Struts, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), pp. 1583 *et seq.*
† Weighted average. The other averages of critical values are simple. ‡ The critical value of one of the three specimens of Type 5 Light was only 22.3 which is abnormal. § The minimum full-sized test of Type 8 Heavy is abnormal, but is included in the average. ‖ Percentage omitted because of abnormality of one of three tests included in average.

umns than those computed from alternative reasonable possibilities. For short columns alternative reasonable possibilities are crucial.

In practice nearly all columns or compression members of structures are connected to adjacent members by full riveted joints, butt joints, or pins; and the joints have sufficient stiffness to transmit flexure between the members, except in the case of pin-ended members under intentional flexure of sufficient magnitude to overcome friction on the pins.

For short columns with butt joints, such as are common in practice, if the abutting faces are not machined to the proper angles a part of the load (sufficient to deform the columns to a point where the faces are in suitable contact) will act with great eccentricity, and the shorter the column the greater will be that part.* Another matter of interest in considering short columns is the fact that even when fairly well designed they are liable in some details to local secondary stresses or reduction in stiffness, which may control their utilizable capacity. In consideration of these facts it is recommended that in practice the utilizable capacity of a short column should be considered as only 90% of the U. L. P. indicated by critical values of specimen tests or by tests of short columns of the same steel and cross-section. Where a large girder or floor-beam, liable to heavy loading, is riveted to a column, the flexure transmitted to the column may be so great that its utilizable capacity to resist direct compression may be reduced to less than 90%; but such cases are special and should receive particular consideration.

Let,

u = the U. L. P. of a tension member or short column, as determined by specimen tests as described in Part 5; or as determined by full-sized tests of tension members or short columns of the same material and cross-section; and

c = the utilizable capacity per unit of cross-sectional area of a column.

For short columns,

$$c = 0.9\,u \qquad (1)$$

The mathematical analyses which follow were made for columns with pivoted or frictionless hinged (but not free) ends, and result in Equation (6) for long columns and Equation (11) for medium columns. Following these analyses, it is shown that it is advisable to apply these equations modified or unmodified, according to conditions, to columns with pin, flat, or riveted ends.

For convenience of analysis and discussion the moments and bending stresses corresponding to the deviation of the line of force from the axis of a structural member before deflection are termed primary moments and stresses, while those corresponding to the deflection (additional deviation) are termed secondary moments and stresses. For further convenience of analysis the curve by which the axis is assumed to deviate initially from the line of force, is assumed to be a sinusoid. Even with this further assumption the analysis is somewhat complicated, and its results are difficult to apply with precision to medium columns; but following the analysis, simple empirical formulas for

* "Imperfect Butt-Joints in Columns, and Stress in Lattice-Bars," *Engineering News*, October 3, 1907, p. 368.

TABLE 12.—Characteristic Useful Limit Points, u, of Columns, Beyond Which Plastic Deformation Is Seriously Detrimental. Limiting Ratios, $\frac{l}{r}$, Between Which Columns Are "Medium"; Below Which They Are "Short"; and Above Which They Are "Long". Utilizable Capacity, c, of Long Columns Beyond Which Elastic Deflection Is Liable to Be Excessive (More Than $\frac{l}{300}$).

For short columns, the utilizable capacity, $c = 0.9\,u$ ..... (1)

For medium columns, $c = u - \left(n_1 \frac{l}{r} + n_2 \frac{l^2}{r^2}\right)$ ..... (11)

For long columns, $c = \frac{207\,260\,000}{\left(\frac{l}{r}\right)^2}$ ..... (6)

Assuming initial eccentricity = $\frac{l}{700}$; ends pivoted.

u, in pounds per square inch.	Medium Columns.			Long Columns.		u, in pounds per square inch.	Medium Columns			Long Columns.	
	$\frac{l}{r}$.	n_1.	n_2.	$\frac{l}{r}$.	c, in pounds per square inch.		$\frac{l}{r}$.	n_1.	n_2.	$\frac{l}{r}$.	c, in pounds per square inch.
19 505	41.6	38.4	0.205	154	8 739	30 493	39.6	50.3	0.680	114	15 948
19 691	41.5	38.6	0.211	153	8 854	30 906	39.5	50.6	0.705	113	16 232
19 880	41.5	38.9	0.217	152	8 971	31 328	39.4	50.9	0.730	112	16 523
20 071	41.5	39.2	0.223	151	9 090	31 760	39.4	51.2	0.756	111	16 822
20 264	41.4	39.4	0.229	150	9 211	32 203	39.3	51.5	0.781	110	17 129
20 465	41.4	39.7	0.235	149	9 336	32 657	39.2	51.7	0.806	109	17 445
20 665	41.3	40.0	0.242	148	9 462	33 121	39.2	52.0	0.837	108	17 769
20 870	41.3	40.3	0.249	147	9 591	33 599	39.1	52.2	0.868	107	18 103
21 079	41.3	40.5	0.256	146	9 723	34 088	39.0	52.5	0.899	106	18 446
21 291	41.2	40.8	0.263	145	9 857	34 590	38.9	52.7	0.930	105	18 799
21 509	41.2	41.1	0.270	144	9 995	35 105	38.8	53.0	0.968	104	19 162
21 729	41.1	41.4	0.278	143	10 135	35 634	38.8	53.2	1.006	103	19 536
21 956	41.1	41.6	0.287	142	10 279	36 177	38.7	53.4	1.044	102	19 921
22 184	41.1	41.9	0.295	141	10 425	36 735	38.6	53.6	1.082	101	20 318
22 417	41.0	42.2	0.304	140	10 574	37 307	38.5	53.8	1.120	100	20 726
22 655	41.0	42.5	0.312	139	10 727	37 895	38.4	53.9	1.169	99	21 147
22 898	40.9	42.8	0.322	138	10 883	38 499	38.3	53.9	1.218	98	21 580
23 146	40.9	33.1	0.331	137	11 043	39 120	38.3	54.0	1.267	97	22 028
23 398	40.8	43.5	0.341	136	11 206	39 762	38.2	54.0	1.316	96	22 490
23 653	40.8	43.8	0.350	135	11 372	40 417	38.1	54.1	1.364	95	22 964
23 917	40.7	44.1	0.360	134	11 543	41 095	38.0	54.2	1.421	94	23 456
24 216	40.7	44.4	0.372	133	11 717	41 791	37.9	54.3	1.478	93	23 968
24 456	40.6	44.7	0.384	132	11 895	42 511	37.8	54.3	1.540	92	24 488
24 732	40.6	45.0	0.396	131	12 076	43 250	37.7	54.2	1.604	91	25 029
25 019	40.5	45.2	0.408	130	12 264	44 011	37.6	54 2	1.673	90	25 588
25 309	40.5	45.5	0.420	129	12 455	44 798	37.5	54.1	1.752	89	26 167
25 604	40.4	45.8	0.434	128	12 650	45 604	37.3	54.0	1.831	88	26 763
25 906	40.4	46.1	0.448	127	12 850	46 442	37.2	53.9	1.910	87	27 383
26 214	40.3	46.5	0.462	126	13 055	47 305	37.1	53.8	1.989	86	28 024
26 530	40.3	46.8	0.476	125	13 265	48 194	37.0	53.7	2.068	85	28 687
26 852	40.2	47.1	0.490	124	13 480	49 112	36.9	53.3	2.173	84	29 373
27 181	40.1	47.4	0.507	123	13 700	50 068	36.8	52.8	2.279	83	30 086
27 516	40.1	47.7	0.524	122	13 925	51 045	36.7	52.4	2.384	82	30 824
27 859	40.0	48.0	0.541	121	14 156	52 062	36.5	51.9	2.490	81	31 591
28 210	40.0	48.4	0.558	120	14 393	53 110	36.4	51.4	2.596	80	32 384
28 569	39.9	48.7	0.575	119	14 636	54 197	36.3	50.6	2.736	79	33 209
28 936	39.8	49.0	0.595	118	14 885	55 325	36.1	49.8	2 877	78	34 067
29 313	39.8	49.4	0.615	117	15 141	56 489	36.0	49.0	3.017	77	34 956
29 697	39.7	49.7	0.635	116	15 403	57 700	35.9	48.2	3.158	76	35 883
30 090	39.6	50.1	0.655	115	15 672	58 955	35.7	47.4	3.298	75	36 847

medium columns are given as alternatives and shown to be nearly equivalent as regards results. (See Equation (11) and Table 12.)

For the benefit of those who desire to follow the general argument without following the mathematical analyses by which Equation (11) for medium columns and Equation (6) for long columns were derived, it may be noted that all medium and long columns are assumed to have an unintentional eccentricity of $\frac{1}{700}$ of their length between pins or points of contrary flexure and that the utilizable capacities of long columns are limited to the loads which would produce deflections of $\frac{1}{300}$ of such length.

Let,

l = length.
I = moment of inertia.
r = radius of gyration.
p = load per unit of cross-sectional area of column.
e = eccentricity of the line of force relative to the axis of the column at the center before deflection.
Δ = deflection at the center of the column.
f = the intensity of the combined stress from direct compression and bending at the center of the column in the extreme fiber on the concave side.

$$E = \text{modulus of elasticity} = \frac{\text{direct stress intensity}}{\text{proportionate elongation or shortening}}.$$

For steel, considering the pound as the unit of stress and the inch as the unit of length, $E = 30\,000\,000$.

Let,

v = the distance from the axis of the column to the extreme fiber on the concave side.

$$q = \frac{\pi^2 E r^2}{l^2}$$

which is Euler's formula.

The curve of the primary bending moment will be a sinusoid. It is a peculiarity of a sinusoidal moment diagram that the deflection corresponding to it will be a sinusoid; hence, the secondary bending moment will likewise be a sinusoid.

From the theory of flexure, when the moment diagram is a sinusoid,

$$\Delta = \frac{\text{Bending moment} \times l^2}{\pi^2 E I} = \frac{p e l^2}{\pi^2 E r^2} + \frac{p \Delta l^2}{\pi^2 E r^2} = \frac{p e}{q - p} \quad \ldots\ldots (2)$$

$$f = p + \frac{p e v}{r^2} + \frac{p e v}{r^2}\left(\frac{p}{q - p}\right) \quad \ldots\ldots (3)$$

Let,

$\frac{l}{x}$ = the unintentional eccentricity.

The utilizable capacity of long columns is limited to the load which can be supported without excessive deflection.

Let,

$\frac{l}{y}$ = the greatest deflection which is not excessive.

Substituting these values in Equation (2),

$$\frac{l}{y} = \frac{p \frac{l}{x}}{q - p} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

from which,

$$p = q \frac{x}{y + x} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Assuming that $\frac{l}{x} = \frac{l}{700}$, which seems ample for unintentional eccentricity, and that, for the condition ($p = c$), $\frac{l}{y} = \frac{l}{300}$ (which although great, is not, for a column loaded to its utilizable capacity, an excessive deflection) then, in pounds per square inch,

$$c = 0.7\, q = \frac{207\,260\,000}{\left(\frac{l}{r}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Equation (6) gives the utilizable capacities, in pounds per square inch, of long steel columns of good workmanship with pivoted or frictionless hinged ends. It is independent of u or of any other critical stress and gives values for c which vary solely as the inverse of $\left(\frac{l}{r}\right)^2$. If a column qualifies as a "long" one, its utilizable capacity will in nowise depend on what U. L. P. a short column of similar steel and cross-section would possess.

Medium columns are classified according to the U. L. P.'s that short columns of similar steel and cross-section would possess. In this paper the U. L. P. that determines the series is termed the "characteristic U. L. P.", or "characteristic u".

For medium columns the utilizable capacity, c, may be considered to be the load that will produce a maximum stress intensity, f, equal to the characteristic u. The dividing line between medium and long columns is the $\frac{l}{r}$ at which the load that will cause a deflection, $\frac{l}{300}$, will also cause a stress, u, in the critical extreme fiber. At this line, from Equations (3) and (6), for $e = \frac{l}{700}$ and $\frac{v}{r} = 1.68$,

$$u = c + \frac{1.68\, c\, l}{700\, r} + \frac{1.68\, c\, l}{700\, r} \times \frac{7}{3} = c + \frac{0.008\, c\, l}{r} \quad \ldots\ldots\ldots\ldots (7)$$

Equation (7) determines the characteristic u of the series; it is useful in compiling tables.

For ratios of v to r other than 1.68, Equation (7) is approximate; but the error is not great. In most cases the ratio is less than 1.68 and, there-

fore, the dividing line between short and medium columns occurs at a slightly greater value of $\frac{l}{r}$ than that for a ratio of 1.68.

From Equation (1) the dividing line between short and medium columns is the $\frac{l}{r}$ at which a load of $0.9\ u$ will produce a stress, f, equal to u.

For the conditions, $e = \frac{l}{700}$, $\frac{v}{r} = 1.68$, and $p = 0.9\ u$,

$$\frac{p\ e\ v}{r^2} = \frac{0.9\ u\ e\ v}{r^2} = \frac{0.9\ u\ \frac{l}{700}\ 1.68\ r}{r^2} = 0.00216\ u\ \frac{l}{r} \quad \ldots\ldots\ldots (8)$$

$$u = 0.9\ u + 0.00216\ u\ \frac{l}{r} + 0.00216\ u\ \frac{l}{r}\left(\frac{0.9\ u}{q - 0.9\ u}\right) \quad \ldots\ldots\ldots (9)$$

and, solving,

$$\frac{l}{r} = \frac{46.296}{1 + \frac{0.9\ u}{q - 0.9\ u}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

From Equation (10), the $\frac{l}{r}$ that marks the dividing line between short and medium columns of any series can be found; it has to be solved by trial, which is tedious but does not impair its usefulness in compiling tables.

The limits expressed in terms of the $\frac{l}{r}$ within which columns are "medium" are given in Table 12. In compiling these limits, the dividing lines between medium and long columns were chosen, and the characteristic u was determined for each series from Equation (7). The limiting $\frac{l}{r}$ for each series as between medium and short was then determined from Equation (10).

At the limits of the "medium" columns in each series, the utilizable capacity can be found from Equations (1) and (6). For intermediate values of $\frac{l}{r}$ an exact equation can be derived from Equation (3), but it would be far too complicated for ordinary use. However, within the limits of medium columns, the loci of the exact equation can be approximated so closely by an equation of the second degree that the differences between the accurate and approximate loci are of no practical importance. Such an equation has the form:

$$c = u - \left(n_1 \frac{l}{r} + n_2 \frac{l^2}{r^2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which, n_1 and n_2 are numerical factors the values of which are given in Table 12 for each series. They were determined by reduction after placing Equation (11) equal to Equations (1) and (6) at the respective values of $\frac{l}{r}$ for the dividing points between medium and long columns of each series.

Table 12 gives for medium columns the limiting values of $\frac{l}{r}$ and values of n_1 and n_2 for eighty successive characteristic U. L. P.'s from $u = 19\,505$ lb. per sq. in. to $u = 58\,955$ lb. per sq. in. It also gives the utilizable capacity, c, of long columns from $\frac{l}{r} = 75$ to $\frac{l}{r} = 154$, inclusive.

For the assumed conditions that unintentional eccentricity, e, $= \frac{l}{700}$ and $\frac{v}{r} = 1.68$, the writer has made a considerable number of computations to develop the maximum error involved in using Equation (11) for determining the utilizable capacity, c, in place of determinations by trial from Equation (3). The maximum error discovered was an excess in capacity indicated by Equation (11) of 2% in the case of a column with a u of 20 071 lb. per sq. in. and an $\frac{l}{r}$ of 120. The greatest errors occur in the series having the smallest values of u. All the errors are probably well within 3 per cent.

To summarize: From Table 12 any column with pivoted or frictionless hinged ends can be readily classified as short, medium, or long, when its characteristic U. L. P. is known; if short, its utilizable capacity can be found from Equation (1); if medium, from Equation (11) with suitable values of n_1 and n_2 taken from Table 12; and if long, from Equation (6), or directly from Table 12 if its $\frac{l}{r}$ is any whole number from 75 to 154, inclusive.

The series of tests of columns with flat ends made by the U. S. Bureau of Standards for the Special Committee on Steel Columns and Struts, and the numerous series of tests of centrally loaded pin, flat, spherical, and fixed-ended columns made since 1880 by U. S. Government Engineers at the Watertown Arsenal, developed in all but one series a much smaller reduction in utilizable capacity with increase in length than that indicated by Equations (6) and (11) (as shown in Tables 11 and 13).

In tests of pin-ended columns, the friction between the pins and their bearings restrains the ends and causes the column to act as if it were fixed or flat-ended until the tendency (induced by the primary and secondary bending moments) of the ends to turn on the pins is sufficient to overcome the friction. In all but one case the records of the tests of pin-ended columns cited in Table 13, for which $\frac{l}{r}$ was greater than 96, show that the ends were restrained up to the moment of failure and that the columns then deflected suddenly, thus indicating that the ends suddenly turned on the pins. In the one case, in which the record makes no mention of sudden deflection, the unintentional eccentricity from an initial curve in the axis $\left(\frac{\text{length}}{550}\right)$ was greater than the assumption of $\frac{\text{length}}{700}$, and the deflection at the U. L. P. $\left(\frac{\text{length}}{241}\right)$

exceeded the permissible deflection of $\frac{\text{length}}{300}$, on which Equation (6) is based. The ends in this one case probably turned on the pins before the U. L. P. was reached, and the load at the "permissible deflection" was probably less than the 9 200 lb. per sq. in. indicated by Equation (6).

The additional value of c, if any, which a medium or long column in service possesses over and above the value indicated by Equations (6) or (11), by reason of its having pin, flat, or riveted ends, depends in each case on the conditions.

In laboratory tests the pin and flat ends of columns are restrained by a fairly rigid testing machine, whereas in practice the ends are restrained simply by the stiffness of adjoined more or less flexible members of a deflected structure, the latter, probably, being much less effective. In pin-ended members the ability of adjacent members to stiffen a member is contingent on friction on the pins, which will be overcome if the bending moment is large enough. For long columns and, ordinarily, for medium columns, it is advisable not to place any reliance on friction but to classify as "frictionless pin or hinged-ended" all columns with pin ends and all continuous compression chords of trusses having pin connections to the web members. For long and medium columns with riveted end connections to adjacent members some allowance may be made for the restraining influence of the end connections by letting the l of the equations and of Table 12 represent from three-quarters to two-thirds of the length, according to the conditions, instead of the whole length as in the case of columns with pivoted or frictionless hinged ends. In other words for such columns l represents the distance between points of contrary flexure assumed by judgment. Under ideal conditions, l for columns with fixed ends equals one-half the length, and for columns with one free and one fixed end twice the length; but ideal conditions are not achieved even in the laboratory.

To illustrate the use of Table 12 and Equation (11), a nickel steel column with a characteristic u of 47 700 lb. per sq. in. and an $\frac{l}{r}$ of 100, well connected to adjacent fairly stiff members by riveted joints, might be assumed to have an $\frac{l}{r}$ between points of contrary flexure of 66.7 and, from Table 12, would be a medium column with an n_1 of 53.7 and an n_2 of 2.033 and, from Equation (11), would have a utilizable capacity, c, of 35 100 lb. per sq. in. The test of such a column with flat ends (cited in a footnote to Table 6) showed under laboratory conditions very little variation from perfect elasticity up to 47 000 lb. per sq. in. and a failure load of 48 910 lb. per sq. in.

The equations and Table 12 given in this Part 6 are based on the assumption of no intentional eccentricity between the line of thrust and the axis of the column, whether produced by eccentric application of the longitudinal load or by a combination of longitudinal and transverse loads. Where there is intentional eccentricity, allowance should be made as described in Part 9. The weight of horizontal and inclined columns causes intentional eccentricity.

TABLE 13.—Comparison of Useful Limit Points and Utilizable Capacities, c, in Full-Sized Column Tests.

(Sets 1, 2, 3, and 4 were compiled from Watertown Arsenal Reports for 1908, 1909, 1910, and 1883; Set 5 from *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20).

All U. L. P.'s are in 1 000 lb. per sq. in. All deflections (except one) were less than $\frac{\text{length}}{300}$. In most cases the average is the average U. L. P. of three columns.

For all pin-ended columns (except one) with $\frac{l}{r}$ greater than 96, the reports record sudden lateral springing at maximum load.)

(*a*) Lap-Welded Bessemer Steel Tubing, 5 Inches in Nominal Diameter; 5.02 to 5.36 Square Inches in Sectional Area; Average Characteristic U. L. P. per Square Inch = u = 81 000 Pounds per Square Inch.

Spherical Ends.				Pin Ends.				Flat Ends.				Fixed Ends.				From Table 12.	
$\frac{l}{r}$.	Minimum.	Maximum.	Average.	$\frac{l}{r}$.	Minimum.	Maximum.	Average.	$\frac{l}{r}$.	Minimum.	Maximum.	Average.	$\frac{l}{r}$.	Minimum.	Maximum.	Average.	$\frac{l}{r}$.	c.
47	31.0	32.0	31.7	47	31.0	33.0	32.3	50	28.0	31.0	29.5	44	29.0	32.0	30.3	47	27.9
72	32.0	34.0	33.3	72	29.0	31.0	30.3	75	27.5	32.0	30.5	69	29.0	35.0	31.7	72	23.7
97	28.0	31.0	29.3	97	22.5	31.0	27.5	100	27.0	30.0	28.7	93	28.0	32.0	30.0	97	19.7
150	23.0	27.0	25.3	150	25.0	25.0	25.0	150	23.0	26.0	25.0	146	24.0	30.0	26.3	150	9.2
175	24.0	25.0	24.7	175	19.0	23.7	20.6	175	23.0	27.0	25.3	...	....	....	....	175	6.8

TABLE 13.—(*Continued.*)

(*b*) I-BEAMS BUILT OF ONE 10 INCH BY ⅝-INCH PLATE AND FOUR 4 INCH BY 3 INCH BY ⅝-INCH ANGLES.

SO-CALLED 60 000-LB. STEEL: u = 30 000 LB. PER SQ IN.								NICKEL STEEL: u = 47 700 LB. PER SQ. IN.					SO-CALLED 70 000-LB. STEEL: u = 33 000 LB. PER SQ. IN.				
$\frac{l}{r}$.	PIN ENDS.			FLAT ENDS.			c from Table 12.	$\frac{l}{r}$.	Minimum.	Maximum.	Average.	c from Table 12.	FLAT ENDS.				c from Table 12.
	Minimum.	Maximum.	Average.	Minimum.	Maximum.	Average.							$\frac{l}{r}$.	Minimum.	Maximum.	Average.	
50	30.0	30.0	30.0	30.0	31.0	30.3	25.9	25 Flat ends	47.0	48.0	47.7	42.9	25	33.0	33.0	33.0	29.7
75	28.0	31.0	29.3	32.0	32.0	32.0	22.6	100 Flat ends	45.0	47.0	46.0	20.7	100	32.0	32.0	32.0	19.4
100	30.0	31.0	30.7	30.0	31.0	30.7	18.4	117 Pin ends	40.0	40.0	40.0	15.1	...	....	....	....	....
150	27.0	28.0	27.7	27.0	29.0	28.3	9.2	187 Pin ends	24.0	27.0	25.7	5.9	...	....	....	....	....
175	13.0	26.0	20.7	23.0	26.0	24.7	6.8	214 Pin ends	19.0	19.0	19.0	4.5	...	....	....	....	....

(*c*) STEEL 6 BY 6-IN., ROLLED H-SECTIONS WITH NOMINAL SECTIONAL AREA OF 7 SQ. IN.: u = 26 700 LB. PER SQ. IN.

$\frac{l}{r}$.	PIN ENDS.			FLAT ENDS.			c from Table 12.
	Minimum.	Maximum.	Average.	Minimum.	Maximum.	Average.	
25	26.0	26.0	26.0	27.0	28.0	27.7	24.0
50	26.0	27.0	26.3	26.0	27.0	26.7	23.1
75	27.0	27.0	27.0	26.0	27.0	26.7	20.5
100	23.0	26.0	24.7	24.0	25.0	24.7	17.2
150	10.0*	20.0	15.7	23.0	23.0	23.0	9.2

(*d*) WROUGHT-IRON BUILT I-BEAMS BETWEEN CHANNELS: u = 29 500 LB. PER SQ. IN.

$\frac{l}{r}$.	PIN ENDS.			c from Table 12.
	Minimum.	Maximum.	Average.	
68	29.5	29.5	29.5	23.2
102	21.0	23.5	22.3	17.9
136	16.0	19.0	17.5	11.2

The ratios, $\frac{l}{r}$, are 3⅜ of the actual ratios for comparison with Table 12 for steel.

(*e*) TWO STEEL CHANNELS AND ONE I-BEAM TYPE 4 LIGHT OF TABLE 11: u = 31 600 LB. PER SQ. IN.

$\frac{l}{r}$.	FLAT ENDS.			c from Table 12.
	Minimum.	Maximum.	Average.	
50	29.9	33.0	32.0	27.2
85	30.4	31.5	31.1	21.9
120	28.2	31.6	30.0	14.4
155	21.0	23.0	22.3	8.6
....	....	....	....	....

* Initial curve in axis, $\frac{\text{length}}{550}$; deflection, $\frac{\text{length}}{241}$; when load was reduced to 1 000 lb. per sq. in., it reduced the deflection to 0.03 in.; on reloading, column failed by deflection; maximum load, 10 100 lb. per sq. in.

It is essential to the applicability of Equations (1), (6), and (11) that the details shall be able to develop the capacity indicated by the equations. For example, tests of forty-two columns by the U. S. Bureau of Standards, for the American Railway Engineering Association's Committee on Steel and Iron Structures, included comparative tests of columns with lacing and columns with batten-plates, and the report of that Committee states:*

"In all cases the columns with batten-plates failed at an ultimate strength of from 6 000 to 9 000 lb. per sq. in. less than the corresponding column with lacing, and, in one case, the heavy section with 120 $\frac{l}{r}$ as much as 15 000 lb. per sq. in. less."

In some cases the details of a column include lattice-bars for one or two open sides. Even under ideal conditions lattice-bars have to possess considerable potential resistance to shear in order to give the column about as much stiffness against deflection from shear as from bending moment. In actual conditions the lattice-bars actually have to resist shear. Lattice-bars also have to take some of the direct compression and should provide some resistance to possible accidental abnormal transverse loads.

Let,

S = the shear when the column is loaded to its utilizable capacity; and
A = the cross-sectional area of column.

The assumption made for short columns that the column is subject to equal eccentricities in the application of the load on opposite sides of the axis at opposite ends sufficient to produce a bending stress of 0.1 u, and the assumption made for short, medium, and long columns that $\frac{v}{r} = 1.68$, together lead to the equation:

$$S = \frac{u\,A}{8.4\,\frac{l}{r}} \qquad (12)$$

The assumption for medium columns that the stress from bending equals $u - c$, and the assumption for both medium and long columns that the column is initially bent in a sinusoidal bow with unintentional eccentricity of $\frac{l}{700}$, together lead to the equation for medium columns:

$$S = \frac{\pi\,(u - c)\,A}{1.68\,\frac{l}{r}} \qquad (13)$$

The assumption for long columns loaded to their utilizable capacity that the deflection is limited to $\frac{l}{300}$, and the assumption for long columns previously cited, together lead to the equation for long columns:

$$S = \frac{\pi\,c\,A}{210} \qquad (14)$$

* *Proceedings*, Am. Ry. Eng. Assoc., Vol. 19, 1919, p. 960.

The following general rule is suggested for proportioning lattice-bars of columns without intentional bending moment: Proportion the lattice-bars of each open side for a stress of $\frac{0.5\ S}{F}$, in which, F is the factor of safety given by the working load used for the column, and S is obtained from the empirical formula:

$$S = 0.02\ u\ A \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (15)$$

Equation (15) gives a value of S varying from 2.1 to 4.5 times as great as either of Equations (13) or (14); and from 0 $\left(\text{when } \frac{l}{r} = 6\right)$ to 6.4 $\left(\text{when } \frac{l}{r} = 38.4\right)$ times as great as Equation (12); hence, in all cases except extremely short columns, this rule allows considerable margin, for considerations other than the shear indicated by the assumptions on which Equations (12), (13), and (14) are based.

In 1891, the writer suggested the following:*

"The lattice shall be so spaced that each channel between lattice connections shall be stronger than the column considered as a whole, and their size shall be obtained by treating the column as a lattice girder loaded at the middle with a load equal to 3% of the total compression on the column."

He now modifies this suggestion in conformity with the analysis given to the following:

"The lattice shall be so spaced that each channel between lattice connections shall be stronger than the column considered as a whole; and its size shall be obtained by treating the column as a latticed girder loaded at the middle to the amount of 4% of the total compression that would be allowed on the column if short."

Part 7.—Utilizable Capacity of Tension Members and of Members Subjected to Alternate Stresses; and the Results of Loading Tension and Compression Members Beyond Their Useful Limit Points

Let,

t = the utilizable capacity per unit of net cross-sectional area.

Tension members, like columns, are liable to unintentional eccentricities between their lines of force and their axes, and their connections are of such stiffness that flexure can be transferred to them from the members to which they are connected. Unlike columns, however, they are not liable to eccentricity from imperfect butt joints, and their flexure, instead of being increased by the secondary moments, is decreased. Another important consideration is the fact that ruinous deformation in most cases probably will not cause failure in a short period, with its resultant danger. Logically, this last consideration affects the factor of safety rather than the utilizable capacity, but to avoid complication, the utilizable capacity is considered, instead, as slightly increased and no recommendation made for any reduction in the factor of safety. After weighing all these facts, the writer considers that, except as

* *Lehigh Quarterly*, June, 1891, p. 159.

noted, $t = u$, u being the useful limit point of a full-sized tension or short compression member as described in Part 5.

The exception is the case of a tension member to which a large girder or floor-beam, liable to heavy loading, is riveted; which, similarly as for columns, is a matter in each case for particular consideration.

There are no published results of tests of full-sized tension members other than eye-bars from which U. L. P.'s can be obtained and no published comparisons of the weighted averages of the U. L. P.'s of specimen tests with the U. L. P.'s of tension members. Such tests are desirable. In their absence it is reasonable to infer that the U. L. P.'s of tension members (per unit of net area), made of single symmetrical shapes or built of shapes and plates, are as great as those of short columns of the same cross-section (per unit of gross area), as inferred from specimen tests.

There are no published comparisons between the U. L. P.'s of specimen tests and those of eye-bar tests. For specimen tests to be directly critical as regards U. L. P.'s it would seem that the bars from which specimens are cut should be first annealed, as for eye-bars. In inferring U. L. P.'s of eye-bars from the U. L. P.'s of unannealed specimens allowance should be made for the effect of annealing.

For alternate tension and compression, consider a test piece with a cross-sectional area of 1 sq. in.

Let,

R = the maximum elastic range of stress which can be developed therein.

u = the value of R when the range is from zero to a maximum. For this range, u is the critical value of the specimen.

u' = the greater stress plus x times the lesser stress.....(16)

in which, x is a factor, constant for similar test pieces, but differing for the many grades and kinds of steel.

Let,

y = one-half the stress by which tension exceeds compression or compression exceeds tension, so that,

$$\text{Greater stress} = \frac{R}{2} + y;$$

and

$$\text{Lesser stress} = \frac{R}{2} - y$$

Then,

$$\frac{R}{2} + y + x\left(\frac{R}{2} - y\right) = u'$$

and, solving,

$$R = \frac{2}{1+x}(u' + xy - y) \quad \text{.....(17)}$$

$$x = \frac{2u' - 2y - R}{R - 2y} \quad \text{.....(18)}$$

By careful scrutiny and trial, approximate values of u' and x were inferred from Table 1 for Lots 2 and 3, and the elastic ranges were computed by use of Equation (17) and then compared with Table 1 to test the assumption that x "is constant for similar test pieces," as shown in Table 14.

In considering Table 14 it should be remembered that, in the absence of published figures, the writer obtained the developed elastic limits cited in Table 1 by scaling them from Bairstow's diagrams.

In the latter part of Part 5 it is specified that the critical value of each specimen shall be limited to 60% of its tensile strength and to 87% of its U. L. P. If these limitations govern, the value of u for the specimens of Bessemer steel will be limited to 60 000 lb. per sq. in. and x will not be greater than 62%; and for all the specimens cited in Tables 1 and 2, tested with 800 or less "cycles per minute", having a tensile strength of less than 70 000 lb., the value of x will be well within 50 per cent.

There is a clause in some specifications to the effect that members subject to alternate tension and compression shall be proportioned for both tension and compression separately considered, but that each kind of stress shall be considered as increased by 50% of the lesser stress. If the stresses are those obtaining when members are stressed to their utilizable capacity and the metal is 70 000 lb., or less than 70 000 lb., steel, this specification seems to be ample. It is probably ample for all steel, but for steel of more than 70 000 lb. and up to 100 000 lb. maximum tensile strength the writer considers it best to specify percentages of more than 50 and up to 65 per cent.

TABLE 14.—Comparison of Computed with Actual Ranges of Stress to Show in Cases of Alternate Stresses that x in Equation (17) is Constant for Steel of the Same Kind and Grade; y = One-half the Stress by Which Tension Exceeds Compression or *Vice Versa;* and u = Range from Zero to Maximum Stress.

Unannealed Axle Steel (All Stresses are in Pounds per Square Inch; Tensile Strength, 85 600; u = 48 000; x = 64.4 Per Cent.)			Unannealed Bessemer Steel (All Stresses are in Pounds per Square Inch; Tensile Strength, 99 900; u = 66 500; x = 79.2 Per Cent.)		
y.	Ranges:		y.	Ranges:	
	Computed.	From Table 1.		Computed.	From Table 1.
0	58 400	.....	0	74 200	.....
1 000	58 000	58 000	1 250	73 900	73 900
12 000	53 200	54 000	23 500	68 800	69 000
24 000	48 000	.....	31 500	66 900	67 000
27 500	46 500	47 000	33 250	66 500	.....

To the writer's knowledge all endurance tests of steel whether by repeated direct tension, by repeated direct compression, or by alternate direct tension and direct compression, are of material that has not been treated by cold-rolling, by cold-drawing, or by heating and quenching. As far as this paper is concerned, the important problem of the utilizable capacity of treated steel eye-bars is not considered.

A superficial consideration of the report of the Special Committee on Steel Columns and Struts would give the impression that the U. L. P. adopted is the *Ultima Thule* of the structural value of steel. The writer has used it as the limit to the application of mathematical analysis based on the assumption of elasticity. This "point" does indicate the "limit" within which steel is predominantly elastic, and it is very "useful"; the term, "Useful Limit Point", should be thus interpreted, but it does not necessarily indicate a limit beyond which steel ceases to have structural value.

Tests 4135* of a 5 by 1-in. eye-bar can be considered as typical of the nineteen eye-bars referred to in Part 5 as tested at the Watertown Arsenal. The results of the test of this bar, in a gauged length of 260 in., near its U. L. P. are as shown in Table 15.

As the gauge was set at zero for 1 000 lb., 0.0091 in. (the elastic elongation for 1 000 lb.) has been added to each elongation taken from the report.

A load about 5% greater than the U. L. P. caused a plastic elongation (which would result in permanent set) of about one-third the elastic elongation; and a load 10% greater caused a plastic elongation of more than 500% of the elastic. Under this latter load, the plastic elongation in 25 ft. 8 in. (the length, center to center, of pins) was 2.3 in. Such a phenomenon in a diagonal bar of a bridge panel 18 ft. high by 18 ft. long would permanently distort the panel 4.6 in.; that is, the vertical drop in the lower pin would be 4.6 in. greater than in the upper one. Such distortion would be ruinous.

TABLE 15.—Elastic and Plastic Deformation of a Typical Eye-Bar.

Pounds per square inch.	Elongation.			
	Total, in inches.	Elastic, in inches.	Plastic.	
			In inches.	In percentage of elastic.
33 920	0.3262	0.3185	0.0077	2.4
35 000	0.3391	0.3283	0.0108	3.3
35 500 U. L. P.	.....	.....	.....	.....
36 000	0.3580	0.3374	0.0206	6.1
37 000	0.4231	0.3465	0.0766	22.1
37 200 Approximate	.....	.....	.....	33.3
38 000	0.8591	0.3557	0.5034	145.2
39 000	2.1491	0.3648	1.7843	501.6
69 330 Maximum load	.....	.....	.....	.....

The eye-bar had a sharp and pronounced yield with a small increase in load just above the U. L. P. Other members the cross-sections of which consist of one piece are liable to sharp and pronounced yields. For instance, an H-section, pin-ended column,† had a U. L. P. of 26 000 lb., a linear compression of 0.0306 in. in 20 in. at 31 000 lb. per sq. in. and of 0.23 in. in 20 in. at 32 000 lb. per sq. in. Built members are usually composed of pieces having different U. L. P.'s and yield points, and are subjected to severe local

* Watertown Arsenal Report, 1886, p. 1574.

† *Loc. cit.*, 1909, p. 755.

strains in fabrication. They rarely if ever have a pronounced yield that is sharply marked until they are near the point of failure.

To show the behavior of columns when loaded beyond their U. L. P.'s Tables 16 and 17 have been compiled. Table 18 gives data referred to in Table 17. The type, section, and other data concerning these columns are given in Tables 3 and 18. (The columns listed in Table 18 are the largest ever tested, are of many varieties of steel, and the data regarding them are very interesting and valuable as criteria.) The forked ends of the columns indicated as Items 5 and 6 in Table 17 were beyond question critical as regards the maximum loads sustained. It appears from these tables that under laboratory conditions short columns with suitable details, to become permanently shortened by an increment of one-third their elastic shortening under the same load, would on an average require loads about 10% greater than at their U. L. P.'s.

TABLE 16.—Small Built Columns, Loaded Beyond Their Useful Limit Points.

R = ratio of plastic to elastic deformation at the U.L.P.
L = load which caused plastic deformation equal to one-third the elastic deformation. After removal of load, plastic deformation becomes permanent set.

References.*	Page.	Description.	Ends.	$\frac{l}{r}$.	U.L.P., in thousands of pounds per square inch.	R, approximate percentage.	L.		Maximum.	
							In thousands of pounds per square inch.	Percentage of U.L.P.	In thousands of pounds per square inch.	Percentage of U.L.P.
(1)	802	Average of three carbon.....	Pin	25	30.3	5	34.1	113	37.8	125
(1)	778	" " " "	Flat	25	30.0	15	32.3	108	36.3	121
(1)	781	" " " "	"	50	30.3	3	33.0	109	34.6	114
(2)	154	Average of three 70 000-lb. carbon.	"	25	33.0	5	36.2	110	40.6	123
(2)	145	Average of three nickel.....	"	25	47.7	17	51.3	108	54.5	114
(3)	2452	Type 2 Light (Table 11).....	"	50	27.0	15	30.0	111	32.6	121
(3)	2452	" " " "	"	50	29.0	12	32.0	110	34.9	120
(3)	2452	" " " "	"	50	27.5	7	31.0	113	33.1	120
(3)	1628	Type 4 A Heavy (Table 11).....	"	50	23.0	19	25.5	111	30.3	132
(3)	1628	" " " "	"	50	22.5	14	26.5	118	29.0	129
(3)	1628	" " " "	"	50	22.0	15	25.5	116	28.0	127
(3)	1619	Average of 54 columns of 18 types	"	50	27.3	.....	.....	.....	32.7	120
(3)	1619	" " " " " "	"	85	27.2	.....	.....	.....	30.3	111
(3)	1619	" " " " " "	"	120	25.8	.....	.....	.....	27.7	107
(1)	808	Average of three carbon.....	Pin	75	29.3	.....	.....	.....	32.4	111
(1)	787	" " " "	Flat	100	30.7	.....	.....	.....	32.4	106
(1)	176	Average of three nickel.....	"	100	46.0	.....	.....	.....	47.6	103

* (1) Watertown Arsenal Report, 1909; (2) Watertown Arsenal Report, 1910; (3) *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1583 *et seq.*

The question arises whether for short built columns and built tension members a total plastic deformation not to exceed one-third of the total elastic deformation would not be a better limit to adopt for critical deformation than the U. L. P. The writer's opinion is decidedly in favor of the U. L. P. Actual conditions are not those of the laboratory; plastic deformation equal to one-third of the elastic deformation would produce excessive

permanent distortion of structures and would vitiate the results of computations based on the elasticity of steel.

Part 8.—Beams and Plate Girders under Transverse Loads

Simple tension or compression in elastic material has its maximum shearing component in any plane making an angle of 45° with the direction of the force. From the theory of mechanics the intensity of this maximum shearing component is one-half that of the simple tension or compression. The microscope has shown that when the elastic limit is reached in malleable metals under simple tension or compression the yield takes place by the sliding within grains of their crystalline layers one over another; which indicates that the material is really yielding under the shearing component of the simple tension or compression. When, at the elastic limit, iron or steel yields, at an indicated simple tension or compression of, say, 30 000 lb. per sq. in., it actually yields under a shearing component of 15 000 lb. per sq. in. It is a fair inference that under compound stress the elastic limit in iron or steel is reached when the shearing intensity reaches one-half the intensity of simple tension at the elastic limit; and the weight of experimental evidence seems to support this inference. The predominant practice in dealing with beams and plate girders is to permit a shear per unit of gross area of six-tenths the permissible tension per unit of gross area, but the writer prefers five-tenths instead of six-tenths and is confirmed in this preference by the fact that ample webs are advisable for stiffness and reduce somewhat the magnitude of the errors involved in the ordinary theory of flexure. He also considers it best in important cases to compute the maximum shear by the theoretical formula given in textbooks on Mechanics* instead of proportioning for a mean intensity of shear considered as carried exclusively by the web. In rolled I-beams it does not make much difference, but for plate girders the maximum intensity of the shear in the web indicated by theory is ordinarily 10 to 25% greater than the mean indicated by the ordinary method.

That there are "faults in the theory of flexure" has been shown by Bach, Love, Saint Venant, and others, and by the writer in a paper,† thus entitled, and its discussion, to which references are made in this Part 8.

One of these faults is the neglect of the influence of shear on cross-sections, by reason of which originally plane cross-sections do not remain plane during flexure (as ordinarily assumed), but are forced into reversed curves somewhat like a long $\int$ only much less pronounced, as illustrated in Figs. 3 and 4 of the writer's paper previously mentioned.

In consideration of the fact that because of the fault cited the ordinary theory of flexure indicates for the extreme fibers intensities of stress that are too small, an approximate method was devised by the writer‡ and improved

* It is given in the Carnegie Pocket Companion, 1913 Edition, p. 180, and 1923 Edition, p. 151.

† *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), pp. 895–981.

‡ *Loc. cit.*, pp. 896–897.

by J. P. J. Williams, M. Am. Soc. C. E.,* for computing the percentage, P_e, which on account of this fault should, theoretically, be added to the intensity of the stress as computed from the ordinary theory. This method has been adapted to the purposes of this paper.

For steel beams of rectangular cross-section uniformly loaded and simply supported†:

$$P_e = 8.88 \frac{d^2}{x\,(l - x)} \qquad (19)$$

in which, l, is the length; d, the depth between extreme fibers; and x, the distance from the end to the point at which the percentage to be added, P_e, is desired.

TABLE 17.—Large Built Pin-Ended Columns, Loaded Beyond Their Useful Limit Points.

R = ratio of plastic to elastic deformation at the U. L. P.
L = load which caused plastic deformation equal to one-third the elastic deformation. After removal of load, plastic deformation becomes permanent set.

Item.	Pages*.	Description.	$\frac{l}{r}$	U. L. P., in thousands of pounds per square inch.	R, approximate percentage.	L		Maximum.	
						In thousands of pounds per square inch.	Percentage of U. L. P.	In thousands of pounds per square inch.	Percentage of U. L. P.
1	23, 64	Nickel steel	25	49.0	15	54.0	110	61.7	126
2	26, 65	" "	37	52.0	14	58.5	113	61.4	118
3	32, 67	Mayari steel	43.5	†	....	¶	....	61.5	....
4	42, 73	" "	38.5	52.0	12	¶	....	64.7	124
5	54, 79	" "	27.7	‡	....	¶	....	47.2	....
6	58, 81	" "	34.5	‡	....	¶	....	49.4	....
7	62, 83	" "	27.1	§	....	¶	....	57.7	....
8	36, 69	Silicon steel	43.5	39.0	17	42.0	108	52.8	135
9	46, 75	" "	38.5	41.0	21	44.0	107	51.7	126
10	38, 71	Chrome steel	43.5	35.0	19	38.0	109	49.2	141
11	48, 76	" "	38.5	36.0	22	40.3	112	51.8	144
12	29	Carbon steel	43.5	30.0	20	32.6	109	38.4	128
13	34, 68	" "	43.5	31.0	21	34.5	111	41.7	135
14	40, 72	" "	38.5	‖	....	¶	....	45.3	....
15	44, 74	" "	38.5	33.0	20	37.0	112	46.6	141
16	52, 78	" "	28.7	28.5	17	¶	....	34.3	120
17	56, 80	" "	36.1	27.0	19	30.0	111	31.0	115
18	60, 82	" "	27.6	23.0	22	26.0	113	32.4	141

* This column refers to pages in *Technologic Paper No. 101*, U. S. Bureau of Standards. Additional information regarding these columns is given in Table 18. The term, U. L. P., does not appear in connection with these tests in *Technologic Paper No. 101*, but it was scaled by the writer from graphs or determined from data given on the pages cited.

† Exceeds 47.0, which was the load at the last measured deformation, and, from the general trend of the curve, is probably 50.

‡ Had not been reached at 45.0. Photographs of column after failure show that failure was due to insufficient stiffness at forked ends.

§ Greater than 50.0.

‖ Greater than 35.0.

¶ Deformations indicative of L were not observed.

By substituting different constant factors in place of 8.88 (Equation 19), it can be applied to I-shaped beams and plate girders.

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), pp. 932–953.

† *Loc. cit.*, p. 940.

TABLE 18.—Tests of Large Steel Columns.

(Made by J. H. Griffith, Associate Engineer Physicist, and J. G. Bragg, Associate Assistant Physicist. Compiled from *Technologic Paper No. 101*, U. S. Bureau of Standards.)

The U. L. P.'s are not stated in *Technologic Paper No. 101*, U. S. Bureau of Standards, but have been taken from the stress-strain diagrams for tests where the diagrams reached this point. The ratios, $\frac{l}{r}$, are different for different axes. The greatest ratio is used. See *Technologic Paper No. 101*, p. 22, for chemical analyses and maximum tensile stress for specimen; pages 20 and 21 for cross-sections; and the pages cited in first column for the remaining data. The columns in each group had nominally identical cross-sections.

Page.*	Name of steel.	Average Chemical Composition, Percentage.								Cross-Section.			U. L. P., in pounds per square inch.	Maximum load stress, in pounds per square inch.	Maximum tensile stress for specimen, in pounds per square inch.	$\frac{l}{r}$
		Ni.	C.	P.	Mn.	S.	Si.	Cr.	Cu.	Area, in square inches.	Thickest pieces.	Thinnest pieces.				
23	Nickel.....	3.51	0.29	0.014	0.59	0.031	....	....	....	88.8	11/16-in. plate	1/4-in. angle	49 000	61 700	93 300	25
26	Nickel.....	3.59	0.32	0.015	0.59	0.033	....	....	....	110.3	29/32-in. plate	1/4-in. angle	52 000	61 400	93 100	37
29	Carbon....	....	0.36	0.012	0.43	0.030	....	....	....	74.6	1-in. plate	3/8-in. angle	30 000	38 400	76 700	43.5
34	Carbon....	....	0.28	0.017	0.64	0.031	....	....	....	76.9	1-in. plate	3/8-in. angle	31 000	41 700	73 400	43.5
32	Mayari....	1.12	0.34	0.010	0.68	0.037	0.10	0.53	0.10	74.9	1 in. plate	3/8-in. angle	+	61 500	100 600	43.5
36	Silicon	....	0.35	0.017	0.83	0.039	0.38	....	....	76.9	1-in. plate	3/8-in. angle	39 000	52 800	87 700	43.5
38	Chrome...	....	0.31	0.017	0.49	0.028	0.13	0.66	....	76.9	1-in. plate	3/8-in. angle	35 000	49 200	82 700	43.5
40	Carbon....	....	0.36	0.012	0.40	0.030	....	....	....	75.4	11/16-in. plate	3/8-in. angle	+	45 300	75 900	38.5
44	Carbon....	....	0.28	0.017	0.64	0.031	....	....	....	77.2	11/16-in. plate	3/8-in. angle	33 000	46 600	74 400	38.5
42	Mayari....	1.12	0.34	0.010	0.68	0.037	0.10	0.53	0.10	75.3	11/16-in. plate	3/8-in. angle	52 000	64 700	101 200	38.5
46	Silicon	....	0.35	0.017	0.83	0.039	0.38	....	....	77.2	11/16-in. plate	3/8-in. angle	41 000	51 700	88 300	38.5
48	Chrome...	....	0.31	0.017	0.49	0.028	0.13	0.66	....	77.2	11/16-in. plate	3/8-in. angle	36 000	51 800	83 600	38.5
52	Carbon....	....	0.23	0.012	0.49	0.027	....	....	....	56.0	7/16-in. plate	1/4-in. plate	+	34 300	65 400	28.7
54	Mayari....	1.58	0.33	0.016	0.64	0.027	....	0.37	....	41.7	7/16-in. plate	3/8-in. angle	+	47 200	99 800	27.7
56	Carbon....	....	0.23	0.012	0.49	0.029	....	....	....	95.8	11/16-in. plate	1/4-in. plate	27 000	31 000	65 400	36.1
58	Mayari....	1.58	0.33	0.016	0.64	0.027	....	0.37	....	66.6	17/32-in. plate	1/4-in. plate	+	49 400	99 800	34.5
60	Carbon....	....	0.23	0.012	0.49	0.029	....	....	....	118.2	3/4-in. plate	1/2-in. angle	23 000	32 400	65 400	27.6
62	Mayari....	1.58	0.33	0.016	0.64	0.027	....	0.37	....	76.5	19/32-in. plate	1/2-in. angle	+	57 700	99 800	27.1

* *Technologic Paper No. 101*, U. S. Bureau of Standards.

† Shortening in gauged length not measured at point high enough in stress to indicate the U. L. P.

Substituting C_c, a factor constant for similar cross-sections, in place of 8.88,

$$P_e = C_e \frac{d^2}{x\,(l - x)} \qquad (20)$$

At the center of the beam or girder,

$$P_e = \frac{4\,C_e\,d^2}{l^2} \qquad (21)$$

For instance, if $C_e = 10$, $d = 30$ in., and $l = 80$ in., the percentage to be added is 5.625.

When the load instead of being distributed from end to end is confined within and uniformly distributed over a distance, $2a$, at the center, the percentage of increase at the center* is:

$$P_e = \frac{C_e\,d^2}{a\,(l - a)} \qquad (22)$$

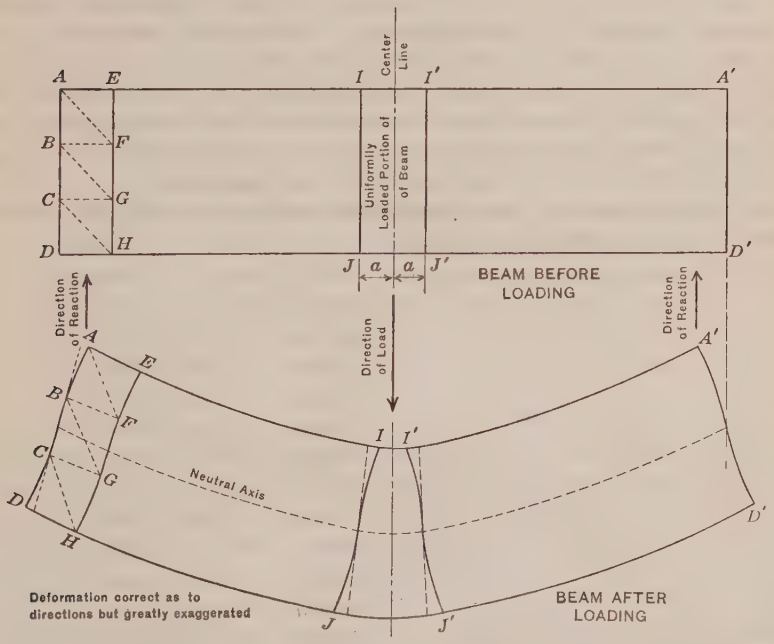


FIG. 4.—DEFORMATION OF BEAM LOADED ON LENGTH $2a$ AT CENTER. SHEAR AND, THEREFORE, DISTORTION IS GREATEST AT NEUTRAL AXIS AND GRADUALLY DECREASES UNTIL IT BECOMES ZERO AT EXTREME TOP AND BOTTOM FIBERS. TO ILLUSTRATE DISTORTION FROM SHEAR, ORIGINALLY STRAIGHT VERTICAL LINES ARE CONCEIVED AS SCRIBED ON BEAM.

C_e has been determined by Williams for rectangular cross-sections as 8.88 (as noted); for a standard 12-in., 31.5-lb. I-beam, as 11.32; and for a 12-in., 28.5-lb. I-beam, as 12.5. From certain determinations by Williams and from others by the writer, the following empirical rules for obtaining the approximate value of C_e for plate girders and I-beams have been devised.

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 952.

Let,

"Ratio" = the ratio of web cross-sectional area to total cross-sectional area, considering the web as of full depth.

Then, for I-beams:

$$C_e = \frac{1.96}{\text{Ratio}} + 7.0$$

For plate girders:

With "Ratio" between 0.28 and 0.36.... $C_e = \frac{7.17}{\text{Ratio}} - 4.7$

With "Ratio" between 0.36 and 0.54.... $C_e = \frac{3.32}{\text{Ratio}} + 6.0$

With "Ratio" between 0.54 and 0.71.... $C_e = \frac{5.72}{\text{Ratio}} + 1.6$

In consideration of the initial imperfections and limitations to the elasticity of beams and plate girders and the tendency of steel overstrained in bending to become adjusted to the conditions, as hereinafter discussed, it seems advisable, in the interest of simplicity, not to add any percentage to the computed extreme fiber stresses unless the percentage, P_e, as indicated by Equations (21) and (22) is more than 5, and to avoid values greater than 5, where practicable, either by changing the ratio of the web area to the total area, by spreading the load, or by a combination of these methods.

For simply supported beams and girders uniformly loaded from end to end the percentage, P_e, will be less than 5:

For	lengths	of	more	than	2.67 d	when	C_e =	8.88
"	"	"	"	"	2.83 d	"	C_e =	10
"	"	"	"	"	2.97 d	"	C_e =	11
"	"	"	"	"	3.10 d	"	C_e =	12
"	"	"	"	"	3.23 d	"	C_e =	13
"	"	"	"	"	3.35 d	"	C_e =	14
"	"	"	"	"	3.47 d	"	C_e =	15
"	"	"	"	"	3.58 d	"	C_e =	16
"	"	"	"	"	3.69 d	"	C_e =	17
"	"	"	"	"	3.80 d	"	C_e =	18
"	"	"	"	"	3.90 d	"	C_e =	19
"	"	"	"	"	4.00 d	"	C_e =	20
"	"	"	"	"	4.10 d	"	C_e =	21

If P_e is 5 and 2 a (the distance over which a concentrated uniformly distributed load at the center is spread) is taken as $\frac{d}{n}$, then, as developed from Equation (22):

$$l = \left(0.4\ C_e n + \frac{0.5}{n}\right) d \qquad (23)$$

For simply supported beams and girders, uniformly loaded over a distance or spread of $\frac{d}{n}$, the percentage, P_e, will be less than 5:

For lengths of more than $\left(3.55\ n + \frac{0.5}{n}\right) d$, when $C_e = 8.88$

" " " " $\left(4.0\ n + \frac{0.5}{n}\right) d$, " $C_e = 10$

" " " " $\left(4.4\ n + \frac{0.5}{n}\right) d$, " $C_e = 11$

" " " " $\left(4.8\ n + \frac{0.5}{n}\right) d$, when $C_e = 12$

" " " " $\left(5.2\ n + \frac{0.5}{n}\right) d$, " $C_e = 13$

" " " " $\left(5.6\ n + \frac{0.5}{n}\right) d$, " $C_e = 14$

" " " " $\left(6.0\ n + \frac{0.5}{n}\right) d$, " $C_e = 15$

" " " " $\left(6.4\ n + \frac{0.5}{n}\right) d$, " $C_e = 16$

" " " " $\left(6.8\ n + \frac{0.5}{n}\right) d$, " $C_e = 17$

" " " " $\left(7.2\ n + \frac{0.5}{n}\right) d$, " $C_e = 18$

" " " " $\left(7.6\ n + \frac{0.5}{n}\right) d$, " $C_e = 19$

" " " " $\left(8.0\ n + \frac{0.5}{n}\right) d$, " $C_e = 20$

" " " " $\left(8.4\ n + \frac{0.5}{n}\right) d$, " $C_e = 21$

By care in designing, the deficiency in the computed intensity of the extreme fiber stress can be reduced to less than 5% in all but very short I-beams and girders. For instance, if $C_e = 20$, $d = 24$, and $2a = 8$, then, for $P_e = 5$, $n = 3$, and $l = 58$. If l is less than 58, but not too small, P_e can be kept down to 5 or less by increasing either $2a$, or the ratio of web area to total area, or by both methods.

The values of P_e given by Equations (20), (21), and (22) are too large. The error increases as the indicated value of P_e increases, but is very slight up to $P_e = 10$; however, when these equations indicate P_e as 50, P_e is probably not more than 35; and when they indicate P_e as 100, P_e is probably not more than 50.

The manner of spreading the load depends on the way it is applied to the beam or girder. If the web has a surplus of cross-section above the requirements of shear and is sufficiently stiff, and if the load is conveyed to it by adequate means, it will spread the load to some extent beyond the limits of the actual contact.

The ordinary theory of flexure is developed on the tacit assumption that the loads and reactions are applied to beams and girders so that each increment

of each horizontal layer, at or vertically in line with each increment of load, receives directly just that share of the load increment necessary to produce the theoretical changes in its shear without any part having been conveyed to it by compression or tension through adjacent layers. This tacit assumption is sometimes approximated, but is never fully realized; consequently, vertical stresses in the web occur at, and inclined stresses in the web originate at, points of loading and reaction, and these inclined stresses have horizontal components, all of which affect the web and flanges although they are neglected in the ordinary theory of flexure. These uncomputed stresses, especially if the load is concentrated, make it advisable to provide web section and web reinforcement in excess of the requirements of ordinary theory, and, in girders with flange cover-plates, to require that cover-plates of the bottom flange extend beyond the nominal theoretical limits when the load is applied at the top and similarly for the top flange when the load is applied at the bottom.

Web section and reinforcement in excess of the requirements of ordinary theory may reasonably be expected to mitigate the effects of concentration much the same as would an increase in the length of direct application.

The theory of flexure is based on ideal material perfectly elastic within limits, but the theory stops at these limits. The effects of the differences between ideal steel and the actual steel of commerce as applied to beams and plate girders under transverse loads require special consideration.

Rolled I-beams are peculiarly liable to auto-stress* and to variations in the elastic resistances of their different parts. The U. L. P.'s in pounds per square inch, as scaled from the diagrams of tests of specimens from the webs, flanges, and roots of certain I-beams tested by the late Edgar Marburg, M. Am. Soc. C. E.†, are given in Table 19.

TABLE 19.—Specimen Tests from Web, Flange, and Root of Large I-Beams.

Beam.	Web, U. L. P., in pounds per square inch.	Flange, U. L. P., in pounds per square inch.	Root, U. L. P., in pounds per square inch.
15-in., 38-lb. I-beam	42 500	.....	26 000
24-in., 72-lb. I-beam	46 000	36 000	38 000
24-in., 80-lb. Standard I-beam	30 500	28 400	25 000
30-in., 120-lb. I-beam	37 000	31 500	30 100
15-in., 73-lb. girder beam	39 000	.....	22 000
24-in., 120-lb. girder beam	36 400	33 300	28 000
30-in., 175-lb. girder beam	34 900	28 000	27 000

Fatigue tests of rotating shafts, of which there have been many (some are cited in Table 5), and fatigue tests by Wöhler and others of presumedly square or rectangular bars loaded alternately in opposite directions, show by inference that fields of seemingly perfect elasticity can be developed by alternate bending in opposite directions; and fatigue tests by Wöhler and others, presumedly on square or rectangular bars, under loads which in each case are in the same

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), pp. 913 and 931.

† Reported in "Tests of Standard I-Beams and Bethlehem Special I-Beams, and Girder Beams", *Proceedings*, Am. Soc. for Testing Materials, Vol. IX (1909), p. 378.

direction but repeatedly vary between a given minimum and maximum, show that these fields of seemingly perfect elasticity can be shifted so that the stress in the extreme fiber at the top of the beam varies between computed elastic limits both of which are in compression whereas the stress in the extreme fiber at the bottom of the beam varies between computed elastic limits both of which are in tension.

It has been shown in Parts 5 and 6 that for compression and tension members, loading which would perfect the elasticity to a point much above the U. L. P. is not permissible because there the yield develops so rapidly that a slightly greater load would cause seriously detrimental extension in tension members or shortening in compression members. In beams and plate girders under transverse loads, the plastic deflection develops more gradually than the plastic deformation in tension and compression members; hence if the beams are held against lateral deflection and the metal is not so attenuated as to permit buckling or crippling of the web or flanges, there is greater opportunity for shifting the elastic field. The difference in this respect between the plastic extension of a soft steel eye-bar and the plastic deflection of beams under transverse loads is illustrated by the following comparison:

(*a*).—An eye-bar tested at the Watertown Arsenal* had a U. L. P. of 29 000 lb. per sq. in. The plastic deformation at this point was quite small and of little consequence *per se*, but the rate of extension indicated that a pronounced yield had set in. Between 31 000 and 32 000 lb. per sq. in. there was a definite yield point and at about 20% above the yield point the plastic extension or permanent set became more than sixteen times as great as the computed elastic extension.

(*b*).—Twenty 15-in. I-beams of 21-ft. span and nominally equivalent strength were tested at Ambridge, Pa., loaded at the center†. If in each case these loads had been 36 600 lb. (including one-half the weight of the beam considered as concentrated at the center), and the beams had been perfectly elastic and simply supported, the computed stress in the extreme fiber by the ordinary theory of flexure would have been 39 000 lb. per sq. in., which is about the average yield point indicated by the specimen tests. The actual loads which produced permanent sets of 0.4 in. were all within 7% of their average, which average was 65.7% greater than the load of 36 600 lb. per sq. in., while the permanent set was only a little more than one-fourth the computed elastic deflection. The beams, however, were not "simply supported" as the end connections and supports must have exercised some restraining influence. If this restraining influence amounted to a bending moment at each end of one-third the moment at the center (an exaggerated proportion), the loads which produced the permanent set of 0.4 in. even in that case would have been more than 20% in excess of the 36 600 lb. per sq. in. and the permanent set would not have been much more than one-half the computed elastic deflection.

When I-beams and plate girders, in which the metal is not so attenuated as to permit buckling or crippling of web or flanges, have properly designed

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 968.

† *Loc. cit.*, pp. 917–918.

details, are adequately proportioned for shear, are not too short, and have their flanges held against lateral deflection, it seems reasonable from the foregoing analysis to assume that their utilizable capacity in each case will be as great as the load for which the intensity of the stress in the extreme fiber, computed from the ordinary theory of flexure, equals u, u being the useful limit point of the flange similarly as described in the rule in Part 5 (page 1236), for tension and compression members. It is probable, for I-beams at least, that the utilizable capacity is somewhat greater than this when the load varies between a minimum and maximum of reduced range, provided the minimum is not too close to the maximum. If the minimum is too high it does not afford the plastic metal sufficient opportunity to recover from elastic fatigue but causes the plastic deformation to increase gradually until failure results. A very slow increase in plastic deflection of beams may continue for a long time under heavy static loads, as shown in the case of a pair of 15-in., 50-lb. I-beams.*

Part 9.—Combined Compression and Flexure

When in addition to the direct load a column is subjected to intentional flexure, it follows from the assumptions in Part 6 that the utilizable capacity is determined by one of three following limitations, *A*, *B*, or *C*, whichever gives the smallest capacity:

Limitation A.—The utilizable capacity shall not be considered greater than indicated by Equation (1) for columns subjected to direct load without intentional flexure.

Limitation B.—The greatest intensity of stress, f, from direct load, intentional flexure, and unintentional eccentricity $\left(\text{assumed as in Part 6 as } \frac{l}{700}\right.$ in the form of a sinusoidal bow$\left.\right)$ shall not be greater than u.

Let,

f_1 = maximum intensity of intentional primary flexure.

f_2 = maximum intensity of unintentional primary flexure.

$$f_c = f_1 + f_2 \qquad (24)$$

From Equation (3):

$$f = p + f_c + f_c\left(\frac{p}{q-p}\right) \qquad (25)$$

Equation (25) is exact only when the moment diagram is a sinusoid; but it is a close approximation for all cases in which the maximum bending moment occurs at or near the center of the column. For computing f_2, e, as in Part 6, may be taken as $\frac{l}{700}$ and $\frac{v}{r}$ as 1.68. Then, from analysis similar to that by which Equation (8) was developed,

$$f_2 = 0.00240\, p\, \frac{l}{r} \qquad (26)$$

The direct load per unit of area, p, which in combination with f_1 and f_2, will produce a stress intensity, f, equal to u, can be found from Equation (25)

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 975.

by first assuming p and then making successive trials, but this tedious process is unnecessary as it can be approximated with sufficient closeness from the following empirical formula:

$$p = c - f_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

c having been first found from Equation (11).

To show the approximate accuracy of Equation (27), f has been determined in Table 20 (a) for a large range of cases from Equation (25) using for p the value obtained from Equation (27). If the value obtained for p had been exact, f in all cases would have been the same as u. It differs from u by less than 2% for all cases except those in bold face type.

TABLE 20.—JUSTIFICATION FOR EMPIRICAL FORMULA, EQUATION (27).

(For this comparison, columns in which the ratio, $\frac{v}{r} = 1.68$, were chosen. For smaller ratios the error would be less.)

(a) MAXIMUM STRESS INTENSITY, f, INDICATED BY EQUATION (25) FROM USING THEREIN THE VALUE FOR p INDICATED BY EMPIRICAL EQUATION (27)*.

Criterion u, in pounds per square inch.	$\frac{l}{r}$	When $f = 0.1\,p$, in pounds per square inch.	When $f = 0.2\,p$, in pounds per square inch.	When $f = 0.5\,p$, in pounds per square inch.	When $f = p$, in pounds per square inch.
20 071	20	20 098	20 054	19 935	19 791
20 071	40	20 047	19 996	19 811	**19 530**
20 071	70	19 991	19 685	**19 631**	**19 069**
20 071	120	20 258	19 873	See (b)	See (b)
35 105	20	35 322	35 278	35 121	34 877
35 105	35	35 248	35 251	35 071	34 646
35 105	60	35 055	35 158	34 828	**33 888**
35 105	90	35 115	34 750	See (b)	See (b)
50 063	20	50 600	50 588	50 429	50 095
50 063	35	50 457	50 599	50 499	49 884
50 063	50	50 209	50 522	50 358	49 225
50 063	70	50 230	50 311	See (b)	See (b)

(b) COMPARISONS OF VALUE OF p INDICATED BY EMPIRICAL EQUATION (27) AND RATIONAL EQUATION (31).

Criterion u, in pounds per square inch.	$\frac{l}{r}$	When $f = 0.5\,p$.		When $f = p$.	
		p as per Equation (31), in pounds per square inch.	p as per Equation (27), in pounds per square inch.	p as per Equation (31), in pounds per square inch.	p as per Equation (27), in pounds per square inch.
20 071	120	8 762	8 104†	6 298	6 078†
35 105	90	13 781	14 996	9 430	11 247
50 063	70	20 127	23 467	13 206	17 600

* The value of f as indicated in (a) differs from u by less than 2% in all cases except those recorded in bold-face type.

† For these two cases and for these two only, Equation (27) seems to be critical; in all the other cases Equation (31) seems to be critical; actually, Equation (31) is critical in all the cases. The use of Equation (27) for the two cases in which it seems to be critical is conservative, and the error involved on the side of safety is not excessive.

Limitation C.—The deflection under combined intentional and unintentional bending moments shall not exceed $\frac{l}{300}$.

Let,

M_1 = the intentional primary bending moment;
Δ = the deflection;
A = the cross-sectional area; and,
e_1 = the primary intentional eccentricity.

From the theory of flexure:

$$\Delta = \frac{(\text{Bending moment})\, l^2}{C_2\, E\, A\, r^2} \qquad (28)$$

in which, for beams or columns of constant cross-section with frictionless hinged ends, C_2 equals 9.6, for a uniformly distributed transverse load; 12.0 for a single transverse load concentrated at the middle; or 8.0, when the bending moment is constant from end to end; or π^2, when the moment diagram is a sinusoid.

For combined compression and flexure with $\Delta = \frac{l}{300}$, and unintentional eccentricity $= \frac{l}{700}$,

$$\frac{l}{300} = \frac{\frac{\pi^2}{C_2} M_1 l^2}{\pi^2 E A r^2} + \frac{p A \frac{l}{700} l^2}{\pi^2 E A r^2} + \frac{p A \frac{l}{300} l^2}{\pi^2 E A r^2} \qquad (29)$$

Substituting q (Euler's formula) for its value, as given in Part 6, and reducing,

$$p = 0.7\, q - \frac{210\, \frac{\pi^2}{C_2} M_1}{A\, l} \qquad (30)$$

When the primary bending moment is caused by an intentional eccentricity, e_1, constant from end to end, $M_1 = p\, A\, e_1$ and $C_2 = 8$; whence, by substitution in Equation (30),

$$p = \frac{0.7\, q}{1 + \frac{259.1\, e_1}{l}} \qquad (31)$$

In some cases the value of p, as determined from Equation (31), is somewhat greater than indicated by the empirical Equation (27); but notwithstanding this is slightly less than indicated by Equation (25). In such cases, to be precise, Equation (31) should govern, but this fact can only be ascertained by a tedious determination of p by trial from Equation (25). The error involved in using Equation (27) in such cases is not excessive and is on the side of safety, as shown in Table 20 (*b*).

When the maximum possible value of p within the utilizable capacity approaches or is less than one-half u, it indicates that there is liable to be tension in the convex side of the column.

The compression chords of bridges being usually continuous, it may be advisable in important cases when the transverse load is very heavy to treat them rigidly as continuous girders; but ordinarily the only transverse load on a column is its own weight, and for this case, if the column has riveted connections to adjacent members of sufficient stiffness to exercise a fair restraint on its ends, the l of the tables and formulas may be taken as from three-fourths to two-thirds of the length of the column.

In proportioning for combined compression and flexure the only alternative, except for a few particular cases, is to rely on analysis or guesswork, as there are not enough experiments to indicate general empirical formulas; but the Watertown Arsenal Reports record tests of four full-sized, pin-ended, eccentrically loaded, built columns, and one of a pin-ended, steel tube column subjected to longitudinal and transverse loading, which tests give an opportunity for comparing actual and theoretical results. This opportunity has been utilized by compiling Table 21. The best means of comparison is by deflections.

Let,

Δ_p = the deflection from primary bending moment.

This is readily obtained by the theory of flexure (see text following Equation (28)).

$$\Delta = \Delta_p + \frac{p\,A\,\Delta_p\,l^2}{\pi^2\,E\,A\,r} \qquad (32)$$

By substituting q for its value, $\frac{l^2}{\pi^2\,E\,r^2}$, and reducing,

$$\Delta = \Delta_p \left(1 + \frac{p}{q - p}\right) \qquad (33)$$

This use of the factor, π^2, is not rigid analysis, but for equal eccentricity at both ends and to the same side of the axis and for any symmetrical transverse loading its use in Equation (32) gives results for frictionless hinged-ended columns which for all ordinary cases differ from results of rigid analysis by less than one-half of 1 per cent.

Actually columns under combined compression and flexure are subject to friction on their pins, to unintentional eccentricity, and, as failure is approached, to considerable plastic deformation, none of which is allowed for in Equation (32). The effect of friction is to hinder deflection until the bending moment is sufficient to turn the ends of the column on the pin; plastic deformation increases deflection; and unintentional eccentricity, according to its direction, may either increase or decrease deflection. In these circumstances, the tests are not conclusive as to the accuracy of the theory and assumptions used, but, intelligently compared, the tests are strong evidence of the safety and general soundness of the theory.

Part 10.—Combined Tension and Flexure

An analysis of flexure combined with tension is not as important as that of flexure combined with compression for the reason that the bending moment from direct tension reduces the total stress intensity, whereas in the case of direct compression it increases the total stress intensity.

TABLE 21.—Tests of Pin-Ended Columns Eccentrically Loaded; and Pin-Ended Column with Combined Longitudinal and Transverse Loads.

(Compiled from the Watertown Arsenal Reports for 1883, 1884, and 1909.)

Notation: p = longitudinal load per square inch; f = computed extreme fiber compression per square inch on the concave side of the column from longitudinal load and flexure; Δ_c = computed deflection in inches, Equation (33); Δ_a = actual deflection; l = length, in inches; r = radius of gyration, in inches; I = moment of inertia, in inches; v = distance, in inches, from the center of gravity to the extreme fiber on the concave side of the column; u = U. L. P. of material on the concave side of the column, as gauged by full-sized tests of short columns of similar 10-in. channels, 8-in. channels, and 5-in. lap-welded steel pipes.

The columns were subjected during the tests to friction from pins; they were liable to unintentional eccentricity; and, as failure was approached, they underwent some plastic deformation, none of which was considered in making the computations tabulated.

For Wrought-Iron Columns with Top Cover-Plate and Bottom Lattice-Bars, E = 28 000 000 Lb. per Sq. In.

Test 1632; year, 1883, p. 167; two 10-in. channels; one cover-plate, 13 by 44 in.; actual area, 17.57 sq. in.; l = 300 in.; r = 4.01 in.; $\frac{l}{r}$ = 74.8 in.; eccentric application of load = 1.62 in.; I = 282.9 in.; v = 6.62 in.; u = 31 000 lb. per sq. in.

p.	f.	Δ_c.	Δ_a.
569	.....	0.02	0.00
1 138	.....	0.05	0.01
2 845	.....	0.12	0.10
5 692	.....	0.26	0.22
11 383	21 770	0.60	0.54
14 228	28 440	0.81	0.77
15 936	32 790	0.95	0.95
17 075	35 840	1.05	1.15
17 416*	36 820*	1.09	1.50

Test 1633; year, 1883, p. 168; two 10-in. channels; one cover-plate, 13 by 44 in.; actual area, 17.721 sq. in.; l = 300 in.; r = 4.01 in.; $\frac{l}{r}$ = 74.8 in.; eccentric application of load = 1.64 in.; I = 285.4 in.; v = 6.64 in.; u = 31 000 lb. per sq. in.

p.	f.	Δ_c.	Δ_a.
564	.....	0.03	0.00
1 128	.....	0.05	0.01
2 822	.....	0.12	0.09
5 643	.....	0.26	0.21
11 287	21 670	0.59	0.51
14 108	28 280	0.80	0.74
15 801	32 590	0.94	0.91
16 930	35 630	1.04	1.20
17 825*	36 730*	1.08	1.38

Test 350; year, 1884, p. 54; two 8-in. channels; one cover-plate, 12 by 43 in.; actual area, 12.48 sq. in.; l = 240.2 in.; r = 3.13 in.; $\frac{l}{r}$ = 76.7 in.; eccentric application of load = 1.72 in.; I = 122.5 in.; v = 5.75 in.; u = 32 000 lb. per sq. in.

p.	f.	Δ_c.	Δ_a.
801	.....	0.04	0.00
1 603	.....	0.07	0.03
4 006	.....	0.20	0.14
8 013	.....	0.44	0.37
12 019	29 270	0.73	0.67
14 423	36 900	0.94	0.90
16 026	42 490	1.10	1.20
16 242*	43 280*	1.12	1.40
.....	.....	....	....

Test 351; year, 1884, p. 55; two 8-in. channels, 7.5 sq. in.; one cover-plate, 12 by 39 in., 4.68; computed area,† 12.18 sq. in.; l = 240.2 in.; r = 3.14 in.; $\frac{l}{r}$ = 76.5 in.; eccentric application of load = 1.62 in.; I = 120.3 in.; v = 5.65 in.; u = 32 000 lb. per sq in.

p.	f.	Δ_c.	Δ_a.
821	.....	0.04	0.00
1 642	.....	0.07	0.02
4 105	.....	0.19	0.12
8 210	.....	0.42	0.32
12 315	28 680	0.70	0.58
14 778	36 150	0.91	0.82
16 420	41 620	1.06	1.05
17 093*	44 000*	1.13	1.38
.....	.....	....	....

For Steel, E = 30 000 000 Lb. per Sq. In.

Test 2087; year, 1909, p. 865; 5-in. lap-welded pipe, 0.36 in. thick; actual area, 5.255 sq. in.; l = 230 in.; r = 1.647 in.; $\frac{l}{r}$ = 139.5 in.; transverse uniform load 332 lb.; transverse center load, 1 482 lb.; I = 14.25 in.; v = 2.5 in.; u = 31 000 lb per sq. in.

p.	f.	Δ_c.	Δ_a.
0	1 662	1.00	....
2 000	19 200	1.15	0.84
4 000	21 580	1.36	0.99
6 000	24 360	1.65	1.22
8 000	27 590	2.11	1.57
9 172*	29 850*	2.52	2.21
Load Dropped			
8 000	.....	2.28	2.42
Load Sustained			

* After reaching this maximum, the load dropped and the deflection increased.

† The amount given in the report as actual area was evidently erroneous.

Let,

p_t = the intensity of the direct tension.

Then, for a tension member with frictionless hinged ends, by analysis analogous to that by which Equations (3) and (25) were developed,

$$f = p_t + f_c - f_c\left(\frac{p}{p-q}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (34)$$

When the tension member is loaded to its utilizable capacity, $f = u$, and,

$$p_t = u - f_c + f_c\left(\frac{p}{p-q}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (35)$$

These equations are strictly accurate only when the primary moment diagram is a sinusoid, but they give close approximations in other cases. They are similar to, or possibly identical with, equations now in use. Equation (35) can be solved only by trial, but in many cases the correction involved is worth the trouble.

In some special instances it may be advisable to treat the tension chords of bridges rigidly as continuous girders and, in other cases of tension members with riveted end connections, to take the l of the formulas and tables as from three-fourths to two-thirds of the length of the members; similarly (as outlined at the end of Part 9) for columns.

Part 11.—Variation of the Characteristic Useful Limit Point Between Different Members of Nominally the Same Grade of Steel

Table 22, listing tests of bars forged down from an open-hearth steel ingot in the direction of its length, shows many variations—in tensile strength between 104 800 and 113 000 lb. per sq. in., in U. L. P.'s between 46 000 and 72 000 lb. per sq. in., and in ratios of U. L. P.'s to tensile strength between 42.4 and 65.5 per cent. In this series the range of the ratio of the U. L. P. to the tensile strength is great but not extreme: For example, Table 7 cites a case where this ratio was 84.2%; the weighted average ratio of U. L. P.'s to tensile strength for twelve specimens representative of the extra heavy H-section columns in Table 11 was only 40.8% (with a ratio in one specimen from the end of the flange of 32.6%); and a specimen from the head of an electric furnace steel rail had a tensile strength of 116 600 lb. per sq. in., a U. L. P. of 47 000 lb. per sq. in., and a ratio of U. L. P. to tensile strength of only 40.3 per cent.*

The chief value of Table 22 is not in the ranges of the properties it shows but in the light it sheds on the fact that the reduction in cross-section and the temperature at which reduced are causes of variation between the U. L. P.'s of different specimens of the same steel. Differences in conditions under which shapes and plates cool after rolling, differences in auto-stresses produced in cooling, and other occurrences during manufacture are, probably, further causes of differences between the U. L. P.'s of different specimens.

The specifications for "Structural Steel for Bridges" (not including eyebars and rivets) of the American Society for Testing Materials and of the

* Watertown Arsenal Report, 1909, Vol. 2, p. 681.

American Society of Civil Engineers* permit a variation in tensile strength of 5 000 lb. either way from 60 000 lb. per sq. in., without any corresponding variation in working stresses. If representative specimens for one member of a structure with the lowest permissible tensile strength (55 000 lb. per sq. in.) have a U. L. P. of 40%, and other specimens of another member in the same structure with the highest permissible tensile strength (65 000 lb. per sq. in.) have a U. L. P. of 60% or more, the characteristic U. L. P.'s for the members

TABLE 22.—TESTS OF BARS FORGED DOWN FROM AN OPEN-HEARTH STEEL INGOT IN THE DIRECTION OF ITS LENGTH, VARIOUS TEMPERATURES, AND AMOUNTS OF REDUCTION.

(Compiled from Watertown Arsenal Report, 1909, Vol. 3, pp. 869-896. Diameter of specimens, 0.798 in.; sectional area, 0.50 sq. in.; gauged length, 6 in.)

No. of test.	Marks.	Reductions under hammer, percentage.	Approximate finishing temperature, in degrees Fahrenheit.	Useful limit point, in 1 000 lb. per sq. in.	Tensile strength, in 1 000 lb. per sq. in.	Ratio of useful limit point to tensile strength, percentage.	Elongation, percentage.	Contraction of area, percentage.
8 658	13 *B*	6.0	1 400	64.0	108.0	59.2	14.6	23.0
8 645	13 *T*	9.2	1 400	67.0	109.8	61.0	17.0	25.2
8 657	12 *B*	16.1	1 400	70.0	111.8	62.6	16.1	29.4
8 643	11 *T*	18.9	1 400	72.0	110.0	65.5	13.1	18.6
8 644	12 *T*	32.2	1 400	70.0	111.4	62.8	16.0	29.4
8 648	3 *B*	2.3	1 600	56.0	105.0	53.3	7.1	9.2
8 634	2 *T*	7.2	1 600	58.0	108.0	49.1	9.1	11.6
8 646	1 *B*	11.5	1 600	61.0	109.8	55.6	13.1	16.2
8 647	2 *B*	27.2	1 600	60.0	110.2	54.4	12.6	16.2
8 633	1 *T*	28.4	1 600	69.0	112.6	61.3	15.5	18.6
8 637	5 *T*	2.5	1 800	48.0	104.8	45.8	7.2	9.2
8 636	4 *T*	5.6	1 800	50.0	108.0	46.3	7.6	9.2
8 650	5 *B*	8.0	1 800	50.0	108.4	46.1	9.5	11.6
8 649	4 *B*	14.7	1 800	57.0	113.0	50.4	8.6	9.2
8 635	3 *T*	28.2	1 800	61.0	112.8	54.5	11.3	14.0
8 655	10 *B*	3.6	2 000	48 0	105.8	45.4	7.8	9.2
8 656	11 *B*	6.1	2 000	48.0	108.4	44.3	12.0	11.6
8 642	10 *T*	10.3	2 000	49.0	108.4	45.2	9.8	11.6
8 654	9 *B*	13.6	2 000	51.0	108.2	47.1	12.3	14.0
8 641	9 *T*	26.5	2 000	53.0	110.4	48.0	13.3	16.2
8 640	8 *T*	31.6	2 000	56.0	111.2	50.4	14.0	25.2
8 638	6 *T*	4.3	2 200	46.0	108.4	42.4	10.5	11.6
8 653	8 *B*	14.5	2 200	49.0	110.6	44.3	12.3	14.0
8 652	7 *B*	21.8	2 200	48.0	109.8	43.7	14.0	23.0
8 639	7 *T*	27.0	2 200	51.0	111.0	45.9	15.1	27.0
8 651	6 *B*	29.6	2 200	53.0	113.0	46.9	16.0	27.4

themselves, according to indications given by the rule in Part 5 (page 1236), will vary between 18 700 lb. per sq. in. for the one member and 39 000 lb. per sq. in. for the other; which is quite a wide range for which to use a common unit working stress. The extremes in the characteristic U. L. P.'s of the column, made of 60 000-lb. (desired) steel, tested for the Society's Special Committee on Steel Columns and Struts were 19 000 and 35 000 lb. per sq. in., as shown in Table 11.

* Final Report of the Special Committee on Specifications for Bridge Design and Construction on Specifications for Design and Construction of Steel Railway Bridge Superstructure, *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 487; also Final Report on Specifications for Design and Construction of Steel Highway Bridge Superstructure, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1291.

In 1875, the President of the United States appointed a Board for the Testing of Wrought-Iron, Steel, etc., and this Board appointed a Committee, headed by Commander L. A. Beardsley, U. S. N., to investigate wrought-iron chain. The Committee, as a preliminary, investigated the properties of different kinds or makes of wrought iron. It found for iron of the same grade considerable variation in tensile strength and great variation in what it termed "elastic limit" and described as the first perceptible stretch. Its tests in this regard were conducted slowly and with great care and with the aid of a magnifying glass. After experimenting with material and labor, in a rolling mill loaned without charge for the purpose by its proprietor, the Committee succeeded in rolling nine sizes of bars, varying in diameter from 1 in. to 2 in., with nearly uniform tensile strength and "elastic limit". After making these tests and experiments the Committee made a report* an excerpt from which is as follows:

"* * * as important differences exist in the proportionate strength of different-sized bars made of the same material, which are due entirely to differences in the processes by which they are manufactured, and as the elimination or reduction of such differences would necessitate such a great and expensive change in the system by which the bars are produced that it is not probable that it will be often attempted, it is necessary that these differences should be taken into consideration when estimates of the strength of any structure in which rolled wrought iron of different sizes is introduced are made, and in all tables of strength based upon the strength of such bars."

It is more than forty-six years since the Committee made its report. During this time steel has about supplanted wrought iron as a structural material and great changes have been made in rolling mills. The writer does not undertake to discuss what further changes, if any, in mill practice affecting the matter of differences between the U. L. P.'s of specimens of nominally the same grade of steel are possible, practicable, or probable; but inasmuch as wide differences in characteristic U. L. P.'s exist, he believes that they should be taken into consideration in estimating the utilizable capacity of steel members of structures.

Part 12.—Limitation of the Useful Limit Point and Maximum Tensile Strength, as Criteria of Utilizable Capacity, to Steel of Good and Suitable Quality and Workmanship.

The U. L. P. and maximum tensile strength of specimens of steel are criteria of the utilizable capacity of the steel members of structures only when the quality of the steel in other respects is good and suitable and the fabrication into structural members is properly performed.

This paper does not define what constitutes good and suitable quality and proper fabrication (the opinion of engineers has fairly well crystallized on these points and on the tests, analyses, and inspection by which they are gauged); but it does emphasize the importance of good and suitable quality and workmanship.

* Report of U. S. Board for Testing Iron, Steel, etc., 1881, Vol. 1, p. 45.

Part 13.—Factor of Safety

An exact determination of the utilizable capacity of a steel member of a structure, if achieved, would indicate precisely the greatest load that could statically or repeatedly, or the greatest opposite loads that could alternately, be applied during the lifetime of the structure (as determined by other considerations) without causing failure, ruin, or seriously objectionable deformation (plastic, elastic, or combined plastic and elastic). The utilizable capacity can not, of course, be exactly determined, but it can be approximately determined, as outlined in this paper, from representative specimen tests of the steel. The ratio of the utilizable capacity thus determined to the working load on the member is the member's approximate factor of safety; and the smallest factor of safety of any of the critical members of a structure is the approximate factor of safety of the structure as a whole.

Under the present practice of purchasing steel for structures and of designing and fabricating the steel members, the different members may have factors of safety that vary greatly from each other and from what is considered amply sufficient. The factor or margin of safety, instead of being entirely selected by the intelligent judgment of designers, depends largely on circumstances of manufacture of which designers, as a class, are inadequately informed and of which they usually take little heed.

Would it not be well for steel manufacturers, structural fabricators, physical metallurgists, and structural engineers, considered collectively as "makers of steel members of structures", to investigate the question whether or not it is practicable and advisable to classify the various sizes of steel plates, shapes, and bars, of nominally the same grade of steel into such groups that the variations of the U. L. P. within each group would be much less than the present variation within nominally the same grade of steel embracing all sizes of plates, shapes, and bars; and thus to enable engineers to take into account in proportioning members the U. L. P.'s of the different groups according to the sizes used?

DISCUSSION

C. A. P. Turner,* M. Am. Soc. C. E. (by letter).—As a study to discover the fundamental facts regarding the capacity of structural steel members this paper is a failure. It is wanting in a concise statement of vital facts from which definite conclusions may be drawn and it is encumbered with terms of nebulous significance.

Attributing to auto-stresses both the defects and improvements of elasticity will prove more confusing to the average reader than blowing hot and cold with the same breath did to the satyr in the fable. It is recognized that stresses in general may be normal or tangential, positive or negative; one wonders in reading the paper to what gender or classification these auto-stresses may be assigned, what they are, how are they generated, and what is their reaction.

Under tension or compression the position of a piece of steel in the Antimony-Bismuth scale changes in proportion to the stress applied within the elastic range of stress, in one direction for tension and in the opposite direction for compression, so that it is possible to measure dead stress by thermo-electric means. When the elastic range has been exceeded, however, the residual effect as to position is a change characteristic of the opposite kind of external stress to that causing the permanent set. This residual effect has been termed a "permanent strain" by some writers, associating it with the permanent set which accompanies but is not proportional to it. By others this effect has been termed "residual strain" or a strain that is left after exceeding the true elastic limit of the piece and is in equilibrium with, or balanced by, elastic strain.

Having opposite physical characteristics it is natural to expect opposite reactions or changes of shape when the piece is cut so that the elastic and residual strain is thrown out of equilibrium. Take several bars $\frac{3}{8}$ in. square, bent cold in horseshoe form, and measure the out-to-out dimension. Then place one in a vise and cut saw kerfs $\frac{1}{8}$ in. deep at $\frac{1}{2}$-in. intervals on the outside of the bend. Release from the vise and note the inward spring of the ends. Cut another in a similar manner on the inside and again measure the inward spring of the ends. Take a third sample, cut both sides, and measure the outward spring of both ends, then the reaction of elastic strain as opposed to the contrary reaction of the residual strains in the outer fibers in the first and second experiments. Again, take a steel bar as it has passed through the rolls and saw kerfs as before on either top or bottom and, in general, the bar will assume a slight curvature. Cut on the sides and no change results. In like manner, cutting a keyway in a piece of cold-rolled shafting will cause the piece to assume a change in alignment.

A state of permanent strain may be defined as a state of internal stress comprising elastic and residual stress in equilibrium, from which no change in shape results unless this equilibrium is disturbed by forces supplying energy from without. Energy to change the internal state of equilibrium may be introduced as follows: (1) By temperature; (2) by applied forces; and (3) by

* Cons. Engr., Minneapolis, Minn.

cutting away a part of the symmetrically strained body. Such removal or cutting away of symmetrical parts of an over-strained elastic material permits the permanent strains that have been thrown out of equilibrium to re-assert themselves and change the form of the body in a manner comparable to applied external forces.

Gradual recovery from residual strain is sometimes termed elastic afterworking and the lag or slow increase of deformation under stress is called hysteresis. These phenomena are more readily observed by twisting a fine wire suspended from a fixed support and having a small mirror attached to the lower end, the position of which can be observed in the usual way by means of a telescope and scale. If the lower end of the wire is twisted through a moderate angle and then left to itself, the mirror makes oscillations the extent of which may be read on the scale. These oscillations diminish more rapidly than when the only retarding force is the resistance of the air, showing that the force of torsion in the wire must be greater when the twist is increased than when it is diminishing.

According to Maxwell:*

"If we begin by keeping the wire twisted (at not too great or too small an angle) say, for a minute or an hour, and then leave it to itself, we find that the point of temporary equilibrium is displaced in the direction of twisting, and that this displacement is greater the longer the wire has been kept twisted. But this displacement of the point of equilibrium is not of the nature of a permanent set, for the wire, if left to itself, creeps back towards its original position, but always slower and slower. This slow motion has been observed going on for more than a week, and also if the wire was set in vibration the motion of the point of equilibrium was more rapid than when the wire was not in vibration.

"We may produce a very complicated series of motions of the lower end of the wire by previously subjecting the wire to a series of twists. For instance, we may first twist it in the positive direction, and keep it twisted for a day, then in the negative direction for an hour, and then in the positive direction for a minute. When the wire is left to itself, the displacement, at first positive, becomes negative in a few seconds, and this negative displacement increases for some time. It then diminishes and the displacement becomes positive and lasts a longer time till it too finally dies away."

The phenomena here described represent an angle of distortion greater than the true elastic limit but less than the yield point of the metal. Kelvin has treated these phenomena as the "viscosity" of elastic solids. Experiments on the torsional vibration of wires detailed by Kelvin,† indicate elastic fatigue for a considerable range of stress within the limit of elasticity of shape. The experiments were as follows:

About April 26, two equal and similar lead weights or vibrators were hung up by equal and similar copper wires, the upper and lower ends of the two wires being similarly fixed by soldering. Wire No. 2 was kept vibrating as continuously as possible from day to day, and the other kept at rest except when vibrated once a day in an experiment to test the comparative viscosity of the wires, but no exact comparison was made until May 15. Then Wire No. 1 in 97 vibrations subsided from an initial range of 20 to a range of 10, while

* Encyclopedia Britannica, "Constitution of Bodies."
† *Loc. cit.*, "Elasticity."

Wire No. 2 gave the same subsidence in 98 vibrations. During the greater part of May 16 and 17, Wire No. 2 was kept vibrating and Wire No. 1 quiet, and experiments were made with the result that Wire No. 1 subsided from an initial range of 20 to 10 in 98 to 99 vibrations of 2.4 sec. each, while Wire No. 2 gave the same subsidence in from 57 to 60 vibrations of 2.4 sec. each. It was supposed from these experiments that true elastic fatigue occurred for all stresses within the limit of elasticity of shape until a similar and more extensive series of experiments conducted by Tomlinson* showed that no such fatigue of elasticity is felt in steel, iron, or copper, provided the stress does not exceed a certain well-defined limit.

The "micro-flaw" theory of fatigue advanced by Johnson† is untenable because of its failure to account for observed phenomena. Thus, the elastic fatigue in Thompson's experiments depending on the immediately previous molecular condition, whether of quiescence or recurring changes of shape, cannot be accounted for by micro-flaws which are necessarily permanent and not temporary. When a bar of steel is loaded a number of times alternately in tension and compression to a point a little beyond the yield points in tension and compression, these points are thereby both reduced to about six-tenths of their original values (called by Bauschinger the "natural elastic limits" of the material). When the material is repeatedly strained in tension from zero to a value above the natural elastic limit in tension, but within the yield point, the yield point value in tension is raised by such treatment and, in like manner, when similar stresses above these natural elastic limits, but within the yield point values, are applied alternately from tension to compression, the normal yield point in tension is lowered. These phenomena raise the question whether the temporary strain which accounts for the elastic after-working is of the same nature as that produced by a permanent set, and what the limits of its accumulation may be under repeated stress in working of the material, without giving it an opportunity to recover by elastic after-working between consecutive loadings.

These questions would seemingly be more readily investigated by considering the thermal stress curves of the material under load. During elongation soft steel grows uniformly cooler until a stress of about five-eighths the yield point is reached, when the heat of imperfect elastic deformation causes the curve to depart from the straight line. This continues until the yield point is reached when there is a sudden evolution of heat. In like manner, under compression the straight-line relation of temperature increase is maintained until a stress of about five-eighths the yield point is reached, then a similar phenomenon occurs.

The identity of cause (that is, thermal effect), would indicate an identity of result. In other words, for the range of imperfect elasticity within the yield point under repeated working of the material beyond its natural elastic limits, the residual strain accumulates with each repetition of the applied load even if no apparent set or plastic deformation is noticeable. As the apparent yield

* *Philosophical Transactions*, Royal Soc., 1886.

† "Materials of Construction."

point is gradually raised by such treatment, the range of stress may be gradually increased until the ultimate resistance of the material is reached without noticeable plastic deformation. The same piece of steel under a single application of the load would stretch 20% of its length before "necking down" and finally breaking.

The fact that a residual strain is of contrary sign to the internal stress opposing the force which produced it leads at once to the question whether a change in the yield point is a phenomenon of residual strain or of change in structure. Microscopic examination fails to reveal any change in the material that has undergone a sufficient number of repetitions above its natural elastic limits to nearly wear it out or cause it to approach the point of rupture.

The behavior of iron and steel under fluctuating or repeated stresses was the subject of long research by Wöhler and later by Bauschinger. The result of these experiments are claimed to show:

1.—That resistance to fracture under stresses fluctuating within certain limits depends on the difference between the maximum and minimum applied stresses rather than on the maximum stress alone.

2.—That reversed stresses (tensile and compressive), much below the static breaking stress, are sufficient to cause fracture if repeated a great number of times.

Experiments made by the Phœnix Company on axle iron having an ultimate strength in tension of 46 000 lb., with an elongation of about 20%, gave limiting maximum stresses under limited ranges as shown in Table 23.

TABLE 23.—Tests on Axle Iron by Repeated Stress.

Kind of repeated load.	Stress, in Thousands of Pounds per Square Inch.	
	Limiting maximum.	Limiting range.
Completely reversed	15.2	30.4
From maximum to zero	30.56	30.56
From maximum to half load	41.02	About 20

The number of repetitions of stress without breaking, from which these limiting ranges were determined, amounted to millions. The specimens unbroken at an alternating stress of 15 200 lb. per sq. in. were subjected to 132 000 000 repetitions without breaking. This limiting range, which allows nothing for dynamic effect, is nearly six-tenths of the normal yield point of the material; if the dynamic effect of the rapidity of repetition of stress amounts to 10%, the limiting range would be about two-thirds of the normal yield point.

The ultimate strength of the material was 46 000 lb. per sq. in. and its normal yield point was about 24 000 lb. per sq. in. The limiting maximum stress of 42 000 lb. per sq. in. for repetitions of half load in Table 23 is, therefore, at least 18 000 lb. more than the normal yield point, and since whenever the normal yield point is exceeded a large deformation results, the manner in which these repeated stresses were applied is of interest. To avoid the large deformation

produced when the normal yield point is exceeded, the stresses first applied in these experiments were less than the normal yield point; this point was then gradually raised until its ultimate position was above the maximum stress considered in the range noted as the limiting range.

In a practical structure the conditions of loading are not such as to raise the yield point of the material and thus increase its resistance, as was done in these experiments, and failure would result from only a few repetitions beyond the yield point. Consequently, the limiting ranges of intensity of stress which the practical structure can withstand cannot be deduced from experimental results where such artificial assistance to the resistance of the material is given.

As advocates of the fatigue formulas of proportioning agree that, if it be established that Wöhler's results are accounted for by the effects of stress beyond the true limits of elasticity, they have no relation to the logical determination of working stresses, this matter will now be considered from the standpoint of the three determinants of ideal elasticity, to wit:

1.—Proportionality of temperature change to the load applied;
2.—Proportionality of elastic elongation to the applied stress or load; and
3.—The reversible cycle of mechanical work during loading and unloading.

No bar of sound metal has ever been broken by Wöhler's treatment under ranges of stress of one kind or of both kinds, that is, from tension to compression within the limits of thermal proportionality. *Per contra* the range of stress from tension to compression within the natural elastic limits was found by Bauschinger to improve the metal in elasticity, tenacity, and ductility instead of injuring it.

The radical improvement in the strength of cast test bars and the suggestion that they shall not be "rumbled"* (treated in a tumbling barrel for cleaning) if their modulus of rupture be taken to represent the average condition of the casting, is a striking confirmation of the beneficial effect of stress ranging from tension to compression in reducing and eliminating residual states of stress arising from shrinkage in cooling.

The theory of residual strain shows that the range of stress of one kind is not as efficient in eliminating residual strain as range of stress from tension to compression. Where the range is from tension to compression extending equally beyond the natural elastic limits the reaction of the accumulation of residual strain is neutralized because of the opposite signs of its constituents, accompanied by increased internal disturbance and more rapid approach to rupture under repeated stress. Where the limiting range was tension from 20 000 to 42 000 lb. per sq. in., the systematic work on the specimen effected an accumulation of approximately 19 000 lb. of residual strain, which, added to the yield point of 24 000 lb., would give a maximum resistance of 43 000 lb. per sq. in. without plastic deformation. In other words, the effect of a tensile stress of 42 000 lb. per sq. in. on a bar under a state of residual tension of 19 000 lb. or more could not cause plastic deformation in a specimen the yield point

* "A. B. C. of Iron and Steel", Penton Pub. Co.

of which is 24 000 lb., since the algebraic sum of these is only 23 000 lb. and, therefore, less than the yield point of the bar.

The fact that rupture of ductile metal under Wöhler's treatment takes place without plastic elongation has been in the past erroneously regarded as in some way indicating or establishing the fact that all ranges of stress without shock tend to deteriorate or injure sound metal. The law of conservation of energy indicates that the same amount of energy is required to produce rupture whether it is expended in small quantities at different times, or at one time in a single application. When rupture is accompanied by nearly negligible ultimate deformation, as in endurance tests under repeated stresses of intensity lower than the ultimate strength of the material, there appears to be no good reason to suppose that the total energy transformed into heat in small amounts under successive repetitions of the load in endurance tests differs from that when this deformation is as large as in the case of a single load developing the maximum strength of the specimen.

From the foregoing discussion have been drawn the following conclusions:

1.—Steel of good quality has definite limits of elasticity which can be lowered but not raised.

2.—Strain beyond these limits cannot occur without performing work on the metal tending to wear it out.

3.—Strain within these limits tends to improve the metal in elasticity, ductility, and tenacity, by elimination of the residual strains from rolling and cooling.

4.—The limit of possible improvement by repeated stress within the limits of elasticity is readily computed and accords satisfactorily with the investigations of Bauschinger and with his conclusions that, far from being fatigued by repeated stress from tension to compression within the natural elastic limits, sound metal is improved in elasticity, ductility, and tenacity.

5.—In the temporary structure the yield point is the critical limit of stress, for, if the yield point is exceeded under quiescent stress, failure is liable to result from the dynamic effect of the load in motion. The plastic deformation, moreover, may so change the configuration of a truss that failure may result. That bridge structures frequently stand up when counters are over-strained and stretched may be accounted for by the momentary duration of the maximum stress due to the moving load.

From the theorem that volume is a kinetic invarient of elastic strain in an isotropic homogeneous solid,* the moduli, K (of lateral contraction) and F (shear), may be computed as $0.4141e$ and $0.3535E$, respectively (before the temperature changes from the applied stress are dissipated by radiation). Since for over-strain and small set the residual strain will be developed along the curve of greatest displacement and no over-strain takes place along the equipotential curves, F, for small over-strain, will remain numerically constant and only K and E will vary. Hence, the limit of increase in E may approximate 5% by the elimination of residual strains through repetition of stress from tension to compression within the natural limits of elasticity.

* "Elasticity and Strength of Materials," Turner, Section 1, p. 75; see, also, Unwin, "Strength of Materials."

The conclusions which should guide in fixing the magnitude of working stresses the writer would summarize as follows:

1.—The working stress for a permanent structure should be kept below the natural limits of elasticity allowing properly for the effect of impact, unsymmetrical distribution of stress across the section due to faulty workmanship, and irregular form.

2.—These working stresses and proper allowances need be no lower for a range of stress from tension to compression than for stress of one kind alone.

3.—It is highly irrational to use higher working stresses for variation of stress of one kind only than for variation of stress of both kinds, because it is impossible in the actual structure for the loads to be applied after the manner of the fatigue experiments of the laboratory by varying the stresses back and forth below the normal yield point until there is temporarily an accumulation of residual strain. This procedure, contrary to any conceivable application of load in the engineering structure, constitutes the hocus-pocus of fatigue formulas; it causes needless waste in the design of bridge members subject to alternating stress, the adoption of unbalanced sections, and a deficiency of strength where the range of stress is small.

Bearing in mind that this natural elastic limit or limit of thermal proportionality hardly exceeds 22 000 lb. per sq. in for soft steel of 55 000 to 60 000 lb. ultimate strength, or 26 000 lb. per sq. in. for medium steel from 65 000 to 75 000 lb. ultimate strength, the desirability of using medium steel rather than soft steel is obvious.

In respect to columns, a fundamental defect of nearly all published column formulas lies in the disregard of twisting. An angle strut* having a ratio of $\frac{l}{r}$ equal to 140 and fitted with ball and socket bearings, is only one-half as strong as a pipe section of the same cross-sectional area and the same radius of gyration. In other words, the ordinary column formula for such a strut is liable to be in error 100% in respect to the strength of these struts if the ratio, $\frac{l}{r}$, is 140 or greater. The inferior showing of steel Z-bar columns as compared with wrought-iron columns in Strobel's tests may be accounted for by the fact that Strobel made one radius of gyration much more than the other, thereby forcing the columns to fail by simple bending instead of by combined bending and twisting as in the U. S. Bureau of Standards tests.

A more careful consideration of the facts determined by test will profit the Engineering Profession, while blind acceptance of ingenious algebraic formulas founded on assumptions which disregard the vital facts of experiment may lead to serious error, lack of economy, and occasional failure.

Mr. Prichard states there are faults in the theory of flexure, and correctly points out that shear strain is disregarded in the ordinary theory of flexure.

* A pipe strut is equally strong whether pin ended or with ball and socket bearing, whereas an angle is 1¾ times as strong with pin as with ball and socket bearing when $\frac{l}{r}$ is greater than 140 although calculated the same by present theory.

This disregard does not increase the maximum fiber stress at the points of greatest deflection as he appears to suppose, nor does the exact theory substantiate the supposition of Marburg that the modulus, E, of Bethlehem beams is any lower for a given grade of steel than that of standard beams. Far from increasing the maximum fiber stress as computed by the ordinary theory of flexure in the restrained beam, the maximum tension is reduced both over and between the supports and a similar reduction holds true for continuous beams. The resistance to external shear may be two to four times as great per inch of section in the restrained beam as in the simply supported beam of the same cross-section. Again, as the neutral plane of the simply supported beam drops downward toward the end of the beam approximately one-tenth of the depth according to the exact theory of flexure, how can this theory be applied to a column, assuming that the neutral plane follows the axis of the piece when it does not in the beam? Moreover, since the principal tensions in the column are at right angles to the axis or cross-wise of the piece until the stage of incipient failure is approached, how can the bending of a column be compared with the bending of a beam, in which the greatest tensions are nearly parallel to the axis of the piece? Is not much of the mathematical discussion of the paper based on assumptions which are unrelated to the facts?

H. F. Moore,* Esq. (by letter).—This interesting paper is a welcome attempt to meet the question "What is an allowable stress for structural steel?" The author recognizes that this is very far from being a simple matter of taking some value from a set of handbook tables and applying it in a standardized formula.

One interesting point of Mr. Prichard's paper is his use of the "Useful Limit Point" (U. L. P.) rather than the yield point, or Johnson's apparent elastic limit (which involves an increase of rate of strain over the initial value amounting to 50 rather than 100 per cent. A more extended statement from Mr. Prichard as to the advantages of the U. L. P. over Johnson's elastic limit would be of value.

In connection with Mr. Prichard's suggestion as to the possibility of "remote fatigue", the writer would like to quote results not conclusive, but interesting, from some tests of I-beams made about 1912 in the laboratories of the University of Illinois. Several I-beams were loaded in flexure with loads which stressed the compression flanges to the yield point. The load was continued for 107 days without causing failure of the beams, and after the first few days there was no further material increase in deformation.

Relative to "remote fatigue", the interrelation of localized stress and corrosion are parts of a subject on which there has been very little investigation. From certain experiences with boilers, the writer believes that this is a very important subject for research by the structural engineer.

In connection with the author's discussion of fatigue tests, the writer would call attention to the results reported† by Mr. Paul L. Irwin, of the Westinghouse Laboratories. Mr. Irwin developed a method of gripping the specimen in a

* Research Prof. of Eng. Materials, Univ. of Illinois, Urbana, Ill.

† *Proceedings*, Am. Soc. of Testing Materials, 1925, p. 53.

repeated stress test under cycles of tension and compression, by which accidental bending action was reduced to a very small value. For all the metals tested thus far, Mr. Irwin finds that the fatigue limits determined by such tension-compression tests and by tests in reversed flexure are almost the same. Previous investigators from Wöhler on (including the writer) have been rather prone to assume that a tension test specimen is under pure tension. As a matter of fact it is very difficult, even under laboratory conditions, to avoid appreciable bending action, especially on a short tension test piece.

G. B. Woodruff,* M. Am. Soc. C. E.—If this paper did no more than call attention to the uncertainties of present knowledge as to the behavior of structural steel specimens and structural members under stress, and the relation of the strength of the member to that of the specimen, it should prove valuable in pointing out the need of further investigations of these matters.

In so far as safe unit stresses are concerned, the problem may be divided into three main phases:

1.—What is the "utilizable capacity" of a structural member?

2.—What requirements may be made of the test specimen that will give the most definite information as to the capacity of the structural member?

3.—What factors may be applied to the results of specimen tests to give the safe load in a structural member?

According to the author's definition, it would be permissible to stress a member to its "utilizable capacity". Evidently this is not his interpretation since he later introduces the "factor of safety," and by this factor apparently destroys his definition. Considered alone, the knowledge of the U. L. P. of a structural member would not seem to be especially valuable. Admitting that the term "proportional limit" is to a certain extent a misnomer, the deviation of the stress-strain curve from a straight line, as generally understood, is so small as to be negligible. Structural theory, especially in the case of indeterminate stresses, is predicated on the proportionality of stress and strain. It would seem that the structural designer is more interested in the proportional limit than in any other characteristic of the behavior of the members. If this proportional limit could be positively predetermined and the design based on the maximum possible combination of loads, no factor of safety would be required.

In the speaker's opinion, therefore, the desired result of the specimen tests is to obtain the best possible index of the proportional limit of the fabricated member. Perhaps the U. L. P. of the specimen will furnish this index, but further investigation of this point is required. That the yield point "as determined in ordinary commercial practice" gives far from satisfactory results is generally admitted. The question here is whether the trouble lies with the yield point as such or with "ordinary commercial practice." The data presented in Table 9 merely indicate errors in the operation or calibration of the testing machines. There seems to be no better indication of the proportional limit of a fabricated member than the ultimate strength of the test specimen.

* Asst. Engr., Modjeski & Moran, New York, N. Y.

The speaker interprets the results of the most careful tests as indications that the proportional limit of fabricated short columns and fabricated tension members approximates 30 and 40%, respectively, of the ultimate strength of the test specimens. He believes it to be conservative when designing for maximum loads to use unit stresses of 25 and 35% of the ultimate strength of the specimens.

The proposal to vary unit stresses with the thickness of the material is a step in the right direction. Quite likely the variable in this case, however, is the amount of working the material receives in the rolling. The thickness is only a partial indication of this variable.

The author's specification for lattice is similar to that in some recent specifications and probably results in as good design as is possible within the limits of present knowledge. Some tests, however, indicate that the reaction from the light lattice in vogue before 1907 has gone too far and that it is possible to use lattice too stiff and with too rigid connections for the maximum strength of the columns.

In conclusion, the speaker would urge that the arguments developed by the author be given full consideration, especially by those urging a revision in working stresses.

R. VON FABRICE,* ASSOC. M. AM. SOC. C. E. (by letter).—An examination of the various tests quoted shows that the yield point is a variable quantity if found from specimens taken from the same plate but tested in different laboratories (see Table 9). Evidently, the true elastic limit can be correctly defined only by means of the closest scientific observations.

Furthermore, Mr. Prichard's references show that the elastic limit is not a constant but a variable quantity and can be changed at will by adopting different conditions of straining. Therefore, it is evident that none of these points should be used as a criterion for the strength of any members.

The author's suggestion of basing practical formulas on the U. L. P. basis, as adopted by the Society's Special Committee on Steel Columns and Struts, must be recognized as a step forward toward the solution of the problems dealing with the strength of structural steel.

The elevation, to higher limits, of the elastic limit shows that steel, as well as other material, will adjust itself to variable conditions and survive severe over-straining by readjustment of the grains or fibers. This behavior of steel may be compared with the characteristics otherwise attributed to the muscular development of an athlete, in whom stiff joints and muscular defects may be compared with the causes underlying auto-stresses in a piece of steel, and with whom careful working and manipulation will cause these defects to disappear as weak fibers or grains in steel recover and remain elastic after being subjected to a certain cycle of stresses. In both instances the elasticity has been raised or increased. If, however, the training or cycle of stresses is beyond the U. L. P., negative results will be obtained and permanent fatigue or failure

* Designing Engr., Public Service Production Co., Newark, N. J.

will be the result in man and material. This behavior of structural steel under loading, adjusting itself to severe loading, accounts for the existence and apparent stability of structures which according to the recognized mathematical calculations should have failed.

Mr. Prichard shows that large variations in elastic limit exist. It would be advisable to determine by tests if a lowering of the lower elastic limit in compression is possible and if it is accompanied by a lowering of the upper elastic limit in tension; in other words, if by a cycle of stresses the elastic limit of the material under compression can be raised thereby losing a part of its resistance against tensile stresses.

The fibers of a piece of metal under tension undoubtedly have a tendency to align themselves in the direction of pull, and thus produce the increased elastic resistance (Fig. 5 (*a*) and Fig. 5 (*b*)).

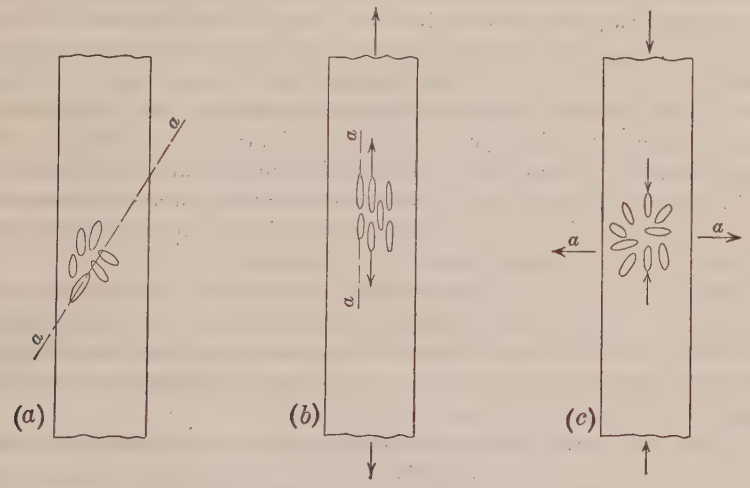


FIG. 5.—ALIGNMENT OF METAL FIBERS UNDER STRESS.

Under compression the fibers tend to flow or squeeze sidewise, which causes transverse shear and consequent buckling of the specimen (Fig. 5 (*c*)). Therefore, it is fair to conclude that under tension all fibers or molecules unite in aiding each other, while under compression they tend to work against each other and squeeze and flow apart. If this conclusion is correct the elastic resistance of steel under compression cannot be increased by a cycle of stresses.

Tables 9 and 19 show great variation in the quality of the material taken from different parts of the plate or I-beam. For example, a 15-in., 38-lb. I-beam gives the U. L. P. for the web as 42 500 lb. per sq. in., while for the root it is 26 000 lb. per sq. in., and, further, the tests for the other beam sections in Table 19 show that the flange has a still different U. L. P. This condition may be due to the process of rolling whereby the web becomes more compacted than the flanges.

The variation in U. L. P. in the various parts of a structural shape leads to the conclusion that not only the elastic resistance, but also the moduli of elasticity of the various parts of any section may vary. Tables and references

show a variation from 28 000 000 to more than 31 000 000 in moduli of elasticity for various compositions of steel; but is it not possible that the modulus of elasticity will vary in the same structural section within these limits?

The theory of flexure is based on ideals, that is, perfectly elastic material with uniform elastic limits throughout the entire member. This condition does not exist in practice. The theory is further based on the erroneous assumption that plane sections remain plane after bending. This is not correct since the influence of the shear (which is usually ignored) prevents the originally plane cross-section from remaining plane. Further, this deformation of the section must become more pronounced if the elastic elongations of the various parts of the section vary in unforeseen and incalculable dimensions.

The rule for finding the U. L. P. of built-up members from specimen tests (page 1236) may give an approximately correct value for tension members and short columns not subjected to flexure. Even tension members may be composed of materials of varying elasticity in such a manner that the resultant of the elastic properties of the different parts does not coincide with the axis of the member. This will result in incalculable auto-stresses since the internal stresses are not balanced around the axis of the section. Mr. Prichard brings this out forcefully in his appeal for an investigation to classify the various sizes of structural shapes into groups having minimum variations in their U. L. P. values. To reduce auto-stresses to the minimum and to manufacture steel uniform in quality and in elastic properties is an ideal which can be hoped for but which probably never will be attained.

To classify the various groups of steel according to the variations of the U. L. P., it is necessary first to establish by a series of tests the variation of auto-stresses, U. L. P.'s and other elastic properties in various rolled shapes and in their different parts, and also to give due consideration to the different rolling processes used by the various steel mills.

From Mr. Prichard's investigations it does not seem possible that built-up members will ever consist of materials of entirely the same class or quality. Fortunately, however, in built-up members the materials with greater elastic resistance seem automatically to go into the extreme fiber; for instance, a Bethlehem H-column or a girder reinforced by two plates will have the more reliable material in the plates. The same holds true for plate girders with cover-plates; Table 9 shows that the edge portions of plates are stronger than the interior ones.

Beams.—To consider the influence of shear, Mr. Prichard recommends adding a percentage, P_c, to the intensity of stress as computed from the ordinary flexure theory and advocates the use of Equations (20), (21), and (22). He then gives the possible error of these equations as up to 50 per cent. It is difficult to understand why the old and approved theory of flexure should be revised for an empirical formula afflicted with 50% factor of error.

As even good steel is not a perfectly elastic medium; as there is the possibility of variable elastic properties of different parts of one and the same shape; and as the designers of steel structures are not assured of the elastic properties of the various parts of built-up members, it is logical to assume

that all these unknown factors probably bring about a greater pronounced warping of plane cross-sections than the hitherto neglected influence of the shear.

Shear.—Of course, for short beams with heavy concentrated loads provision should be made to bring this load safely over the supports without distortion of the local parts. In such cases the flanges should be supported by milled stiffeners. Web sections and reinforcement in excess of ordinary theory should be provided as recommended by Mr. Prichard.

Columns.—Under Paragraph (*m*) of the "Synopsis" it is stated that the U. L. P. of a light H-column was 35 000 lb. per sq. in., while that of an extra heavy H-column of the same material was only 19 000 lb. per sq. in. According to Table 11, twelve tests were made with these extra heavy columns and only three with light ones. The writer does not believe that these tests should be used for final deductions.

All the tension tests of Table 11 show an increase of unit strength for the heavier columns with the exception of tests for No. 5 which show the reverse. It is recommended that light and heavy H-columns be re-tested in order to determine whether the cross-section of the heavier columns was properly proportioned and the stress evenly distributed over the entire area, rather than to arrive at the conclusion that the U. L. P. of a heavy column is only about 60% that of a light column of practically the same shape as shown in Paragraph (*m*) of the "Synopsis".

Heavier sections of the same or nearly the same over-all dimension have a smaller radius of gyration, which confirms the conclusion that an increase of area without an increase of outer dimensions is not an economical solution. Therefore, Mr. Prichard is correct in his statement that heavier columns possess a lower U. L. P. than lighter ones; but the variation from 19 000 to 35 000 lb. per sq. in. seems to be too great in view of the tests recorded in Table 11.

By his research work Mr. Prichard has added most valuable material as to the theory of columns, but his suggestion to classify columns into three divisions—short, medium, and long—does not simplify the already very complex problem of column design. What is needed for designers is simplification, not theoretical refinements and consequent complications as to the selection and classification of columns.

Equations (1), (6), and (11) have been derived assuming frictionless hinges at the ends, a condition which does not exist in practice. For columns restrained at both ends, the recommendation is made to use as l of the equation a value from 0.67 to 0.75 of the actual length. The wide range of this recommendation eliminates all refinements gained in developing Equations (6) and (11).

The author's classification of columns cannot be termed a happy solution since the first two, short and medium, are made dependent on the U. L. P., while the long columns are independent of their U. L. P.'s or of any other critical stress mentioned.

Table 12 cannot be readily understood, nor do the illustrations of its use shed any light on this subject. It would be helpful if the author would add

further discussion of this part to clarify his data and increase the usefulness of the paper.

F. E. SCHMITT,* M. AM. SOC. C. E.—Hitherto, the relation between the ultimate strength and the working strength of structural members has of necessity been a matter of judgment based on structural experience, illuminated and supplemented by the data of tests on parts and entire structures. It remains so to-day, and the art of testing, in spite of its high development, has not yet progressed to the point of allowing the engineer to utilize it for definite verification of his design procedure or selection of his working stresses. In large part, this shortcoming is due to the virtual impossibility of determining service requirements and translating them into quantities commensurate with test values. The line of progress is strongly in the direction of placing safety margins and working stresses on a rational basis, however, and the author's scholarly paper is of value in inviting closer attention to this objective.

Views long abandoned are brought forward anew in the reversion to a fatigue or endurance basis for structural design. It is now commonly accepted that steel structures are free from fatigue action, and no evidence to the contrary has been brought forward. Occasional tests of wrought iron and steel taken from structures long in service have in no case shown recognizable weakening or reduction of ductility of the material. Moreover, the large-scale experiments of recent years on fatigue endurance have established that in steel and other ferrous metals there is no fatigue at stresses lower than a certain limiting amount, the endurance limit, which is not far below the yield point even in case of complete reversal during the repetitive stressing, and probably very near to the yield point for one-directional stress repetition. Stresses permitted in structures are well below these limits, and the evidence is clear that such stresses may be repeated an indefinite number of times without effect on the metal. Thus, it would seem that there is no ground for the reversion to the concept of fatigue in connection with studies of working stress.

Further, some of the data on endurance phenomena contained in the paper are not in harmony with experimental results. The investigations carried out by Messrs. H. F. Moore and his associates, at the University of Illinois, D. J. McAdam, at Annapolis, Md., R. R. Moore, at Dayton, Ohio, and others, give no support to such an anomaly as the terminal drop of the stress-cycle curves in Fig. 3, attributed to Bauschinger.

Certain unusual terms introduced by the author contribute to the difficulty of following an intrinsically complex argument. Such expressions as "remote fatigue" or "auto-stress" are unfamiliar. Even the term used in the title of the paper, "utilizable capacity," is not wholly clear in spite of a carefully phrased definition.

With respect to the latter term, few structural members are subject to only a single kind of loading, and it is, therefore, hardly possible to consider capacity, that is, the limits of service, without at the same time considering the conditions of service. The definition given and the term itself seem to

* Associate Editor, *Engineering News-Record*, New York, N. Y.

have the sole effect of substituting for the concepts of breaking strength and yield point the unaccepted concept of structural endurance limit.

The revival of the term "Useful Limit Point", coined eight or ten years ago by the Society's Special Committee on Steel Columns and Struts, seems unfortunate. The term was the outgrowth, apparently of an attempt to displace the yield point by a compromise between yield point and proportional limit. It differs a little from the "apparent elastic limit" of the late J. B. Johnson, M. Am. Soc. C. E., an ingenious approximation to the proportional limit devised to give a uniform, unambiguous definition of that limit. No good reason has been advanced for modifying the Johnson limit and introducing still another point and name into the already confused terminology of the elastic behavior of structural steel—a material whose yield point is well marked and whose resistance to load is largely dependent on the yield. With respect to structural behavior, the establishment of an arbitrary point some distance below the yield point has no demonstrated justification.

Some interesting considerations suggest themselves in connection with "auto-stress." This new term presumably covers mainly initial and residual stresses. In such parts as gusset-plates and other vital details these stresses may be important even for uniformly ductile steel, especially when the load stresses are compressive. This fact constitutes one reason for estimating stresses in details rather liberally and maintaining a relatively large safety margin, since such concealed stresses are not determinable by analysis or test, but require empirical allowance. On the other hand, similar stresses within the solid body of a member do not appear to be important. In reasonably good fabrication and assembly they are certain to disappear before the strength of the member is overtaxed. This may be deduced consistently from large-scale tests, in which a uniformity of action is displayed that could not be realized if the local constraints arising from rolling, fabrication, and other sources of "auto-stress" were not ironed out by the growing load. It has been held by many authorities that initial stresses constitute one important element of the necessary margin between calculated working stress and testing machine load; with respect to main members, this is open to doubt.

The treatment of reversed-stress members which are also considered by the author, has long been a doubtful point in specifications and structural practice. Increased section is customarily provided for such members, but it is questionable whether there are tests or data from experience to support this rule; it rests rather on tradition. In so far as it seeks to provide resistance against the greater fatigue effect of alternating stress as compared with repeated one-directional stress, it cannot be defended in the light of modern endurance investigation. The loosening effect of reversed stress on connections should have consideration, and, as common rivet loads exceed the lower limit of frictional bond between the connected parts, the provision of excess strength in the end details of reversed-stress members is justified; but the members themselves ought not be penalized on this score.

It may be hoped that the author's study will bring about further attempts to define structural strength more precisely than has yet been done. This

strength, it would seem, cannot be a fixed single quantity, but must be interrelated with service. The view that one point should determine the strength of a member is not in harmony with practical appraisal of service conditions. It is certain that some service resistances are related to the ultimate strength, that is, to the actual point at which a member breaks and other resistances to earlier stages of the elastic behavior of the material.

The understanding of these relations is slowly progressing. Several attempts have been made in recent years to approach the determination of factors of safety or of working stresses rationally, although so far without marked success. There are still too many variables, too many indeterminates, in the problem to permit of full rational analysis. Yet, even to-day, it is possible to set down some general conclusions from known facts and structural experience as a starting point. The following items may be noted, for example:

(1) Under steady, slow, and frequently occurring load, a member is likely to be deformed to destructive degree if the load passes its yield point.

(2) Stress intensification at points of changing section, holes, and flaws does not appear to be significant in ductile steel as long as the mean stress remains below the yield point, but at a higher stress it presents the possibility of failure, so that such stress has an importance greater than that due only to its deforming or crippling effect on the structure.

(3) The small, high-frequency variations or oscillations of stress which are present in all or nearly all steel structures should be considered as a possible tax on fatigue endurance when the main stress is beyond a certain limit. This limit, which is not yet known with certainty but probably is at or somewhat above the yield point, therefore is to be rated as the ultimate strength for some groups of stress.

(4) Plastic readjustment at the yield point acts to redistribute load in most structures to a degree that makes this point not definitely critical, in addition to providing warning of over-stress.

(5) Under single, suddenly applied loading, a member has unimpaired resistance nearly up to its breaking strength.

(6) Stresses set up in elastic distortion, or secondary stresses, are in most (but not all) instances self-limiting in character, tending to disappear by a smoothing out of the inequalities in stress when yield is approached and before they can contribute to failure.

Statements of this kind seem capable of providing a starting point for an approach to the desirable end of rational determination of working stresses and safety factors. For the present, the best that can be said is that the problem is not amenable to mathematical calculation but must be tested by experience of long service, and to this extent the work of the immediate future seems likely to depart from the course outlined by the author.

Henry S. Prichard,* M. Am. Soc. C. E. (by letter).—The writer's reason for introducing the new term, "auto-stresses", is that there is no other term that indicates precisely those stresses and only those stresses within a body

* Analytical Engr., Am. Bridge Co., Pittsburgh, Pa.

which, considered by themselves, without regard to others from external forces, form a balanced system. The term, "initial stresses", is frequently used to designate auto-stresses but is not suitable. Strictly, initial stresses are those which exist in a body before an experiment or before an episode of loading. They include the auto-stresses existing beforehand, but they also include stresses from the body's own weight and those from slight loads applied in advance to take up slack but not considered in comparing measured stress with measured strain. Initial stresses, from the fact that they are initial, cannot include any changes that occur in auto-stresses during an experiment or episode. "Residual" ("remaining after a part has been taken away"), as applied to stresses, does not precisely take the place of "auto". The writer does not know of any other term that has been used or suggested for "auto-stresses".

The writer regrets that he has failed to enlighten Mr. Turner regarding the fundamental facts on which the utilizable capacity of steel members of structures depends. He is confident that to most of the intelligent and thoughtful readers, and hopes that to the average reader, the presentation of the facts regarding auto-stresses will not (using Mr. Turner's phraseology) "prove more confusing * * * than blowing hot and cold with the same breath did to the satyr in the fable". Be this as it may, the facts themselves can not be altered because of confused ideas or preconceived notions; but it may be helpful to quote the previous clear presentation, or rather anticipation, of some of the facts by an eminent engineer and physicist.

In 1848, Professor James Thomson (the elder brother of Lord Kelvin) published* the paper from which the following excerpts were taken. He assumes a case in which a round bar of malleable iron is twisted beyond its elastic resistance and then proceeds:

"* * * and during the progress of this process the outer particles will yield in virtue of their ductility, those toward the interior successively reaching their elastic limits * * *.

"If, after this, all external stress be removed from the bar, it will assume a position of equilibrium, in which the outer particles will be strained in the direction opposite to that in which it was twisted, and the inner ones in the same direction as that of the twisting, the two sets of opposite couples thus produced among the particles of the bar balancing one another. * * * Principles quite similar to the foregoing, are applicable in regard to beams subjected to cross strain * * *.

"When malleable iron has received its final conformation by the process termed *cold swaging,* that is, by hammering it till it is cold, the outer particles exist in a state of extreme compression and the inner ones in a state of extreme tension. The same seems to be the case in cast iron when it is taken from the mould in which it has been cast. The outer portions have cooled first, and have therefore contracted, while the inner ones still continued expanded by heat. The inner ones then contract as they subsequently cool, and thus they, as it were, pull the outer ones together. That is, in the end the outer ones are in a state of compression and the inner ones are in the opposite condition.

"The foregoing principles may serve to explain the true cause of an important fact observed by Mr. Eaton Hodgkinson in his valuable researches in regard to the strength of cast iron (Report of the British Association for 1837,

* *Cambridge and Dublin Mathematical Journal.*

p. 362). He found that contrary to what had been previously supposed, a strain, however small in comparison to that which would produce rupture, was sufficient to produce a set, or permanent change of form, in the beams on which he experimented. * * *

"It appears to me that the defects which he (Hodgkinson) has shown to occur even with very slight strains, exist only when the strain is applied for the first time, or, in other words, that if a beam has already been subjected to a considerable strain, it may again be subject to any smaller strain in the same direction without taking a permanent set."

This remarkable prediction has been supported by the subsequent experiments of Wöhler, Bauschinger, Bairstow, Moore, and others, except that it was found that while the first subjection "to a considerable strain", within limits, causes a great improvement in elasticity, it requires a cycle of such applications to perfect it. Professor Thomson proceeds by a pure effort of scientific reasoning, and without any tests on the subject to guide him:

"The considerations adduced seem to me to show clearly that there really exist *two elastic limits* for any material, between which the displacements or deflexions, or what may in general be termed the changes of form, must be confined, if we wish to avoid giving the material a set, or, in the case of variable strains, if we wish to avoid giving it a continuous succession of sets which would gradually bring about its destruction; that these two elastic limits are usually situated one on the one side and the other on the opposite side of the position which the material assumes when subject to no external strain, though they may be both on the same side of this position of relaxation; and that they may therefore with propriety be called the *superior* and the *inferior limit* of the change of form of the material for the particular arrangement which has been given to its particles; that these two limits are not fixed for any given material, but that, if the change of form be continued beyond either limit, two new limits will, by means of an alteration in the arrangement of the particles of the material, be given to it in place of those which it previously possessed."

Lord Kelvin thought so highly of his brother's paper that he incorporated it in its entirety in his own notable discussion on "Elasticity".*

Ewing, Bairstow, Staunton, Rosenhain, Beilby, Howard, and other eminent physicists and physical metallurgists (none of whom is mentioned by Mr. Turner) having the advantage of the previous investigations of Maxwell, Kelvin, Tomlinson, Wöhler, and Bauschinger (all of whom are mentioned by Mr. Turner), advanced the science of physical metallurgy considerably beyond the point reached by the earlier investigators. Some of Mr. Turner's statements regarding fatigue and the effects of strain are far from orthodox. Experiments by the foregoing eminent investigators will refute his views; some of their results, already quoted by the writer in a previous paper,† prove their contentions.

Mr. Turner states "that rupture of ductile metal under Wöhler's treatment takes place without plastic elongation". In cases of rupture under alternate compression and tension in bending, like that described previously by Ewing, rupture occurs with very little plastic deformation, but where a bar is ruptured

* In Encyclopædia Britannica, 9th Edition, Vol. VII, pp. 798–800.

† From writings of Bairstow, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1512; of Rosenhain, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1520. See, also, explanation of Ewing, Encyclopædia Britannica, 11th Edition, Vol. XXV, p. 1019.

by cycles of stress in tension there often is considerable plastic deformation. For instance, in Mr. Bairstow's valuable paper* on the range of the elastic limits, "experiments on an axle steel with a yield point of 24.9 tons per sq. in. showed that in repeated loadings from 0 to 23.2 tons per sq. in. no sign of want of elasticity occurred until after 6 000 loadings when a permanent extension in the order of a yield took place."

The writer wishes to go on record emphatically against Mr. Turner's statement that "these working stresses and proper allowances need be no lower for a range of stress from tension to compression than for one kind only." Intensity of stress that could be repeated indefinitely in tension without causing either rupture or undue permanent set is likely to cause rupture if it occurs frequently in alternate tension and compression. Moreover, alternate stresses are especially severe on end connections. The writer has seen pinholes that have been enlarged ¼ in. in members subjected to alternate stresses. The matter of alternate stresses should be considered in connection with possible increase in the live loads that produce them.

Mr. Turner is correct in his contention that columns are liable to twisting. The writer has seen a column that was perceptibly twisted by reason of its load; but for centrally loaded columns Mr. Turner attaches too much importance to this consideration. Columns should be designed with forms and proportions of cross-section that offer considerable resistance to torsion; but beyond this point tests do not seem to indicate that superior resistance to torsion appreciably adds to the resistance to direct compression for equal ratios of length to radius of gyration. The writer is pleased that "form of section" is included as a sub-division of the subject of "steel column research" outlined in the Progress Report of the Special Committee on this subject.†

The last paragraph of Mr. Turner's discussion, the one in which he discusses faults in the theory of flexure, contains the amazing statement that the writer appears to suppose that the disregard of shear strain increases the maximum fiber stress at points of greatest deflection. In dealing with faults in the theory of flexure the writer did not mention or refer directly or indirectly or by implication to a point or points of greatest deflection.

The supposition, imagined to be the writer's, appeared to Mr. Turner and he combatted it with ideas regarding constrained and continuous beams; and when he had disposed of it to his satisfaction, he inquired, "Is not much of the mathematical discussion of the paper based on assumptions which are unrelated to the facts?" The writer's analysis was confined to beams simply supported and uniformly loaded either over the entire length or over any shorter distance symmetrical with respect to the center line; and for such beams both the maximum fiber stress and the maximum deflection occur at the center and by correct theory both are greater than indicated by the ordinary theory.

No attempt was made in the paper to analyze the true theory of flexure of restrained or continuous beams. The reactions indicated by the true theory

* *Philosophical Transactions*, Royal Soc., Vol. 210, p. 35; see, also, discussion by Mr. T. E. Stanton, *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), p. 1482.

† *Transactions*, Am. Soc. C. E., Vol. 89 (1926), see Contents.

would differ somewhat from the reactions indicated by the ordinary theory and in some cases of restrained and continuous beams this would tend to reduce the bending moment.

Mr. von Fabrice discusses "Beams" in a paragraph in which he states: "It is difficult to understand why the old and approved theory of flexure should be revised for an empirical formula afflicted with 50% factor of error."

The ordinary theory of flexure obviously neglects the influence of shear on cross-sections. An analysis of beams and girders simply supported was made to ascertain whether, in consequence of this neglect, the ordinary theory was in any such cases seriously in error and, if so, in what cases.

Believing such an analysis and its results would be of interest to engineers it was published. The method of analysis was to determine approximately the percentage, P_e, which theoretically should be added to the intensity of the stress, as computed from the ordinary theory, to make it correct. Equations (20), (21) and (22) give approximately the value of P_e, for all well-designed beams and girders except very short ones.

The writer's attitude regarding P_e, as applied to very short beams, is stated in the paper. That a theory has long been approved and has stood the test of long experience is a good and strong argument for continuing to use it; but, as regards concentrated loads and short beams, the ordinary theory has not been approved, and as regards these points, it is theory supplemented by good practice which has stood the test of experience. In this regard Mr. von Fabrice has himself provided the answer to his criticism in his discussion of "Shear."

With respect to that part of Mr. von Fabrice's discussion which has to do with his Fig. 5 (*a*), (*b*), and (*c*), the writer refers those interested (and all engineers should be interested) to a discussion* of a paper entitled "Effects of Straining Steel and Wrought Iron." This discussion contains descriptions of some instructive microscopic examinations of strained iron. It was shown that plastic elongation took place almost entirely by successive diagonal slippages over one another, to right and to left, of the layers within the crystalline grains, of which the metal is composed, without appreciably altering the inclination of the grains. There is no reason why such slippages should not similarly take place in plastic shortening under compression, and there is good reason to suppose that they do similarly take place; in other words, the physical phenomenon of plastic deformation under compression is in all probability similar to that under tension, except that each grain becomes shorter and wider under compression instead of becoming longer and narrower, as under tension.

Professor Moore, after pointing out that Johnson's apparent elastic limit involves an increase in rate of strain over the initial value amounting to 50 rather than the 100% of the U. L. P., suggests that, "a more extended statement from Mr. Prichard as to the advantages of the U. L. P. over Johnson's elastic limit would be of value." Mr. Woodruff inquires, "What is the 'utilizable capacity' of a structural member?" and he adds, "according to the author's definition, it would be permissible to stress a member to its 'utilizable

* *Transactions*, Am. Soc. C. E., Vol. LXXX (1916), pp. 1503–1522, especially p. 1522.

capacity'." As a basis for discussing these points, suppose that a steel works which rolls shapes and plates and is known as "the Mill", retains a consulting engineer to design a sample column, of a given length, the purpose being to build a considerable number for testing and, if the tests are satisfactory, to build a large number, all like the sample, for a customer.

The Mill desires that when the columns are loaded with (*a*) a gradually increasing load of twice the working load; or (*b*) an oft-repeated load from zero to twice the working load; or (*c*) a long-continued static load of twice the working load; it will not (1) fail; or (2) acquire an undue permanent set; or (3) be seriously injured; or (4) deflect unduly. The Mill also desires that the columns shall have as little excess section as practical.

The consulting engineer is free to select all the material required from which to make specimen tests, and he is afforded every opportunity for making the specimen tests in any way he chooses. Doubtless he would make a number of specimen tests and a few tentative designs before finally deciding on the design from which the test columns will be built, and he would doubtless adopt or originate a method of procedure.

If in testing the full-sized columns, twice the working loads are applied under Conditions (*a*), (*b*), and (*c*) with the eccentricities to which they are liable in service and if they stand the tests as regards all the Requirements (1), (2), (3), and (4); and if they are again tested under slightly increased loads and fail to meet one or more of the Requirements (1), (2), (3), and (4), the consulting engineer will have accomplished his purpose, and the Mill will supply its customer with columns that, as nearly as can be foreseen, have a factor of safety of two.

The attention of engineers in general, and Mr. Woodruff in particular, is called to the fact that a factor of safety is necessary (although it may not be the factor of two used for illustrative purposes in the assumed case) for the reasons that in spite of the best intentions, the actual loading may exceed the assumed working loading, and in spite of the consulting engineer's precautions, his determinations may be deficient.

The writer's reasons for preferring the U. L. P. to Johnson's apparent elastic limit, in determining utilizable capacity, are that he considers that the additional load permitted by the U. L. P. (as qualified in the paper) will not cause failure or serious injury and that the additional permanent set permitted is not undue.

In general, the consulting engineer's most difficult problem will be to determine the eccentricity of application of the loads (restraint of the ends being considered in terms of eccentric application), as it is a problem which, ordinarily, does not admit of close mathematical determination. Such being the case, it does seem to be somewhat irrational, as Mr. von Fabrice points out, to specify precise formulas for the influence of length on utilizable capacity or on permissible loading; but every engineer who writes specifications for the design of structures gives a formula or formulas for columns, and if he distinguishes between steels with different characteristic U. L. P.'s he should make the formulas consistent one with another. In a paper like the writer's, how-

ever, attention should be called to the fact (as it has been) that in determining, as nearly as may be, the utilizable capacity, such formulas should be used with judgment.

The writer considers that Table 12, used with judgment, should be quite useful in determining the utilizable capacities of columns, and he hopes that engineers will find it useful in devising formulas for alloy and other steels with high U. L. P.'s (assuming that they will for a time continue the present practice of specifying the same formula for all steel of the same grade and chemical composition); but he realizes the justice of Mr. von Fabrice's criticism that "Table 12 cannot be readily understood"; hence he has supplemented the table with a diagram, Fig. 6, which he trusts will make it easier to understand.

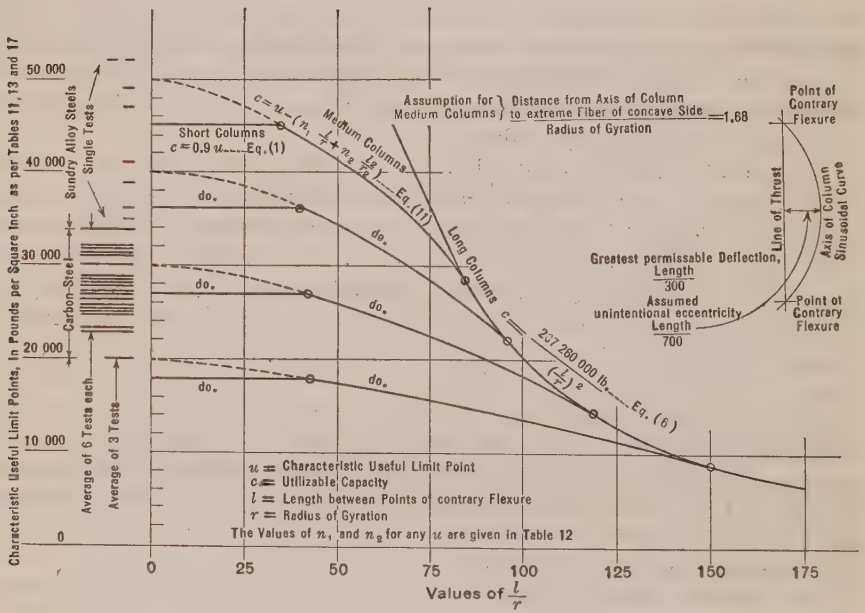


FIG. 6.—THE RELATIONSHIP OF LENGTH OF COLUMN, CHARACTERISTIC USEFUL LIMIT POINT, AND UTILIZABLE CAPACITY.

After carefully considering Mr. Schmitt's discussion it seems best to explain some matters in and regarding the paper. The first four parts were originally written as a monograph on the subject, "Perfecting of Elasticity of Steel Bodies by Modification of Auto-Stresses, and the Development, by Repeated Cycles of Stress, of Fields of Seemingly Perfect Elasticity of Chosen Range and Position". It was the original intention to present the monograph to the Society as a contribution to pure science without regard to its application to engineering practice. Its incorporation as a part of a paper on "Utilizable Capacity of Steel Members of Structures" was an afterthought. Tables 1, 2, 3, 4, and 5 therein give a collection of the results of experiments by others, and show for various cases the limits within which elasticity has been perfected. The explanation of the perfecting of elasticity by modification

of auto-stresses is the writer's adjustment of the ideas, previously referred to, of James Thomson to the discoveries of Beilby, Ewing, Rosenhain, and others as to the nature of plastic deformation of metals.

Fig. 1 illustrates the various ranges of seemingly perfect elasticity which can be developed in different specimens from the same piece of steel by repeatedly straining them in direct tension or compression through extreme ranges of stress with different upper and lower elastic limits. If the ranges were any greater and the upper limits any higher the specimens would eventually break. The terms and expressions used in the first four parts of the paper were chosen with reference to close reasoning and definite meaning.

Among the experiments of Bauschinger, as given by Professor Unwin,* there were some, numbered 3, 6, 9, 11, 12, and 13 on Fig. 3, which indicate a strange anomaly in the diagram in the way of a decided terminal drop after seemingly perfect elasticity had been indicated through millions of repetitions. This is not in accord with the general indications of the other tests in the tables nor with the writer's explanation of the perfecting of elasticity. He considers it highly probable that there were special circumstances which caused this anomaly but he could not find information to confirm this view, so he included the tests in the paper, with qualifications as described therein. He was in hopes that some reader could contribute information regarding them. To distinguish the superficial indications of this anomaly from permanent fatigue, as previously defined, he used the term "remote fatigue".

After it was decided to incorporate the first four parts in a paper on utilizable capacity, the lines termed, "Critical Deformation Point in Stress", were added to Fig. 1. This critical deformation point occurs somewhere in the zone of the yield which extends from the point, where the strain ceases to be closely proportional to the stress, to the yield point, when there is one. Sometimes the zone of yield is of zero length. Sometimes the yield shown in pulling tests is deceptive. In an extreme case (Test 7934, Table 10) soft steel with a maximum tensile strength of 48 240 lb. per sq. in. seemed, in a careful pulling test, to be perfectly elastic up to 40 000 lb. per sq. in. and had a U. L. P. of 40 600 lb. per sq. in., or 84.2% of the tensile strength; and then the load dropped to 30 000 lb. per sq. in. Valuable information on this subject was obtained in one of Bairstow's experiments on axle steel, with a yield point of 49 800 lb. per sq. in., which in repeated loadings from 0 to 46 400 lb. per sq. in., showed no lack of elasticity until after 6 000 loadings, when a permanent extension of the order of the yield took place. The first yield point observed was not a true criterion as the yield point under repeated stress was 3 800 lb. per sq. in. lower.

Some engineers consider the limit to approximate proportionality to be the critical deformation point, others consider the yield point as the critical point in deformation, whereas the Society's former Special Committee on Steel Columns and Struts adopted the U. L. P. (the point where the plastic deformation just equals the elastic deformation) as critical. For specimens used as criteria for full-sized members the writer adopted 87% of the U. L. P., or 60% of the tensile strength, whichever is the smaller.

* "Testing of Materials of Construction."

After Wöhler's experiments were published Launhardt and others developed formulas to indicate the loci of the upper elastic limit, under cycles of stress, beyond which failure would take place. In the past many engineers used these formulas as criteria on which to base working stresses, and some continue to use Professor Cain's modified Launhardt's formula. The writer for many years has consistently opposed such a practice and has advocated the use of the critical deformation point as the criterion, and this paper is in harmony with his record in this regard, but, in the light of experience, he now places the position of the point somewhat lower than formerly. If the critical deformation point is chosen somewhat too high a member under repeated one-directional stress from zero to the point chosen may fail from permanent fatigue, while stress from a minimum tension or compression of appreciable intensity, to the point chosen as critical, may cause undue deformation. By adopting his method of selecting the critical deformation point, the writer considers it likely that he has escaped both these possibilities.

In the tests recorded in Table 2, it did not seem to make much difference whether the speed was 2 cycles per min., or 800. Above 800 the range and upper elastic limit were reduced. Regarding repetitions of stress to capacity under the conditions of actual service, it is conjectural whether the range and upper elastic limit would differ from those recorded, but unless the range would be less, it would not affect the criteria advocated in the paper for members subjected to compression only or tension only.

For alternate stresses attention is invited to the well-known specification:

"Members subject to alternate tension and compression shall be proportioned for both kinds of stress separately considered, but each kind shall be considered as increased by x per cent. of the lesser stress."

Suppose that, for purposes of analysis, the factor of safety is discarded; and that unit stresses for the various kinds of steel and iron termed Lots 1, 2, and 3 in Table 1 are chosen as follows: For charcoal iron, 32 500 lb. (it cannot be the 40 000 lb. indicated in the table, as that is greater than the yield point, which is 32 500 lb.); for unannealed axle steel, 48 000 lb.; and for unannealed Bessemer steel, 66 500 lb.; and that a different value of x is chosen for each kind of metal, as indicated in Table 24; and suppose further that the bars are proportioned for total stresses just equal to the stresses per square inch shown in Table 1; the tension per square inch plus the compression per square inch, which is the "range", will then be in each case up to, or nearly up to, the greatest range which can be indefinitely repeated without causing failure or exceeding the limits within which elasticity can be perfected as shown in Table 24. It should be noted that the analysis of the experiments recorded in Table 1, indicates, for every case of alternate stresses, (a) that an addition has to be made to the greater stress before the proper cross-sectional can be determined; (b) that the percentage of the lesser stress to be added varies in some way with the tensile strength; and (c) that, with the percentages of increase determined by experiments, the "well known specification" seems to be well supported by the experiments. The writer does not know who originated the "well known specification" or how he derived it, but it is shown in Table

24 that there are tests to support it. It is generally recognized that reversed stresses are severe on end connections of members. It seems reasonable to infer that they are also severe on the connections between the crystalline layers of the grains of the steel.

TABLE 24.

Lot.	x, in percentage.	Tensile strength, in pounds per square inch.	Yield point, in pounds per square inch.	Specified unit stress allowing no factor of safety, in pounds per square inch.	Total Stresses.		Tension plus x% of compression, in pounds.	Area required, in square inches.	Tension plus compression, in pounds per square inch.	Greatest possible range, tension plus compression, from Table 1, in pounds per square inch.
					Tension, in pounds.	Compression, in pounds.				
1	38.8	43.900	32.500	32.500	22.000	20.600	30.000	1.00	42.600	42.600
2	64.0	85.000	55.800	48.000	30.000	28.000	47.900	1.00	58.000	58.000
2	64.0	85.000	55.800	48.000	39.000	15.000	48.600	1.01	53.400	54.000
3	79.2	99.900	71.700	66.500	38.200	35.700	66.500	1.00	73.900	73.900
3	79.2	99.900	71.700	66.500	58.000	11.000	66.700	1.00	73.900	73.900
3	79.2	99.900	71.700	66.500	65.000	2.000	66.600	1.00	67.000	67.000

The writer is aware of the tendency to provide no additional cross-section to tension and compression members on account of reversed stress. When many engineers very properly discarded Launhardt's and similar formulas, some of them also discarded (as the writer considers, without good reason) the "well known specification" cited.

The writer does not know of any modern endurance investigation that explains away the tests by direct stresses recorded in Table 1. Most if not all modern endurance tests are by bending, for which stress strain diagrams are very different from those of tests by direct stresses. Bending tests are so affected by auto-stresses developed in testing that inferences from them as to the behavior of members under direct stresses carry little weight as compared with tests by direct stresses. Such of the pertinent bending tests as the writer has analyzed, however, tend to support the conclusions reached, notably the extensive series of fine tests given in Table 5.

The writer would emphasize the fact that utilizable capacity can not be gauged from one "point in stress". The U. L. P. is sometimes so high that it can not be regarded as critical and some percentage of tensile strength should be substituted; the liability of failure from permanent fatigue is critical as regards members subjected to direct alternate stresses, but the liability to undue deformation is the consideration which limits the capacity of members subjected to stresses in only one direction. In some cases the degree of stiffness limits capacity. Details are supposed to be stronger than the member as a whole, but eternal vigilance is advisable in this regard. Some allowance is made for inevitable defects in workmanship, but good practice and intelligent inspection are highly important. Of course, long endurance is dependent on protection of structural members from corrosion.

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Paper No. 1593

THE HEXAGONAL SLAB DESIGN OF CONCRETE PAVEMENT*

By Lewis A. Perry,† Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. E. G. Hooper, S. Herbert Hare, Jacob Feld, H. O. Root, Wesley Vandercook, and Lewis A. Perry.

Synopsis

This paper records the results of experiments conducted by the writer to learn what relation the shape of a pavement slab has on its strength.

Although these experiments were made in a laboratory with miniature specimens, the condition of moment, stress, and manner of loading was such that the results may be assumed to be representative of the behavior of pavement slabs in actual use. The uniform behavior of the test specimens lends weight to this assumption.

With the record of these experiments, a study of moments, as developed in various shaped slabs, is reported. This study is confirmed by the results of the experiments and presents some pertinent facts apparently hitherto unrecognized or considered unimportant.

The paper also discusses current practice in the light of information gained from these studies. Pavements of correct slab shape, as determined from these studies, are illustrated and described. Finally, the writer has attempted to define the limitations and essential features of an economic design.

Corner Weakness of Four-Sided Slabs.—A general failure of a concrete pavement is invariably a progressive destruction following corner fractures developed under flexural stress. Such corners may be right-angled corners

* Published in November, 1925, *Proceedings*.

† Constr. Engr., The Longview Co., Longview, Wash.

at the intersection of joints, or joint with edge, or may be corners formed at the intersection of temperature cracks in an over-sized slab.

Proof that the 90° corner is susceptible to fracture under loading that is safe for the mid-portion of the slab is found in a study of Fig. 1 (*a*), (*b*), and (*c*), which shows a stress analysis of a slab, 16 ft. square and 6 in. thick. The section, *Y-Y*, is arbitrarily fixed at about 1 ft. less than the breadth of the section. The depth of 6 in. has also been arbitrarily chosen. The section modulus then established defines the moment, using 300 lb. per sq. in. as the allowable fiber stress. Knowing L and the moment, the loads, W and W', are found, which, of course, are much less than the actual safe loads because the sub-grade offers substantial resistance to flexure. The diagram, however, is still proportional and representative. The computed sections at x for each foot along L result in the curved figure the area of which outside the square is shaded.

The equation defining the moment at any section is:

$$M = W x \left(\frac{1}{2} - \frac{2 x^2}{31^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

As the section modulus is $\frac{b d^2}{6}$, and $d = 6$, the section, $A B$, is defined by Equation (2):

$$A B = \frac{M}{6 f} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$= W x \frac{\left(\frac{1}{2} - \frac{2 x^2}{31^2}\right)}{6 \times 300} = 10\,000 \frac{6\left(\frac{1}{2} - \frac{2 x^2}{31^2}\right)}{6 \times 300}$$

$$= \frac{(60\,000)\,(0.4532)}{1\,800} = 15.1 \text{ ft.}$$

Thus, it is determined that 15.1 ft. of section is needed at AB and the right-angled corner is only 12 ft. at this point, or only 80% efficient.

In his paper entitled "Highway Research in Illinois",* Clifford Older, M. Am. Soc. C. E., emphasized this weakness by stating that corner failures could be prevented by proportioning the thickness of the edge to the thickness at mid-slab as $\sqrt{\frac{3\,W}{S}}$ is to $\sqrt{\frac{W}{S}}$. When W is 8 000 lb. and S is 300 lb., the respective thicknesses should be 8.9 in. and 5.2 in.

Tests of Laboratory Specimens.—In order to determine the relative strength of small specimen slabs, a simple apparatus shown in Fig. 2 was constructed. Square and hexagonal-shaped laboratory specimens, ¾ in. thick, 144 in. in area, and of 1 : 3 sand mortar were made and tested after having been cured 7 days and 5 months, respectively, in open air.

As shown in Fig. 2, the specimens were tested on a bed of dry fine sand, the loads being applied at two points, each 1 in. inside the actual corner, and on

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1180.

a line joining opposite corners. The loads were applied in units of 5 lb. until the specimens were fractured, the time of each test to failure occupying about 10 min.

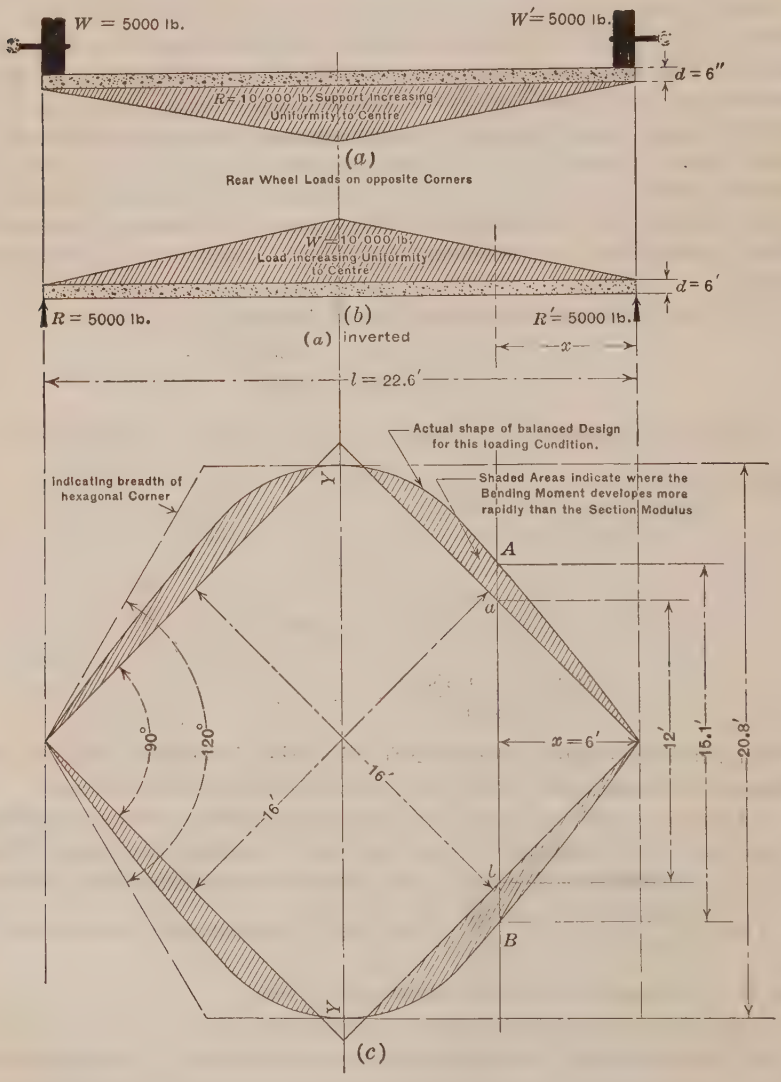

FIG. 1.

Table 1 records the results of the tests on specimens that had been aged 7 days, of secondary tests on the fragments of the 7-day specimens, and the loadings necessary to produce primary fractures on the 5-month specimens.

The broken specimens and the relative breaking loads under primary and secondary loading are shown in Figs. 3 and 4. The compound fracture of

Hexagon No. 4 (Fig. 3) was caused by splinters from broken specimens that had become hidden in the sand bed. The compound fractures of Hexagons Nos. 6 and 9 (Fig. 4) were caused by the falling of the bar after primary fracture had occurred. It will be noted from Fig. 3 that all square specimens fractured at the corners and that the hexagonal specimens under primary loading (see Table 1) broke into figures of uniform shape and size.

Present Methods of Compensating Corner Weakness.—The practice of using a center joint in highway and street pavements is logical provision against the development of longitudinal cracks. Such joints, or cracks, form right-angled corners at their intersection with transverse joints, or cracks. To preclude fracture of these right-angled corners without the use of shear bars, or other such compensation, the thickness at the edge must be greater than that required at the mid-portion of the slab. This means that a uniform thickness is either insufficient at the corners or extravagant at mid-slab. The additional concrete required to deepen the edge along the center joint is an important item of cost.

The shear-bar connection is probably the most widely used provision against corner breakage along the center joint, but it is far from ideal for several reasons. Neither shear bars that are placed at their closest effective spacing, nor the so-called tongue-and-groove type of joint, can entirely compensate corner weakness because diagonal shear of concrete is the limiting factor.

Under adverse conditions, such as unstable subgrades and frost heaving, where the slabs vary seriously in coincidence, spawling of the concrete destroys the bond of the shear bar. The bar is then rendered ineffective, and the slab has suffered a reduction in section at what may be a critical point.

In time, corrosion at the exposed part, and the crystallization caused by vibration, must disqualify the shear bar as an agency of support, and shorten the life of the pavement, which otherwise would be limited only by wear. Corrosion increases the volume of steel about 90 per cent. This increase in volume produces pressure, and a shear bar in a relatively thin pavement may be expected to spawl off a part of the slab, thereby weakening it.

The cost of shear bars, or metal tongue-and-groove joints, although not the governing consideration, is nevertheless important.

Structural Advantages of Hexagonal Slabs.—In view of the foregoing it would seem that an arrangement of plain concrete slabs, the interior corners of which were of sufficient breadth to preclude corner breakage without costly compensation, would be a logical design. This may be accomplished by arranging a plurality of hexagonal or fractional hexagonal slabs, the interior corners of which are substantially 120°, the 90° corners occurring along the outer edge being compensated by additional thickness of concrete. Such an arrangement is illustrated in Figs. 5 and 6.

Fig. 5 shows part of a pavement, of which each slab is a true hexagon, 314 sq. ft. in area and 6 in. in thickness. This pavement is in use continuously for heavy traffic and storage and has shown no evidence of failure. Fig. 6

shows an adaption of hexagonal slabs for street pavement. As shown by Figs. 3 and 4, the corner was the pronounced place of weakness in the four-sided laboratory specimens, which is likewise true of four-sided roadway slabs.

TABLE 1.—Test Data on Square and Hexagonal Shapes of Equal Area and Thickness—Loading, in Pounds, to Produce Failure.

(Both the square and the hexagonal slabs were constructed of 1 : 3 sand mortar, ¾ in. thick and 144 sq. in. in area.)

	Specimen Numbers.											Total load, in pounds.	Number of specimens.	Average load, in pounds.	Comparative percentage of loads.
	1	2	3	4	5	6	7	8	9	10	11				
Primary Loading, Specimens Seven Days Old.															
Hexagonal slabs.....	672	496	464	687	560	626	560	530	590	590	530	6 305	11	573.1	145
Square slabs.....	316	300	464	432	368	464	432	400	432	438	400	4 346	11	395.0	100
Secondary Loading, Specimens Seven Days Old.															
Hexagonal slabs.....	1 526	1 086	896	.....	1 180	496	898	962	.....	1 086	1 086	9 216	9	1 024.0	187
Square slabs.....	242	590	530	656	338	464	496	464	432	960	866	6 038	11	549.0	100
Primary Loading, Specimens Five Months Old.															
Hexagonal slabs.....	1 330	1 690	1 600	1 660	1 840	1 870	1 660	1 630	1 660	1 480	1 870	18 296	11	1 663.0	167
Square slabs.....	1 090	1 030	670	1 090	1 210	970	940	970	.....	.....	.....	7 970	8	994.0	100

A study of Figs. 7 and 8 will reveal the marked structural advantages of the hexagonal slab. It will be noted that the areas of these two slabs are practically equal, yet the greatest diagonal length of the hexagon is only nine-tenths that of the square. This reduces the bending moments to a gratifying extent. It is also found that the hexagonal corner develops 1.732 times the section width of the square corner, as shown by comparing the line, *a-b*, in Fig. 7, with the line, *A-B*, in Fig. 8. It will be noted that the perimeter of the hexagon is 4 ft. less than that of the square. In localities where extremes of temperature occur and prepared expansion joint material is used in generous thickness, this saving may amount to as much as 3 cents per sq. yd.

An important virtue of the hexagonal slab pavement is that the greater breadth of corner accumulates the greatest possible area of support near the

FIG. 2.—APPARATUS FOR TESTING LABORATORY SPECIMENS. NOTE SAND BED.

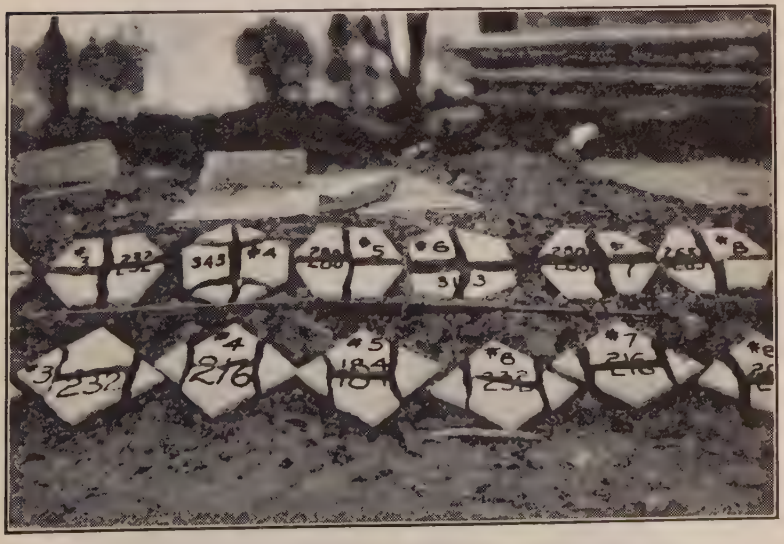

FIG. 3.—GROUP OF TEST SPECIMENS AFTER SECONDARY LOADING. HEXAGON SPECIMENS BROKEN INTO PIECES OF UNIFORM SHAPE AND SIZE.

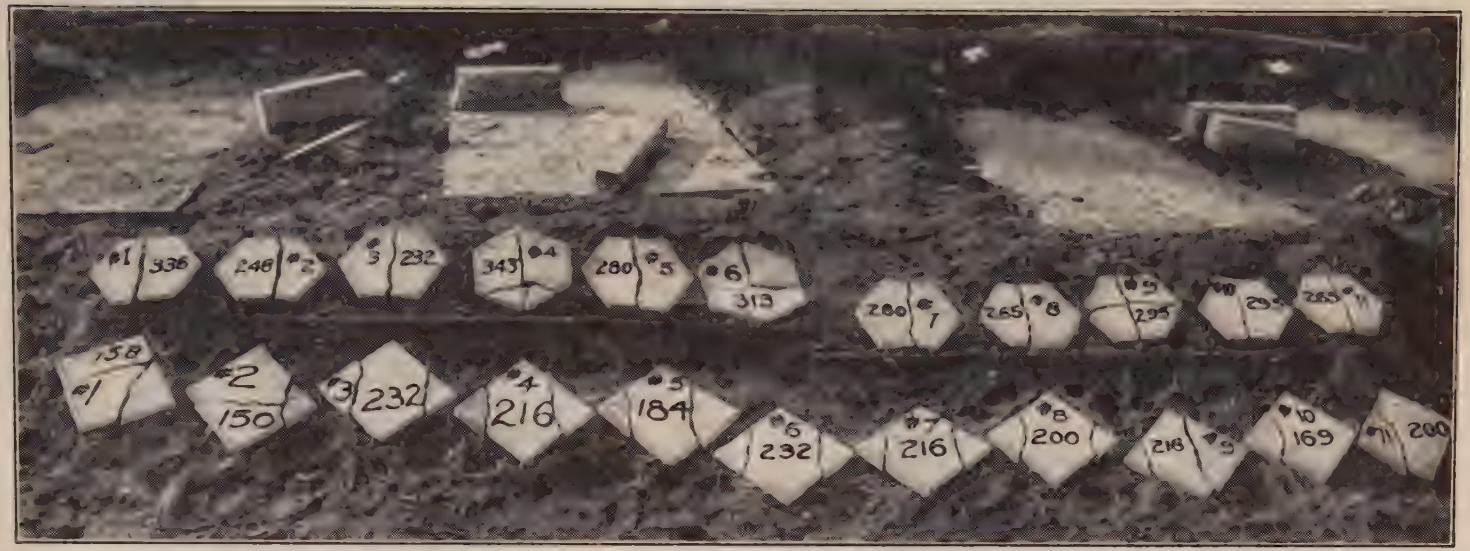

Fig. 4.—View Showing Actual Breaks of Test Specimens and Relative Breaking Loads Under Primary Loading. Note That All Square Specimens Developed Corner Fractures.

FIG. 5.—VIEW OF PAVEMENT OF HEXAGONAL SLAB DESIGN CONNECTING LONG-BELL WEST FIR MILL WITH DOCK "A". UNDER CONTINUAL HEAVY TRAFFIC AND STORAGE.

FIG. 6.—VIEW OF FINISHED STREET BEFORE TRIMMING OF EXPANSION JOINTS.

point of application of the load. In Fig. 9, which is a panel design of a highway or narrow street pavement, the hexagon or fractional hexagon most closely

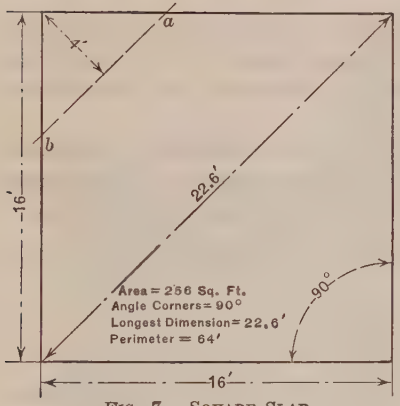

FIG. 7.—SQUARE SLAB.

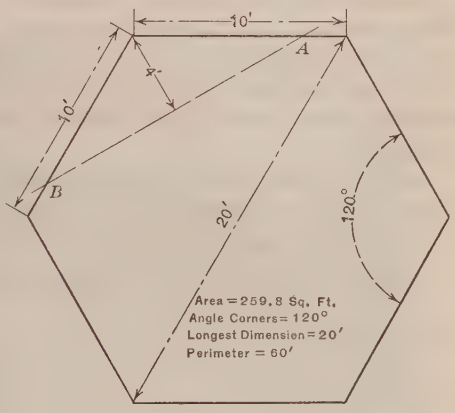

FIG. 8.—HEXAGONAL SLAB.

approaches the circle, or semi-circle, of any figure that can be grouped with similar figures. The circle is the most efficient slab shape by reason of the absence of projecting corners. All sections through it are maximum sections. The hexagonal slab, therefore, offers the maximum resistance to fracture under flexure.

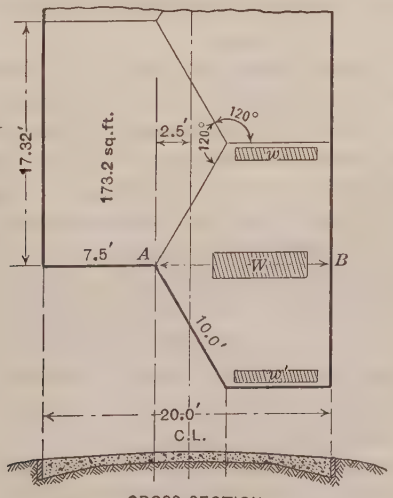

FIG. 9.—PANEL DESIGN OF HIGHWAY OR NARROW STREET PAVEMENT.

Of importance, second only to corner protection, is the virtue of the fractional hexagonal slab, as illustrated in Fig. 9, in which the maximum transverse bending occurs on the line, *A-B*. This is true regardless of whether the loading is at *w*, *w'*, or *W*. Assuming the width of the pavement to be 20 ft. and

the staggered joints at the center to be 10 ft., the distance, *A-B*, then becomes 12.5 ft., or 25% greater than the width of a slab with a straight center joint.

Some idea of the greater strength of the hexagonal slab may be obtained from a study of Table 1, and Figs. 3 and 4.

It will be seen from Fig. 1 (*c*) that although the 90° corner provides only 12 ft. of section along the line, *a-b*, and the actual requirement is 15.1 ft., the hexagonal corner would have a breadth of 20.8 ft., or nearly 38% greater than required.

As the section modulus of any member is a correct measure of its strength, the economic possibilities of the hexagonal design are shown by comparing a square slab corner, 7 in. thick, with a hexagonal slab corner, 6 in. thick.

Referring to Fig. 1 (*c*); when x is 3 ft., the section modulus, $\left(\frac{b\,d^2}{6}\right)$, at this section becomes,

$$\frac{(72)\ (49)}{6} = 588$$

Compare this section with that of a hexagonal corner 6 in. thick, the section modulus of which is:

$$\frac{(124.56)\ (36)}{6} = 747$$

it is seen that the hexagonal slab 6 in. thick has a strength 27% greater at that point than the square slab 7 in. thick.

Another fact deserving mention is that the quality of the concrete itself is likely to be better in the hexagonal corner than in the square corner owing to its better placement in a wider space.

Adaptability of Hexagonal Design to Concrete Pavement.—Before the construction of the work illustrated, the writer discussed with contractors and fellow engineers, a pavement design involving the principle of the hexagonal, or fractional hexagonal, slab. The majority of critics commended the theory of this design, but doubted its practical application with reasonable labor costs. Rarely did one meet with the opinion that the additional cost of constructing such a design could be neglected, yet this is true.

As applied to street and highway pavement, the methods of construction and use of tools employed in the laying of the hexagonal slab design are typical of construction of any design incorporating a plurality of units of uniform area. The fact that the center joint is staggered rather than straight is immaterial to the workman. The total length of transverse and center joints is less than it would be with a straight center joint.

All joint strips were left flush with, or beneath, the surface of the pavement to allow unrestricted use of the rod, roller, limber float, and hand floats. The division boards supporting the joint strips were pulled immediately after the rodding. The test straight-edge and all finishing tools were used continuously over the unbroken surface of the concrete, regardless of the location of slab joints, as shown in Fig. 10, which is a view of hexagonal slab pavement being placed at a street intersection. The joints are embedded ½ in., which allows unrestricted use of finishing tools. In this manner the continuity of true contour and the coincidence of adjacent slabs was assured.

FIG. 10.—HEXAGONAL DESIGN IN STREET INTERSECTION. JOINTS EMBEDDED ½ INCH, ALLOWING UNRESTRICTED USE OF FINISHING TOOLS.

FIG. 11.—EDGING SLABS AFTER JOINT MATERIAL HAS BEEN PULLED.

Before final edging and marking (see Fig. 11), and while the concrete was still plastic, the joint strips were pulled with flat-billed tongs. This pull was perpendicular and was intended to raise the top of the joint strip $\frac{1}{4}$ to $\frac{1}{2}$ in. above the surface of the pavement.

The construction of the pavement illustrated in Fig. 5 required a somewhat different procedure. A false form system was used for alternate tiers of slabs, the filler tiers being rodded and finished from the existing concrete and with no forms other than division boards. In both these types of improvement it was found that familiarity of workmen with the particular design was not as important as good organization by the contractor. It was also found that the skilled workman will adapt himself to any logical design and go the designer "one better" in developing practical methods of construction.

The writer has designed and supervised the construction of about 376 000 sq. yd. of hexagonal slab pavement during 1924 and 1925 in Longview, Wash. The question of excessive labor costs due to the irregularity of design is answered by the fact that better progress has been consistently made with this work than with the square panel design constructed during 1923. This comparison was made with equal depths of pavement, and it is worthy of note that the finish and riding quality of the hexagonal slab pavement is, if different, superior to that of the old design.

Credit is due H. O. Root, Principal Assistant, for valuable assistance in design and field supervision of the work illustrated in this paper. Henry G. Niblett, President of Olympic Construction Company, who directed the construction of this work, is deserving of compliment for his co-operation in successfully adapting the design to practice.

Conclusions.—It is incorrect to compensate a pronounced weakness in design, when that weakness may be eliminated.

The right-angled corner is a pronounced weakness and has no rightful place in a concrete pavement, except where it must occur along the outer edge.

The cost to compensate such weak corners successfully is not justified in the light of a more economic design.

Harmful temperature cracks which intersect and form corners in an oversized slab may be avoided by a reasonable reduction of the slab area.

The necessity of forming reasonably small slabs to avoid these cracks may be met without the formation of interior right-angled corners.

The writer believes that the economical design is a grouping of slab units of plain concrete, each unit sized so as to prohibit temperature cracks and excessive bending moments, and entirely independent of, and disconnected with, adjacent slabs, which prevents injury to any one slab by reason of the displacement or failure of its neighbor, and, further, that each slab be cast in the ideal shape—the hexagon.

In this grouping the corner ceases to be a weakness and the general design is logical and economical, with a uniform thickness, except where right-angled corners occur of necessity. This reasoning is presented as the writer's interpretation of simple, natural laws.

DISCUSSION

E. G. HOOPER,* ASSOC. M. AM. SOC. C. E. (by letter).—The problem of pavement design has pre-empted the thought of many practicing and research engineers ever since the automobile became such an important economic factor. Many novel ideas have been developed and presented as the solution of that problem, but as yet none has proved more than partly satisfactory in meeting demands under widely different and varying conditions. A full appreciation of the limitations of the several more generally adopted plans, together with a thorough knowledge of the loads, structural material, and soil conditions in a given locality, makes it feasible to select a reasonably satisfactory type. The outstanding factor militating against a satisfactory general design is a widely varying undependable supporting sub-grade. Until some means is found to reduce materially the number of different critical conditions that can develop under a pavement no satisfactory structural study can be made, as that requires an established maximum loading, a knowledge of the behavior of structural material and type of structure, and a certainty of permanency in the nature and positions of the supporting reactions. The only thing the writer can imagine that will obviate this fundamental difficulty is to provide supports that will carry the reactions to a soil level below that subject to disturbance. A design has been evolved along this line that is practical, the cost of which is not materially above that of present pavements. This discussion, however, will be confined to points of agreement and disagreement in Mr. Perry's paper.

The writer's comment, elaborated subsequently, may be summed up briefly, as follows: It is agreed that (1) corners of pavement slabs are the seats of greatest weakness; (2) corners of hexagonal slabs are stronger than those of rectangular slabs; and (3) improperly planned doweling across joints may produce serious spalling. On the other hand, it is objected that (1) increase in the number of corners by use of hexagonal slabs may offset to a degree the advantage of individual corner strength; (2) the position of corners with reference to traffic lines in the hexagonal slab arrangement proposed subjects these weak places to unusual punishment; (3) the regular zigzag line or joint making an angle of 30° with the center line of the road is a possible source of confusion to drivers; (4) temperature curling at corners that are coming continuously and suddenly under load is likely to aggravate conditions beyond normal; (5) absence of shear connections with adjoining units permits at times relative movement, or displacement of a permanent nature, at joints so that impact, otherwise negligible, may become a formidable factor; and (6) on soil which very little water can reach and from which that little is quickly drawn, as in built-up and sewered communities, the weaknesses of any of the slab designs would not develop to any great degree.

As Mr. Perry states in his excellent presentation the common rectangular slab construction develops its greatest weakness at the corners. The importance of strengthening these corners is definitely recognized, more particu-

* Asst. Prof., Civ. Eng., New York Univ., New York, N. Y.

larly since the publication of results of, and conclusions from, the "Bates" and the "Pittsburg" road tests. Various means have been adopted for making, directly or indirectly, such points as strong as the remainder of the unit. Mr. Perry approves of thickening the edges as he has incorporated that idea where he, of necessity, has right angles; but he objects to the use of dowels or tongue-and-groove joints for transferring some of the load to adjoining members.

Objection to dowels may be partly justified. An insufficient number and size of bar together with an open joint will inevitably produce spalling which may eventually nullify the expected benefits from the steel. However, as such spalling is due to excessive bending of bars across an open joint, or is caused by closure of the joint by temperature, where a dowel is bent or out of alignment it may be corrected by use of bars of proper size and number, to make the bending negligible and the face-bearing area adequate. The writer believes expansion from corrosion is negligible because air and moisture must have access to the embedded metal; even the sliding ends of bars are usually wound with an adequate protective covering.

Tongue-and-groove joints are effective in transferring shear or load from one unit to another and the percentage thus transmitted is a function of the relative deformation of the adjoining units. Exact calculations cannot be made because of the ever-present bugaboo—variable and indeterminate soil resistance—which renders it impossible to predict slab deformation. Numerous service tests have given favorable impressions of carefully designed and made joints of this kind, and until other acceptable tests prove otherwise they may logically be accepted as satisfactory.

It is readily acknowledged that full hexagonal slabs are stronger at the corners than rectangular ones, nevertheless those corners remain a weakness. The use of half hexagons for pavements, as recommended by Mr. Perry, of necessity, retain two of the taboo right angles at critical points. In place of two other right angles along the center line, he substitutes three 120° angles, all three of which are situated so that the left-hand wheels of every vehicle traveling the right-hand side of the road will traverse the ones on that side, and in such a manner as to apply a force comparable with a critical load in case of heavy trucks. Vehicles traveling down the center will likewise traverse the corners, so that the weakest points of the pavement are suffering punishment all the time, a condition not duplicated in the usual rectangular construction. It is conceivable, therefore, that, in spite of the apparently greater strength, the hexagonal slab would break down under as light or lighter traffic than the rectangular slab.

The ordinary "geology-topography-climate" combination for any particular locality produces such a great variation in the supporting value for the same soil that a number of critical design conditions develop. Unless all these critical conditions have been reproduced or approximated for the given case no set of tests can be said to prove the fitness of a design, even for the one case, much less for general use. Mr. Perry's tests show the comparative strength of

hexagons and rectangles with the conditions of load and support shown and described in his paper. As pointed out before, in practice few loads will pass over rectangle corners as compared with those of hexagons in the plan proposed. The test results, therefore, may not be taken as fairly indicative of relative service values.

Absence of ties between the pavement units makes a collection of free slabs, which, at the worst, where soil conditions are very unsatisfactory, is comparable with a field of floating ice cakes. As already stated the best sub-grades serve equally well for any of the better slab types; it is the poorer sub-grades that require the special treatment so many engineers are trying to develop. The action of a soft soil under pressure is to an extent comparable with water. To develop the illustration, if we step on one corner of a floating ice cake that point will go down and the opposite side will rise; in other words, it will tip. The amount of tip, the amount of depression of the corner, is greater for the smaller cakes. If we step from one cake to the next, the one tends to come back as that next cake tips. There is a difference in level between adjoining units during transit of loads, a condition that would produce impact with a rolling load. In the case of pavement units similar vertical movement is bound to occur, its magnitude depending on the stability of the particular sub-grade, the size of the slab, and the number of load repetitions. Although such movement may be insignificant, repetition may develop permanent difference in level. Moreover, the impact developed as a horizontal component against an elevated edge will most probably work the slabs apart laterally even if reasonably good shoulders are maintained. For these reasons the writer considers independent action of slabs highly undesirable.

Another point of minor importance is the comparative amount of joint filler for the two types of pavement. For a given area the perimeter or joint length of a hexagon is the less, and the same may be said of the half hexagon. Were the usual transverse joint spacings the same as those required in the "hex" design, it would be correct to claim a saving in joint material. As a matter of fact, however, there are enough fewer such joints in the usual rectangular design to eliminate any such saving. This is another instance where comparisons under different conditions modify or change conclusions.

An advantage of the small units, offsetting to a degree the disadvantage, is greater freedom from cracks developed in the early stages of setting when volume changes occur due to moisture variation and temperature. As such small cracks are fruitful sources or beginnings from which develop the large disruptive breaks, it is obviously very desirable to reduce them to a minimum.

In discussing at such length the practicability of hexagonal slab design for highway pavements, the writer has had in mind that he should make his viewpoint clear. Although many of the premises, or conclusions, or both, may not be approved, he believes that a consideration of them all will be helpful in the final analysis.

S. Herbert Hare,* Esq. (by letter).—The writer has had an opportunity to watch the development of this type of slab under Mr. Perry's supervision in

* Kansas City, Mo.

street paving at Longview, Wash., and has been much interested in the results. His interest, however, is not primarily in the strength, economy, or efficiency of this type of slab, which points are so ably discussed in the paper and which are so vital to the engineer, but in the appearance of the paving as laid on the streets. Any longitudinal joint, whether continuous or broken, appears very conspicuous in the perspective view as one drives down the street. The hexagonal joints, having no sides parallel to the curb, soon lose themselves from view. In most cases they cannot be seen farther than 150 to 200 ft. away. Beyond that the pavement appears as one unbroken slab.

Although often secondary from the point of view of the engineer, the esthetic effect of any paving is quite important to those who use the street, especially when such paving is used on boulevards and parked roads, as in Longview.

Jacob Feld,* Assoc. M. Am. Soc. C. E. (by letter).—The author plainly states his thesis and proves it. The right-angled corner is a pronounced weakness in a pavement. Both experimentally and theoretically, he shows why right angles should find no rightful place in a pavement design. Then, as a solution to the problem, he recommends the use of hexagonal units, noting as special advantages the decreased danger from temperature changes and the elimination of the right-angled corners. However, as is plainly seen in Fig. 6, the proposed type does not eliminate the right-angled corners along the outer edges of the pavement. It is along these edges that the greatest stresses occur and to allow for these, Mr. Perry recommends greater thickness of the slab along the edges—the same remedy as is used in present-day construction.

There is no special advantage in having the intersections of the joints with the edges staggered; nor is the elimination of the square joints from the center line of great importance. The advantage gained from the use of smaller units can be just as easily obtained by increasing the number of expansion joints in the square arrangement. The elimination of the straight center joint takes away the common traffic line, separating the streams of travel. There is no doubt that hexagonal slabs are of special value in paving large areas, as shown in Fig. 5, and should be used in such cases, especially since their cost is no more than that of square slabs.

A useful application of the hexagonal idea for soil bearing is for column footings. A hexagonal footing contains the same volume of concrete as an equivalent square footing, with somewhat less steel reinforcement and a smaller area of forms. The cost is practically the same. Hexagonal footings are used only where space limitations govern, or for extremely large loads as, for example, high stacks.

The author has brought out a novel idea and has proven its advantages; but the writer cannot see where its use can be justified in the ordinary type of pavement.

H. O. Root,† Esq. (by letter).—The theory of the hexagonal design of concrete pavement is simple and has been ably handled in this original paper. There may be, however, some question in the minds of many engineers as to

* Cons. Engr., New York, N. Y.

† Los Angeles, Calif.

its practicability. The writer, having had two years of field experience in laying the hexagonal design, feels that a detailed description of field methods would be a fitting contribution to this discussion.

Narrow Streets and Highways.—When the street or highway is of such width that it can be screeded successfully from edge to edge and the design is a single staggered center joint, the placing of the interior joints is very simple.

The start is a matter of special plan in each case, and, if made from an existing straight pavement edge, will result in one smaller and one larger slab, the end of the first length of center joint being brought to a point off center the required distance at the end of the first transverse joint.

Assume, for illustration, that the pavement is to be 24 ft. wide and that the design calls for a single staggered center joint of 10-ft. lengths. These will form 120° angles with intersections 2.5 ft. off center, and the transverse joints from these points to the edge will be 9.5 ft. in length.

The contractor provides metal "saws", or strips, in both the 9.5 and 10-ft. lengths. One end of a 10-ft. saw is secured to the first fixed angle point established, its free end is brought to an intersection with a 9.5-ft. saw and the latter adjusted to be at right angles with the edge. The location of each 10-ft. center length is, therefore, automatically established by bringing it to meet the end of the next 9.5-ft. transverse joint. The transverse joints can be kept square with the edge by eye and should fall at intervals of 17.32 ft. on each edge.

Intersections of streets must be carefully planned in advance, and it may be necessary to shorten or lengthen two or three panels at the end of a block so as to join in correctly with the intersection plan. Thin boards are used instead of metal saws where odd lengths are required. When panel plans of both block and intersection are supplied to the field engineer and the inspector, there need be no delay to the mixer while constructing irregular layouts.

The fact that the intersection of the center joint with the transverse joint falls off center gives the joint setter more freedom of action, for he is not required to work directly under the boom of the mixer, and the to-and-fro movement of the bucket does not interrupt him, nor does his work in any way interfere with the rapid deposit of concrete by the mixer operator.

Experience proved that it was more efficient to move the mixer and pull up the power sub-grade drag only far enough to allow one length of center joint and one transverse joint to be set at a time. These short moves can be made without loss of mixing time and tend to simplify the organization of the work on the sub-grade and joints between the mixer and the line of deposit.

Actual experience covering two seasons' work and a yardage of more than 400 000 seems to warrant this assertion, that the hexagonal design actually makes for higher efficiency and, therefore, for better progress and lower costs than is usually attained with the standard rectangular design.

Wide Streets.—When the improvement is of such width that it cannot be laid completely at one trip, it is necessary to set one straight form outside the

line of hexagon points to be laid in the first operation. The sub-grade may then be brought to correct crown and grade by a template drag riding on this false form and the curb, or edge, form.

The contractor provides a supply of 3-in. form lumber, of a width equal to the thickness of the pavement. These pieces are cut so that the ends have a bevel of 60°, the two bevel cuts being parallel, and the face length equal to the designed length of the joints. The straight false form is marked off in panel-length intervals and the extreme ends of two beveled pieces are brought to these marks; their adjacent ends are then brought together on the grade and secured by staking. The setting of these forms in pairs to the marks on the false form is a simple operation and increases only slightly the whole cost of form-setting.

The concrete is then struck off, tamped, and finished, all from the false form; the beveled forms are merely trued up to the rod as it passes over them. Longitudinal expansion joint material is placed with the first strip, flush with the form, and secured to the concrete by small nails driven through flat tin flashers, adequately spaced and located near the top of the pavement. This holds the material firmly against the concrete and prevents loosening and flopping after the forms are removed.

A long straight-edge should be used on the projecting points of concrete during the finishing process. It is important that these areas be reasonably true as to surface contour, for they must fit the crown of the tamping rod as it idles over them during the laying of the adjacent strip.

The beveled forms may be removed and used elsewhere the following day, but the false form should be left in place. The remainder of the sub-grade, or such part of it as will be required for the next run, may be brought to its true crown by means of the template drag sliding on this form. The open triangles between the form and the old concrete must be smoothed up by hand. After the sub-grade has been struck off and before the concrete is placed, this false form should be removed and all tamping and finishing done from the hardened surface of the old concrete.

Here, again, it is important that the long straight-edge be used carefully where the triangles of new concrete mesh in with the old. A series of humps or hollows, even if slight, occurring as they do at regular intervals, will seriously affect the riding qualities of the pavement. As the difference in cost between a smooth finish and a rough one is probably less than 1 cent per sq. yd., it is the writer's opinion that the public would gladly pay this difference and secure the better riding quality.

Wide Areas.—Alternate tiers of hexagons may be laid between false forms as described previously, the filler tiers being screeded and finished to the surface of the old tiers, with no forms other than division boards.

General.—Complete separation of the slabs from one another is important. Careful edging and complete separation will prevent spalling and the subsequent raveling at corners and along joints. The longitudinal joint material

may be run continuous through the three-way intersection and all splices made to occur at least 2 ft. from the angle. The concrete deposited in the angle of the continuous joint tends to keep it butted firmly against the end of the transverse joint. It is possible that some metal clip device could be used to hold the end of the transverse joint in true position.

Where manholes occurred on or near the line of joints, the joints were broken and turned to intersect them squarely. An extra thickness of concrete was put in to compensate for any section reduction caused by manholes, inlets, or other features.

An instrument crew can be kept steadily employed with each paving mixer, staking and checking forms, laying out intersections, and checking crowns on both sub-grade and finished concrete. Close co-ordination in this respect is reflected in the progress, quality, appearance, and cost of the finished work.

Conclusions.—The laying of hexagonal slabs is not as difficult as appears at first glance. The radical change in design is not confusing to workmen and even ceases to be a novelty after the first few panels have been laid. The design permits of a highly efficient organization of the labor, which, in turn, results in rapid progress and low costs for the contractor. The item of increased cost soon disappears under the pressure of competitive bidding.

The methods here detailed will no doubt be improved by other engineers and contractors when the use of the hexagonal design becomes more general. It is with this thought in mind that the writer presents this discussion, more as a basis for further refinement in practice than as a set of hard and fast rules.

Wesley Vandercook,* Esq. (by letter).—The fact that such widely different views are held concerning the essentials of concrete pavement design should promote research of the character reported in Mr. Perry's paper. Even if no definite refinement of design resulted from this work, the subsequent investigations on a larger scale (which, unfortunately, are not reported in the paper) and the better acquaintance with the causes and character of typical failures in pavement slabs have completely justified the effort made.

It seems necessary to the intelligent discussion of the paper to note that the designer of this type of pavement is among those who believe in segregating a pavement into certain definite slab units and eliminating cracking by meeting Nature "half way". To this school of thought the paper should be of interest, if not of value. There still are, however, adherents to the custom of constructing pavement in slabs of full width and in excessive or indefinite lengths, who rely on Nature to use her best judgment about cutting them up into smaller units.

Serious consideration of the paper pre-supposes that the former group of designers is thinking rightly, which leads to the question of slab sizes. Since the structural effect is of prime importance, probably the soundest argument for moderate slab sizes is the fact that the expansion of over-sized slabs, with insufficient expansion provision, sets up compressive stresses which express

* Chf. Engr., The Long-Bell Lumber Co., Longview, Wash.

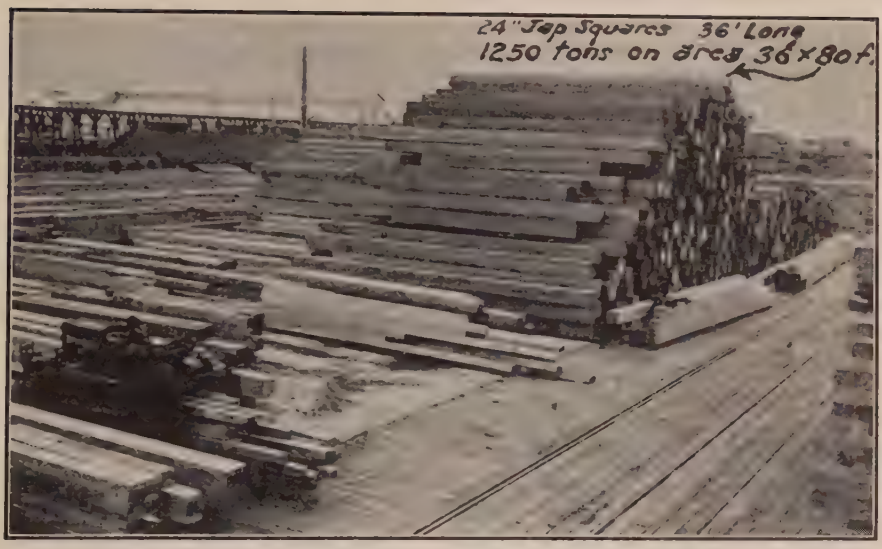

Fig. 12.—View of Heavy Storage of Lumber on Hexagonal Slab Pavement, Long-Bell Lumber Company, Longview, Wash.

Fig. 13.—View of Loading and Traffic on Hexagonal Slab Pavement. Same Construction as Shown on Author's Fig. 5 and in Fig. 12.

themselves in transverse flexure. This becomes harmful when sympathetic transverse loading is applied and probably accounts for much pavement breakage. Random transverse cracks often follow local lines of least resistance and frequently terminate at the edge with an acute corner, which suffers under cantilever loading even more heavily than the right-angled corner. In the case of city streets, of course, this is not as serious as on highways, where the traffic is concentrated along the edge.

Probably the best argument in favor of the relatively long slab is the apparent saving in first cost of expansion joint material. The writer, however, questions whether within a few years, this saving, and more, has not been lost in the "hot pot" by the maintenance crew. Certainly the appearance of pavements, especially of city streets, is highly important, even if secondary to utility, and, in the writer's mind, cracks in a pavement, whether raw and raveled or treated with asphalt, are decidedly unsightly. It is unfortunate that the logical asphalt treatment serves to accentuate these flaws.

The writer, therefore, endorses the practice of segregating pavements into slabs, sized as nearly as may be to the ideal unit. Probably the size will vary with conditions, in fact, all the conditions having a bearing on the correct slab size may not be known as yet. Some of these conditions are range of temperature, character of subsoil, width of roadway, position of loading concentrations (if defined), and the correlated consideration of slab shape, which the paper discusses.

Of these conditions perhaps the most important is temperature. A survey of existing pavements in Longview, Wash., indicates that the maximum slab area may be fixed by the formula:

$$A = d\,y$$

in which,

A = the area of the slab, in square feet;
d = the depth of the pavement, in inches; and

$$y = 100 - \left(\frac{\text{temperature range, in degrees Fahrenheit}}{2}\right)$$

For a local temperature range of 100° this would set the maximum size of a 7-in. pavement slab at 350 sq. ft., and for a range of 140° at 210 sq. ft. Subgrade conditions in any given locality, of course, enter into this determination. The foregoing values seem logical for the local conditions, and this rule should be followed only when the slab is compact in shape, preferably hexagonal. In Longview the minimum slab area is fixed by a minimum weight of 7 000 lb. It is not to be expected that these empirical rules will hold good for any or all localities, but they may suggest a logical measure of safe dimensioning whereby slabs may be intelligently designed rather than merely jointed according to some custom that may have been established under dissimilar conditions.

Professor Hooper probably would not have considered his objections to this type applicable or important had he had an opportunity to watch closely its development, and the apparently highly successful performance of the resulting improvement. His comment seems to be directed chiefly at this design as

applied to narrow street or highway pavements, as shown in Fig. 6, and has apparently missed the importance of the increased transverse section at mid-slab. Professor Hooper does not comment on the advantages of this principle for a street three, four, or five panels in width. Certainly, the wider the street the greater will be the number of corners and the greater the value of the design.

Fig. 5 was taken immediately after construction; Figs. 12 and 13 show the normal heavy traffic and storage on this pavement. It is 6 in. thick and was built on a hydraulic sand fill of from 6 to 35 ft. in depth. The slabs are slightly in excess of the allowable area as determined by the formula, being 314 sq. ft. Since this improvement shows no signs of structural defect after twenty months of this usage, it is considered reasonable proof of the fitness of the design. The writer doubts whether any other type of pavement at the same, or nearly the same, cost would have proven as satisfactory.

The author compares the 6-in. hexagonal corner with the 7-in. rectangular corner, which no doubt is correct reasoning. The writer, however, cannot agree with the implication that it is good practice to minimize the thickness of pavement of hexagonal design in the interest of first cost economy. It is thought that, except in rare cases where maximum loading is fixed for all time, it is better practice to maintain depths heretofore considered standard and enjoy the increased value of the improvement by reason of the design, rather than to hold to capacities of existing standards at reduced cost. In this connection it may be remembered that the structural value of the slab itself increases with the square of the depth, whereas the cost does not quite vary directly with the depth.

The hexagonal design has passed the experimental stage, has proved to have structural merit peculiar to its shape, and is of considerable economic value. The 120° corner has been found approximately 50% stronger than the 90° corner under cantilever loading on sub-grades. It is doubtful whether this value can be ignored by engineers. While it is questionable whether this type will find immediate acceptance by designers, because conservatism in the engineer is a virtue, the writer ventures the prediction that this principle will find favor when it has been investigated in theory and practice as thoroughly as it has been by Mr. Perry.

Lewis A. Perry,* Assoc. M. Am. Soc. C. E. (by letter).—A review of the discussion indicates that the writer has wrongly made the paper too brief, assuming the reader's understanding of facts and theories implied, rather than presenting them.

Fig. 14 will serve to illustrate the reference to the added breadth at mid-slab of the fractional hexagonal slab, shown in Fig. 6. It is surprising that this point has been missed or considered unimportant by some members. Slab (*a*), 10 by 20 ft., is typical of the design of the excellent pavements built by the Highway Department of the State of Washington; Slab (*b*) is identical with those shown in Fig. 6. In the design of the theoretical slab, indicated

* Constr. Engr., The Longview Co., Longview, Wash.

by the upper diagrams of Fig. 14, end concentrated loading was assumed, the slabs then being inverted and computed as simple beams. In both cases the sub-grade is assumed to be uniform, and resistance to flexure is not considered. There is also a fallacy of compensating bearing values at the ends, but as these conditions are common to both slabs, and as the loading and depths are equal, the resulting wide difference in the failure areas is a true indication of the superior value of Slab (*b*). This value, of course, will be realized first when "curling" occurs, and contact with the sub-grade at the ends of the slab is lost. The fact that the distance between the loads is not identical has no bearing as the distance between concentrations varies in actual practice. It should be borne in mind that these loads are not actually maximum safe loads, but are theoretical values used to compute moments. The actual safe load of Slab (*b*) is probably four times the load used here. In these computations plain concrete is assumed to be stressed to 300 lb. per sq. in. To simplify computations deepened outer edges are omitted, although in practice both designs include this feature; also, shear bar connections along the inner edge of Slab (*a*) are not considered because they do not serve the companion slabs when both are similarly loaded, and, further, because the effective life of shear bars, under these conditions, is limited.

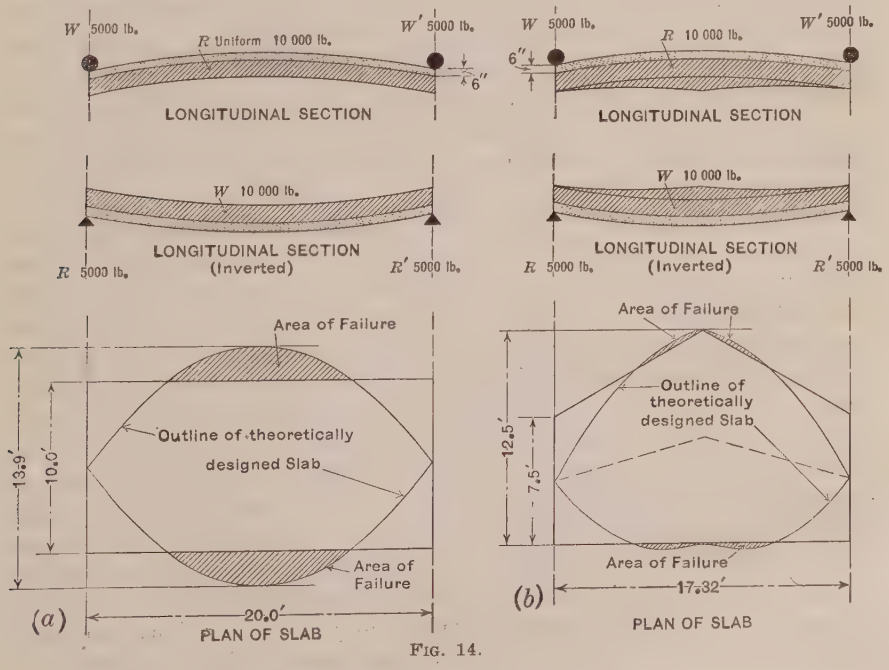

FIG. 14.

It should be noted that the dimensions of Slab (*b*) might be varied to benefit the slab structure. It is a good rule, within reasonable limits, to make the length of the diagonal center joint 1 ft. less than one-half the roadway width. In this case these joints would become 9 ft. instead of 10 ft. The writer has constructed some 20-ft. pavement by each method but as practi-

cally no cracking or breakage has occurred in either it is not possible to state which will show the better behavior. The shorter slab, with "dummy joints," is preferred by the writer.

The lengths of joint filler in these two designs are:

Slab (*a*).....0.900 lin. ft. per sq. yd.
Slab (*b*).....0.909 lin. ft. per sq. yd.

Thus, Professor Hooper's objection concerning the quantities of joint filler is correct only to this extent.

The question of the advisability of a grouping of free slabs has been discussed at considerable length by Professor Hooper, and the writer values his opinion, except that in his illustration buoyancy of water has been used to illustrate sub-grade fluidity or elasticity. This comparison does not seem fitting.

The writer believes that something more satisfactory than either the steel bar tie, or absolute slab freedom, exists in the "dummy joint" developed by Messrs. W. H. Tiedeman and H. F. Faulkner, Assistant City Engineer and Engineer of Tests of the City of Seattle, Wash., respectively. This "dummy," which is a narrow strip of joint filler, is inserted, with the aid of a T-bar template, into the plastic concrete after finishing, and penetrates about one-third of the depth of the slab; subsequent cracking is then controlled and does not show on the surface, and there is a frictional connection between slabs that the writer considers very valuable. Complete joints are placed at regular intervals to provide for expansion. This type of joint may be finished easily and is excellent in appearance; the writer is now using it in connection with the hexagonal design.

Professor Hooper has evidently meant by his reference to the "geology-topography-climate combination," that, in his opinion, the tests reported do not define the fitness of the slab design, because these tests were not correlated with sub-grade studies. The writer cannot think of anything that would have detracted more seriously from the value of the paper than to have confused slab structure with soil mechanics. The latter subject is a comprehensive study in itself, which is being investigated by some of the ablest engineers. It is to be hoped that they will arrive at something more definite and scientific than reliance on human judgment. The writer is, as yet, unversed in the science of soil mechanics, and the assumption that this ignorance condones illogical slab sizes and shapes would be evidence of unfitness for the tasks before the engineer. The questions of slab design and soil mechanics are equally important. A good slab will fail on a poor sub-grade; a poor slab will fail on a good sub-grade; and a properly designed slab has a much better chance, on any sub-grade, than one of illogical shape, depth, and size.

Professor Hooper believes:

"It is conceivable, therefore, that, in spite of the apparently greater strength, the hexagonal slab would break down under as light or lighter traffic than the rectangular slab."

Except in the rare cases where street widths, and other conditions defining traffic lanes, are fixed for all time, load concentrations must be considered to be generally distributed as to transverse location; and that even if traffic is temporarily localized ("traffic" is generally understood to mean standard motor vehicles up to 12 tons, which, however frequent, should have no effect on a well-designed and well-built concrete pavement*), the great majority of initial failures are caused by the movement of occasional excessive loads, such as heavy construction equipment. These initial failures then progressively expand and become increasingly evident under standard motorized traffic so that the layman and sometimes the engineer fails to fix the responsibility on the right kind of "traffic". The point is that such occasional excessive loads do not necessarily follow, but, in fact, frequently avoid, traffic lanes, and if the improvement is designed with assumptions regarding locality of traffic—even admitting that a Utopia exists where the American motorist even temporarily drives where he is wanted, and the power shovel goes somewhere else, which is to be desired—the designer's plans have seriously miscarried.

The writer has in mind an instance that may be illustrative. In July, 1924, a 12-ton, gasoline-propelled road roller was moved three city blocks along 21st Avenue, Longview, Wash. This pavement was 24 ft. wide and 6 in. thick, having straight longitudinal joints, the slabs being 12 by 20-ft. (rectangular) and the concrete about 12 months old. Cars were parked next to the curbs in places, which forced the movement to the center line and brought concentrations on slab corners. Thirty-six corner fractures developed, which under subsequent standard traffic suffered to the extent that repairs were necessary. Later, the same load was moved over an adjacent fractional hexagonal slab pavement, also 24 ft. wide, but only 5 in. thick, and about 3 months old. The same conditions of loading occurred and the subgrades on the two streets were identical In the latter case no fractures of any character have resulted, either from this loading or the subsequent standard traffic. Since that time many such instances, permitting comparisons of the two types, have occurred, which fix in mind three facts, namely: First, the occasional excessive loads that amount to probably only 1% of the traffic tonnage and 0.1% of the traffic count cannot be relied on to follow fixed traffic lanes; second, that such loads do probably 95% of the damage, which in most cases is initiated by simple corner fractures; and, third, that the substitution of the 120° corner for the common 90° corner materially reduces, if not practically eliminates, this breakage.

* The writer has determined to his own satisfaction, from experiment, that a 6-in. hexagonal pavement slab will fail at about 12 tons cantilever loading on each corner, or 24 tons total restrained load.

Conditions bearing upon this determination are:

Sub-Grade.—Uniform and sandy and not less than 1 500 lb. per sq. ft. bearing value, without deflection.

Concrete.—5 U. S. gal. per sack of cement (about 4 400 lb. per sq. in. compression).

Curing.—10 days under water, 18 days drying in air. Average daily temperature 60° Fahr.

Size of Slab.—16 ft. across opposite corners.

By stressing concrete one-third its ultimate strength this type of improvement is eminently fitted for 8 000-lb. wheel loads, but the writer would recommend heavier slabs under abusive traffic conditions and, under all conditions, careful sub-grade treatment.

It may be true that State or city laws should be enforced to prohibit abuses. On the other hand it is also true that the building of cities requires the use of certain heavy tools, which must go from one task to another, and while it is manifestly unjust to build with public funds improvements suitable for 0.1% of the traffic, it is also improper to disregard design refinements which materially increase loading capacities without additional cost.

Figs. 15 and 16 illustrate how unwise it would be for the designer to make assumptions concerning the transverse location of traffic on a wide street. The situation changes with parking restrictions, density of traffic, crown of street, and future widenings. It is correct, therefore, to assume that any weak spot between curbs will be found and punished; and to try to eliminate those weaknesses in design rather than to repair them later at considerable cost, and with sacrifice of street appearance. Fig. 15 also illustrates that, under the conditions shown, a rectangular slab pavement frequently comes under the objections raised by Professor Hooper to hexagonal slabs, namely, that the corners are generally in the path of traffic.

A pointed illustration of the fallacy of Professor Hooper's objection to the number of corners of hexagonal slabs would be to consider a circular slab (which is admittedly an ideal shape) the radius of which is, say, 8 ft. and the perimeter of which is cut into 1-ft. chords. It remains essentially a circle, with all its virtues, but with about fifty corners. If, however, a limited number of corners were desired, and the designer went to extremes, he might make slabs of equilateral triangles, having only three, instead of fifty, corners. A slab of this type, and of an area equal to the circle, would have sides about 21.6 ft. long, the corners of which develop less than 16% of the breadth of the corners of the former shape, yet this shape would use 27% more joint material. It is seen, therefore, that the number of corners and their locations are immaterial as long as each corner is structurally fit, and it is well to remember that breadth as well as depth of members fix their section moduli.

Professor Hooper has referred to the "regular zigzag line or joint" as "a possible source of confusion to drivers." It is to be assumed that this observation was made without due consideration. It has been necessary to photograph the slab layout from a great height to show the joints.

The question regarding curling at corners due to temperature raised by Professor Hooper, but left "up in the air" by him, is important. The curl is proportional to the slab length. The "aggravated condition" is roughly proportional to the curl, and inasmuch as, for the hexagonal slab, the extreme cross-corner length is about 90% that of a square slab of equal area, it follows that the hexagonal slab suffers less than 90% of the "aggravated condition" of the square slab.

The question of appearances raised by Mr. Hare is, as he states, "secondary" to that of utility, but is still important. In perspective, from the driver's eye, this design takes on a suggestion of uniform texture from curb to curb that is quite pleasing, and not possible with straight center points. On highways, and elsewhere, where lines for traffic division are desirable,

FIG. 15.—VANDERCOOK WAY, LONGVIEW, WASH., EXAMPLE OF 8-INCH RECTANGULAR SLAB PAVEMENT.

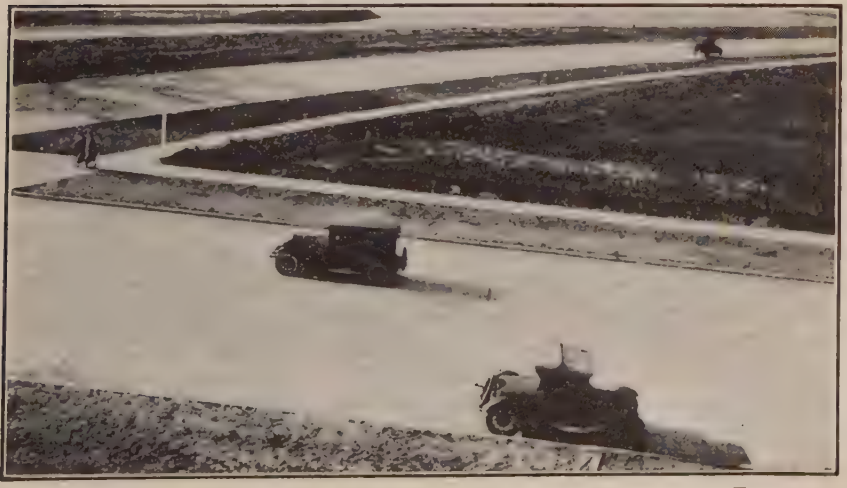

FIG. 16.—WASHINGTON WAY AND KESSLER BOULEVARD, LONGVIEW, WASH., EXAMPLE OF 6-INCH HEXAGONAL SLAB PAVEMENT.

they must be frequently re-painted for clearness, which is also true of the hexagonal design.

The writer thinks that Mr. Feld might revise his statement that "there is no special advantage in having the intersections of the joints with the edges staggered," after careful consideration of Fig. 14; and that he intended the statement, "nor is the elimination of the square joints from the center line of great importance," to apply to highways or narrow streets. Even with this qualification the writer cannot agree. It is thought that each street width presents a new application of the principles, all of which have not been considered by Mr. Feld. The writer regrets that Figs. 14, 15, and 16 were not submitted with the paper for the better information of members in discussion.

Mr. Feld has referred to the "ordinary type of pavement" meaning probably highway pavements, and his approval of the hexagonal design for "large areas" leaves the writer wondering what his judgment would be of widths "in between," such as a street pavement from 30 to 100 ft. in width. Surely all common widths are "ordinary," and many are what the writer would call "large areas."

Mr. Root's discussion has fittingly been a description of construction methods. This should be of great importance to engineers, especially since, in the construction of the work illustrated in the paper, some enviable progress records have been made, and the general consistent weekly averages have exceeded any others known to the writer.

It is thought that Mr. Root's reference to "complete separation" is meant to imply approval of moderate slab size without steel bar connection. The writer believes that the "dummy joint" will fulfill Mr. Root's requirement of complete separation. Certainly, the existence of joint filler for the top 2 in. of the slab will serve to "prevent spalling and the subsequent raveling" of the surface. It is not probable that the bottom of the controlled crack will spall; in the event this did occur, no harm will be done.

Mr. Vandercook has a geographical advantage over some other discussors, in that he has been in intimate contact with the work described, from its first consideration, and in that the same and many other objections and questionings that have been raised in the discussions, were carefully considered by Mr. Vandercook before he sanctioned the writer's extensive use of the new design.

Mr. Vandercook's reference to the inadvisability of reducing the thickness, by reason of the hexagonal design, is the expression of a valuable opinion. This is a question of economic policy, to be determined by the local conditions, such as community finances, the traffic anticipated, and the nature of the sub-grades; and it is probable that the general trend in pavement construction will be toward higher safety factors, because in few cases can it be said that loading limits are fixed, or even anticipated, "for all time."

The "subsequent investigation" referred to by Mr. Vandercook was a series of tests of square and hexagonal pavement slabs under cantilever loading of corners.* Briefly, the average results may be reported, as follows: (*a*) 5-in.

* *Engineering News-Record*, October 8, 1925.

hexagonal slabs bore 23% greater loading than 6-in. square slabs of equal area; and (*b*) 5-in. hexagonal slabs bore 48% greater loading than 5-in. square slabs of equal area. The slabs were cast *in situ* on a water-settled sand subgrade. The individual breaks were remarkably uniform.

The purpose of the paper has been to draw attention to the possibility of enhancing the value of this logical type of road surface by intelligent consideration of the slab shapes. It pleases the writer to think that, in a measure, this has been done; and to thank those engineers whose discussions have furthered this purpose.

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Paper No. 1594

STREAM POLLUTION

A SYMPOSIUM*

PAGE

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Quantitative Studies of Bacterial Pollution and Natural Purification in the Ohio and the Illinois Rivers.

By J. K. Hoskins, Esq. 1365

With Discussion by Messrs. Richard H. Gould, J. Frederick Jackson, Harrison P. Eddy, H. A. Whittaker, Kenneth Allen, William Firth Wells, and J. K. Hoskins.

* Presented at the meeting of Sanitary Engineering Division, Cincinnati, Ohio, April 23, 1925.

A REVIEW OF THE WORK OF THE UNITED STATES PUBLIC HEALTH SERVICE IN INVESTIGATIONS OF STREAM POLLUTION

By W. H. Frost,* M. D.

In March, 1901, Congress provided for the erection of a laboratory by the United States Public Health Service, "for the investigation of infectious and contagious diseases and matters pertaining to the public health," and in the same year a Division of Scientific Research was organized in the Bureau of the Public Health Service. Therefore, the year 1901 may be said to mark the establishment of systematic and continued scientific investigation as a recognized function of the Public Health Service. Considering the rôle which sewage-polluted drinking water was playing at that time in the spread of typhoid fever and other infectious diseases, and recalling that the membership of the Hygienic Laboratory Advisory Board included the great leader in sanitary science, Professor William T. Sedgwick, it was inevitable that attention should have been directed at once to the importance of comprehensive studies of stream pollution in relation to disease. That this was true is evidenced by frequently recurring references in the Annual Reports of the Director of the Hygienic Laboratory during its early years; but the number of other urgent problems was so great and the resources of the Laboratory so limited, that, for several years, work in this field was of necessity limited to occasional studies of local water supplies, undertaken usually in connection with investigations into the causes of the epidemic or endemic prevalence of typhoid fever in various localities.

In 1910, the first systematic investigation of the status and effects of sewage pollution in any large area was begun by the assignment of A. J. McLaughlin, Surgeon, U. S. Public Health Service, to make a survey of cities in the Great Lakes region, with instructions to investigate the extent of the pollution of their water supplies and its relation to the prevalence of typhoid fever and other water-borne diseases, and to examine State and municipal ordinances relating to its control. Upon the completion of these surveys and of the reports thereon, which were published as *Bulletins* of the Hygienic Laboratory, Dr. McLaughlin was assigned, by request of the health authorities of States bordering on the Missouri River, to make a survey of the sewage pollution of that stream. In this work, which was carried out during the summer of 1912, Dr. McLaughlin for the first time had the assistance of another officer of the Service and was enabled, through the co-operation of the health authorities of the States concerned and of certain cities on the river, to establish several laboratories and make a rather extensive series of bacteriological examinations.

* Surgeon, U. S. Public Health Service, in Chg. of Stream Pollution Investigations, Baltimore, Md.

By the time this work had been brought to a close, the International Joint Commission, established under the treaty between the United States and the Dominion of Canada, had taken up the question of regulating the pollution of international boundary waters; and on request of the Commission, Dr. McLaughlin was granted leave of absence from the Service to accept appointment as Chief Sanitary Expert and Director of field work in investigations undertaken by the Commission. These studies, although undertaken independently by the International Joint Commission, may, in a certain sense, be considered as an extension and continuation of the survey of Great Lakes cities previously undertaken by Dr. McLaughlin for the Public Health Service.

In the meantime, by an Act approved August 14, 1912, Congress had extended the function of the Public Health Service to include, among other added duties, that of investigating "the diseases of man and conditions influencing the propagation and spread thereof, including sanitation and sewage and the pollution, either directly or indirectly, of the navigable streams and lakes of the United States"; and in 1913 made a special appropriation, which has since been continued annually, for carrying out these provisions. The Public Health Service was thus enabled for the first time, in 1913, to establish field laboratories at such points in the United States as might be most suitable for special purposes and to employ a scientific personnel especially qualified to conduct investigations in various fields of research.

It was under this extended authority that, in the summer of 1913, a group of sanitary engineers, chemists, biologists, and bacteriologists was assembled and a beginning made on a concerted plan of investigations relative to stream pollution. As originally organized, the work undertaken comprised the following main divisions.

1.—Studies of the bio-chemistry of sewage and industrial wastes were undertaken at the Hygienic Laboratory under the direction of Mr. Earle B. Phelps, who was appointed in that year as Chief of the Division of Chemistry in the Laboratory. These studies were devoted especially to testing and developing the application of biological oxygen demand determinations to the measurement of the potential polluting effect of sewage and the capacity of streams for its oxidation, a field of research to which Mr. Phelps had already made notable contributions.

2.—Intimately connected with these was a series of studies, likewise under the direction of Mr. Phelps, but carried on for the most part at various points outside Washington, D. C., attempting, by means of experimental installations, to devise better methods for the treatment of various important industrial wastes, for which economical and effective processes had not previously been evolved.

3.—Under the direction of H. S. Cumming, Surgeon, U. S. Public Health Service, the present Surgeon-General of the Service, a study of the pollution and natural purification of the Potomac River was undertaken. The Potomac was selected as a type of tidal stream and special attention was

paid, in this study, to the effect of sewage from the City of Washington on the waters near the mouth of the river where important shellfish beds are situated. This investigation, which was completed in the summer of 1914, was then extended and continued as a survey of the sewage pollution of various coastal waters, with special reference to the contamination of shellfish.

4.—At the same time, in the summer of 1913, work was begun on a study of the pollution and natural purification of the Ohio River, which was selected as a typical large inland stream, receiving sewage, usually without treatment, from all cities on its water-shed, and, at the same time, being used by many of these cities as their source of water supply. Headquarters for this work were established in Cincinnati, Ohio, with subsidiary temporary laboratories at five other points along the river.

These several studies, although conducted by working parties organized into separate units, were closely knit together by being all under the direction of the Division of Scientific Research in the Bureau of the Public Health Service and by the intimate relations which were maintained between those in charge of the several organizations. In fact, they were considered and pursued, not as separate studies, but as interdependent parts of a common and general plan. They were all continued, substantially as originally organized in 1913, until 1917 when it was necessary to discontinue them in order to utilize their personnel in various other more urgent duties during the period of the World War.

By the latter part of 1919, when it was possible to resume the investigations, the original personnel had become much dispersed by necessary assignments to other duties and by resignations. Likewise, the funds available for these investigations, although not actually reduced to any great extent, were relatively diminished by the material increase in all scales of cost, so that in the re-organization it was necessary to discontinue the investigations of coastal waters, which had been brought to a fairly definite conclusion, and to re-establish the other work at a single base in Cincinnati, which has since served as central headquarters for experimental studies of stream pollution and as the base from which parties have been sent out for work in the field.

Shortly after this re-organization, the Surgeon General, recognizing the need for authoritative advice in the planning and conduct of these investigations, requested Dr. Stephen A. Forbes, Professor Emeritus of Biology at the University of Illinois, and Director of the Illinois State Natural History Survey; Dr. Edwin O. Jordan, Professor of Hygiene and Bacteriology at the University of Illinois; Langdon Pearse, M. Am. Soc. C. E., Sanitary Engineer of the Sanitary District of Chicago; and Earle B. Phelps, Consulting Sanitary Engineer, of New York, N. Y., to serve as consultants in studies of stream pollution. These consultants, meeting once or twice each year with the staff engaged in the investigations, and keeping in close touch with the progress made, have rendered generous and valuable assistance in shaping plans, devising methods, and interpreting results. Subsequently, Joseph W.

Ellms, M. Am. Soc. C. E., consented to serve as special consultant in studies of water purification processes and has had an active share in the development of investigations along this line.

Since 1919, the principal field investigations undertaken from this base have been:

1.—A study of the pollution and natural purification of the Illinois River, undertaken chiefly to check and extend observations previously made on the Potomac and the Ohio Rivers relative to the laws governing natural purification in streams.

2.—A survey of representative municipal sewage disposal plants in various parts of the United States, to collect information as to their efficiency and cost in actual operation.

3.—A collective study of municipal water purification plants, chiefly rapid sand filters, as operated in a number of cities on the Ohio River and elsewhere, with a special view to ascertaining more precisely the relations between pollution of the raw water and quality of the effluent under varying processes and conditions of operation.

Along with these field studies, experimental investigations have been consistently pursued in the Cincinnati Laboratory, chiefly along the following lines:

(*a*).—An attempt has been, and is being made, so far without notable success, to reproduce on a small scale, adapted for intensive experimental study, the phenomena of bacterial purification which are now known to take place in natural streams. This has included as a necessary item rather extensive research into the biology of various plankton forms in relation to bacterial purification.

(*b*).—Studies of the biological oxygen demand of sewage, industrial wastes, and polluted river waters have been continued in the endeavor to establish more definitely the laws governing the natural processes of oxidation in streams and to check and improve the precision of methods for making the determinations required.

(*c*).—As an extension of the collective study of municipal filter plants which was completed in 1924, experimental studies are now being made of the relation of the pollution of raw water to the quality of effluent obtainable by rapid sand filtration and chlorination, utilizing an experimental plant on the laboratory grounds which is designed so that the conditions of loading and of operation can be varied at will through a wide range.

In addition to these studies, which have been pursued at Cincinnati, work has been going on for several years at the Hygienic Laboratory, under the direction of Dr. William Mansfield Clark, in a study of the physical chemistry of coagulation, with special reference to applications in water purification.

It would be impossible within a brief space and is, moreover, not pertinent to this paper to relate in more detail the history of the various undertakings which have been outlined, nor will any discussion of the results be attempted. As far as they have matured, they have already been made generally available

in a considerable number of publications,* and some of them with the addition of some more recent data, have been discussed in the papers by Messrs. Theriault, Streeter, and Hoskins, which follow.

In conclusion, it will be more appropriate to review briefly the broad general considerations which have determined the scope and direction of such studies as the Public Health Service has undertaken in this field since it has been in a position to make and pursue any general plan, that is, since 1913.

The first consideration, of course, has been the limitation of available resources, which have sufficed in most years for the maintenance of a staff not exceeding six to twelve workers in the higher grades, enough to form a compact group for consistent work on definite lines, but obviously not sufficient to permit of any wide dispersion. The governing considerations in deciding on the use to be made of these resources have been: The existing status and trend of conditions with respect to sewage pollution in the waterways of this country; the status of sanitary science as applied to devising the remedial measures necessary to meet present and future conditions; and the facilities available through State and municipal organizations, independent institutions for research, and the Engineering Profession at large, for conducting such further investigations as may be required.

With respect to sewage pollution, the status in the United States was, in 1913, and is to-day, that the greater part of the sewage from cities, probably not less than 85 to 90% of it, is discharged without treatment into the most convenient stream. Where the dilution is insufficient for prompt oxidation and removal of the sewage, the result is the establishment of a gross nuisance in the immediate vicinity, offensive to the sense of decency and frequently injurious to the financial interests of the community responsible for the pollution. The remedy for this, however, is at hand, as the ingenuity of sanitary engineers, chemists, and biologists has already devised effective means for the treatment of sewage at reasonable cost, and self-interest may be relied upon to impel cities which suffer nuisance from their own sewage to avail themselves of this remedy. The abatement of such gross nuisance is usually a local matter, requiring no broad plan of concerted action between widely separated communities; and as the principles of the required treatment are already well established, such special investigation as is required is usually a matter of detail, to ascertain the particular process or combination of processes which will serve most economically and effectively in the particular case. Obviously, such investigations are the business of the State and local authorities and of the practicing engineers retained by them rather than of a Federal agency.

The more usual and more serious result, where dilution and current are sufficient to prevent immediate gross nuisance from the discharge of untreated sewage, is to contaminate the water supplies of other cities taken from the same river system at down-stream points, or, in the case of tidal waters, to dangerously contaminate waters from which shellfish are taken. In

* A list of the more important of these publications is given in the Bibliography appended.

the case of public water supplies necessarily taken from such polluted sources, the immediate remedy is artificial purification of the supply. For this, again, sanitary science has already provided the means in various processes of treatment, economically practicable and of such efficiency that they may be relied upon to give safe effluents from water which is highly but not indefinitely polluted. In 1913 there were, to be sure, a number of cities using dangerously polluted water supplies, but in every instance the remedy—installation of adequate water-purification works—was obvious, and such investigations as were required were not general, to ascertain the practicability of a remedy, but local and special, to decide upon the details of the installation best adapted to apply established principles to the problem at hand. It is clear that these local investigations, like those required for local sewage treatment installations, are not the function of the Public Health Service.

In general, the situation has been that, notwithstanding the customary practice of discharging raw sewage into streams, those cities which have had to take their water supplies from the rivers thus polluted have almost invariably been able, by applying established processes of artificial water purification, to secure water supplies of good, safe quality. This has been true because the volume of the larger rivers is such as to afford great dilution, even for the sewage of the larger cities, and because the distances between the sewer outlets of these cities and the water-supply intakes of other cities down stream are such as to permit of great reduction in pollution by the natural agencies of purification. Similarly, in coastal waters, although they are grossly polluted in the immediate vicinity of cities discharging sewage, there are still great areas sufficiently free from dangerous contamination to be suitable for shellfish culture. Consequently, local measures, namely, the installation of water-purification plants for safeguarding water supplies and the condemnation or local protection of the relatively small areas unfit for shellfish culture, have sufficed for immediate protection against the dangers of sewage pollution. The protection has not been perfect, but it has tended to become progressively better in recent years, as evidenced by the enormous decrease in prevalence of sewage-borne diseases.

Looking to the future, the conditions foreseen and the remedies which must eventually be applied, become more complex. With the growth of urban population, which still continues at a rapid rate, the sewage pollution of streams and coastal waterways must increase; and sooner or later, in the absence of anticipatory control, it seems inevitable that eventually the pollution will become such that water-purification plants of the highest attainable efficiency will not be able to deliver consistently safe effluents. To guard against this condition it will be necessary, perhaps in the near future, to limit the pollution of such inland streams as are necessary sources of water supply by such measure of sewage treatment as will suffice to keep the pollution at water-works intakes within definite bounds.

This, however, is an extraordinarily complex matter, not only from the administrative point of view, with which this presentation is not concerned, but equally from the scientific viewpoint. It implies a concerted plan of

control applied to an entire river system as a unit, a plan in which, presumably, each community will be required to limit its contribution of sewage pollution, not in the interests of its own citizens, but for the protection of other communities down stream, usually including cities in several States. Safety demands that the measure of control exercised be adequate; justice demands that it be distributed among the communities on some definite and equitable principle, and economy demands that it be not more rigid than is actually necessary to insure the requisite protection to health.

The data needed for laying out any such comprehensive plan for controlling the pollution of an entire river system, with due regard for the considerations of safety, equitable distribution of the burden of control, and economy, are as follows:

First.—It is necessary to have established some quite definite and objective criterion of the quality which is to be maintained in the water supplies taken from the river, as they are delivered to the consumers after artificial purification. This criterion or standard must be in terms of measurable characteristics, determinable by quantitative bacteriological or chemical examinations. It must be rigid enough to insure safety beyond any reasonable question, but not much more rigid than is actually necessary, lest it impose an excessive burden of costs.

Second.—It is necessary to have a fairly precise knowledge of the reliability and efficiency of such purification processes as can be applied at a reasonable cost to purification of the raw water available at the best practicable intake, for it is this efficiency, taken in connection with the standards set for the final effluent, that determines the upper limits of the pollution which may be tolerated at the intake.

Third.—It is necessary to know what proportionate part each of the sewered communities, situated at varying distances up stream, contributes to the pollution existing at any given intake, for otherwise it is impossible to estimate what effect elimination or reduction of the pollution from any single community will have in reducing the pollution in the intake zone. This, in turn, implies a fairly precise quantitative knowledge of the laws governing the processes of natural purification, and of how they may vary in different types of streams in relation to various climatic, seasonal, and hydrographic conditions, for it is only through such knowledge that these great protective processes which Nature has provided may be used most effectively; and not to use them is to waste a natural resource of enormous economic importance.

Unfortunately, sanitary science has not furnished such full and precise knowledge as will be required on any of these points, especially in regard to the natural agencies which tend so greatly and rapidly to reduce bacterial contamination and which constitute one of the main reliances for protection of health. Moreover, it seems unlikely that it will be possible to borrow this knowledge from the experience of other more densely populated countries, as the writer knows of no other country having similar problems in the control of stream pollution on a comparable scale and for a similar

purpose, that will probably have to be studied successfully before a solution becomes necessary for some of the great river systems in the United States.

It is with these considerations in view that the Public Health Service, with the advice of its consultants, has consistently directed its investigations of stream pollution along the lines described, devoting a large part of its effort to such undertakings as the attempt to improve technical methods for laboratory determinations, to evaluate the efficiency of filtration plants under the adverse conditions of loading which may be anticipated in the future, and to add something to the present scanty knowledge of the laws of natural purification. Information of this kind, even if it may seem at this time to be more or less academic, will be essential to sound sanitary engineering practice in the future. Moreover, it appears to be pre-eminently the kind of information that a Federal agency should collect, because it is of general, not local, application, and because it involves such long-continued and laborious investigations as are not likely to be undertaken by private agencies, or even by State and municipal organizations, busy as they are with more immediate administrative work, and with the necessary local studies incident to it.

However, while the Public Health Service is confident that this general policy is sound, it can not, of course, feel equally confident that the sequence which is being followed in the development of these studies is the best possible or that the methods which are being applied are always the most effective. For guidance in such matters the Service relies primarily on its special consultants, but, in addition, it always has sought and sincerely desires the criticism and constructive advice of the entire Sanitary Engineering Profession.

APPENDIX

BRIEF BIBLIOGRAPHY RELATING TO STUDIES OF STREAM POLLUTION, SEWAGE, AND WATER SUPPLIES.

The following is a list of the publications of the U. S. Public Health Service, relating to studies of stream pollution, sewage, and water supplies. The list includes only publications containing original data, omitting numerous articles which present general discussions of various topics.*

Sewage Pollution of Interstate and International Waters, with Special Reference to the Spread of Typhoid Fever: I, Lake Erie and the Niagara River.
A. J. McLaughlin. (*H. L. B. No. 77* (1912); 169 pp.)

Sewage Pollution of Interstate and International Waters, etc.: II, Lake Superior and St. Marys River; III, Lake Michigan and the Straits of Mackinac; IV, Lake Huron, St. Clair River, Lake St. Clair, and the Detroit River; V, Lake Ontario and the St. Lawrence River.
A. J. McLaughlin. (*H. L. B. No. 83* (1912); 296 pp.)

* The abbreviations used in the Bibliography are as follows: "H. L. B.", *Bulletin*, Hygienic Laboratory, U. S. Public Health Service; "P. H. B.", *Public Health Bulletin*, U. S. Public Health Service; and "P. H. R.," Weekly Public Health Reports, U. S. Public Health Service. The reprint number is given where article has been reprinted separately. All these publications are issued from the Government Printing Office, Washington, D. C.

Sewage Pollution of Interstate and International Waters, etc.: VI, The Missouri River from Sioux City to Its Mouth.
A. J. McLaughlin. (*H. L. B. No. 89* (1913); 84 pp.)

Investigation of the Pollution and Sanitary Condition of the Potomac Watershed, with Special Reference to Self-Purification and the Contamination of Shellfish in the Lower Potomac River.
Hugh S. Cumming, with Contributions by W. C. Purdy and Homer C. Ritter. (*H. L. B. No. 104* (1916); 231 pp.)

Investigation of the Pollution of Tidal Waters of Maryland and Virginia, with Special Reference to Shellfish-Bearing Areas.
Hugh S. Cumming. (*H. L. B. No. 74* (1916); 199 pp.)

Artificial Purification of Oysters.
William F. Wells. (P. H. R., July 14, 1916; Reprint No. 351; 4 pp.)

Investigation of the Pollution of Certain Tidal Waters of New Jersey, New York, and Delaware.
Hugh S. Cumming. (*P. H. B. No. 86* (1917); 147 pp.)

Stream Pollution: A Digest of Judicial Decisions and a Compilation of Legislation on the Subject.
Stanley D. Montgomery and Earle B. Phelps. (*P. H. B. No. 87* (1917); 408 pp.)

Treatment and Disposal of Creamery Wastes.
Earle B. Phelps. (P. H. R., December 6, 1918; Reprint No. 496; 5 pp.)

Studies on the Treatment and Disposal of Industrial Wastes: I, The Treatment and Disposal of Strawboard Waste, by Harry B. Hommon; **II, The Determination of Biochemical Oxygen Demand of Industrial Wastes and Sewage,** by Emery J. Theriault and Harry B. Hommon. (*P. H. B. No. 97* (1918); 56 pp.)

Studies on the Treatment and Disposal of Industrial Wastes: III, The Purification of Tannery Wastes.
Harry B. Hommon. (*P. H. B. No. 100* (1919); 133 pp.)

Studies of Methods for the Treatment and Disposal of Sewage: Treatment of Sewage from Single Houses and Small Communities.
Leslie C. Frank and C. P. Rhynus. (*P. H. B. No. 101* (1919); 117 pp.)

A Further Study of the Excess Oxygen Method for the Determination of the Biochemical Oxygen Demand of Sewage and Industrial Wastes.
Emery J. Theriault. (P. H. R., May 7, 1921; Reprint No. 594; 11 pp.)

Studies on the Treatment and Disposal of Industrial Wastes: IV, The Purification of Creamery Wastes.
Harry B. Hommon. (*P. H. B. No. 109* (1921); 87 pp.)

Studies on the Treatment and Disposal of Industrial Wastes: V, The Purification of Tomato-Canning Wastes.
Harry B. Hommon. (*P. H. B. No. 118* (1921); 58 pp.)

Hypochlorite Process of Oyster Purification (Experimental).
F. A. Carmelia. (P. H. R., April 22, 1921; Reprint No. 652; 10 pp.)

The Loading of Filter Plants.
H. W. Streeter. (P. H. R., March 24, 1922; Reprint No. 737; 13 pp.)

A Study of the Pollution and Natural Purification of the Ohio River: I, The Plankton and Related Organisms.
W. C. Purdy. (*P. H. B. No. 131* (1923); 78 pp.)

Sewage Treatment in the United States: Report on the Study of Fifteen Representative Sewage Treatment Plants.
H. H. Wagenhals, E. J. Theriault, and H. B. Hommon. (*P. H. B. No. 132* (1923); 260 pp.)

An Experimental Study of the Relation of Hydrogenion Concentrations to the Formation of Floc in Alum Solutions.
Emery J. Theriault and William Mansfield Clark. (P. H. R., February 2, 1923; Reprint No. 813; 20 pp.)

Indicators for p^H Control of Alum Dosage.
Barnett Cohen. (P. H. R., April 6, 1923; Reprint No. 828; 2 pp.)

On the Composition of the Precipitate from Partially Alkalinized Alum Solutions.
Lewis B. Miller. (P. H. R., August 31, 1923; Reprint No. 862; 10 pp.)

A Study of the Pollution and Natural Purification of the Ohio River: II, Report on Surveys and Laboratory Studies.
W. H. Frost, H. W. Streeter, J. K. Hoskins, and R. E. Tarbett. (*P. H. B. No. 148* (1924); 343 pp.)

Adsorption of Aluminium Hydrate Considered as a Solid Solution Phenomenon.
Lewis B. Miller. (P. H. R., June 20, 1924; Reprint No. 932; 14 pp.)

A Study of the Pollution and Natural Purification of the Ohio River: III, Factors Concerned in the Phenomena of Oxidation and Re-aeration.
H. W. Streeter and Earle B. Phelps. (*P. H. B. No. 146* (1925); 75 pp.)

The Determination of Dissolved Oxygen by the Winkler Method.
Emery J. Theriault. (*P. H. B. No. 151* (1925); 43 pp.)

Some Preliminary Observations from a Study of Water Purification Plants Along the Ohio River.
H. W. Streeter. (P. H. R., January 30, 1925.)

A Study of the Effects of Anions Upon the Properties of "Alum Floc".
Lewis B. Miller. (P. H. R., February 20, 1925.)

THE RATE OF DEOXYGENATION OF POLLUTED WATERS

By Emery J. Theriault,* Esq.

The biochemical oxygen demand test to be discussed in this paper, although at present it enjoys a certain measure of renewed interest, is by no means new. The earliest record of such a procedure is probably to be found in a report published in 1870 by a British Rivers Pollution Commission. In France, oxygen demand determinations were made as long ago as 1885 in a study of the pollution of the Seine. In Germany, extensive series of experiments were conducted on the test from 1900 to 1911. In the United States, a modified procedure appears to have been used in the early experiments at the Lawrence Experiment Station, although it is only since 1915 that the method now in use has been more or less generally adopted.

It is significant both of the intrinsic merit of the biochemical oxygen demand test and, it must be admitted, of the numerous difficulties which arise in its practical application, that, in a recent bibliographical review, no less than 150 references were found which dealt directly with the subject. The consensus of opinion appears to be that the test is valuable. In fact, for the purposes of stream pollution studies, it is frequently the only chemical procedure which can be used to advantage. As a measure of the relative strength of various organic wastes and as a guide in estimating the efficiency of particular methods of treatment, the test also appears to possess decided advantages over the usual chemical procedures.

General Considerations

As regards the theory underlying the test, it is a well established fact that a polluted water containing bacteria, if exposed to air, tends to become completely purified. It has been repeatedly demonstrated that definite quantities of dissolved oxygen are absorbed during this self-purification process. It follows that the quantity of oxygen required for the complete stabilization of a polluted water may be taken as a measure of its organic matter content. In the simplest case, two glass-stoppered bottles are completely filled with the sample under examination. The initial dissolved oxygen content is found by analyzing one of these sub-samples at the beginning of the test. The other sub-sample is placed in a constant temperature chamber at 20° cent. After an arbitrarily selected time, preferably five days, the sample is removed from the incubator and its oxygen content is re-determined. If bacteria and organic matter were present, a decrease in the oxygen content is invariably observed. This decrease is then reported as the 5-day oxygen demand of the sample at 20° cent.

A limitation of this test as outlined lies in the fact that the saturation value for the dissolved oxygen content of water at 20° cent. is only 9 parts

* Associate Chemist, U. S. Public Health Service, Cincinnati, Ohio.

per million, corresponding to the 5-day oxygen demand of a highly purified effluent or a highly polluted water. With sewage effluents of average quality, a 5-day oxygen demand value of about 20 parts per million may be expected. Before the test can be applied, it is necessary, therefore, to dilute such effluents with five or ten volumes of fully aerated distilled water or tap-water of good quality. For raw sewages, the 5-day oxygen demand is generally greater than 100 parts per million, so that the samples must be diluted about fifty times in order to provide a sufficient supply of oxygen throughout the course of the test. Tannery and abattoir wastes possess oxygen demand values which range from 1 000 to 10 000 parts per million. With unusual trade wastes, 5-day oxygen demand values of 50 000 parts per million have been obtained. At the other extreme, the 5-day oxygen demand of good tap-water is about 0.5 part per million.

Various other methods of procedure have been proposed for determining the oxygen requirements of heavily polluted waters without resorting to dilution. The "excess-oxygen" method just described, inasmuch as it depends on the volumetric determination of dissolved oxygen, using ordinary glass-stoppered bottles, possesses the merit of extreme simplicity. Extensive series of experiments conducted at the Cincinnati Laboratory of the U. S. Public Health Service have amply demonstrated that the precision attainable leaves little to be desired even if it is necessary to dilute the samples before conducting the test. With suitable laboratory facilities, the dilution technique is simple.

A more serious limitation, and a limitation which is inherent in any method of procedure, is the necessity for interpreting the results in the light of time and temperature relationships. Owing to the fact that the rate of absorption of oxygen by a polluted water is exceedingly slow, it is generally desirable to extend the incubation period over several days. Again, as the reaction is purely biochemical, the temperature at which the test is conducted must be carefully controlled. In order to correlate the laboratory results with the ever-changing time of flow and temperature conditions of a stream, it is necessary, therefore, to obtain reasonably accurate formulas by which the oxygen demand of a sample after any interval of time at any specified temperature may be calculated from the values obtained under standardized conditions.

The experiments herein described, were undertaken, primarily, for the purpose of confirming the validity of the various time and temperature correction formulas which have thus far been proposed. The discussion will be limited to the formulas developed in the course of the Ohio River investigation.* These experiments have also demonstrated that factors other than time and temperature must be considered before a valid interpretation of the highly consistent results obtained with the "excess-oxygen" method can be made. In particular, the condition of a sample with respect to its state of oxidation and, possibly, the nature of the micro-organisms present, both exert a marked influence on the magnitude of the observed oxygen demand values.

* H. W. Streeter and E. B. Phelps, U. S. Public Health *Bulletin No. 146.*

Experimental Procedure

For the purpose of securing representative samples, a large vessel was first filled with Ohio River water or, in some instances, with sewage suitably diluted. After the sample had been thoroughly mixed, it was siphoned into bottles with capacities of 350 cu. cm. The initial oxygen content was then determined and the remaining sub-samples were incubated at 9°, 20°, or 30° cent. In the course of experiments which have extended somewhat more than a year, twelve separate series of observations have been made. In most cases, the course of the deoxygenation was followed for at least one month. As a rule the experiments were conducted in duplicate and, in several instances, comparative data were obtained at three different temperatures.

Precision of Base Data

The agreement between duplicate samples was excellent even when the incubation period extended over several months. In one series of experiments in which a large number of sub-samples were titrated after an incubation period of 96 days at 20° cent., the average deviation from the mean was found to be less than 0.2 part per million. The findings in this respect are of considerable analytical interest.

General Course of Deoxygenation Curve

Given the precision of the base data, the next step has been to plot the observed average oxygen demand values against the period of incubation. The type of curve obtained in a typical series of observations is illustrated by Fig. 1. The data plotted in this chart are probably unique in so far as they all refer to the same sample incubated at different temperatures over prolonged periods. It is also to be noted that the oxygen demand determinations were made at relatively short intervals, so that the general course of the deoxygenation curve is reasonably well defined. At 9° cent. (lower curve), there was a slight lag in the establishment of bacterial equilibrium. In other respects, however, there is a striking parallelism between the results obtained at different temperatures.

Considering only the results obtained at 20° cent. (middle curve), it is evident that the rate of deoxygenation decreased very uniformly during the first nine or ten days. Relatively small quantities of oxygen were absorbed during the next five or six days. After the sixteenth day, the rate of deoxygenation suffered a marked acceleration. It is also noteworthy that, contrary to a generally accepted notion, appreciable quantities of dissolved oxygen continued to be absorbed even after the twentieth day. As the same phenomenon has been observed with fully aerated samples, this secondary increase in the rate of deoxygenation can hardly be ascribed to the approaching exhaustion of dissolved oxygen. In fact, within wide limits, the rate of deoxygenation is quite independent of the quantity of dissolved oxygen present.

The evidence accumulated thus far is very favorable to a view emphasized by Adeney and other British experimenters, namely, that under aerobic condi-

tions the stabilization of organic matter proceeds in two distinct and strictly consecutive stages; the carbonaceous matter, etc., is first oxidized; then, and only then, does nitrification set in. The second point of inflection on the deoxygenation curve, therefore, marks the onset of the nitrification stage. It will be convenient to discuss these two distinct stages separately.

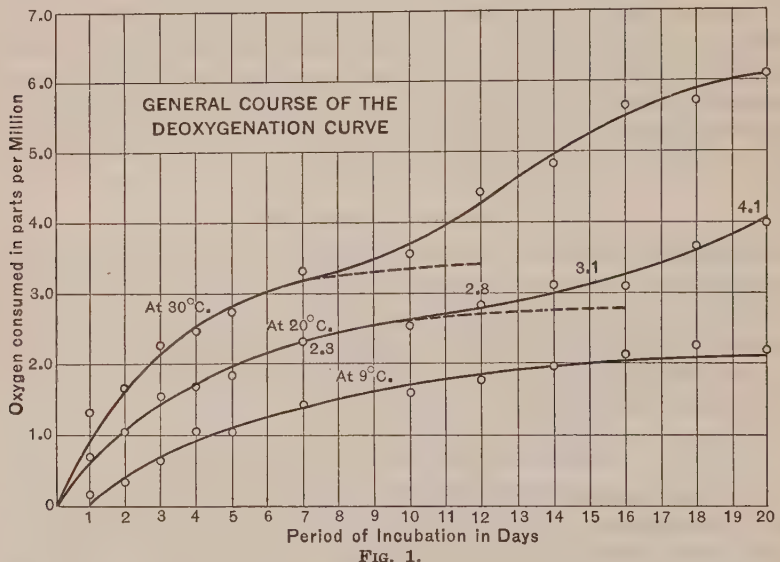

FIG. 1.

RATE OF DEOXYGENATION FORMULA

Considering only the average oxygen demand values corresponding to the first or carbon-oxidation stage, an attempt was next made to determine whether these results conformed with reasonable accuracy to a formula proposed some years ago by Phelps. The formula in question is based on the assumption that the rate of deoxygenation at any instant is directly proportional to the amount of organic matter present in a sample. In the differential notation:

$$\text{Rate of deoxygenation} = \frac{d\,(L_a - L)}{d\,t} = \frac{-d\,L}{d\,t} = K'\,L \quad \ldots\ldots\ldots (1)$$

in which,

L_a = oxygen absorbed during the first stage.

L = oxygen requirement of the sample at the time, t.

K = a constant at a given temperature.

The integration of this expression leads directly to the equation:

$$\log \frac{L_a}{L} = \log \frac{L_a}{L_a - X} = K\,t \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which,

X = oxygen absorbed in t days (the value generally reported as the oxygen demand of the sample).

$K = 0.4343\,K'$ = the deoxygenation constant.

Solving for X in Equation (2), the following expression is obtained:

$$X = L_a (1 - 10^{-Kt}) \quad \ldots\ldots (3)$$

By the aid of tables giving the value of the term, $1 - 10^{-Kt}$, the validity of the Phelps formula may readily be tested. It is only necessary to observe whether a value of L_a exists which satisfies the condition imposed by Equation (3). The agreement between the observed and the computed values is represented graphically by the data plotted in Fig. 2, where the average values obtained in twelve separate series of observations have been recorded. In order to place all values on a comparable basis, and for the sake of avoiding a multiplicity of charts, the results have been plotted, not in parts per million, but as a percentage of the oxygen absorbed during the first stage of the deoxygenation. At each temperature, the line drawn through these average results is simply the graph of the expression:

$$X = L_a (1 - 10^{-Kt})$$

For periods of incubation of less than 8 days at 30° cent., 10 days at 20° cent., or 15 days at 9° cent., the agreement between the observed and the computed percentage values is excellent.

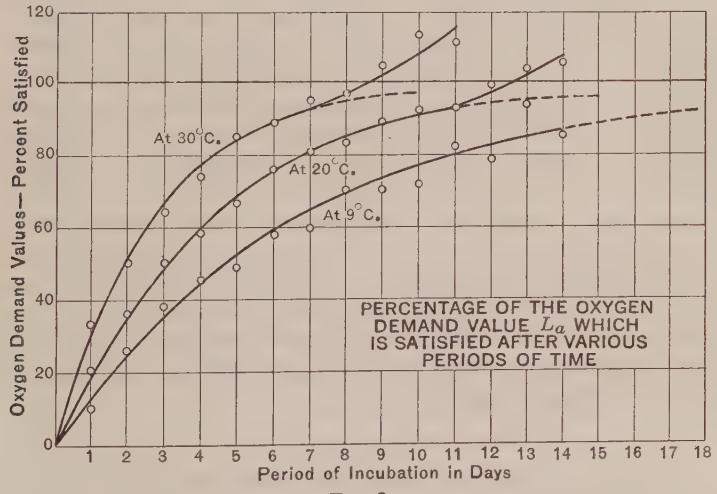

FIG. 2.

TEMPERATURE CONVERSION FORMULAS

a.—The Value of K at Different Temperatures.—It is also to be noted that in plotting the theoretical curves the value of K was computed by the equation:

$$K_T = K_{20} (1.047^{T-20}) \quad \ldots\ldots (4)$$

in which,

K_T = the deoxygenation constant at $T°$ cent.

K_{20} = the deoxygenation constant at 20° cent. = 0.100.

The indication is that, in the interval from 9° to 30° cent., the deoxygenation constant is accurately defined in terms of Equation (4).

b.—The Value of L_a at Different Temperatures.—One further point to be considered in connection with Fig. 2 is the value of L_a at different temperatures. Denoting the value of L_a at 20° cent. by 100, the value of L_a at 9° cent. becomes 78 ± 5. Similarly, the relative value of L_a at 30° cent. is 120 ± 7. These values may be represented empirically by the equation:

$$(L_a)_T = (L_a)_{20} (0.02\ T + 0.60) \quad \ldots\ldots(5)$$

in which,

$(L_a)_T$ = value of L_a at T° cent.
$(L_a)_{20}$ = value of L_a at 20° cent.

The failure to correct for this variation in the oxidizability of a sample with a change in the temperature of incubation does not lead to serious error when the temperature differences are small. In extreme cases a suitable correction can readily be applied.

Applicability of Formulas to Stream-Pollution Problems

Within certain limits, therefore, the possibility exists of converting an oxygen value obtained at any temperature over any period of incubation into terms of the oxygen demand value which would have been obtained under any other given set of conditions. It is to be borne in mind, however, that the applicability of the formulas is restricted to heavily polluted waters, such as raw river water or recently diluted sewage. By inspection of the data plotted on Fig. 1, it is obvious that an entirely different type of deoxygenation curve would be obtained if samples in a more advanced state of oxidation were to be selected. As it is seldom necessary to consider periods of flow exceeding five or ten days below a point of fresh pollution, these limitations are of little consequence in stream-pollution studies. On the whole, it appears safe, therefore, to conclude that, when the various formulas discussed in this paper are applied to the average values corresponding to reasonably large groups of observations on recently polluted water, the cumulative error should not exceed 10 per cent. For the purposes of stream-pollution studies, this degree of precision is entirely satisfactory.

Applicability of 5-Day Oxygen Demand Test to Sewage Treatment Problems

From the foregoing discussion, it may be inferred that for highly polluted waters the oxygen demand values obtained over relatively short periods of incubation possess a clear-cut significance, so that the interpretation of such results offers no difficulty. Attention will now be directed to samples which have reached a higher state of oxidation. The discussion will be conducted with particular reference to sewage treatment problems.

Considering the data plotted in Fig. 1, and assuming that the 5-day oxygen demand of the sample at 20° cent. had been determined only after a preliminary conditioning period of seven days, corresponding to the relatively flat portion of the deoxygenation curve, the observed depletion would have

been about (2.8 — 2.3) = 0.5 part per million. However, if the examination had been delayed for fifteen days, so that nitrification was about ready to start, the observed loss of oxygen would have been about (4.1 — 3.1) = 1.0 part per million. Referred to a sewage effluent which had been diluted fifty times before conducting the test, the two oxygen demand values obtained would have been 25 or 50 parts per million, depending on the amount of preliminary purification which the sample had received. It is noteworthy that under these special conditions the 5-day oxygen demand of the more highly oxidized sample was apparently twice as great as that of the same sample in a less highly purified state. In part, the discrepancy arises from the fact that one set of values has been selected from the relatively flat portion of the deoxygenation curve (8 to 14 days at 20° cent.).

The findings in this respect have a direct bearing on the calculation of the percentage removal of organic matter effected by a treatment plant, and on similar problems in connection with the operation or the comparison of various types of treatment plants. The usual procedure is to base such calculations on the 5-day oxygen demand value of the influent and effluent wastes. In the extreme case in question, it is obvious that the percentage values obtained would stand in inverse relation to the purification actually accomplished. It is not inconceivable that a good measure of the efficiency commonly attributed to Imhoff tanks and similar treatment devices may be due to an effect of this nature. For filter effluents, however, the maximum effect produced by the abrupt change in the slope of the curve may generally be discounted, because the nitrification stage should be fully established when such samples are examined. The possibility of error from this source is nevertheless to be borne in mind.

As regards the time required under laboratory conditions to effect the complete oxidation of the organic matter in a polluted water, definite conclusions can hardly be drawn. On the basis of nitrite, nitrate and free ammonia determinations, it is probably safe to conclude that, at 20° cent., the oxidation of the purely nitrogenous impurity is virtually completed after forty or fifty days. Appreciable quantities of dissolved oxygen, however, continue to be absorbed even after several months of incubation at 20° cent. (See Fig. 3.) The absorption of oxygen beyond the sixtieth day is probably due to the slow oxidation of cellulose-like materials. As it would be impractical to conduct routine tests over such extended periods, it is obviously necessary to conclude that the ultimate oxygen demand of a sample is an indeterminate quantity.

Continuing the discussion of the results derived over long periods of incubation, it appears that when a stage of oxidation has been reached corresponding to that which obtains when a sample of raw sewage is incubated for 30 days at 20° cent., the deoxygenation curve is approximately a straight line. (See Figs. 1 and 3.) The 5-day oxygen demand of a given type of waste, therefore, should be a constant when a sufficiently high degree of purification is reached. It follows that the percentage purification figures computed on the basis of the 5-day oxygen demand test should also tend to be constant when samples in an advanced state of oxidation are examined.

The findings in this respect are in satisfactory accord with the direct observation that the removal of organic matter effected by a representative group of treatment plants was always approximately 90% when partly nitrified effluents only were considered. In view of wide variations in the strength of the raw sewages, in the nature of the treatment devices, and in the methods of operation, this approximate constancy* of the percentage purification values obtained was an unlooked-for result.

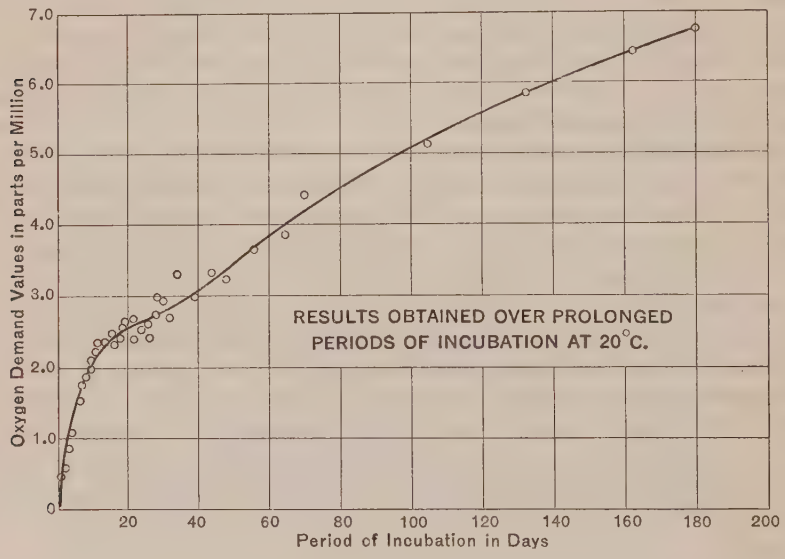

FIG. 3.

Finally, it need hardly be pointed out that a statement to the effect that the 5-day oxygen demand of a sample is, say, 20 parts per million, is of little significance unless a great deal is known concerning the nature or, more precisely, the state of oxidation of the sample. Thus, a 5-day oxygen demand value of 20 parts per million could be referred, with equal reason, to the middle or relatively flat portion of the deoxygenation curve, corresponding to a highly polluted sample, or to the last portion when the nitrification stage has been virtually completed.

CORRESPONDENCE BETWEEN ANALYTICAL DATA AND OBSERVED NATURAL CONDITIONS

The results thus far presented, although indicative of great uniformity, could hardly be referred to natural conditions without further supporting data. Evidence to the effect that the oxygen demand values obtained during the first stage of the oxidation are directly related to the quantity of organic matter present is given in Table 1. Using the 5-day oxygen demand of a raw sewage as a measure of its organic matter content, and given the contributing popula-

* "Sewage Treatment in the United States", *Public Health Bulletin No. 132*, p. 29.

tion and the total flow of sewage, the per capita contribution of organic matter has been computed for places where fairly accurate data were available. The average per capita oxygen requirement is 51.1 grammes per day, with an average deviation from this figure of 5 grammes. The high value obtained at Columbus, Ohio, is probably due to the presence of relatively large quantities of industrial wastes. Omitting the Columbus result, the average per capita oxygen demand is 48.8 ± 3.1 grammes per day. The constancy of the per capita values is remarkable and leads to the conclusion that the 5-day oxygen demand of a raw waste is directly proportional to the concentration of organic matter present. Moreover, it is apparent that the rate of deoxygenation of diluted raw sewage is not subject to extreme variations; otherwise, the per capita values derived with different sewages would not be consistent.

TABLE 1.—Per Capita Oxygen Demand Values.

(Base data from *Public Health Bulletin No. 132,* p. 115.)

Locality.	Results, in Parts Per Million.			
	5-day oxygen demand actually observed.	Per capita oxygen demand daily.	Deviation from mean, d_1.	Deviation from mean, d_2.
Alliance, Ohio	92	45.6	5.5	3.2
Baltimore, Md.	120	45.1	6.0	3.7
Canton, Ohio	213	51.6	0.5	2.8
Columbus, Ohio	190	67.6	16.5	....
Fitchburg, Mass.	155	51.6	0.5	2.8
Lexington, Ky.	144	48.5	2.6	0.3
Reading, Pa.	118	45.1	6.0	3.7
Rochester, N. Y.	104	53.9	2.8	5.1
Average*	....	51.1	±5.0	....
Average †	....	48.8	....	±3.1

* To include all observations.

† Omitting the Columbus results.

As regards the general course of the oxidation of organic matter under natural conditions, it is well established that, in sewage treatment, nitrification does not begin until considerable preliminary purification has been effected. Moreover, it has been demonstrated in experiments conducted at the New Jersey Agricultural Experiment Station that, even in a filter bed, the onset of the nitrification stage is sharply defined. In the Illinois River investigation, nitrification was not observed until a point far removed from the source of initial pollution had been reached. The exhaustive studies of the Royal Commission on Sewage Disposal of Great Britain also afford instances where the deoxygenation curve represented by Fig. 1 was clearly reproduced in streams. Similar curves were also obtained using undiluted sewage. It appears reasonable to assume, therefore, that the phenomena observed in the laboratory actually correspond to natural conditions.

Conclusions

As a result of the foregoing, the following conclusions have been reached:

1.—The Phelps formula holds with reasonable accuracy when applied to samples recently polluted with organic matter.

2.—For periods of incubation of less than ten days, it is possible to refer the results obtained under standardized laboratory conditions to the actual times of flow and temperatures of a stream.

3.—Under aerobic conditions, the stabilization of organic matter apparently proceeds in two distinct stages.

4.—The rate at which a polluted water is deoxygenated depends largely on the condition of the sample with respect to its state of oxidation.

5.—It is necessary to exercise considerable caution in interpreting the results of analyses when the nitrification stage has almost been reached.

6.—Absolute values for the purification accomplished by a treatment plant cannot be obtained without resorting to protracted incubation.

7.—A complete solution of the problem probably depends on the development of methods whereby the state of oxidation of a sample may be determined more readily.

THE RATE OF ATMOSPHERIC REAERATION OF SEWAGE POLLUTED STREAMS

By H. W. Streeter,* M. Am. Soc. C. E.

Introductory

In all problems of stream sanitation involving the maintenance of an adequate reserve supply of dissolved oxygen for the preservation of fish life or the prevention of nuisance, there are two major factors to be considered as determining the limiting degree of pollution of streams which is consistent with satisfying a given reserve oxygen requirement. One of these factors is the rate of biochemical deoxygenation of the stream water, proceeding in accordance with laws which have been described by Mr. Theriault.† The other element is the rate and extent of replenishment of its oxygen supply from three natural sources:

(*a*) Dilution water entering the stream through the medium of tributaries and local inflow.
(*b*) Biological reoxygenation through the activities of certain oxygen-producing plants.
(*c*) Atmospheric reaeration, or absorption of oxygen directly from the atmosphere.

Of these three sources of oxygen, atmospheric reaeration is by far the most important in freely flowing streams, and this paper is limited to this subject.

It has been widely recognized that atmospheric reaeration is an important factor in the recovery of dissolved oxygen by flowing streams subjected to progressive deoxygenation but, as far as is known, the first effort to evaluate its effects quantitatively as observed under natural conditions, and to correlate such measured effects with the various physical elements which modify them, was made in connection with a survey of the pollution and self-purification of the Ohio River, by the U. S. Public Health Service, in 1914, 1915, and 1916. The results obtained from this phase of the survey, which have been published in the form of a separate report,‡ have served as a basis for a further study of stream reaeration by the Service in connection with a survey of the pollution of the Illinois River, in 1921 and 1922. Although a full analysis of the reaeration data obtained from the Illinois River study has not been completed, it has been carried forward sufficiently to suggest wherein the conclusions reached from the Ohio River study concerning the laws and factors underlying this phenomenon appear to be confirmed and wherein they may require modification. In this paper it is proposed to indicate what both studies have shown, of interest to engineers, as bearing on the theory of stream reaeration and its applications to problems of river sani-

* San. Engr., U. S. Public Health Service, Cincinnati, Ohio.

† See p. 1341.

‡ "Studies of the Pollution and Natural Purification of the Ohio River, Pt. III: Factors Concerned in the Phenomena of Oxidation and Reaeration", by H. W. Streeter and E. B. Phelps, *Public Health Bulletin No. 146*, U. S. Public Health Service, Washington, D. C.

tation. For the sake of brevity, the term, "reaeration", will be used hereafter in referring to this phenomenon.

The Nature of Stream Reaeration

The reaeration of flowing streams is governed primarily by the laws controlling the absorption of moderately soluble gases by unsaturated liquids kept in a continuous state of agitation. These laws have been studied by a group of chemists, the results of whose observations have been published in the form of a Symposium.* In a paper included in this Symposium, Mr. H. G. Becker† states in the following general form the law of gas absorption which underlies stream reaeration: When a liquid and a moderately soluble gas are allowed to come in contact and the liquid is thoroughly mixed, "the rate of solution of the gas varies directly as the degree of unsaturation of the liquid." In the report on studies of reaeration in the Ohio River, to which reference has been made, it was stated that the rate of solution of oxygen at the surface is directly proportional to the existing saturation deficit (which is merely another way of stating the same law), and it was shown that results obtained by Dibdin and by Adeney and Becker afford experimental confirmation of this principle.

Expressed in terms of stream reaeration, the law thus stated signifies that in each successive unit of time a constant percentage of the remaining deficit in the dissolved oxygen content of the stream below the saturation point, will be satisfied by absorption of oxygen from the atmosphere. The percentage will vary with conditions affecting the rate of absorption, but will remain constant for a given condition. This is analogous to the law of deoxygenation discussed in Mr. Theriault's paper, except that in the latter case the rate of progress of the action is a direct function of the biochemical oxygen demand, rather than the oxygen saturation deficit of the stream water.

In the Ohio River studies, the law of oxygen absorption was formulated thus:

Let,

D_a = the initial oxygen saturation deficit, in terms of concentration;
D = the oxygen deficit at any time, t, expressed in similar terms; and
K_2 = a coefficient defining the rate of reaeration.

Then,

$$\frac{d\,D}{d\,t} = -\,K_2\,D$$

whence,

$$\log \frac{D}{D_a} = -\,K_2\,t \qquad (6)$$

On referring to Mr. Theriault's paper, it will be noted that this expression is exactly similar to that which defines the rate of deoxygenation, that is,

$$\frac{d\,L}{d\,t} = -\,K_1\,L$$

* *Journal of Industrial and Engineering Chemistry*, December, 1924, pp. 1215–1230.

† "Mechanism of Absorption of Moderately Soluble Gases in Water", *Journal of Industrial and Engineering Chemistry*, December, 1924, pp. 1220–1224.

whence,

$$\log \frac{L}{L_a} = - K_1 t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

except that, in this case, the biochemical oxygen demand, L, replaces the oxygen deficit, D, and the coefficient of deoxygenation, K_1, replaces the coefficient of reaeration, K_2.

The coefficient of reaeration, K_2, defining the rate of absorption of oxygen, when expressed in terms of oxygen concentration in the stream, has been found, in the Ohio River study, to be modified by stream depth and by various physical conditions which influence the turbulence of flow, among which are the velocity of the current and the slope and irregularity of the channel. In the Ohio River, these relations were found to be governed by a simple equation:

$$K_2 = c\ V^n \times H^{-2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which, V, represents the velocity of flow; H, the depth; and c and n, the constants for a particular river stretch, the values of which depend in part on the channel slope and irregularity. In most cases, it has been found that the value of K_2 is very nearly inversely proportional to the discharge of the stream, which term, multiplied by a proper reducing constant, may be substituted for the square of the depth in Equation (8).

The rate of reaeration is further modified by the water temperature, being accelerated at the higher and diminished at the lower temperatures. The controlling element in this temperature effect appears to lie in the fact that the rate of absorption of oxygen at the surface is limited by the process of diffusion, which, as shown by Black and Phelps,* is governed by a similar temperature relation. It was found in connection with the Ohio River study that when observed values of the reaeration coefficient, K_2, are corrected in accordance with the factors developed by Black and Phelps, the corrected values are more closely correlated with the other stream conditions which have been noted than the uncorrected ones. A few results obtained from the Illinois River study have indicated that the rate of reaeration of this stream does not appear to be influenced as much by seasonal changes in temperature as connections based on the diffusion factors developed by Black and Phelps would imply. However, the results of the recent experiments by Becker, previously mentioned, and by Haslam, Hershey, and Keen,† carefully conducted under physical conditions closely approaching those of flowing streams, have confirmed the earlier findings of Black and Phelps in respect to the direction, and, roughly, to the extent of the temperature effect. As these experimental results are based on far more carefully controlled observations than would be possible under natural conditions, they must be interpreted, for the present at least, as affording a reasonably accurate index of the influence of temperature variations on the rate of reaeration of streams. From a plot of the data compiled by Becker, converted to terms of the reaeration

* W. M. Black and E. B. Phelps, Report on Discharge of Sewage into New York Harbor, to the Board of Estimate and Apportionment, New York City, 1911.

† *Journal of Industrial Engineering Chemistry*, December, 1924, pp. 1224–1230.

coefficient, K_2, the following temperature correction equation has been derived:

$$K_{2\,(T^\circ\text{ cent.})} = K_{2\,(20^\circ\text{ cent.})} \times [1.0159^{\,(T-20)}] \ldots\ldots\ldots\ldots (9)$$

This equation is proposed tentatively as probably representing most nearly, from available data, the effect of temperature variations on the value of the reaeration coefficient, K_2, under natural stream conditions. In Fig. 4 is shown a plot of this temperature function as compared with a similar plot of temperature correction factors affecting the rate of deoxygenation, which was developed in connection with the Ohio River studies and has been discussed in Mr. Theriault's paper.

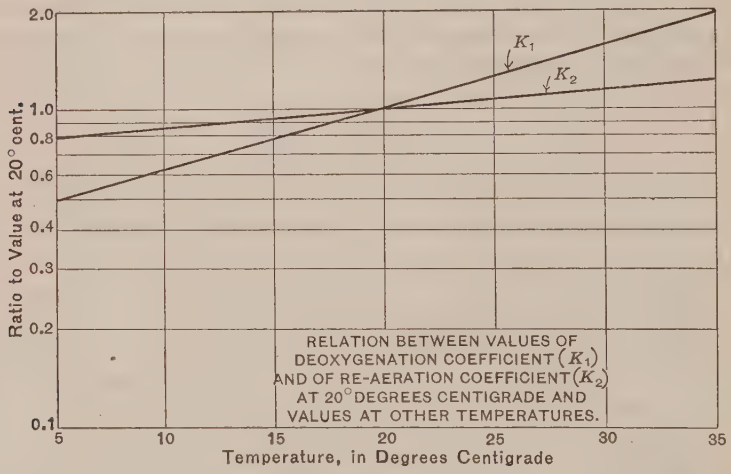

FIG. 4.

EMPIRICAL MEASUREMENT OF THE REAERATION RATE

From what has been stated concerning the extent and modes of action of atmospheric reaeration in streams acting as receivers of community wastes, it is fairly obvious that no even reasonably accurate estimate can be made of the ability of a particular stream to maintain a specified minimum of reserve oxygen supply under a given degree of pollution without a definite knowledge of its capacity for reaeration. This thought leads to a consideration of available means for measuring the reaeration capacities of streams.

Owing to the fact that the rate of reaeration is influenced by a complexity of natural conditions, such as have been noted, methods of laboratory study that have been found suitable for determining the deoxygenation rate are not applicable in this case; hence recourse must be had to measurements in the stream.

If a sufficient number of representative streams could be found in which progressive deoxygenation was not a complicating element, the solution of this problem would be comparatively simple, involving merely the observation of the rate of increase in the dissolved oxygen content of a river between two or more sampling points located at known time intervals of flow from each other. Unfortunately, such a condition never exists, for reasons which are obvious.

The true rate of reaeration, then, is always masked, as far as its observable effect on the dissolved oxygen is concerned, by having superimposed on it a rate of deoxygenation acting simultaneously in the opposite direction.

In order to take account of this condition, an equation was devised during the Ohio River studies, whereby the resultant effect of two given rates, one of deoxygenation and the other of reaeration, on progressive changes in the dissolved oxygen content of a stream can be calculated. This equation was derived by combining the differential expressions, Equations (6) and (7), into a differential equation and integrating it to a variable time, t. The equation thus derived is:

$$D = \frac{K_1 L_a}{K_2 - K_1} (10^{-K_1 t} - 10^{-K_2 t}) + D_a \times 10^{-K_2 t} \quad \ldots\ldots\ldots (10)$$

in which,

D_a = the initial dissolved oxygen saturation deficit, in terms of concentration;
D = the dissolved oxygen deficit after time, t, in similar terms;
L_a = the initial biochemical oxygen demand;
K_1 = the coefficient of deoxygenation; and
K_2 = the coefficient of reaeration.

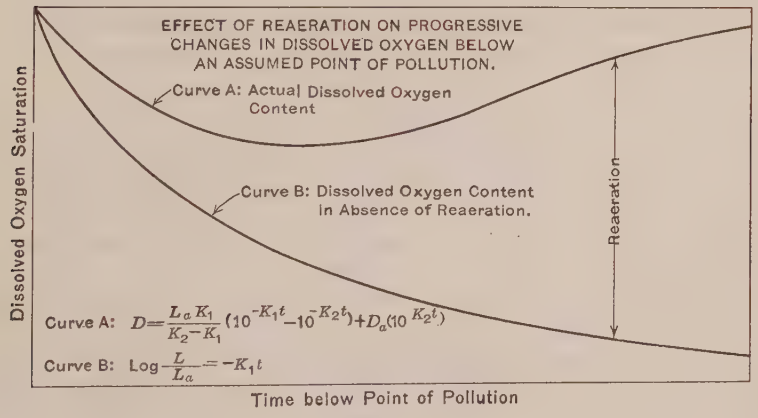

FIG. 5.

The type of curve defined by this equation is shown by Curve A in Fig. 5, which has been reproduced from the report of the Ohio River studies to which reference has been made. For comparison with Curve A, is shown Curve B, representing the progressive deoxygenation which would occur in the absence of reaeration. Curve A is characteristic of progressive changes in the dissolved oxygen content of streams which frequently have been observed in streams below points of major pollution, for example, in the Illinois River below the outlet of the Chicago Drainage Canal; also, in the White River below Indianapolis, Ind. Curve B is characteristic of conditions occasionally occurring in highly polluted streams when covered by a continuous ice sheet, temporarily cutting off reaeration.

By substituting in Equation (10) known or observed values of all terms except that of the reaeration coefficient, K_2, the latter can readily be computed for a given river stretch. A large number of calculations of this kind were made for a series of stretches of the Ohio River, based on observations of the dissolved oxygen and the oxygen demand at the terminals of each river section and on assumed values of the deoxygenation coefficient, K_1, derived from laboratory studies such as have been described by Mr. Theriault, and corrected to the stream temperature by the equation discussed in his paper. A limited number of parallel computations also have been made for a few stretches of the Illinois River. In Table 2 are shown, for comparison, values of the reaeration coefficient derived in this manner from observations in three stretches of the Ohio River and two stretches of the Illinois River presenting, approximately, similar flow and channel characteristics. The results in both cases cover the summer seasonal period, May to September, inclusive. A marked similarity is shown between values of K_2 thus derived in the two streams. It is also noteworthy that the rates of reaeration observed in these five river stretches are approximately double the corresponding rate of deoxygenation as measured by the laboratory value of the coefficient, K_1; thus, the mean value of K_2 is approximately 0.24, whereas that of K_1, at the average river temperature for the given period, is about 0.12.

TABLE 2.—Measured Values of the Reaeration Coefficient, K_2, in Three Stretches of the Ohio River and Two Stretches of the Illinois River.

(May to September, inclusive).

Month.	Values of Reaeration Coefficient, K_2.				
	Ohio River.			Illinois River.	
	Stations 11–19.	Stations 23–65.	Stations 104–349.	Stations 263–240.	Stations 148–122.
May	0.25	0.20	0.18	0.31	0.47
June	0.19	0.33	0.27	0.31	0.28
July	0.29	0.23	0.21	0.21	0.20
August	0.22	0.26	0.21	0.19	0.27
September	0.14	0.19	0.17	0.31	0.14
Mean	0.22	0.24	0.21	0.27	0.27

The locations of river stretches are, as follows:

Ohio River (River miles below confluence of Allegheny and Monongahela Rivers):

Stations 11- 19.....Below Pittsburgh, Pa.
" 23- 65.....From above mouth of Beaver River to above Steubenville, Ohio.
" 104-349.....From below Moundsville, W. Va., to above mouth of Scioto River.

Illinois River (River miles above mouth):

Stations 263-240.....From opposite Morris to opposite Ottawa, Ill.

" 148-122.....From Pekin to Havana, Ill.

Under some conditions, as, for example, where a stream flows rapidly over a shallow "riffle", the rate of reaeration may become greatly accelerated owing to the diminished depth and increased turbulence of flow. An instance of this kind is found in a short stretch of the Des Plaines River immediately below Joliet, Ill., where the channel is steep and rough and a series of shallow rapids is formed. Calculations of the value of K_2 for this section of the river, based on daily observations extending over a period of ten months, from August, 1921, to April, 1922, inclusive, have given indicated rates of reaeration roughly ten times those observed in deeper and less turbulent stretches of the Illinois River down stream. During the period of December to April, when conditions were most favorable for measuring the true rate of reaeration in this stretch of the river, the following values of K_2 were obtained:

December	2.42
January	2.63
February	2.70
March	2.83
April	2.25
Mean	2.57

The average value of K_2 for the full 10-month period was 2.00.

In general, optimum conditions for determining empirically the value of the reaeration coefficient exist where a stream contains a measurable quantity of dissolved oxygen and where the channel bottom is relatively free from unstable and readily oxidizable sludge deposits. When a stream is wholly or nearly depleted of dissolved oxygen and its channel contains any considerable quantities of decomposing sludge, a very sizeable proportion of the atmospheric oxygen absorbed by such a stream may be withdrawn from solution almost immediately and thereby fail to be accounted in terms either of reserve oxygen or of biochemical oxygen demand. Under such circumstances, the measured value of the reaeration coefficient may be widely in error and always will be lower than the true value. Where an excessively polluted stream contains a measurable supply of oxygen and is relatively free from sludge deposits during a part of the time, measurements of its reaeration capacity should be made when it is in this condition.

Applications

The most important applications of the theory outlined in this paper are found in the estimation of dilution or sewage treatment requirements to be met at specified points along excessively polluted streams to avoid over-taxing their capacities for maintaining a specified reserve oxygen supply, or, conversely, in the calculation of the future limiting permissible degree of pollution

of streams now in a satisfactory condition from this standpoint. Both cases are similar in that they involve the prescription of a limiting biochemical oxygen demand of a stream at certain critical points. As the rate of deoxygenation is accelerated during the summer season to a greater proportionate extent than the rate of reaeration (the latter often is actually retarded during this season owing to a greatly diminished stream flow), conditions during the summer ordinarily are the most critical to be considered in this connection.

In Fig. 6 is given an example showing the effect of temperature variations on progressive changes in the dissolved oxygen as calculated by Equation (10), assuming an initial oxygen demand, L_a, of 20 parts per million and an initial oxygen saturation deficit of zero. The values of the deoxygenation and reaeration coefficients, K_1 and K_2, have been assumed to be 0.10 and 0.20, respectively, at 20° cent. and have been corrected for temperature in accordance with the factors shown in Fig. 4. The time required to attain the maximum oxygen deficit is shown to vary from about 2 days at 30° cent. to 5 days at 5° cent.

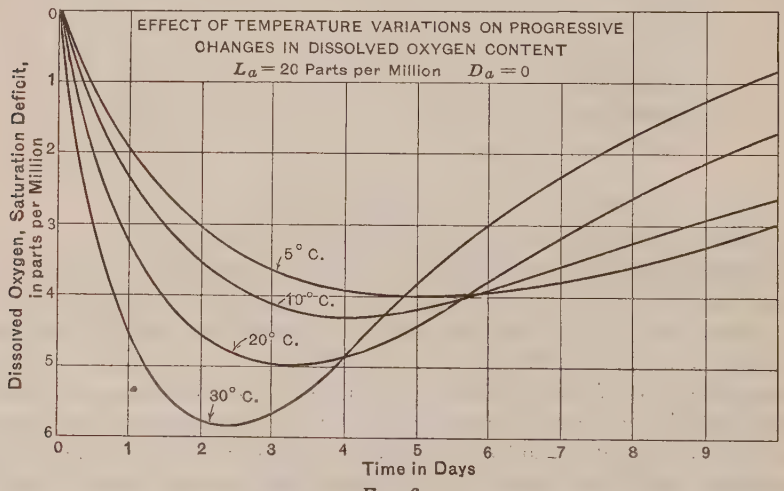

Fig. 6.

The effect of variations in the initial oxygen demand, L_a, on the dissolved oxygen content of a stream below a point of pollution is illustrated by the curves in Fig. 7, computed for a temperature of 20° cent. and with an assumed initial oxygen deficit of 1.0 part per million. In Fig. 8 is a plot of the maximum oxygen deficits and the times required to attain the maximum, as indicated by the curves in Fig. 6, the plotted quantities being calculated, however, by a formula developed by differentiating Equation (10) and placing the resulting expression equal to zero. In this case it is noted that although the maximum deficit varies almost as a straight-line function of the initial oxygen demand, the time to attain the maximum lies within a comparatively narrow range, that is, between 2 and 3 days.

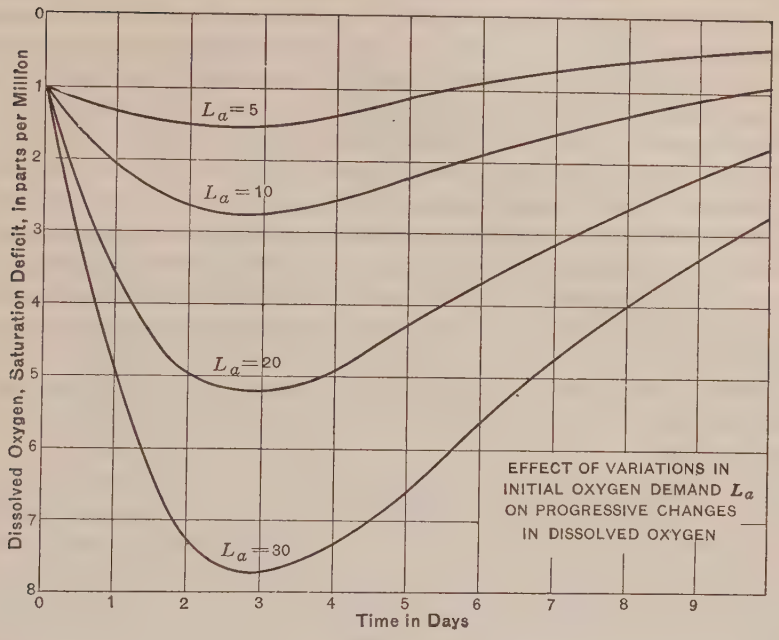
Dissolved Oxygen, Saturation Deficit, in parts per Million
Time in Days
$L_a = 5$
$L_a = 10$
$L_a = 20$
$L_a = 30$
EFFECT OF VARIATIONS IN INITIAL OXYGEN DEMAND L_a ON PROGRESSIVE CHANGES IN DISSOLVED OXYGEN

FIG. 7.

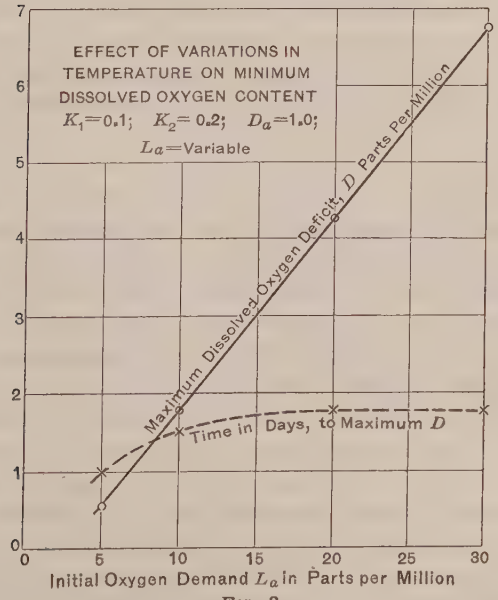
EFFECT OF VARIATIONS IN TEMPERATURE ON MINIMUM DISSOLVED OXYGEN CONTENT
$K_1 = 0.1$; $K_2 = 0.2$; $D_a = 1.0$;
L_a = Variable
Maximum Dissolved Oxygen Deficit, D Parts Per Million
Time in Days, to Maximum D
Initial Oxygen Demand L_a in Parts per Million

FIG. 8.

It thus appears that the points of maximum dissolved oxygen depletion in polluted streams normally should lie within comparatively short distances, as measured by time, below major sources of pollution, and that their positions should be affected to a much less extent by variations in the initial oxygen demand than they are by seasonal changes in temperature. Observations on numerous streams, both in the United States and abroad, have confirmed this statement in so far as it applies to streams which are not grossly polluted. If the pollution of a stream is so great, however, as to over-tax its capacity for reaeration, zones of complete deoxygenation, indefinite in length, may be established at certain seasons of the year, notably during periods of dry-weather flow in summer. A condition of this kind is frequently aggravated by the tendency of grossly polluted streams to deposit a sludge mat in the bottom of the channel which may greatly augment the oxygen demand of the stream proper during critical seasons. Under these circumstances, the equations previously noted are not applicable and special methods of analysis must be used.

A good example of such a condition is found in the stretch of the Upper Illinois River, extending from Joliet down stream for approximately 110 miles to the head of Peoria Lake, which receives at its up-stream end the sewage of Chicago, discharged into it through the Drainage Canal and a stretch of the Des Plaines River channel. During eight months of the year, October to May, inclusive, this stretch of the river contains a measurable, although in places low, reserve supply of dissolved oxygen. During the four summer months, June to September, its dissolved oxygen content is practically exhausted throughout its entire length, owing, in part, to the lower dilution provided by the river and its tributaries, to the effect of the higher summer temperatures in causing an accelerated rate of deoxygenation as compared with that of reaeration, and to the greatly added deoxygenating effect of the dense mat of decomposing sludge with which the bottom of the river channel is covered.

Following the method previously outlined, an effort was made to calculate values of the reaeration coefficient, K_2, from observations made in the Illinois River during the summers of 1921 and 1922. Owing to the conditions at that time, previously noted, an accurate calculation was found to be impracticable, the values of the coefficient derived being obviously too low and, in some cases, negative. A similar calculation based on observations during the two months, October, 1921, and May, 1922, when the river temperatures approached those of summer and measurable quantities of dissolved oxygen were found in the river, gave results reasonably consistent both as to their agreement with each other and as to their relation to known physical conditions in the several river stretches. From these results the following values of K_2, converted to their equivalents at 20° cent., were derived for the five river stretches forming the upper section of the Illinois River between the limits stated (the station numbers referring to the locations, in stream-miles, above the mouth of the Illinois River):

River stretch.	Value of K_2
Stations 286-263	0.68 (mean of October and May)
" 263-240	0.33 (" " " " ")
" 240-227	0.15 (" " May)
" 227-196	0.23 (mean of October and May)
" 196-179	0.14 (" " May)

Although it is likely that the values thus derived (especially the lowest two) are affected to some extent by excessive and unaccountable deoxygenation due to sludge deposits, they are believed to be as nearly representative of the true rates of reaeration prevailing in the several river stretches as any other figures obtained from the present very incomplete series of calculations.

With the foregoing derived values of K_2 as a basis, and using the resultant oxygen Equation (10), a computation has been made of the progressive changes in the dissolved oxygen content of the Upper Illinois River occurring in the stretch extending from Station 286, below Joliet, to Station 179, located 107 miles down stream, during each one of the four months, October, 1921, and May, June, and July, 1922. In making the calculation (details of which are omitted for the sake of brevity), the value of the deoxygenation coefficient assumed was based on the laboratory figure in every instance except that of the river stretch from Station 286 to Station 263, for which the mean of the rates of deoxygenation observed in the stream during the two months, October and May, was used. The values of the reaeration coefficient assumed were the same as those just given, corrected to the river temperature. The calculated dissolved oxygen figures at each station are compared with the corresponding results of observation in Table 3 and illustrated graphically in Fig. 9.

TABLE 3.—Comparison of Calculated and Observed Dissolved Oxygen Contents of Upper Illinois River at Successive Sampling Stations.

Station.	Dissolved Oxygen Saturation Deficit, in Parts per Million.							
	October, 1921.		May, 1922.		June, 1922.		July, 1922.	
	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.
263	8.4	9.1	8.6	8.0	8.6	8.8	6.2	8.6
240	6.4	7.1	7.0	6.3	6.4	7.6	4.8	8.0
227	6.5	7.4	7.0	6.4	6.5	8.1	4.9	8.0
196	4.3	5.3	4.4	4.3	3.6	5.9	3.0	7.5
179	4.9	6.6	3.6	4.6	2.6	6.3	2.7	8.1
	Dissolved Oxygen, Percentage of Saturation.							
263	20	14	11	19	3	1	28	2
240	39	32	28	35	28	15	44	6
227	38	29	28	34	27	9	43	8
196	59	50	55	61	60	31	65	12
179	63	37	63	52	71	26	69	9

On referring to Fig. 9, it is noted that the calculated and observed figures agree with each other closely for May and reasonably well for October, but they diverge widely for June and July. The divergence probably is due largely to the effect of sludge decomposition in the channel during the summer months, as it represents the excess of dissolved oxygen, unaccounted for in terms of reaeration or normal deoxygenation, which has disappeared from the

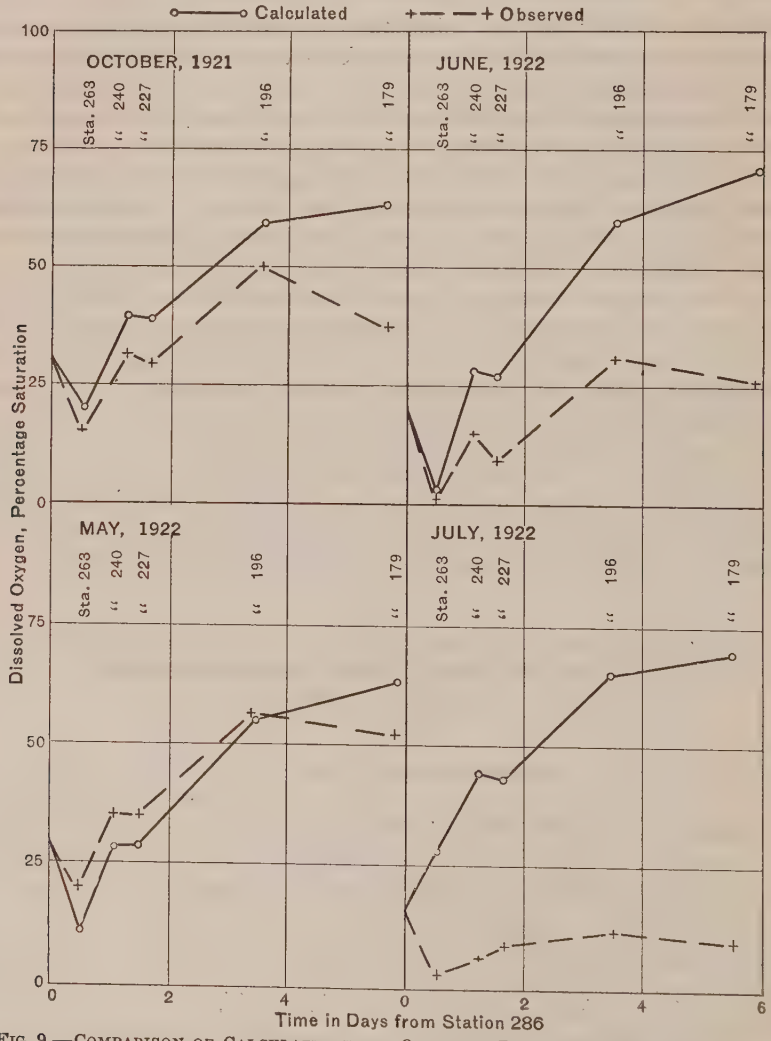

FIG. 9.—COMPARISON OF CALCULATED WITH OBSERVED DISSOLVED OXYGEN CONTENTS AT STATIONS IN UPPER ILLINOIS RIVER. (PLOT OF DATA IN TABLE 3.)

stream in passing from the uppermost to the lowest station and can be accounted for only as oxygen absorbed by the bottom sediments. The deoxygenating power of sludge deposited in the channel is thus indicated as having been sufficient, in July, 1922, to cause an absorption of a quantity of dissolved oxygen equivalent to 60% of the saturation value in a river distance of

107 miles. Although it is hazardous to indulge in speculation in a problem as complex as that presented by the Illinois River, it seems fairly evident that the mere elimination of sludge deposits from the channel of this stream would go far toward restoring the effectiveness of its powers for self-purification.

The density of pollution of the stream proper, however, is fully as important a factor as its condition in respect to sludge deposits in determining its ability to recover its reserve supply of oxygen. To illustrate this point, a series of curves is given in Fig. 10, showing calculated progressive changes in the dissolved oxygen content of the Upper Illinois River with various assumed quantities of initial oxygen demand, the calculation being based on observed conditions at Station 286, below Joliet, during May, 1922. The figures from which Fig. 10 have been plotted are given in Table 4. The comparison is not valid except for purposes of illustration, as any lowering of the initial oxygen demand at Station 286 would necessarily entail improved conditions up stream, which, in turn, would cause an increased oxygen saturation at the point of departure, or *vice versa*. The comparative trends of the curves merely serve to give a rough illustration of the improvement which would be expected if the pollution of a stream at a given point were diminished, without any change occurring in its oxygen status above that point.

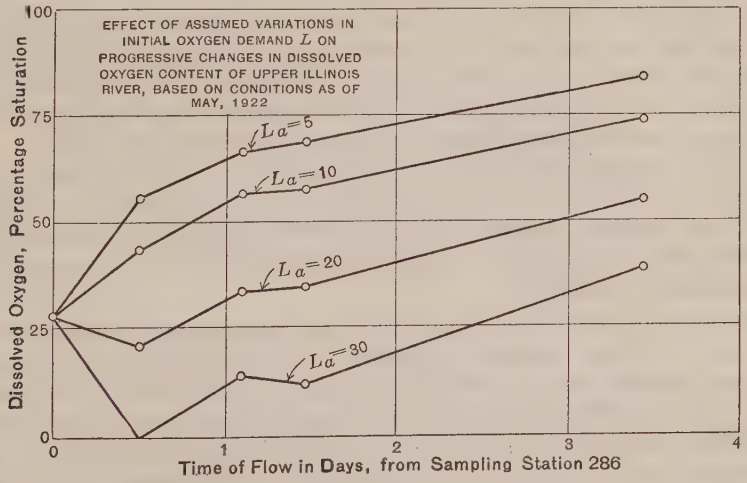

FIG. 10.

In general, it is evident that in almost any given instance where systematic measures are undertaken to relieve excessive stream pollution, a reduction in the oxygen demand of the stream proper and an improvement in its condition with respect to sludge deposits should go hand in hand. This point is an important one to be borne in mind in forecasting the extent of beneficial results to be obtained from extensive stream-cleaning activities. The illustrations given in this paper err considerably on the side of conservatism in this respect, as this fact has not been taken into account in deriving them.

TABLE 4.—Calculated Percentages of Dissolved Oxygen Saturation at Stations on the Upper Illinois River, Assuming Different Initial Oxygen Demand Values, L_a, at Uppermost Station.

(Based on conditions as of May, 1922.)

Station.	Time of flow, in days.	Calculated Percentage of Dissolved Oxygen Saturation, with Initial Oxygen Demand, L_a, Assumed as:			
		5 parts per million.	10 parts per million.	20 parts per million.	30 parts per million.
286	0.00	28	28	28	28
263	0.49	56	44	21	1
240	1.08	67	57	35	12
227	1.46	69	58	35	12
196	3.44	84	74	55	39

Conclusions

From the studies briefly described in this paper, the following tentative conclusions appear to be justified:

1.—The reaeration of flowing streams proceeds substantially in accordance with physical laws which have already been described.

2.—Its rate at any time is controlled mainly by the temperature, turbulence, and oxygen saturation deficit of the stream.

3.—The empirical method of measuring rates of reaeration which has been described, involving the use of the resultant oxygen Equation (10) and the substitution therein of quantities derived by observations in the stream made under proper circumstances, gives results which appear to be consistent with known facts concerning the physical conditions influencing such rates.

4.—By a proper combination of predetermined rates of reoxygenation and of reaeration, using Equation (10), a reasonably accurate calculation may be made of the resultant progressive changes in the dissolved oxygen content of a stream under any given or assumed condition of flow, temperature, and initial degree of pollution.

The studies of stream reaeration thus far made along lines indicated in this paper have been confined to the Ohio and Illinois Rivers, surveys of which have offered the only sufficiently extensive and properly co-ordinated data thus far available for this purpose. A much more comprehensive analysis of the Illinois River data, as yet to be completed, probably will give a more satisfactory basis for judgment as to the wider applicability of the results of these studies than it has been practicable to establish within the limited scope of this paper. Some features of the present theory of stream reaeration and its method of application doubtless will require further modification as more experience is gained in testing it against specific problems. The studies thus far completed, however, have indicated that the theory in question, applied with due consideration to its practical limitations, offers a working hypothesis for a much more rational treatment of stream sanitation problems involving the prevention of conditions contributing to nuisance and to the destruction of fish life in streams than hitherto has been available.

QUANTITATIVE STUDIES OF BACTERIAL POLLUTION AND NATURAL PURIFICATION IN THE OHIO AND THE ILLINOIS RIVERS

By J. K. Hoskins,* Esq.

The U. S. Public Health Service has been engaged for some years in studies of various phenomena concerned with the pollution and natural purification of streams. One general purpose of these studies has been to evaluate the intensity of bacterial pollution to be expected from known populations discharging sewage into streams of known discharge and velocity of flow. With this end in view detailed bacteriological data have been collected from two streams of quite different types, the Ohio and Illinois Rivers. Published observations on the Ohio River† covered a period of three years, and those of the Illinois River were continued for a complete year, so that in each instance information was obtained throughout an entire seasonal cycle.

From a consideration of the data of these studies some general tendencies in bacterial changes are indicated, which may be of assistance to sanitary engineers in forming an estimate of the effect, both immediate and prolonged, of adding sewage, from a definite population, to a watercourse of determined hydrometric characteristics.

The degree of bacterial pollution contributed by cities, about which information is most generally desired, may be separated into two principal cases. The first is concerned with the intensity of bacteria that will result in the stream in the zone of highest pollution below the point at which the sewage is discharged. The second and sometimes more important consideration, is the proportion of such contributed bacteria that will remain in the stream at a known distance, or time of flow, below the point at which they were added.

Discussion

Due to fluctuations in discharge and inflow of all streams, the bacterial concentration resulting from a constant rate of contribution may vary widely. It is essential, therefore, for a comparative study of results that not only the concentration of bacteria be considered, but that the actual quantities of organisms be taken into account as well. The quantities of bacteria present in, or added to, a watercourse can be expressed most conveniently in terms of a unit in which is combined the elements of volume, time, and bacterial concentration. Such a unit, called the "quantity unit," has been used for this purpose. The quantity unit may be defined to be the product of the discharge of 1 cu. ft. per sec. and a concentration of 1 000 bacteria

* San. Engr., U. S. Public Health Service, Cincinnati, Ohio.

† "A Study of the Pollution and Natural Purification of the Ohio River, Part II: Report on Surveys and Laboratory Studies" (1924), *Public Health Bulletin No. 143*, U. S. Public Health Service, Washington, D. C.

per cu. cm. Hence, the number of quantity units of bacteria in a stream equals:

$$\frac{\text{Discharge, in second-feet} \times \text{bacteria per cubic centimeter}}{1\,000}$$

Obviously, this unit is convertible into bacterial numbers per unit of time, such as the day. Thus, an average of 1 000 bacteria per cu. cm. in a flow of 1 sec-ft. for 1 day, or 86 400 sec., is equivalent to 28 317 (= number of cubic centimeters in 1 cu. ft.) $\times$ 1 000 $\times$ 86 400, or 2 446 589 000 000 bacteria per day in one quantity unit.

Immediate Pollution

In observations of the effect of pollution by sewered communities, it has been noted consistently that the zone of greatest bacterial density in the receiving stream does not occur immediately below the sewer outfalls, but at a point 10 to 30 hours down stream from the place where such pollution is added. Moreover, the location of this maximum zone seems to be influenced by seasonal temperatures, being farthest down stream during the winter months. Whether an actual multiplication of organisms in the stream takes place until this maximum is reached, or whether the observed increase in bacterial numbers is due to the physical separation of organic matter, has not been definitely determined, although the evidence seems to point to the former assumption as the most logical explanation.

Observations extending over the entire seasonal cycle have been made of the numbers of bacteria per capita added to the stream by the sewage pollution from Cincinnati, Ohio, Louisville, Ky., Chicago, and Peoria, Ill. In each instance the numbers appear to vary with seasonal temperature conditions, being considerably greater in summer than in winter. These seasonal fluctuations are shown for each of the four cities, both in terms of quantity units per capita and in billions of bacteria per capita per day, in Table 5, wherein the values for summer, for winter, and the averages for the entire year, are presented.

By combining the yearly per capita contributions of gelatin, agar, and *B. coli* counts, respectively, of all the four cities, a general average is obtained which may be considered to be roughly representative of the annual average quantity units of the respective types of bacteria contributed to these streams per capita of the sewered population.

The variation from month to month in the numbers of bacteria contributed is, in general, reasonably consistent, increasing quite rapidly to a maximum in June or July and declining again gradually until October and then more rapidly to the lower numbers found throughout the winter season. These changes in the contribution of *B. coli* from each of the four cities are shown in more detail in Table 6, in which the figures represent the ratio of the count each month to the annual average, the latter being taken as equivalent to 100. The averages of the ratios for these four cities for corresponding months represent what might be considered a general measure of the degree of change from month to month in numbers of *B. coli* contributed by urban sewered population. Similar averages have been derived for the monthly

TABLE 5.—SEASONAL CHANGES IN NUMBERS OF BACTERIA ADDED TO STREAMS BY SEWERED POPULATIONS OF CINCINNATI, OHIO, LOUISVILLE, KY., AND CHICAGO AND PEORIA, ILL.

Added by:	Quantity Units of Bacteria per Capita.*			Billions of Bacteria per Capita per Day.		
				Growing on:		
	Gelatin.	Agar.	*B. coli.*	Gelatin at 20° cent. for 48 hours.	Agar at 37° cent. for 24 hours.	*B. coli.*
Chicago:						
Summer	10.148	10.485	0.175	24 828	25 652	428
Winter	1.740	0.346	0.017	4 257	847	42
Year	6.252	4.566	0.094	15 296	11 171	230
Peoria:						
Summer	6.447	10.480	0.0912	15 773	25 640	231
Winter	0.869	3.125	0.0577	2 126	7 646	141
Year	4.518	7.894	0.0763	11 054	19 313	187
Cincinnati:						
Summer	5.764	7.486	0.238	14 102	18 314	583
Winter	1.058	0.410	0.0486	2 588	1 002	119
Year	4.811	5.009	0.1463	11 770	12 256	358
Louisville:						
Summer	5.544	6.475	0.1189	13 564	15 842	291
Winter	3.008	0.3707	0.0789	7 359	907	193
Year	4.431	3.254	0.0907	10 841	7 962	222
Averages:						
Summer	6.976	8.731	0.1565	17 067	21 362	383
Winter	1.669	1.063	0.0507	4 083	2 600	124
Year	5.003	5.181	0.1018	12 240	12 676	249

*One quantity unit = 2 446 589 000 000 000 of bacteria per day.

TABLE 6.—MONTHLY VARIATION IN WATER TEMPERATURE AND IN NUMBERS OF *B. coli* ADDED TO STREAMS BY SEWERED POPULATIONS OF CITIES. (Average for year = 100.)

Month.	Cincinnati, Ohio.		Louisville, Ky.		Chicago, Ill.		Peoria, Ill.		Average.	
	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.
January	2.4	29	2.6	60	0.3	12	0.6	2	1.5	26
February	3.3	39	3.3	109	1.0	16	0.4	77	2.0	60
March	4.1	32	4.3	91	3.7	13	4.9	228	4.3	91
April	10.9	53	11.3	69	8.6	33	9.6	13	10.1	42
May	17.9	93	18.5	123	15.0	136	18.6	158	17.5	128
June	23.2	116	23.2	79	20.9	200	24.6	494	23.0	222
July	26.2	185	26.8	174	22.4	393	25.8	51	25.3	201
August	25.5	151	26.3	122	23.8	110	25.5	24	25.3	102
September	22.4	111	23.4	174	20.1	123	23.5	64	22.3	118
October	16.7	252	17.5	108	13.8	98	14.5	66	15.6	131
November	9.3	84	11.0	26	7.4	35	7.6	23	8.8	42
December	3.9	56	5.0	66	3.2	30	4.2	0	4.1	38
Year		100		100		100		100		100

variations in numbers of bacteria growing on gelatin at 20° cent. in 48 hours and an agar at 37° cent. in 24 hours, all of which are assembled in Table 7, and plotted in Fig. 11. For purposes of comparison, the average monthly river water temperatures are also given in Tables 6 and 7.

TABLE 7.—SEASONAL VARIATION IN NUMBERS OF BACTERIA CONTRIBUTED TO STREAMS BY SEWERED POPULATIONS.
(Yearly average = 100.)

Month.	Temperature, in degrees centigrade.	PERCENTAGE OF ANNUAL AVERAGE.		
		Gelatin count.	Agar count.	*B. coli.*
January	1.5	27	13	26
February	2.0	52	20	60
March	4.3	31	22	91
April	10.1	77	32	42
May	17.5	162	107	128
June	23.0	186	227	222
July	25.3	115	168	201
August	25.3	134	188	102
September	22.3	132	161	118
October	15.6	155	166	131
November	8.8	91	69	42
December	4.1	59	27	38
Year		100	100	100

If these average values as derived are representative of the bacterial changes in streams in general, as brought about by sewage pollution, they supply a ready means of estimating the maximum concentration of bacteria to be expected in a stream of known discharge resulting from the sewage of a known population. The bacteria per cubic centimeter thus added can be computed at once by the relationship:

$$\text{Bacteria per cubic centimeter added} = \frac{\text{Population} \times \text{quantity units per capita}}{\text{Discharge, in thousands of second-feet}}$$

As an example, the average yearly numbers per cubic centimeter of *B. coli* contributed by Cincinnati (with a sewered population of 494 300) to the Ohio River, where the mean annual flow is 97 500 sec-ft., is:

$$\frac{494\,300 \times 0.1018}{97.5} = 516$$

Similarly, the concentration for any season or month may be estimated by applying to the yearly average the proper seasonal factor given in Table 7. Thus, in July, when the discharge of the Ohio River is 19 000 sec-ft., the estimated concentration of *B. coli* per cubic centimeter contributed by Cincinnati would be:

$$\frac{494\,300 \times 0.1018 \times 2.01}{19\,000} = 5\,320$$

Applying this method of estimation to the four cities of Cincinnati, Louisville, Chicago, and Peoria, it is possible to compare directly the com-

puted concentrations of bacteria to be expected in each case with the densities actually observed to have resulted from the sewage pollution contributed. Such a comparison of computed and observed concentrations of *B. coli* is presented in Table 8, together with the average monthly rates of stream

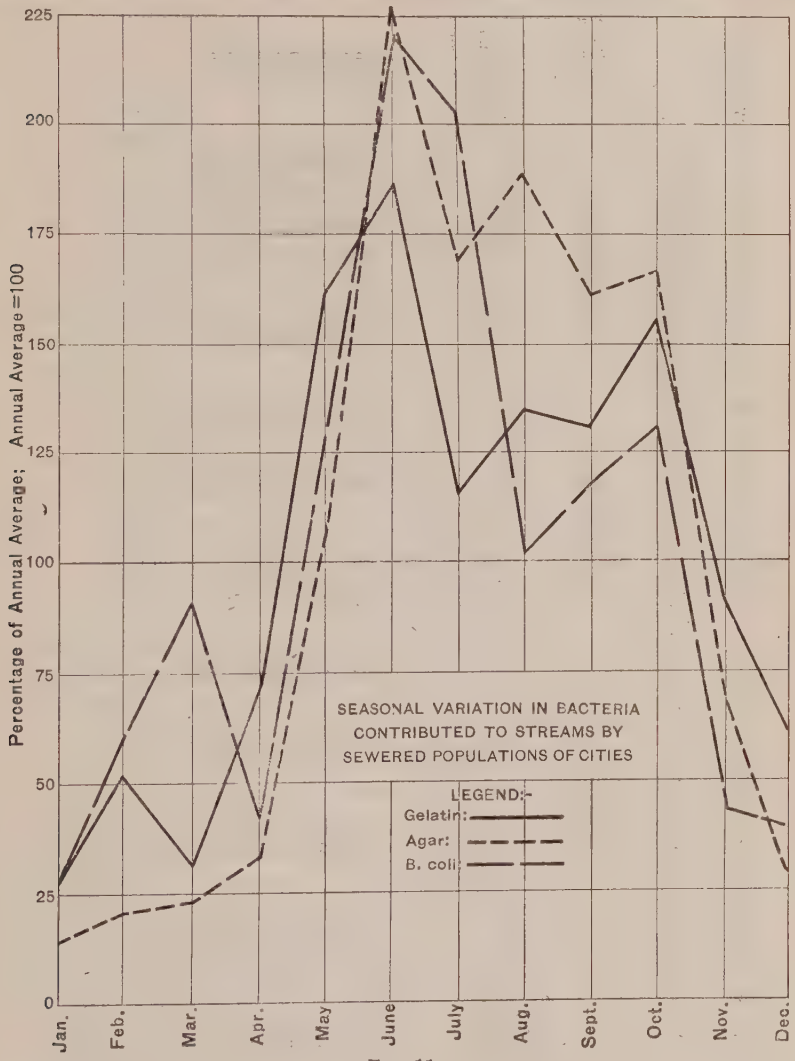

FIG. 11.

discharge and other related data. It will be noted that, although the estimated concentrations for some individual months differ quite widely from those actually observed, yet in the main the two sets of values are comparable and generally of the same degree of magnitude. Bearing in mind that the actual enumeration of *B. coli* is subject to considerable variation, it would appear that the use of the formula as a rough working model may be justified.

TABLE 8.—Comparison of Computed and Observed Concentrations of *B. coli* Contributed by Sewered Populations.

$$\left(B.\ coli \text{ per cubic centimeter (computed value)} = \frac{\text{Population} \times 0.1018 \times \text{Factor}}{\text{Discharge (Thousands of Second-Feet)}}.\right)$$

Month.	Temperature range, in degrees centigrade.	Factor.	Cincinnati,* Ohio, Population, 494 300.			Louisville,† Ky., Population, 179 800.			Chicago, Ill., Population, 2 834 000.			Peoria, Ill., Population, 76 500.		
			Discharge, in thousands of second-feet.	*B. coli* per cubic centimeter.		Discharge, in thousands of second-feet.	*B. coli* per cubic centimeter.		Discharge, in thousands of second-feet.	*B. coli* per cubic centimeter.		Discharge, in thousands of second-feet.	*B. coli* per cubic centimeter.	
				Computed.	Actual.		Computed.	Actual.		Computed.	Actual.		Computed.	Actual.
January	1.5	0.26	195.8	67	106	108.0	44	91	8.31	9 030	3 890	20.50	99	4
February	2.0	0.66	212.7	142	131	221.0	50	80	8.38	20 600	5 200	16.10	290	265
March	4.3	0.91	149.7	306	156	192.0	87	77	8.74	30 000	4 050	23.40	303	541
April	10.1	0.42	165.5	128	232	297.0	26	38	8.48	14 300	10 400	47.80	68	15
May	17.5	1.28	92.9	693	720	147.0	159	136	8.91	41 400	40 500	24.40	409	358
June	23.0	2.22	68.3	1 640	1 230	28.8	1 410	445	9.37	68 400	56 800	16.90	1 020	1 622
July	25.3	2.01	47.1	2 150	2 830	23.1	1 590	1 230	8.85	65 500	118 000	11.70	1 340	241
August	25.3	1.02	38.5	1 330	2 830	18.3	1 020	1 090	8.52	34 500	34 500	10.10	786	131
September	22.3	1.18	29.4	2 020	2 720	20.9	1 030	1 360	8.23	41 400	39 600	11.90	772	297
October	15.6	1.31	37.2	1 770	4 900	22.4	1 070	786	8.54	44 300	30 500	12.90	790	284
November	8.8	0.42	29.5	716	2 070	12.3	625	340	8.87	13 700	10 400	16.10	203	78
December	4.1	0.38	100.3	191	403	96.5	72	111	8.67	12 600	9 200	25.10	118	.....
Year	.....	1.00	.....	929	1 530	.....	599	482	....	33 000	30 300	.....	517	349

* Averages for three years, 1914, 1915, and 1916.

† Data for 1914.

Rates of Decrease in Bacterial Pollution

Quite extensive observations of the decrease of bacteria in polluted waters indicate that such changes follow a fairly regular course, modified by variations in environment, such as temperature and other factors, but yet having an orderly arrangement of reduction. Just what agency is primarily responsible for the death of such bacteria has not been definitely determined. However, there is considerable evidence suggesting that plankton activity rather than lack of food supply is the dominant influence in bacterial diminution.*

A simple and direct method for determining the rates of bacterial decrease in streams, if it were practicable, would be to observe the changes occurring in stored samples of the water under consideration. Unfortunately, the decreases in such stored samples do not correspond invariably with the natural rates occurring in the stream. Long-continued efforts—still in progress—to place the study of bacterial death rates on such an experimental basis, have thus far not been successful. Resort must then be made to the observation of natural purification occurring in streams. Under such conditions, all modifying factors are impossible of accurate control and in many cases corrections for them can be applied only in an approximate manner. Therefore, rates of decrease thus determined must necessarily be interpreted with these limitations clearly understood.

Studies of natural purification of the Ohio River† indicate that changes in the bacterial content between Cincinnati and Louisville are quite orderly and that the rates of decrease can be represented in a general way by empirical curves and formulas. Similar observations on the Illinois River have tended to confirm this conclusion and have indicated that such changes may be of general occurrence, rather than confined to these two streams. The rates of decrease in all instances are not directly comparable, however, and, as stated in the *Public Health Bulletin* referred to, these rates must be considered as approximate only, since they apparently are modified by other factors, such as density or concentration, and perhaps, also, by relative age or staleness of the sewage contributed. It is certain at least that the rates of bacterial decrease from the point of maximum concentration in the Ohio River are quite different from those observed in the Illinois. However, when the disparity in initial concentrations is taken into account and comparisons made at points of equal bacterial density, the rates coincide much more closely. This condition is perhaps best illustrated by the summer rates of decrease in bacteria growing on agar at 37° cent., as observed in the Ohio, in the Illinois River below Chicago, and, again, below Peoria, the base data of which are given in Table 9. Fig. 12 shows these rates plotted from the same origin, and Fig. 13 shows the same curves shifted so that at the points of maximum concentration they coincide with the corresponding density of the Upper Illinois River curve.

Although such an adjustment according to maximum density brings the rates into closer harmony, characteristic preliminary decreases, probably due

* "The Effect of Plankton Animals upon Bacterial Death Rates," by W. C. Purdy and C. T. Butterfield, *American Journal of Public Health*, Vol. VIII, No. 7, July, 1918, pp. 499–505.

† Presented in detail in *Public Health Bulletin No. 143.*

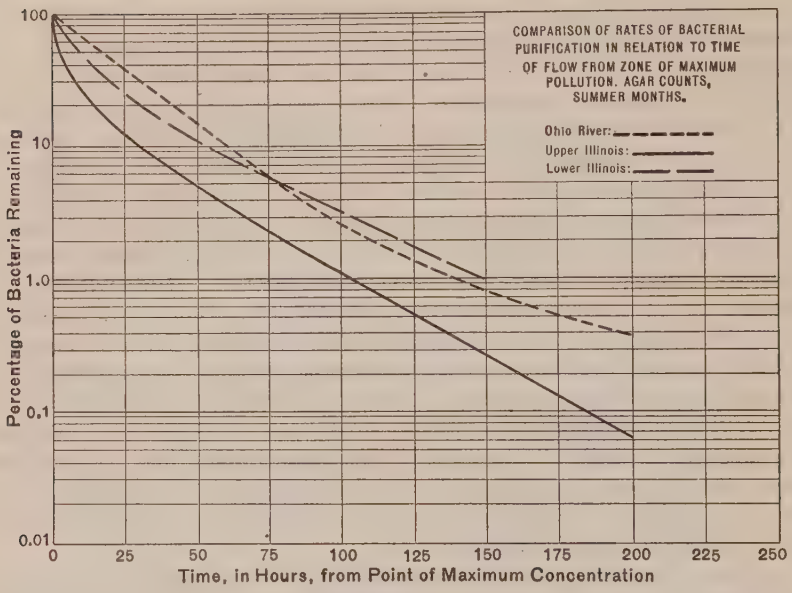
COMPARISON OF RATES OF BACTERIAL PURIFICATION IN RELATION TO TIME OF FLOW FROM ZONE OF MAXIMUM POLLUTION. AGAR COUNTS, SUMMER MONTHS.
Ohio River:
Upper Illinois:
Lower Illinois:
Percentage of Bacteria Remaining
100
10
1.0
0.1
0.01
0
25
50
75
100
125
150
175
200
225
250
Time, in Hours, from Point of Maximum Concentration

FIG. 12.

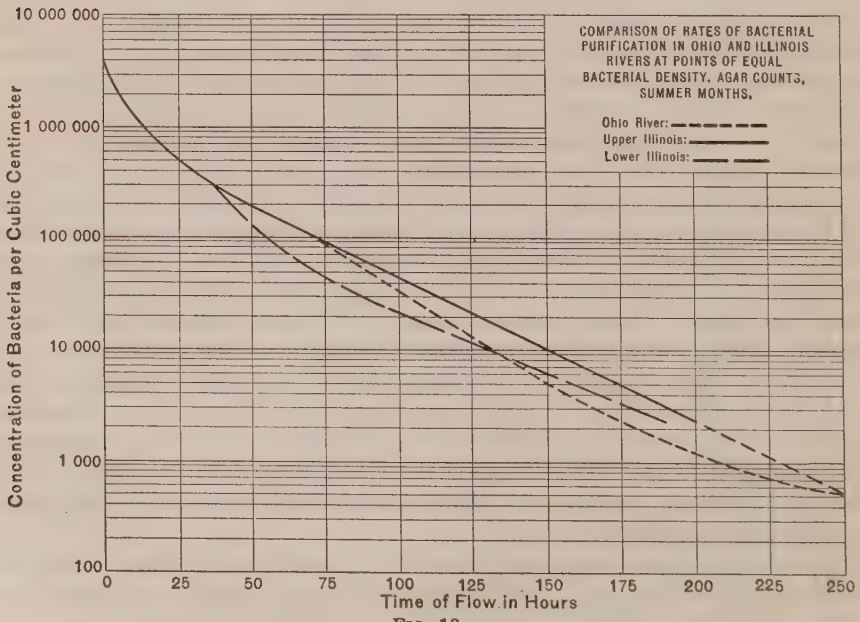
COMPARISON OF RATES OF BACTERIAL PURIFICATION IN OHIO AND ILLINOIS RIVERS AT POINTS OF EQUAL BACTERIAL DENSITY. AGAR COUNTS, SUMMER MONTHS.
Ohio River:
Upper Illinois:
Lower Illinois:
Concentration of Bacteria per Cubic Centimeter
10 000 000
1 000 000
100 000
10 000
1 000
100
0
25
50
75
100
125
150
175
200
225
250
Time of Flow in Hours

FIG. 13.

to other unknown factors, are still evident in each of the curves. It may finally prove to be impracticable, therefore, to develop a composite expression or curve defining accurately the general rate of bacterial decrease in all streams. A series of such expressions or curves, taking into consideration various modifying factors, may be found to best portray the actual rates of most probable change.

TABLE 9.—Decrease in Agar Counts, Summer Season.
(As read from curves.)

Time from maximum, in hours.	Ohio River.		Upper Illinois River.		Lower Illinois River.	
	Per cubic centimeter.	Percentage of maximum.	Per cubic centimeter.	Percentage of maximum.	Per cubic centimeter.	Percentage of maximum.
0	99 300	100.00	3 890 000	100.0	248 000	100.0
10	66 800	67.26	1 180 000	30.3	121 000	48.7
20	45 100	45.37	640 000	16.45	75 000	30.2
30	30 500	30.71	410 000	10.55	52 000	21.0
40	20 800	20.90	275 000	7.07	38 000	15.3
50	14 200	14.31	197 000	5.06	28 200	11.4
70	6 840	6.89	106 000	2.72	16 500	6.65
100	2 540	2.56	43 000	1.10	7 860	3.14
125	1 290	1.30	20 500	0.526	4 200	1.69
150	755	0.76	9 600	0.247	2 150	0.87
175	497	0.50	4 500	0.116	.....	.....
200	357	0.36	2 150	0.055	.....	.....

However, the decreases in bacterial numbers are in a broad way quite similar, and it is possible, from the available data collected, to indicate the general trend of such decrease. Such a general rate of purification, if applicable to a specific case, may assist in forming an estimate of the relative numbers of bacteria that may be expected to survive in the stream after any definite interval of time.

TABLE 10.—Decreases in *B. coli* from Various Maximum Concentrations, in Relation to Time of Flow, Summer Season.

Ohio River Curve.*		Upper Illinois River.		Lower Illinois River.	
Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.
0	2 280	0	65 600	0	3 550
10	1 459	10	22 200	10	1 300
20	934	20	11 800	20	620
30	599	30	6 700	30	340
40	385	40	4 100	40	190
70	105	70	1 100	70	40
100	32	100	385	100	11
125	14	125	170	125	4
150	8	150	76	.....	.....
175	5	175	34	.....	.....
200	4	200	15	.....	.....

*Table No. 125, *Public Health Bulletin No. 143.*

Such general rates of decrease for *B. coli,* under both summer and winter seasonal conditions, have been outlined by the data of the Ohio and Illinois

TABLE 11.—Decreases in *B. coli* from Various Maximum Concentrations, in Relation to Time of Flow, Summer Season.

Ohio River Observations Grouped by Initial Concentrations.*

Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.
0	8 237	0	4 623	0	2 557	0	1 684	0	1 211	0	607	0	239	0	165
.....	.....	.....	.....	4.7	1 746	.....	.....	4.6	1 091	2.3	325	3.8	138	2.5	77
.....	.....	.....	.....	7.3	1 560	7.3	961	8.9	728	6.0	286	6.3	100	.....	.....
.....	.....	14.9	1 741	.....	.....	11.5	1 264	.....	.....	.....	.....	12.5	70	.....	.....
17.7	1 460	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	18.2	70
24.9	1 548	22.8	1 953	.....	.....	.....	.....	23.5	336	23.6	417	.....	.....	.....	.....
28.6	1 359	29.2	951	.....	.....	25.9	197	.....	.....	27.3	174	28.3	25	.....	.....
.....	.....	36.4	604	.....	.....	38.2	178	38.7	396	.....	.....	.....	.....	35.4	51
.....	.....	43.5	450	49.4	325	45.1	402	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	62.8	429	64.2	88	35.1	73	30.8	98	.....	.....	.....	.....
.....	.....	96	122	.....	.....	.....	.....	84.5	70	.....	.....	.....	.....	.....	.....
110	94	.....	.....	118	21	101	35	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	131	328	136	18	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	159	9	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	255	5	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

* Data from Table No. 114, *Public Health Bulletin No. 143.*

River studies, and are assembled in Tables 10, 11, and 12. The rates of decrease for the Illinois River were derived from observations on the Upper Illinois River in which the sewage of Chicago is the agency of pollution, whereas in the Lower Illinois River, the major pollution is contributed by the Metropolitan District of Peoria. Daily observations at successive down-stream sampling stations were averaged over both the summer and winter seasons, and smooth curves defining the rates of natural purification were drawn through these experimentally determined results, plotted on semi-logarithmic paper. The method of obtaining the Ohio River rates of decrease is described in *Public Health Bulletin No. 143,* referred to previously. In addition to these general curves of the Ohio, data defining rates of purification at different maximum densities of bacterial content are presented in Table No. 114 of that publication, wherein are presented observations of bacterial numbers at successive sampling stations grouped according to volume of discharge of the river. By grouping these data according to initial concentration and by times from origin, average densities for each group have been obtained at various average intervals of time in hours from the point of maximum density. The results of such grouping for different initial densities are also presented in Tables 10, 11, and 12 for summer and winter, respectively, and define average rates of decrease of *B. coli* starting from the various maximum concentrations.

TABLE 12.—DECREASES IN *B. coli* FROM VARIOUS MAXIMUM CONCENTRATIONS, IN RELATION TO TIME OF FLOW, WINTER SEASON.

OHIO RIVER CURVE.*		UPPER ILLINOIS CURVE.		LOWER ILLINOIS CURVE.		OHIO RIVER OBSERVATIONS GROUPED BY INITIAL CONCENTRATIONS.†					
Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.
0	260	0	7 200	0	180	0	368	0	231	0	173
...	...	...	.....	...	...	4.6	62	3.4	115	2.3	92
...	...	...	.....	...	...	7.4	133	6.6	98	6.0	86
10	156	10	4 220	10	111	12.8	175	....	...	12.5	130
...	...	...	.....	...	...	....	...	....	...	18.8	66
...	...	...	.....	...	...	....	...	20.5	117	21.6	70
25	78	25	2 500	25	68	....	...	26.1	40	27.3	70
...	...	...	.....	...	...	38.3	46	33.9	86	....	...
...	...	...	.....	...	...	44.3	54	....	...	49.7	41
50	31	50	1 400	50	37	59.3	25	64.2	33	51.6	35
75	17	75	890	75	22	83.3	18	....	...	....	...
100	12	100	610	100	14	104	8	....	...	....	...
125	9	125	422	...	...	131	11	....	...	....	...
...	...	150	300	...	...	...	...	....	...	....	...
...	...	175	210	...	...	...	...	....	...	....	...
...	...	200	150	...	...	...	...	....	...	....	...

* Table No. 127, *Public Health Bulletin No. 143.*

† Data from Table No. 114, *Public Health Bulletin No. 143.*

If these data are plotted on semi-logarithmic paper, in which the concentrations of *B. coli* are plotted as ordinates on the logarithmic scale and the times

from maximum concentration as abscissas on the plain scale, it will be observed that these points define, in a general way, fairly smooth curves, all of which have the same general trend. It may be noted also that the curves of winter purification are of much flatter slope than are those of summer, indicating that natural purification proceeds at much slower rates during cold weather. It is entirely possible, therefore, regardless of the generally lower numbers of bacteria contributed during the winter season, that the critical or most severe period of bacterial pollution to be expected from a given sewage discharged up stream may occur during the winter, rather than during the summer. This is, of course, the reverse of what would be expected of the critical oxygen depletion condition resulting from the same sewage pollution.

Having obtained the general rates of decrease in *B. coli* for various initial concentrations, it is possible to interpolate intermediate rates of decrease, starting from definite initial concentrations. Such interpolated rates should indicate, in a general way, the numbers of *B. coli* that may be expected to survive after definite intervals of time have elapsed beyond the point of greatest bacterial density in the stream. Such rates of decrease are for convenience placed in tabular form for ready reference in Tables 13 and 14, Table 13 representing summer conditions and defining numbers of bacteria remaining after definite intervals of time, starting from various densities at the maximum point, and Table 14 presenting the same data for winter months.

TABLE 13.—NUMBERS OF *B. coli* PER CUBIC CENTIMETER REMAINING AFTER STATED TIMES OF FLOW FROM POINT OF MAXIMUM CONCENTRATION, SUMMER SEASON.

Initial maximum concentration of *B. coli*.	*B. coli* PER CUBIC CENTIMETER REMAINING AFTER INTERVAL OF:								
	10 hours.	25 hours.	50 hours.	75 hours.	100 hours.	125 hours.	150 hours.	175 hours.	200 hours.
75 000	30 000	9 800	2 900	1 000	420	190	84	37	17
60 000	20 000	7 900	2 300	800	350	150	70	32	15
40 000	14 000	5 900	2 000	600	270	120	57	27	14
20 000	7 600	3 000	1 070	420	190	94	47	25	13
10 000	4 000	1 600	640	270	130	68	36	20	12
5 000	2 300	1 100	410	170	78	42	24	15	12
1 000	440	210	80	30	13	5	....	....	....
500	260	130	70	25	8	....	....	....	....
100	40	20	12	....	....	....	....	....	....

However, before such estimates can be accepted with complete confidence, it is obviously necessary that they be checked by observations on a considerable number of streams of different physical characteristics. The empirical results herein presented outline what the observations thus far have indicated to take place, and endeavor to suggest their practical application. The explanation of the phenomena concerned in such changes must await additional research.

TABLE 14.—Numbers of *B. coli* per Cubic Centimeter Remaining After Stated Times of Flow from Point of Maximum Concentration, Winter Season.

Initial maximum concentration of *B. coli*.	*B. coli* per Cubic Centimeter Remaining After Interval of:								
	10 hours.	25 hours.	50 hours.	75 hours.	100 hours.	125 hours.	150 hours.	175 hours.	200 hours.
10 000	6 000	3 500	2 000	1 200	840	600	420	300	200
5 000	3 000	1 800	960	600	400	300	200	140	100
1 000	520	280	140	80	54	38	26	.....	.....
500	240	120	60	32	21	15	.....	.....	.....
100	62	40	20	12	7	.....	.....	.....	.....

Summary

Quite extended observations of the pollution of the Illinois and Ohio Rivers have indicated that the numbers of bacteria contributed per capita by the sewered populations of various cities are reasonably constant; these numbers change, however, with seasonal temperature, being much greater in summer than in winter. Such bacteria tend to increase in numbers in the receiving stream for a short period and then decrease at orderly rates as the time from the point of maximum density is increased. These rates of decrease were found to be affected by water temperature and apparently by concentration, being most intensive during the warmer months and under conditions where the density of bacteria was greatest.

Having established definite quantitative relationships from these observations, and assuming that they are fairly representative of stream conditions in general, a method is suggested for estimating the maximum concentration of *B. coli* in streams of known volume of flow that may be expected to result from pollution contributed by known sewered populations. Furthermore, the concentration of such organisms remaining at any point down stream may be estimated, providing the velocity of flow is ascertained.

If the observations are representative of general biological laws, they are of practical value for estimating the increasing burden placed on streams receiving the sewage of growing communities and, consequently, the added loads that water-purification plants must be prepared to handle where such polluted watercourses are used as sources of water supply.

DISCUSSION

RICHARD H. GOULD,* M. AM. SOC. C. E. (by letter).—The methods outlined by Mr. Streeter will be of assistance to those working on stream pollution problems. The mathematical treatment is interesting and is evidence of much careful work. It must be confessed, however, that for one a little rusty in mathematics it is somewhat difficult to visualize fully all the aspects disclosed and to obtain a clear understanding of the relative importance of the factors involved. This does not detract, however, from the merit of the paper. The results secured are well worth the extra effort involved in following these new methods. The work on the Ohio and Illinois Rivers and the studies reported are of great importance, and the men who are responsible for the results deserve great credit.

A number of points as to the practical application of the methods involved will bear comment. Some of these are brought out in the paper, but it may be well to emphasize them further. One of the assumptions in this method of computation is that a load of pollution entering a stream will exert a demand on the dissolved oxygen of that stream in accordance with certain definite laws of biochemical action. The demand exerted by this polluting material proceeds in an orderly manner, depending on the time elapsed, temperature, etc. The duration of contact with the waters to any down-stream point is presumed to be that of the time of flow of the water in the stream. This assumption is essentially correct where the polluting matter is free from settleable solids so that all material producing a demand on dissolved oxygen moves down stream with the water.

A serious error would appear to be introduced if parts of the polluting matter can separate out in the form of sludge on the bed of the stream. This latter condition more often than not seems to be the one actually met. Very few natural streams receiving polluting matter maintain velocities at all places and times that will prevent deposition of sewage solids. Depositions occur most readily where the stream is sluggish or the discharge is into tidal waters with reversing currents and slack water. The problem is thus complicated in most cases by the condition that, whereas part of the polluting matter will remain in suspension or solution and, consequently, its total oxygen demand within a definite time of flow can be calculated with a satisfactory degree of accuracy, the other part will exert its demand, within a particular section, for an uncertain period of time.

Mr. Streeter has discussed the effect of sludge deposits and states that the methods proposed are not accurate where such deposits exist. As a practical consideration, it is probable that the effect of settleable solids is nearly always present to a greater or less extent when raw sewage is discharged into natural watercourses. Even where no deposition occurs, the slow rolling of solid particles along the bottom may thus lengthen their period of contact and consequent oxygen demand within a given stretch of river. The relative importance of these two classes of material producing oxygen demand will

* Prin. Asst. Engr., James H. Fuertes, New York, N. Y.

vary in any particular case. Not much is known about the demand exerted by sludge and sludge deposits, but enough is known to indicate that this demand may have a considerable effect.

Another factor which adds to the importance of separate consideration of settleable solids is that solids separating out in the stream during the winter may lie more or less dormant, in regard to exertion of an oxygen demand in the stream. With the coming of the warmer summer temperatures, the rate of decomposition will increase and the decomposition of accumulated deposits will add to the oxygen demand exerted by the current summer solids. The maximum rate will be exerted at the most critical time of the year as far as the condition of the stream is concerned. The extent of this effect will depend on conditions in the stream.

The relative importance of the separate consideration of settleable and non-settleable solids may be indicated very roughly by a hypothetical case as follows: If it is assumed, for example, that raw sewage is being discharged into a sluggish stream whose time of flow to a large volume of diluting water is two days, the oxygen demand of the sewage on a per capita basis can be calculated as follows:

If the 20-day oxygen demand of raw sewage is 0.24 lb. per capita, at a water temperature of 23° cent., about 42% of the 20-day demand, or 0.10 lb. of oxygen per capita per day, would be exerted within a 2-day period by the decomposing matter, assuming that both the settleable and non-settleable solids were affected at the same time. In the particular case under consideration, however, it is assumed that the sluggish stream will permit the deposition of sewage solids, and that these will remain on the stream bed for a long time. Based on the ordinary oxygen demand test, about one-third of the oxygen demand of raw sewage is removed by sedimentation. The demand of the liquid parts of the sewage within the section considered would be then 66% of the figure for raw sewage, or 0.066 lb. per capita per day. The 20-day demand of the current settleable solids, if all were deposited and remained on the stream bed, would be one-third of 0.24 lb., or 0.08 lb. per capita. If this were increased 50%, a purely arbitrary figure, owing to the demand of old sludge banks in the stream becoming more active under the warmer summer temperatures, the total sludge demand would then be 0.12 lb. per capita, and would all be exerted in the section under consideration. The total of both the liquid and the sludge parts would be a demand of 0.186 lb. per capita per day, which is 86% above that derived on the assumption that no separation of solids took place. Absolute values will vary with existing conditions, but it would seem essential in most cases to give separate consideration to settleable and non-settleable matter.

A few engineers have felt for some time that the difference between the demand of raw and settled sewage may not be entirely brought out by the 5-day incubation test for oxygen demand. It has seemed that settled sewage would exert a higher percentage of its total demand in a period of 5 days than raw sewage with its coarser particles. How important this distinction is if there is a difference is simply a matter of speculation. In this connection, it is interesting to note in the paper by Mr. Theriault that even after

20 days a very considerable oxygen demand may be exerted, that is, the 0.24 lb. per capita oxygen demand which has been mentioned, may not represent the total, and the solids deposited in the stream may exert more of a demand than is now estimated.

The influence of sludge deposits is indeed a disturbing factor and extends to the values derived for the aeration factor, K_2. This factor, it is understood, expresses the net aeration taking place in a stream, that is, it is the total aeration taking place through the water surface less the oxygen demand of the sludge deposits in the stream. This being so, coefficients derived from rivers of the same degree of turbulence, cross-sectional areas, depths of flow, velocities, etc., might be entirely different, depending on the relative amounts of oxygen demand exerted by the sludge deposits in the different streams. If the full significance of this coefficient is clear to the writer, it would thus appear that a coefficient derived from a sluggish stream receiving raw sewage pollution might not apply to this same stream when this sewage had been treated by sedimentation or otherwise.

Definite values for the effect of sludge deposits and of the amount of aeration are difficult to determine. There is one equation with two unknowns. If the rate of true aeration in a particular case was known, the demand of the sludge could be calculated, or *vice versa*. It would seem that it might be easier to derive values for reaeration rather than for sludge deposits of uncertain age and extent. In most problems where complete data are lacking, aeration values would be the easier to apply. The three fundamental factors would be: The area of water surface, which is easily obtained; the percentage of saturation of the water, which may be measured without difficulty; and the degree of turbulence of the water. The degree of turbulence would be the most uncertain item, but it would seem that in the course of time a sufficient number of limiting values could be obtained to be of considerable aid to judgment in selecting the proper value.

The basing of aeration figures on the area of water surface rather than on the time of flow or other considerations involving the depth of water, velocity of flow, and stream characteristics, has the advantage that it may be applied both to flowing streams and to lakes and harbors where the effect of currents is not so noticeable. In most natural waters, particularly flowing streams and tidal estuaries, there seems to be little evidence that stratification of dissolved oxygen exists to any considerable extent. Where the methods outlined by Mr. Streeter are not applicable it would simplify computations if all considerations other than the area of water surface and percentage of saturation could be omitted. The particular reference in mind is that the consideration of depth of water as affecting the rate of diffusion of oxygen to the bottom layers does not seem essential. The area of water surface is readily obtainable, whereas mean depths and velocities are more difficult to secure. A convenient unit, such as pounds of oxygen absorbed per day per 1 000 sq. ft. of water surface, might be used.

An interesting approximation is suggested by the material presented by Mr. Streeter in connection with rates of reaeration. It is brought out that the rate of solution of oxygen at the surface is directly proportional to the

existing saturation deficit. It is also shown that the saturation deficit of water at 5° cent. is about 50% greater than the saturation deficit at 25° cent. when both liquids are completely devoid of dissolved oxygen. According to this law, it would be expected, neglecting the effect of temperature on the rate of solution, that oxygen would be absorbed by the colder water at a rate about 50% greater than by the warmer water. Mr. Streeter, however, gives curves, Fig. 4, showing the effect of temperature on the rate of oxygen absorption. These corrections to a large measure counteract the effect of the greater saturation deficits that are possible in colder temperatures. The relative figures for three selected temperatures are given in Table 15.

TABLE 15.—Variation of Rates of Aeration with Temperature.

Temperature, in degrees centigrade.	Saturation deficit at 0%, in parts per million.	Ratio of aeration rate to that at 20° cent.	Column (2) times Column (3).	Figures in Column (4) expressed in percentage of value at 20° cent.
(1)	(2)	(3)	(4)	(5)
5	12.80	0.79	10.1	110
20	9.17	1.00	9.17	100
25	8.38	1.10	9.22	100.3

The figures in Table 15 indicate that the net variation in the rate of absorption of the waters from the highest ordinary summer temperatures to those closely approaching the lowest in winter is not much more than 10 per cent. The difference in the range in which engineers are mostly interested is considerably less and, in any event, within the limits within which the values of aeration and oxygen demand of the waters in a stream can be determined. With the present knowledge of the values involved, would it not be permissible to express values for reaeration in terms of percentage of saturation and neglect temperature corrections, except as they are involved in determining the percentage of saturation of the liquid?

The intent of this discussion is not to detract from the importance of the work which is described by Mr. Streeter. This work marks a distinct advance in the science of the study of problems relating to the condition of polluted waters. The application of the methods outlined, however, must be made with caution in order that the fundamental assumptions in the theory developed may be in accord with the actual conditions in the particular problem to be solved.

J. Frederick Jackson,* M. Am. Soc. C. E. (by letter).—Following Mr. Hoskins' suggestion, a comparison of the results obtained by him with some of those obtained on the Naugatuck and Hockanum Rivers in Connecticut is herein attempted.

Taking the seasonal variation, the total count for bacteria in summer below Chicago, Ill., was 24 828 billions and for *B. coli,* 428 billions per capita per day; in winter, it was 4 257 billions and 42 billions, respectively. The results

* Cons. Engr., New Haven, Conn.

for Peoria, Ill., Cincinnati, Ohio, and Louisville, Ky., are similar, that is, the number of both total bacteria and *B. coli* was much larger in summer than in winter.

Below Waterbury, Conn., on the Naugatuck River, for summer the total bacteria were 4 839 billions and the *B. coli,* 1 068 millions per capita per day; and for winter the count was total bacteria, 429 billions, and *B. coli,* 1 334 millions. The results below Torrington and Ansonia, Conn., show the same relation in numbers of bacteria for summer and winter. While the results obtained on the Naugatuck River show higher numbers of bacteria in summer than in winter, the percentage increase is much less than on the Ohio and Illinois Rivers and the *B. coli* actually were greater in number in winter than in summer.

The differences are shown more clearly in the following maximum and minimum monthly counts of bacteria per cubic centimeter:

Below Torrington:	
In June: Maximum total bacteria	162 000
B. coli	37
In December: Maximum *B. coli*	1 000
Total bacteria	25 000
Below Waterbury:	
In June: Maximum total bacteria	347 000
B. coli	28
In December: Maximum *B. coli*	622
Total bacteria	23 000
Below Ansonia:	
In October: Maximum total bacteria	985 000
B. coli	8
In March: Maximum *B. coli*	280
Total bacteria	34 000

These results show a consistently higher number of *B. coli* for the winter than for the summer months. Mr. Hoskins states:

"Such bacteria tend to increase in numbers in the receiving stream for a short period and then decrease at orderly rates as the time from the point of maximum density is increased. These rates of decrease * * * being most intensive during the warmer months and under conditions where the density of bacteria was greatest."

On the Naugatuck River, the results in bacteria per cubic centimeter were:

Above Torrington:	
Total bacteria	6 000
B. coli	103
3 miles down stream below the city:	
Total bacteria	123 000
B. coli	289
5.6 miles farther down stream:	
Total bacteria	208 000
B. coli	289

Above Waterbury:
Total bacteria 30 000
B. coli 92
4.7 miles down stream below city:
Total bacteria 302 000
B. coli 164
2 miles farther down stream:
Total bacteria 424 000
B. coli 189
Above Ansonia:
Total bacteria 504 000
B. coli 28
5 miles down stream below city:
Total bacteria 479 000
B. coli 121

These results do not confirm those obtained on the Ohio and Illinois Rivers in relation to the time factor and the decrease in number of bacteria or as to the largest rate of decrease being during the warmer months.

Applying the formula proposed by Mr. Hoskins for computing the number of *B. coli* per cubic centimeter contributed to a stream by sewered populations (page 1368), and using the mean annual flow of the Naugatuck River and the coefficients used by him, the results shown in Table 16 are obtained. The difference between the computed and the actual numbers of bacteria is so great as to cast suspicion on the general applicability of the formula.

In order to determine whether industrial conditions on the Naugatuck River were responsible for the differences in the seasonal variation and the number of bacteria as computed and as actually found, the same computations were made for Rockville, Manchester, and East Hartford, Conn., on the Hockanum River where the industrial wastes entering the river are not such as should affect materially the bacteria or plankton.

TABLE 16.—Comparison of Computed and Actual Number of Bacteria.

	Total Bacteria.		*B. coli.*	
	Computed.	Actual.	Computed.	Actual.
Torrington, Conn.	1 175 000	123 000	23 088	289
Waterbury, Conn.	1 302 179	302 000	25 590	164
Ansonia, Conn.	167 721	479 000	3 295	121

Above Rockville the maximum total bacteria was, in August, 9 800, and the maximum *B. coli* was, in March, 10; 5.4 miles down stream below the city the maximum for both was, in March, total bacteria, 4 000 000 and *B. coli,* 100 000.

Above Manchester, the maximum for both was, in June, total bacteria, 765 000 and *B. coli,* 1 000; 4.6 miles below Manchester, the maximum for both was, in March, total bacteria, 620 000 and *B. coli,* 10 000; 3.5 miles down stream

at the mouth of the river, the maximum total bacteria was, in March, 780 000 and for *B. coli,* in May, 1 000.

The figures for bacteria computed according to Mr. Hoskins' formula and those actually occurring are as given in Table 17.

TABLE 17.—Comparison of Computed and Actual Number of Bacteria.

	Total Bacteria.		*B. coli.*	
	Computed.	Actual.	Computed.	Actual.
Rockville, Conn.	938 000	877 000	19 100	17 600
Manchester, Conn.	247 000	246 000	5 040	7 400
South Manchester, Conn.	70 042	150 000	12 400	2 200
East Hartford, Conn.	469 000	102 000	9 543	396

The seasonal variation compares more favorably with results on the Ohio and Illinois Rivers, and there is a fairly close agreement between the computed and the actual numbers of bacteria below Rockville and Manchester both of which have sewage disposal plants which, however, are in very poor condition. Below South Manchester and East Hartford the results vary considerably. South Manchester has a sewage disposal plant, but it is not in operation. The dye wastes discharged from the silk mills may have an inhibiting action on the bacteria. Opportunity for sedimentation in the pond above the sampling stations may be another explanation. The samples were collected above the sewer outfall at East Hartford, where the river is affected by tidal action, but this would not explain entirely the differences obtained.

While Mr. Hoskins has evidently established a relation between sewage population and *B. coli* in streams receiving wastes, the general application of his formula is apt to give quite misleading results. It is certain that it does not apply to streams of the character of the Naugatuck River, and its application to a stream similar to the Hockanum would lead to wrong conclusions unless one were familiar with conditions likely to effect its application.

The advisability of using the numbers of bacteria as an index where results are so erratic is questionable. At the best, the count of bacteria is only an estimate and the chance for error by using the prevailing technique is very great. Further, the magnitude of the number entering into the argument is so great that the variations between different places are relative only and the actual differences are hardly comprehensible.

Harrison P. Eddy,* M. Am. Soc. C. E.—The members of the Sanitary Division are indebted to Dr. Frost and the other investigators of the U. S. Public Health Service for the fundamental scientific work which they have been doing over a period of many years. Doubtless, they enjoy some satisfaction and encouragement in knowing that their investigations have provided much of the supporting data and evidence used by the Engineering

* Cons. Engr. (Metcalf & Eddy), Boston, Mass.

Board of Review in its study and in the preparation of its report on the disposal of the sewage of the Sanitary District of Chicago.

Thus far, the sewage has been disposed by dilution, only an insignificant part having received treatment prior to the discharge into the Canal System. Among the interesting conditions is the unique fact that in the summer the Des Plaines-Illinois River into which the sewage is discharged is nearly as large at its source as at its mouth. This is due to the great volume of sewage and diluting water discharged from the Main Drainage Canal of the Sanitary District into what may be termed the head-waters of the Des Plaines-Illinois River the natural flow of which is relatively small at that season of the year. At the point of confluence this flow is usually a few hundred cubic feet or less per second, whereas the discharge from the Main Drainage Canal has been as large as 10 000 cu. ft., of which about 10% was sewage.

Another matter of great interest developed during the investigations of the Board of Review is the effect of the temperature. Mr. Theriault's paper shows how the temperature has a marked effect on the volume of diluting water required to maintain certain conditions in the river. The discharge of polluting matter or sewage into a river during the cooler season of the year may be compared with refrigeration. Meat is placed on ice to prevent its decomposition and change. If sewage is discharged into a river during the winter the rate of change will be extremely slow, with the result that the volume of diluting water required to maintain a certain percentage of saturation with oxygen will be relatively small. This condition has a very important bearing in many disposal problems and advantage may be taken of it to reduce the cost of treatment of industrial wastes and sewage in many cases in the northern part of the United States. This has been done notably by certain tanneries and paper mills in Massachusetts. It is practicable for such industries to reduce the treatment of their wastes to the removal of settleable solids during the cool season of the year, whereas during the period of high temperature it may be necessary to carry the treatment much further. This principle of more fully utilizing the conditions of Nature in the economical disposal of sewage and industrial wastes undoubtedly will occupy a more important place in the operation of treatment plants in the future than it has in the past.

Another interesting subject, mentioned by Mr. Streeter, is the effect of sludge deposits in rivers. The study of the Board of Review showed that this is an important factor in the disposal of the Chicago sewage by dilution, as it has been in many stream-pollution problems in other parts of the country.

The effect of temperature upon the oxygen demand exerted by organic matter in the water is not different from the effect upon the oxygen demand exerted by the organic matter in sludge deposits. Solids suspended in the water constitute a traveling load. In winter, those which are carried by the currents from the source to the mouth of a river may not exert a great oxygen demand and they may then finally be disposed of with similar solids which may be deposited on the bed of the river. They also will exert only a relatively small oxygen demand in winter, but as soon as the temperature reaches the point where decomposition proceeds rapidly the rate at which

they exert their oxygen demand is greatly increased. Thus, a larger proportion of the oxygen demand of the year's deposit of solids is exerted in the two or three months during which the oxygen demand of the flowing load is also highest. This illustrates the importance in many cases of always removing the suspended matter from sewage and industrial wastes even if the oxygen supply in the river water during certain seasons of the year may be high.

H. A. Whittaker,* Esq. (by letter).—Dr. Frost presents in his paper a clear picture of the past and present scientific investigations of the U. S. Public Health Service on stream pollution and certain allied problems. The program of investigations in progress, which include intensive studies of the natural purification of streams, the subject of sewage treatment, and water purification, covers the fundamentals of the problem, and embraces most of the agencies involved in the protection of man against water-borne diseases. The fact that these investigations are being conducted under both experimental and natural conditions increases the possibility of obtaining accurate information on the basic laws relating to these subjects and their application to practical conditions. Since the results of these investigations have a nation-wide application, this field of study is logically one for the Federal Government to sponsor although certain problems may be undertaken in co-operation with the various States that are equipped to carry on similar activities.

It is now recognized that where public water supplies are involved, there is only one attitude toward this question and that is that the pollution of streams and lakes used as water supplies must be so limited as not to impair the safety or palatability of any water supply obtained from the same stream or lake. The other condition, that the stream or lake is not used as a source of water supply, is a phase of the problem which is more difficult of solution. Here, on the one hand, is the value of streams and lakes for recreational purposes—boating, bathing, fishing, etc.—admittedly of great value to the public but very difficult to place on a monetary basis; and the use of streams and lakes for commercial fishing, the value of which can be quite definitely established. On the other hand, is the relatively great expense of constructing and operating sewage treatment works to which municipalities and industries would be subjected if all sewage and waste previously discharged into inland waters were to be treated.

It must be admitted quite candidly that except where water supplies and bathing beaches are affected, or cattle are exposed to sewage polluted streams and lakes, the health aspect of this problem is relatively slight. In most cases the cost of constructing and operating sewage treatment works greatly exceeds the value of commercial fishing in inland waters. In many instances bathing beaches can be established in waters not subject to sewage pollution, or such facilities can be provided by the construction of artificial bathing places. Cattle may be affected by contact with sewage polluted waters, but the fact remains that these same cattle are probably in greater danger of contracting disease from infected animals that are pastured up stream on the same watershed than from municipal sewage. It would seem that, in most cases, cattle

* Director, Div. of Sanitation, Minnesota State Board of Health, Minneapolis, Minn.

can be more surely protected against all forms of water-borne diseases by fencing off the streams and providing the animals with clean drinking water from other sources than by the installation of sewage treatment works.

Apparently, where public or industrial water supplies are not affected, it must either be frankly admitted that the recreational value of streams and lakes is not commensurate with the cost of sewage treatment and the lakes and streams not used as sources of water supply must be considered simply as places for the natural disposal of sewage, or the intrinsic worth of preserving these lakes and streams in their natural condition must be set so high that there can be no question as to the soundness of a policy of treating practically all sewages and industrial wastes to such a degree that the condition of the stream or lake will be practically what might be expected if no sewage or wastes were discharged into it. Just what will be the final outcome only future developments will reveal. Already powerful organizations, such as the Isaac Walton League, are endeavoring to mould public opinion and create a public demand for protective measures against stream pollution.

The Minnesota Department of Health has taken the attitude that no drastic legislation should be enacted until a comprehensive sanitary engineering survey has been undertaken on the streams and lakes throughout the State, and until rather detail information relative to actual conditions has been obtained. After this has been done, it should be possible to outline a policy which would have a remedial effect so far as the State of Minnesota is concerned.

Minnesota contains the head-waters of several important rivers which flow into or adjacent to other States or provinces. Two adjoining States have already entered formal complaint regarding the pollution of interstate rivers. The Wisconsin State Legislature, during its 1925 session, appointed a committee to take up with the Minnesota Legislature, then in session, the question of the pollution of the Mississippi River. As a result of this action the Minnesota Legislature also appointed a committee to investigate the pollution of the boundary waters between Minnesota and Wisconsin. The two Committees have held a joint meeting and have decided to co-operate in their study of the problem which is of common interest to both States. The Committees have jointly requested the U. S. Public Health Service to detail a sanitary engineer to co-operate with the sanitary engineers of the Departments of Health of Wisconsin and Minnesota in working out a plan of scientific study of the Mississippi River in relation to stream pollution. The Cities of Minneapolis and St. Paul have also offered to co-operate in this study which is of interest to them in determining a future course to follow in regard to sewage disposal. The interests of the parties involved are so varied that any co-operative work, to be acceptable to all, must necessarily be directed by some disinterested and thoroughly competent agency such as the U. S. Public Health Service.

It is hoped that the splendid work already being conducted by Dr. Frost and his associates in the Service will be continued and expanded so as to insure the just and equitable solution of the stream pollution problem in the United States in all its phases.

KENNETH ALLEN,* M. AM. SOC. C. E. (by letter).—The four papers by Messrs. Frost, Theriault, Streeter, and Hoskins present an admirable summary of the notable investigations carried on by the U. S. Public Health Service on stream pollution which constitute a distinct advance in studies of this subject.

The pollution of bodies of salt water presents similar problems and although the principles controlling oxidation and reaeration are the same, the rates at which these take place and the products of decomposition are, because of the salinity of the water, quite different. Moreover, the tides present conditions as to velocity, direction of flow, and degree of dilution, that are quite at variance with those found in inland waters.

These modifying conditions as related to the problem of New York Harbor are thoroughly discussed in the three reports—issued in 1910, 1912, and 1914—of the Metropolitan Sewerage Commission and in the excellent paper† by H. de B. Parsons, M. Am. Soc. C. E. This discussion is confined to a résumé of the studies and work looking to a better control over the pollution of the waters of New York Harbor that have been carried out since 1914.

Dissolved Oxygen.—Chemical examinations to detect pollution have been practically confined to tests for dissolved oxygen,‡ as this has been found the best criterion of pollution for which field tests can be readily made. Beginning with the results obtained by the Metropolitan Sewerage Commission in 1909 there is, with the exception of 1910, a continuous record of warm-weather saturations.§

The average and minimum percentages found at certain representative stations and the average for the main branches of the harbor in 1909 and 1924 are given in Tables 18 and 19. About one-half the samples were taken from near the surface and one-half from near the bottom so that the averages furnish a fair measure of the general condition of the waters.

The general average summer saturations in the main branches of the harbor are shown in Table 20.

The most striking loss of dissolved oxygen is seen to be in the Lower East River, where the degree of saturation in 1924 was only 40% of that of 1909.

In the summer of 1921 exceptionally low saturations were noted:

At Throgs Neck, an average of 49 and a minimum of 24 per cent.
At the Narrows, an average of 35 and a minimum of 15 per cent.
In the Hudson River, an average of 30 and a minimum of 12 per cent.
In the Lower East River, an average of 16 and a minimum of 0 per cent.‖
In the Harlem River, an average of 15 and a minimum of 0 per cent.

Remembering that a few days of complete depletion in warm weather would result in a very serious condition it is significant that from past experience minimum percentages may be expected any summer, not exceeding about 20%

* San. Engr., Office of Chf. Engr., Board of Estimate and Apportionment, New York, N. Y.

† "Tidal Phenomena in the Harbor of New York," *Transactions*, Am. Soc. C. E., Vol. LXXVI (1913), p. 1979.

‡ A limited number of oxygen demand, oxygen consumed, and hydrogen-ion tests have also been made in connection with special studies.

§ Published in 1917 and subsequently in the Annual Report of the Chief Engineer of the Board of Estimate and Apportionment, New York, N. Y.

‖ Off 42d Street, September 13, 1921.

in the Hudson River and 10% in the East River, whereas in the Harlem River complete depletion is noted at times every summer.

TABLE 18.—Saturations of Dissolved Oxygen at Selected Stations, 1909 and 1924.

Station.	Saturation.			
	Minimum percentage and date.		Number of samples and average percentage.	
	1909.	1924.	1909.	1924.
East River:				
Throgs Neck	88%-June 23	39%-July 17	25- 96%	14-56%
42d Street	57%-July	14%-Aug. 6	4- 63%	30-23%
23d Street	52%-July 2	15%-Aug. 6 " 21	8- 62%	30-24%
Pier 10	43%-June 25	12%-July 10	20- 62%	32-31%
Hudson River:				
Mt. St. Vincent	60%-July 7	30%-July 8	4- 70%	14-59%
Spuyten Duyvil	55%-July 7	26%-July 16	4- 71%	14-53%
155th Street	69%-July 26	27%-Sept. 2	1- 69%	14-47%
42d Street	65%-Sept. 7	22%-Aug. 7	5- 74%	18-40%
Pier A	57%-July 22	25%-July 8	11- 67%	38-41%
Harlem River:				
Morris Heights	46%-July 15 (University Heights)	10%-Sept. 9	4- 53% (University Heights)	16-39%
Willis Avenue	32%-July 15 (Third Avenue)	0%-July 8 Aug. 6	16- 56% (Third Avenue)	16-11%
106th Street	21%-July 15 (109th Street)	1%-July 30 Aug. 21	6- 41% (110th Street)	16-14%
Upper Bay:				
Bell Buoy 26	60%-July 16	28%-Aug. 13	14- 73%	48-42%
Robbins Reef	62%-July 22	31%-June 26	8- 70%	111-52%
The Narrows	62%-June 24	42%-July 10	16- 78%	16-73%
Kill van Kull				
Shooters Island	78%-Sept. 16	35%-July 2	2- 78%	16-46%
Arthur Kill:				
Opposite Fresh Kills	71%-Sept. 16	34%-Aug. 28	4- 85%	16-71%
Tottenville Ferry	100%-Sept. 21	61%-Aug. 28	2-101%	12-88%
Jamaica Bay:				
Barren Island	78%-Sept. 17	81%-Aug. 15	2- 80%	8-93%
Beach Channel and Long Island Railroad	80%-June 29	84%-July 25	2- 80%	8-96%
Bergen Beach	67%-Sept. 17	63%-Sept. 12	2- 68%	1-63%
Mouth of Paerdergat Creek	.....	20%-July 1	.....	5-49%

Population and Sewage Flow.—The chief cause of harbor pollution is the sewage from the population of the Metropolitan District,* now estimated at 8 700 000.

The population of New York City was 5 620 048, or, excluding sailors on ships and those living on islands in the East River, about 5 586 500, in 1920, and it is estimated that by 1960 this figure will have increased to 11 900 000, as indicated in Table 21.

* Including White Plains, N. Y., on the north, the mouth of Raritan River on the south, the New York City limits on the east, and Paterson, Summit, and Perth Amboy, N. J., on the west; in all about 700 sq. miles, within a range of from 15 to 20 miles from New York City Hall, Report, Metropolitan Sewage Comm., 1910.

TABLE 19.—Dissolved Oxygen—Averages of Samples Analyzed Between June 1 and October 1 for Each of the Main Branches of New York Harbor.

Year.	Total number of samples.	Average Percentage of Saturation.						
		Hudson River below Spuyten Duyvil.	Harlem River.	Upper East River.	Lower East River.	Upper Bay.	Kill van Kull.	The Narrows.
1909*	404	72	55	86	65	67	79	83
1911*	861	62	42	69	54	72	70	76
1912*	150	58	..	..	49	64	65	71
1913*	880	57	29	..	43	66	65	69
1914	473	50	30	50	40	71	..	68
1915	245	43	28	..	33	72	..	78
1916	176	46	24	..	26	64	..	63
1917	288	42	22	47	29	50	..	63
1918	54	54	23	50	21	56	..	61
1919	320	36	29	30	24	51	35	58
1920	264	44	23	50	27	43	42	52
1921	258	30	15	37	16	33	38	35
1922	280	44	26	51	26	51	51	60
1923	354	37	27	38	22	47	43	57
1924	627	44	26	45	26	48	48	73

* By Metropolitan Sewerage Commission.

In the entire Metropolitan District, as defined by the Metropolitan Sewerage Commission,* there were 5 834 857 inhabitants in New York State and 1 863 577 in New Jersey—a total of 7 698 434—in 1920, and by 1960 it is believed these figures will be increased to 12 312 000 in New York and 3 377 000 in New Jersey, or a total of 15 689 000. The distribution of these populations is given by counties in Table 22.

TABLE 20.—Dissolved Oxygen—Average Percentage of Saturations in Main Branches of New York Harbor, June 1 to October 1.

Locality.	1909.	1914.	1924.	Percentage of 1909 value.
The Narrows	78	68	75	94
Upper Bay, Robbins Reef	70	71	52	74
Hudson River	72	50	44	61
Harlem River	55	30	26	47
Upper East River	86	50	45	52
Lower East River	65	40	26	40

The volume of sewage entering New York Harbor from New York City is estimated at 940 000 000 gal. per day for 1920 and 1 757 000 000 gal. per day for 1960, and the quantity from each Borough to each of the main divisions of the harbor is given in Tables 23 and 24. From these tables it will be seen that about four-fifths of the total quantity is received above The Narrows,

* Including White Plains, N. Y., on the north, the mouth of Raritan River on the south, the New York City limits on the east, and Paterson, Summit, and Perth Amboy, N. J., on the west; in all about 700 sq. miles, within a range of from 15 to 20 miles from New York City Hall, Report, Metropolitan Sewage Comm., 1910.

through which it flows to the Lower Bay. These estimates are based on the water supply, which averaged 131 gal. per capita of the residential population in 1920 and is estimated to average 130 gal. per capita in 1960, to which is added a small volume due to infiltration of ground-water.

Aside from the sewage of New York City there is that from the outlying population. If this is taken as averaging 80 gal. per capita daily, it amounts

TABLE 21.—Population of New York, N. Y., Recorded in 1920 and Expected in 1960.

Tributary in 1920 to	Manhattan.	The Bronx.	Queens.	Brooklyn.	Richmond.	Total.
Hudson River	793 000	2 000	.....	.....	.....	795 000
Harlem River	607 000	406 000	.....	.....	.....	1 013 000
Upper East River	.....	323 000	127 000	.....	.....	450 000
Lower East River	857 000	.....	186 000	942 000	.....	1 985 000
Upper Bay	.....	.....	.....	696 000	33 000	729 000
Kill van Kull	.....	.....	.....	.....	48 500	48 500
Arthur Kill	.....	.....	.....	.....	18 000	18 000
Jamaica Bay	.....	.....	156 000	373 000	.....	529 000
Lower Bay	.....	.....	.....	.....	17 000	17 000
Long Island Sound	.....	2 000	.....	.....	.....	2 000
Total	‡2 257 000	*733 000	*469 000	†2 011 000	116 500	5 586 500
Tributary in 1960 to						
Hudson River	1 499 000	511 000	.....	.....	.....	2 010 000§
Harlem River	421 000§	631 000§	.....	.....	.....	1 052 000
Upper East River	.....	962 000	850 000	.....	.....	1 812 000
Lower East River	869 000	.....	401 000	1 094 000	.....	2 364 000
Upper Bay	.....	.....	.....	1 523 000	153 500	1 676 500
Kill van Kull	.....	.....	.....	.....	408 500	408 500
Arthur Kill	.....	.....	.....	.....	280 000	280 000
Jamaica Bay	.....	.....	849 000	1 183 000	.....	2 032 000
Lower Bay	.....	.....	.....	.....	258 000	258 000
Long Island Sound	.....	7 000	.....	.....	.....	7 000
Total	2 789 000*	2 111 000*	2 100 000	3 800 000	1 100 000	11 900 000

* Marble Hill (population 1 000 in 1920; 11 000 in 1960) included for sewerage studies in The Bronx.

† Sailors on ships, 7 000.

‡ Population on outlying lands and sailors on ships, 26 000.

§ Includes population on some areas at present draining to Harlem River, but intended eventually for the Hudson River (295 000 from Manhattan and 292 000 from Jerome Avenue area, The Bronx).

to 166 000 000 gal. per day from a population of 2 078 386 in 1920 and will amount to 303 000 000 gal. per day from 3 788 996 persons in 1960. The total volume then produced in the Metropolitan District will be 2 060 000 000 gal. per day.

Causes of Pollution.—As already mentioned, the chief cause of harbor pollution is the sewage from the 8 700 000 inhabitants of the Metropolitan District. That entering each main branch of the harbor from each Borough of New York in 1920 is given in Table 23 and as forecast for 1960 in Table 24. With a uniform increase the volume in 1925 may be taken at 1 050 000 000 gal. per day, and assuming a daily volume of 80 gal. per capita from the

2 300 000 persons in the outlying sections of the Metropolitan Area the total volume now produced, in gallons per day, is,

$$1\,050\,000\,000 + (80 \times 2\,300\,000 = 184\,000\,000) = 1\,234\,000\,000$$

TABLE 22.—Population of Metropolitan District, Recorded in 1920 and Expected in 1960.

	1920.	1960.
New York:		
New York City	5 620 048	11 900 000
Westchester County	212 172	405 924
Nassau County	2 637	6 053
Total	5 834 857	12 311 977
New Jersey:		
Bergen County	148 741	365 953
Essex County	634 946	1 152 794
Hudson County	629 154	1 037 042
Middlesex County	55 130	111 384
Passaic County	239 217	375 021
Union County	156 389	334 825
Total	1 863 577	3 377 019
Total in Metropolitan District	7 698 434	15 688 996

TABLE 23.—Sewage Discharged Into New York Harbor from the City of New York in 1920.

Source.	Million Gallons Per Day.					
	Manhattan.	The Bronx.	Brooklyn.	Queens.	Richmond.	Total.
Hudson River	159	1	....	....	....	160
Harlem River	68	72	....	....	....	140
Upper East River	....	65	....	46	....	111
Lower East River	137	....	103*	27*	....	267*
Upper Bay	....	....	88	....	9	97
Kill van Kull	....	....	....	....	15	15
Total above The Narrows						790
Lower Bay	....	....	....	....	17	17
Jamaica Bay	....	....	59	52	....	111
Arthur Kill	....	....	....	....	22	22
Total below The Narrows						150
Grand total						940

* Allowing for 14 000 000 gal. per day diverted from the Borough of Queens to the East River at North 12th and South 5th Streets, Brooklyn.

Of the sewage received from outside of New York City by far the most important contribution is from the Passaic Valley Sewer District. The popu-

lation of this District is estimated at 995 000 and the sewage delivered to the outlet is understood to approximate 135 000 000 gal. per day. No such volume is received at any other outlet in the Metropolitan District, the 1 050 000 000 gal. per day from the City of New York being discharged through about 465 outlets, distributed as follows:

Manhattan	190
The Bronx	30
Brooklyn	100
Queens	95
Richmond	50

TABLE 24.—Sewage Discharged Into New York Harbor from the City of New York in 1960.

Source.	Million Gallons Per Day.					
	Manhattan.	The Bronx.	Brooklyn.	Queens.	Richmond.	Total.
Hudson River	248.08*	76.83†	.....	.....	.....	324.91
Harlem River*	.....	.....	.....	.....	.....	.....
Long Island Sound	.....	1.27	.....	.....	.....	1.27
Upper East River	.....	150.14	.....	104.85¶	.....	254.99
Lower East River	212.14§	92.83‖	160.04‡	55.07‡¶	.....	520.08
Upper Bay	.....	.....	194.08	.....	21.16	215.24
Kill van Kull	.....	.....	.....	.....	54.49	54.49
Total above The Narrows						1370.98
Lower Bay	.....	.....	.....	.....	43.48	43.48
Jamaica Bay	.....	.....	158.45	133.40	.....	291.85
Arthur Kill	.....	.....	.....	.....	51.09	51.09
Total below The Narrows						386.42
Grand total						1757.40

*Assuming 48 750 000 gal. per day diverted from the Harlem River to the Hudson River.

†Assuming 43 260 000 gal. per day diverted from the Harlem River to the Hudson River.

‡Assuming 13 970 000 gal. per day diverted from Queens to Brooklyn.

§Assuming 69 000 000 gal. per day diverted from the Harlem River to the Lower East River.

‖Assuming 92 830 000 gal. per day diverted from the Harlem River to the Lower East River.

¶Assuming 25 170 000 gal. per day diverted from the Upper East River to the Lower East River.

Besides the domestic sewage there is more or less industrial waste. This is an important consideration in the Passaic Valley Sewer District where there are large woolen, silk, dye, chemical, and metal works and where their waste constitute a material part of the total flow. In New York there are large gas and oil works and a limited number of other plants, such as packing houses, dye works, etc., while along the New Jersey shores of the Arthur Kill there are important chemical, copper, oil, asphalt, and fertilizer works. Within the New York area industrial wastes, aside from oil, do not materially affect general conditions. Although it is admitted that, in spite of the required

use of separators at garages, large quantities of gasoline and case oil reach the sewers and are the cause of occasional explosions, the total domestic sewage is so much larger in proportion that oil from these sources is not recognizable at most sewer outlets. The fields of sleek and oil found on the open water are mainly due to the surreptitious pumping out of oil tanks on vessels, from careless or accidental spills at refineries and gas works, and from the wastes received or washed off by the tide at ship-repair yards.

Sludge beds increasing the depletion of oxygen in the outlying waters are prevalent in the slips, in Wallabout Basin, Gowanus Canal, Newtown Creek, Newark Bay, off Hoboken, Bergen Point, and Elizabethport. The channels of the Lower East River and the Jersey Flats are relatively free from deposits.

Disregarding the depletion of dissolved oxygen due to sludge deposits or other than domestic sewage and assuming 0.22 lb. per capita, the writer roughly estimated the biological oxygen demand of the harbor waters in 1920 at 1 558 700 lb. per day.

To offset the effect of this demand on the oxygen of the harbor there is the replenishment of oxygen due to that brought in by the tides and fresh-water streams, that absorbed from the air, and a certain amount set free by aquatic vegetation. In some streams the latter may be the controlling factor, and at times this is probably true of Jamaica Bay, where samples frequently show supersaturation. In New York Harbor, however, as a whole, one would not be warranted in relying on so uncertain and temporary a supply. Disregarding the production of oxygen by algæ and other vegetation, and making certain rather crude assumptions as to average saturations, salinity, temperature, and stream flow, the oxygen supplied by incoming tidal and upland waters was estimated* at 1 000 600 lb., and that due to surface absorption at 863 100 lb. per day, leaving an excess available for purposes of purification of 305 000 lb. per day, provided diffusion were complete and no other factors were involved. Where the several basic conditions are in a state of such constant flux and where so many assumptions must be made on incomplete supporting data, estimates of this kind must be taken with reservation and considered merely as indicating possible conditions.

The importance of recognizing the effect of reaeration in any forecast of conditions is made very clear by Mr. Streeter and from his able analysis of the results secured in studying the Ohio River such forecasts may be made in the case of fresh-water streams of known characteristics with reasonable confidence. The reaeration of quiescent sea water is probably at a slower rate than that of fresh water although diffusion is as a rule more rapid. When disturbed by winds, eddies, or vessels, the rate of absorption is increased by bringing the less highly oxygenated water to the surface. Owing to the limited depth affected by these causes the increased rate is less in deep than in shallow waters.

These influences are fully discussed by W. M. Black, M. Am. Soc. C. E., and Mr. Earle B. Phelps in Chapter II of their report to the Chief Engineer of the Board of Estimate and Apportionment of New York City dated Feb-

* *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 718.

ruary 16, 1911. In this report the attempt is made to find an average rate of absorption for the waters of the harbor after making due allowance for salinity, depth, temperature, and the mixing action due to tides, vessels, and wind. They say:

"The action of wind and tide reverses the conditions found in quiescent water, in that the rate of absorption of oxygen per million gallons increases slightly with increasing depth down to 30 ft. With the added action of boats this effect is not so noticeable, and in the final results the combined effect results in a very even rate of absorption so that special treatment of the 12-ft. depths need no longer be considered and for all practical purposes the average rate of absorption in the Bay and Lower Hudson may be taken at 0.045, in the East River at 0.050, and in the Upper Hudson at 0.019 lb. per hour per million gallons of water in all depths less than 40 ft. The effect of aeration in depths greater than 40 ft. may safely be disregarded."

During February, March, and April, 1916, experiments were made by Warrent R. Borst, Assistant Engineer, under Theodor S. Oxholm, M. Am. Soc. C. E., then Engineer in Charge, Bureau of Engineering, Borough of Richmond, to determine the absorption of atmospheric oxygen by dilutions of sewage in fresh and salt water.* The liquid was placed in one of four tanks 25 ft. long, 6 ft. wide, and 6 ft. deep, and tested at stated intervals for dissolved oxygen (*a*) when quiescent; (*b*) when slightly agitated by two electric fans; and (*c*) (in the saline mixtures) when mechanically agitated by a strip of furring extending across the tank forming waves about 4 in. high. Temperatures were maintained at about 20° cent. by exhaust steam and simultaneous samples were taken at different depths.

As a control, tightly stoppered bottles of the mixture were kept immersed in the tank during agitation, by which the oxygen demand could be determined and allowed for. The absorption was then computed by adding the oxygen demand to the increase of dissolved oxygen in the tank.

These experiments were never carried out to such an extent that specific rates of absorption could be confidently stated, but they warranted the acceptance of certain general conclusions. With twenty dilutions of sewage with fresh water and a ½-in. ripple the absorption of oxygen after the fourth hour was 0.18 parts per million per hour, whereas with a like dilution with salt water it was 0.085 parts per million per hour. With a salt-water dilution devoid of oxygen at the start and a 4-in. wave the dissolved oxygen at the end of 4 hours at all depths was 6.56 parts per million—an increase of 1.64 parts per million per hour. After 4 hours' agitation and 20 hours' rest, the dissolved oxygen had decreased to 4.50 parts per million. In general, it was found that "reaeration is decidedly rapid and uniform throughout the body of water."

Experiments along somewhat similar lines were made by Professor Edward Ellery, of Union College, and the results given in evidence before the Supreme Court in the Passaic Valley sewer case.† These comprised studies with pol-

* Report on Sewage Experimental Investigations at West New Brighton, Staten Island, N. Y., by Warren R. Borst, 1919.

† Transcript of Record, October Term, 1916, No. 3, Original. The People of the State of New York *vs.* the State of New Jersey, and the Passaic Valley Sewerage Commissioners, Vol. 3, p. 2824, and Vol. V, p. 4603.

luted and unpolluted fresh and salt water, both quiescent and in motion. The results reached by Professor Ellery may be summarized as follows: When it is 40 to 60% saturated with dissolved oxygen, water of 60 to 70% as much salinity as sea water absorbs atmospheric oxygen at about the same rate as fresh water. This rate varies from between 0.18 cu. cm. and 0.306 cu. cm. per liter per day in quiescent water to 0.515 cu. cm. in water flowing at the rate of 1 ft. per sec. It diminishes as the humidity increases and increases with increased pollution. Under conditions of low humidity a permissible dilution of sewage in water quiescent or moving less than 1 ft. per sec. is 1 to 100, while a dilution of 1 to 60 is not permissible, resulting in an appreciable lowering of the oxygen content.

Standard of Cleanness.—Suggested standards of cleanness for New York Harbor have naturally laid stress on the dissolved oxygen content. Maximum values have varied from 70% saturation, proposed by Messrs. Black and Phelps in order to avoid detriment to major fish life, to 25% proposed by the late Rudolph Hering, M. Am. Soc. C. E., who stated: "I consider a standard of 25% with frequent sludge removal from the harbor bottom a safe protection against any nuisance arising from the water." Opinions of other authorities range between these limits. In a review of the subject* the writer gave as his opinion:

"That to ensure a supply of dissolved oxygen at all points of a stream or harbor a liberal margin is required in the main body of the water, which, in the case of New York, should not be less than from 30% to 50% saturation, depending upon the intensity of local pollution due to sewage, the septicity of the sewage, the temperature of the air and on the amount of the sludge deposits on the bottom."

The essential matter is to provide that at no point shall the water be deprived of all its dissolved oxygen. Owing to the difficulty in expressing this in definite terms that might not be misconstrued, the Metropolitan Sewerage Commission finally omitted reference to any specific degree of saturation from its recommended standard which was as follows:

"Proposed Standard of Cleanness

(1) "Garbage, offal, or solid matter recognizable as of sewage origin shall not be visible in any of the harbor waters.

(2) "Marked discoloration or turbidity, effervescence, oily sleek, odor or deposits, due to sewage or trade wastes, shall not occur except perhaps in the immediate vicinity of sewer outfalls, and then only to such an extent and in such places as may be permitted by the authority having jurisdiction over the sanitary condition of the harbor.

(3) "The discharge of sewage shall not materially contribute to the formation of deposits injurious to health or navigation.

(4) "The quality of the water at points suitable for bathing and oyster culture should conform substantially as to bacterial purity to the drinking water standard. It is not practicable to maintain so high a standard in any part of the harbor north of The Narrows, or in the Arthur Kill."

No further attempt has been made to formulate a standard.

* Annual Report, Chief Engineer, Board of Estimate and Apportionment, New York, N. Y., 1917.

Plans for Remedy.—The general principles underlying any reasonable plan of main drainage are set forth in Part II, Chapter I, of the Final Report of the Metropolitan Commission, which proceeded to describe in some detail a plan that it could recommend. Similarly, Messrs. Black and Phelps, in their report to the Chief Engineer of the Board of Estimate and Apportionment of New York, previously mentioned, presented a general plan for the collection and disposal of the sewage, while, in 1920, the writer also presented a report to the Chief Engineer of the Board reviewing the entire subject and submitted a tentative revision of the general plan.* Based on these earlier studies and taking advantage of the more recent developments in methods of treatment, the tentative plan as now proposed to meet 1960 conditions may be briefly stated, as follows:

Where a high degree of stability as well as freedom from solids is demanded, the activated sludge process promises to afford the most desirable form of treatment, and plants of considerable size are contemplated: (1) At Wards Island, to handle 187 000 000 gal. per day of sewage; and (2) south of Jamaica to handle 114 000 000 gal. per day. These, it is believed, will control the serious pollution that would otherwise occur in the neighboring parts of the East River and Jamaica Bay.

Large populations on other areas, tributary to the East River, Jamaica Bay, and the waters of Coney Island, will make it necessary to remove a much larger quantity of polluting material than can be accomplished by screening, and sedimentation plants will probably be installed in the East River at Old Ferry Point, Tallman's Island, Rikers Island, Welfare Island, near Sheepshead Bay at the precipitation plant now known as Caisson No. 4, and on Rockaway Point.

For other localities it is believed that fine screening will suffice. Of these screening plants the most important now contemplated will be at West 155th Street, Manhattan, to which about 92 000 000 gal. per day will be diverted from areas now tributary to the Harlem River and carried to a submerged outlet 1 000 ft. or more from shore. If this plan should be carried out it would be equivalent to a reduction in population on the tributary drainage areas roughly estimated at 1 180 000 in 1960, or 33% of the resident population, as follows:

Assuming a removal of 5% of the putrescible matter by fine screening, 33⅓% by sedimentation, and 85% by the activated sludge process, the results would be:

5%	of 1 992 000 persons by screening	100 000
33⅓%	of 342 000 persons by sedimentation	181 000
85%	of 1 052 000 persons by activated sludge.....	899 000
	Equivalent to a reduction in population of.....	1 180 000

Stated another way, the result would be the same as if the population were decreased from 3 586 000 to 2 406 000. The estimated cost of carrying out this

* Annual Report, Chief Engineer, Board of Estimate and Apportionment, New York, N. Y., 1920.

project, including interception, is $33 442 000, or $28 341 for the elimination of the pollution resulting from each 1 000 persons.

Most of these projects have never advanced to the point of authorization or even of general discussion and acceptance. It is probable that they will, in large measure, be adopted and the works built piecemeal as demanded, but modified as to detail to meet the situation. It is hardly open to argument (*a*) that to protect the waters of the Lower East River and Jamaica Bay, either a large volume of the sewage must be entirely removed or else some method of treatment equivalent to the activated sludge process must be used; and (*b*) that, for the further protection of the large volumes of sewage that will eventually reach the East River, other large volumes will have to undergo some more complete treatment than fine screening, but for the majority of the smaller installations this will be all that can be reasonably required. Chlorination is desirable where bathing beaches or shellfish are to be protected, but otherwise would appear to be an unwarranted expense.

Progress already made in carrying out this program includes the following items:

On the Hudson River, a fine-screening plant with two 14-ft. Riensch-Wurl screens, chlorination, and submerged outlet has been installed at Dyckman Street, Manhattan.

A fine-screening plant with three 6 by 55-ft. Rex screens and chlorination has been installed at Canal Street, Manhattan, for the Clarkson Street sewage brought to the plant by a 54-in. intercepting sewer.

On the Upper East River, a fine screening plant with two 14-ft. Riensch-Wurl screens, chlorination, and a 36-in. submerged outlet, 3 280 ft. long, has been installed at 25th Street, near North Beach, Queens.

On Staten Island, a fine-screening plant with two Dorrco screens, 6 ft. in diameter by 4-ft. face, with chlorination and a submerged outlet, was installed in 1923. This is operated in the summer for the protection of bathers at South Beach and Midland Beach.

A fine-screening plant with two Link-Belt screens, chlorination, and a submerged outlet to protect a bathing beach near Great Kills is under construction.

On the shores of Jamaica Bay at the old 26th Ward Plant, Brooklyn, two 14-ft. Riensch-Wurl screens which have been in operation for several years and two new 26-ft. screens of the same type have been installed in the building that heretofore has housed the tanks of a precipitation plant.

In Queens, a plant comprising two 26-ft. Riensch-Wurl screens is being constructed south of Jamaica to serve the Fourth Ward of that Borough (north of Jamaica Bay) and one with two 22-ft. Riensch-Wurl screens at Hammel to serve the Fifth Ward (The Rockaways). The former constitutes the first installation of an activated sludge plant to treat 120 000 000 gal. per day of sewage from the entire Fourth Ward of Queens.

These fine-screening plants (not to mention several temporary hand-operated screens in Queens) probably handle about 85 000 000 gal. per day of sewage—a small quantity, considering the outlay—but at most of the plants the increase in volume will be rapid. In the greater part of the city the main lines of development are fairly well determined. The two major remaining problems

are those relating to the sewage tributary to Wards Island and that of South Brooklyn. Wards Island, although belonging to Manhattan, is leased to the State for institutional purposes until 1963, but it is hoped to obtain a release of about 50 acres through legislative action. This will clear the way for initiating the work of interception and treatment—a measure of great sanitary importance to the city.

The entire subject of handling the sewage of Brooklyn and Queens tributary to Jamaica Bay was treated in a report submitted by the writer to the Chief Engineer* in 1917. At that time a plan originating with the Metropolitan Sewerage Commission was recommended, conveying all the sewage tributary to the Bay from the north to a treatment plant on Barren Island. Since then developments at Barren Island and Rockaway Point to the south and a greater confidence in the reliability and inoffensive character of the activated sludge process have resulted in a modification of the original plant,† whereby the sewage of the Fourth Ward, Queens, will be treated locally by this process and the sewage of Brooklyn—at least from Coney Island to Canarsie—will be brought to an old existing precipitation plant near Sheepshead Bay, known as Caisson No. 4, for some suitable method of treatment. The sewage tributary to the 26th Ward Plant may later be treated by activated sludge or, more probably, brought to Caisson No. 4 for treatment.

To provide a central authority to co-ordinate interborough problems of sewage disposal and the control of harbor pollution a Committee on Main Drainage has been appointed, consisting of the President of each of the five Boroughs of New York and the Chief Engineer of the Board of Estimate and Apportionment, Arthur S. Tuttle, M. Am. Soc. C. E. In a broad sense, these problems are of general concern, affecting the welfare of the city as a whole, and by the agency of this Committee the interests of the different Boroughs not only may be presented, but their importance to the entire subject of the city's sanitation may be correlated and unified.

William Firth Wells,‡ Esq. (by letter).—Consideration of the natural changes which pollution undergoes in water is as essential in controlling the quality of water as artificial processes of purification. Engineering practice has been principally concerned with artificial purification, but the present administrative tendency to treat water quality as a natural resource has shown the need of further study of the phenomena of self-purification. From this viewpoint the U. S. Public Health Service studies are particularly valuable. They separate the basic factors, develop methods of measure, determine the relations existing, and lead to a simple and logical system of expression.

The fundamental conditions treated are: (1) Infection, the power of water to convey living forms which may directly affect persons, studied bacteriologically; (2) pollution, the condition produced by the decomposition of dead organic matter, determined biochemically; and (3) contamination, the altera-

* "The Main Drainage and Sewage Disposal of the Area Tributary to Jamaica Bay."

† The plan as proposed in 1917 for the Fifth Ward, comprising Far Rockaway and the Rockaway Peninsula, has since been adopted and is now under construction with a temporary screening plant and outlet at B. 87th Street, Hammel.

‡ Biologist and Sanitarian, New York State Conservation Comm., Glen Cove, N. Y.

tion in quality by contact with foreign substances, tested chemically, as in the studies of trade wastes.

The oxygen studies have been discussed and the importance of the bacteriological work should also be recognized. From data representing years of work under diverse conditions, the determinations of the average daily number of *B. coli* contributed per capita, with the seasonal variation and the rate of disappearance from the streams, make a valuable addition to quantitative bacteriology. The utilization of logarithms in presenting the results is significant. Many advantages can be pointed out: First, it would be impractical to plot directly the ordinary variations in bacterial numbers, whereas the logarithm of the number per liter gives a convenient unit throughout the entire range, from drinking water to the most intense pollution; second, self-purification expressed in logarithmic terms approximates a straight-line function; third, Wolman and Streeter have shown that the ratio of the logarithm of the bacterial content for the influent to that for the effluent in purification processes is nearly constant; fourth, the distribution of these logarithms approximates the normal frequency law, and the average logarithm, therefore, approximates the median sample; fifth, the recognized scale of water quality is more directly represented by the logarithm than by the actual number of bacteria, and water-borne typhoid is more nearly proportional to the logarithm than to the actual number of *B. coli* in a water supply; and, sixth, the dilution method used in bacteriology actually gives a logarithmic scale by the simple numbering of dilutions.

The direct use of the highest positive dilution number, therefore, gives immediately the advantages of the logarithm in a rational system of quantitative bacteriology.

J. K. Hoskins,* Esq. (by letter).—Mr. Jackson has brought out a point that should be emphasized, namely, that excessive quantities of industrial wastes, especially those of a toxic nature, such as acids, metal salts, dyes, etc., materially affect the normal biological regimen of streams into which they are discharged. In such special cases obviously it is impossible to apply any general method of quantitative estimation of the bacterial content.

The Naugatuck River in Connecticut is seriously polluted with copper and brass factory and acid wastes practically throughout its length. Below Torrington, the uppermost city on the stream, where such pollution is less intense, the average concentration of bacteria for the summer months is 248 000 and for the winter 20 800. At points farther down stream, higher counts in winter than in summer might be expected to result because of the greater dilution of the toxic wastes effected by higher stream flows.

On the Hockanum River, above sources of serious industrial waste pollution at Rockville and Manchester, computed estimates, as presented by Mr. Jackson, compare quite favorably with actual observations (Table 17). However, at South Manchester, where silk mill wastes constitute 78% of the total sewage, the lack of correlation of computed and actual results is evident. Furthermore, the observations of the stream at East Hartford were

* San. Engr., U. S. Public Health Service, Cincinnati, Ohio.

made above the sewer outlet and as stated in the published report* of this work, "the results have not indicated the presence of this sewage in the samples collected." This would appear to justify the elimination of East Hartford results from consideration.

This same bactericidal effect of toxic wastes was observed during the course of the studies of Ohio River pollution, notably in the vicinity of Pittsburgh, Pa., where the Monongahela River carries large quantities of acid mine drainage. Here, the weighted average bacterial content of the Allegheny and Monongahela Rivers above Pittsburgh was found to be 4 120 per cu. cm., and of the Ohio River, 11 miles below the city, after receiving the sewage of 827 000 people, only 1 900 per cu. cm., or a reduction of 2 200 per cu. cm.

In closing, the writer desires to state again that justification for the acceptance of the empirical formulas presented in his paper must depend on the results of further observations on numbers of different streams.

* First and Second Biennial Reports of the Industrial Wastes Board, State of Connecticut, Department of Health, Vol. I, 1918-21.

AMERICAN SOCIETY OF CIVIL ENGINEERS
INSTITUTED 1852

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Paper No. 1595

NOTES ON SHEAR IN COMPRESSION MEMBERS*

By Ralph E. Goodwin,† Assoc. M. Am. Soc. C. E.

With Discussion by Messrs. C. A. P. Turner, E. G. Walker, Henry S. Prichard, Anders Bull, and Ralph E. Goodwin.

Synopsis

The object of this paper is to apply the principles of Mechanics to the subject of shear in compression members. A method is presented by which given conditions can be analyzed and those which produce the greatest shear can be determined, thus enabling the engineer to design for the most dangerous conditions that he considers it advisable to assume.

The relationship between shear and the slope of the column axis herein given has already been published elsewhere, but as far as the writer is aware, the interesting possibilities of this relationship have not previously been developed.

In spite of the general discussion concerning shear in compression members, which took place following the collapse of the first Quebec Bridge in 1907, the fundamental principles governing shear are not so generally understood by engineers as could be desired, in view of the importance of the subject. On account of the difficulty of applying these principles directly to practical problems, their significance appears to have been overlooked by the Engineering Profession. As the slope of the column axis is a controlling condition, but is seldom known, a theoretical analysis might seem at first glance to be of no practical value. Such a conclusion, however, would be far from the truth. Closer study shows that it is impossible to make safe assumptions without a knowledge of the fundamental principles involved. Without such knowledge the engineer can not even know what possibilities exist, and, therefore, can not

* Published in December, 1925, *Proceedings*.

† Instr., Civ. Eng., Coll. of the City of New York, New York, N. Y.

design for the worst possible condition. It will be shown that the actual shear in Member A-9-L of the first Quebec Bridge must have been approximately double that indicated by the beam analogy which is used by some of the standard engineering reference books and present-day specifications. This fact could not have been determined or explained without a knowledge of the fundamental principles involved in the problem.

It is not claimed that the analysis given in this paper reduces the computation of shear to a mathematical certainty, but it does furnish a scientific basis for the necessary assumptions, and leads to the following interesting conclusions:

1.—The minimum possible shear for a given deflection, instead of the maximum value, which is desired in designing, is given by the time-honored assumption that, in respect to shear, a column is similar to a simple beam with uniform load. If the maximum deflection occurs anywhere except at the middle of the column, the maximum shear will be greater than that indicated by the beam analogy.

2.—Purely theoretical deflection curves which show the maximum deflection occurring at the middle of the column, also give the minimum possible shear for a given deflection.

The reason the actual shear in Member A-9-L of the first Quebec Bridge was twice that indicated by the beam analogy is that it deflected in an **S**-curve (see Fig. 2), and, in general, an **S**-shaped deflection curve will produce double the shear indicated by the beam analogy or by the commonly assumed deflection curves.

Fig. 1 shows a column with hinged ends deflected under load. The shear at the end of this column is obviously ab, the component of the load which is normal to the axis of the column, or

$$\text{Shear} = ab = P \sin G$$

For a parabola, $cd = f$, so that:

$$\tan G = \frac{2f}{\frac{L}{2}} = \frac{4f}{L}$$

As the sine of small angles is approximately equal to the tangent:

$$\sin G = \tan G \text{ (approximately)} = \frac{4f}{L} \text{ (approximately).}$$

Therefore,

$$\text{Shear} = P \sin G = \frac{4Pf}{L} \text{ (very nearly).}$$

As Pf is the maximum bending moment:

$$\text{Shear} = \frac{4M}{L}$$

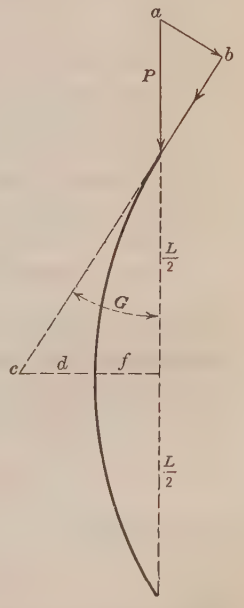

FIG. 1.—COLUMN WITH HINGED ENDS DEFLECTED UNDER LOAD.

The illustration chosen is one in which the maximum shear occurs at the end of the column. In

certain other cases the maximum shear will be elsewhere, but it will always be at a point where a tangent to the deflection curve makes the maximum angle with the direction of the applied load. This last statement may be checked from a different point of view. From the principles of Mechanics it is known that the greatest shear will occur at the point where the bending moment changes most rapidly. For a column, this will be where the tangent to the deflection curve makes the largest angle with the direction of the applied load, as stated previously.

In applying the equations given, it should be noted that the theoretical deflection curve shown in Fig. 1 produces the minimum possible shear. If the same maximum deflection occurred at the quarter-point, the inclination of the axis at the end would be greatly increased, and the shear would be increased proportionately. If an **S**-shaped deflection curve were assumed, the shear would be double that produced by the curve of Fig. 1, or $\frac{8\,M}{L}$. The expression,

shear $= P \sin G$, however, will always be true, if G is the angle between a tangent to the deflection curve at the point in question and the direction of the applied load.

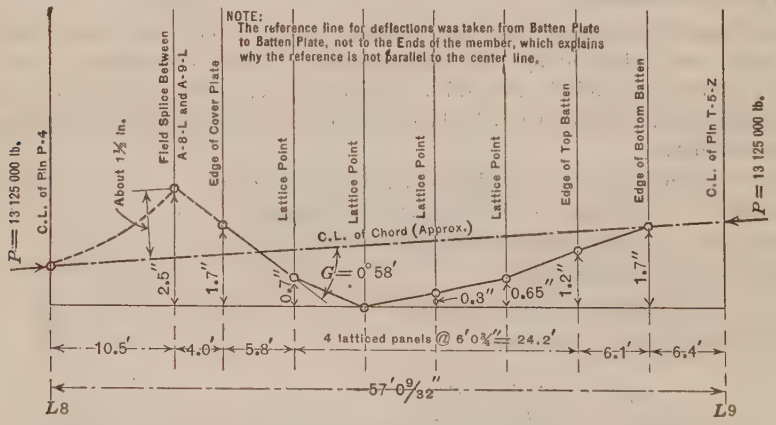

FIG. 2.—DEFLECTION OF CHORD MEMBER A-9-L, FIRST QUEBEC BRIDGE.

A frequently used allowance for shear in columns is 0.02 P. Exactly what this means can be found by applying the principles set forth. It means that the maximum value of the angle, G, must not exceed 1° 09′, if the column is to be safe. For the theoretical case shown in Fig. 1, it also means that:

$$\text{Shear} = \frac{4\,P\,f}{L} \text{ must not exceed } 0.02\,P$$

and, therefore,

$$f \text{ must not exceed } \frac{0.02\,P\,L}{4\,P} = 0.005\,L$$

The application of these principles to the much discussed chord member, A-9-L, of the first Quebec Bridge, which collapsed, is interesting. The

latticing of this member was double, at 45 degrees. There were two $\frac{7}{8}$-in. rivets in the end of each lattice-bar. According to the report of the Royal Investigating Commission, the ultimate strength of the rivets in single shear was 50 960 lb. per sq. in., and the total compressive load, P, on the chord at the time of failure was 13 125 000 lb. The maximum total shear indicated by the expression, $P \sin G$, for conditions just prior to failure (Fig. 2), is 223 125 lb., computed as follows:

$$\sin G = 0.017 \text{ (found by scaling Fig. 2)}$$

$$P \sin G = 13\,125\,000 \times 0.017 = 223\,125 \text{ lb.}$$

The total shear indicated by the beam analogy for a maximum deflection, $f = 1\frac{1}{2}$ in. (the maximum deflection reported just before failure) and a length, $L = 57.02$ ft. $= 684.3$ in., is:

$$\text{Shear} = \frac{4\,P\,f}{L} = 115\,100 \text{ lb.}$$

The ultimate total shear necessary to cause failure of the latticing on this member, computed from the value in single shear of the end rivets in the lattice-bars, is:

$$8 \times 0.601 \times 50\,960 \times \sin 45^\circ = 173\,224 \text{ lb.}$$

The method here used for computing the shear corresponding to a given stress in the lattice-bars is the approximate one that assumes the total transverse shear to be resisted by the lattice-bars. The conclusions are not changed, however, when the shear is computed by the rigorous method; that is, the actual shear (as indicated by the deflection curve and the load) was greater than the shear theoretically necessary to cause failure, and was nearly twice as great as the shear indicated by the beam analogy.

As the results just given speak for themselves, extended comment is unnecessary. The method herein described is based on the fundamental conditions of equilibrium and eliminates all assumptions as to the similarity between columns and beams. The difference between practical and purely theoretical cases has been explained and a means furnished by which engineers can design for shear in columns intelligently and with complete understanding of the significance of their assumptions. For purely theoretical deflection curves, it is not difficult to find the maximum shear corresponding to the bending allowances of the standard column formulas. It is highly important, however, to bear in mind that in practical problems the deflection curve may assume a shape which produces much greater maximum shear than the theoretical curve, without causing any increase of bending moment. The example shown in Fig. 2 illustrates such a condition.

A fixed-ended column is acted upon by end couples in addition to the applied load. These couples do not cause any shear because they produce constant bending moment and no shear results from a constant bending moment. For the purely theoretical deflection curve usually assumed, having points of contraflexure at the quarter-points, the maximum shear is found to be $\frac{4\,P\,f}{L}$, which is the same as that derived for the example represented by Fig. 1. In this

case, however, the maximum bending moment is only $\frac{P f}{2}$, owing to the counter moment of the couples, so that the maximum shear for the purely theoretical deflection curve will be $\frac{8 M}{L}$.

In this paper, the term "applied load" signifies the total force acting at the end of a column. If there is a lateral reaction in addition to the load supported (for example, the case of a column fixed at one end and hinged at the other), the applied load will be the resultant of the two forces.

An eccentric load does not produce any greater shear than is produced by a concentric load of the same amount, provided the eccentricity is the same at both ends of the column and provided the deflection curve is the same in the two columns. In the eccentrically loaded column the bending moment will be the sum of the bending moment which exists in the concentric column plus a constant bending moment due to eccentricity; but a constant bending moment does not produce any shear. Therefore, the shear in the two columns will be the same. If, however, the deflection is increased by the added bending moment due to eccentricity, the shear will be increased also, for shear is always equal to $P \sin G$.

In practical problems the eccentricity will ordinarily not be the same at the two ends, and lateral reactions will be introduced which must be considered in the computation for shear. Fig. 3 shows a column which bears on its left-hand edge at the top and on its right-hand edge at the bottom. P_1 is the load supported; P_2 is the lateral reaction, the value of which is $\frac{P_1 e}{L}$; and P_r is the resultant of the two forces. The shear at the middle of the column is $P_r \sin G_r$, but it is not necessary to compute the direction and amount of the resultant force, P_r, because the shear will be $P_2 + P_1 \sin G_1$ (very nearly). The error involved in assuming that P_2 is normal to the direction of the column axis at its middle point is negligible. For this column:

$$\sin G_1 = \frac{8 f}{L} \text{ (approximately), and shear} = P_2 + \frac{8 P_1 f}{L}$$

As this condition is usually considered dangerous, but indeterminate, it will be interesting to examine it somewhat in detail. Let it be required to find the maximum shear in a standard latticed channel column, 25 ft. long, composed of two 10-in. by 30-lb. standard channels.* It will be assumed that the conditions are similar to Fig. 3, and that no allowance was made for eccentricity, as central loading was assumed and the eccentricity resulted from poor workmanship:

Safe total load, by the formula,† $\frac{P}{a} = 19\,000 - 100 \frac{L}{r}$, = 180 000 lb.

8 in. = value of e (Fig. 3), assumed.

* Carnegie Pocket Companion, 1923, p. 225.

† Column formula, American Bridge Company.

$$P_2 \text{ (Fig. 3)} = 180\,000 \times \frac{8}{300} = 4\,800 \text{ lb.}$$

$$\text{Shear} = P_2 + \frac{8\,P_1\,f}{L}$$

In order to avoid exaggeration, f will be assumed to be only 0.1 of 1% of the length. It was 0.22 of 1% for the member, A-9-L, of the first Quebec Bridge. Therefore:

$$\text{Shear} = 4\,800 + \frac{8 \times 180\,000 \times 0.3}{300} = 4\,800 + 1\,440 = 6\,240 \text{ lb.}$$

Therefore, for the conditions assumed, the maximum shear would be 0.035 P_1.

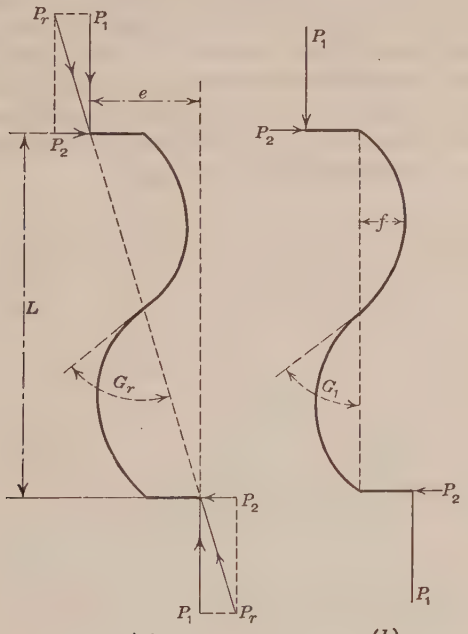

FIG. 3.

No one, least of all the writer, would claim that this result represents actual field conditions with a high degree of precision. The method, however, does furnish a rational basis of attack. It is applicable, not so much to actual design, as to the determination of standard allowances to be included in specifications and rules for standard practice.

Throughout this paper, the theoretical deflection curve has been assumed to be a parabola. Engineers who demand a more rigorous treatment will find that the difficulty of the problem is increased little, if any, by the assumption of a sine curve. A sine curve indicates a theoretical shear about 20% less than a parabola, and is, therefore, less conservative.

In closing, a word of caution is necessary against drawing unwarranted conclusions from the statements that have been made. It has been stated that, for a given deflection, an **S**-shaped deflection curve will produce twice the shear indicated by the beam analogy. It does not necessarily follow from this, however, that the shear allowance for a given column should be twice that indicated by the beam analogy. For example, a centrally loaded column, having round ends and an **S**-shaped deflection curve, is theoretically equivalent to two columns of one-half the length placed end to end. The bending effect (assuming a straight-line column formula) is only one-half what it would be if the deflection curve were a bow, and the theoretical shear to be allowed for in such a case is the same, whether the curve is a bow or **S**-shaped. The fact remains, however, that the beam analogy is a false and misleading concept. Applied to Member A-9-L of the first Quebec Bridge, it not only gives a result approximately one-half the true value, but also fails to show at what point in the member the maximum shear occurred. It gives results too small for columns designed as fixed-ended, and for all columns that develop the full contemplated bending moment at any point except the middle.

Acknowledgment is due to J. P. J. Williams, M. Am. Soc. C. E., for valuable suggestions in the preparation of this paper.

DISCUSSION

C. A. P. TURNER,* M. AM. SOC. C. E. (by letter).—The difficulty in adapting beam theory to the column is that such a theory, like a playful pup, chases its tail in a circle without practical progress. Thus, the fiber stress caused by bending varies as the bending moment; the deflection varies as the fiber stress due to bending; but for the same load, the bending moment varies as the deflection. Hence, the magnitudes of the fiber stress, deflection, and bending moment are theoretically unstable under the applied beam theory and the assumption that the bending is parabolic does not help in stabilizing sin G on which the author's calculations depend.

The amount that a column under load bends or deflects has no direct proportion to the amount of the load; nor, in fact, to the apparent eccentricity of the stressed area at mid-length; nor to the ratio of the mean compressive stress, u, to the maximum, f. However, when $\frac{f}{u}$ equals unity the eccentricity, e, is zero and the deflection also is zero.

When $\frac{f}{u}$ at mid-length equals 2 the area of fiber stress distribution is triangular and entirely compressive, that is, zero on the convex side and increasing uniformly to a maximum on the concave side. Relative deflections for a given ratio of $\frac{L}{R}$ may be plotted to illustrate typical column action, rating as unity† the deflection when $\frac{f}{u}$ equals 2.

Such a curve of relative deflections shows that the solid column of uniform material obeys Hooke's law in its elastic lateral deflections, since these deflections are proportional to the product of two factors, namely, (1) the eccentricity, e, at mid-length of the total stressed area (the lever arm); and (2) the magnitude of the triangular area (the force).

The lateral deflection of the column is a continuous function. The fact that the deflection is almost negligible at the critical load (under which the stress area changes from total compression to part tension) and reduces to but a minute fraction of this almost negligible amount under working loads, has given rise to the erroneous supposition that the deflection is a discontinuous function absolutely instead of being approximately zero under ordinary working loads.

In test specimens, except those in which the slenderness ratio, $\frac{L}{R}$, is large, no tension parallel to the axis will exist in any of the fibers until the column may be said to have failed. The common conception of the bending of a column is erroneous, being founded on the impression obtained from the curve of flexure used in deriving the formula, which indicates a considerably bent member, with a tension and compression zone representing the conditions

* Cons. Engr., Minneapolis, Minn.

† "Elasticity and Strength of Materials," Turner, Section IV, p. 8.

long after the maximum load has passed and the deflection has greatly increased under reduced load in the follow-up action of the testing machine, thus giving a false impression of the true action.

Because the safe unit stress decreases as the slenderness ratio increases various unsymmetrical rolled sections, such as angles, T's, or double angles, are used in roof trusses and H-sections, Z-bar columns, plate and channel sections, etc., for larger sections in building and bridge work. If one of these hollow unsymmetrical sections be divided in longitudinal strips, each presents a slenderness ratio of a far higher order than that of the section as a whole and hence the total resistance becomes some function of the particular combination of strips presented by the configuration of the section, requiring investigation of different typical forms to determine how the formulas for the solid section should be modified for application in design.

A strip at the extremity of an angle, T, or Z-section, is relatively lacking in rigidity laterally and will tend to buckle out of line normal to the direction in which the strut tends to bend under axial compression. Such a buckling tendency gives rise to the twisting action presented by the Z-column and other sections when tested to destruction. The extent of this torsional stress should be investigated in addition to the bending action typical of the column of solid symmetrical section.

Twisting of the column about its axis under axial compression cannot occur in a solid shaft, such as a round or a square column, because the resulting deformation increases the length of the shaft, which, under a force tending to shorten it, is contrary to Hooke's law. The failure of the solid column, therefore, is one of pure crushing and bending; and the same reasoning applies to the hollow cylinder. In the unsymmetrical section or hollow section other than a cylinder the phenomenon of twist occurs, and for a given twisting (in this case the lateral buckling tendency of the strips) the angle of distortion increases as the free length of the shaft twisted. For a column with round ends the full length of the shaft may contribute to this deformation, but if the ends be pin-connected or clamped they are fixed against torsional rotation at the ends and the free length for the summation of the twist is diminished to one-fourth the length of the column.

Christie's tests* show the strength of an angle-iron strut with ball-and-socket ends having a slenderness ratio, $\frac{L}{R}$, greater than 140, to be four-sevenths as great as one with pin ends; but the pipe column has identically the same strength whether it is pin-ended or has ball-and-socket bearings.† For the same ratio, $\frac{L}{R}$, a pipe of the same area and the same material is substantially one-seventh stronger than a corresponding angle, pin-ended, and with ball-and-socket bearings the pipe is twice as strong as an angle when the equal slenderness ratios are greater than 140. Calculated according to column

* "Elasticity and Strength of Materials", Turner, Table, Section IV, p. 27, and "Mechanics of Materials," Merriman, 1916 Edition, p. 198.

† See, "Columns," by E. H. Salmon, p. 219, Fig. 70, showing Watertown Arsenal Experiments Nos. 1915 and 1947 (1909–10).

theories which have been derived from the erroneous application of beam theory the three kinds of struts have the same theoretical strength although they differ 75 to 100% in actual strength.

No wonder that German engineers, after adopting sections for their bridge structures which would readily twist, strove to find a new theory of action of the main members that would account for the failures. They accordingly developed a theory of secondary stress instead of rectifying their theory of primary stress. They have inflicted on the profession the illogical secondary stress computations which have no practical value when the primary stresses are rationally provided for and when suitable triangular elements are adopted for the frame.

Only the minimum radius of gyration of a member is considered in the ordinary column formula, yet the maximum radius of gyration has a marked effect as regards the manner in which the section fails under compression. The Z-bar column having the same radius of gyration in both directions, if it is of minimum section, twists like a corkscrew before failure. On the other hand, the same Z-section, if the Z's are spread so that one radius of gyration is much greater than the other, will fail as they did in Strobel's tests,* by pure bending and crushing.

In large columns the symmetrical section will develop far higher resistance than those commonly used in American bridge design. The column sections in the Forth Bridge present a satisfactory distribution of metal, but the work involved was excessive. To obviate this difficulty the writer has devised a stave column, octagonal in form, with covers inside and outside the apices, using cast-steel rings at intervals to maintain the symmetrical shape of the section. Such a section is more readily spliced than those in ordinary use. It lends itself readily to the connection of riveted web members and lateral struts, and can be made accessible for painting and inspection inside as well as out.

The collapse at Quebec was the last step in the practice of saving weight of pin-plates by concentrating the metal in the cross-section of the web. The chord sections differed little from those of the Blackwell Island Bridge and of other bridges except in the seemingly conservative increase in the slenderness ratios, but this increase gave greater opportunity for twisting and buckling of the built channels, which were sadly lacking in stiffness in comparison with the rolled sections usual in the laced channel compression bridge member. The laminated webs were not comparable to the solid web of the rolled channel while the flanges, the function of which is stiffening the web against buckling, were of negligible area and small width. The need of correspondingly substantial bracing and connections of the bracing was assumed to be unnecessary. Thus, a forceful object lesson in the need of applying the principles of proportion from models of known strength and resistance is brought home by this failure.

E. G. Walker,† M. Am. Soc. C. E. (by letter).—The author has done service not only in applying mechanical principles to the determination of

* *Transactions*, Am. Soc. C. E., Vol. XVIII (1888), p. 103.

† Care, Maxted & Knott, Ltd., London, England.

shear stresses in compression members, but also in emphasizing the necessity for calculating such stresses in many cases that occur in practice. This necessity is overlooked frequently; in many sound textbooks on the mechanics of structures no treatment of the problem is attempted or even mentioned.

The basis of the treatment developed in the paper is not new. The writer has found it in textbooks with the modification that the deflection curve of the compression member has been taken as a sinusoid instead of a parabola. There does not appear to be much difference in the two treatments except that the author's assumption produces some simplification in the equations and arithmetic.

The author is right in deprecating the method of determining shear by calculations based on the assumption of an equivalent uniformly distributed lateral load. There seems to be no connection between the internal forces set up in a compression member and in the presumed equivalent beam except that the two have the same maximum direct fiber stress. Surely it is making a very broad assumption indeed, to consider that because of these equalities the maximum shear is the same in both cases. It is time that such a basis of calculation should be abandoned especially as the rational method is so easy of application.

There is some analogy between the present problem and the kindred one of determining the web stresses of a plate girder. Various standard specifications give different empirical ways of calculating the shear which is to be taken as an average and which must fall below a prescribed maximum. The quantity thus calculated is not a maximum stress and there is no reason why the actual maximum shear stress in a girder should not reach an unsafe value although the calculation should fulfill the specification; it is only a question of distribution of material. It is easy to calculate the actual shear stress at various critical points of the cross-section by rational methods and by the application of principles that are explained at an early stage in the teaching of structural mechanics. There is little justification, therefore, for the use of empirical or semi-empirical methods in this case, and the same argument applies to the problem considered in the paper.

The author's illustrative calculation is possibly the most useful feature of the paper, as it draws attention in a very definite manner to the risks inherent in the use of the equivalent beam method of calculation and cuts away the support often given to that method by illustrative examples, in which the shear calculated by it is shown to be considerably greater than that from direct calculation. The author shows that the reverse may easily be the case to a dangerous extent. The writer hopes, therefore, that this paper will draw attention to the defects of the equivalent beam method of estimating shear stress in compression members and cause more exact methods to be substituted in standard specifications.

HENRY S. PRICHARD,* M. AM. SOC. C. E. (by letter).—The "frequently used allowance" mentioned by the author is one of the requirements in the

* Analytical Engr., Am. Bridge Co., Pittsburgh, Pa.

"frequently used specifications", of a particular company, as follows: "The latticing of compression members shall be proportioned to resist a shearing stress equal to 2% of the direct stress."* The writer does not know who was the first to specify the 2%, or whether any one has attempted to give the rationale of this allowance, or whether by any mathematical ingenuity it can be made to link the "beam analogy" of a uniform transverse load with any particular column formula, as was done for a certain rule for lattice and a form of the "straight line" column formula in many editions of "Modern Framed Structures";† he does know, however, that it requires 33⅓% greater allowance for shear than, but is otherwise identical (although differently worded) with, a rule specified and used by the New Jersey Steel and Iron Company prior to 1891, referred to as Prichard's rule in the report of the Royal Commission on Collapse of Quebec Bridge,‡ which reads as follows:

"The lattice shall be so spaced that each channel between lattice connections shall be stronger than the column considered as a whole, and their size shall be obtained by treating the column as a lattice girder loaded at the middle with a load equal to 3% of the total compression on the column."

In their very able report on the collapse of the Quebec Bridge, the Commissioners state:§

"The angle of inclination may vary from point to point along the column owing to the curvature of the column. This curvature may be due to original bends or to the action of the load or to both combined. If the curvature is sufficiently small the variation of inclination due to it will be negligible. There remains, however, the original inclination or obliquity which is due to the method of application of the loads at the ends of the column. If the eccentricity of application is the same at each end and in the same plane with the axis of the column, there will be no obliquity other than that arising from the curvature of the webs which may be negligible. If, however, the eccentricities at the opposite ends are different or in different directions the obliquity may be of considerable amount. If the curvature of the column be negligible the obliquity arising from the eccentricity will be the same at every point. This obliquity causes a transfer of load throughout the whole length of the column from one web to another. This transfer of load is accompanied by longitudinal shearing stresses in the lattice system. The obliquity also causes transverse shearing stresses at every cross-section of the column."

Before the writer published his rule for lattice he realized in a general way all these facts (but he did not formulate or mathematically analyze any of them until shortly after the collapse of the Quebec Bridge when he wrote a paper on "Imperfect Butt Joints in Columns and Stress in Lattice Bars"‖). From experience and observation (which included the exciting spectacle of a violent collision on a through span of a railroad pin-connected bridge which stood the shock), he knew that columns are liable to slips and falls in handling in shop and field, to impacts in transportation, to accidental side blows in service, as indicated by telltale dents and other injuries, and to usages for which no specific provision is made in their design. After considering the

* Carnegie Pocket Companion, 1923, p. 98.

† "The Theory and Practice of Modern Framed Structures," 1896, p. 255.

‡ Vol. I, 1908, p. 126; originally published in the *Lehigh Quarterly*, June, 1891, p. 159.

§ Vol. I, 1908, p. 123.

‖ *Engineering News*, October 3, 1907.

sundry and various possibilities, liabilities, and probabilities, he concluded, regardless of theory, that columns should possess substantial resistance to shear and torsion. From all these considerations and a diligent study of specifications and designs of other engineers, Prichard's rule was devised. It was not the formulation of a fine spun theory but a simple judgment, and probably represents about the average of the best practice of the time when it was devised. Later, in the "frequently used allowance", 33⅓% was added, presumedly for good measure.

The practice with respect to lattice at the time Prichard's rule was published remained about constant up to the date of the collapse of the first Quebec Bridge, except for a growing antipathy to batten plates at intervals in place of lattice, with little or no reason to regard the sizes of the lattice bars as inadequate.

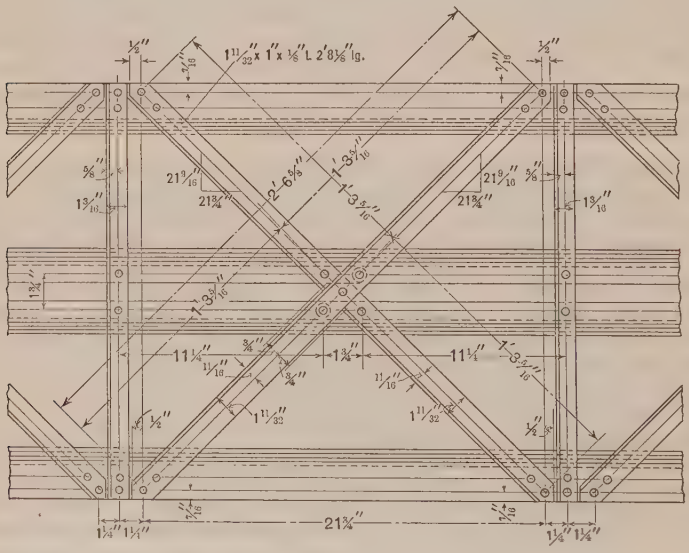

FIG. 4.

It is the consensus of opinion that the latticing of the compression chords of the first Quebec Bridge was too light. The results of an analysis of the "frequently used allowance" in connection with the "frequently used" specifications, as applied hypothetically to Member *A*-9-*L*, is strong evidence that it is conservative for a large range of ratios of length to diameter of column. Member *A*-9-*L* had a cross-sectional area of 780 sq. in.; ⅞-in. rivets; a radius of gyration at right angles to the webs of 16.1 in. and parallel to the webs of 19.7 in.; an extreme width in the plane of the lattice of 67.5 in.; and a length of 684 in.; all as given in the report of the Royal Commission.* The arrangement of cross-section and lattice bars was as indicated in Figs. 4 and 5 (which are reproductions of a portion of a drawing of a model chord, the length and

* See pp. 130 and 199.

all the linear dimension of the portion shown being one-third of those of the full-sized member):*

"The latticing between the center webs is inefficient. Intermediate latticing should have been used on these webs. One of the bars between the center webs in every panel on the upper face of the chord, *A*-9-*L*, has a net section of only 1.15 sq. in. at the center and 1.5 sq. in. for a length of about 4 in., whereas between the center web and the outside web the section is 2.48 sq. in. (gross)".†

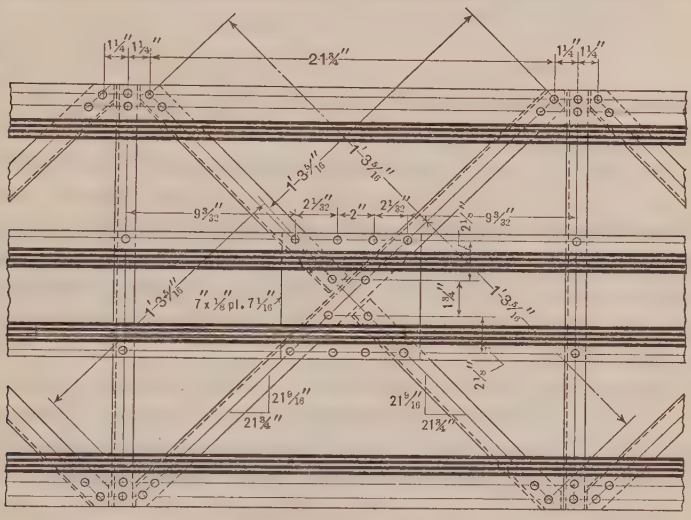

FIG. 5.

Between each panel of latticing, the chords of the member were connected by a cross-tie consisting of a 3½ by 3 by ⅜-in. angle. For this case, the "frequently used" specification with its "frequently used allowance" for shear requires each of the main lattice bars to be connected to each outside built channel by ten rivets instead of two,‡ as shown by the following computation.

The member is composed of four units (built of web-plates and angles) of equal or nearly equal cross-sectional area; and the distance of the center of gravity of each outside unit from the center of gravity of the member is 4.5 times as great as the similar distance of each inside unit; hence the bending stress in the outside units is 4.5 times as great as in the inside units, and, therefore, the shear in the lattice between the outside and inside units, is $\frac{4.5}{5.5}$ of that between the inside units (assuming intermediate latticing between the inside units, to meet the criticism of the Commission).

The limiting load on the member is:

$$13\,000 \text{ lb.} \times 780 = 10\,140\,000 \text{ lb.}$$

* Report of the Royal Commission on Collapse of Quebec Bridge, p. 112.

† *Loc. cit.*, p. 133.

‡ Correspondingly, the "frequently used allowance" requires larger lattice, but the number of rivets affords the simplest comparison between the "frequently used allowance" and the actual provision.

The member should be proportioned for shear of 2%, or 202 800 lb.; $\frac{4}{5}$ of 202 800 lb. between outside and inside channel equals 166 000 lb.

The lattice bars, of which there are four per panel or four cut by a cross-section, make an angle with the channels of slightly more than 45°, with a secant of 1.42:

The direct stress in each bar from the shear in the member = (166 000 × 1.42) ÷ 4 =	58 900 lb.
The direct stress in the member transferred to each lattice-bar as explained hereinafter =	9 200 "
Total	68 100 lb.

The number of rivets required = 68 100 ÷ 7 200 (the permissible shearing value of each rivet) = 9.46 or, say, 10.

The two rivets by which each lattice bar was actually connected to each outside unit, or web, did not shear until the computed load on the member exceeded an intensity of 16 800 lb. per sq. in. The lattice intersected at points well outside the center of gravity of the outside webs, thereby producing bending stresses in the webs and lattice and tending to increase the shear on the rivets, but the amount of the increase would doubtless have been trivial in a 10-rivet connection.

The effect of the tie at the panel points of the latticing was to force the lattice to take part in the direct compression in the member. For each lattice bar (4 by 3 by $\frac{3}{8}$-in. angle), each tie ($3\frac{1}{2}$ by 3 by $\frac{3}{8}$-in. angle), and each lattice making an angle of 45° with the webs, the writer has shown, in a discussion dealing with latticing,* that the theoretical intensity of the stress in the lattice from this cause is 28.4% of the intensity of the direct compression in the member, or 28.4% of 13 000 lb., which, applied to the gross area, 2.48 sq. in., gives 9 200 lb. as already used.

The fact that a member of the first Quebec Bridge, with a 2-rivet connection at each end of each lattice, collapsed after the computed compression in the member exceeded 16 800 lb. per sq. in., is no reason for doubting the adequacy of a rule which, as applied to the lattice bars of the member, requires a 10-rivet connection.

The other important questions which the author raises are: (*a*) The position of the eccentricities which will induce the greatest lateral reactions and, therefore, shears from lateral reactions; (*b*) the amount of unintentional eccentricity to which a member is liable from "poor workmanship"; and (*c*) the increase which deflection causes in shear.

In those cases in which the ratio of shear to direct compression is greatest, the computed increase which deflection causes in shear is small in comparison with the shear from the lateral reaction, even when the deflection, instead of being correctly computed, is arbitrarily chosen much too great, and "the theoretical deflection curve has been assumed to be a parabola", as illustrated by the author's computation of 4 800 lb. of shear from the lateral reaction and

* *Engineering News*, October 3, 1907, p. 368.

1 400 lb. increase from deflection "in a standard latticed channel column, 25 ft. long, composed of two 10-in. by 30-lb. channels."* The author has high authority for assuming a parabola† but for the purpose of determining the increase in shear, it does not give a close approximation. By correct theory the increase is much smaller than that computed by the author and may be neglected without serious error, as will be shown.

For a definite longitudinal load and definite eccentricities in application at opposite ends of the column, the lateral reactions and shears therefrom are greatest when the eccentricities are on opposite sides of the axis and are as indicated by a simple formula given by the author, which may be expressed as follows:

$$\text{Lateral reaction} = \frac{\text{Load} \times \text{sum of eccentricities}}{\text{Length of column}}$$

This simple formula strikingly shows that, other things remaining constant, the lateral reaction, and, therefore, the shear from the lateral reaction, increases inversely as the length of the column.

The unknown quantities in the formula are the eccentricities. There may be eccentricities caused by unsymmetrical variations in cross-sectional area, slight unsymmetrical variations in the modulus of elasticity, and slight curves or bends in the axis, all of which are unintentional and may be important; they will be referred to again after a discussion of eccentricities caused by "poor workmanship". The discussion of eccentricities from poor workmanship is restricted to cases in which the longitudinal load, or the major portion of it, is received, through contact of surfaces intended to be plane and intended to make a known angle, usually 90°, with the axis. The poor workmanship, where it occurs, consists in the surfaces not being plane, or in the angles not being true, or in unspecified lateral forces being applied so as to open or to tend to open the joints between the ends of the columns.

Contracts for fabrication and erection of framed structures should be awarded only to concerns which are properly equipped to face the ends of columns so that they will be smooth and square or true to angle and to properly erect the structures of which the columns are a part, and which employ engineers and superintendents who understand the importance of these matters, including the proper position and setting of the shoes and base-plates on the foundations, and the proper design, fabrication, and placing of the pieces which connect the columns. Structural engineers probably understand and are alive to these matters to a greater extent than the author realizes. For instance, certain large compression members of a comparatively recent structure were designed and faced so that they were in contact for about the middle-third and only about the middle-third of their depth. This was accomplished by a slight flare from the middle-third toward the edges.

* Carnegie Pocket Companion, 1923, p. 225.

† The late C. C. Schneider, Past-President, Am. Soc. C. E., in his report, "Quebec Bridge Inquiry," 1908, p. 195.

It is not disputed that it would be well to exercise more discretion in the award of contracts, more intelligence in their execution, and greater care in inspection, but the writer does not see the need for modifying specifications to provide for any such unintentional eccentricities as the author assumes in the suppositious case of "a standard latticed channel column" (previously referred to) to examine the matter "somewhat in detail." He assumes an eccentricity of 4 in. at each end of the column and to opposite sides of the axis at opposite ends. For this, say, the first case (with P A = 180 000 lb., L = 25 ft.; A = 17.6 sq. in.; I = 323.25 in.^{4}; R = 4.29 in.; and V = 6.75 in.),* using the method of computation and significance of symbols to be indicated subsequently in this discussion, the computed maximum shear (which occurs at the middle of the column) is 5 180 lb., or 2.88% of the load, as compared with the "frequently used allowance" of 2%, and a computed maximum stress in the extreme fiber (which occurs at the ends) of 25 260 lb. per sq. in., as against 16 000 lb. permitted by the "frequently used" specifications. If the 4-in. eccentricities were on the same side of the axis for, say, the second case, the computed maximum shear (which, for this case, occurs at the ends) is 2 349 lb. and the computed maximum stress (at the center) is 29 070 lb. per sq. in. It is almost needless to say that none of the well-known specifications provides for such eccentricities, and structures are not designed for them, yet failures have been exceedingly rare notwithstanding that loads on bridges have steadily increased.

Structures and the methods of erecting them vary so greatly that no extended discussion of erection will be attempted and attention will be called to only a few points.

A compression member in the course of its erection, held in the approximate position while connections with enough temporary bolts are made to hold it, stands more or less chance, as there is clearance in the holes, of being placed with, and temporarily held by, the friction of bolt-heads and tightly screwed nuts, with a slight V-shaped opening where it joins the adjacent member, a condition which may seem alarming to a novice; but such bolts will not hold much load without some slippage.

Even the best workmanship is not made with the precision required in astronomical and other scientific instruments. It is not surprising, therefore, that after the successive columns in buildings and chord members in bridges have come to a bearing they are likely, collectively considered, to form a slightly inclined, or bent, or wavy or curved line. The extent to which this occurs may perhaps be judged by the provision in the "Code of Standard Practice", October 1, 1924, of the American Institute of Steel Construction:†

"In the setting or erecting of structural steel work, the individual pieces shall be considered plumb or level where the error does not exceed 1 to 500.

"For exterior columns and columns adjacent to elevator shafts or multiple-story buildings, the error from plumb shall not exceed 1 to 1 000 for the total height of the column."

* Carnegie Pocket Companion, 1923, pp. 109, 113, 156.

† See p. 11.

Unsymmetrical yielding of foundations under eccentric pressure, elastic deformation of columns, and, on occasion, plastic deformation of small portions of their outer and nearly outer fibers and crushing of their bearing edges, may be useful in effecting adjustments which bring adjacent surfaces to a bearing. In a short properly proportioned column with unintentional eccentricities to opposite sides at opposite ends, buckling of the column as a whole will not occur, shortening from deformation will be trivial, and plastic deformation will relieve the column of the excessive stress which caused it. Elastic deformation of the lattice-bars will also help and plastic deformation may help; but, on the other hand, excessive stress in the lattice may shear the rivets, or break or severely cripple the bars.

For short columns particular attention should be given to the problem of shear. Columns subject to heavy loads in buildings are usually built with cover-plates instead of lattice; hence the problem of the sufficiency of the latticing in very short columns in buildings does not often arise. In columns and compression members in outdoor construction exposed to the elements it is desirable that the interiors shall not be inaccessible for painting. It is also highly important that accessibility for purposes of fabrication, erection, and painting shall be achieved in such a way that an employee engaged in such service, if overcome by exhaustion, accident, heat, cold, fumes, or otherwise, will not be in an almost closed interior from which he can not be readily rescued. To make adequate provision for the greatest possible shear in short members requires a high degree of practical as well as engineering knowledge. Concerns that understand the subject in all its phases take particular pains in such cases to reduce the possible shear from poor workmanship to a minimum.

The engineer may ask how does he know that the designedly abutting surfaces of compression members actually come to a good bearing. The fact is that, except for the cases he has personally examined and the testimony of others regarding the cases they have personally examined, he does not really know; but failures of such members are exceedingly rare, and he can infer from this fact that sufficient bearing is achieved to keep the stresses from the actual loads within safe limits; and he knows, if he is experienced, that in many cases the actual loads sustained by structures in service have been greater than those for which they were designed. The writer knows of only one failure, the first Quebec Bridge, where either the latticing or the butt joints are under suspicion. In this bridge, the bottom chord, *A*-8-*L*, extended beyond the panel point 10.5 ft. (almost one-fifth of the panel length) to a butt joint with the member, *A*-9-*L*. This long distance to the joint was occasioned by a strut which ran from the panel point making an acute angle with *A*-8-*L*. The details of *A*-8-*L* between the panel point and the butt joint are not shown in the report of the Commission previously mentioned, but it seems to the writer from the data in the report, that the out-of-line distance of 1.5 in. observed at the joint before the collapse should be regarded as the individual outward bending of insufficiently connected webs loaded at their ends rather than the deflection of a compression member between panel points.

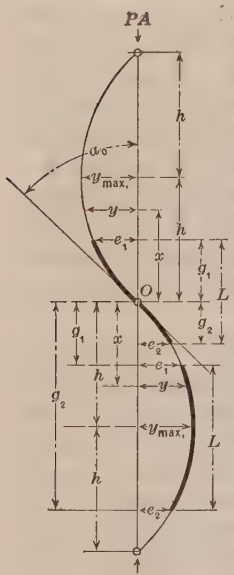

FIG. 6.

For short columns, the possible unintentional eccentricities, from unintentional variations in cross-section, slight variation in modulus of elasticity, and slight bends or curves in the axis, are all small as compared with the eccentricity assumed by the author for the "standard latticed column 25 ft. long", and do not require further discussion.

For short columns, the theoretical maximum shear, from eccentricities to opposite sides at opposite ends, is only slightly greater, than the theoretical lateral reactions (the computation of which is much simpler). To justify the formulas on which this statement is based, and from which the writer's computations were made, and to clarify ideas regarding the theory of eccentrically loaded columns, the following general analysis is given.

Fig. 6 shows the axis of a centrally loaded, continuous, symmetrical column of constant cross-section (made of perfectly elastic steel) with three points, of no flexure, one at each end and one at the middle, where it is held against lateral movement by frictionless pins which offer no resistance to the rotary movements developed in the column in deflecting. Owing to the continuity of the two halves the deflections in each half have to be equal and to opposite sides of the axis; otherwise, each half behaves precisely like an independent column and will support the same loads and have the same elastic line.

Let A = the cross-sectional area;
$2h$ = the length of each half; and
E = the modulus of elasticity.

For steel, considering the pound as the unit of stress and the inch as the unit of length, $E = 30\,000\,000$.

Let p = the load per unit of area; and
R = the radius of gyration.

As shown by Euler and numerous mathematicians since Euler, and as demonstrated in nearly all textbooks on Mechanics, the curve of incipient deflection of such a column is a sinusoid and the load which will cause incipient deflection is given by Euler's "formula":

$$p = \frac{\pi^2 E}{\frac{4h^2}{R^2}} \quad \ldots\ldots(1)$$

$$h = R\sqrt{\frac{\pi^2 E}{4p}} \quad \ldots\ldots(2)$$

In Fig. 6, which shows the column deflected, the deflection is shown greatly exaggerated for convenience in drawing and clearness in illustrating, but,

for cases which have either a direct or indirect application in engineering, the curvature, even of what is considered a large deflection, is very slight. The errors involved, in considering that the arc is the same length as the chord, that the curve is a precise sinusoid, and that the load which causes incipient flexure will hold the column in equilibrium in any deflected position, are so small that they do not seriously impair the accuracy of the computations. This was shown to be the case by William Cain, M. Am. Soc. C. E.*

For any originally straight, perfectly elastic, eccentrically loaded, symmetrical column under a given load, a certain continuous, centrally loaded, column, as shown in Fig. 6, can be conceived as held in such a deflected position by the given load that a particular part of it, the centrally loaded column, will be precisely similar to the eccentrically loaded column, as regards length, elastic line, relative position of axis and line of thrust, stresses, and shears. These characteristics under a given load of such an eccentrically loaded column, therefore, can be determined by first finding from Equation (2) the quarter length, h, of the centrally loaded continuous column of the same cross-section which can be held in any deflected position under the given load, and then, with the continuous column held in the particular position corresponding to the elastic line of the eccentrically loaded column, finding the characteristics of the part corresponding to the eccentrically loaded column.

Let y equal the ordinate from the line of thrust to the sinusoid formed by the axis of the centrally loaded column, and x the corresponding arc measured from the point of contrary flexure, O, at the middle, considered as the origin.

Let y_m = the maximum ordinate;
L = the length of the eccentrically loaded column;
g_1 = the distance from the origin point, O, to its upper end;
g_2 = the distance from the origin point, O, to its lower end;
e_1 = the eccentricity at its upper end;
e_2 = the eccentricity at its lower end;
a = the angle which a tangent to the column axis at any point makes with the line of thrust;
a_1 = the angle which a tangent to the column axis at the upper end of the eccentrically loaded column makes with the line of thrust;
a_2 = the angle which a tangent to the column axis at the lower end of the eccentrically loaded column makes with the line of thrust;
a_0 = the angle which a tangent to the column axis, at origin point, O, makes with the line of thrust;
S = the transverse shear at any cross-section;
S_1 = the transverse shear at a given point the ordinate of which is e_1 and the abscissa, g_1;
S_2 = the transverse shear at a given point the ordinate of which is e_2 and the abscissa, g_2;
S_0 = the transverse shear at origin point of contrary flexure, O;

* *Transactions*, Am. Soc. C. E., Vol. XXXIX (1898), p. 96.

f = the stress intensity in the outer fiber at any cross-section;
f_1 = the stress intensity in the outer fiber at the upper end of the eccentrically loaded column;
f_m = the maximum stress intensity;
A = the cross-sectional area of the column; and
v = the distance from the axis to the extreme fiber.

Then,

$$y_m = \frac{e_1}{\sin\left(\frac{\pi g_1}{2h}\right)} = e_1 \operatorname{cosec}\left(\frac{\pi g_1}{2h}\right) \quad \ldots\ldots(3)$$

$$\tan \alpha = \frac{dy}{dx} = \frac{\pi y_m}{2h} \cos\left(\frac{\pi x}{2h}\right) \quad \ldots\ldots(4)$$

$$\tan \alpha_1 = \frac{\pi y_m}{2h} \cos\left(\frac{\pi g_1}{2h}\right) \quad \ldots\ldots(5)$$

$$\tan \alpha_0 = \frac{\pi y_m}{2h} \quad \ldots\ldots(6)$$

$$S_1 = p A \tan \alpha_1 \quad \ldots\ldots(7)$$

$$S_0 = p A \tan \alpha_0 \quad \ldots\ldots(8)$$

$$f_1 = p + \frac{p e_1 v}{R^2} \quad \ldots\ldots(9)$$

$$f_m = p + \frac{p y_m v}{R^2} \quad \ldots\ldots(10)$$

Calling eccentricities to opposite sides of axis at opposite ends of column, Case I; and eccentricities to the same side of the axis at opposite ends of column, Case II; then,

For Case I, for $e_2 = e_1$,

$$g_2 = g_1 = \frac{L}{2} \quad \ldots\ldots(11a)$$

and for $e_2 = 0$,

$$g_1 = L \quad \ldots\ldots(12)$$

For Case II, for $e_2 = e_1$,

$$g_1 = h - \frac{L}{2} \quad \ldots\ldots(11b)$$

For any other relative value of e_1 and e_2, the values of g_1 and g_2 have to be obtained by trial, which can be readily accomplished by using Equations (13) and (14a) or (14b), with the aid of a table of sines:

$$\frac{e_1}{e_2} = \frac{\sin\left(\frac{\pi g_1}{2h}\right)}{\sin\left(\frac{\pi g_2}{2h}\right)} \quad \ldots\ldots(13)$$

For Case I:

$$\text{Angle}\left(\frac{\pi g_2}{2h}\right) + \text{angle}\left(\frac{\pi g_1}{2h}\right) = \text{angle}\left(\frac{\pi L}{2h}\right) \quad \ldots\ldots(14a)$$

For Case II:

$$\text{Angle}\left(\frac{\pi g_2}{2h}\right) - \text{angle}\left(\frac{\pi g_1}{2h}\right) = \text{angle}\left(\frac{\pi L}{2h}\right) \quad \ldots\ldots(14b)$$

Euler, and other mathematicians, who have developed Equations (1) and (2), neglected the influence of shear distortion.

Let t = the deflection due to a shear of unity in a length of unity.

The writer has previously shown* that when t is considered, Equations (1) and (2) become Equations (15) and (16):

$$p = \frac{\pi^2 E}{\frac{4 h^2}{R^2} + \pi^2 E A t} \quad \ldots\ldots (15)$$

$$h = R\sqrt{\frac{\pi^2 E}{4 p} - \frac{\pi^2 E A t}{4}} \quad \ldots\ldots (16)$$

Equations (15) and (16) clarify theory by bringing it into line with the common sense idea that even under ideal conditions potential resistance to shear is essential to capacity to support load and is really an element in determining the maximum load; but, for all cases which arise in practice, the effect of substituting Equation (16) for Equation (2) is so small it may be neglected.

The special case in which the eccentricities to the same side of the axis are equal at opposite ends, has been developed by a number of engineering authors, including Anson Marston, M. Am. Soc. C. E.,† who agree in their results. The general analysis here given leads to Marston's equation as a special case.*

For the 10-in. channel column, 25 ft. long, previously cited, the results recorded were obtained, by applying Equations (2), (3), (5), (6), (8), (9), and (10), as follows:

For both the cases previously cited:

From Equation (2), $h = 364$ in.

For Case I:

From Equations (11a) and (3), $y_m = 6.633$ in.

From Equation (5), $\tan a_1 = 0.02283$

From Equation (6), $\tan a_0 = 0.02862$

From Equation (8), maximum shear = 5 160 lb. (which is 360 lb. greater than the 4 800 lb. lateral reaction obtained by both the author and the writer).

From Equation (9) the stress in the outer fiber at the ends (which for this case is the maximum) = 25 260 lb.

For Case II:

From Equations (11b) and (3), $y_m = 5.0144$ in.

From Equation (5), $\tan a_1 = 0.1305$

From Equation (7), the shear at the end (which for this case is the maximum) = 2 349 lb.

From Equation (10), the maximum stress in the outer fiber, f_m = 29 070 lb., which is the same as would have been obtained by applying Marston's formula.

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* *Engineering News*, February 25, 1909, p. 206.

† *Transactions*, Am. Soc. C. E., Vol. XXXIX (1898), p. 108.

For Case I with the eccentricities on opposite sides of the axis, which the author chose to "examine somewhat in detail", he assumed, and illustrated in his Fig. 3, the vertices to the elastic line as occurring about half way between the ends of the column and the point, *O*, of contrary flexure, whereas the writer's analysis shows them to occur at 364 in., or 30 ft. 4 in., from the point, *O*, in prolongations of the elastic line 17 ft. 10 in. beyond the ends. If the length of the column were, say, 63 ft. instead of 25 ft., and all other conditions, including load, were the same, the vertices would lie between the point, *O*, and the ends of the column; 30 ft. 4 in. from Point *O* and 1 ft. 2 in. from the ends.

Anders Bull,* Esq. (by letter).—To apply the method outlined by Mr. Goodwin to column-lattice design, a value must first be assumed for the deflection, or, what amounts to the same thing, for the eccentricity due to imperfect workmanship or other causes, the former being a function of the latter.

The beam analogy furnishes a simple empirical rule by which this unknown factor is taken into account, whereas Mr. Goodwin would entrust its evaluation to the judgment of the designer. This would seem to be dangerous practice because it would leave too much room for guesswork. The deflection is too complex a function of loading conditions, length, and section characteristics to be divined, even by the most experienced of designers; it is far more involved than the allowable unit stress, which is always determined by a column formula.

Neither do the examples given by Mr. Goodwin support his claims. For the column shown in Fig. 3, the deflection, f, is a function of the distance, x, from the end, or, approximately,

$$f = \frac{P_1 e}{4 E I}\left(\frac{L x}{3} - x^2 + \frac{2 x^3}{3 L}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (17)$$

in which, E is the modulus of elasticity and I the moment of inertia of the column section, the other notations being those used by Mr. Goodwin.

The graph of Equation (17) is a line the curvature of which increases evenly from the middle section toward the ends. By equating to zero the first differential, the maximum deflection is found to occur at a distance of 0.211 L from either end. Substituting this value for x in Equation (17), the maximum is found to be:

$$f_{\max.} = 0.008 \frac{P_1 e L^2}{E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

The angle of inclination of the column axis to the vertical is greatest at the ends, where:

$$\tan G = \frac{P_1 e L}{12 E I} \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

the inclination at the middle section being just one-half as great.

* Asst. Engr., Board of Transportation of the City of New York, New York, N. Y.

Substituting in Equation (18) the values for the column selected,

$$f_{max.} = 0.008 \frac{180\,000 \times 8 \times 90\,000}{30\,000\,000 \times 323} = 0.106 \text{ in.}$$

This is only 0.035% of the length, as compared with the 0.1% which Mr. Goodwin takes as his starting point.

The shear, S_1, due to flexure, at the middle of the column, is:

$$S_1 = P_1 \tan G_1 = 180\,000 \frac{180\,000 \times 8 \times 300}{24 \times 30\,000\,000 \times 323} = 335 \text{ lb.}$$

as against the 1 440 lb. obtained by Mr. Goodwin.

The flexure shear at the ends is twice as large, but acts in the opposite direction and, therefore, counteracts the shear, S_2, due to eccentricity. The latter shear being 4 800 lb., there results accordingly:

$$\text{Total shear} = S_2 + S_1 = 4\,800 + 335 = 5\,135 \text{ lb.}$$

Before comparing this result with that obtained by the beam analogy it should be noted that an eccentricity of 4 in., as assumed by Mr. Goodwin, is too large for the column section selected, giving a combined stress of:

$$\frac{180\,000}{17.64} + \frac{180\,000 \times 4 \times 7}{323} = 10\,200 + 15\,600 = 25\,800 \text{ lb. per sq. in.}$$

If a maximum stress of only 19 000 lb. per sq. in. is permitted, as indicated by the column formula,

$$\frac{P}{a} = 19\,000 - c\frac{L}{r}$$

the eccentricity must be reduced in the proportion, 8 800 : 15 600, which gives:

$$\text{Total shear} = 5\,135 \times \frac{88}{156} = 2\,895 \text{ lb.}$$

This is equal to only 0.016 P_1, as compared with Mr. Goodwin's figure, 0.035 P_1.

The beam analogy applied to the same column formula, gives for a column depth, d, of 14 in.:

$$\text{Shear} = \frac{8\,c\,a\,r}{d} = \frac{8 \times 100 \times 17.64 \times 4.28}{14} = 4\,315 \text{ lb., or } 0.024\,P_1$$

Being thus 50% in excess of the actual amount as found previously, the result is well on the safe side.

The Quebec Bridge failure cannot be ascribed to any shortcoming of the beam analogy, as Mr. Goodwin seems to imply, but to a number of concurrent causes, among which the most fatal were: (1) The excessive unit stresses specified; (2) failure to make allowance for added weight; and (3) the inadequate provision made for eccentricity in designing the compression members.

In regard to the stresses in the chord member, A-9-L, it must be considered that the member was in the act of collapsing when the measurements given in Fig. 2 were made. Care should be exercised in drawing any con-

clusions from these observations, since there is no way of determining how much of the computed stress was transferred at that time to the opposite chord member, *A*-9-*R*.

Ralph E. Goodwin,* Assoc. M. Am. Soc. C. E. (by letter).—In closing, the author desires to summarize the purpose of his paper as it relates to the discussions which have been submitted. It has been his experience that many engineers have only a specialized understanding of shear and fail to comprehend it when it assumes a disguised form. This is especially true in the case of columns for which the axis may be curved and the direction of the shear not constant. The purpose of the paper was, therefore, first, to show that the applied shear is the normal component of the applied load; and, second, to call attention to the satisfactory mathematical foundation which exists as a basis for the determination of empirical shear allowances. That such allowances must be empirical is due to the fact that the initial curvature and accidental eccentricity of a column are unknown quantities, for which values corresponding to the most dangerous shear condition likely to occur in practice must be assumed. For determining what conditions of curvature and eccentricity will produce the most dangerous shear condition, the principles reviewed in the paper will be found useful.

Since Mr. Walker states that the basis of treatment is not new, the writer would call attention to his acknowledgment of this fact in the Synopsis.

Mr. Prichard objects to the shape of the deflection curve and to the value of the deflection assumed for the case illustrated by the writer's Fig. 3. Since the objection is based on theoretical grounds the writer wishes to state with emphasis that the application of the method should not be limited to theoretical deflections and theoretical deflection curves. The shear arising from initial curvature may be greater than that due to deflection caused by bending, and no method of analysis can be considered satisfactory which fails to make allowance for this factor. The general method (shear $= P \sin G$) is applicable to any curvature, regardless of the cause which produced it. The deflection assumed in the illustrative problem may be considered as arising from a combination of initial curvature with bending deflection, and as such does not appear to be unreasonable. Any deflection and any curvature that may occur in a practical column are legitimate cases for investigation.

Using the method presented in Mr. Prichard's discussion, the writer finds the maximum bending deflection (measured from the dotted line axis of Fig. 3(*b*)) to be about 0.11 in., leaving 0.19 in. to be accounted for by initial curvature. This is $0.0006\,L$, or $0.0013\,\frac{L}{2}$. As Mr. Prichard shows, the shear due to deflection is small in comparison with that due to eccentricity. Nevertheless, it is the amount of deflection assumed by the writer which forms the basis of his criticism. The eccentricity and direct stress are admittedly high in the illustration cited, however, and, as Mr. Prichard states, the case falls outside the limits set by specifications.

* Instr., Civ. Eng., Coll. of the City of New York, New York, N. Y.

Mr. Prichard's criticism in regard to the position of the vertex of the elastic line, in the same illustration, is somewhat misleading. When he states that the vertex lies 364 in. from the point of contrary flexure, he is speaking of the vertex determined by the axis of his equivalent centrally loaded column, which axis is inclined at an angle of about 1½° to the dotted-line axis of the writer's Fig. 3(*b*). If the writer's axis is applied to Mr. Prichard's elastic line, the point of maximum column deflection is found to be 85 in. from the point of contrary flexure, or 10 in. beyond the middle point of the half length, and Fig. 3(*b*), which alone formed the basis for computations, is, therefore, essentially correct in this respect, even when the possible effect of initial curvature is omitted from consideration. Fig. 3(*a*) was obtained by applying the inclined thrust line to Fig. 3(*b*), and its distortion with respect to that line results unavoidably from the exaggerated curvature adopted for the sake of clearness in illustrating.

The matter of changing present specifications, if this should be found necessary, may well be left in the hands of the Society's Special Committee on Steel Column Research. It is to be hoped that this Committee will include a study of the initial curvature of practical columns and of the curvature of loaded columns in its program.

Mr. Bull's discussion is essentially similar to that of Mr. Prichard in using a theoretical deflection curve without making any allowance for original lack of straightness, and in criticizing the writer's illustrative problem. The need of making an allowance for initial curvature has already been mentioned. It can be seen more readily if a centrally loaded column is considered. The ideal centrally loaded column has no deflection until subjected to the Euler load. Since practical columns carry less than the Euler load, no basis for shear computation exists, in this case, until allowances have been made for initial curvature and accidental eccentricity. Mr. Bull states that the deflection is a function of the eccentricity due to imperfect workmanship or other causes. This statement is based on the assumption of a column originally straight, whereas Turneaure states that a column may have an original deflection of 0.001 L, and Godfrey, one of 0.0025 L. Even in the extreme case used by the writer as an illustration an initial deflection of 0.001 $\frac{L}{2}$ proves to be greater than the deflection due to bending. The total deflection from all sources should be used for the computation of shear.

It is difficult to reconcile the objections raised by Mr. Prichard and Mr. Bull to the writer's illustrative problem on practical grounds with their purely theoretical treatment of deflection, from which the practical consideration of initial curvature is omitted. Possibly the oversight is traceable to the practice of treating the deflection of a crooked column as equivalent to an equal amount of eccentricity in a straight column. The maximum bending moment will be the same in both cases, but not the shear, owing to the fact that shear depends on the inclination of the axis. Shear due to initial deflection can not be treated on the basis of equivalent eccentricity.

Mr. Bull undertakes to bring the writer's illustrative problem within the limits of good practice by reducing the eccentricity until the extreme fiber stress is reduced to a reasonable figure. For this purpose he assumes that the sum of the average stress and the bending stress may be made 19 000 lb. per sq. in., apparently overlooking the fact that the column formula of the American Bridge Company is limited to a maximum of 13 000 lb. per sq. in.

Again, in computing the beam analogy, he assumes that the bending allowance of the column formula is $100\,\frac{L}{r}$. The writer is not familiar with any specification which defines the use of the beam analogy in connection with this particular column formula, but he is quite certain that the term, $100\,\frac{L}{r}$, is not all bending allowance. The formula is limited to a maximum of 13 000 lb. per sq. in., due to the fact that the elastic limit is the practical ultimate value in compression. This particular formula, then, assumes that the elastic limit controls the design up to $\frac{L}{r} = 60$, and that bending controls from that point on. It is more nearly correct, therefore, to consider the bending allowance to be $100\left(\frac{L}{r} - 60\right)$.

These, however, are minor details. The main point of interest lies in the idea of a treatment that is fundamentally sound and applicable alike to practical and theoretical curvatures. Owing to the empirical factors inherent in the problem, rules and specifications will always be necessary in the designing room, but in order that the designer may design intelligently and with understanding of the assumptions involved, he should be familiar with the correct theory of shear.

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INSTITUTED 1852

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Paper No. 1596

CO-ORDINATION OF IRRIGATION AND POWER*

By William Kelly,† M. Am. Soc. C. E.

With Discussion by Messrs. J. D. Galloway, Fred. H. Tibbetts, Barry Dibble, Markham Cheever, M. C. Hinderlider, C. S. Jarvis, and William Kelly.

Synopsis

Irrigation and power are both of great importance in the development of the Western States. Both interests require regulation of stream flow, but to different degrees, which may bring them into conflict. This paper gives a general presentation of the factors entering into the problem, illustrates the difficulties, and points out certain solutions by describing some of the specific cases that have come before the Federal Power Commission.

General

The co-ordination of irrigation and power is a problem found on nearly every stream in the arid West. The arid West is a region in which the mountains exercise a marked influence on climatic as well as on physical conditions. The annual precipitation in this region, except in the extreme Northwest, varies from 2 in. to about 15 in., whereas in the non-arid parts of the country the annual precipitation exceeds 25 in. The larger part of the precipitation in the West occurs on the mountain ranges often in the form of snow. Most of the streams get their principal supply from high up on the ranges, and the water in running down to the lower levels where farming is

* Presented at the Summer Meeting, Salt Lake City, Utah, July 8, 1925.

† Col., Corps of Engrs., U. S. A.; Director of Eng., National Electric Light Assoc., New York, N. Y.

practicable offers opportunity for power development. The area west of the Mississippi River contains about 72% of the potential water power of the United States.

There is a well-established principle in the Western States that the use of water for irrigation is superior to any other except domestic use, and the laws of those States reflect that principle. It is not the purpose of this paper to discuss the merits of that principle. States have a right to establish such principles and the Federal Government always has and always will recognize that right so long as it does not infringe on the rights of others under the Federal Constitution.

The principle must be borne in mind, however, in considering co-ordination of irrigation and power because to a considerable extent it sets aside the laws of economics and often causes co-ordination to take the form of adjusting power development so that it will not curtail irrigation development. It should also be borne in mind that, as a rule, power development has proved more profitable to investors than irrigation, and is, therefore, more easily financed. While irrigation is often seeking State or Federal aid, power does not need such aid; on the contrary, it frequently gives aid to irrigation without recompense.

Development of storage to regulate flow is more likely to be financially practicable for a power project than for an irrigation project, and most of the successful irrigation projects that have developed storage have produced power as a by-product to help carry the cost of the storage. The irrigation use of water is distinctly a seasonal use. With a few exceptions, such as the Imperial Valley, the season lasts from four to nine months and in most cases the peak demand occurs in June and July. Fortunately for farming, the maximum flow of most Western rivers being due to melting snows, occurs at the time of the maximum irrigation demand, thus reducing materially the storage capacity needed.

The power use of the water, on the other hand, is fairly uniform throughout the year, and, therefore, requires sufficient storage for more or less complete seasonal regulation. Secondary or seasonal power has a limited value. To the extent that power can be used for irrigation pumping, the power demand for water can be made to conform to the irrigation demand. Generally, the shortage of water for irrigation occurs in the latter part of the irrigation season; storage for power increases the discharge during this season, and to that extent is beneficial to irrigation. Figs. 1 to 6, inclusive, show graphically the relation between the irrigation demand and the monthly discharge for a high, average, and low-water year on the following rivers: Colorado, Columbia, Snake, San Joaquin and principal tributaries, Sacramento and principal tributaries, and Kings.

Examination of Fig. 1, Colorado River, shows that there is not much need for storage for irrigation for the Lower Basin provided it is practicable to keep the demand down to 5 acre-ft. per acre per year at the point of diversion. For the Upper Basin, however, storage is essential to provide for the irrigation demand from July to October, inclusive. The irrigation demand and its dis-

tribution through the year is taken from an unpublished report by the United States Bureau of Reclamation.

Examination of Fig. 2, Columbia River, indicates that the stream flow is more than ample to take care of irrigation at any and all times. As explained later, however, the irrigable lands lie so high above the main river that gravity diversion is possible only on the tributaries at points where the discharge is not adequate without storage.

TOTAL DEMAND

Irrigable Area - Lower Basin 1 000 000 Acres at 5.0 Acre Ft. = 5 000 000 Acre Ft.
Irrigable Area - Upper Basin 4 000 000 Acres at 1.5 Acre Ft. = 6 000 000 Acre Ft.

MONTHLY DEMAND

		Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Total
Percentage	Lower Basin	4	5	8	10	11	12	12	11	10	8	5	4	100
	Upper Basin	—3⅓	—3⅓	—3⅓	—3⅓	10	20	33⅓	33⅓	20	6⅔	—6⅔	—6⅔	100
Thousands of Acre Feet.	Lower Basin	200	250	400	500	550	600	600	550	500	400	250	200	5000
	Upper Basin*	—200	—200	—200	—200	600	1200	2000	2000	1200	400	—400	—200	6000
	Net Demand	0	50	200	300	1150	1800	2500	2550	1700	800	—150	0	11 000

*Includes Return Flow

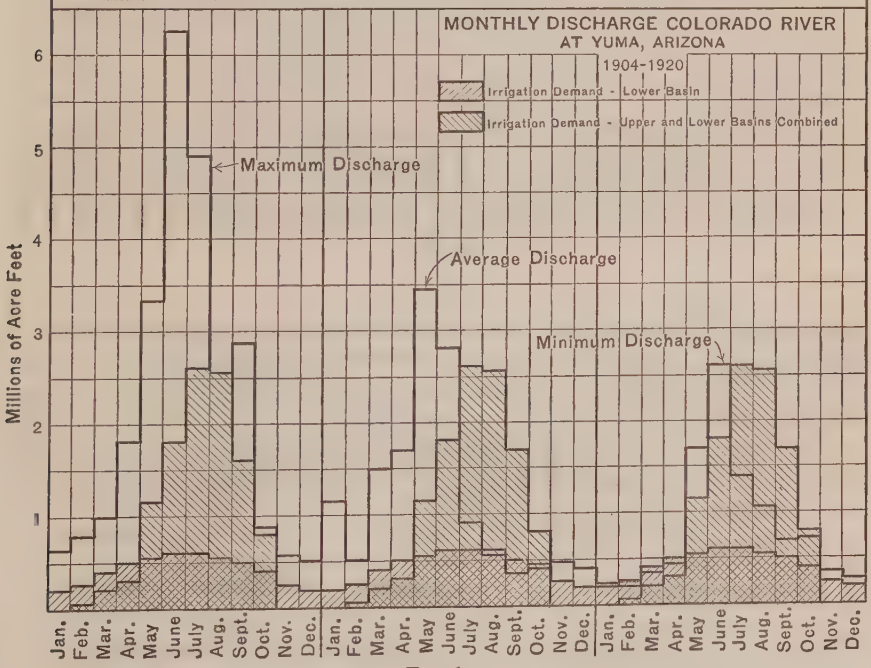

FIG. 1.

Fig. 3 shows that storage for irrigation on Snake River is needed from June to September, inclusive. The Snake River in Idaho presents an interesting example of co-ordinated irrigation and power. (See Fig. 7.) At several places on the Snake all the low-water flow is diverted for irrigation, and yet a short distance below such diversion the river flow is revived by underground contributions. In Southern Idaho the river flows through a lava formation and between the more or less horizontal layers of lava are water-

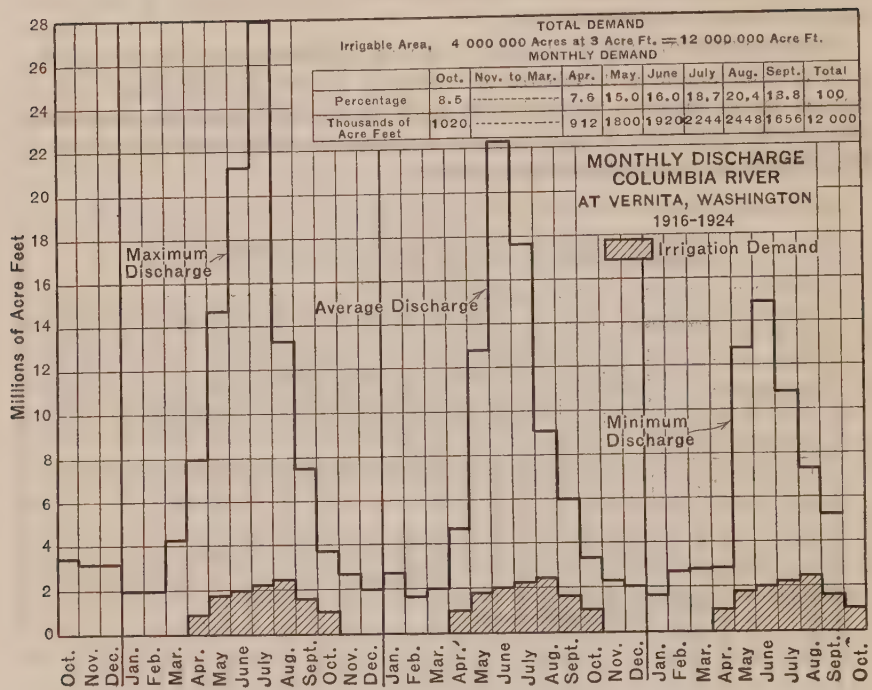
TOTAL DEMAND
Irrigable Area, 4 000 000 Acres at 3 Acre Ft. = 12 000 000 Acre Ft.
MONTHLY DEMAND
Oct. | Nov. to Mar. | Apr. | May | June | July | Aug. | Sept. | Total
Percentage | 8.5 | ---------- | 7.6 | 15.0 | 16.0 | 18.7 | 20.4 | 13.8 | 100
Thousands of Acre Feet | 1020 | ---------- | 912 | 1800 | 1920 | 2244 | 2448 | 1656 | 12 000
MONTHLY DISCHARGE
COLUMBIA RIVER
AT VERNITA, WASHINGTON
1916-1924
Irrigation Demand
Maximum Discharge
Average Discharge
Minimum Discharge
Millions of Acre Feet
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28
Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct.

FIG. 2.

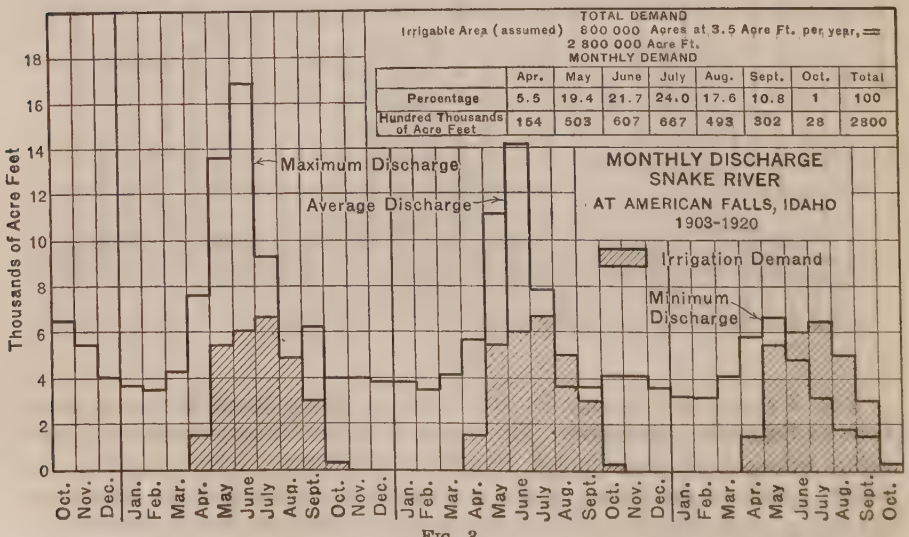
TOTAL DEMAND
Irrigable Area (assumed) 800 000 Acres at 3.5 Acre Ft. per year, =
2 800 000 Acre Ft.
MONTHLY DEMAND
Apr. | May | June | July | Aug. | Sept. | Oct. | Total
Percentage | 5.5 | 19.4 | 21.7 | 24.0 | 17.6 | 10.8 | 1 | 100
Hundred Thousands of Acre Feet | 154 | 503 | 607 | 667 | 493 | 302 | 28 | 2800
MONTHLY DISCHARGE
SNAKE RIVER
AT AMERICAN FALLS, IDAHO
1903-1920
Irrigation Demand
Maximum Discharge
Average Discharge
Minimum Discharge
Thousands of Acre Feet
0 2 4 6 8 10 12 14 16 18
Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct.

FIG. 3.

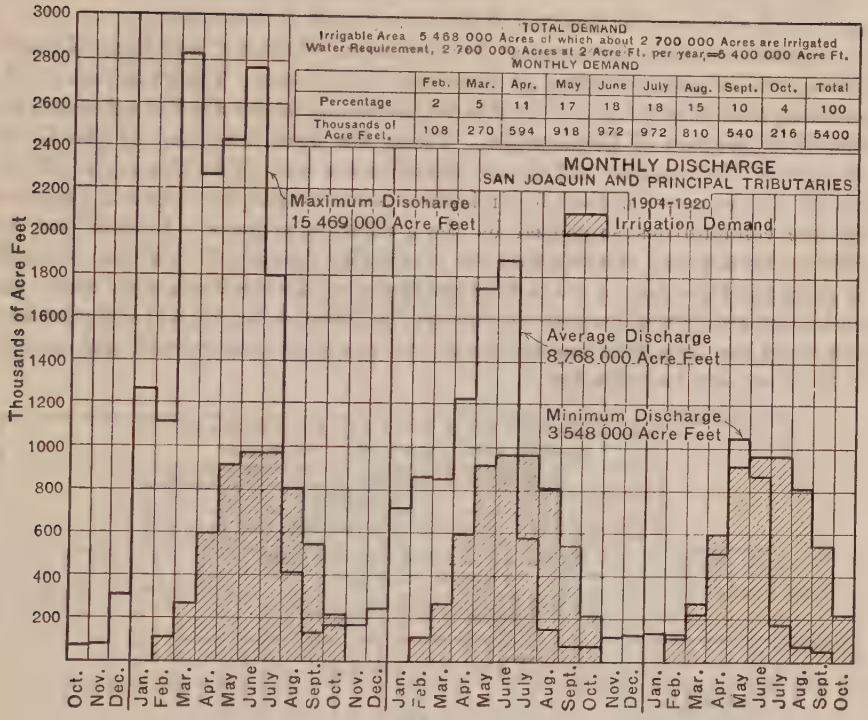
TOTAL DEMAND
Irrigable Area 5 468 000 Acres of which about 2 700 000 Acres are Irrigated
Water Requirement, 2 700 000 Acres at 2 Acre Ft. per year,=5 400 000 Acre Ft.
MONTHLY DEMAND
Feb. Mar. Apr. May June July Aug. Sept. Oct. Total
Percentage 2 5 11 17 18 18 15 10 4 100
Thousands of Acre Feet. 108 270 594 918 972 972 810 540 216 5400
MONTHLY DISCHARGE
SAN JOAQUIN AND PRINCIPAL TRIBUTARIES
1904-1920
Irrigation Demand
Maximum Discharge
15 469 000 Acre Feet
Average Discharge
8 768 000 Acre Feet
Minimum Discharge
3 548 000 Acre Feet
Thousands of Acre Feet
3000
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct.

FIG. 4.

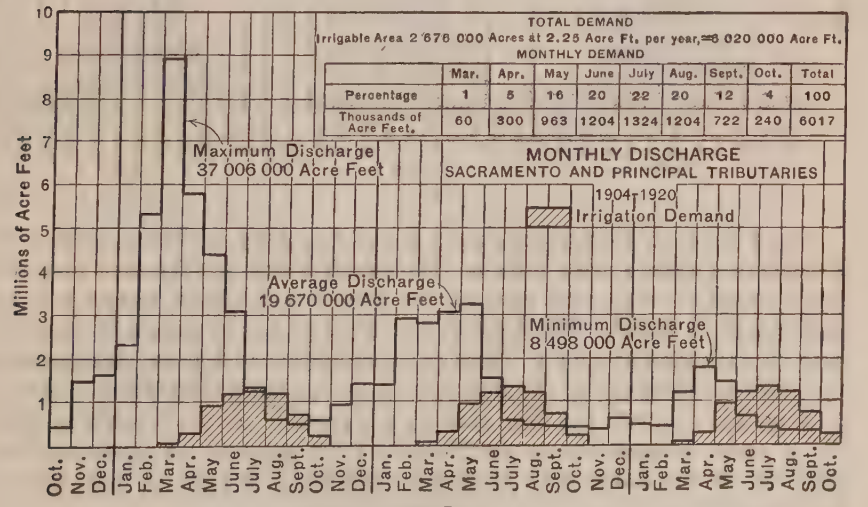
TOTAL DEMAND
Irrigable Area 2 676 000 Acres at 2.25 Acre Ft. per year,=6 020 000 Acre Ft.
MONTHLY DEMAND
Mar. Apr. May June July Aug. Sept. Oct. Total
Percentage 1 5 16 20 22 20 12 4 100
Thousands of Acre Feet. 60 300 963 1204 1324 1204 722 240 6017
MONTHLY DISCHARGE
SACRAMENTO AND PRINCIPAL TRIBUTARIES
1904-1920
Irrigation Demand
Maximum Discharge
37 006 000 Acre Feet
Average Discharge
19 670 000 Acre Feet
Minimum Discharge
8 498 000 Acre Feet
Millions of Acre Feet
10
9
8
7
6
5
4
3
2
1
Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct. Nov. Dec. Jan. Feb. Mar. Apr. May June July Aug. Sept. Oct.

FIG. 5.

bearing strata. Diversion for irrigation greatly exceeds consumptive use and much of the water diverted finds its way back to the river.

Storage has been developed at Jackson Lake, Wyoming, and Lake Walcott, Idaho, and is being developed at American Falls, Idaho. The irrigation developments have been made both by districts organized under the State law and by the U. S. Bureau of Reclamation. Power has been developed by the U. S. Bureau of Reclamation and by the Idaho Power and Light Company. Practically all the power is used for irrigation pumping and for domestic purposes by the communities supported by the irrigation projects. In this part of Idaho, farming and stock-raising are practically the only industries. In such a situation there can be no serious conflict over the relative use of water for irrigation and power, because the market for power depends on the success and extent of the irrigation developments, and power is economically and practically secondary to irrigation.

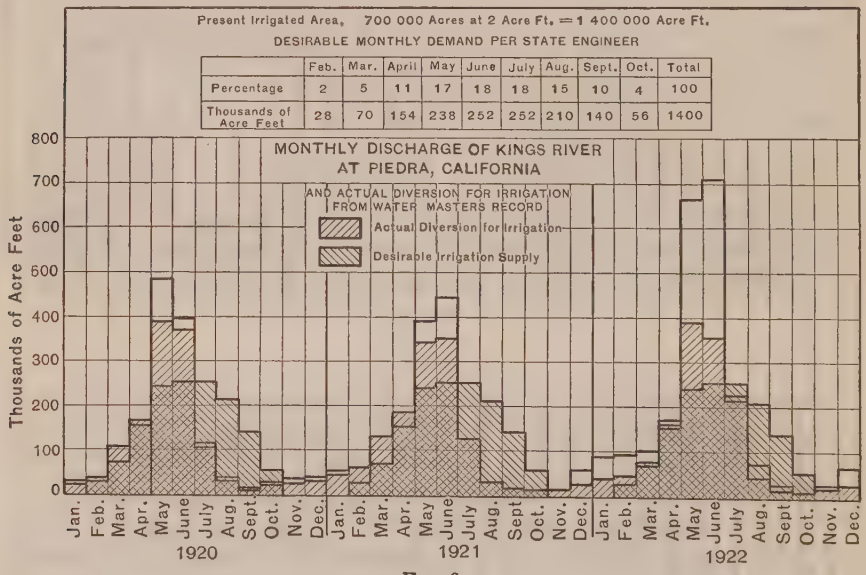

FIG. 6.

Fig. 4 shows that the water supply in the San Joaquin Valley, California, is deficient generally from July to October, inclusive, for the area now irrigated, which is only about one-half the area that would be irrigable if the water could be found. The deficiency is more or less overcome by pumping from the underground supply. Storage in connection with power developments has aided by building up the flow during the period of deficiency. Storage by irrigation interests has been undertaken recently on the Tuolumne River and is under construction on the Merced. Such projects are under consideration also on the Mokelumne, Stanislaus, and Kings Rivers. A more detailed description of the Kings River situation follows.

Fig. 5 shows that there is more than enough water in the Sacramento River to irrigate all the land suitable for irrigation, but that the natural flow

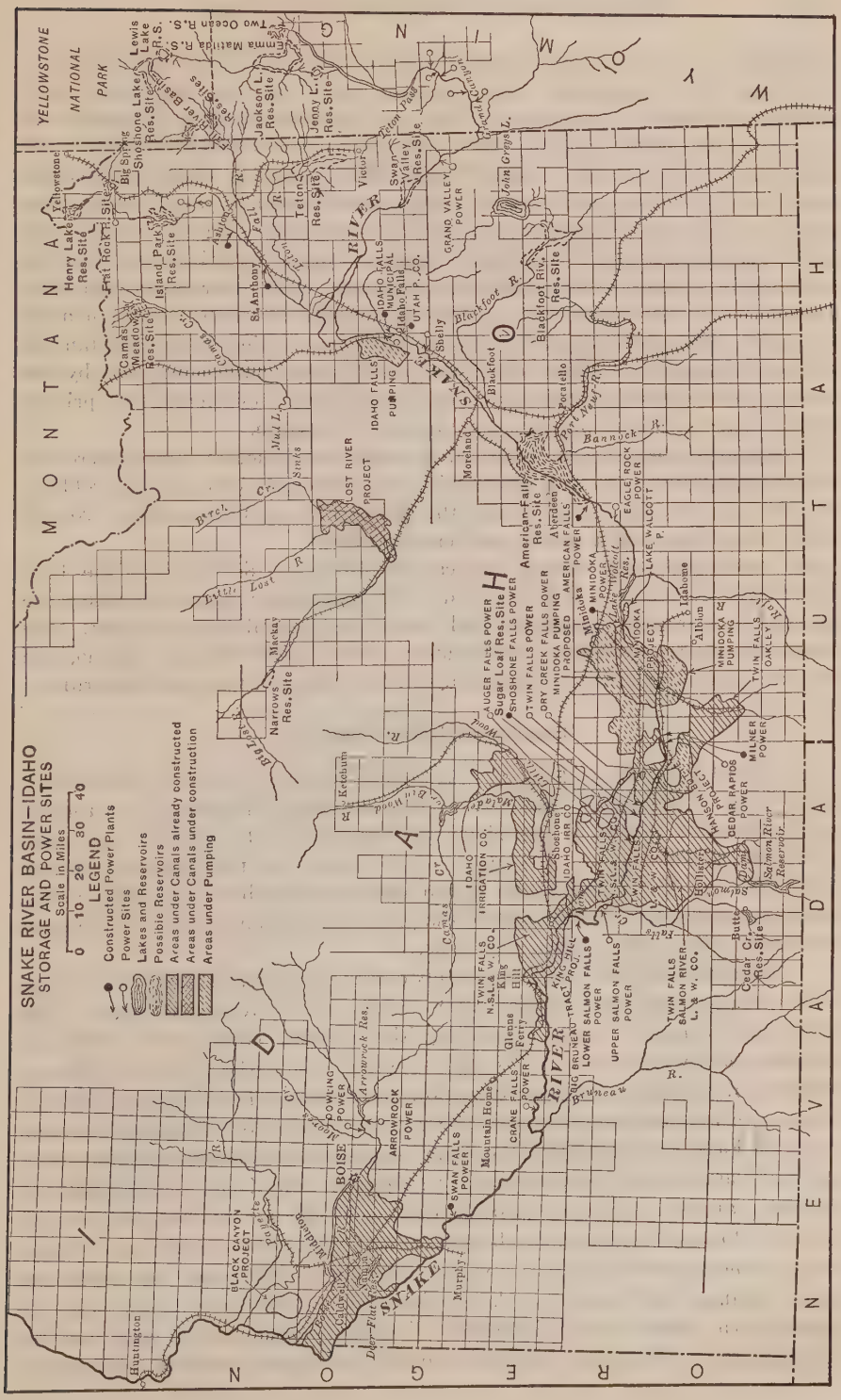
SNAKE RIVER BASIN—IDAHO
STORAGE AND POWER SITES
Scale in Miles
0 10 20 30 40
LEGEND
Constructed Power Plants
Power Sites
Lakes and Reservoirs
Possible Reservoirs
Areas under Canals already constructed
Areas under Canals under construction
Areas under Pumping
MONTANA
WYOMING
UTAH
NEVADA
OREGON
YELLOWSTONE NATIONAL PARK
Henry Lake Res. Site
Island Park Res. Site
Camas Meadow Res. Site
Jackson L. Res. Site
Jenny L. Res. Site
Swan Valley Res. Site
Teton Res. Site
Lewis Lake R.S.
Two Ocean R.S.
Emma Matilda R.S.
Big Springs Res. Site
Shoshone Lake Res. Site
Yellowstone
Fall R.
Ashton
St. Anthony
Teton
Victor
Teton Pass
Grand Canyon
John Grays L.
GRAND VALLEY POWER
Blackfoot R. Res. Site
Blackfoot
Shelly
Poratello
Port Neuf R.
Bannock R.
Moreland
IDAHO FALLS PUMPING
IDAHO FALLS
Idaho Falls
UTAH P. CO.
SNAKE RIVER
LOST RIVER PROJECT
Mud L.
Birch Cr.
Little Lost R.
Mackay
Narrows Res. Site
Big Lost R.
Ketchum
Wood R.
Malad R.
Camas Cr.
Camas Prairie
Aberdeen
American Falls Res. Site
AMERICAN FALLS POWER
EAGLE ROCK POWER
LAKE WALCOTT P.
Idaho
Albion
MINIDOKA PROJECT
MINIDOKA PUMPING
TWIN FALLS OAKLEY
MILNER POWER
HANSON BUTTE PROJECT
CEDAR RAPIDS POWER
Salmon River Reservoir
Hollister
Butte
Cedar Cr. Res. Site
Falls R.
Minidoka
Minidoka Res.
MINIDOKA POWER
MINIDOKA PUMPING PROPOSED
DRY CREEK FALLS POWER
TWIN FALLS POWER
SHOSHONE FALLS POWER
AUGER FALLS POWER
Sugar Loaf Res. Site
SNAKE RIVER
Shoshone
IDAHO IRR. CO.
TWIN FALLS N.S.L. & W. CO.
TWIN FALLS SALMON RIVER L. & W. CO.
TWIN FALLS SALMON RIVER L. & W. CO.
UPPER SALMON FALLS POWER
LOWER SALMON FALLS POWER
KING HILL PROJECT
BIG BRUNEAU TRACT
King Hill
Glenns Ferry
GRANE FALLS POWER
Mountain Home
Bruneau R.
SWAN FALLS POWER
Murphy
Caldwell
Nampa
Middleton
Payette R.
Boise R.
BOISE
DOWLING POWER
Arrowrock Res.
ARROWROCK POWER
BLACK CANYON PROJECT
Huntington
Deer Flat Res.
Snake R.

FIG. 7.

is deficient for the irrigation demand in the months of July to September, inclusive. The Sacramento River presents an interesting and complicated problem of conflicting interests of up-river and down-river irrigationists in the use of the water, and the effect of such use on navigation and on the infiltration of salt from the ocean. There is also a problem of flood protection which has been fairly well solved. Power development requiring head-water storage, building up the late summer and fall flow, and using the streams only in the mountain sections, is generally beneficial to all other interests, and not particularly difficult to bring into co-ordination. The Sacramento and San Joaquin Rivers have dependent on them the greatest irrigation development in the world. The problems of these rivers are being studied by the substantial, practical irrigators to whose efforts the present developments are due, by the various power producers, and by various Federal, State, municipal, and private experts, and continued progress and success are certain.

Perhaps the best way to bring out in detail the difficulties encountered in co-ordinating irrigation and power will be to describe some of the problems of this kind that have come before the Federal Power Commission. Nearly every stream presents a special problem of its own.

In connection with certain applications for power development the Federal Power Commission found a need of comprehensive studies to determine the relations between irrigation and power on the following streams: The Deschutes River, in Oregon; the Columbia River, in Washington, Idaho, and Montana; and the Trinity, American, and Stanislaus Rivers, in California.

Deschutes River, Oregon

A Board consisting of J. B. Cavanaugh, Colonel, Corps of Engineers, U. S. A., D. C. Henny, M. Am. Soc. C. E., Consulting Engineer, U. S. Bureau of Reclamation, and F. F. Henshaw, M. Am. Soc. C. E., District Engineer, U. S. Geological Survey, was appointed to report on the Deschutes River. (See Fig. 8.) The report of the Board was published by the Federal Power Commission in 1922, and has shaped the policy of the Commission with respect to the Deschutes River since that time.

The Deschutes Basin lies immediately east of the Cascade Range in Oregon, and a considerable part of the water supply comes from that range. The river flows generally north paralleling the Cascade Range and enters the Columbia a few miles east of The Dalles. From its mouth to Crooked River, a distance of 112 miles, the Deschutes flows in a canyon from 1 000 to 2 000 ft. deep, with a river slope of about 12.8 ft. per mile. From Crooked River to Benham Falls, about 70 miles, the slope increases to an average of about 38 ft. per mile, and the depth of the canyon gradually diminishes. Above Benham Falls the gradient suddenly flattens to about 1.5 ft. per mile for the next 36 miles. Storage is practicable above Benham Falls either in the main valley or in numerous small mountain valleys and lakes higher up. The rapid fall below Benham Falls and the relative shallowness of the canyon for about 20 miles down stream render irrigation diversion comparatively easy, and all present gravity diversions take place in this stretch of the river. Diversions

COLUMBIA RIVER
Hood River
The Dalles
Rimrock
Moody
Lockit
Reclamation Power Site
Sinamox
Oak Brook
Sherar Falls
Oak Springs
Maupin
Frieda
White Horse Rapids
No. 5
No. 4
Coleman
Mecca
No. 3
NORTH
No. 2
White Water
Pelton
Metolius
No. 1
Madras
UNIT
Jefferson
Riggs
Culver
Jacks Cr.
Suttles L.
Lone Pine
Res.
Ochoco
Prineville
Redmond
WEST UNIT
NORTH CANAL UNIT
CENT. ORE.
SOUTH UNIT
CROOKED RIVER
Post
Post Res. Site
Bend
ARNOLD
Green L.
Tumalo Cr.
Dry River Bed
Crane Prairie Res. Site
Benham Falls Res. Site
Paulina L.
Davis L.
Odell L.
Crescent Cr.
Crescent L.
Walker Basin
DESCHUTES RIVER OREGON
IRRIGATION AND POWER SITES
Scale of Miles
0 5 10 15 20 25
LEGEND
Power Sites
(Numbers Refer to Application of Columbia Valley Power Co.)
Lands Irrigated
Lands Previously Proposed for Irrigation
CANADA
WASHINGTON
OREGON
IDAHO
CALIFORNIA
NEVADA
PACIFIC OCEAN
Seattle
Spokane
Olympia
Portland
The Dalles
Eugene
Bend
Baker
Roseburg
Medford

FIG. 8.

more than 25 miles below Benham Falls are impracticable on account of the depth of the canyon.

The flow of the Deschutes River is remarkably uniform. At the mouth, it generally fluctuates between 5 000 and 8 000 sec-ft. The lowest flow during the last 14 years was about 3 700 sec-ft., and short-time flood peaks usually do not exceed 16 000 sec-ft. This is due to the very large inflow of underground water from the porous volcanic formation above Crooked River as well as to the well-sustained low-water surface flow from the high Cascades. The minimum monthly mean flow of the Deschutes at Metolius, Ore., is 3 250 sec-ft. This minimum will not be materially affected if all the surface water reaching Benham Falls is diverted for irrigation. If all the Upper Deschutes water is dedicated to irrigation, the present irrigated area of about 81 000 acres can be expanded to 191 000 acres, and 555 000 h.p. can be developed on the Lower Deschutes.

There are at present six small hydro-electric developments on the Deschutes, all above the mouth of Crooked River. Their aggregate installed capacity is 3 230 h.p. The water rights of these developments conflict with irrigation expansion and will have to be extinguished for full irrigation development. The expansion of irrigation is likely to be delayed by the fact that all the irrigable areas are more than 4 000 ft. above sea level, so that only the hardier crops can be grown, and also by the fact that much of the area is now used more or less successfully for dry farming. This river presents an unusual condition in that all the irrigable areas lie above the main fall in the stream, and the peculiar physical and geological formation makes it possible to develop practically all the power in the lower river without curtailing irrigation development. Much of the power in the lower river can be developed at a very low cost and it undoubtedly will be developed whenever a market can be found.

On June 25, 1924, the Federal Power Commission issued a license to the Columbia Valley Power Company for developments at the Pelton and Metolius, Ore., sites on Deschutes River, with an ultimate installed capacity of 185 000 h.p. The Company proposed to sell power to the utility companies in Portland, Ore.

On November 28, 1922, the Federal Power Commission issued a preliminary permit to the Pacific Power and Light Company for a development at the Reclamation site on Deschutes River, with a proposed installed capacity of 54 000 h.p.

On November 13, 1924, the Federal Power Commission issued a preliminary permit to the Pringle Falls Electric Power and Water Company for a project near Bend, Ore., with a proposed installed capacity of 5 400 h.p.

No other applications are now before the Commission for developments on the Deschutes.

Columbia River

A Board consisting of J. B. Cavanaugh, Colonel, Corps of Engineers, U. S. A., D. C. Henny, M. Am. Soc. C. E., Consulting Engineer, U. S. Bureau of Reclamation, F. F. Henshaw, M. Am. Soc. C. E., District Engineer, U. S.

Geological Survey, C. S. Heidel, M. Am. Soc. C. E., State Engineer of Montana, W. G. Swendsen, Assoc. M. Am. Soc. C. E., Commissioner of Reclamation, Idaho, and Marvin Chase, M. Am. Soc. C. E., Supervisor of Hydraulics, State of Washington, was appointed to make a study of the Columbia River (see map, Fig. 9), from Flathead Lake, Montana, to the mouth of Snake River; to report on the character and extent of present and prospective uses of the river for navigation, irrigation, and power, with a discussion of the relative merits of each and of their relation to each other; and to outline a program of development that would harmonize the conflicting uses in such manner as to secure, in the long run, the greatest combined benefit from all.

After nine months' investigation, the Board submitted a report which was published by the Federal Power Commission in 1923. The report was concurred in by all members of the Board except the representative of Washington, who concurred in all conclusions and recommendations except those with respect to the Columbia Basin project.

The Columbia drains an area of about 259 000 sq. miles segregated by stream basins, as follows:

	Square miles.	Percentage.
Columbia River above Clark Fork in Canada..	34 000	13.1
Clark Fork	26 000	10.0
International Boundary to Snake River.....	43 000	16.7
Snake River	109 000	42.1
Columbia River below Snake River.....	47 000	18.1
Total.....	259 000	100.0

The mean annual precipitation of the Columbia Basin varies according to location, from 6 to more than 100 in. Where the precipitation exceeds 20 in. the country is generally forested and mostly mountainous, and where it is less than 15 in. the country is arid. The areas with low rainfall lie in two broad belts, one comprising East-Central Washington and extending northward up the Okanogan Valley into British Columbia, the other lying in the inter-mountain valley of Montana and British Columbia. Most of the land sufficiently smooth for agricultural development lies in the arid area.

The mean annual discharge at various points from Flathead Lake to the mouth of the Snake River is shown in Table 1.

TABLE 1.

Station.	Mean annual discharge, in acre-feet.	Drainage area, in square miles.
Flathead River, Polson, Mont.....	8 108 800	7 200
Clark Fork, Plains, Mont.....	14 914 400	19 900
Clark Fork, Metaline Falls, Wash.....	19 982 400	25 100
Columbia River, International Boundary.....	72 841 200	60 000
Columbia River, Pasco, Wash.....	94 699 200	103 000

The Upper Basin of the Columbia abounds in lakes, nearly all of which present possibilities of water storage. The three principal lakes are:

Flathead Lake, in Western Montana, at an elevation of nearly 2 900 ft., which has a surface area of 120 000 acres. The lake has a fairly steep shore

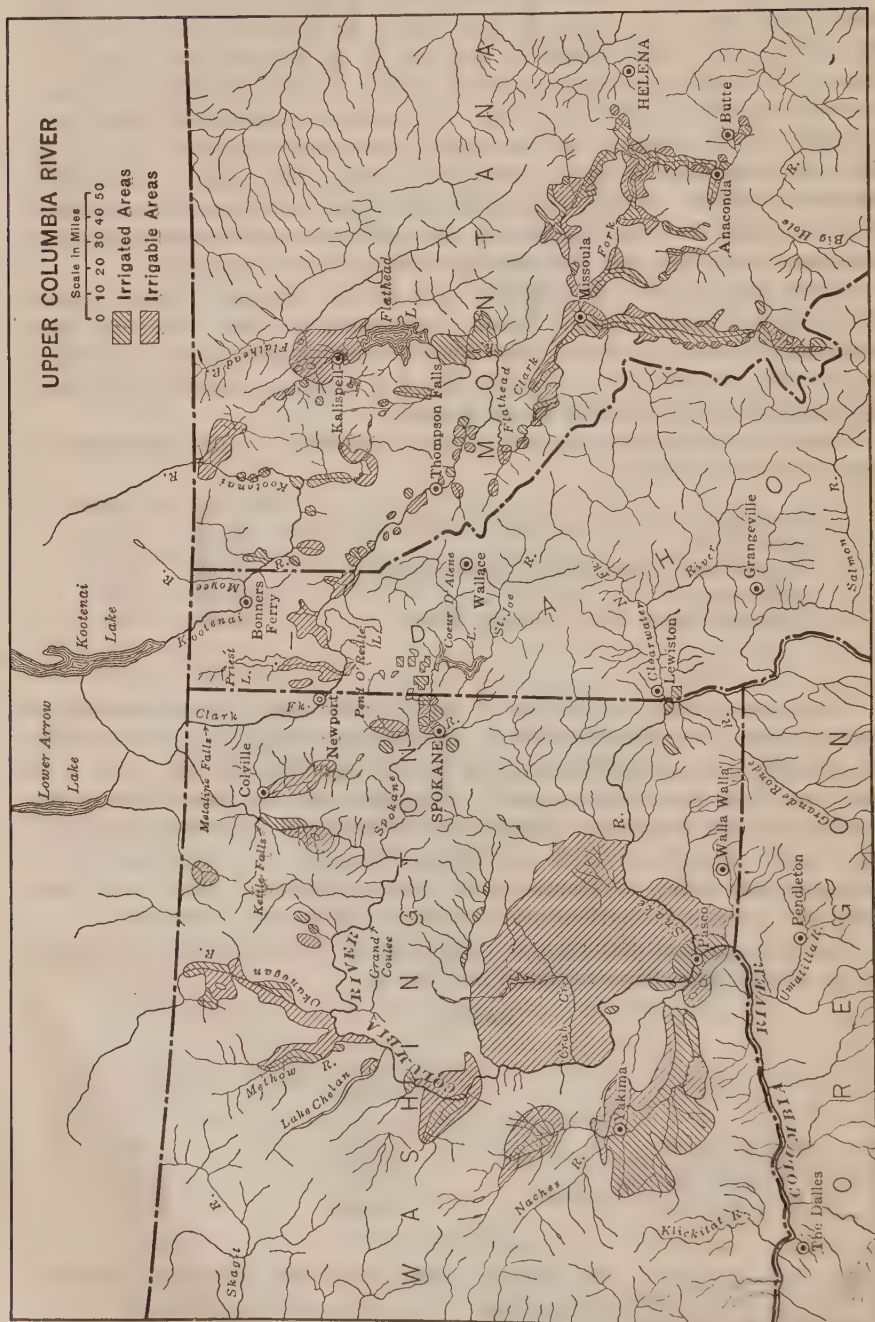
UPPER COLUMBIA RIVER
Scale in Miles
0 10 20 30 40 50
Irrigated Areas
Irrigable Areas
Lower Arrow Lake
Kootenai Lake
Kootenai R.
Moyee R.
Bonners Ferry
Priest L.
Clark Fk.
Newport
Colville
Metaline Falls
Kettle Falls
Okanogan R.
Methow R.
Lake Chelan
Skagit R.
Grand Coulee
COLUMBIA RIVER
Spokane R.
SPOKANE
Pend O'Reille L.
Coeur D'Alene L.
Wallace
St. Joe R.
Thompson Falls
Kalispell
Flathead L.
Flathead R.
Kootenai R.
Missoula
Clark Fork
Flathead
Anaconda
Butte
HELENA
Big Hole R.
Lewiston
Clearwater R.
Grangeville
Salmon R.
Walla Walla
Grande Ronde R.
Pendleton
Umatilla R.
Pasco
Snake R.
Yakima
Naches R.
Klickitat R.
The Dalles
WASHINGTON
IDAHO
MONTANA
OREGON

FIG. 9.

except at its upper end where the Upper Flathead River has built up a large delta. A considerable part of this delta is flooded at ordinary high water, but the remainder is used for agriculture which constitutes the principal support of the country around Kalispell. The outlet of the lake is through a rocky gorge below Polson offering satisfactory foundation for a dam. Storage to the amount of 1 600 000 acre-ft. can be obtained without serious damage from flooding.

Lake Pend Oreille, in Northern Idaho, has a surface area of 86 000 acres at low water and 98 000 acres at high water. The shores are generally steep, but much of the western portion is bordered by gravel beaches. The meadow lands at the head of the lake have the characteristics of a delta which the river overflows in flood. The outlet of the lake is between rocky banks and at Albany Falls the river crosses a dike of hard rock affording satisfactory foundation for a controlling dam. A storage capacity of 1 750 000 acre-ft. can be obtained without serious damage from flooding.

Priest Lake forms an enlargement of Priest River which discharges into Clark Fork about five miles above Albany Falls. It has an area of about 24 000 acres. The average run-off is about 862 000 acre-ft. The shores are fairly steep and the area subject to flooding by a raise of water surface is not large. The outlet is over a bed of boulders between banks of sand and gravel. The fall is about 30 ft. in the first 2 miles to a point where the river passes through a narrow defile with rock exposed on both sides to a considerable height. A dam can be built either at the gorge or at the outlet. Storage capacity of 600 000 acre-ft. can be obtained without serious damage from flooding.

Fig. 2 shows the monthly discharge of the Columbia River at Vernita, Wash., for high, average, and low years. It also shows the maximum irrigation demand above Vernita based on the irrigation of 4 000 000 acres, which figure was determined by using the largest estimates that could be found. It will be seen from this that the natural flow is more than ample in the main river. The irrigable lands, however, lie at such a height above the main river that, if high-lift pumping projects are to be avoided, diversion must be made on tributaries where the natural flow is inadequate without storage. The figures in Table 2 were furnished by the State representatives as to present and possible future irrigable areas in the Columbia Drainage Basin above Pasco, Wash.

The only project included in Table 2 which will have any material effect on the stream flow of the main river is the Columbia Basin project proposed by the State of Washington. This project contains about 1 750 000 acres in what is known as the Big Bend Country along the east side of the Columbia, north of Pasco and west of Spokane, Wash. The gross diversion required by this project as estimated by the State of Washington is shown in Table 3.

Two methods of diverting the water for this project have been studied: First, by gravity, diverting just above Albany Falls at the outlet of Lake Pend Oreille; and, second, by pumping from the river near the head of Grand Coulee. If the first method, which is favored by State authorities, is adopted, storage will be necessary.

Table 4 shows the storage that would have been required during the period, 1903-21, under the three different methods of regulating the flow.

Method I is based on a storage of 2 350 000 acre-ft. in Pend Oreille and Priest Lakes controlled for irrigation, with natural flow from Flathead Lake.

TABLE 2.

State.	Acres Irrigable.		
	Present.	Future.	Total.
Montana	274 000	425 000	699 000
Idaho	0	150 000	150 000
Washington	95 000	1 870 622	1 965 622
Total	369 000	2 445 622	2 814 622

TABLE 3.

Month.	Acre-feet.
April	561 000
May	1 152 000
June	1 152 000
July	1 152 000
August	1 092 000
September	685 000
October	436 000
Total	6 228 000

Method II is based on a storage of 2 350 000 acre-ft. in Pend Oreille and Priest Lakes controlled for irrigation, with 1 600 000 acre-ft. in Flathead Lake controlled for power.

Method III is based on a storage of 3 350 000 acre-ft. in Flathead and Pend Oreille Lakes controlled for irrigation.

It will be seen that 1919 was the only year when there would have been a shortage under any of the methods of regulation described. This shortage under Method II amounts to about 10% of the seasonal needs after July 1, and, being the only shortage in a cycle of 19 years, is not serious.

The Board concluded, therefore, that storage in Flathead Lake is not essential for the Columbia Basin project. If the second method is adopted, no storage for irrigation is necessary. The total lift from the river to Coulee is 630 ft. and a dam 200 ft. high would leave a remaining height of 430 ft. as pump lift. There is a good dam site at Grand Coulee with bed-rock at 60 ft. below low water. A pumping lift of 430 ft. is considerably greater than that of any successful pumping project now in existence, and its economic feasibility has been challenged. Whether or not economically feasible, a study of the pumping project made for the State by Mr. W. T. Batcheller, Electrical

and Mechanical Engineer, Seattle, Wash., shows that the cost per acre of a pumping project would be less than that of a gravity project, and this fact was confirmed by estimates made by the Columbia River Board.

TABLE 4.

Year.	Method I.	Method II.	Method III.
1903	649 000	1 005 000	1 118 000
1904	1 584 000	1 451 000	1 830 000
1905	2 195 000	2 389 000	2 603 000
1906	2 185 000	2 083 000	2 561 000
1907	160 000	225 000	349 000
1908	1 098 000	1 011 000	1 451 000
1909	965 000	1 132 000	1 388 000
1910	1 914 000	2 091 000	2 310 000
1911	1 226 000	1 211 000	1 461 000
1912	649 000	822 000	823 000
1913	846 000	828 000	1 113 000
1914	1 568 000	1 329 000	1 695 000
1915	1 124 000	1 162 000	1 362 000
1916	21 000	50 000	78 000
1917	1 046 000	463 000	1 508 000
1918	1 505 000	1 389 000	1 678 000
1919	2 787 000	2 706 000	3 360 000
1920	1 080 000	1 165 000	1 348 000
1921	1 593 000	1 515 000	2 051 000

A new study of the Columbia Basin project is being made by the U. S. Department of the Interior, the results of which are not yet available. All authorities appear to agree that the soil and climatic conditions are excellent, but there is a wide difference of opinion as to the probable costs. The costs have been variously estimated at from $145 to $240 per acre for the water system, including storage, diversion, main canals, main laterals, and distribution system.

The fall in the river from Flathead Lake to Pasco is 2 567 ft. Data with respect to dam sites are not complete, but based on the dam sites that look feasible, 1 398 ft. of the fall can be developed for power. Table 5 shows the all-year power possibilities under various assumed conditions.

From Table 5 it is evident that a decision between the methods of irrigating the Columbia Basin affects not only the project itself, but also future power possibilities. The selection of the gravity method will destroy nearly 650 000 all-year effective horse-power.

There is a tendency to under-estimate the value of power in a region such as the Northwest where the potential power is great and present demand relatively small. The demands of a rapidly increasing power market are being felt all over the United States. Twenty years ago California had a great excess of water power, but it is now evident that before many years additional sources of power will have to be sought, and the Northwest seems certain to follow the same trend.

The conclusions of the Columbia River Board were that:

(*a*) Freedom should be given to fullest irrigation expansion in Montana, Idaho, and Washington, and no rights should be allowed to accrue to the lower

interests, which would legalize limitation of or interference with irrigation above.

(*b*) The Columbia Basin project is the most important single item to be considered in the uses to be made of Columbia River water above the mouth of Snake River.

(*c*) The Columbia Basin project can be irrigated by gravity from the Clark Fork at Albany Falls, or by pumping from the Columbia River at Grand Coulee.

(*d*) The Columbia Basin gravity project can be supplied adequately by the aid of storage in Pend Oreille and Priest Lakes in Idaho, conditioned on the maintenance of natural flow from Flathead Lake when needed.

TABLE 5.—All-Year Power on Flathead River, Clark Fork, and Columbia River.

Stream.	Location.	Usable head at dam sites, in feet.	No storage. All year horse-power.	Flathead Storage, 1 600 000 Acre-Ft. for Power; Pend Oreille and Priest, 2 300 000 Acre-Ft. for:	
				Columbia Basin gravity project, all-year horse-power.	Power, all-year horse-power.
Flathead River:					
Clark Fork..	Flathead Lake to mouth of Flathead River.....	303	69 000	165 000	165 000
No. 1.....	Mouth of Flathead River to Idaho line.....	140	75 000	106 000	106 000
No. 2.....	Idaho line to Lake Pend Oreille.....	77	43 000	59 000	59 000
No. 3.....	Lake Pend Oreille to International Boundary..	250	232 000	176 000	435 000
No. 4.....	International Boundary to mouth of Clark Fork.	265	234 000	178 000	440 000
Columbia River:					
No. 1.....	Mouth of Clark Fork to foot of Kettle Falls.....	75	166 000	178 000	225 000
No. 2.....	Foot of Kettle Falls to Grand Coulee.....	198	612 000	640 000	665 000
No. 3.....	Grand Coulee to foot of Priest Rapids.....	90	294 000	294 000	348 000
Total.....		....	1 725 000	1 796 000	2 443 000

(*e*) The Columbia Basin project can be supplied with practically no shortage without the aid of storage, with a dam at Grand Coulee about 156 ft. above low water.

(*f*) Information on which to base a final decision between a gravity and a pumped supply for a Columbia Basin irrigation project is not complete and should be completed. Any decision should take into account the effect on potential power.

(*g*) Pending such decision no permanent rights of storage should be granted in Pend Oreille and Priest Lakes.

(*h*) If ultimately the decision is in favor of a Columbia Basin gravity project, storage rights in Pend Oreille and Priest Lakes should be granted

to such project, but should be limited to storage of inflow in excess of 7 000 sec-ft.

(*i*) If ultimately the decision is in favor of a Columbia Basin pumping project, storage rights in Pend Oreille and Priest Lakes should be granted to the joint interests of power on the Lower Clark Fork and the Columbia River subject to limitations contained in Section (*a*).

(*j*) Storage rights in Flathead Lake should be granted to Flathead power interests, subject to the fullest development found to be practicable after complete investigation. Prior to such development, rights may be advantageously granted to power interests on the Clark Fork subject to conditions protecting ultimate control for Flathead power and natural flow release when needed for the protection of the Columbia Basin project if built on the gravity plan.

(*k*) Storage control at Flathead, Pend Oreille, and Priest Lakes should be under impartial supervision.

(*l*) A permit to develop power at the Grand Coulee site, or at any point on the Columbia River as far down stream as the Foster Creek site, should not be granted to power interests until it is known that such site will not be needed by the Columbia Basin project. In case the Columbia Basin project shall require a power site for irrigation and power on this stretch of the Columbia River, a permit should be granted to it after the best location and height of dam shall have been determined from the standpoint of the project and the public interest.

These conclusions appear to be sound and equitable and are, therefore, likely to be followed by the Federal Power Commission.

The following is the status of applications to the Federal Power Commission for power projects on the Upper Columbia:

Application for preliminary permit by the Rocky Mountain Power Company, of Butte, Mont.: A project on Flathead River below Flathead Lake with five dams and a proposed installed capacity of 272 000 h.p. Action indefinitely suspended awaiting completion of investigation of Columbia Basin irrigation project.

Application for license by the Washington Irrigation and Development Company, of New York, N. Y.: A project on the Columbia River at Priest Rapids with a proposed installed capacity of 300 000 h.p. License authorized March 2, 1925.

Application for preliminary permit by Hugh L. Cooper, M. Am. Soc. C. E., of New York, N. Y.: A project at Z Canyon, Clark Fork, with a proposed installed capacity of 350 000 h.p. Action indefinitely suspended awaiting completion of investigation of Columbia Basin irrigation project.

Application for preliminary permit by the Washington Water Power Company, of Spokane, Wash.: A project on the Columbia River at Kettle Falls with a proposed installed capacity of 153 400 h.p. Permit issued August 26, 1922.

Application for preliminary permit by Mr. Hugh L. Cooper, New York, N. Y.: A project on the Columbia River at Grand Coulee with a proposed

installed capacity of 800 000 h.p. Action indefinitely suspended awaiting completion of investigation of Columbia Basin irrigation project.

Trinity River

A Board consisting of W. F. McClure, M. Am. Soc. C. E.,* State Engineer of California, D. C. Henny, M. Am. Soc. C. E., Consulting Engineer, U. S. Bureau of Reclamation, U. S. Grant, 3d, Major, Corps of Engineers, U. S. A., Assoc. M. Am. Soc. C. E., and E. W. Kramer, M. Am. Soc. C. E., Hydraulic Engineer, U. S. Forest Service, was appointed by the Federal Power Commission to study the Trinity River (see map, Fig. 10), in California, in order to indicate the most beneficial uses that can be made of its waters and especially to determine whether diversion to the Sacramento River will be to the general benefit. The Board submitted its report in February, 1924. The report will probably not be printed.

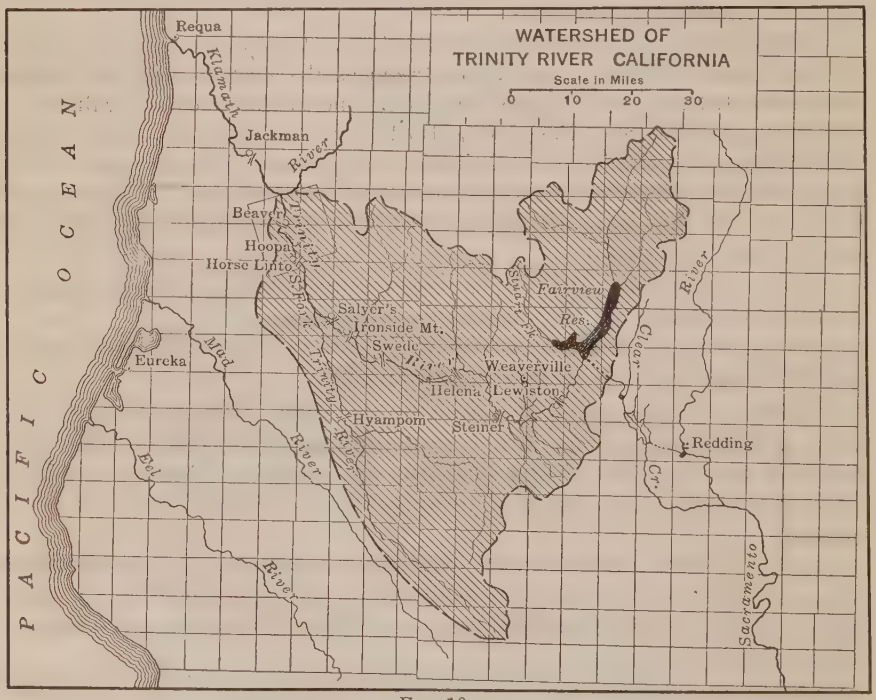

Fig. 10.

The following are the main facts, conclusions, and recommendations set forth in the report:

Trinity River is the main tributary of Klamath River and drains 2 925 sq. miles of mountainous and wooded area on the west slope of the Coast Range in Northern California. The annual precipitation, dependent on altitude and nearness to the ocean, ranges from 35 to 80 in. Its average for the water-shed is about 50 in. The only town of consequence on the water-

* Mr. McClure died June 22, 1926.

shed is Weaverville, an old mining town which derived some importance from hydraulic mining operations that have practically ceased. Agricultural development is small and is confined mostly to the Hoopa Valley Indian Reservation at the lower end of the stream. There are not more than 6 000 acres of irrigable land in the Trinity Basin. There is a great seasonal variation in the flow of the river. At Lewiston, Calif., the mean flow is about 1 600 sec-ft., the maximum is 25 000 sec-ft., and the minimum 52 sec-ft. The areas available for irrigation are so small that the minimum flow is more than enough to irrigate them. It is evident, however, that for successful power development large storage is essential.

There are reservoir sites on Stuart Creek, which enters the Trinity six miles above Lewiston, that have sufficient capacity to regulate the flow of that creek. On the main stream there are only two opportunities for large storage above Lewiston, one at Trinity Center and one at Fairview. The latter is probably the more favorable, because it takes in the flow of Stuart Creek. It can be developed by a dam 270 ft. high to a capacity of 600 000 acre-ft., is at an elevation of more than 2 000 ft., and receives the flow from the best yielding part of the drainage area.

Fig. 11 shows the run-off of Trinity River past Fairview Dam site and the contents of Fairview Reservoir on the basis of maintaining a regulated flow of 1 053 sec-ft. The suitability of foundations and abutments for a dam 270 ft. in height above the stream bed at this site has not been established by borings.

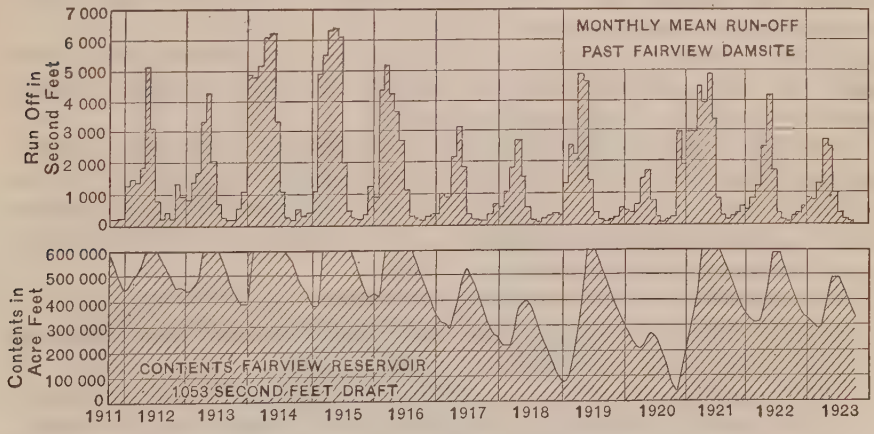

Fig. 11.

The average elevation of the surface of Fairview Reservoir would be 2 138 ft. above sea level. From this point to tail-water at the Jackman site, on Klamath River, the fall is 2 091 ft. A reconnaissance of the Trinity, made by Mr. E. W. Kramer, of the U. S. Forest Service, indicates that dam sites can be found, that will permit development for power of about 1 738 ft., or 83% of this fall. On this basis and with 560 000 acre-ft. of Fairview storage, the all-year power that can be developed on the Trinity is about 250 000 h.p.

It is possible to divert the Trinity at a point just below Fairview into Clear Creek, a tributary entering the Sacramento River 7 miles below Redding, by a tunnel 6.3 miles long. The tunnel would discharge at a height of 487 ft. above Clear Creek, giving an opportunity for power development. The tail-water from the power-house would flow into a reservoir on Clear Creek with a dam, at Whiskeytown, 210 ft. high and a storage capacity of 135 000 acre-ft. From this reservoir a conduit and pressure tunnel could conduct the water to a power-house with a head of about 660 ft. on the Sacramento River 3 miles above Redding. The usable head in case of diversion is about 1 268 ft. as against 1 738 ft. on the Trinity, and the all-year power in case of diversion would be about 115 000 h.p. as against 250 000 h.p. on the Trinity. After diversion, it would still be possible to develop about 105 000 all-year horse-power on the Trinity. The net loss of power by diversion is, therefore, about 30 000 h.p. This loss would be reduced by about 7 000 h.p. if the Iron Canyon project on the Sacramento River were built.

Irrigation on the Trinity will not be injured by diversion of Trinity water at Fairview, whereas about 160 000 acres can be irrigated in the Sacramento Valley by such diversion. It is well recognized that the water supply in the Central Valley of California is sufficient to supply only a part of the irrigable land in the Valley. The shortage is in the San Joaquin or southern half of the Valley, but any water supplied to the west side of Sacramento Valley can be made to release water from the Sacramento to supply the northern part of the San Joaquin. In other words, water diverted from the Trinity will increase the irrigated area in the Central Valley by about 160 000 acres, but with the loss of about 30 000 h.p.

Before a decision can be definitely reached the cost of the diversion project must also be considered, and, unfortunately, the data available are not sufficient to permit of satisfactory estimates.

The Federal Power Commission has recently granted a preliminary permit for the diversion project to Mr. W. H. Sampson, of Corning, Calif., with conditions that require him to make investigations and procure the data necessary to determine whether the project is economically feasible. If it is found to be feasible, it will undoubtedly be built.

The conclusions of the Board were, as follows:

(*a*) The full use of Trinity River water makes essential large storage at its head-waters where required storage facilities exist.

(*b*) Potential power of Trinity River water regulated by available storage is slightly greater if flowing down its natural course than if partly diverted.

(*c*) Irrigation possibilities in the Trinity Basin are relatively small and will not be adversely affected by diversion.

(*d*) The Sacramento River water supply and the potential means of regulating it are sufficient for the irrigation of the entire Sacramento Valley.

(*e*) The joint water supply of the Sacramento and San Joaquin Rivers and means of regulating it are insufficient for the irrigation of both valleys.

(*f*) Diversion of Trinity River water will permit more complete irrigation development in the joint valleys than is otherwise possible, involving a potential addition of at least 160 000 acres of irrigated land.

(*g*) Trinity River water, if diverted, is likely to be used on the west side of the Sacramento Valley, thereby permitting later Sacramento storage development to supply part of the Lower San Joaquin Valley.

(*h*) The only important industrial use made of Trinity River water is in connection with gold dredging. Diversion will not interfere with this industry. Storage above diversion may flood prospective dredging lands in the reservoir site, which fact does not justify delay in reservoir construction.

(*i*) Navigation is confined to Klamath River below the mouth of the Trinity. It may be interfered with to a small extent by diversion which, on the other hand, may benefit Sacramento navigation.

(*j*) The advantages of diversion greatly outweigh its disadvantages.

In harmony with the conclusions, the Board made recommendations as follows:

(*a*) That no power rights be granted to prospective Trinity River water users, that will interfere with ultimate diversion of Trinity water to the Sacramento Valley.

(*b*) That any permission to divert water from the Trinity River to the Sacramento Valley provide for the maintenance of a flow of at least 20 sec-ft. in the Trinity River below the point of diversion.

American River

A Board consisting of D. C. Henny, M. Am. Soc. C. E., Consulting Engineer, U. S. Bureau of Reclamation, W. F. McClure, M. Am. Soc. C. E., State Engineer of California (represented by Paul Bailey, M. Am. Soc. C. E.), U. S. Grant, 3d, Major, Corps of Engineers, U. S. A., Assoc. M. Am. Soc. C. E., and E. W. Kramer, M. Am. Soc. C. E., Hydraulic Engineer, U. S. Forest Service, was appointed by the Federal Power Commission to study the American River, in California (see map, Fig. 12), in order to indicate the most beneficial uses that can be made of its waters and especially to determine whether diversion from one to another tributary will be to the general public benefit. The Board submitted its report in April, 1924. The following is abstracted from the report and the records of the Federal Power Commission:

The American River System with its three main Forks—South, Middle, and North—drains the mountainous area lying to the east and northeast of the City of Sacramento. The steep river gradients descending about 6 000 ft. in the upper 50 miles of the stream courses favor economical power development. This is offset to some extent, however, by the wide fluctuations of stream flow throughout the year, by the occurrence at intervals of unusually dry years, and by the paucity of storage facilities at high elevations.

Some agricultural land is found on the ridges between the branches of the rivers at elevations less than 3 500 ft., but the only tract that is feasible of irrigation is on Georgetown Ridge between South and Middle Forks. This tract contains 25 000 acres of irrigable land. The only practicable source of supply for this land is Rubicon Creek, and 50 000 acre-ft. will be required, about 30 000 acre-ft. of which must be supplied from storage. It will be

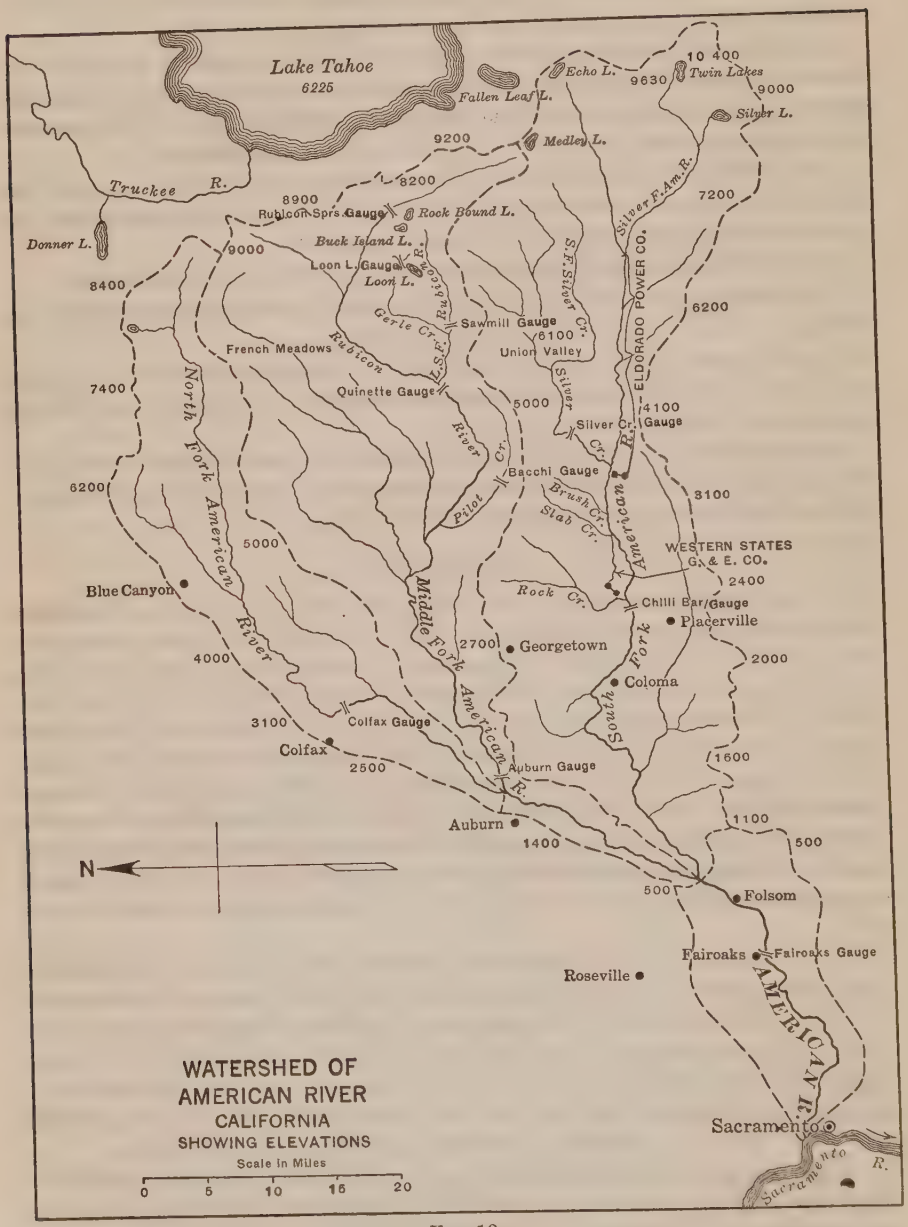
Lake Tahoe
6225
Fallen Leaf L.
Echo L.
Twin Lakes
Silver L.
Medley L.
Truckee R.
Donner L.
Rubicon Spgs. Gauge
Rock Bound L.
Buck Island L.
Loon L. Gauge
Loon L.
Rubicon R.
Gerle Cr.
L.S.F. Silver Cr.
S.F. Silver Cr.
Silver F. Am. R.
ELDORADO POWER CO.
Sawmill Gauge
Union Valley
French Meadows
Rubicon River
Quinnette Gauge
Silver Cr. Gauge
Pilot Cr.
Bacchi Gauge
Brush Cr.
Slab Cr.
North Fork American River
Middle Fork American R.
American R.
WESTERN STATES G. & E. CO.
Rock Cr.
Chili Bar Gauge
Placerville
Blue Canyon
Georgetown
Coloma
South Fork American R.
Colfax Gauge
Colfax
Auburn Gauge
Auburn
Folsom
Fairoaks
Fairoaks Gauge
Roseville
AMERICAN R.
Sacramento
Sacramento R.
N
WATERSHED OF AMERICAN RIVER
CALIFORNIA
SHOWING ELEVATIONS
Scale in Miles
0 5 10 15 20

FIG. 12.

possible to irrigate about 400 000 acres adjacent to the American River and below Folsom on the floor of Sacramento Valley with water from the American River, provided storage can be developed to the amount that appears practicable.

The flood period in the American River generally extends from February to June and during this period about 80% of the annual run-off is discharged. The dry seasonal flow becomes very small in August, September, and October. At times, the September outflow from the entire basin averages as low as 250 sec-ft. The average annual run-off of the American River since 1905 has been more than 3 000 000 acre-ft., but in dry years it has been as low as 1 250 000 acre-ft.

The Coloma Reservoir site at a river elevation of about 600 ft. on the main stream of the South Fork, has a possible storage capacity of at least 300 000 acre-ft. The flat gradients and narrow gorges at corresponding elevations on the Middle and North Forks make it probable that ample storage may be found at low elevations on these Forks to regulate the flow so as to meet the seasonal demands of irrigation on the floor of the Sacramento Valley. Such storage is all up stream from Folsom Dam on the main American River which would serve as a logical diversion point for distribution on the Valley floor. The storing of water for irrigation on the lower reaches of the stream will permit the use of the limited up-stream storage for power, except for 30 000 acre-ft. on the Rubicon, needed for Georgetown Ridge. Storage in the upper drainage area is believed to exist to the following extent:

North Fork	0 acre-ft.
Middle Fork:	
Main stream: French Meadows	64 000 " "
Rubicon: Rock Bound and Buck Island Lakes	24 000 " "
Hell Hole	32 000 " "
Parsley Bar	13 000 " "
Loon Lake on Gerle Creek	47 000 " "
Middle Gerle Creek	8 500 " "
Upper Pilot Creek	5 600 " "
South Fork:	
*Silver Creek: Ice House	45 000 " "
Union Valley	165 000 " "
Main stream: Medley and Echo Lakes	11 000 " "
*Silver Fork: Twin and Silver Lakes	50 000 " "
Alder Creek and Plum Creek	31 000 " "
Total	496 100 acre-ft.

The following are the existing power developments on American River:

Folsom Project, Pacific Gas and Electric Company: Main stream, head, 55 ft.	4 000 h.p.
Western States Gas and Electric Company: South Fork, head, 572 ft.	6 500 "
El Dorado Plant, Western States Gas and Electric Company: South Fork, head, 1 905 ft.	21 500 "
Total	32 000 h.p.

* Silver Creek must not be confused with Silver Fork.

The Federal Power Commission has before it the following applications for power development on American River:

American River Water and Power Company:

Diversion of Rubicon River to French Meadows and development of power on Middle Fork.

City of Sacramento:

Development of storage on Rubicon River and diversion to Silver Creek.

Development of storage and power on Silver Creek.

Development of power on South Fork below Silver Creek and diversion of water from Coloma to Sacramento for domestic and irrigation supply.

Rubicon River diversion was disapproved and preliminary permit for Silver Creek and South Fork, American River, development was issued October 25, 1924.

Western States Gas and Electric Company:

Additional development of storage and power on Silver Fork and South Fork. License issued February 23, 1922.

There have also been applications, now canceled, to divert Middle Fork into North Fork and North Fork into Middle Fork. Middle Fork and Silver Creek have more than enough storage capacity on their head-waters to equalize their stream flow. Rubicon River and South Fork have sufficient head-water storage to regulate their flow in part. North Fork, so far as is known, has practically no head-water storage. It is on account of this condition that projects have been proposed for diverting North Fork and the Rubicon to Middle Fork and for diverting the Rubicon to Silver Creek. On South Fork the head-water storage is controlled by water rights held by the Western States Gas and Electric Company, and that Company proposes to proceed with complete development above Silver Creek as fast as its power demand permits.

The conflicting applications on the other tributaries caused the Federal Power Commission to appoint the Board to determine the best plan of development.

Mr. E. W. Kramer, of the U. S. Forest Service, made a study of various plans of development, the results of which are given in Table 6. Credits from irrigation have been arbitrarily based on an estimate of $20 per acre.

The Federal Power Commission has adopted a general rule, that diversions from one stream to another shall be permitted only when positive advantages outweighing all disadvantages can be shown. The studies of the Board show that such advantages exist in the diversion of the North Fork of American River to the Middle Fork, and that they do not exist in the case of any of the other proposed diversions.

The Board also investigated the use of American River water for navigation in the Sacramento, and for improving salt-water conditions below

Sacramento, and the use of American River storage for flood control. On the basis of its studies, the Board concluded:

(*a*) That navigation and salt-water conditions in the Sacramento River below Sacramento will not be injuriously affected by any now anticipated uses of the American River water, including domestic water supply, irrigation, and power.

(*b*) That storage facilities in the American River Basin should be dedicated to irrigation and power primarily, since their economic value for these purposes is too great to justify their developments solely for flood control.

TABLE 6.

Plan of development.	All-year horse-power.	Total cost, exclusive of transmission.	Credit for Irrigation.		Net cost of power.	Cost per all-year horse-power.
			Acres.	Amount.		
1.—Diversion, North Fork to Middle Fork:						
Diversion, Upper Rubicon to Silver Creek:						
North and Middle Forks	80 000	$20 120 000	36 000	$ 720 000	$19 400 000	$242
Rubicon	37 000	7 960 000	26 000	520 000	7 440 000	201
Silver Creek	136 000	32 099 000	87 000	1 740 000	30 359 000	223
Total	253 000	$60 179 000	149 000	$2 980 000	$57 199 000	$226
2.—Diversion, North Fork to Middle Fork:						
North and Middle Forks	87 000	$21 250 000	36 000	$ 720 000	$20 530 000	$236
Rubicon	99 000	20 509 000	77 000	1 540 000	18 969 000	192
Silver Creek	79 000	19 250 000	45 000	900 000	18 350 000	232
Total	265 000	$61 009 000	158 000	$3 160 000	$57 849 000	$218
3.—No Diversion:						
North Fork	0					
Middle Fork	45 000					
Rubicon	99 000			Not computed		
Silver Creek	79 000					
Total	223 000	.....	.....	.....	.....	....
4.—Diversion, North Fork to Middle Fork:						
Diversion, Upper Rubicon to Middle Fork:						
Middle Fork	118 000					
Rubicon	0			Not computed		
Silver Creek	79 000					
Total	197 000	.....	.....	.....	.....	....

(*c*) That the fullest practicable irrigation of the cultivable lands of the American River Basin is in the interest of the public and requires that any power or irrigation projects developing upper storage reservoirs in the Rubicon drainage area under Federal Power license, shall be obligated to supply at reservoirs, shortage in irrigation water for approximately 25 000

acres on the Georgetown Ridge, at a price based on investment and cost of service, and not on value of power lost by supplying this irrigation need.

(*d*) That until investigations show that large storage for Valley irrigation cannot be feasibly developed on the lower reaches of the North and Middle Forks below River Elevation 1 150, it is inadvisable to permit power development which would interfere with irrigation storage below this elevation.

(*e*) That the Coloma Reservoir has sufficient capacity and is located so that it can regulate for the benefit of irrigation, almost the entire flow of the South Fork of the American River below power developments. Its primary value is for irrigation storage.

(*f*) That the Folsom Dam site admits raising the dam to a considerable additional height and that this site is located at the logical point for diverting American River water for all lower gravity irrigation.

(*g*) That the potential power of the North and Middle Forks of the American River can be best developed by the diversion of the Upper North Fork drainage to French Meadows Reservoir on the Middle Fork.

(*h*) That the diversion of Rubicon water to the Middle Fork would be wasteful of power.

(*i*) That the diversion of Rubicon water to Silver Creek would prevent maximum power utilization and would tend to increase the total power cost.

(*j*) That the municipal power, domestic water supply, and irrigation requirements of the City of Sacramento and the Sacramento Municipal Utility District, for a reasonably extended future, can be satisfactorily supplied from Silver Creek without Rubicon diversion.

Stanislaus River

The Federal Power Commission also requested the Board which reported on the Trinity and American Rivers in California to report on a proposed enlargement of the present development on the North Fork of Stanislaus River, California, with a view to determining the best method of development. The report was submitted December 1, 1924, and will probably not be printed. The questions involved are probably not of sufficient general interest to merit discussion herein.

Kings River

The San Joaquin Light and Power Corporation has been planning to undertake the complete development for power of North Fork, Kings River (see plan, Fig. 13), since 1913. After many delays, on July 28, 1922, the Corporation was issued a Federal Power Commission license for the entire project. Section 9 (*b*) of the Federal Water Power Act, reads, as follows:

"Satisfactory evidence that the applicant has complied with the requirements of the laws of the State or States within which the proposed project is to be located with respect to bed and banks and to the appropriation, diversion, and use of water for power purposes and with respect to the right to engage in the business of developing, transmitting, and distributing power, and in any other business necessary to effect the purposes of a license under this act."

On account of this section, however, the license was made subject to the following proviso:

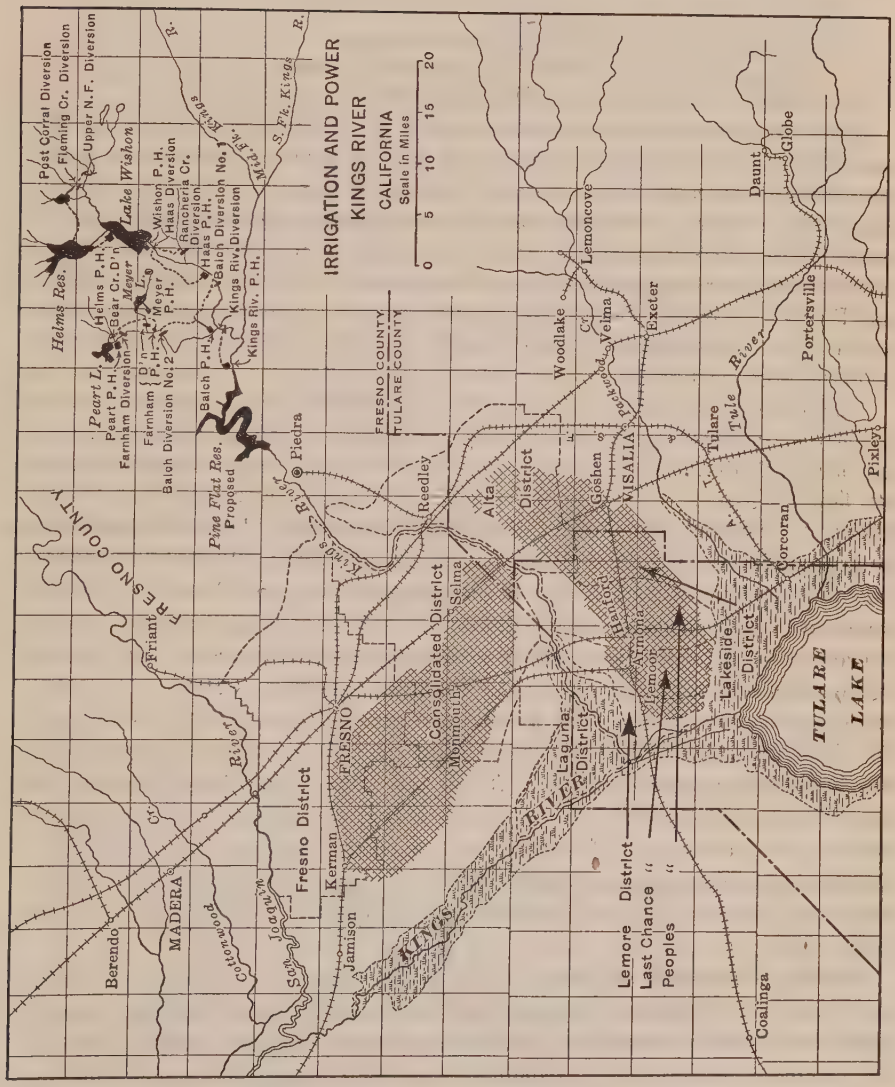
IRRIGATION AND POWER
KINGS RIVER
CALIFORNIA
Scale in Miles
0 5 10 15 20
Post Corral Diversion
Fleming Cr. Diversion
Upper N.F. Diversion
Lake Wishon
Helms Res.
Helms P.H.
Bear Cr. D'n
Meyer L.
Meyer P.H.
Wishon P.H.
Haas Diversion
Rancheria Cr. Diversion
Haas P.H.
Balch Diversion No. 1
Kings Riv. Diversion
Kings Riv. P.H.
Pear L.
Pear P.H.
Farnham Diversion
Farnham D'n P.H.
Balch Diversion No. 2
Balch P.H.
Pine Flat Res.
Proposed
Piedra
FRESNO COUNTY
TULARE COUNTY
Kings River
Mid. Fk. Kings R.
S. Fk. Kings R.
Friant
Cottonwood Cr.
San Joaquin River
Berendo
MADERA
Fresno District
Kerman
Jamison
FRESNO
Consolidated District
Monmouth
Selma
Reedley
Alta District
Laguna District
Lemore District
Last Chance "
Peoples "
Hanford
Armona
Lemoor
Goshen
VISALIA
Woodlake
Lemoncove
Velma
Exeter
Packwood Cr.
Tulare
Tule River
Corcoran
Lakeside District
TULARE LAKE
Pixley
Portersville
Daunt
Globe
Coalinga
KINGS RIVER

FIG. 13.

"Provided, that this license shall not become effective with respect to the following separable parts of the above described project works until the Licensee has complied with the requirements of the laws of the State of California with respect to appropriation, diversion, and storage of water contemplated in the development of said separable parts:

(*a*) Wishon Dam, Reservoirs and Power Plant
(*b*) Meyer Dam and Reservoir
(*c*) Peart Dam, Reservoir and Power Plant
(*d*) Rancheria Creek Diversion Dam and Tunnel
(*e*) Bear Creek Diversion Dam and Canal."

The Corporation was issued water permits by the State for the use of the natural flow of North Fork, Kings River, but its applications to store water were held up on account of protests from the irrigation interests that use Kings River water. The Division of Water Rights, State Department of Public Works of California, notified the Commission, however, that it was expected that water rights for the entire project of the San Joaquin Light and Power Corporation would be granted as soon as action could be taken on the Pine Flats Reservoir project, application for which was made to the State on January 1, 1916.

The Pine Flats project proposes the construction of a reservoir with a capacity of 600 000 acre-ft., located at the outlet of Kings River Canyon, just above Piedra, Calif. The primary object of the project is to regulate the flow of Kings River for irrigation. The area irrigated varies from 500 000 to 700 000 acres, and it is estimated that about 1 000 000 acres can be irrigated when the Pine Flats project is completed. With the possible exception of Kern River, Kings River presents more complications for the organization of a comprehensive irrigation development than any other stream in California. The are twenty-three different groups of interests, with ten or twelve additional subsidiary interests using Kings River water. The water rights are very complicated and, as might be expected, are more or less in conflict. The interest having the last water right can take no water except when the flow in the river exceeds 9 500 cu. ft. per sec. The Kings River territory is one of the largest and most successful irrigation developments in the United States. The present use of water is not as economical as it should be, and the advantages of expanding the development by construction of the Pine Flats project are so great that undoubtedly the difficulties will eventually be overcome.

The low-water flow of North Fork, Kings River, is so meager and the cost of power development is so great that the power project of the San Joaquin Light and Power Corporation is not economically feasible without storage. The complete development will include four reservoirs, as shown in Table 7.

The power capacity of the project under the Federal Power Commission's definition will be about 215 000 h.p., and the proposed installed capacity about 400 000 h.p.

There are also considerable power possibilities on the South and Middle Forks of Kings River, but the region is very rugged and difficult of access.

and the projects will be relatively expensive so that some years may elapse before they are built. After the construction of the Pine Flats project, the development of power on South and Middle Forks will benefit irrigation development, but the data available are so meager that the ultimate relation between power on these Forks and irrigation cannot now be determined.

Since 1917 the diversion of water from Kings River for irrigation has been supervised by a Watermaster appointed by the State Division of Water Rights. Table 8 has been compiled from the records of the Watermaster. Table 9 has been compiled from the discharge records of the Watermaster and the irrigation demands set forth in a report on "Irrigation Requirements of California Lands."* Table 9 shows that the unregulated discharge is deficient in July, August, September, and October properly to supply the irrigation need, the deficiency running from 318 000 acre-ft. to 660 000 acre-ft. Table 8 indicates that this deficiency is made up as far as practicable by over-irrigation in May and June. It is also partly made up by pumping underground water, and with this in view water is diverted, when available, to build up the underground supply. Table 8 shows also that during the period of record there was each year water unused for irrigation which might have been stored for power, the quantity varying from 111 000 acre-ft. to 842 000 acre-ft., which would be sufficient to make the power project feasible. Use of this excess water for power has been prevented thus far by the irrigation interests near Tulare Lake, who claim this undiverted water is necessary to underground storage from which they pump. Fig. 6 shows graphically the relation between discharge, present diversions, and desirable irrigation supply.

TABLE 7.

	Height of dam, in feet.	Elevation of crest, in feet.	Capacity, in acre-feet.
Wishon Lake	250	6 545	128 000
Meyer Reservoir	190	6 845	21 000
Peart Reservoir	315	5 645	54 600
Helms Reservoir	297	8 177	102 000
Total capacity	....	.....	305 600

The effect of the power storage on the irrigation supply will depend on the rate of release of stored water for power. Under any practicable method of operation, however, the power storage will reduce the capacity needed in Pine Flats Reservoir for seasonal regulation, and to that extent will increase its capacity for hold-over storage and will thus benefit irrigation.

The Pine Flats Reservoir will make possible the full use of the waters of Kings River for both power and irrigation.

Green River Above Flaming Gorge

On January 24, 1921, the Utah Power and Light Company made application to the Federal Power Commission for a preliminary permit for power

* *Bulletin No. 6*, State Dept. of Public Works, California, 1923.

TABLE 8.—MONTHLY DISCHARGE OF KINGS RIVER AT PIEDRA AND ACTUAL TOTAL DIVERSION FOR IRRIGATION, IN THOUSANDS OF ACRE-FEET.

(From records of Watermaster.)

Year.		January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Water unused for irrigation available for power storage.
1918	Discharge	12	22	117	195	329	499	92	26	30	86	32	35	
	Diversion	7	15	52	172	301	328	92	26	17	47	23	30	
	Water unused	5	7	65	23	28	171	0	0	13	39	9	4	364
1919	Discharge	31	46	75	205	459	150	55	20	11	14	10	27	
	Diversion	30	43	75	195	370	150	55	20	11	11	9	23	
	Water unused	1	3	0	10	89	0	0	0	0	3	1	4	111
1920	Discharge	22	35	104	163	485	392	113	38	15	28	35	37	
	Diversion	20	28	104	163	388	370	105	35	13	25	29	33	
	Water unused	2	7	0	0	97	22	8	3	2	3	6	4	154
1921	Discharge	52	62	131	183	390	444	128	28	16	14	12	56	
	Diversion	45	62	131	183	342	350	128	28	14	12	11	27	
	Water unused	7	0	0	0	48	94	0	0	2	2	1	29	183
1922	Discharge	89	90	100	164	664	709	226	72	27	15	24	70	
	Diversion	38	42	75	160	390	356	217	44	16	12	21	27	
	Water unused	51	48	25	4	274	343	9	28	11	3	3	43	842
Average		....	....	....	....	....	....	....	....	....	....	....	....	331

TABLE 9.—Monthly Discharge of Kings River at Piedra and Desirable Monthly Supply for Present Irrigated Area.

(700 000 Acres at 2 Acre-Feet per Year = 1 400 000 Acre-Feet.)

Year.		January.	February.	March.	April.	May.	June.	July.	August.	September.	October.	November.	December.	Totals, in Thousands of Acre-Feet.	
															Storage required.
1918	Discharge..	12	22	117	195	329	499	92	26	30	86	32	35		
	Demand...	....	28	70	154	238	252	252	210	140	56	....	....		
	Difference.	+12	— 6	+47	+ 39	+ 91	+247	—160	—184	—110	+30	+32	+35	+ 533	—460
1919	Discharge..	31	46	75	205	459	150	55	20	11	14	10	27		
	Demand...	....	28	70	154	238	252	252	210	140	56	....	....		
	Difference.	+31	+18	+ 5	+ 91	+221	—102	—197	—190	—129	—42	+10	+27	+ 403	—630
1920	Discharge..	22	35	104	163	485	392	113	38	15	28	35	37		
	Demand...	....	28	70	154	238	252	252	210	140	56	....	....		
	Difference.	+22	+ 7	+34	+ 9	+247	+140	—139	—172	—125	—28	+35	+37	+ 531	—464
1921	Discharge..	52	62	131	183	390	444	128	28	16	14	12	56		
	Demand...	....	28	70	154	238	252	252	210	140	56	....	....		
	Difference.	+52	+34	+61	+ 29	+152	+192	—124	—182	—124	—42	+12	+56	+ 588	—472
1922	Discharge..	89	90	100	164	664	709	226	72	27	15	24	70		
	Demand...	....	28	70	154	238	252	252	210	140	56	....	....		
	Difference.	+89	+62	+30	+ 10	+426	+457	— 26	—138	—113	—41	+24	+70	+1118	—318

development at the Flaming Gorge power site on Green River, Utah (see Fig. 14). A preliminary permit was granted for a period of three years on August 15, 1923. The permit contains the following special conditions:

"If license is issued for said proposed project it * * * may contain the following special conditions or provisions:

"*A*. The rights of the Licensee to appropriate, store, divert, and use the waters of said Green River in connection with said project shall be subject to and be limited by the provisions of the 'Colorado River Compact' signed at Santa Fé, New Mexico, on November 24, 1922, if, when, and as said Compact is ratified.

"*B*. Said rights to appropriate, store, divert, and use waters of Green River shall be subordinate to all present and future rights to the use of the waters of said river above said dam for purposes of irrigation or of domestic water supply."

The second of these special conditions was inserted by the Federal Power Commission to meet objections by the State of Wyoming which, with the other arid States, is following a policy of giving the use of water for irrigation unqualified precedence over its use for power.

The Utah Power and Light Company has completed its investigation of the power site and prepared plans for the project structures, but has not yet applied for a license. It is understood that the Company feels that it can not finance the project if Special Condition *B* is included without qualification in the license, and therefore it has modified its construction program and will extend its development on Bear River to meet immediate market demands in the hope that sentiment in Wyoming will change so that the reservation of water for irrigation use on Green River may be definitely limited to what will be needed to meet the probable growth of irrigation. A brief review of the physical features of the situation will show the principal factors involved.

The power project proposed consists of a dam about 220 ft. high in Horse Shoe Canyon, about 40 miles by air line below Green River, Wyo.; a tunnel, 3½ miles long, leading from the reservoir above the dam through Bear Mountain; and a power-house on the river, 15 miles by river below the dam. The latter part of the project may be modified on further study, but the power capacity will not materially change. The reservoir capacity is 3 476 000 acre-ft., but probably only the top 50 ft., with a capacity of 1 710 000 acre-ft., will be drawn upon as that amount of storage is sufficient to equalize the flow of the stream. The average power capacity of the project continuously available will be about 75 000 h.p. and the installed capacity will be more than 100 000 h.p. There appear to be no serious construction difficulties, and although the site is about fifty miles from the nearest railroad and the project is not a cheap one, it probably can be built at a cost that will not raise present power rates. The Company now furnishes power to most of the State of Utah, and this project would be connected to and supplement its power system.

There is considerable difference of opinion as to the amount of irrigation development that is feasible in Green River Valley, Wyoming. Some believe that there will be little increase over that of the present while others believe that the entire stream flow above Green River, Wyo., will be needed for irrigation. The following information is taken mostly from unpublished reports by

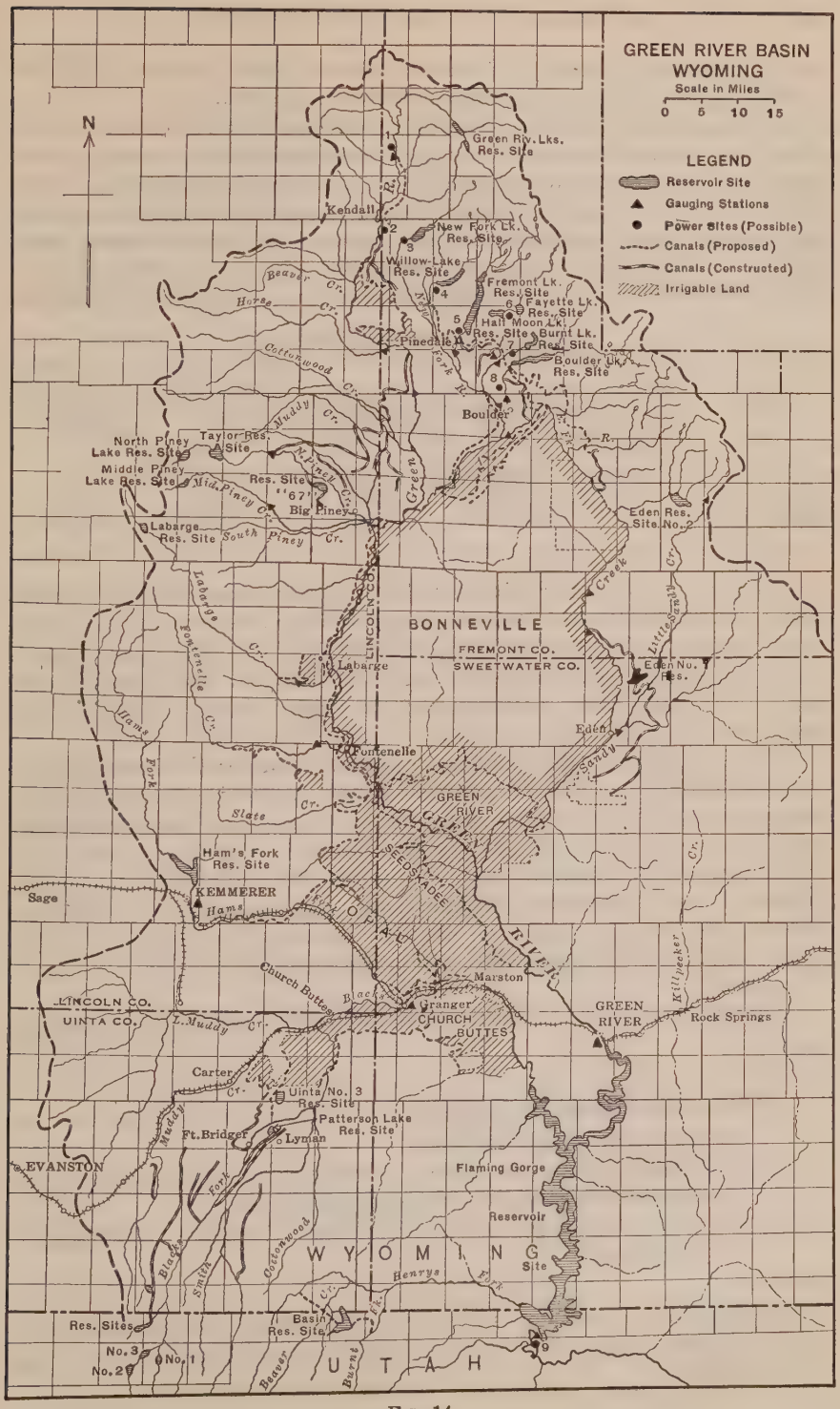
GREEN RIVER BASIN
WYOMING
Scale in Miles
0 5 10 15
LEGEND
Reservoir Site
Gauging Stations
Power Sites (Possible)
Canals (Proposed)
Canals (Constructed)
Irrigable Land
N
Green Riv. Lks. Res. Site
Kendall
New Fork Lk. Res. Site
Willow Lake Res. Site
Fremont Lk. Res. Site
Fayette Lk. Res. Site
Hall Moon Lk. Res. Site
Burnt Lk. Res. Site
Pinedale
Boulder Lk. Res. Site
Boulder
North Piney Lake Res. Site
Taylor Res. Site
Middle Piney Lake Res. Site
Mid. Piney Res. Site "67"
Big Piney
Labarge Res. Site
Eden Res. Site No. 2
Eden Res. Site No. 1
BONNEVILLE
FREMONT CO.
SWEETWATER CO.
LINCOLN CO.
Labarge
Fontenelle
Eden
GREEN RIVER CREEK
Ham's Fork Res. Site
KEMMERER
Sage
OPAL
Marston
Granger
CHURCH BUTTES
Church Buttes
LINCOLN CO.
UINTA CO.
GREEN RIVER
Rock Springs
Carter
Uinta No. 3 Res. Site
Patterson Lake Res. Site
Ft. Bridger
Lyman
EVANSTON
Flaming Gorge Reservoir
WYOMING
Henrys Fork
Basin Res. Site
Res. Sites
No. 3
No. 1
No. 2
UTAH

FIG. 14.

Mr. R. R. Woolley, of the U. S. Geological Survey, dated February, 1923, and by the U. S. Bureau of Reclamation, dated February, 1924.

The Green River Basin in Wyoming covers an area of about 15 000 sq. miles. All the irrigable land lies at an elevation of 6 000 ft. or more above sea level. The average precipitation in the valley ranges from 6 in. in the south to 11 in. in the north with much higher precipitation on the mountains. The average annual temperature varies from about 42° Fahr. at Green River, Wyo., to 34° Fahr. in the northern part of the valley. The growing season varies from 105 days to 30 days. Frosts may be expected any month (Table 10).

TABLE 10.—Frost Data.

Station.	Number years of record.	Average date of first killing frost, fall.	Average date of last killing frost, spring.	Earliest date of killing frost, fall.	Latest date of killing frost, spring.
Green River.....	10	September 4	June 10	August 11	June 23
Daniel.....	13	August 13	June 22	July 7	July 9
Pinedale.....	7	August 17	June 29	July 31	July 16
Manila.....	8	September 8	June 22	August 25	July 10

Coal mining and stock-raising are the principal occupations. The population is about 20 000, more than 65% of which live in towns along the Union Pacific Railway. The irrigable land lies from 10 to 100 miles from the railroad. The present irrigated area is largely made up of bottom-lands developed by individual effort to supplement the stock industry. The principal crop is wild hay.

Diversion from the tributaries is not difficult, but for the large projects, with lands away from the river, long main canals are required because of light river grades. Under the Carey Act, many projects have been outlined by surveys and much development proposed and several projects have been partly built. A total of about 500 000 acres is covered by permits for appropriation of water. Diversion demands range from 2.5 acre-ft. to 3.0 acre-ft. per year, with a net consumption of water averaging 1.5 acre-ft. Lack of reservoir sites at strategical points will prevent the use of the entire run-off of the basin.

Table 11 gives the U. S. Reclamation Bureau's classification of irrigable lands.

Class A.—

1.—Undeveloped areas of constructed or partly constructed projects not requiring undue expenditures.

2.—New projects concerning which no serious construction difficulties are known and for which water supply seems ample.

3.—Increase in area under small ditches already built or anticipated to be built shortly.

Class B.—

Developments similar to those of Class A but by reason of relatively higher cost not likely to be carried through in the immediate future.

Class C.—

Developments possibly feasible from the standpoints of construction and water supply, but out of the question in the near future by reason of excessive cost.

The average annual discharge of Green River at Flaming Gorge, deducting present irrigation use and probable evaporation losses in the reservoir, is about 1 810 000 acre-ft., giving about 2 500 cu. ft. per sec. available for power.

According to the estimate of the U. S. Bureau of Reclamation the depletion of flow caused by future development per Table 11 is:

Class *A* projects,	279 000 acre-ft. per year	equivalent to	385 cu. ft. per. sec.
Class *B* " ,	255 000 " " "	" "	352 " " " "
Class *C* " ,	236 000 " " "	" "	326 " " " "
Total.....	770 000 acre-ft. per year	equivalent to	1 063 cu. ft. per. sec.

TABLE 11.—Classification of Projects.

Project.	Project area, in acres.	Acres irrigated in 1922.	Class.		
			A.	*B.*	*C.*
Above Green River City :					
Fontenelle Creek	4 000	.....	.....	4 000	.....
La Barge Project	3 000	.....	3 000	.....	.....
Big Piney-La Barge	6 000	.....	6 000	.....	.....
"67" Reservoir	2 200	.....	2 200	.....	.....
Cottonwood Development Company	40 000	2 000	38 000	.....	.....
Uinta-Fremont	14 000	1 000	13 000	.....	.....
Apex Project	4 000	.....	4 000	.....	.....
Fremont Lake	7 000	1 500	5 500	.....	.....
Boulder	8 600	6 000	2 600	.....	.....
East Fork Canal	7 800	2 000	5 800	.....	.....
Willow Lake	1 000	.....	1 000	.....	.....
Paradise	3 500	.....	3 500	.....	.....
Bertram	900	500	400	.....	.....
Eden	40 000	7 000	33 000	.....	.....
Small ditches	180 000	120 000	30 000	30 000	.....
Green River	60 000	.....	.....	60 000	.....
Seedskadee	93 000	.....	.....	27 000	66 000
Bonneville	69 000	.....	.....	.....	69 000
Total	544 000	140 000	148 000	121 000	135 000
Hams Fork :					
Opal project	40 000	.....	.....	40 000	.....
Small ditches	17 000	12 000	5 000	.....	.....
Total	57 000	12 000	5 000	40 000	.....
Blacks Fork :					
Lyman ditches	63 000	31 000	32 000	.....	.....
Small ditches	44 000	29 000	10 000	5 000	.....
Total	107 000	60 000	42 000	5 000	.....
Henrys Fork :					
Small ditches	20 000	8 000	7 000	5 000	.....
Grand total	728 000	220 000	202 000	171 000	135 000

At the Flaming Gorge project 1 cu. ft. of water will produce about 30 h.p., and the average continuous horse-power will be:

With present water supply.....	75 000	h.p., or	492 750 000	kw-hr.	per year
After Class *A* developments.....	63 450	" "	416 866 000	" "	" "
" Class *B* "	52 890	" "	347 487 000	" "	" "
" Class *C* "	43 110	" "	283 233 000	" "	" "

The estimated cost of the Flaming Gorge project is $24 300 000. The power at the switchboard will cost about:

With present water supply.....	$0.00589 per kw-hr.
After Class *A* developments.....	0.00697 " " "
" Class *B* "	0.00836 " " "
" Class *C* "	0.01026 " " "

Present power rates in Utah will hardly justify a cost of $0.006 per kw-hr. for power at this plant. It is possible that as the load grows and more head can be developed below Flaming Gorge the regulated flow through the greater head will reduce the costs so that the project will be feasible with a reduced flow, but, at present, any considerable reduction in flow will make the project impossible.

Any prediction as to rate of growth of irrigated area is perhaps hazardous, but a study of the rate of growth of Green River Valley, Wyoming, is interesting.

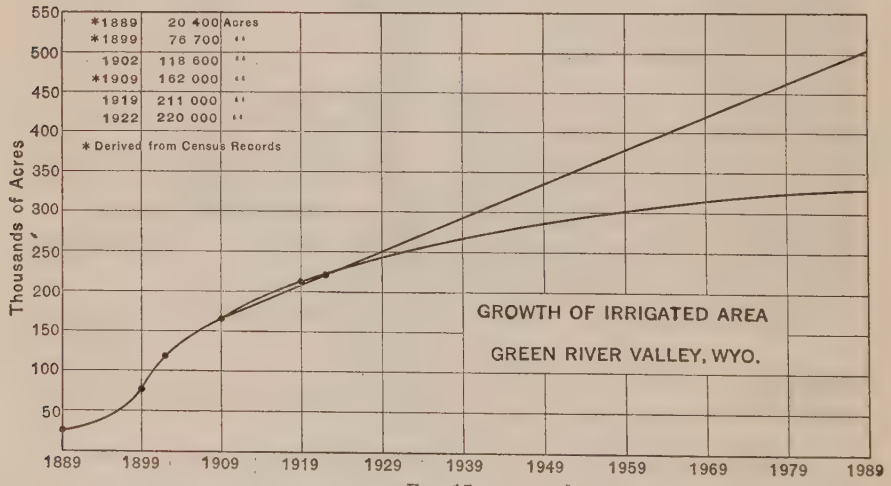

FIG. 15.

Fig. 15 shows the growth of the irrigated area from 1889 to 1922. It also shows what the future growth may be: First, by projection of the curve; and, second, by projection of a straight line through the points for 1909 and 1922. Considering the very short growing season, the inaccessibility of the region, and the expense of developing the larger projects, there is reason to believe that the irrigated area will not increase faster than is indicated by projection of the curve, and it certainly is unlikely that it will increase faster than the

straight-line projection. In other words, 50 years hence, the irrigated area may have increased to 320 000 acres, and it is unlikely that it will have increased to 445 000 acres. Successful agriculture in this region of high altitude is likely to be limited in the future, as in the past, to the raising of winter feed for stock that can be grazed on the many miles of adjoining range. Such irrigation is usually confined to individual effort on small ditches. Referring again to the classification of the U. S. Reclamation Bureau, it will be seen that, under Class *A*, there are 52 000 acres that can be irrigated with small ditches.

Irrigation is much better known than it was twenty years ago. Private capital will be difficult to obtain for any large project in Green River Valley, Wyoming. Strong efforts are still made from time to time to get the Federal or the State Governments to undertake projects that are considered infeasible for private interests, but fairly strong opposition generally is met, and of late such undertakings have been very rare.

The probabilities are that irrigation in Green River Valley, Wyoming, will not seriously interfere with the Flaming Gorge power project but, in order to finance that project, it may be necessary to set a schedule specifying the rate at which the water supply may be depleted for irrigation.

Salt River Valley

Since 1912, Frank G. Baum, M. Am. Soc. C. E., of San Francisco, Calif., has had under consideration and, since 1914, has been trying to obtain Federal authority for a power project on Black River, a tributary of Salt River, in Arizona. (See Fig. 16.) In December, 1920, he made application to the Federal Power Commission for a preliminary permit. The project included three reservoirs with a combined capacity of 35 000 acre-ft. at an elevation about 8 000 ft. above sea level. The power-house would have a head of about 870 ft., and an installed capacity of about 10 000 h.p. It was proposed to find a market for the power in Globe, Clifton, Miami, Jerome, or other mining centers.

The Salt River Valley Water Users' Association made a strong protest against the project on the ground that storage of water on Black River in low-flow years would deplete the water supply for about 200 000 acres of irrigated land in the Salt River project by at least 10 000 acre-ft., and would also cause a power loss of about 700 000 kw-hr. per year which, if available, would pump about 10 000 acre-ft. of ground-water. The total loss of water, it was claimed, would amount to from 20 000 to 30 000 acre-ft. It was stated that the seepage losses in the main canals of the Salt River project amounted to about 200 000 acre-ft. per year, and that the loss due to the Baum project could be compensated for by lining some of the canals or by paying for pumping the seeped water from the ground-water supply. Mr. Baum claimed that his project would cause no loss to the Salt River Valley Water Users' Association, but on the contrary would be beneficial. The data available are too meager to determine accurately what increased evaporation loss might be due to the maintenance of a fairly uniform flow throughout the year in the 125 miles of river channel between the Baum project and Roosevelt Reservoir,

instead of sending most of the water down on the flood flows, as at present. From such data as are available studies were made by engineers of the Federal Power Commission. These studies indicate that for about 5 years in the past 35, the Baum project would have diminished the available water supply by perhaps as much as 2 000 acre-ft., out of a total of about 900 000 acre-ft., and would have reduced the mean head at the power plant at Roosevelt Dam by perhaps 2 ft. out of 80 ft. For 17 years it would have increased the water supply and head for power, and, during the other 13 years, its effect would have been negligible. As the detrimental effect during the 5 years of low flow can be remedied by lining canals or by supplying power for pumping ground-water, the problem is one of determining the cost of such operations and balancing that cost against the value of extra power in the 17 years during which the Salt River project would have benefited. It is doubtful whether the balance would be in favor of, or against, the Baum project, but it is fairly certain that the amount involved would be small.

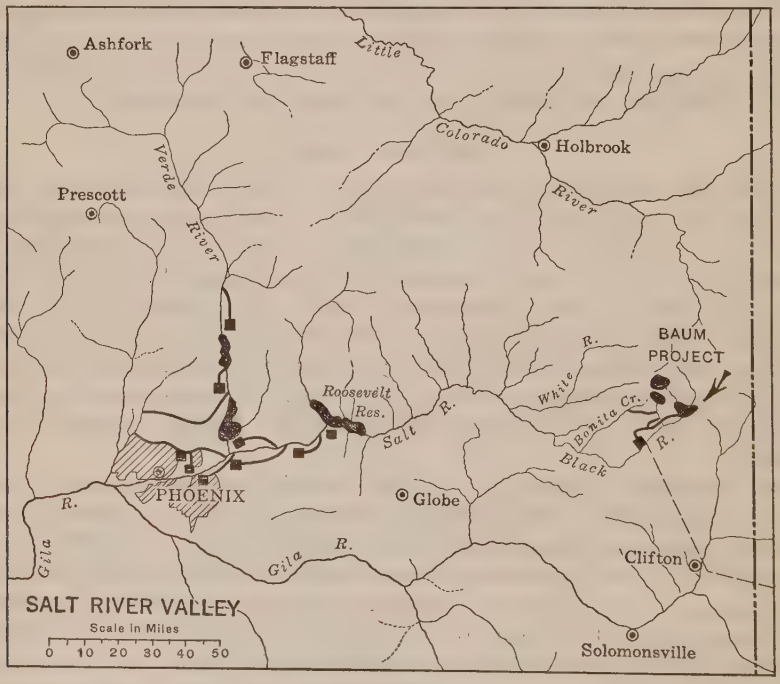

FIG. 16.

The drainage area above Roosevelt Dam contains about 6 000 sq. miles and is capable of economically developing about 75 000 continuous h.p. whenever a market offers, provided head-water storage can be obtained. The State of Arizona has adequate laws to protect the existing water rights of the Salt River Valley Water Users' Association, but practically the entire State supported the Association's protest and the Federal Power Commission, wishing to avoid putting the Association to the expense of a lawsuit, decided to

attempt to adjudicate the question itself, and, therefore, inserted in the Baum permit the following conditions:

"For the purpose of safeguarding the interests of the United States and the Salt River Valley Water Users' Association, the permittee shall, prior to the issue of a license, amend the notices of appropriation of the waters of the tributaries of Salt River so as to eliminate therefrom all reference to appropriations for irrigation purposes, and the license, if issued, shall contain a condition that the licensee shall not use said waters for the purposes of irrigating lands, and that the licensee shall:

"1. Permit the total normal flow of Black River to pass continuously through the power plant or through by-passes at the power plant, except when the flow of Salt River at the diversion dam at the head of the Roosevelt Power Canal of the Salt River project is greater than 250 cu. ft. per sec.

"2. Return all waters diverted or stored for any purpose to the natural channel of Black River immediately below the power plant.

"3. Replace to the satisfaction of the Commission any net loss of water to the Salt River Valley Water Users' Association caused by evaporation from the storage reservoirs and other project works of the licensee by an equal amount of water for the use of said Association or by compensation for such loss in an amount approved by the Commission. Provided that, if satisfactory settlement be made between the permittee and said Association during the period of the permit, this provision will be omitted from the license or modified as required by such settlement."

Mr. Baum was unwilling to accept the conditions in Section 3, and asked reconsideration. Before anything was done, Mr. Dell M. Potter, of Clifton, Ariz., who had protested Mr. Baum's water rights, won a suit establishing his own water rights. Mr. Baum did not contest Mr. Potter's claim and the Federal Power Commission thereupon decided to rescind its authorization of Mr. Baum's permit and to reject his application.

The present demand for power in the head-water region of the Salt River probably does not justify development, but some day it will. The claims of the Salt River Valley Water Users' Association appear to be exaggerated, but the Association has indicated quite clearly that it will not willingly permit anybody to go in on the drainage area above it.

The Salt River project is one of the most successful of the Reclamation projects, and it should be and is fully protected. On the other hand, it should not be permitted to prevent any development in the 6 000 sq. miles of country above it which can be made without infringing upon its rights.

Although this case presents an example of conflict between power development and irrigation, the Salt River Valley Water Users' Association is demonstrating on the same stream the benefits that can be derived by co-ordinating irrigation and power. The Association is developing power itself wherever such development is practicable in its district, and the returns from the sale of power are contributing materially toward paying the cost of irrigation.

These cases illustrate fairly well the problems of the Federal Power Commission on the subject of co-ordinating irrigation and power. The function

of the Commission in this respect is prescribed by Section 10 (*a*) of the Federal Water Power Act, which reads as follows:

"That the project adopted, including the maps, plans, and specifications, shall be such as in the judgment of the Commission will be best adapted to a comprehensive scheme of improvement and utilization for the purposes of navigation, of water-power development, and of other beneficial public uses; and if necessary in order to secure such scheme the commission shall have authority to require the modification of any project and of the plans and specifications of the project works before approval."

This function has been regarded by the Commission as one of the most important with which it is charged.

DISCUSSION

J. D. Galloway,* M. Am. Soc. C. E. (by letter).—This paper is a comprehensive and general review of the irrigation and power problems that exist on most rivers of the Western States. The diversity of the problems is illustrated by the contrast between physical conditions that exist on different rivers. In one case, as in the streams of the Sierra Nevadas, all power development is on the mountain section of the river and practically all irrigation on the broad plains below. In another case, such as the Deschutes River, or in the Green River Valley, all irrigation is above and the power development below. However, some general principles are applicable to all situations and it is of interest to discuss, in some detail, a few of the points mentioned in the paper.

The writer believes that too much emphasis has been placed on the apparent conflict between power and irrigation. This idea has been fostered by demagogues and politicians. The underlying cause has not been so much an inherent difference between the two uses of water as it has been that irrigation development, at least in recent years, has become a public function, while power development has been made by private corporations. Ever since the beginning of the hydro-electric industry, it has been the target for all forms of State and National regulation and restriction, beyond that of any other industry with the possible exception of the railroads. No real difference exists between the two agencies. Both are made up of groups of individuals organized under the laws of the State to perform certain necessary functions. It will be admitted that, as a matter of necessity, the production of food is more important than the generation of energy, but in any ordinary community, or in the United States as a whole, there is ample food, and all communities require power.

Under this conception, it is as important to develop power as it is to provide irrigation as a factor in the cycle of food production. Both are required in any community and it is a false principle to favor one as against the other, merely because of a difference in type of organization. The situation in Green River Basin, Wyoming, illustrates this point. When the Utah Power and Light Company applied for a license to develop power at the Flaming Gorge site, it is to be presumed that additional power was required in the territory served by the Company. Yet Restriction *B* of the preliminary permit, as given by the author (page 1460), was practically a prohibition against building the power plant. No capital, private or public, should ever be put into power development for which the water supply may be progressively reduced by irrigation until the stream is nearly dried up. It would seem that under such conditions a balance should be established between the power requirements and those for irrigation, as both are necessary to a community. This point becomes self-evident when, as in many cases, much power is used for irrigation pumping or for drainage. To prohibit power development, as in the case cited, is to injure irrigation. It is to be hoped that some day the American people will become sane on this subject and repeal laws that definitely place any

* Cons. Engr., San Francisco, Calif.

irrigation project, no matter how dubious its merits, above a very necessary power development.

In most cases there is no conflict between irrigation and power; both are mutually beneficial. The writer has already set forth this view at length.* Herewith, is reproduced Fig. 17,† a hydrograph of a stream on which is superimposed an irrigation demand curve and a power demand curve. This form is applicable to most rivers where power is developed in the mountains and irrigation on the plains below. To the extent that reservoirs for power are installed, there is assistance to irrigation during the irrigation season of any one year. In addition, if there is an irrigation reservoir on the lower reaches of the stream, water discharged from power system reservoirs in the period after the irrigation season is caught by the irrigation reservoir and retained for the next year's irrigation. The regulation of the stream over more than one year is thus accomplished. Where, as in California, the annual variation in total discharge may range from minimum to maximum in the ratio of 1 : 4 or 1 : 5, the regulation of the stream over more than one year is of especial benefit to irrigation in years of deficient discharge.

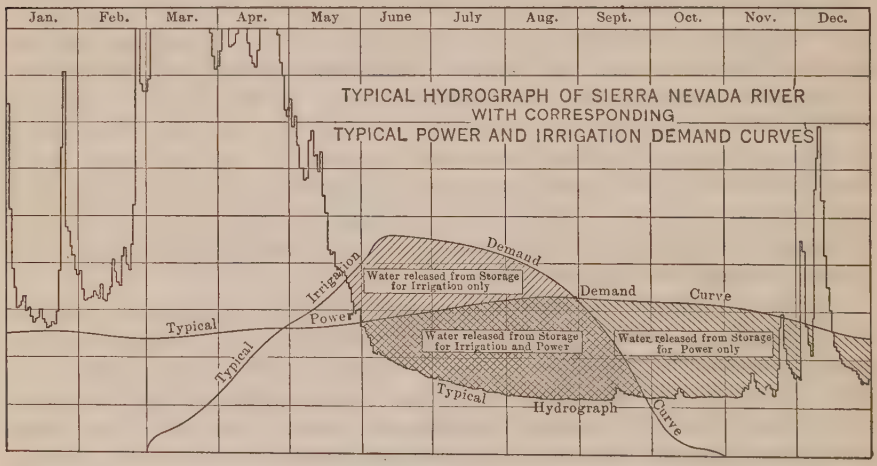

FIG. 17.

This solution is not so simple in cases such as in the Green River Valley or on the Deschutes River, where the irrigated lands lie above the power development. Here, a compromise such as that just suggested is necessary. It is aided by the fact that proper irrigation would have some reservoirs above the irrigated region and, to the extent that water was returned to the stream either by seepage or drainage, the power project would benefit. Such examples are rather unusual, and in the case of the Deschutes the natural regulation of the stream by the lava formation counteracts to a great extent the effects of irrigation diversions on the flow. All power developments on that stream will be by low-head plants. The power demand curve in the Portland, Ore., region

* "Hydro-Electric Developments on the Pacific Coast." *Transaction*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 803, and *Journal of Electricity*, May 15, 1924.

† *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 814, Fig. 25.

is fairly uniform throughout the year, and as a result of these two factors the power plants will generally be limited in size by the stream flow during the low-water season. The winter excess flow can be stored in the numerous irrigation reservoirs, and this, in time, will act to increase the low-water discharge.

It is true, as mentioned by the author, that in some cases, as on Kings River, California, there is so much irrigation that, in many years, all the flood flow is taken by irrigation ditches with varying priority to the water. Even in such an extreme case, the construction of the Pine Flat Reservoir would make more certain the water supply for all irrigation ditches and would also make possible the development of power on the upper reaches of the river.

The statement by the author that the power demand throughout the year is fairly constant, is only partly correct. On the Pacific Coast there is a wide variation in the type of annual load curve. In rainy regions, such as at Portland, Ore., or at Vancouver, B. C., Canada, the annual peak and mass of maximum demand is in the winter. In California, the reverse is true. In the northern part of the State the power demand is fairly uniform throughout the year, with the peak in the summer. In Central and Southern California the mass of energy is used in summer, the peak load occurs in June or July, and cases exist where the July demand is twice that of February. This is caused by the large irrigation and drainage load and the industries that are of the seasonal agricultural character.

A further correlation between power and irrigation lies in the need of power for drainage pumping. The writer believes that in many, if not most, cases, the drainage of low-lying lands in an irrigation region must be done by pumping, as open ditches are, in most cases, a failure for obvious reasons. They cannot be dug deep enough to produce the hydraulic gradient required to cause the water to run into them.

Many of the projects for irrigation in the West cannot be justified on account of cost and physical conditions. Irrigation has reached its most extensive development in California. On account of the climate, many of the products are of the character that could be termed luxuries—oranges, figs, raisins, and other fruits. These crops bring high prices, but, as a rule, general irrigation in California that costs more than $75 per acre, capital cost, will not pay. It is difficult to see how irrigation can pay in such regions as on the Upper Deschutes, or in the Green River Valley, where killing frosts occur in eight or ten months of the year. Crops are automatically limited to forage, grains, and hardy fruits. It would not seem that the proposed irrigation on the Columbia River at costs mentioned by the author could ever be justified on a financial basis. It must be remembered that such lands, producing wheat and similar crops, must compete with other lands, not only in America but in other countries, where there is no irrigation cost. The failure of the Reclamation projects to pay or to be settled is primarily due to the fact that with the burden of the costs of the irrigation systems the farmers could not compete with more favored regions.

Mention is made of this phase of the subject because it has a bearing on the rulings of the Federal Power Commission. The power exercised by the

Commission arises from the ownership of public lands by the Federal Government. It is conceded that water is in the control of the States. It is believed that there is too great a tendency on the part of the Commission to extend its rule into realms not justified by the divided ownership of the land and the water. The writer is not familiar with the Green River Valley in Wyoming mentioned by the author, but on the statements as made it would seem to be a wide extension of the Commission's powers to prohibit practically a present power development because at some future time there might be an irrigation development of an uncertain extent, either by the State or by individuals. The Commission might have given a license to use public lands and have allowed the State of Wyoming to adjust the use of water as between power and irrigation, that being the function of the State which owns or controls the water, the vital element in both cases.

The problems arising from the dual use of water are complex and grow more so. If treated without prejudice as to either, it can generally be found that power and irrigation are mutually interdependent and in most cases can be made to work together. Colonel Kelly is to be congratulated in having presented a paper in which the most important problems of a wide region are set forth in a clear and comprehensive manner.

Fred. H. Tibbetts,* M. Am. Soc. C. E. (by letter).—This paper is of great and timely interest in developing the viewpoint of the engineers of the Federal Power Commission regarding the principles of co-ordination of irrigation and power. However, the author's statements regarding the superiority of the use of water for irrigation and particularly his statement that this well-established principle in the Western States "sets aside the laws of economics" is but a reluctant and half-hearted admission of the limitations under which the engineers of the Federal Power Commission must of necessity work when irrigation and power interests are in conflict.

Of the total estimated installed horse-power covered by licenses of the Federal Power Commission outstanding as of June 30, 1924, 47% was included in projects lying within the State of California. The somewhat different viewpoint of the State of California has already been reported to the Society† by Everett N. Bryan, Assoc. M. Am. Soc. C. E., whose position as Office Engineer of the Division of Water Rights of the Department of Public Works of the State of California, closely corresponds to that formerly held by Colonel Kelly with the Federal Power Commission. The very title of Mr. Bryan's paper, "Hydro-Electric Power as a By-Product of Agricultural Storage", epitomizes the great difference in the two viewpoints. Power has no rights in California when in conflict with irrigation or with the domestic use of water. The author refers to the well-established principle in Western States that the use of water for irrigation is superior to any other except domestic use, and states that to a considerable extent it sets aside the laws of economics. In the writer's opinion this is a superficial viewpoint.

Empire building in an arid section is dependent wholly on the use of water for irrigation. Four thousand years ago the greatest civilizations of antiquity

* Civ. Engr., San Francisco, Calif.

† *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 910.

were built in arid regions and as soon as they ceased to use water for irrigation, the countryside became a desert and the cities uninhabited heaps of ruins.

The ultimate development of the arid West and the extent of the development of its population, its cities, and its industries, depend wholly on the number of acre-feet of water that it devotes to agricultural production. If the State of California allowed 100 acre-ft. of water to run into the ocean each year unused for agricultural production, there would ultimately be one less family tilling the soil and two less families in the cities.

In California, this principle is a fundamental feature of the organic law, having been embodied in a forceful declaration of policy by enactment of the Legislature, confirmed by popular vote. The Water Commission Act approved by a referendum vote of the people on November 3, 1914, and as amended since then, contains the following statement:

"It is hereby declared to be the established policy of this State that the use of water for domestic purposes is the highest use of water and that the next highest use is for irrigation. In acting upon applications to appropriate water the Commission shall be guided by the above declaration of policy."

The policy so emphatically stated has been continuously observed by the State authority having jurisdiction over the distribution of the State's water resources and has been supported on all occasions for ten years by the State Department of Engineering and Irrigation.

The trend of development of the State's water resources in accordance with this policy is indicated in Mr. Bryan's paper, in which he described several new agricultural developments in California where power is to be produced in connection with storage works created primarily for irrigation supply. Reservoir releases are to be regulated entirely in accordance with irrigation requirements. Such power as is produced, although of great importance and value, will be merely incidental to the main purpose.

It is the writer's belief that the intent as well as the letter of the Act by which Congress organized the Federal Power Commission, makes it mandatory upon the Commission to incorporate in its permits the same policy. Section 9 (*b*) of the Water Power Act reads as follows:

"That each applicant for a license hereunder shall submit to the Commission * * * satisfactory evidence that the applicant has complied with the requirements of the laws of the State or States within which the proposed project is to be located with respect to bed and banks and to the appropriation, diversion, and use of water for power purposes and with respect to the right to engage in the business of developing, transmitting, and distributing power, and in any other business necessary to effect the purposes of a license under this Act."

Practically all the important new irrigation developments in California are by irrigation districts which are municipally organized as State agencies and hence it is also important to note that Section 7 of the Federal Power Commission Act states:

"That in issuing preliminary permits hereunder or licenses where no preliminary permit has been issued and in issuing licenses to new licensees under Section 15 hereof the Commission shall give preference to applications therefor by States and municipalities * * *."

Finally, Section 27 of the Federal Power Commission Act states:

"That nothing herein contained shall be construed as affecting or intending to affect or in any way to interfere with the laws of the respective States relating to the control, appropriation, use or distribution of water used in irrigation or for municipal or other uses, or any vested right acquired therein."

Under these provisions of the Federal Power Act and of the Water Commission Act of the State of California, the writer can see no possible interpretation of the laws other than a mandatory direction to the Federal Power Commission to adopt the State's policy of irrigation superiority and, when irrigation and power applications are in conflict, especially irrigation applications by the State or political subdivision thereof, to cease attempts to determine relative values of kilowatt-hours and acres of irrigated or arid land. The apparent intention of the Federal Power Commission to act only in cooperation with the State authorities has been further assured by the Commission* as follows:

"The development of water power on the lands of the United States requires the approval of both the State and Federal Governments, the former granting the right to use the water, the latter the right to use the land. Under the Federal Water Power Act a license may not be issued until an applicant has secured its water rights from the appropriate State authority. Under such circumstances it is apparent that development can proceed only when the State and the Federal Governments take corresponding action."

The second and principal part of the author's paper is a series of examples of action taken by the Commission on various streams, three of which are California streams. It is of chief interest to the writer because of the conspicuous absence of the Yuba River on which is the most important example of conflicting irrigation and power interests and with regard to which the Executive Secretary of the Federal Power Commission on June 6, 1925, stated his purpose to recommend the rejection of the irrigation applications in favor of the power applications. This recommendation, in the writer's opinion, is in complete violation of all the principles stated in the first part of the paper and this, quoting the words† of Arthur P. Davis, Past-President, Am. Soc. C. E., regarding the Colorado River, "in the face of the mandatory provision of the law that the Commission shall give preference to applications from municipalities" and in "defiance of law" (Federal Power Commission Act).

In the three examples of California streams discussed by the author the Trinity and American Rivers were thoroughly investigated by a Special Board which included a representative of the State Department of Engineering and Irrigation and which, therefore, embodied in its findings the State's irrigation policy. In the third case, the Kings River, action was postponed until such time as the applicant had complied with the requirements of the laws of the State of California with respect to appropriation, diversion, and storage of water. In all three cases, the action of the Commission was in thorough accord with the adopted policy of the State.

The strategic position of the Yuba River makes it one of the most important streams in California. It is in a region of the heaviest precipitation in

* Fourth Annual Rept., Federal Power Comm., 1924, p. 10.

† *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 393.

the Sierra Nevada Mountains (in the central part of the State), is near the main centers of urban and manufacturing development, and lies opposite large bodies of the best, and as yet undeveloped, agricultural land in the State. It has excellent reservoir sites so high up in the mountains as to be particularly valuable for power development. It was the scene of the spectacular fight fifty years ago by which the agricultural interests of California in the Supreme Court of the United States checked hydraulic mining, the then leading industry in the State; and for the past four years it has been the battle ground of an exceedingly bitter fight between power and agricultural interests. The power and mining interests were represented by the Excelsior Water and Mining Company and by the Yuba River Power Company, the latter in particular a strongly organized and well conceived project for developing in a comprehensive way a water-shed of about 1 200 sq. miles, with applications before the Federal Power Commission for the development of 227 000 h.p. The agricultural interests were represented by the Nevada Irrigation District municipally organized under the laws of the State of California and proposing to irrigate about 150 000 acres of foothill lands lying, in the main, between elevations of 1 000 and 3 000 ft.

The Irrigation District by co-operation with the Pacific Gas and Electric Company, the largest going concern in Central California, proposes to develop as much of the power resources of the river as it is able and still deliver the water at the relatively high elevation at which its lands are situated. Obviously, it will get less kilowatt-hours per acre-foot of water stored at the head of the Yuba River watershed than the Power Company, whose point of delivery is below the District's lands. This latter project is mentioned in Mr. Bryan's paper,* where the irrigation features, if any, of the Yuba River Power Company are referred to as a by-product of power, as follows:

"They [referring to by-product power plants] are even distinctly different from such composite developments as the project of the Yuba River Power Company on the Yuba River, where irrigation, mining, and power are to benefit jointly, and in which case if the benefit to either enterprise may be considered a by-product, it is the benefit from stored water delivered for irrigation."

The conflict centers particularly around the Jackson Meadows Reservoir site on the Middle Yuba River at an elevation of 5 800 ft., tributary to which there is an average run-off of 146 300 acre-ft. To the Yuba River Power Company this storage site is of even greater advantage than its water yield would indicate because of its high elevation and its efficiency in regulating the run-off from the lower part of the water-shed. The Irrigation District claims that without this water-shed, from which it derives more than one-half its mountain water supply, it can never obtain sufficient water to irrigate its lands.

The Irrigation District has already contracted with the Pacific Gas and Electric Company for the sale of its power potentialities in a contract (approved by the State Railroad Commission of California and by the Irrigation District Bond Certification Commission) by means of which it will

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 911.

derive a revenue of about $400 000 per year. This contract and this revenue are dependent chiefly on the Jackson Meadows waters in conflict and the District is economically unfeasible and cannot even begin development without these waters. The Power Company on the other hand, while demonstrating that the waters in conflict would be of disproportionately great value to its project, has repeatedly admitted that they were not absolutely essential.

All these facts are matters of record during four years of hearings, largely joint hearings between the State Division of Water Rights and the Federal Power Commission. As the result of these hearings and after the most thorough investigation, the State Division of Water Rights on May 19, 1925, granted the waters in contest to the Irrigation District, confidently assuming that the Federal Power Commission, as by law required, would immediately follow suit.

The District has voted bonds to the extent of $7 250 000 for the construction of its works; its financial soundness is insured by its contract for the disposal of its power resources; its economic feasibility and necessity have been testified by formal resolution of the State Bond Certification Commission; and it was actually ready to proceed when it received on June 6, 1925, the most astonishing news that the Executive Secretary of the Federal Power Commission proposed to recommend the rejection of its application for storage reservoir sites or diversion from the Middle Yuba of waters which had already been granted it by the State of California.

The writer believes that the principles involved in this conflict of Federal and State authority are of vastly more far-reaching importance than either of the two other projects to members of the Society, to the State of California, and to the Nation. Governmental developments within the last twenty years have taken the form of steadily multiplying officials and commissions with overlapping authority. A National Cabinet officer, Herbert Hoover, Hon. M. Am. Soc. C. E., has probably foreseen more clearly than any one else the fateful results of this tendency if it is allowed to proceed unchecked. He is also one of the few men in politics big enough to recommend decreasing the authority of his own Department. In recent speeches on the Pacific Coast he has come out as strongly opposed to further Federal regulatory authority and has stated that the greatest need at Washington is to decentralize the National Government and to pass back into the hands of the people the responsibilities which have been gradually taken from them. The impassé which has been so long foreseen and dreaded by students of this tendency may have at last arrived in California. The Nevada Irrigation District, a political subdivision and an executive agency of the sovereign State of California, the rights of which to its water resources have never been questioned, has been awarded a portion of those public resources by State authority and the Federal Power Commission is denying that State agency the use of Government lands. Unless this conflict can be quickly resolved, the Federal Power Commission will have lost its usefulness and the development of important resources, at least in California, will be threatened with the same stagnation that existed prior to the formation of the Power Commission.

BARRY DIBBLE,* M. AM. SOC. C. E.—No argument is required to establish that agriculture is the basic industry of civilization. Before the beginnings of history, the density of population in some areas had become so great that men could not depend on wild supplies of food. The earliest civilizations that have left any record were founded on cultivation of food crops. In Mesopotamia, the staples were wheat, oats, and barley; in China, rice; in Mexico, Indian corn and maize; in Peru, the potato. It is noteworthy that much of this primitive agriculture was under irrigation.

It is in no sense a disparagement of present-day accomplishments in engineering and industry to class most modern devices as luxuries. If all electrical devices were destroyed to-morrow, mankind would continue to live and function and soon adapt itself to the changed conditions as it did before it had electricity at its service. Strange as it may be, people are willing to and do pay greater rewards to the producer of luxuries than to the producer of necessities. The more nearly a product or a service becomes a necessity, the more critical they are of its price and the profits.

So it is that the farmer thinks he has been overlooked in the distribution of prosperity; so it is that should food prices rise, tariffs will be removed or the farmer's profits in some way brought down; so it is that hydro-electric power development has proved more profitable than irrigation farming.

Nor should it be overlooked that food production in the back country has much to do with the growth of manufacturing industries and centers of population. The electric business feeds on the centers and the centers feed on agriculture. Perhaps it is a law of compensation that justifies Western agriculture in collecting some toll when irrigation water is used for power production.

Many irrigation projects in their development and growth present favorable opportunities for power production—often as a by-product. Usually, the water users are alive to the possibilities and if a market is available the question presents itself whether the power house should be constructed by the project or whether the power rights should be sold to a power company. In some cases, most or all of the power can be efficiently used for irrigation pumping on the project land. Generally, however, the power must be sold and the price at which it can be sold governs the feasibility of the power development. The primary objective of the water users is then to obtain the greatest net revenue without interfering with the application of water to irrigation.

If the irrigation district sells its power rights, its investment is reduced and the financial risk correspondingly lessened; the district is kept out of the power business, with its more or less technical requirements. On the other hand, the purchaser of power rights expects to make a profit and will discount the risks and uncertainties of the future as far as he can see them. Such an arrangement must be based on the contract in which the rights of both parties are set forth. The district will try to protect the water rights of the irrigationists; the power company will try to obtain a free hand to use water when most needed, for power purposes.

* Cons. Engr., Redlands, Calif.

Cycles of wet and dry years, changing conditions in irrigation and power demands, may give important and unanticipated economic advantage to one or the other of the parties. The power production may either be far greater than expected or the power company may find itself paying for something it does not get.

Bitterness and jealousies between the contracting organizations are likely to be aroused, and the contract will probably find its way into the Courts. If the power house is built and operated by the water users and the energy sold on a production basis, they immediately become interested in securing the largest and most saleable output from the water available. When the pinch comes, in years of drought, they will be far more ready to co-operate to get the best results than if they receive no benefit. It is much easier to draft a contract that will prove satisfactory, when the interests of the parties run along parallel lines.

The modern hydro-electric station, properly designed and installed, is not difficult to operate efficiently and satisfactorily. Often, the irrigation district can handle the plant with an organization not much different from that which would be required to care for the irrigation work alone. The hydro-electric construction involves the irrigation district in larger investment and in certain risks of the business. However, the district is in better position than any one else, to use the investment to the best advantage. The risks must be taken by some one. If the right to develop power is sold, they come back to the district by lowering the consideration paid for those rights.

The Minidoka Project, in Idaho, and the Salt River Project, in Arizona, are examples of the better co-ordination that can be secured between power and irrigation, when the power is owned and generated by the irrigation interests. In each instance, the use of structures and regulation of flow have been materially modified to meet the needs for power, and to increase the revenue. Irrigation has not suffered and yet it is certain that the water users would bitterly have fought the proposal that some outside company be allowed to do the things they have done.

Generally, the best results from both economic and other standpoints will be secured where the ultimate water users themselves are directly interested in seeing the water put to the fullest beneficial use.

Markham Cheever,* Esq.—It is eminently fitting that a discussion of this paper should take place in Utah where pioneer work has been done both in irrigation and in the generation and distribution of hydro-electric energy, and where both uses of water have been co-ordinated to such a great extent as to provide a high degree of beneficial utilization of the water resources.

The early settlers in Utah were the first in the United States to practice irrigation on an extensive scale and the general condition of prosperity throughout the irrigated areas is evidence of the economic soundness with which the projects were conceived. Likewise, much valuable pioneer work in the development of hydro-electric energy from water power was done in Utah. Approximately thirty years ago construction was commenced on hydro-electric

* Gen. Supt. and Chf. Engr., Utah Power & Light Co., Salt Lake City, Utah.

plants in Provo, Ogden, and Big Cottonwood Canyons, and, in 1897, the first commercial transmission line in the country to operate at so high a voltage as 40 000 was placed in service to distribute energy from the Provo Canyon plant. Since that time, the development of water power has been on a substantial scale and an adequate supply of hydro-electric energy has been made available over wide areas for lighting cities and towns, operating irrigation pumping plants, and developing industries. It is especially interesting to note that electrical service has been made available to a larger proportion of the farms in Utah than in any other State. According to the census of 1920, which represents the latest authoritative source of information, Utah was at the head of the list with more than 34% of the farms receiving electrical service as against 7% representing the average for the whole United States.

There is in the vicinity of Salt Lake City a striking example of successful co-ordination between irrigation and power development. Bear River is the largest stream flowing into Great Salt Lake. It rises on the northerly slope of the Uintah Mountains, in Northeastern Utah, flows northward through Wyoming, enters Southeastern Idaho, and after making a turn around the northerly end of the Bear River Range flows southward into Utah and discharges into Great Salt Lake. In Southeastern Idaho it flows near, but not through, Bear Lake, a body of water having an area of 110 sq. miles. By means of canals and other extensive diversion and control works, including a pumping plant, the lake has been converted into a reservoir of capacity adequate to equalize completely the flow of the river below that point, not only during any one year, but also throughout a long series of years.

The total fall between Bear Lake and Great Salt Lake in a distance by river of approximately 200 miles is 1 720 ft. The river flows alternately through open valleys and canyon sections. The valleys are irrigated and in the canyons it has been possible to utilize for power development a substantial part of the total fall in the stream. The use of Bear Lake Reservoir has enabled the production of a large amount of firm power, and at the same time all irrigation requirements in the river basin have been met. During the exceptionally dry year of 1919 full crops were raised in the Bear River Basin, and it was estimated by a representative of the United States Department of Agriculture that in that year the value of crops saved by the release of stored water from Bear Lake amounted to between $4 000 000 and $5 000 000.

Although the vital importance of irrigation to the arid States is unquestioned, the value of an adequate and reliable supply of electrical energy must not be overlooked. In addition to the obvious advantageous use of electricity for pumping irrigation water, the widespread availability of electrical service from large interconnected systems has stimulated industries which depend on agriculture and which, in turn, create a market for farm products. Electrical energy has revolutionized industrial processes, has assisted greatly in conserving labor and has produced domestic conveniences which have now become necessities. New and enlarged uses for energy are constantly arising, and it is inevitable that the electric power industry will become of increasingly great importance in the economic development of the United States.

Difficulties in co-ordinating irrigation and power projects are in many cases imaginary rather than real. Misunderstandings arise because of lack of reliable and adequate data, or because of failure to analyze the information correctly. If, in each case, the full facts are made available and are viewed disinterestedly from the standpoint of economic practicability of the projects involved and of best results in the general development of the territory served, the solution of the problem of co-ordination will not be difficult. It should be remembered, however, that no two situations are alike in either physical or economic characteristics and no general formula can be previsioned and thereafter applied to all future problems of co-ordination if the greatest beneficial results are to be obtained from developing the water resources for both important uses.

M. C. HINDERLIDER,* M. AM. SOC. C. E.—The subject of co-operation in irrigation and power development is not of prime importance to Colorado at this time and this is largely true of the States in the Rocky Mountain region, due primarily to three factors:

First.—The wide monthly and yearly fluctuations in run-off in the streams of the Rocky Mountain region would require the use of extensive storage systems for the proper regulation of flow to meet the demands of power development or to justify expenditures for such purposes on an economical scale.

Second.—Although an extensive system of storage reservoirs with an aggregate capacity of 2 400 000 acre-ft. is in use in Colorado, such reservoirs are utilized almost wholly for irrigation and cannot be converted to power use without secondary storage which, under present conditions, would not be economically feasible.

Owing to the necessities of irrigation which is of paramount importance in the semi-arid States, stream run-off over periods of many months must be stored and held in reserve against the demands occurring later in the summer, which demands usually cover only two or three months.

Additional difficulties arise over the system of laws and administration wherein priority of claim for irrigation use must be observed regardless of where such claims occur on a stream. The time is anticipated, however, when through proper legislation present priority requirements for irrigation uses will be so modified as to remove some of the obstacles now faced by the promoter of hydro-developments.

Third.—Such developments are not more extensive in the Rocky Mountain region (1) because of the great deposits of cheap coal which are found throughout this part of the United States; and (2) because of the attendant cost of transmission over sparsely settled areas. In Colorado, for the present at least, investigations have shown that power may be supplied more economically by means of steam-generating plants than from available water power sources.

These conditions will doubtless be modified with the passing of time, when due to greatly increased density of population and to attendant demands, together with reduced cost of transmission, hydro-power possibilities, aggregat-

* State Engr. of Colorado, Denver, Colo.

ing several million horse-power, will be utilized through proper legislative provisions, permitting of a high state of co-ordination and co-operation between the dominant irrigation interests and power possibilities. Such co-ordination must of necessity come about rather slowly in order that existing corporate and individual rights may not be unduly disturbed in the process of giving way to the greater demands of the public.

C. S. Jarvis,* M. Am. Soc. C. E. (by letter).—The record of the Federal Power Commission's rulings as reported by the author goes far to prove both the need for such an authority and the benefits that have already accrued under its administration.

When irrigation and power interests are in opposition, only a non-partisan, deliberate, and independent estimate of the whole situation will lead to an equitable adjustment. It appears that many cases of litigation have been avoided by the intensive analysis of each problem with a view not only to the recognition of initiated or vested rights, but also to the ultimate beneficial use under other projects barely conceived and not yet launched.

Experience indicates that the general policy should be to promote projects for stream regulation and diversion on the head-waters concurrently with plans for similar enterprises in the intermediate and lower valleys. In no other way can there be utilized (*a*) the maximum head for hydro-electric power plants; (*b*) the maximum irrigable acreage and variety of products; (*c*) the maximum underground storage with return seepage increasing the natural flow—an important factor during years of drought; and (*d*) the full measure of potential benefits.

An excellent example of co-ordinated power and irrigation development is furnished by the Salt River Project, in Arizona. Formerly, the power plants at the Roosevelt Dam could not be operated during the non-irrigation season, because the discharge from the turbines was lost to the system. This situation has been relieved by the supplemental storage provided between the Roosevelt Dam and the intake of the main irrigation canals. This immediately replaced the intermittent power generation with constant, dependable service, brought the canal and reservoir systems into more intimate relation so that the flow at the head-gates could be varied on a few hours' notice, and made possible additional power developments nearer the place of use. As a considerable part of the demand is for pumping during the irrigation season, there is a special value in the secondary power at the lower sites.

Such a favorable solution is readily attainable where both power and irrigation interests are under the same ownership. Progress toward harmonizing the control of streams has resulted from the attitude of the Federal Power Commission to the effect that, in the last analysis, the welfare of the citizenry in each drainage basin should govern the development of water resources. This seems to be the spirit and intent of the Federal Water Power Act. Wisely administered, it should provide the unified control so essential to thorough co-ordination of power and irrigation interests, and should promote the normal activities which make for sound industrial growth.

* Care, U. S. Bureau of Pubic Roads, Washington, D. C.

WILLIAM KELLY,* M. AM. SOC. C. E. (by letter).—A few closing remarks seem to be called for by Mr. Tibbetts' discussion. The Yuba River was not mentioned in the paper because no final action had been taken on it by the Federal Power Commission at that time and, therefore, it could not be cited as an example. The Federal Power Commission has since granted the application of the Nevada Irrigation District in which Mr. Tibbetts is interested.

The District lies in the foothills of Nevada County, California. It contains about 200 000 acres, of which, according to the State Engineer's estimate, not more than 75 000 acres are suitable for irrigation. About 3 000 acres are now under irrigated cultivation. The principal industries in Nevada County up to the present have been gold mining and cattle grazing. The greater part of the land proposed for irrigation is covered with a growth of timber and brush and it is conservatively estimated that the average cost of clearing will be as much as $100 per acre. The irrigable lands are scattered over the district so that to supply them with water will require practically the same length of ditch as if the entire 200 000 acres were to be irrigated. The project could be developed to the extent of about 15 000 acres by the construction of the Scotts Flat Reservoir in Deer Creek Valley, using the run-off of the water-shed within which the District lies.

For greater development, water must be diverted from adjacent water-sheds. The project which has been approved contemplates the development of storage on the head-waters of the Middle and South Forks of the Yuba and its diversion by tunnel and conduit into Bear River and a part thence to Deer Creek.

The Pacific Gas and Electric Company has entered into a contract with the District to take the diverted water for use in existing and proposed power plants and finally to release it near the boundary of the Nevada Irrigation District. The schedule governing the release is based mainly on the power values and, therefore, only about 36% of the water supply will be delivered for use during the irrigation season. The contract is a good one for the Power Company, and the promoters of the Nevada District claim that returns from the power use of the water will pay all carrying charges on the investment required for the storage and diversion works. In other words, if the estimates are not exceeded, the District will get a small initial water supply, adequate for about 20 000 acres, at its boundaries practically without cost, except for the maintenance and operation of the mountain works. It should be noted, however, that the property owners of the District and not the Power Company assume all the risks.

Mr. Tibbetts' criticism that the engineers of the Federal Power Commission were balancing kilowatt-hours against acres is hardly justifiable. The question is not one of balancing kilowatt-hours against acres, but of balancing a possible but doubtful 75 000 acres in the Nevada District, plus a certain amount of power, against a greater irrigable acreage in the Sacramento Valley, plus about 40 000 kw. more power, plus incidental benefits to

* Col., Corps of Engrs., U. S. A.; Director of Eng., National Electric Light Assoc., New York, N. Y.

flood protection on the Yuba, Feather, and Sacramento Rivers, plus the possibility of renewing hydraulic mining on the Yuba where it is estimated that more than $50 000 000 of gold lies still unmined.

In other words, using the water from the head-waters of the Yuba in its own water-shed would increase and not decrease the acreage of land economically irrigable in California. Moreover, diversion of water from the Bowman water-shed, combined with the development of local sources, would supply all the lands that are probably irrigable in the Nevada District and leave the water of the Middle Fork to be developed in combination with the remainder of the Yuba River. It was the diversion of water from the Middle Fork to the Nevada District to which the engineers of the Federal Power Commission particularly objected.

Again Mr. Tibbetts is not correct when he attempts to point out a great difference in viewpoint between Everett N. Bryan, M. Am. Soc. C. E., and the writer. In his paper, "Hydro-Electric Power as a By-Product of Agricultural Storage,"* Mr. Bryan deals only with foothill reservoirs similar to the proposed Pine Flats Reservoir on the Kings River, described by the writer. Power at such reservoirs is and should be purely incidental to irrigation, but one of the important functions of such reservoirs is that they leave the river above them free to be developed primarily for power. Such reservoirs, when properly combined with head-water storage, accomplish the maximum use of water for both power and irrigation.

Investigation of the Nevada Irrigation District, in its relation to the Yuba River, was made jointly by engineers of the State and of the Federal Power Commission. There was no material difference of opinion between them as to physical facts or economics. The State engineers, however, operate under State laws which grant rights on the basis of priority of filing, and they could give but little regard to the broad economics of the problem, whereas the engineers of the Federal Power Commission are charged with determining whether or not any project applied for makes the fullest possible use of the water for all purposes.

In granting the Nevada District application, the Federal Power Commission certainly has demonstrated its intention to co-operate with the States to the fullest extent possible in administering the Federal Water Power Act. The Nevada District case illustrates some of the factors which the writer had in mind when he stated that practice in the Western States "sometimes sets aside economic laws". The California law, providing for the organization of Irrigation Districts, has proved itself a good one, but no law has ever been so good that it could not be used in ways not intended.

The Nevada Irrigation District was organized through the efforts of energetic and competent promoters. Its formation was made possible by the votes of many non-property owning individuals. Owners of nearly 50% of the property in the District opposed the mountain development project. The owners of property in the Lower Yuba Valley, who have perhaps a better right than the Nevada District to look to the Yuba River for water, have not

* *Transactions*, Am. Soc. C. E., Vol. 88 (1925), p. 910.

organized and thus stand to lose their rights. If the Yuba water-shed and the Nevada District combined were being developed by one owner as a business proposition, looking to ultimate results as well as to earning expenses during development, it is inconceivable that the diversion of all the head-waters of the Middle Yuba to Deer Creek Valley would form the initial step.

Mr. Tibbetts' statement that "empire building in an arid section is dependent wholly on the use of water for irrigation", is rather beside the point in the Yuba controversy because, leaving aside all other factors, diversion of the head-waters of the Middle Fork to the Nevada District will result either in less land irrigated than if that water were used in its own water-shed, or in greater cost if water for the Sacramento Valley lands tributary to the Yuba is sought elsewhere.

Such a diversion may result in the gain of a valuable asset by the Nevada Irrigation District, but it will be at the expense of reduced irrigation and power development in the State of California as a whole.

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Paper No. 1507

PROGRESS REPORT OF THE SPECIAL COMMITTEE ON STEEL COLUMN RESEARCH*

To the Board of Direction,
American Society of Civil Engineers:

1.—Preliminary Statement.—This Committee was authorized in August, 1923, and held its first meeting in October of that year. At this meeting, it was unanimously agreed that the first work of the Committee should be to make a thorough study of all existing data before undertaking any further tests. It was thought that it would doubtless be desirable to do further experimental work, but plans for any such work should be based on a careful study of all existing data. In order to carry out this program, it was obviously necessary to employ considerable assistance, and it was therefore arranged that M. O. Withey, Professor of Mechanics at the University of Wisconsin, should devote a large part of his time, during the year 1924-25, in assisting the Committee in this work. This plan was carried out, and the material thus brought together has been reviewed by the Committee at three meetings during the current year (1925). In addition to this review of existing data, a few preliminary tests have been made on six columns, with a view to developing the best method of testing the efficiency of lacing and other lateral connections. These tests have been made on the 1 000 000-lb. testing machine at the U. S. Forest Products Laboratory at Madison, Wis., and the University of Wisconsin has contributed to some extent in carrying on this program. These tests were made to assist the Committee in formulating any further program of experimentation. The Committee also desires to acknowledge the interest and financial assistance of Engineering Foundation in its work.

Although the Committee is not prepared to make a final report on its study of existing data, it has carried this work far enough to make it seem desirable to present some of the results in the form of a progress report.

* Presented to the Annual Meeting, January 20, 1926.

2.—Division of the Subject.—In making a thorough study of the strength of structural steel columns, it appears to the Committee that the consideration of the subject may profitably be divided into four fairly distinct parts, as follows:

(A) Strength of relatively short columns $\left(\frac{l}{r} = 30 \text{ to } 100\right)$ of good design as compared to the elastic properties of the metal, as shown by compression and tension specimen tests.

(B) Results of all tests of columns heretofore made where failure was not due to weakness of detail. These results would give information particularly on the effect of slenderness ratio and end conditions.

(C) Effect of various details, form of section, and method of manufacture, on the strength of columns.

(D) A study of the column as a part of a structure, especially with reference to end conditions and relation to columns as tested; column theories; working formulas.

The first part (A) would include the very important questions of the significance of the elastic limit or yield point or the useful limit point of test specimens; relative value and reliability of mill tests; the tension test compared with the compression test; the variation of quality of steel over the cross-section; methods of determining the proper average quality; and, to some extent, the effect of form of section and grade of steel.

The second part (B) attempts to bring together all existing data in convenient form for discussion. Even if additional tests appear to be necessary, it is thought that the Committee's study on existing data should be presented without waiting for further tests.

The third part (C) would include a multitude of questions involved in the general problem of design, such as form of section, lacing, diaphragms, thickness of unsupported lengths and widths, tie-plates, pin-plates, riveting, etc. It should be the aim of this investigation to determine the requirements necessary to produce a column which would act substantially as a unit the strength of which could be estimated closely by a suitable column formula based on integral action.

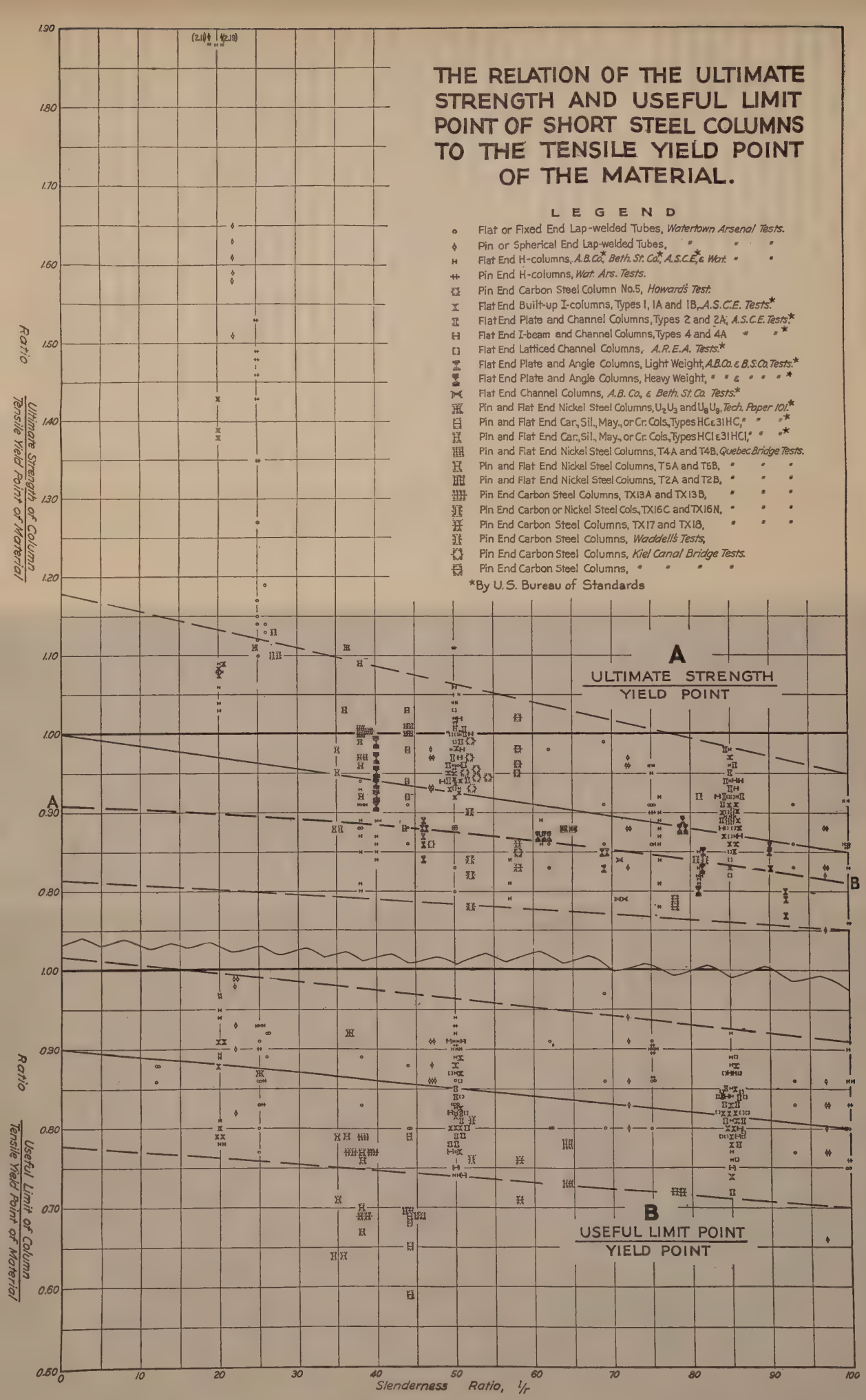
The fourth part (D) would be more or less theoretical in character, but essential to the consideration of column formulas or factors of safety. The column in the testing machine acts under very different conditions from those which obtain in most structures.

Although a discussion of the various items under these four divisions will necessarily overlap somewhat, it is thought that better progress will be made by subdividing the material in this way.

This report covers Parts (A) and (B) and some items under Part (C).

(A) Relation of Strength of Relatively Short Columns $\left(\frac{l}{r} = 30 \text{ to } 100\right)$ to the Elastic Properties of the Metal

3.—Whatever formula may be used to express column strength, it is agreed that for axially loaded columns of slenderness ratio up to 50 or 60, at



VOL. 89, PAPER No. 1597.
PROGRESS REPORT OF SPECIAL COMMITTEE
ON STEEL COLUMN RESEARCH.

THE RELATION OF THE ULTIMATE STRENGTH AND DEFLECTION OF SHORT REINFORCED CONCRETE BEAMS UNDER STATIC BENDING

BY [illegible]

least, the effect of length is small, and that the strength of the column, if properly designed, is primarily a function of the elastic strength of the metal. A constant value of unit stress can well be used for columns of this length, provided the character of the metal is uniform; and such value, once determined, is of much significance in arriving at the column formula needed for the greater lengths. In a study of test data with respect to this phase of the question, a great many uncertainties are confronted, especially with reference to the quality of the material. In most cases the information is inadequate, and in some cases very unreliable or given in terms of doubtful meaning. After much study, it was decided that the most useful information could be obtained by a study of the ratio of the ultimate strength of columns of different length ratios up to $\frac{l}{r} = 100$ to the tensile yield point strength of the material. The Society's Special Committee on Steel Columns and Struts made use of what was named by it the "Useful Limit Point" (U. L. P.)* as being of more significance in some respects than the ultimate strength of the column or the yield point of the material. Although there is good ground for adopting this point in the stress-deformation diagram, as the best measure of column strength, it is not practicable to adhere to this method in a study of all available data. Information does not exist to determine the U. L. P. in a great many of the tests that have been made; but nevertheless it is believed that the results of such tests are valuable and cannot be neglected. As regards the material tests, the most reliable information appears to be the values of the tensile yield point, but in many cases this information is not very satisfactory.

4.—Ratios of Ultimate Strength and Useful Limit Point of Columns to Tensile Yield Point of Material.—The Committee has examined all available tests of steel columns, and has shown in Plate X and Table 1 the results of all tests for $\frac{l}{r}$ up to 100 where failure appeared to be due primarily to a general over-stressing of the material, and where the information regarding the yield point strength of the material was fairly reliable. The results are plotted in Plate X (*A*) as ratios of ultimate strength to yield point strength. Each point represents an individual result. The legend shows what test data have been used; and it will be noticed that pin-ended columns are included, also alloy steels, as well as carbon steels. In using a simple ratio of strength of column to strength of metal, it is recognized that the slenderness ratio has some effect below $\frac{l}{r} = 100$, and, therefore, the results for the hard alloy steels are not quite comparable to the structural steel columns for the higher values of $\frac{l}{r}$; but the difference is relatively small, and it was thought desirable to use all values up to 100 in this study. Columns which failed because of some weak detail, or because of purposely eccentric loading, are omitted. The general make-up of the columns is shown by the symbols on the diagram.

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1618.

TABLE 1.—THE RELATION OF THE STRENGTH OF SHORT COLUMNS TO THE YIELD POINT OF THE METAL.

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 Lb. per Sq. In. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	Ratios: U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
						Data from Tests of Metals and Other Materials, Watertown Arsenal.					
1908	Flat	Steel tubing	1895 1896 1897	5.2	12	36.2 (Avg. of three tests)	67.0 66.4 66.2	32.0 32.0 31.0	1.85 1.83 1.83	0.88 0.88 0.86	Bulged at ends. " " " " " "
								Ave.	1.84	0.87	
1909	Flat		1891 1892 1893 1898 1899 1900	5.2	25	Called elastic limit in report.	41.7 40.6 39.8 46.0 42.3 41.4	33.0 30.0 29.0 31.0 31.0 28.0	1.15 1.12 1.10 1.27 1.17 1.14	0.91 0.83 0.80 0.86 0.86 0.77	Triple flexure after ultimate. " " " " " " " " " " " " " " " "
								Ave.	1.16	0.84	
1909	Flat		1901 1902 1903	5.2	38	36.2	34.0 32.0 31.7	32.0 30.0 27.0	0.94 0.88 0.88	0.88 0.83 0.75	Triple flexure after ultimate. " " " " " " " "
								Ave.	0.90	0.82	
1909	Flat		1888 1889 1890 1904 1905 1906	5.2	50	36.2	28.8 32.0 32.0 30.0 32.0 32.0	.... 30.0 30.0 28.0 31.0 30.0	0.80 0.88 0.88 0.83 0.88 0.88	.... 0.83 0.83 0.77 0.86 0.83	Triple flexure after ultimate. " " " " " " " " " " " " " " " "
								Ave.	0.86	0.82	

TABLE 1.—*(Continued.)*

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: $\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S'}{S_y}$	$\frac{\text{U. L. P.}}{\text{Yield point}} = \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Tests of Metals and Other Materials, Watertown Arsenal.											
1909	Flat		1907	5.2	62	36.2	35.6	33.0	0.98	0.91	Triple flexure after ultimate.
			1908				31.0	29.0	0.86	0.80	" " " "
			1909				30.0	29.0	0.83	0.80	" " " "
								Ave.....	0.89	0.84	
1909	Flat		1894	5.2	75	36.2	33.0	31.0	0.91	0.86	Triple flexure after ultimate.
			1885				33.0	33.0	0.91	0.91	" " " "
			1886				33.0	31.0	0.91	0.86	" " " "
			1887				31.0	29.0	0.86	0.80	" " " "
								Ave.....	0.90	0.86	
1909	Flat		1882	5.2	100	36.2	31.0	27.0	0.86	0.75	Triple flexure after ultimate.
			1883				31.0	27.0	0.86	0.75	" " " "
			1884				31.0	29.0	0.86	0.80	" " " "
								Ave.....	0.86	0.77	
1909	Fixed		1977	5.2	26	36.2	41.4	33.4 S	1.14	0.92	Scaling triple flexure after ultimate.
			1978				41.0	32.4 S	1.13	0.92	" " " " "
			1979				43.0	32.3 S	1.19	0.89	" " " " "
								Ave.....	1.15	0.91	

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength. in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios. Ultimate / Yield point $= \frac{S'}{S_y}$	U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Tests of Metals and Other Materials, Watertown Arsenal.											
1909	Fixed		1974	5.2	44	36.2	32.0	29.0	0.88	0.80	Triple flexure after ultimate.
			1975				33.0	32.0	0.91	0.88	" " " "
			1976				31.0	29.0	0.86	0.80	" " " "
								Ave.	0.88	0.83	
1909	Fixed		1971	5.2	69	36.2	33.0	31.0	0.91	0.86	Triple flexure after ultimate.
			1972				36.0	35.0	0.99	0.97	" " " "
			1973				31.0	29.0	0.86	0.80	" " " "
								Ave.	0.92	0.88	
1909	Fixed		1968	5.2	93	36.2	31.0	30.0	0.86	0.83	Triple flexure after ultimate.
			1969				30.0	28.0	0.83	0.77	" " " "
			1970				32.8	31.0	0.91	0.86	" " " "
								Ave.	0.87	0.82	
1909	Spherical		1929	5.2	22	36.2	57.2	33.6 S	1.58	0.93	Bulging at ends and triple flexure.
			1930				57.5	35.9 S	1.59	0.99	" " " " " "
			1931				58.9	35.7 S	1.63	0.99	" " " " " "
								Ave.	1.60	0.97	

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 Lb. per Sq. In. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios. Ultimate Yield point $= \frac{S'}{S_y}$	U. L. P. Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Tests of Metals and Other Materials, Watertown Arsenal.											
1909	Spherical		1926	5.2	47	36.2	35.0	32.0	0.97	0.88	Deflection small at ultimate, rapidly increasing after maximum load.
			1927				33.6	31.0	0.93	0.86	
			1928				33.8	31.0	0.93	0.86	
								Ave.....	0.94	0.87	
1909	Spherical		1923	5.2	72	36.2	34.6	33.0	0.96	0.91	.....
			1924				35.0	31.0	0.97	0.86	.....
			1925				34.7	34.0	0.96	0.94	.....
								Ave.....	0.96	0.90	
1909	Pin		1920	5.2	97	36.2	32.0	30.0	0.88	0.83	.....
			1921				30.2	28.0	0.83	0.77	.....
			1922				29.7	29.0	0.82	0.80	.....
								Ave.....	0.84	0.80	
1909	Pin		1950	5.2	22	36.2	58.2	32.7 S	1.61	0.90	Bulging at ends and deflection.
			1951				51.5	29.6 S	1.51	0.82	Deflected normal to pins.
			1952				59.6	35.3 S	1.65	0.98	Bulging at ends and triple flexure.
								Ave..	1.59	0.90	

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$.	U. L. P. / Yield point $= \frac{S''}{S_y}$.	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Tests of Metals and Other Materials, Watertown Arsenal.											
1909	Pin		1947	5.2	47	36.2	33.5	31.0	0.93	0.86	Little deflection until after the maximum load had been reached, then columns deflected normal to pins except No. 1945, which deflected in plane of pins.
			1948				35.0	33.0	0.97	0.91	
			1949				35.6	33.0	0.98	0.91	
								Ave.	0.96	0.89	
1909	Pin		1944	5.2	72	36.2	30.0	29.0S	0.83	0.80	
			1945				32.0	...	0.88	....	
			1946				31.8	30.0	0.88	0.83	
								Ave.	0.86	0.81	
1909	Pin		1941	5.2	97	36.2	27.0	24.0	0.75	0.66	.....
			1942				30.0	28.0 (?)	0.83	0.77	.....
			1943				32.0	30.0 (?)	0.88	0.83	.....
								Ave.	0.82	0.75	

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: $\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S'}{S_y}$	$\frac{\text{U. L. P.}}{\text{Yield point}} = \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Tests of Metals and Other Materials, Watertown Arsenal.											
1908-09	Flat		1855	7	25	30.1 (Average of four flange specimens called elastic limit in report.)	43.1	28.0	1.43	0.93	Flanges buckled.
			1856				40.7	28.0	1.35	0.93	" "
			1857				44.7	28.0	1.48	0.93	" "
								Ave.	1.42	0.93	
1908-09	Flat		1858	7	50		29.0	27.0	0.96	0.90	Triple flexure after ultimate.
			1859				28.6	27.0	0.95	0.90	" " " "
			1860				29.0	27.0	0.96	0.90	" " " "
								Ave.	0.96	0.90	
1908-09	Flat		1861	7	75		29.0	27.0	0.96	0.90	Triple flexure after ultimate.
			1862				29.0	27.0	0.96	0.90	" " " "
			1863				28.6	27.0	0.95	0.90	" " " "
								Ave.	0.96	0.90	
1908-09	Flat		1864	7	100	30.1	28.0	27.0	0.93	0.90	Triple flexure after ultimate.
			1865				26.0	24.0	0.86	0.80	" " " "
			1866				28.0	26.0	0.93	0.86	" " " "
								Ave.	0.91	0.85	

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 Lb. per Sq. In. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$.	Ratios: U. L. P. / Yield point $= \frac{S''}{S_y}$.	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Tests of Metals and Other Materials, Watertown Arsenal.											
1908-09	Pin		1995	7	25	30.1	45.0	27.0	1.49	0.90	Flanges buckled.
			1996				44.0	25.0	1.46	0.83	" "
			1997				46.0	26.0	1.53	0.86	.....
								Ave.....	1.49	0.86	
1908-09	Pin		1992	7	50	30.1	28.8	25.0 27.0	0.96	0.83	Little deflection until maximum load was applied then column rapidly deflected in plane normal to webs (and pin axes).
			1993				33.4	28.0	1.11	0.90	
			1994				30.8		1.02	0.93	
								Ave.....	1.03	0.89	
1908-09	Pin		1989	7	75	30.1	28.0	27.0	0.93	0.90	.....
			1990				28.0	27.0	0.93	0.90	.....
			1991				28.6	26.0	0.95	0.86	.....
								Ave.....	0.94	0.89	
1908-09	Pin		1986	7	100	30.1	24.0	23.0	0.80	0.76	.....
			1987				26.0	25.0	0.86	0.83	.....
			1988				27.0	26.0	0.90	0.86	.....
								Ave.....	0.85	0.82	

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: $\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S'}{S_y}$	$\frac{\text{U. L. P.}}{\text{Yield point}} = \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Waddell Tests.*											
1909	Pin.....		1	17.4	27	35.4	38.9	34.0	1.10	0.96	Carbon steel. Short columns failed by buckling of flanges and webs; long columns by bending in plane normal to pins.
			2				39.8	32.0	1.13	0.90	
			3				38.9	29.0	1.10	0.82	
								Ave.....	1.11	0.89	
1909	Pin.....		1	17.4	81	35.4	29.6	....	0.84	....	.....
			2				32.4	29.0	0.92	0.82	.....
			3				29.6	28.0	0.84	0.79	.....
								Ave.....	0.87	0.80	
Data from Howard Tests.†											
1911	Pin.....		5	90.0	47	35.4	30.5	....	0.86	....	.....

* *Transactions*, Am. Soc. C. E., Vol. LXIII (1909), p. 101.
† *Loc. cit.*, Vol. LXXIII (1911), p. 429.

TABLE 1.—(*Continued.*)

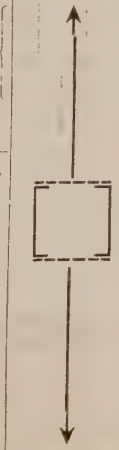
Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from American Railway Engineering Association Tests.											
1915	Flat		12	9.6	50	Specifications	37.9	....	1.00	....	
			57	....	..	38.0	39.3	32.6	1.03	0.86	Deflected south and up.
			74	....	..	±1.0	38.2	33.0	1.00	0.87	" " " down.
			7	12.5	50	....	38.5	....	1.01	....	Triple flexure.
			72	....	..	....	37.7	32.0	0.99	0.84	Buckling down and north.
			73	....	..	....	35.4	31.0	0.93	0.82	" " " "
								Ave.....	0.99	0.85	
1915	Flat		17	9.6	85	....	32.9	30.0	0.87	0.79	Triple flexure.
			58	....	..	....	35.0	34.0	0.92	0.89	Triple flexure, buckling at center.
			61	....	..	....	34.1	33.0	0.90	0.87	Deflected north and up.
			20	12.5	85	....	34.1	32.0	0.90	0.84	Triple flexure, buckling at center.
			60	....	..	....	34.9	32.0	0.92	0.84	.....
			62	....	..	....	34.0	30.0	0.89	0.79	.....
			191	9.6	85	....	33.9	30.0	0.89	0.79	.....
			192	....	..	....	33.9	33.0	0.89	0.87	.....
			197	....	..	....	33.9	31.0	0.89	0.82	.....
			199	9.6	85	....	33.6	32.0	0.88	0.84	.....
			200	....	..	....	33.5	32.0	0.88	0.84	.....
			204	....	..	....	34.2	31.0	0.90	6.82	.....
			201	12.5	85	....	32.1	31.0	0.84	0.82	.....
			202	....	..	....	31.0	29.0	0.82	0.76	.....
			203	....	..	....	32.4	30.0	0.85	0.79	.....
								Ave.....	0.88	0.82	

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: $\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S'}{S_y}$	$\frac{\text{U. L. P.}}{\text{Yield point}} = \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from American Bridge Company and Bethlehem Steel Company Tests.											
....	Flat		ABC	36	40	40.6	35.0	....	0.86	....	
						39.8	35.6	....	0.89	....	
						41.1	35.8	....	0.87	....	
			B	36	40	36.2	32.1	....	0.89	....	
						37.0	31.5	....	0.85	....	
						38.4	32.2	....	0.84	....	
								Ave.	0.87	....	
....	Flat		ABC	35.5	61	40.1	35.8	....	0.89	....	
						40.9	35.0	....	0.86	....	
						40.5	35.3	....	0.87	....	
								Ave.	0.87	....	
....	Flat		ABC	35.5	81	44.8	37.9	....	0.85	....	
						42.9	35.3	....	0.82	....	
						45.1	36.8	....	0.82	....	
			B	35.5	81	38.4	32.0	....	0.83	....	
						38.0	31.1	....	0.82	....	
						37.2	30.1	....	0.81	....	
								Ave.	0.82	....	
....	Flat		ABC	83	38	34.9	28.2	....	0.81	....	
						30.5	25.8	....	0.85	....	
						32.3	25.9	....	0.80	....	
			B	83	38	38.1	34.0	....	0.89	....	
						39.4	36.0	....	0.91	....	
						40.1	35.0	....	0.87	....	
								Ave.	0.86	...	

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of columns.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 Lb. per Sq. In. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate Yield point $= \frac{S'}{S_y}$	U. L. P. Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data From American Bridge Company and Bethlehem Steel Company Tests											
....	Flat.....		ABC	83	57	33.6	26.7	....	0.79	....	
						30.0	25.3	....	0.84	....	
						33.1	26.7	....	0.81	....	
								Ave.....	0.81	....	
....	Flat.....		ABC	83	76	35.1	28.4	....	0.81	....	
						31.4	24.5	....	0.78	....	
						33.4	28.7	....	0.86	....	
			B	83	76	38.8	34.4	....	0.89	....	
						39.1	36.0	....	0.92	....	
						40.0	33.6	....	0.84	....	
								Ave.....	0.85	....	
....	Flat.....		B	85.4	38	43.3	39.4	....	0.91	....	
						44.8	41.0	....	0.91	....	
						43.9	41.0	....	0.93	....	
								Ave.....	0.92	....	
....	Flat.....		B	85.4	76	44.2	39.7	....	0.90	....	
						44.1	38.6	....	0.88	....	
						43.8	38.1	....	0.87	....	
								Ave.....	0.88	....	

TABLE 1.—(*Continued.*)

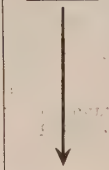
Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from American Bridge Company and Bethlehem Steel Company Tests.											
.....	Flat		ABC	37.0	46	41.0	36.6	.....	0.89	....	.....
						41.0	35.8	.....	0.87	....	.....
						41.2	36.3	.....	0.88	....	.....
			B	35.8	46	38.1	33.7	.....	0.88	....	.....
						38.0	32.0	.....	0.84	....	.....
						38.2	32.9	.....	0.86	....	.....
								Ave.....	0.87		
.....	Flat		ABC	37.3	69	39.3	32.7	.....	0.83	....	.....
						39.3	33.3	.....	0.85	....	.....
						38.1	32.4	.....	0.85	....	.....
								Ave.....	0.84		
.....	Flat		ABC	37.2	92	39.3	31.6	.....	0.80	....	.....
						40.0	30.9	.....	0.77	....	.....
						39.3	31.2	.....	0.79	....	.....
			B	35.8	90	37.2	31.9	.....	0.86	....	.....
						36.3	30.9	.....	0.85	....	.....
						36.0	30.0	.....	0.83	....	.....
								Ave.....	0.83		
.....	Flat		ABC	35.0	40	38.6	37.6	.....	0.97	....	.....
						38.2	36.3	.....	0.95	....	.....
						37.4	37.1	.....	0.99	....	.....
			B	33.1	40	33.9	31.8	.....	0.94	....	.....
						35.1	31.8	.....	0.91	....	.....
						33.6	31.8	.....	0.92	....	.....
								Ave.....	0.95		

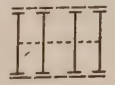
TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: $\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S'}{S_y}$	$\frac{\text{U. L. P.}}{\text{Yield point}} = \frac{S''}{S_y}$	Remarks.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from American Bridge Company and Bethlehem Steel Company Tests.											
.....	Flat		ABC	85	61	35.7	31.2	.....	0.87	....	.....
						35.3	30.7	.....	0.87	....	.....
						35.8	31.3	.....	0.87	....	.....
								Ave.....	0.87		
.....	Flat		ABC	85	81	39.8	32.9	.....	0.83	....	.....
						35.9	30.5	.....	0.85	....	.....
						37.6	30.2	.....	0.80	....	.....
			B	83	79	34.1	30.3	.....	0.89	....	.....
						34.0	30.0	.....	0.88	....	.....
						34.0	30.0	.....	0.88	....	.....
								Ave.....	0.86		
.....	Flat		ABC	39.9	71	42.8	34.0	.....	0.79	....	.....
						42.8	33.9	.....	0.79	....	.....
						42.8	35.8	.....	0.84	....	.....
								Ave.....	0.81		

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Tests in *Technologic Paper No. 101*, U. S. Bureau of Standards.											
....	Pin and flat		$U_2\,U_3$	88.8	25	55.5	61.7	48.0	1.11	0.87	Nickel-steel angles buckled near end.
			$U_8\,U_9$	88.8	37	55.2	61.4	51.0	1.11	0.92	Angles buckled near end.
								Ave.....	1.11	0.90	
....	Pin and flat		HC	74.6	44	40.7	38.4	28.0	0.94	0.69	Carbon steel. Buckled at center.
			31HC	76.9	44	42.9	41.7	29.0	0.98	0.68	Carbon steel. Deflection normal to pin.
								Ave.....	0.96	0.68	
....	Pin and flat		HC1	75.3	36	44.1	45.3	35 +	1.03	0.79	Carbon steel. Deflection parallel to pin.
			31HC1	77.2	38	42.7	46.6	32.5	1.09	0.76	Carbon steel, center web buckled. Column deflection normal to pin.
								Ave.....	1.06	0.78	
....	Pin and flat	See HC	32S	76.9	44	57.2	52.8	37.0	0.92	0.65	Silicon steel. Center web buckled and column deflected normal to pin.
		See HC1	32S1	77.2	38	56.5	51.7	39.5	0.92	0.70	Silicon steel. Center web buckled and column deflected normal to pin.
....	Pin and flat	See HC	MY	74.9	44	59.9	61.5	47.0	1.03	0.79	Mayari steel. Deflection normal to pin.
		See HC1	MY1	75.3	38	62.1	64.7	48.0	1.04	0.77	Mayari steel. Deflection normal to pin.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: $\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S'}{S_y}$	Ratios: $\frac{\text{U. L. P.}}{\text{Yield point}} = \frac{S''}{S_y}$	Remarks
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Tests in *Technologic Paper No. 101*, U. S. Bureau of Standards.											
....	Pin and flat	See HC See HCI	34CS 34CS1	76.9 77.2	44 38	55.3 54.2	49.2 51.8	33.0 36.5	0.88 0.96	0.59 0.67	Carbon steel. Center web and lacing buckled. Column deflection normal to pin.
Data from Quebec Bridge Tests.											
....	Pin and flat		T4A T4B	45.8	64	58.0	50.8 51.3	45.5* 42.3*	0.88 0.88	0.78 0.73	Nickel steel. Deflection failure. Deflection failure.
								Ave.....	0.88	0.76	
....	Pin and flat		T5BL T5AL	52.3	35	58.0	51.0 51.1	37.0* 37.0*	0.88 0.88	0.64 0.64	
								Ave.....	0.88	0.64	
....	Pin and flat		T2A T2B	40.2	44	58.0	57.8 58.3	40.1* 40.1*	1.00 1.01	0.69 0.69	Nickel steel. Buckling near transverse diaphragm.
								Ave.....	1.00	0.69	
....	Pin and flat		T5AS T5BS	27.6	35	58.0	55.0 57.0	42.0* 45.6*	0.95 0.98	0.72 0.79	
								Ave.....	0.96	0.76	

*Signifies that the recorded values were designated elastic limits in the report of the Quebec Bridge tests. Since the U. L. P. could not be determined from the stress-strain curves of these columns, these elastic limits were inserted herein. They are somewhat lower than the U. L. P. values.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: $\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S'}{S_y}$	$\frac{\text{U. L. P.}}{\text{Yield point}} = \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
						Data from Quebec Bridge Tests.					
....	Pin.....		TX13A1	68.9	38	40.5	39.3	28.0	0.97	0.69	Carbon steel. Flanges buckled at center. Deflection perpendicular to pins. No failure until two 1-in. holes were bored in middle of each web-plate.
			TX13A2				40.7	31.0	1.00*	0.77	
								Ave.....	0.99*	0.73	
....	Pin.....		TX13B1	69.0	38	40.5	40.5*	31.0	1.00*	0.77	
			TX13B2				40.5*	32.0	1.00*	0.79	
								Ave.....	1.00*	0.78	
....	Pin.....		TX16C1	34.8	52	40.9	36.6	31.0	0.90	0.76	Ribs buckled near center splice.
			TX16C2				33.4	33.0	0.82	0.81	
								Ave.....	0.86	0.78	
....	Pin.....	See TX 16 c	TX16N1	34.6	52	61.2	47.5	....	0.78	....	Nickel steel. Ribs buckled.
			TX16N2				51.6	....	0 84	....	
								Ave.....	0.82	....	
....	Pin.....		TX17-1	41.1	78	41.7	32.6	30.0	0.78	0.72	Carbon steel. Lower flange buckled. Deflection perpendicular to pins.
			TX17-2				33.0	30.0	0.79	0.72	One rib buckled. Deflection perpendicular to pins.
								Ave.....	0.78	0.72	

* Signifies that the recorded values were designated elastic limits in the report of the Quebec Bridge tests. Since the U. L. P. could not be determined from the stress-strain curves of these columns, these elastic limits were inserted herein. They are somewhat lower than the U. L. P. values.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: $\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S'}{S_y}$	$\frac{\text{U. L. P.}}{\text{Yield point}} = \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Quebec Bridge Tests.											
....	Pin.....		TX18-1	56.1	58	40.9	33.8	29.0	0.83	0.71	Deflection perpendicular to pins.
			TX18-2				35.1	31.0	0.86	0.76	
								Ave.....	0.84	0.74	
Data from Special Committee on Steel Columns and Struts Tests *											
919	Flat.....		178	11.4	20	35.1	38.2	32.0	1.08	0.91	Flanges buckled.†
			179				38.4	32.0	1.09	0.91	
			180				38.0	31.0	1.08	0.88	
								Ave.....	1.08	0.90	
1919	Flat.....		175	22.1	20	31.2	43.3	24.5	1.38	0.79	Flanges buckled.†
			176				43.4	25.0	1.39	0.80	
			177				44.5	25.0	1.43	0.80	
								Ave.....	1.40	0.80	
1919	Flat.....		8	11.1	50	35.1	32.4	28.3	0.92	0.80	Flanges buckled.
			38				32.8	29.2	0.93	0.83	" "
			77				33.0	29.0	0.94	0.82	Failed by deflection.
								Ave.....	0 93	0.82	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1634.

† Indicated from photograph.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests.*											
1919	Flat		93	22.2	50	31.2	29.0	25.0	0.93	0.80	Deflection failure.†
			96				28.9	24.0	0.93	0.77	
			97				29.7	25.0	0.95	0.80	
								Ave.....	0.94	0.79	
1919	Flat		18	11.4	85	35.1	32.1	26.0	0.91	0.74	Flanges buckled.
			84				31.4	30.0	0.89	0.85	
			85				30.0	27.5	0.85	0.78	
								Ave.....	0.88	0.79	
1919	Flat		91	22.0	85	31.2	28.0	25.7	0.90	0.82	Deflection failure.†
			94				28.2	24.7	0.91	0.79	
			95				28.0	25.5	0.90	0.82	
								Ave.....	0.90	0.81	
1919	Flat		229	28.4	85	32.5	28.2	26.5	0.87	0.82	Deflection failure.
			230				27.0	26.0	0.83	0.80	
			231				28.0	26.0	0.86	0.80	
								Ave.....	0.85	0.81	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1634.

† Indicated from photograph.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 Lb. per Sq. In. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: $\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S'}{S_y}$	$\frac{\text{Ultimate}}{\text{Yield point}} = \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns And Struts Tests.*											
1919	Flat		80	11.2	120	35.1	28.6	25.4	0.81	0.72	
			56				27.6	26.3	0.79	0.75	
			69				28.7	28.0	0.82	0.80	
								Ave.	0.81	0.76	
1919	Flat		92	22.1	120	31.2	25.0	23.0	0.80	0.74	
			98				25.5	23.5	0.82	0.75	
			99				25.8	23.3	0.83	0.75	
								Ave.	0.82	0.75	
1919	Flat		207	11.7	155	35.1	27.0	26.0	0.77	0.74	
			218				25.6	25.0	0.76	0.71	
			222				26.0	24.5	0.74	0.70	
								Ave.	0.76	0.72	
1919	Flat		205	22.3	155	31.2	22.2	21.3	0.71	0.68	
			206				23.2	22.4	0.74	0.72	
			208				22.7	21.0	0.73	0.67	
								Ave.	0.73	0.69	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20). p. 1634.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 Lb. per Sq. In. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	Ratios: U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests.*											
1919	Flat.....		5	13.3	50	35.0	34.3	30.3	0.98	0.87	
			37				32.7	30.6	0.94	0.88	Failed by buckling of angle legs.
			76				33.2	31.0	0.95	0.89	Failed by deflection.
								Ave.....	0.96	0.88	
1919	Flat.....		15	13.4	85	35.0	34.0	30.8	0.97	0.88	Triple flexure,
			89				30.0	29.0	0.86	0.83	Failed by deflection upward.
			90				30.7	28.5	0.93	0.81	Failed by deflection.
								Ave.....	0.90	0.84	
1919	Flat.....		28	13.3	120	35.0	28.3	28.3	0.81	0.81	.....
			48				26.9	25.9	0.77	0.74	.....
			51				29.0	25.6	0.83	0.73	.....
								Ave.....	0.80	0.76	
1919	Flat.....		107X	9.2	20	38.1	39.6	34.0	1.04	0.89	Flanges buckled.†
			108X				41.0	36.0	1.08	0.94	" " .†
			114X				41.0	37.0	1.08	0.97	" " .†
								Ave.....	1.07	0.93	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1634.

† Indicated from photograph.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	Ratios: U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests.*											
1919	Flat.....		106X	18.3	20	34.7	35.6	31.0	1.03	0.89	
			112X				37.8	33.0	1.09	0.95	
			117X				36.7	33.5	1.06	0.97	55 200 maximum load obtained in emphasizing failure. Flanges buckled.
								Ave.....	1.06	0.94	
1919	Flat.....		181CX	26.6	20	25.8	48.1	21.0	1.88	0.81	Flanges buckled.†
			183BX				54.5	20.0	2.11	0.78	.....
			225AX				56.6	20.2	2.19	0.78	.....
								Ave.....	2.06	0.79	
1919	Flat.....		107	9.6	50	38.1	38.0	35.0	1.00	0.92	Buckling of flanges.
			122				38.0	34.5	1.00	0.91	.....
			123				38.0	34.5	1.00	0.91	Buckling of flanges.
								Ave.....	1.00	0.91	
1919	Flat.....		106	17.8	50	34.7	34.0	30.0	.98	0.87	.....
			120				36.1	31.0	1.04	0.89	.....
			121				36.0	32.5	1.04	0.94	.....
								Ave.....	1.02	0.90	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1634.

† Indicated from photograph

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests.*											
1919	Flat		225	27.4	50	25.8	24.0	19.0	0.93	0.74	
			226				24.6	21.1	0.96	0.82	
			182				27.1	19.0	1.05	0.74	Deflection failure.†
							Ave.		0.98	0.77	
1919	Flat		114	8.6	85	38.1	36.0	34.0	0.94	0.89	
			116	9.6			34.0	32.5	0.89	0.85	
			119	9.6			33.0	32.0	0.87	0.84	Deflection failure.
							Ave.		0.90	0.86	
1919	Flat		115	18.0	85	34.7	33.9	32.0	0.98	0.92	
			117				32.0	30.5	0.92	0.88	
			118				31.0	29.3	0.89	0.84	Failed by buckling south and down.
							Ave.		0.93	0.88	
1919	Flat		227	27.3	85	25.8	22.6	19.6	0.88	0.76	
			228				23.0	20.0	0.89	0.77	
			183				24.8	21.0	0.96	0.81	
							Ave.		0.91	0.78	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1634.

† Indicated from photograph.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate Yield point $= \frac{S'}{S_y}$	Ratios: U. L. P. Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests.*											
1919	Flat.....		110	8.6	120	38.1	33.9	33.0	0.89	0.87	.....
			111				31.0	30.2	0.81	0.79	.....
			108				31.0	30.0	0.81	0.79	.....
								Ave.....	0.84	0.82	
1919	Flat.....		109	17.8	120	34.7	30.0	29.0	0.86	0.84	.....
			112				30.0	30.0	0.86	0.86	.....
			113				30.0	29.0	0.86	0.84	.....
								Ave.....	0.86	0.85	
1919	Flat.....		223	27.5	120	25.8	21.0	19.0	0.81	0.74	.....
			224				21.0	18.5	0.81	0.72	.....
			181				21.8	20.0	0.85	0.78	.....
								Ave.....	0.82	0.75	
1919	Flat.....		6	11.7	50	36.3	35.7	29.9	0.98	0.82	Triple flexure.
			36				36.5	33.0	1.00	0.91	Triple flexure.
			75				38.6	33.0	1.06	0.91	.....
								Ave.....	1.01	0.88	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1634.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	Ratios: U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests *											
1919	Flat.....		102	16.9	50	29.8	30.3	23.0	1.02	0.77	Buckled south and down.
			166				29.0	22.5	0.97	0.75	.....
			169				28.0	22.0	0.94	0.74	.....
								Ave.....	0.98	0.75	
1919	Flat.....		13	11.7	85	36.3	34.0	30.4	0.94	0.84	Triple flexure.
			79				33.9	31.5	0.93	0.87	Deflection failure.
			80				34.0	31.5	0.94	0.87	Deflection failure
								Ave.....	0.94	0.86	
1919	Flat.....		152	16.9	85	29.8	26.0	22.5	0.87	0.75	.....
			153				26.4	23.5	0.88	0.79	.....
			154				27.5	24.0	0.92	0.80	.....
								Ave.....	0.89	0.78	
1919	Flat.....		26	11.7	120	36.3	32.0	30.3	0.88	0.83	.....
			45				30.7	28.2	0.85	0.78	.....
			49				33.1	31.6	0.91	0.87	.....
								Ave.....	0.88	0.83	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1634.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S}{S_y}$	Ratios: U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests.*											
1919	Flat		132	17.0	120	29.8	23.5	21.5	0.79	0.72	
			133				23.9	22.0	0.80	0.74	
			134				24.2	21.5	0.81	0.72	
								Ave.	0.80	0.73	
1919	Flat		211	11.9	155	36.3	24.0	21.0	0.66	0.58	
			212				23.2	23.0	0.64	0.63	
			220				23.5	23.0	0.65	0.63	
								Ave.	0.65	0.61	
1919	Flat		209	16.6	155	29.8	24.0	24.0	0.81	0.81	
			210				23.0	23.0	0.77	0.77	
			213				23.0	23.0	0.77	0.77	
								Ave.	0.78	0.78	
1919	Flat		11	10.1	50	34.3	33.1	27.4	0.97	0.79	
			41				34.0	28.2	0.99	0.82	Triple flexure.
			43				32.4	26.9	0.94	0.78	Buckled at center.
								Ave.	0.97	0.80	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1634.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 Lb. per Sq. In. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratio: Ultimate / Yield point $= \frac{S'}{S_y}$	Ratio: U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests.*											
1919	Flat		101	17.0	50	32.2	32.2	26.0	1.00	0.81	
			124				32.5	27.0	1.01	0.84	
			165				32.3	27.5	1.00	0.85	
								Ave.	1.00	0.83	
1919	Flat		21	9.9	85	34.3	33.7	29.2	0.98	0.85	Buckled at center.
			59				32.3	28.4	0.94	0.83	
			65				31.7	28.6	0.92	0.83	Deflection failure.
								Ave.	0.95	0.84	
1919	Flat		145	17.1	85	32.2	30.7	27.0	0.95	0.84	Buckled at center.
			146				30.0	26.0	0.93	0.81	
			151				31.0	27.0	0.96	0.84	
								Ave.	0.95	0.83	
1919	Flat		32	10.1	120	34.3	28.3	25.6	0.82	0.75	
			54				28.7	26.3	0.84	0.77	
			70				30.9	29.0	0.90	0.85	
								Ave.	0.85	0.79	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1634.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 lb. per sq. in. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	Ratios: U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests.*											
1919	Flat		135	17.0	120	32.2	28.5	26.5	0.88	0.82	
			136				28.8	26.5	0.89	0.82	
			137				27.0	26.0	0.84	0.81	
								Ave.	0.87	0.82	
1919	Flat		215	10.4	155	34.3	26.1	25.7	0.76	0.75	
			217				27.8	26.8	0.81	0.78	
			221				25.6	24.7	0.75	0.72	
								Ave.	0.77	0.75	
1919	Flat		214	16.8	155	32.2	24.7	24.0	0.77	0.75	
			216				25.0	23.0	0.78	0.71	
			219				25.0	25.0	0.78	0.78	
								Ave.	0.78	0.75	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1634.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 Lb. per Sq. In. of: Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ratios: Ultimate / Yield point $= \frac{S'}{S_y}$	U. L. P. / Yield point $= \frac{S''}{S_y}$	Remarks:
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Data from Special Committee on Steel Columns and Struts Tests.*											
1919	Flat		1	8.7	50	35.9	33.8	28.3	0.94	0.79	Triple flexure.
			2				34.3	28.7	0.96	0.80	.....
			42				34.3	27.9	0.96	0.78	.....
								Ave.....	0.95	0.79	
1919	Flat		14	8.7	85	35.9	32.5	26.9	0.91	0.72	Triple flexure.
			87				31.9	29.0	0.89	0.81	.....
			88				32.9	28.0	0.92	0.78	.....
								Ave.....	0.91	0.77	
1919	Flat		24	8.7	120	35.9	30.6	27.9	0.85	0.78	.....
			35				30.2	28.6	0.84	0.80	.....
			47				30.0	27.5	0.84	0.77	.....
								Ave.....	0.84	0.78	

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1634.

TABLE 1.—(*Continued.*)

Series.	End conditions.	Form of column.	Column no.	Area of section, in square inches.	$\frac{l}{r}$	Strength, in 1 000 Lb. per Sq. In. of:			Ratios:		Remarks:
						Metal at yield point, S_y.	Column at ultimate, S'.	Column at U. L. P., S''.	Ultimate / Yield point $= \frac{S'}{S_y}$	U. L. P. / Yield point $= \frac{S''}{S_y}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
					Data from Kiel Ship Canal Bridge Tests.						
....	Pin		I-1	63	51	31.2	29.6	.....	0.95	.....	.....
			I-2	63	51	....	29.4	.....	0.94	.....	.....
....	Pin		II-1	70	58	34.0	32.8	.....	0.96	.....	.....
			II-2	70	58	....	34.7	.....	1.02	.....	.....
			III-1	70	58	34.0	33.2	.....	0.98	.....	.....
			III-2	70	58	....	32.3	.....	0.95	.....	.....
....	Pin		IV	60	52	33.4	31.4	.....	0.94	.....	.....
			V	60	52	....	31.2	.....	0.93	.....	.....
....	Pin		VI-1	64	51	37.6	36.4	.....	0.97	.....	.....
			VI-2	64	51	....	37.4	.....	0.99	.....	.....
....	Pin		VII-1	64	51	40.5	34.5	.....	0.85	.....	.....
			VII-2	64	51	....	38.5	.....	0.95	.....	.....

The results are scattered over a rather wide range of values, the limits, except for $\frac{l}{r}$ less than 30, being indicated closely by the two straight lines shown in Plate X. The central straight line represents fairly well average values. Some of the variation in strength is due undoubtedly to the uncertainty regarding yield point of the material, and probably the high values for the tests of the Special Committee on Steel Columns and Struts* at $\frac{l}{r} = 50$ are due in part to the use of a lower value for yield point than for some of the other tests. There appears to be little effect of form of column, although most of the larger built-up columns are below the center line.

The effect of $\frac{l}{r}$ is quite marked, and is fairly well indicated by the slope of the central line. The high values for $\frac{l}{r}$ less than 30 are due to the well-known behavior of very short compression members, and are of little significance in the consideration of the ordinary column.

Plate X (*B*) gives ratios of the U. L. P. of the column to the yield point of the metal for such tests as furnished the necessary information. Here, again, the general range is well shown by two straight lines. Comparing these results with those of Plate X (*A*), an important difference is seen to be in the flatter slope of the lines, showing a less effect of $\frac{l}{r}$, and the absence of any high results for small values of $\frac{l}{r}$. Comparing the center lines of the two diagrams, it is seen that for $\frac{l}{r} = 0$, they are 10% apart; whereas for $\frac{l}{r} = 100$, they are only 5% apart. That is, for short columns there is a greater margin between the U. L. P. and the ultimate strength than for long columns. This is what would be expected, and for $\frac{l}{r}$ as great as 155 (see Fig. 6), the ultimate strength and U. L. P. values are very nearly equal.

Another noteworthy feature of Plate X (*B*) is the very low values for some of the large built-up sections, all of which come well up among the general group in Plate X (*A*). This result is due to the shape of the stress-deformation curves of these columns from which the U. L. P.'s were determined. These curves show a comparatively low proportional elastic limit, with gradually increasing rate of deformation over a wide range of stress, so that the U. L. P., as defined by the Society's Special Committee on Steel Columns and Struts is low. An explanation of this behavior probably lies in the large number of parts of which the columns were composed, a condition which is apt to result in an appreciable amount of inelastic deformation at comparatively low loads, but with little influence on ultimate strength.

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1583.

In Figs. 1, 2, 3, 4, and 5, the data shown on Plate X have been separated into groups for further study. In these diagrams, the upper and lower values are shown by the arrow-points and the average for the group by the plotted position of the column section. The sloping straight lines are the central lines from Plate X, drawn in as a guide for comparison.

Fig. 1 shows the tests of the Special Committee on Steel Columns and Struts. The ultimate ratios are rather high. Fig. 2 contains results on I and H-sections. The effect of form is hardly noticeable. Fig. 3 gives results on tubular columns. The range of ultimate values is naturally less than in the other groups on account of uniformity of material and form. There is no noticeable difference between pin and flat ends. Fig. 4 gives results for built-up latticed columns of all kinds, including the large columns reported in *Technologic Paper No. 101,* U. S. Bureau of Standards, and the Quebec Bridge tests. Here, the ultimate strength ratio corresponds well with the others; but the U. L. P. ratio is low, as previously noted. Fig. 5 includes the tests on alloy steels of high elastic limit (up to a yield point of 60 000 lb. per sq. in.). The ultimate strength ratios correspond well with the general average; the U. L. P. ratios are low. As far as these tests are concerned, it would appear that for short columns the strength bears about the same relation to the yield point of the material as in the case of ordinary structural steel. Fig. 6 gives the results of the tests of the Society's Special Committee on Steel Columns and Struts in more detail, and up to $\frac{l}{r} = 155$. The heavy lines are rough average curves. Only those types are shown for which values of yield points were closely known. All lines tend to converge for increasing $\frac{l}{r}$, as is to be expected; for the column strength for high values of $\frac{l}{r}$ is more dependent on the modulus of elasticity, E, which is practically the same for all. Type 5, the solid H-section, shows up favorably; Type 2, the narrower box section, is about the best; Type 1, the I-section, is generally the poorest; Type 7, the bulb-angle section, is also rather poor. These relations correspond in general to the deductions in Part (B) of the detailed studies on component eccentricities. For the higher values of $\frac{l}{r}$, the range of variation becomes relatively small.

Table 1 gives detailed results for all the columns used in these diagrams.

5.—The Yield Point Strength of the Metal.—The close relation between the strength of columns and the yield point strength of the material is obvious, and is well shown on the diagrams already discussed; but this relation, as shown by tests, does not appear to be as exact as it should be. This is due to several causes. One is the lack of reasonably exact information as to the material itself; and this is a very important factor in the whole problem of column strength. This kind of structural element is so sensitive to the effect of inelastic deformation, or over-strain, that it is especially important to know the practical elastic limit strength of the material out of which it is made. In the diagrams of Plate X, only those tests have been plotted for

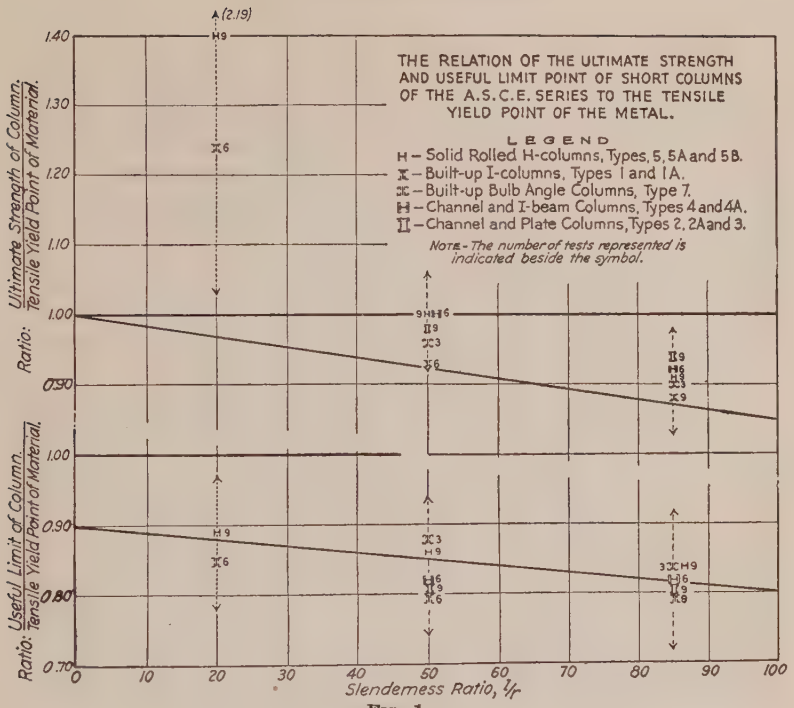

THE RELATION OF THE ULTIMATE STRENGTH AND USEFUL LIMIT POINT OF SHORT COLUMNS OF THE A.S.C.E. SERIES TO THE TENSILE YIELD POINT OF THE METAL.
LEGEND
H – Solid Rolled H-columns, Types, 5, 5A and 5B.
I – Built-up I-columns, Types 1 and 1A.
– Built-up Bulb Angle Columns, Type 7.
H – Channel and I-beam Columns, Types 4 and 4A.
II – Channel and Plate Columns, Types 2, 2A and 3.
Note - The number of tests represented is indicated beside the symbol.
Ultimate Strength of Column. Tensile Yield Point of Material.
Ratio:
Useful Limit of Column. Tensile Yield Point of Material.
Ratio:
Slenderness Ratio, l/r
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0
10
20
30
40
50
60
70
80
90
100

FIG. 1.

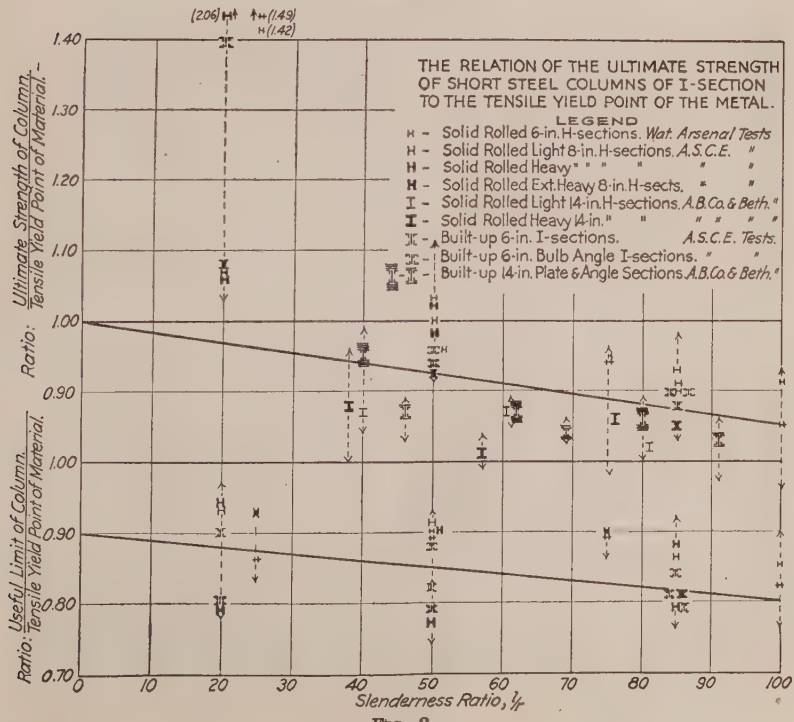

(2.06)
(1.43)
(1.42)
THE RELATION OF THE ULTIMATE STRENGTH OF SHORT STEEL COLUMNS OF I-SECTION TO THE TENSILE YIELD POINT OF THE METAL.
LEGEND
H – Solid Rolled 6-in. H-sections. Wat. Arsenal Tests
H – Solid Rolled Light 8-in. H-sections. A.S.C.E. "
H – Solid Rolled Heavy " " " " " "
H – Solid Rolled Ext. Heavy 8-in. H-sects. " "
I – Solid Rolled Light 14-in. H-sections. A.B.Co. & Beth. "
I – Solid Rolled Heavy 14-in. " " " " " "
I – Built-up 6-in. I-sections. A.S.C.E. Tests.
– Built-up 6-in. Bulb Angle I-sections. " "
– Built-up 14-in. Plate & Angle Sections. A.B.Co. & Beth. "
Ultimate Strength of Column. Tensile Yield Point of Material.
Ratio:
Useful Limit of Column. Tensile Yield Point of Material.
Ratio:
Slenderness Ratio, l/r
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0
10
20
30
40
50
60
70
80
90
100

FIG. 2.

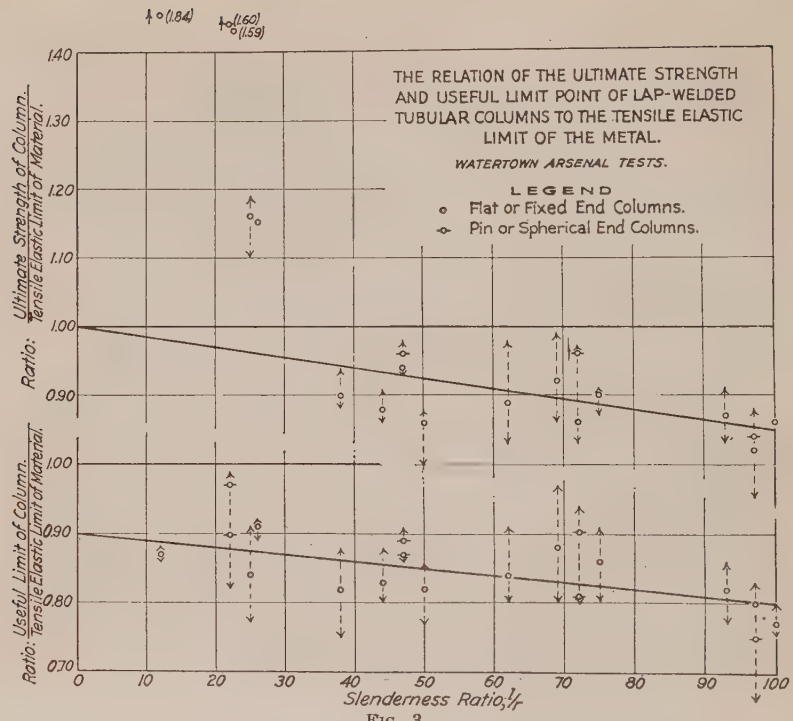

(1.84)
(1.60)
(1.59)
THE RELATION OF THE ULTIMATE STRENGTH AND USEFUL LIMIT POINT OF LAP-WELDED TUBULAR COLUMNS TO THE TENSILE ELASTIC LIMIT OF THE METAL.
WATERTOWN ARSENAL TESTS.
LEGEND
Flat or Fixed End Columns.
Pin or Spherical End Columns.
Ratio: Ultimate Strength of Column / Tensile Elastic Limit of Material
Ratio: Useful Limit of Column / Tensile Elastic Limit of Material
1.40
1.30
1.20
1.10
1.00
0.90
0.80
0.70
0
10
20
30
40
50
60
70
80
90
100
Slenderness Ratio, l/r

FIG. 3.

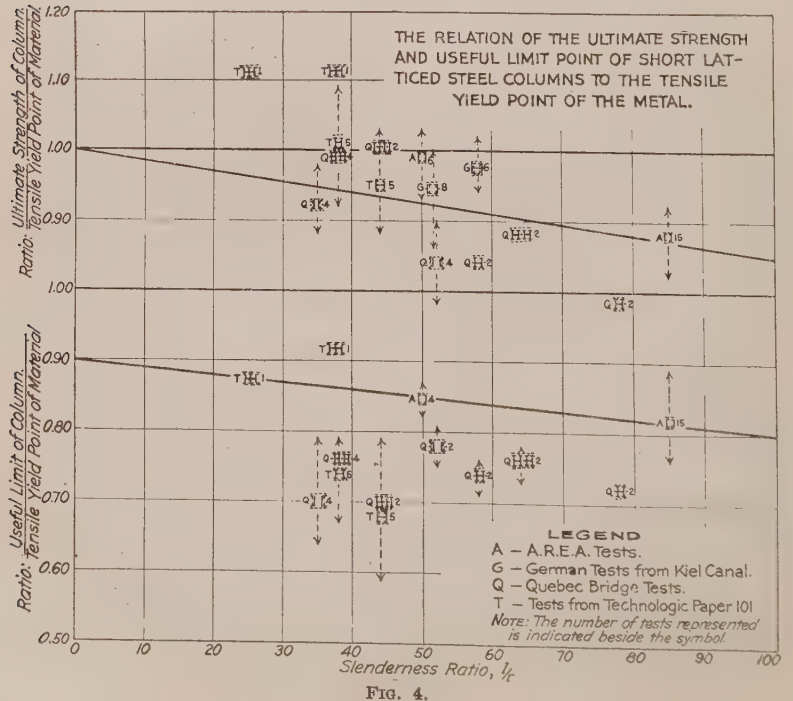

THE RELATION OF THE ULTIMATE STRENGTH AND USEFUL LIMIT POINT OF SHORT LATTICED STEEL COLUMNS TO THE TENSILE YIELD POINT OF THE METAL.
Ratio: Ultimate Strength of Column / Tensile Yield Point of Material
Ratio: Useful Limit of Column / Tensile Yield Point of Material
LEGEND
A — A.R.E.A. Tests.
G — German Tests from Kiel Canal.
Q — Quebec Bridge Tests.
T — Tests from Technologic Paper 101
Note: The number of tests represented is indicated beside the symbol.
1.20
1.10
1.00
0.90
0.80
0.70
0.60
0.50
0
10
20
30
40
50
60
70
80
90
100
Slenderness Ratio, l/r

FIG. 4.

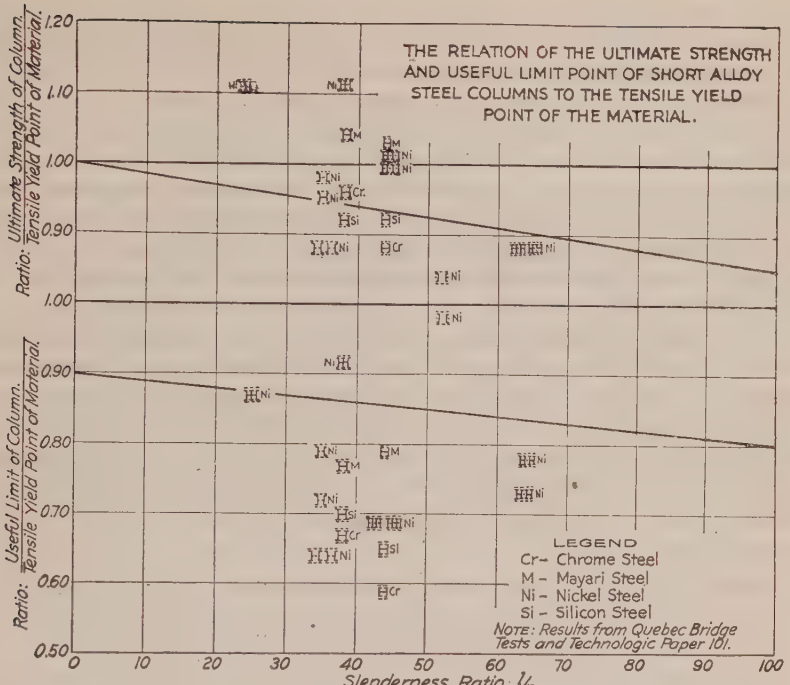

THE RELATION OF THE ULTIMATE STRENGTH AND USEFUL LIMIT POINT OF SHORT ALLOY STEEL COLUMNS TO THE TENSILE YIELD POINT OF THE MATERIAL.
Ratio: Ultimate Strength of Column / Tensile Yield Point of Material
Ratio: Useful Limit of Column / Tensile Yield Point of Material
LEGEND
Cr- Chrome Steel
M - Mayari Steel
Ni - Nickel Steel
Si - Silicon Steel
NOTE: Results from Quebec Bridge Tests and Technologic Paper 101.
Slenderness Ratio, l/r

FIG. 5.

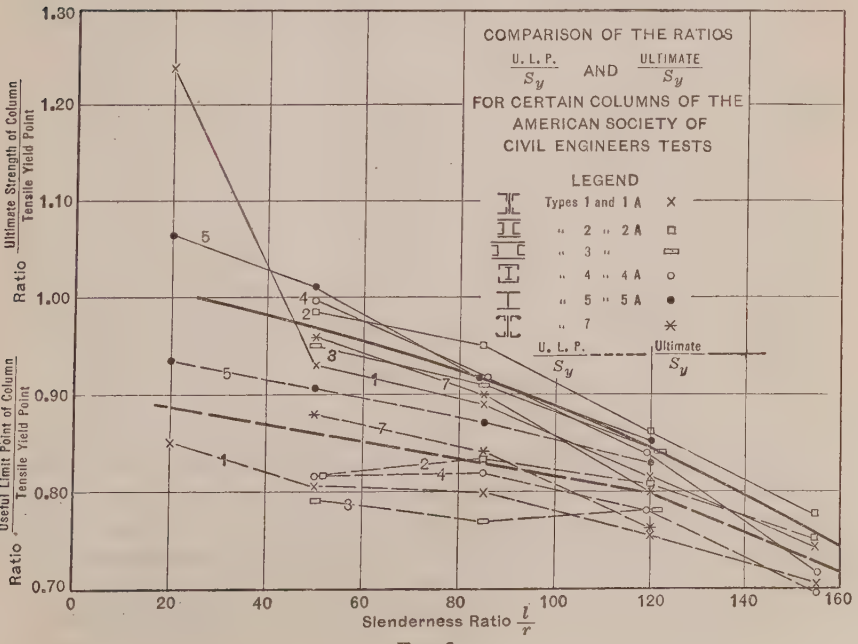

COMPARISON OF THE RATIOS
U. L. P. / Sy AND ULTIMATE / Sy
FOR CERTAIN COLUMNS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS TESTS
LEGEND
Types 1 and 1 A
" 2 " 2 A
" 3 "
" 4 " 4 A
" 5 " 5 A
" 7
U. L. P. / Sy
Ultimate / Sy
Ratio Ultimate Strength of Column / Tensile Yield Point
Ratio Useful Limit Point of Column / Tensile Yield Point
Slenderness Ratio l/r

FIG. 6.

which the tensile yield points appeared to be fairly well known. In the case of the tests of the Special Committee on Steel Columns and Struts, these values were taken from the stress-strain diagrams prepared at the U. S. Bureau of Standards, and were generally from 1 500 to 2 000 lb. per sq. in. higher than the U. L. P.'s given in the report. In other cases, the statements made in the reports were taken at their face values and doubtless did not always accurately represent the material used. It is possible that some such point as the U. L. P. used by the Society's Special Committee on Steel Columns and Struts would be a more accurate reference point than the yield point, but as no other tests give this information, it seemed best to use the yield point instead, and it is believed that the diagrams represent fairly the relations indicated.

The difficulty of securing correct information regarding the elastic properties of the material actually used is well known. The values of yield point as determined by the usual "drop of the beam" method depend largely on the details of testing; the mill tests vary largely from laboratory tests; and the material varies greatly in thin and thick sections, and at different points in the same section. The data reviewed in these studies furnish a great deal of information on some of these points, and some typical results are given in the following paragraphs.

TABLE 2.—Mill Tests and Laboratory Tests for Elastic Limit.

(Final Report of the Special Committee on Steel Columns and Struts of the Society.)

Type of column.	Average mill test, drop of beam, in pounds per square inch.	Average U. S. Bureau of Standards U. L. P., in pounds per square inch.	Average yield point from U. S. Bureau of Standards curves, in pounds per square inch
1	34 000	33 100	35 100
1A	38 300	29 800	31 200
1B	36 400	31 100	32 500
2	36 500	32 500	34 300
2A	36 900	30 200	32 200
4	36 500	33 800	36 300
4A	37 700	27 100	29 800
7	34 600	33 500	35 000
5	.....	37 500	38 100
5A	.....	33 300	34 700
5B	.....	22 600	25 800

6.—Mill Tests vs. Laboratory Tests.—The difficulty in using mill test data for comparative purposes is well shown in the report of the Society's Special Committee on Steel Columns and Struts. In Table 2 are shown the values of yield point (drop of beam) from mill specimen tests; the average U. L. P. from the U. S. Bureau of Standards tests; and the average yield point as calculated from the Bureau of Standards curves and as used in this report.*

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1629.

The figures in Table 2 show not only a wide discrepancy between results for material of the same column group, but also, as is well known, a large variation between light and heavy sections. There is unquestionably a very close relation between the strength of columns of moderate length and the actual yield point of the material; but to determine this relation or to utilize it in designing with any great accuracy requires more than mill test data as now ordinarily supplied.

The great discrepancy which may exist between mill test and laboratory test is also illustrated by the results obtained by the U. S. Bureau of Standards on rolled H-sections. In one case (Bethlehem H-287½ lb.), the average mill test reported was 42 900 lb. per sq. in. as compared to the Bureau of Standards' 31 700 lb. per sq. in. In the other cases, the average values were much closer, generally not more than 2 000 lb. per sq. in. apart.

The difficulty of getting a proper average value even from coupons cut from the piece is not small. This is particularly true of such forms as the H-section. This was especially studied at the U. S. Bureau of Standards, where test specimens were cut from six different zones and a variation of from 35 000 to 45 000 lb. per sq. in. in yield point was found in one case. The tests of Marburg on I-beams* showed extreme variations in elastic limit (proportional), the value for material at the root being as low as 11 000 lb. per sq. in., and in several cases as low as 15 000 to 17 000 lb. At the same time the flanges showed about 25 000 lb. The yield points varied much less, the variation being generally only 3 000 to 4 000 lb. per sq. in. Moore† gives considerable data on beams, the variation of yield point in the same beam being commonly from 3 000 to 5 000 lb., and running as high as 15 000 lb. per sq. in., the large differences being due to especially high values of yield point for the flange material. Hancock‡ gives further data on the same subject. Considering the effect of variation of material across the section, it would appear that the value of the minimum yield point would have a great effect on the strength of the column, and that perhaps a rating based on minimum yield point would be the most significant; but so far as the Committee has been able to observe, the average value is better. No doubt a very low yield point of the material making up the flange of a column would seriously affect its strength, but flange material of any form is not likely to be of a particularly low value, and the difficulty of determining the properties of the actual components of a member make it very doubtful whether anything would be gained by attempting to use minimum rather than average values.

7.—*Tensile vs. Compressive Yield Point of Test Specimens.*—In the foregoing discussion, the tensile yield point of the material has been used in all cases. It is seldom that information is given concerning the compressive yield points, and it is doubtful whether it will be found practicable to secure such information in general if the tensile yield point gives satisfactory results; but, undoubtedly, the compressive yield point is, after all, the significant thing, and it is desirable to compare the two values as far as possible. The tests of

* *Proceedings*, Am. Soc. for Testing Materials, 1909.
† *Loc. cit.*, 1910.
‡ *Loc. cit.*, 1911.

the U. S. Bureau of Standards for the Society's Special Committee on Steel Columns and Struts give some information on this point. Table 3 shows the U. L. P. from tension and compression tests of some of the material used. Average values are given in each case. The values of $\frac{l}{r}$ ranged from 12 to 20.

Considerable other information exists regarding the relation of elastic properties from tensile and compressive tests. Marshall's experiments* on mild steel show slightly higher values of elastic limit in compression than in tension, the average being about 2½ per cent. Seeley and Putnam† give the average results from a considerable number of tests shown in Table 4.

TABLE 3.—Comparison of Tensile and Compressive Useful Limit Points.
(From Final Report of Society's Special Committee on Steel Columns and Struts.)

Type of column.	Test specimen.	Average U. L. P., tension, in pounds per square inch.	Average U. L. P., compression, in pounds per square inch.	Ratio: Tension / Compression.
1	5 by 3 by 5/16-in. angle 6 by 5/16-in. plate	33 300 32 300	33 100 29 800	1.006 1.08
1 A	5 by 3 by 5/8-in. angle	29 800	27 000	1.11
2	8 by ¼-in. plate 6-in. channel	31 500 33 200	28 200 34 000	1.12 0.98
3	5-in. channel	34 100	35 200	0.97
4	8-in. channel 8-in. I-beam	34 700 34 500	36 100 30 800	0.96 1.12
4 A	8-in. channel 8-in. I-beam	26 100 28 000	28 600 28 400	0.91 0.99

Results of tests on normal and heat-treated steels with ultimate strengths from 42 400 to 138 000 lb. per sq. in. by Moore and Kommers‡ gave values for ratio of tensile to compressive yield points ranging from 92 to 116%, and averaging about 101 per cent. These included nickel and chrome steels.

TABLE 4.—Average Results from Tests on the Relation of Elastic Properties of Various Kinds of Steel.

Material.	Tension, ultimate, in pounds per square inch.	Ratio: Tensile to Compressive.		
		Proportional limit.	U. L. P.	Yield point.
Soft steel	55 400	0.784	0.933	1.04
Mild steel	54 900	0.943	.....	.....
Medium steel	78 000	0.99	1.01	0.965
Vanadium steel	109 000	0.950	0.972	0.943
Nickel steel:				
(*a*). Parallel to rolling	87 900	0.989	0.923	0.956
(*b*). Perpendicular to rolling	87 700	1.00	0.985	0.972
Chrome nickel steel	65 600	0.886	0.887	0.942

In general, it would appear that the tensile yield point is a close indication of the compressive yield point, and for the purpose in question probably as

* *Transactions*, Am. Soc. C. E., Vol. XVII (1887), p. 54.
† *Bulletin No. 115*, Eng. Experiment Station, Univ. of Illinois, 1919.
‡ *Bulletin No. 124*, Eng. Experiment Station, Univ. of Illinois. 1921

satisfactory an indication of column strength. An examination of the column test results of the Society's Special Committee on Steel Columns and Struts shows no closer relation to the compressive than to the tensile yield point.

8.—Yield Point Determined by Tests on Short Column Sections.—It has been proposed to make use of short sections of actual columns for the purpose of determining the elastic properties of the combination of shapes used and to base estimates of column strengths upon such information. This is a logical process, and should give superior results. It is doubtful, however, whether such a method is practicable in general. As a matter of fact, the results of the tests made by the Society's Special Committee on Steel Columns and Struts, when measured by the useful limit values from the tests on the very short test pieces $\left(\frac{l}{r} = 20\right)$, show a greater variation than when referred to the average yield point of the material. This is hardly sufficient evidence to condemn this method, but it throws doubt on its special value.

(B) Results of Tests on Columns of All Lengths and Showing Integral Action

9.—There have been assembled in Figs. 7, 8, and 9 the results of all available tests which appear to be pertinent to the present investigation. Tests have been excluded where failure was due to weak details, or where the load was applied eccentrically. The complete list of tests is given herewith, and for the full test data, the reader is referred to the original articles:

Steel Columns.—

American Railway Engineering Association. 30 tests. Latticed channel columns tested at the U. S. Bureau of Standards. Columns with batten-plate connections omitted. (*Proceedings,* Am. Ry. Eng. Assoc., Vol. 16, p. 636; Vol. 19, p. 789).

American Society of Civil Engineers. 99 tests. Tested by the U. S. Bureau of Standards. The following tests are omitted because of unknown or uncertain yield point of material: No. 3 heavy; No. 6; No. 6A; No. 8; No. 8A; Z-bar; No. 10 heavy; and the Carnegie H-columns; also, No. 5, extra heavy for $\frac{l}{r} = 20$. (*Transactions,* Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1583).

American Bridge Company and Bethlehem Steel Company. 69 tests. Tested at the U. S. Bureau of Standards. Report to be published.

Watertown Arsenal Tests of H-Sections and Built-Up I-Sections. 130 tests. Two omitted on account of excessive bends. 94 tests of lap-welded tubing; 12 omitted for $\frac{l}{r} = 12$ to 22. (Report on Tests of Metals and Other Materials, 1908, 1909, 1910, 1912, 1913, 1914).

Waddell's Tests on Built-Up Channel Sections. 12 tests. (*Transactions,* Am. Soc. C. E., Vol. LXIII (1909), p. 246).

Buchanan's Tests on Built-Up I- and Plate- and Channel-Sections. 6 tests. Six omitted on account of eccentric loading or failure of details. (*Engineering News,* Vol. 58, p. 685).

Christie's Tests on Small Angle-Sections. 16 tests. Used only tests for $\frac{l}{r} = 160$ to 300. (*Transactions*, Am. Soc. C. E., Vol. XIII (1884), p. 253).

Kiel Canal Bridge. Built-Up Channel Sections. 12 tests. (*Engineering News-Record,* Vol. 88, p. 877).

Tests on Large Built-Up Columns, U. S. Bureau of Standards. 10 tests. Omitted 2 tests on account of high yield point of material and 6 tests on account of failure in details. (*Technologic Paper No. 101,* U. S. Bureau of Standards).

Quebec Bridge Tests. 18 tests. Omitted 2 tests on account of high yield point of material and others that were tested horizontally without counterpoise. (The Quebec Bridge, 1919, Department of Railways and Canals, Canada; also *Engineering Record,* Vol. 62, p. 564; Vol. 69, p. 333).

Wrought-Iron Columns.—

Bouscaren's Tests. 16 tests. Tests on Keystone and old American Bridge Company's sections omitted. (*Transactions,* Am. Soc. C. E., Vol. IX (1880), p. 447).

Christie's Tests on Small Shapes. Used results for $\frac{l}{r} = 160$ to 300. (*Transactions,* Am. Soc. C. E., Vol. XIII (1884), p. 85).

Buchanan's Tests. 2 tests. Omitted others which failed in details. (*Engineering News,* Vol. 58, p. 685).

Clark Reeves and Company. Phœnix Columns. Omitted 2, $\frac{l}{r} = 2.7$. (Report of Tests of Metals and Other Materials, 1881, p. 263).

Strobel's Tests on Z-Bar Columns. 15 tests. (*Transactions,* Am. Soc. C. E., Vol. XVIII (1888), p. 103).

Watertown Arsenal. 190 tests. Omitted 13 because of eccentric loads or failure in details. Omitted 3 because of excessive bends. (Report on Tests of Metals and Other Materials, 1881, 1882, 1883, 1884).

The results for the steel columns have been grouped as closely as practicable in Figs. 7 and 8, in accordance with the yield point or elastic limit of the material, although in many cases this could not be done very accurately. Tests on the wrought-iron columns have been grouped together in Fig. 9. These general groups will be first discussed, after which certain individual groups of tests will be considered.

10.—Tests on Steel Columns.—In Figs. 7 and 8 it will be noted that both flat-end and pin-end columns have been grouped together. This is done because a careful examination of the data showed that most pin-end columns in these tests behaved as flat-end columns up to the maximum load. A few pin-end columns, due to initial bends, acted more like round-end columns, giving a strength much less than the others, but still much higher than a truly pivoted end column. The number of such results is, however, so few, and the results are so erratic, that they have not been plotted. They will be discussed later. Figs. 7 and 8, therefore, represent results of tests on flat-end columns and on pin-end columns that were essentially flat ended. The points on the dia-

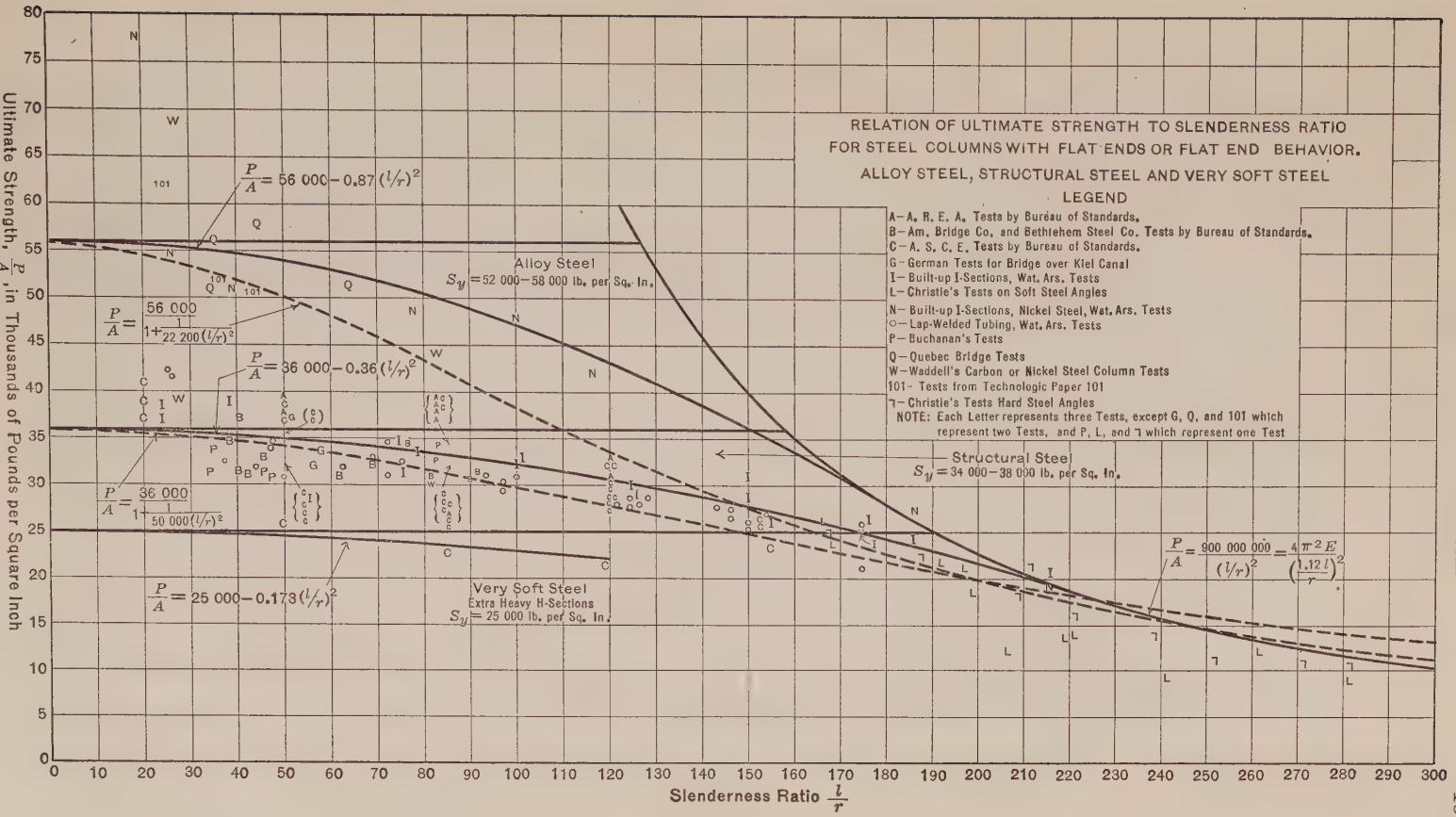

RELATION OF ULTIMATE STRENGTH TO SLENDERNESS RATIO
FOR STEEL COLUMNS WITH FLAT ENDS OR FLAT END BEHAVIOR.
ALLOY STEEL, STRUCTURAL STEEL AND VERY SOFT STEEL
LEGEND
A—A. R. E. A. Tests by Bureau of Standards.
B—Am. Bridge Co. and Bethlehem Steel Co. Tests by Bureau of Standards.
C—A. S. C. E. Tests by Bureau of Standards.
G—German Tests for Bridge over Kiel Canal
I—Built-up I-Sections, Wat. Ars. Tests
L—Christie's Tests on Soft Steel Angles
N—Built-up I-Sections, Nickel Steel, Wat. Ars. Tests
O—Lap-Welded Tubing, Wat. Ars. Tests
P—Buchanan's Tests
Q—Quebec Bridge Tests
W—Waddell's Carbon or Nickel Steel Column Tests
101—Tests from Technologic Paper 101
⌝—Christie's Tests Hard Steel Angles
NOTE: Each Letter represents three Tests, except G, Q, and 101 which represent two Tests, and P, L, and ⌝ which represent one Test
$\frac{P}{A} = 56\,000 - 0.87(l/r)^2$
Alloy Steel
S_y = 52 000–58 000 lb. per Sq. In.
$\frac{P}{A} = \frac{56\,000}{1 + \frac{1}{22\,200}(l/r)^2}$
$\frac{P}{A} = 36\,000 - 0.36(l/r)^2$
Structural Steel
S_y = 34 000–38 000 lb. per Sq. In.
$\frac{P}{A} = \frac{36\,000}{1 + \frac{1}{50\,000}(l/r)^2}$
$\frac{P}{A} = 25\,000 - 0.173(l/r)^2$
Very Soft Steel
Extra Heavy H-Sections
S_y = 25 000 lb. per Sq. In.
$\frac{P}{A} = \frac{900\,000\,000}{(l/r)^2} = \frac{4\pi^2 E}{\left(\frac{1.12\,l}{r}\right)^2}$
Ultimate Strength, $\frac{P}{A}$, in Thousands of Pounds per Square Inch
Slenderness Ratio $\frac{l}{r}$

FIG. 7.

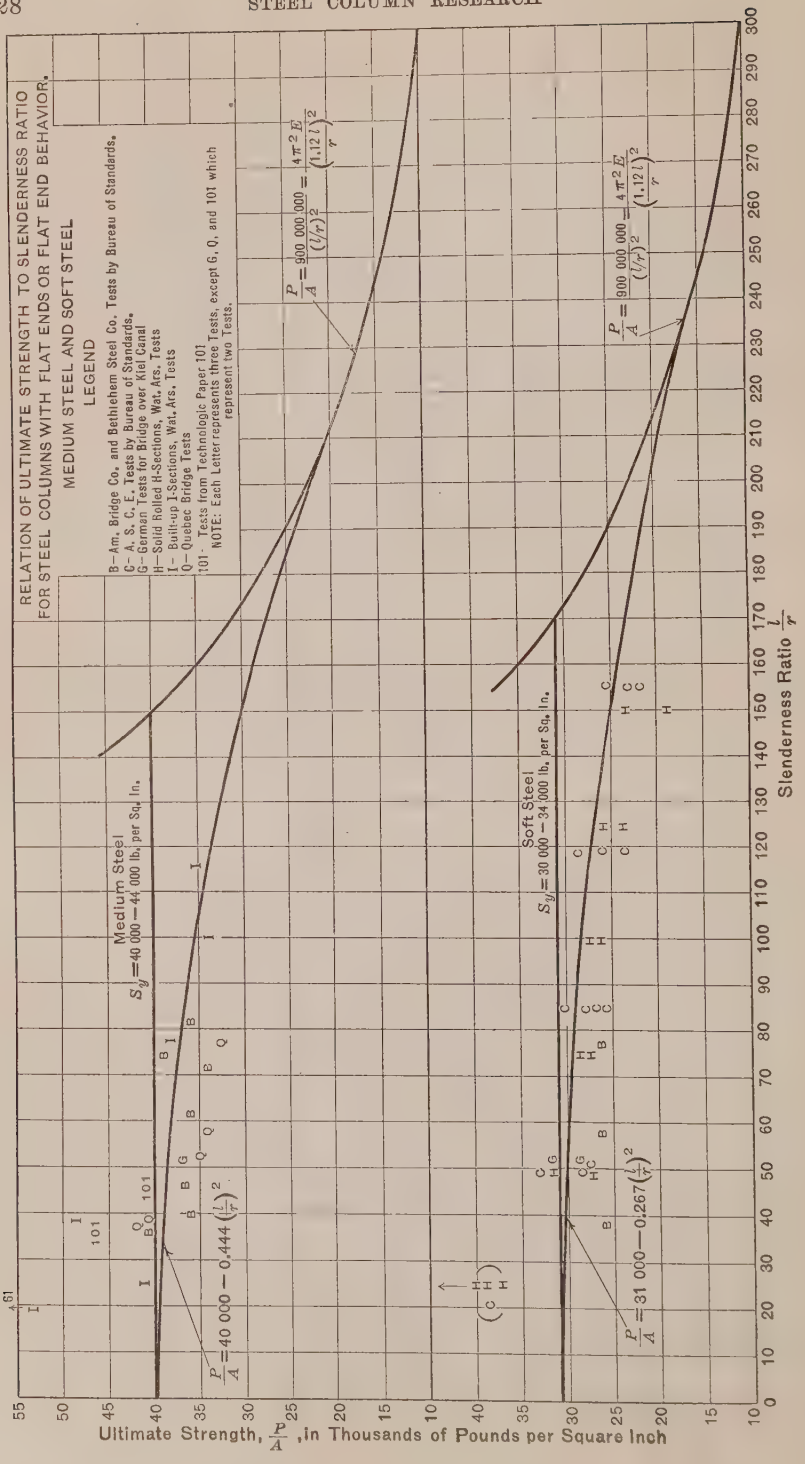

RELATION OF ULTIMATE STRENGTH TO SLENDERNESS RATIO
FOR STEEL COLUMNS WITH FLAT ENDS OR FLAT END BEHAVIOR.
MEDIUM STEEL AND SOFT STEEL
LEGEND
B—Am. Bridge Co. and Bethlehem Steel Co. Tests by Bureau of Standards.
C—A. S. C. E. Tests by Bureau of Standards.
G—German Tests for Bridge over Kiel Canal
H—Solid Rolled H-Sections, Wat. Ars. Tests
I—Built-up I-Sections, Wat. Ars. Tests
Q—Quebec Bridge Tests
101—Tests from Technologic Paper 101
NOTE: Each Letter represents three Tests, except G, Q, and 101 which represent two Tests.
Medium Steel
S_y=40 000—44 000 lb. per Sq. In.
Soft Steel
S_y=30 000—34 000 lb. per Sq. In.
$\frac{P}{A}=40\,000-0.444\left(\frac{l}{r}\right)^2$
$\frac{P}{A}=31\,000-0.267\left(\frac{l}{r}\right)^2$
$\frac{P}{A}=\frac{900\,000\,000}{(l/r)^2}=\frac{4\pi^2E}{\left(1.12\frac{l}{r}\right)^2}$
Ultimate Strength, $\frac{P}{A}$, in Thousands of Pounds per Square Inch
Slenderness Ratio $\frac{l}{r}$

FIG. 8.

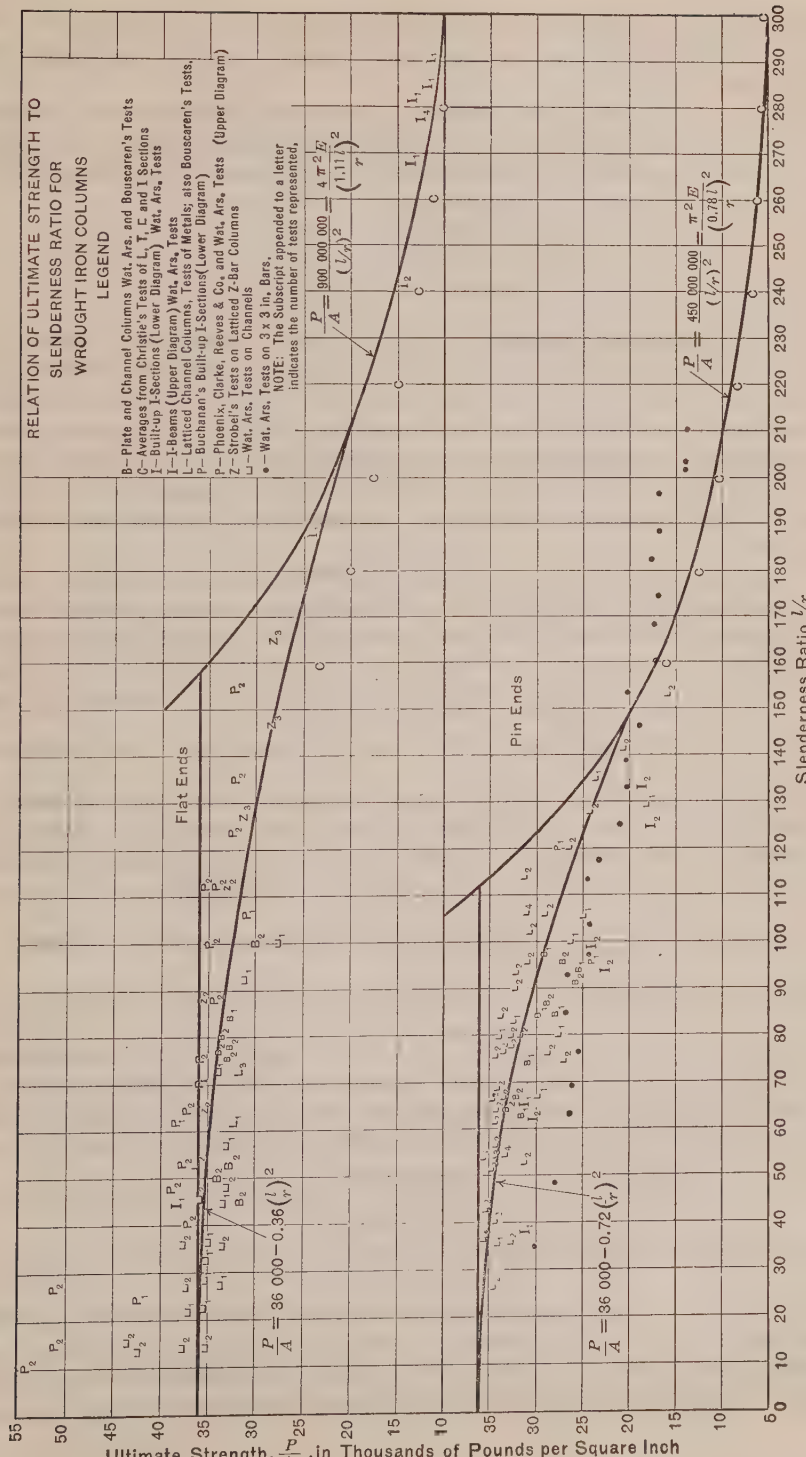

RELATION OF ULTIMATE STRENGTH TO SLENDERNESS RATIO FOR WROUGHT IRON COLUMNS
LEGEND
B– Plate and Channel Columns Wat. Ars. and Bouscaren's Tests
C– Averages from Christie's Tests of L, T, C and I Sections
I– Built-up I-Sections (Lower Diagram) Wat. Ars. Tests
I–I–Beams (Upper Diagram) Wat. Ars. Tests
L– Latticed Channel Columns, Tests of Metals; also Bouscaren's Tests.
P– Buchanan's Built-up I-Sections (Lower Diagram)
P– Phoenix, Clarke, Reeves & Co., and Wat. Ars. Tests (Upper Diagram)
Z– Strobel's Tests on Latticed Z-Bar Columns
L– Wat. Ars. Tests on Channels
• – Wat. Ars. Tests on 3 x 3 in. Bars.
NOTE: The Subscript appended to a letter indicates the number of tests represented.
Flat Ends
Pin Ends
$\frac{P}{A} = 36\,000 - 0.36\left(\frac{l}{r}\right)^2$
$\frac{P}{A} = 36\,000 - 0.72\left(\frac{l}{r}\right)^2$
$\frac{P}{A} = \frac{900\,000\,000}{(l/r)^2} = \frac{4\pi^2 E}{\left(\frac{1.11\,l}{r}\right)^2}$
$\frac{P}{A} = \frac{450\,000\,000}{(l/r)^2} = \frac{\pi^2 E}{\left(\frac{0.78\,l}{r}\right)^2}$
Ultimate Strength, $\frac{P}{A}$, in Thousands of Pounds per Square Inch
Slenderness Ratio l/r

Fig. 9.

grams do not, as a rule, represent individual tests, but the average of two or three results; but in all cases the range of values above or below the average is relatively small.

In order to relate the plotted results to some basis for discussion, a Euler curve has been drawn in Figs. 7 and 8 and a horizontal straight line representing an average value for the yield point of the group. The same Euler curve has been used for the groups, namely,

$$\frac{P}{A} = \frac{900\,000\,000}{\left(\frac{l}{r}\right)^2} = \frac{4\,\pi^2\,E}{\left(\frac{1.12\,l}{r}\right)^2}$$

This is the theoretical fixed-end formula for long columns for a value of l 12% longer than the actual length. This assumes the points of inflection to be 0.56 l apart instead of 0.5 l, as required by theory. As a matter of fact, this corresponds to the observed location of the points of inflection in certain tests described later. A somewhat lower curve would apparently fit Christie's flat-end tests shown in Fig. 7 somewhat better, but a very close agreement will be noted in Fig. 9, and the curve selected will answer well enough for present purposes.

In any effort to represent these results by a formula, it is apparent that such formula, either empirical or rational, should give values for short columns closely related to the tensile yield point strength of the material, which varies with different materials, and for long columns values approaching those given by the Euler law, and which is the same for all grades of material. If the yield point strength is to be taken as the limit for $\frac{l}{r} = 0$, the simplest empirical formula is perhaps the parabola as proposed by the late J. B. Johnson, M. Am. Soc. C. E., drawn tangent to the Euler curve. Such parabolas are shown in the diagrams. They fit the tests well, but the study of short columns, as shown in Plate X, indicates that this curve may not slope rapidly enough between $\frac{l}{r} = 0$ and $\frac{l}{r} = 100$. Curves of the Rankine form are also shown in Fig. 7, arbitrarily drawn through $\frac{P}{A} = 20$ for $\frac{l}{r} = 200$. For high values of yield point, the Rankine curve appears to fall off too rapidly. This is equivalent to saying that the eccentricity (including deflection effect) tacitly assumed in the Rankine formula, increases too fast for columns of moderate length and high elastic limit. For the structural steel (36 000 lb. yield point), the Rankine curve corresponds closely to the median line of Plate X for $\frac{l}{r}$ from 80 to 100, but is considerably higher for shorter lengths $\left(95\%\text{ of yield point for } \frac{l}{r} = 50\right)$.

Probably the simplest and most logical theoretical curve is the so-called secant formula, derived by assuming a certain initial eccentricity based on results of tests, and such a curve can be drawn to fit the tests quite well. Engesser and Karman have modified the Euler formula so as to take account

of the behavior of the metal beyond its elastic limit, but they assume perfectly central loads and thus obtain relatively high results for columns of moderate length. Their curves show an upward turn for very short columns, corresponding to many test results, but this point is hardly of practical value. If the high values for very short columns are considered, the test results can be represented very closely by single straight lines intersecting or tangent to the Euler curve, and if it is desired to exclude the higher values, then two straight lines can obviously be drawn, one horizontal and the other inclined, that will represent the tests within the range of practice about as well as any curve.

It is not proposed, however, here to suggest a working formula, but only to present the test records with some comparison with well-known types of formulas. The tests shown in Figs. 7 and 8 are for flat-end conditions, and can hardly be applied to columns in structures. This subject is reserved for discussion in a later report.

11.—Tests on Wrought-Iron Columns.—Fig. 9 shows results of tests on wrought-iron columns. Here the pin-end tests show a greater variation from the flat-end tests than in the case of the steel columns, and are plotted in a separate group, and a different Euler curve is drawn. Greater differences in straightness and in pin friction are doubtless the cause of so wide a variation in results. A straight line seems to fit these results as well as any curve.

*12.—Watertown Arsenal Tests on Steel Built-Up I-Columns.**—These tests included the variables given in Table 5.

TABLE 5.

Nominal tensile strength of metal, in pounds per square inch.	Number of columns tested.	Kind of ends.†	Range in $\frac{l}{r}$.‡	Dimensions, in Inches.				Nominal area, in square inches.	Least radius of gyration, in inches.
				Web-plate.	Four angles.	Section.			
						Width.	Depth.		
55 000 to 65 000	21	Flat	25 to 175	10 by ⅜	4 by 3 by ⅜	8⅜	10¼	13.7	1.65
55 000 to 65 000	21	Pin	25 to 175	10 by ⅜	4 by 3 by ⅜	8⅜	10¼	13.7	1.65
90 000*	6	Flat	25 and 100	10 by ⅜	4 by 3 by ⅜	8⅜	10¼	13.7	1.65
70 000	6	Flat	19 to 214	6 by ⅜	2½ by 2½ by ⅜	5⅜	6¼	9.17	1.05
60 000	15	Pin	19 to 116	6 by ⅜	2½ by 2½ by ⅜	5⅜	6¼	9.17	1.05
70 000	12	Pin	19 to 116	6 by ⅜	2½ by 2½ by ⅜	5⅜	6¼	9.17	1.05
90 000*	17	Pin	19 to 214	6 by ⅜	2½ by 2½ by ⅜	5⅜	6¼	9.17	1.05

* Nickel steel.

† Where pins were used; they were placed with their axes in the plane of the column web.

‡ Based on length between centers of pins. Pins were 3 in. in diameter.

The results of these tests are plotted in Fig. 10, the points representing generally the average of three tests.

Here, again, it will be observed that the end conditions affected the strengths very little. The pin-end columns of 60 000-lb. steel with a slenderness ratio of 25, were slightly stronger than the corresponding flat-end columns; with $\frac{l}{r}$ greater than 125, the pin ends were somewhat weaker than the flat ends.

* Report on Tests of Metals and Other Materials, 1909–14.

Considering columns of 60 000-lb. steel, it appears that:

(*a*).—Flat-end columns with $\frac{l}{r} = 25$ failed by buckling of the flanges. Longer members failed by triple flexure in a plane normal to the web-plate. Sudden springing laterally occurred in tests of columns with $\frac{l}{r} = 125$, or more, when the maximum load was reached.

(*b*).—The larger sections with pin ends and $\frac{l}{r} = 25$ failed by buckling of the flanges; with $\frac{l}{r} = 50$, by triple flexure and buckling of the flanges; with $\frac{l}{r}$ greater than 50, by deflection normal to the web. The smaller sections with pin ends and $\frac{l}{r} = 19$ failed by buckling of the web and flanges; with $\frac{l}{r} = 39$ or greater, by deflection normal to the pins. Pin-end columns with $\frac{l}{r} = 100$, or greater, exhibited sudden lateral springing at the maximum load.

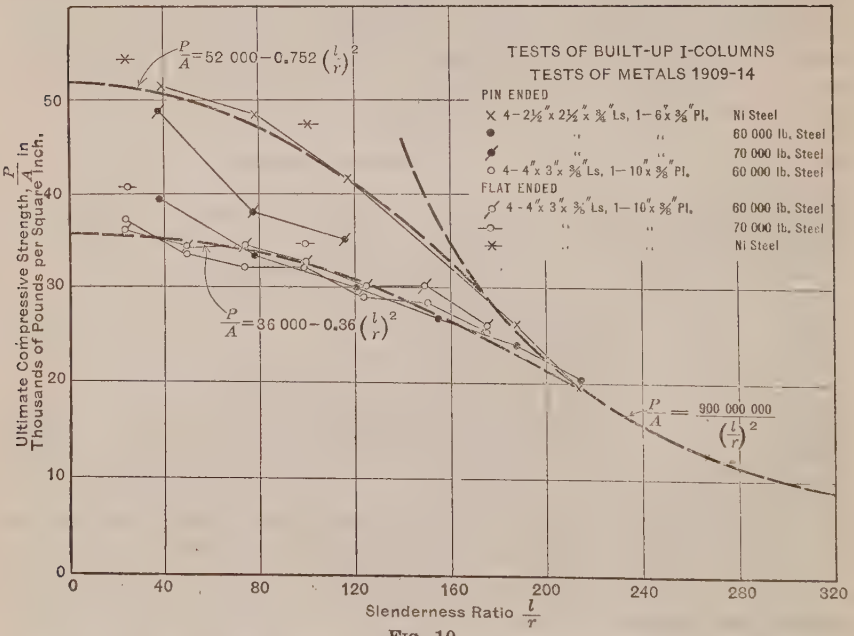


FIG. 10.

(*c*).—As in the tests on tubes and solid-rolled H-columns, so in these tests the pin friction was sufficient in most cases to prevent rotation of the ends of the columns under inherent eccentricities of load. Certain of the longer pin-end columns had initial bends which had an effect sufficient to offset the pin friction.

From the diagram (Fig. 10) it will be noted that the strength of the short lengths, $\frac{l}{r} = 25$ or less, did not run as high as in some other forms. For these short lengths, failure occurred by buckling of the flanges. The ratio of unsupported width to thickness was 10.7 : 1. For purposes of comparison, there have been drawn, on the diagram, the same Euler curve as in Fig. 7, and also tangent parabolas applicable to the 60 000-lb. steel and the nickel steel. The results on the 70 000-lb. steel are not sufficiently numerous to warrant separate consideration. The manner in which the strength of the nickel-steel column approaches that of the structural steel at high values of $\frac{l}{r}$, is quite striking.

13.—Tests on Solid Rolled H*-Columns.**—Results of tests on eighteen solid-rolled H-sections with flat ends and a like number with pin ends are given in the two reports referred to. These H-sections had flanges 6 in. wide, varying in thickness from $\frac{7}{16}$ in. at the edges to $\frac{1}{2}$ in. near the webs. They were 6 in. deep, and had webs $\frac{1}{4}$ in. thick. The area of cross-section was 7 sq. in., and the least radius of gyration was 1.54 in. Tests were made in triplicate for six lengths, ranging between 3 ft. $2\frac{1}{2}$ in. and 19 ft. 3 in. The pins were 3 in. in diameter.

Tensile tests on four specimens cut from the flanges of these members gave an average yield point of 30 140 lb. per sq. in. and an ultimate strength of 56 965 lb. per sq. in. Two web specimens gave an average elastic limit of 31 480 and an ultimate strength of 54 920 lb. per sq. in.

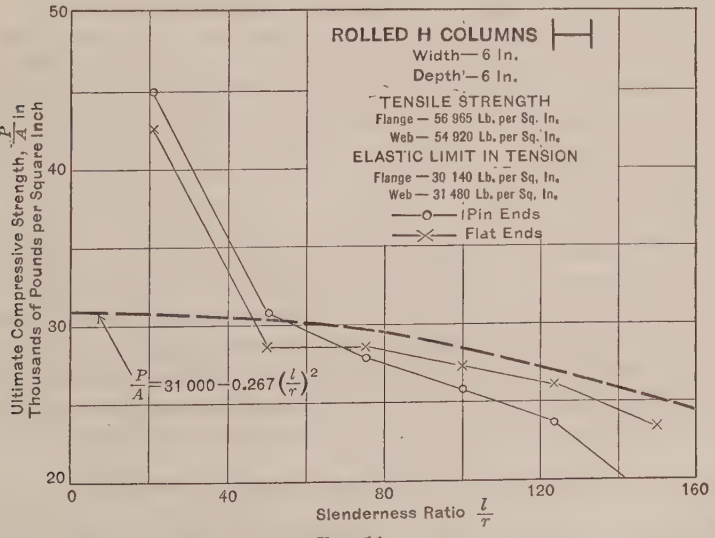


Fig. 11.

The results of these tests are shown in Fig. 11, together with a reference parabola corresponding to the yield point of the material. In this case the

* Reports on Tests of Metals and Other Materials, Watertown Arsenal, 1908–09.

strength of the pin-end columns for $\frac{l}{r} = 100$ or more was considerably less than those with flat ends. For $\frac{l}{r} = 25$ and 50, the pin ends showed the better result, probably due to a more central application of load. Several of the pin-end columns of $\frac{l}{r} = 100$ and 150 had considerable initial bends, in one case as high as 0.42 in. (omitted from the diagram), which reduced their strength materially and prevented flat-end behavior. The data on bends are discussed later. It will be noted that the values for $\frac{l}{r} = 25$ show in a marked degree the excess strength for very short columns, and quite unlike the results shown in Fig. 10. This difference is probably due in large measure to the thicker flange in the H-section, the ratio of unsupported width to the thickness being about 6, as compared to 10.7 in the case of the weaker built-up I-sections of Fig. 9.

*14.—Tests on Bessemer Steel Lap-Welded Tubing.**—One hundred and six tests on tubing, 5 in. in outside diameter, with a nominal thickness of 0.36 in., nominal area of cross-section of 5.25 sq. in., and radius of gyration of 1.645 in., were made at the Watertown Arsenal. The lengths of the specimens ranged from 1 ft. $8\frac{9}{16}$ in. to 24 ft. The slenderness ratio varied from 12.5 to 175. Most of these tubes had local bends at points along their length. Four types of end condition were used in these tests: Flat, fixed, pin, and spherical.

In the fixed-end tests, the tubing was secured at the ends in special castings which terminated in bases 18 in. in diameter, or they were clamped in the jaws of the testing machine. The radius of curvature for the spherical ends was 2.75 in.; for the seats, it was 3.00 in. The diameter of the pins used in the pin-end tests was 3.00 in.; the diameter of the seats for the pins was $3\frac{1}{16}$ in. In nearly all cases columns were tested in triplicate; six were tested in the flat-end series with $\frac{l}{r} = 25$ and with $\frac{l}{r} = 50$. Tensile tests on three specimens cut from these tubes gave an average elastic limit of 36 250 lb. per sq. in. and an average ultimate of 58 310 lb. per sq. in.

The results of these tests are shown in Fig. 12. Here, again, there is very little difference in results for different end conditions, except for the very short lengths, where the pin and spherical ends give the best results, and for the longest specimens, where the flat and fixed ends show best. Several curves are shown in this diagram: The formula (upper curve) proposed by Lilly† for steel tubing; a modified Lilly formula; a parabola, $\frac{P}{A} = 35\,000 - 0.44\left(\frac{l}{r}\right)^2$; and the same parabola and Euler curve as shown in Fig. 7. The Lilly formula is of the Rankine type with special constants. A parabola, or, within certain limits, a straight line, appears to fit these tests satisfactorily.

* Reports on Tests of Metals and Other Materials, Watertown Arsenal, 1908–09.

† *Transactions*, Am. Soc. C. E., Vol. LXXVI (1913), p. 258.

*15.—Strobel's Tests on Z-Bar Iron Columns.**—These consisted of tests on fifteen flat-end columns varying in sectional area from 9 to 10 sq. in. The average yield point of material was 34 200 lb. per sq. in. All columns failed as a unit and the results of tests in duplicate and triplicate were concordant. They are shown in Fig. 13 with the same parabola as that used in Fig. 7. Considering the relatively low yield point, the results may be regarded as rather superior to those obtained on some of the other forms.

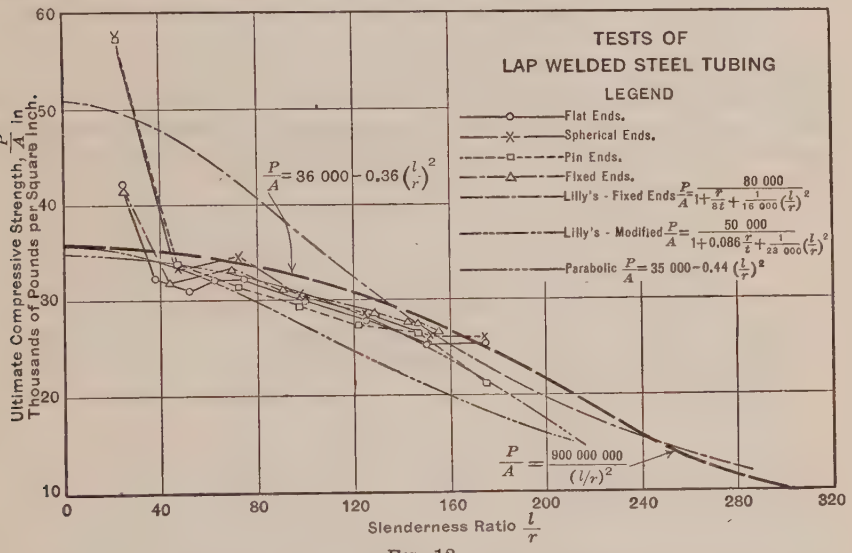


FIG. 12.

16.—Tests on Phœnix Iron Columns.†—These comprise twenty-two tests of flat-end columns of 12 to 18 sq. in. sectional area. No report on the metal composing these columns was made. The results are shown in Fig. 14 and are uniformly good.

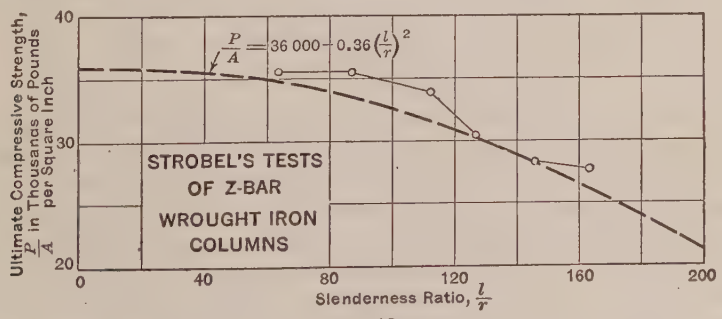


FIG. 13.

17.—Tests on 3-Inch by 3-Inch Square Wrought-Iron Columns.‡—These tests are shown by the solid circles on Fig. 9, pin-end group. The material

* *Transactions*, Am. Soc. C. E., Vol. XVIII (1888), p. 103.

† *Loc. cit.*, Vol. XI (1882), p. 1, and Report on Tests of Metals and Other Materials, Watertown Arsenal, 1881.

‡ Report on Tests of Metals and Other Materials, Watertown Arsenal, 1883.

was of rather low elastic limit (24 000 lb. per sq. in.), and the pins relatively large, 1½ in. in most cases. The result is, for the short lengths, low values compared to other tests on wrought iron; and for the long lengths, rather high values due to resistance to turning on the pins. As a matter of fact, the group of test results can be represented very closely by a single straight line, which, however, is not of much significance in a consideration of the common type of structural column.

The effect of size of pin on the strength of the column is brought out in these tests by a series in which the pin size varied from ⅞ in. to 2¼ in., and $\frac{l}{r} = 139$. The ultimate strength increased steadily with diameter of pin, from 16 300 to 22 250 lb. per sq. in., the latter value being about 92% of the elastic limit of the material. This is a good illustration of the effect of pin friction in tending to fix the ends of columns.

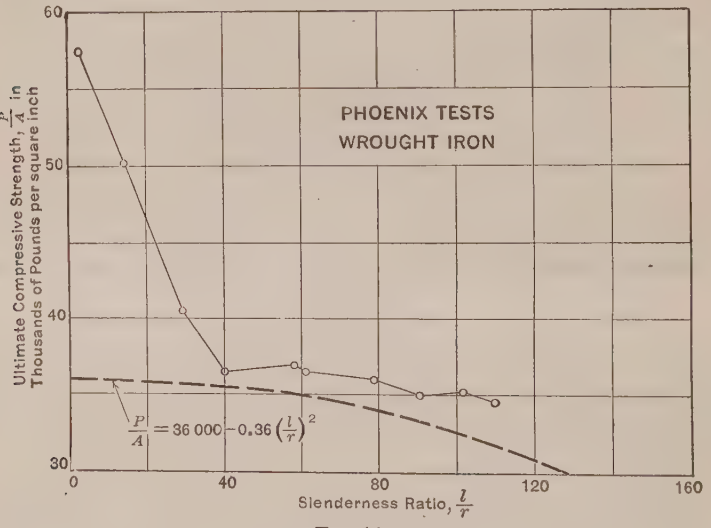


FIG. 14.

*18.—Christie's Test on Wrought Iron and Steel Struts.**—These tests were made on small shapes, generally less than 2 sq. in. in section. The ends were flat, fixed by clamps, hinged and spherical for the wrought iron, and flat only for the steel. In the diagrams of Figs. 7 and 9, only the longer lengths are used, as these are thought to be of some value in reference to failure by deflection. In the case of the longer flat-end tests, it is reported that generally the ends left their bearings at one edge before failure, and in some cases before the maximum load was reached. This action indicated a further deviation from a fixed-end condition than in the case of the tests on larger sections plotted in the diagrams. The flat-end tests, however, have been plotted on Fig. 7, and the fixed and hinged-end tests on Fig. 9. The results are relatively low except in the hinged-end group of Fig. 9.

* *Transactions*. Am. Soc. C. E., Vol. XIII (1884), pp. 253, 285.

(C) Effect of Various Details on Strength of Columns

1.—Inherent Eccentricities and Initial Bends

19.—Some information on this subject may be obtained from some of the test data, but many of the older tests give little or none except what may be inferred from the deflections produced by loads. Eccentricity may be due to bends in the column; to eccentrically applied loads; to irregularities of cross-section; or to unequal elasticity of the column components.

Where loads are presumed to be centrally applied, there may be in effect considerable eccentricity, especially in flat-end tests, where unequal bearings would have the effect of eccentric application of loads. Initial bends will also result in eccentricity of load with respect to some parts of the column. The best information available from the tests are the strain measurements made at different points along the columns, especially near the center, and considerable data to that effect are given later. These data are of some value in giving information regarding the applicability of various column formulas, but it is to be noted that in many cases the columns have been made with greater care than is required in ordinary commercial practice.

20.—Calculation of Eccentricity and Bending Moment from Strain Measurements.—Eccentricities with respect to the two principal axes have been calculated by the formulas:

$$e_m = \frac{V}{X}\frac{r_m^2}{c_m}, \text{ and } e_n = \frac{W}{X}\frac{r_n^2}{c_n} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

in which, e, r, and c represent, respectively, eccentricity, radius of gyration, and distance to remotest fiber, the subscript, m, denoting the axis with respect to which the radius of gyration is a maximum, and the subscript, n, the axis with respect to which the radius of gyration is a minimum; V equals the difference in the sums of the deformations of the two sets of strain measurements at a distance, c_m, from the m-axis; W equals the same with respect to the n-axis; and X equals the sum of deformations of the four remotest fibers.

Bending moments are expressed in terms of the ratio of maximum fiber strain due to bending, to average direct strain. Assuming a constant value of E across the section, this ratio would be equal to the ratio of stresses, and has been so indicated. They have been calculated from the formula:

$$\frac{\text{Bending stress}}{\frac{P}{A}} = \frac{V + W}{X} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

It will be noted that $\frac{V}{X}\frac{P}{A}$ represents bending stress in the direction of maximum moment of inertia, and $\frac{W}{X}\frac{P}{A}$, the bending stress at right angles.

21.—Eccentricities and Bending Stresses in the Am. Soc. C. E. and A. R. E. A. Columns.—Very careful strain measurements were made in these tests, which have been studied from various aspects. An especial effort has been made to determine effect of difference of form in the behavior of the

different columns. It is important also to study the effect of slenderness ratio. Fig. 15 gives the values of eccentricities calculated in the manner already described for a value of $\frac{P}{A}$ (average unit stress) of 15 000 lb. per sq. in., and for all the various types of columns. The light lines represent values for the light sections and the heavy lines for the heavy sections. In most cases the values represent the average of three columns. The average of all values is given in the first set of lines.

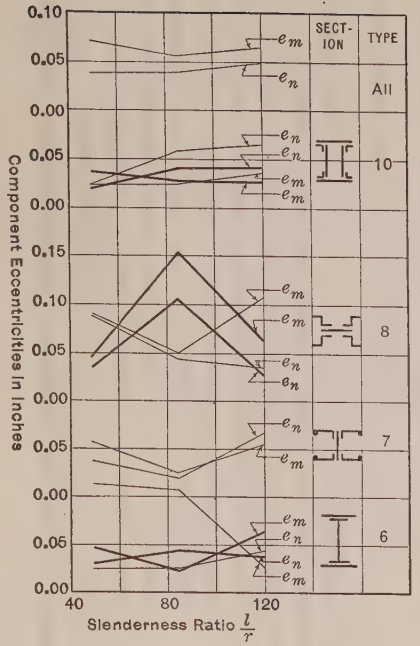
It will be noted that the value of the eccentricity at 15 000 lb. per sq. in. is not appreciably affected by the slenderness ratio. Few of the eccentricities exceed 0.1 in. Of perhaps more significance are the bending stress ratios, as these values include not only the effect of eccentricity, but also take account of variation in r and c.

Fig. 16 shows bending stress ratios averaged for each type of column. Values for all slenderness ratios of 50, 85, and 120 are averaged for this purpose, little variation being due to this factor. Results are given for $\frac{P}{A} = 15\,000$ lb. per sq. in., and also for $\frac{P}{A}$ = highest elastic load. The diagrams show no very consistent variation, although Types 2 and 5 are generally good, and Type 8 is generally poor. Type 1, which shows up rather poorly in Fig. 6, does not reveal large eccentricities. Considering the possible effects of inaccurately fitted end bearings and the relatively small bending stresses (average about 5% at $\frac{P}{A} = 15\,000$), these results are not of great significance in attempting to estimate effect of form.

22.—Eccentricity of Built-Up I-Columns, Watertown Arsenal Tests, 1909-14.—In this series, 33 tests were made with flat ends, and 65 with pin ends. In most cases the end conditions affected the strength but little, the pin friction being sufficient to give the effect of flat ends. In a few cases the initial deflection or bend was sufficient to offset the pin friction and greatly to decrease the strength of the column. The data (Table 6) from six columns of the same length $\left(\frac{l}{r} = 175\right)$ and section show how a rather small eccentricity affected the resistance of certain long columns of this series (60 000-lb. steel).

If the ends of the columns shown in Table 6 had been unrestrained, the Euler load which would have produced failure by lateral bending is 9 370 lb. per sq. in. (assuming $E = 29\,000\,000$ lb. per sq. in.).

These data show that an initial bend of small amount caused a marked reduction in the strength of the pin-end columns, but produced little effect on the strength of the flat-end columns. Apparently, the pin friction was sufficient to prevent Column No. 2036, which was reasonably straight, from rotating at the ends when the Euler load for unrestrained ends was applied. With the load nicely centered, it carried more than the flat-end struts of the



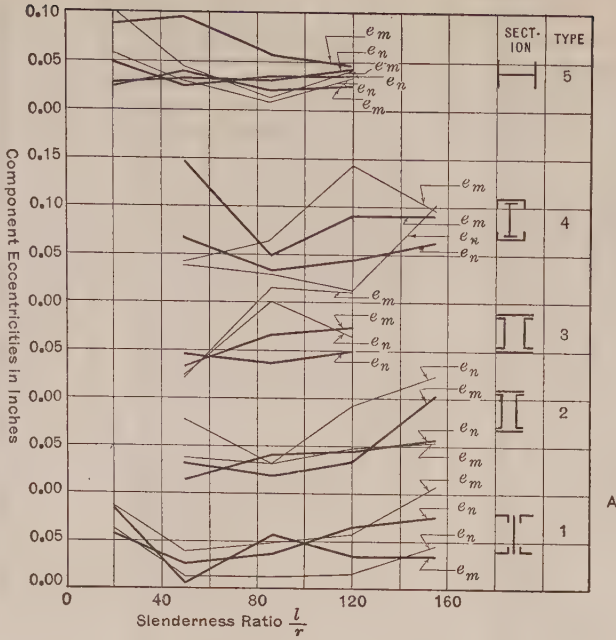
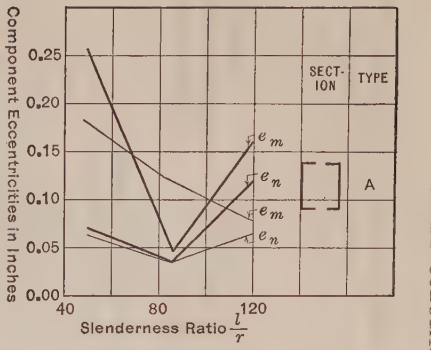


FIG. 15.



COMPONENT ECCENTRICITIES
IN COLUMNS OF
AMERICAN SOCIETY OF CIVIL ENGINEERS
AND
AMERICAN RAILWAY ENGINEERING ASSOCIATION
SERIES AT
$\frac{P}{A}$ = 15 000 LB. PER SQ. IN.

same length and nearly three times the Euler load. On the other hand, Column No. 2035, with an initial bend of 0.15 in. (about 0.09 r), failed at a load less than one-half as large as that sustained by Column No. 2036, and only 40% larger than the Euler load.

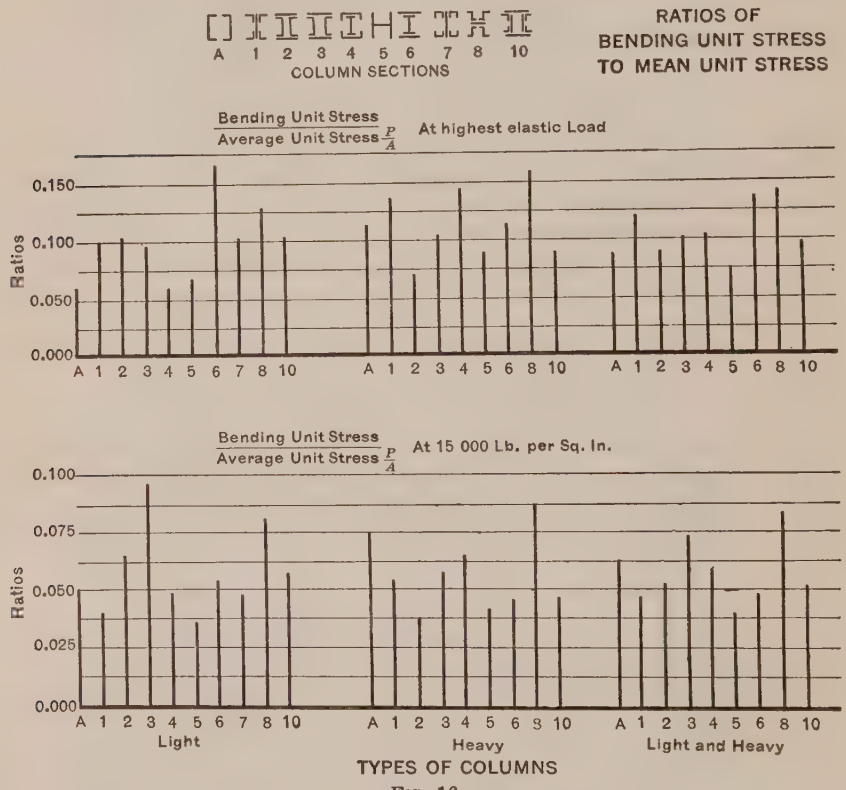


FIG. 16.

Tests of three columns—Nos. 14911-14913* (sectional area, 9 sq. in.; least $r = 1.05$ in.; $\frac{l}{r} = 186$)—are especially interesting, as strain measurements were made along the entire length of the column. The results of the gauge readings showed that two of the columns had two inflection points, the distance between these points being about 55% of the column length, corresponding closely to truly fixed-end columns. The other showed three inflection points. These gauge readings also furnished information regarding eccentricity of load throughout the column and the effect of such eccentricity upon the ultimate strength values. Table 7 gives the calculated eccentricities at the ends and center of these columns and observed deflections for loads of 12 000 and 21 000 lb. per sq. in. The values are in inches.

* Report on Tests of Metals and Other Materials, Watertown Arsenal, 1912.

It will be noted from Table 7 that the calculated eccentricities at $\frac{P}{A} = 12\,000$ are considerably greater than the actual deflections, but at $\frac{P}{A} = 21\,000$ they

TABLE 6.—Effect of Initial Bend on Column Strength.

Column No.	Kind of ends.	Deflection at Center, in Inches.				Ultimate strength, in pounds per square inch.
		Initial.		Increment in deflection under $\frac{P}{A} = 5\,000$ lb. per sq. in.		
		Parallel to pins.	Normal to pins.	Parallel to pins.	Normal to pins.	
2035	Pin	0.14	0.15	0	0.10	13 130
2037	"	0	0.15	0	0.08	23 200
2036	"	0	0	0	0.01	27 010
2032	Flat	None noted		0	0	24 730
2033	"	"	"	0	0.01	26 650
2034	"	0.12	0.10	0	0	26 720

are nearly the same. This is doubtless due to the effect of initial bends and eccentric application of load, which would have more influence in the calculated eccentricities at low loads, than in the observed deflections. In Column No. 14913, the eccentricity of 0.235 at the center gives a value of $\frac{e\,c}{r^2}$ of 0.57, which means a bending fiber stress of 57% of the direct stress, or at $\frac{P}{A} = 21\,000$, the bending stress was 12 000 lb. per sq. in. The ultimate compressive strengths of these columns were, respectively, 24 000, 26 200, and 21 500 lb. per sq. in.

TABLE 7.—Comparisons of Calculated Eccentricities and Observed Deflections at Mid-Length for Columns Nos. 14911, 14912, and 14913.

(Report on Tests of Metals and Other Materials, 1912.)

Column No.	At $\frac{P}{A} = 12\,000$ Lb. per Sq. In.				At $\frac{P}{A} = 21\,000$ Lb. per Sq. In.			
	Mean eccentricity* at		Sum, $e_0 + e_c$.	Observed deflections.	Mean eccentricity* at		Sum, $e_0 + e_c$.	Observed deflection.
	Ends, e_0	Center, e_c.			Ends, e_0.	Center, e_c.		
14911	+0.040	−0.035	0.075	0.04	+0.084	−0.096	0.180	0.17
14912	−0.012	+0.023	0.035	0	−0.004	+0.014	0.018	0.02
14913	−0.085	+0.086	0.171	0.11	−0.237	+0.235	0.472	0.43

* Mean eccentricities are the algebraic averages of values for Gauges 1 and 18 at the ends, and Gauges 8, 9, and 10 at the middle, of the columns.

23.—Initial Bends of H*-Sections, Watertown Arsenal Tests (1909).*—Many of the pin-end columns had initial bends of considerable amounts, the maximum being 0.42 in., several being from 0.15 to 0.20 in. In tests of columns with $\frac{l}{r} = 100$ and 150, the maximum loads per square inch and the initial deflection normal to the pins were as given in Table 8.

TABLE 8.—Effect of Initial Bends on H-Columns.

Column No.	$\frac{l}{r}$.	Maximum load, in pounds per square inch.	Initial deflection, in inches.
1986	100	24 000	0.20
1987	100	26 000	0.15
1988	100	27 000	0.08
1980	150	20 360	0.15
1981	150	17 430	0.16
1982	150	10 100	0.42

By reference to Fig. 7, it will be seen that Column No. 1988, with initial deflection of 0.08 in., gave a result not greatly below the flat-end group, but the results from the best of the columns with $\frac{l}{r} = 150$ fell much below that group. The effect of the large deflection of 0.42 in. is evident in the low value for the ultimate of 10 100 lb. per sq. in. Applying the secant formula for eccentric loads to the column,

$$S = \frac{P}{A}\left(1 + \frac{e\,c}{r^2}\sec\left(\frac{l}{2\,r}\sqrt{\frac{P}{A\,E}}\right)\right)$$

and using $\frac{P}{A} = 10\,100$ lb. per sq. in., $e = 0.42$ in., $c = 3$ in., $E = 30\,000\,000$ lb. per sq. in., and $r = 1.54$ in., the value, $S = 38\,200$ lb. per sq. in., is derived as the extreme fiber stress, with an average load of 10 100 lb. per sq. in.—a value about equal to the yield point of the material. The additional deflection at failure was 1.74 in., giving a central eccentricity of $1.74 + 0.42 = 2.16$ in., and,

$$S = \frac{P}{A}\left(1 + \frac{e\,c}{r^2}\right) = 37\,700 \text{ lb. per sq. in.}$$

a close agreement with the secant formula. The theoretical Euler load on this column for hinged ends is 13 000 lb. per sq. in. It is seen, therefore, that whereas this large initial deflection of 0.42 in. greatly weakened the column as compared to straighter columns, the reduction below the theoretical Euler load for strictly hinged-end columns was not nearly as great. This illustration shows clearly the restraining effect of pin friction in the normal pin-end columns.

24.—Eccentricities of Columns Reported in Technologic Paper No. 101.—Strain gauge readings in this series of tests give some information on the eccentricities observed in testing large columns. Table 9 gives calculated

values of eccentricity, e, and eccentric ratio, $\frac{e\,c}{r^2}$, for several of these columns, and at an average stress of 30 000 lb. per sq. in. All these columns were of symmetrical cross-section, consisting of built-up I or channel-sections connected by a central web and lacing.

As shown in *Technologic Paper No. 101,* of the U. S. Bureau of Standards, the eccentricities in general decreased with increasing load up to 30 000 lb. per sq. in., at least, thus showing that the eccentricities were primarily due to imperfect bearings in the testing machine rather than to bends or inherent eccentricities. The eccentric ratios correspond to the proportionate bending fiber stress, the maximum value of which in Table 9 is 19%, the next highest being 10 per cent. These values range about the same as in the series of tests made by the Society's Special Committee on Steel Columns and Struts.

TABLE 9.—Eccentricities of Columns, Technologic Paper No. 101.

Column.	Area, in square inches.	r perpendicular to pin.	$\frac{l}{r}$.	Eccentricity, e, in inches.	Eccentric ratio, $\frac{e\,c}{r^2}$.
H. C.	74.6	6.73	43.5	0.74	0.19
M. Y.	74.9			0.35	0.09
31 HC	76.9			0.39	0.10
32 S.	76.9			0.40*	0.05*
34 CS.	76 9			0.09	0.02
HC 1.	75.3	7.60	38.5	0.04	0.01
MY 1.	75.3			0.12	0.03
31 HC1.	77.2			0	....
32 S1.	77.2			0.42	0.10
34 CS1.	77.2			0.20	0.05

* Parallel to pin. All others perpendicular to pin.

25.—Other Studies of Eccentricity.—Several studies have been made of the question of eccentricity of loading and that due to bends and irregularities in section, value of E, etc., by many investigators. These studies are concisely summarized by Salmon,* and show a great variety of results, most of them much larger than those found from a study of the latest experimental results determined by strain gauge measurements. The values given are, in general, what may be called inferred eccentricities, determined in some cases by deflection measurements, in others by fitting curves from eccentric-load column formulas to results of tests; and in still others by a detailed consideration of practical conditions in manufacture. The total combined eccentricity due to all causes, expressed as the eccentric ratio, $\frac{e\,c}{r^2}$, ranges from as high as 0.60 to as low as 0.06. Morris† calculated values from deflections in Buchanan's tests of $\frac{e\,c}{r^2}$ from 0.01 to 0.20, averaging 0.07 (perpendicular to

* "Columns", by E. H. Salmon, pub. by Frowde and Hudder & Stoughton, London, England.

† *Engineering News,* Vol. 66.

pin) and much larger values parallel thereto. Prichard* suggests a value, $\frac{e\,c}{r^2} = 0.1 + \frac{l}{700\,r}$, and Basquin† suggests a value, $\frac{e\,c}{r^2} = 0.1 + \frac{l}{1000\,r}$, but this is based on existing column formulas, and not on tests. Salmon has plotted values of initial deflections, or bends, of many test columns, and shows that very few have bends greater than $e = \frac{l}{750}$. These suggested values are considerably greater than those calculated from strain gauge measurements given herein. Comparing them with the values for H-columns mentioned in Paragraph 23, Basquin's formula would give for $\frac{l}{r} = 100$, $\frac{e\,c}{r^2} = 0.20$,. or $e = 0.16$ in., and for $\frac{l}{r} = 150$, $\frac{e\,c}{r^2} = 0.25$, or $e = 0.20$ in. Salmon's suggestion for initial bends would give for $\frac{l}{r} = 100$ ($l = 154$ in.), $e = \frac{154}{750} = 0.20$ in., and for $\frac{l}{r} = 150$ ($l = 230$ in.), $e = \frac{230}{750} = 0.30$ in.

26.—Eccentricities Shown by Relation of Ultimate Strength to Yield Point of Relatively Short Columns.—If the average yield point strength of the material is taken as a basis, some information can be deduced as to equivalent eccentricities from the data in Plate X. Taking no account of the effect of deflection on the stress, the maximum reduction of strength is 25% for $\frac{l}{r} = 100$. This would correspond to an increase of fiber stress due to eccentricity of $\frac{25}{100 - 25} = 33\frac{1}{3}\%$, or $\frac{e\,c}{r^2} = 0.33\frac{1}{3}$ per cent. However, at this length, the effect of deflection is considerable. The line, AB (Plate X), represents the result of applying an eccentric load formula in which $\frac{e\,c}{r^2}$ is taken at $0.1 + 0.001\,\frac{l}{r}$, and the maximum stress, S, at 36 000 lb. per sq. in. Comparatively few results lie below this line, and none of the tests made by the American Railway Engineering Association or the Society's Special Committee on Steel Columns and Struts. As far as these test results go, such a value, therefore, appears to be conservative, even for built-up members of many elements, and includes necessarily the effect of bends, eccentricity of load, variation in value of E, and all other factors. It represents rather a near approach to a maximum than an average effect.

27.—General Conclusion Regarding Eccentricities.—In the eccentricities calculated by the different methods, the various factors have in effect been lumped together in one result. In the case of flat-end tests, an important element is the adjustment of the specimen in the testing machine, or, as it may be called, eccentricity of loading. This information is an aid in judging the significance of test results, but is of little practical value in estimating

* *Transactions*, Am. Soc. C. E., Vol. LXI (1908), p. 174.
† *Journal*, Western Soc. of Engrs., Vol. 18, p. 543.

eccentricities in the column as used in the structure. Eccentricity of the actual loading, at least for any but pin-ended columns, must be determined by a study of the structure and not from testing machine results.

The other principal causes of eccentricity—crookedness and variation of size and elasticity of the elements—are important, and the tests can be considered as giving some information on this point, if indeed somewhat negative in character. That is, the combined effects of these factors can hardly be greater than the total as shown by the tests, as this would be the result with loads perfectly centered. Referring to Fig. 15, it will be seen that the value of e is seldom more than $\frac{1}{10}$ in., and is little affected by length. Considering this to be due entirely to initial bends, it would correspond, (considering the column as fixed) to a bend of 0.2 in., a considerably smaller value than that indicated by most estimates. These columns, however, were fabricated with especial care and were doubtless straighter than the average column.

Of all the factors contributing to eccentricity in the column in most modern structures and inherent in the column itself, it would appear that crookedness is the most important and the only element capable of direct determination. On this subject, information is very scarce, and what is available relates mostly to columns built for testing purposes. It would appear quite possible and feasible to obtain valuable information on this point by numerous careful measurements made in structural shops and on existing structures. This is a line of work well worth consideration. Eccentricity of load on riveted and flat-ended columns is a function of the entire structure, and has little relation to testing machine eccentricity. The column test can give useful information on the effect of crookedness, variation in material, form, and details, and for this purpose, it would appear that the loading should be applied centrally, or with a definite known eccentricity. From such tests, column formulas can be derived suitable for centrally applied loads, or for a certain assumed eccentricity as determined by a study of the action of the column as a part of the structure.

2.—The Unsupported Length of Column Components

28.—Sources of Information.—Although there is no comprehensive series of tests dealing with the effect of the unsupported length of components on the strength of built-up columns, considerable information concerning this subject is obtainable from the tests on wrought-iron columns of the box type reported in Tests of Metals and Other Materials (1881 and 1884), also, from the tests of Bouscaren, Buchanan, the American Railway Engineering Association, Talbot and Moore, and *Technologic Paper No. 101,* U. S. Bureau of Standards. These tests give some information relating to the effect of the unsupported length of cover-plate in channel-plate columns and the unsupported length of channels or ribs; also, some data concerning the effect of unsupported length in segmental columns of the older types.

29.—Unsupported Length of Cover-Plates.—Table 10 has been prepared from tests of wrought-iron columns reported in Tests of Metals and Other Materials (1884). As the pin diameters were 3½ in. and the deflections normal

TABLE 10.—Effect of Unsupported Length of Cover-Plate on Strength of Columns of Box Section Consisting of Two Channels and Two Plates.

(Report on Tests of Metals and Other Materials, 1884.)

Column No.	Length, in inches.	$\frac{l}{r}$ (least).	End conditions.	Maximum unit load, $\frac{P}{A}$, in pounds per square inch.	Calculated maximum fiber stress, in pounds per square inch.	Rivet pitch, p, in inches.	Thickness of plates, t, in inches.	Ratio: $\frac{p}{t}$.	Ratio: unsupported width to thickness.	Kind of failure.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Two 6-Inch Channels with Two 10 by ¼-Inch Plates.										
377	127.9	46	Flat	31 700	32 600	6	0.26	23	31.8	Buckling
378	127.9	46	"	33 600	34 400	....	0.25	24	32.0	"
367	180	65	Pin	32 300	34 100	....	0.27	22	29.6	Deflection
368	180.1	65	"	33 200	35 800	....	0.28	21	28.6	"
356	240	87	"	29 900	34 900	....	0.26	23	31.8	"*
357	240	87	"	29 200	32 400	....	0.26	23	31.8	"*
Two 5½ by ¼-Inch Web-Plates; Four 1½ by 1½-Inch Angles; Two 9⅛ by ¼-Inch Cover-Plates.										
371	119.9	49	Pin	31 300	32 400	6	0.22	27	34.1	Buckling
372	120	49	"	31 700	33 500	....	0.26	23	28.8	"
370	180	74	"	31 600	34 400	....	0.22	27	34.1	Deflection*†
369	180	74	"	30 700	32 800	....	0.23	26	32.6	"*†
354	240	98	"	29 000	34 000	....	0.25	24	30.0	"†
355	240	98	"	29 900	33 000	....	0.25	24	30.0	"

* Buckled between rivets after considerable deflection under maximum load.

† Failed by deflecting in plane of the pins.

TABLE 10.—(*Continued.*)

Column No.	Length, in inches.	$\frac{l}{r}$ (least).	End conditions.	Maximum unit load, $\frac{P}{A}$, in pounds per square inch.	Calculated maximum fiber stress, in pounds per square inch.	Rivet pitch, p, in inches.	Thickness of plates, t, in inches.	Ratio: $\frac{p}{t}$.	Ratio: unsupported width to thickness.	Kind of failure.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
TWO 8-INCH CHANNELS WITH TWO 12½ BY 5/16-INCH PLATES.										
379	167.8	47	Flat	35 000	35 300	....	0.26	23	40.4	Buckling
380	167.8	47	"	35 600	37 000	....	0.30	20	35.0	Deflection
358	240	67	Pin	32 100	35 100	....	0.30	20	35.0	"
359	240	67	"	32 300	37 600	....	0.30	20	35.0	"
TWO 7⅜ BY 5/16-INCH WEB-PLATES; FOUR 2 BY 2 BY ¼-INCH ANGLES; TWO 12 BY 5/16-INCH COVER-PLATES.										
346	167.9	50	Flat	32 800	33 000	6	0.32	19	32.8	Buckling
347	167.6	50	"	35 000	35 300	....	0.31	19	33.8	"
342	247.6	75	"	33 000	33 600	....	0.31	19	33.8	Deflection
344	247.8	75	"	34 500	36 200	....	0.32	18	31.8	Buckling
TWO 7 BY 7/16-INCH WEB-PLATES; FOUR 2 BY 2 BY ¼-INCH ANGLES; TWO 12 BY 7/16-INCH COVER-PLATES.										
348	167.8	51	Flat	33 700	34 400	6	0.44	14	23.8	Buckling
349	167.8	51	"	33 100	37 400	....	0.48	12	21.8	Deflection
341	247.6	75	"	33 000	34 800	....	0.48	12	21.8	"
343	247.6	75	"	33 900	34 800	....	0.50	12	21.0	"

to the pins at maximum loads were small, it was assumed that frictional restraint at the pins was sufficient to prevent rotation. The maximum fiber stresses given in Column (6) of Table 10 were calculated from the deflections recorded in the reports on the assumption of fixed ends. Ratio of rivet pitch to thickness of plate is given in Column (9), and Column (11) shows the behavior of the column at failure, *B* indicating buckling of the cover-plate and *D* indicating general deflection of the column. An examination of this information shows little difference in the behavior of columns of the various plate thicknesses or ratios, $\frac{p}{t}$. In all cases except two, the shortest columns of each group showed plate-buckling at failure, and the longer columns a failure by general deflection. Only one column of $\frac{l}{r}$ exceeding 50 $\left(\frac{l}{r} = 75\right)$ showed a buckled plate at failure. The average maximum fiber stress (Column (6)) for the columns with $\frac{1}{4}$-in. cover-plates was 33 700 lb. per sq. in.; for those with $\frac{5}{16}$-in. plates, it was 35 400 lb. per sq. in., and for those with $\frac{7}{16}$-in. plates, it was 35 300 lb. per sq. in. The ratio of the unsupported length to thickness, $\frac{p}{t}$, for the $\frac{1}{4}$-in. plate was about 24, and for the thicker plates about 20 and 12, respectively. The smaller average maximum stress sustained by the $\frac{1}{4}$-in. plates may be taken as some evidence of too great an unsupported length for these plates. This group shows also four values below 33 000, which was the lowest of the other group. Furthermore, four of the columns of the first group of $\frac{l}{r} = 74$ and $\frac{l}{r} = 87$ showed buckling of plates "after considerable deflection under maximum load."

It seems obvious that in the case of columns of small slenderness ratio, such as 40 or, perhaps, 50, a failure of the column at a load approximately equal to or perhaps greater than the average yield point of the material will be accompanied by a buckling of outside plates or outstanding flanges. Such a column is too short to fail by deflection without wrinkling the component parts on the concave side, that is, the material on the concave side will be strained somewhat beyond its yield point at maximum load, and if not held in place will wrinkle or buckle. This over-strain is well illustrated by the fact that columns of $\frac{l}{r}$ less than 30 quite generally carry loads greater than the yield point strength of the material. It would seem, therefore, that buckling of plates in the case of the shortest columns of this series is not to be taken as evidence of too great unsupported length; the most important factor is whether or not the rivet spacing was sufficiently close to develop the full yield point stress in the material before buckling. Judging by the results obtained with the thicker plates, this was probably not accomplished in the case of the thinner plates with $\frac{p}{t} = 24$, but was accomplished with $\frac{p}{t} = 20$.

or less. As far as these tests are concerned, they indicate that current practice in this regard is conservative.

Considering the plate between rivets as a flat-ended column, its value of $\frac{l}{r} = 3.46 \times \frac{p}{t}$ and for $\frac{p}{t} = 20$, $\frac{l}{r} = 70$. Reference to the values of column strength given in Plate X indicates that the strength of an ordinary column of such slenderness ratio is likely to be 10 to 15% below the yield point strength of the material; but in effect the unsupported length of the plate is considerably less than the rivet pitch so that the effective slenderness ratio is much less than 70.

Respectfully submitted,

F. E. TURNEAURE, *Chairman,*
C. G. E. LARSSON,
B. R. LEFFLER,
G. L. TAYLOR,
S. H. WIDDICOMBE.

October 1, 1925.

MEMOIRS OF DECEASED MEMBERS

GEORGE HENRY BENZENBERG, Past-President, Am. Soc. C. E.*

DIED MAY 31, 1925.

George Henry Benzenberg was born in New York, N. Y., on May 31, 1847. One year later, his parents removed to Detroit, Mich., where he received his early education. He was graduated as Bachelor of Science in Civil Engineering at the University of Michigan in June, 1867.

Following his graduation, Mr. Benzenberg was engaged for two years on the primary triangulation of the Great Lakes, and from 1870 to 1873, he was employed on the location and construction of the Milwaukee Northern Railroad, from Cedarburg to Green Bay, Wis.

From 1874 to 1899, he served the City of Milwaukee, Wis., as Assistant City Engineer, City Engineer, and President of the Board of Public Works. He also served as Consulting Engineer and, in connection with their water-works or sewerage systems, as a member of several engineering commissions for the following cities: Cleveland, Ohio, Kansas City, Mo., New Orleans, La., Omaha, Nebr., Toledo, Ohio, Racine, Wis., and a number of smaller cities, and completed the water filtration plant of Cincinnati, Ohio.

From 1904 to his death, Mr. Benzenberg served as a Trustee of the Northwestern Mutual Life Insurance Company. As Chairman of the Building Committee of the Trustees, he supervised the building of the home of this Company in Milwaukee, which is the outstanding building in that city and cost about $3 000 000. In 1910, he retired from all professional activities, except those connected with the supervision of the construction of the Insurance Company's building.

With the exception of the first six years, Mr. Benzenberg devoted his professional career to municipal work, many monuments of which indicate his outstanding ability as a resourceful engineer.

While building the water intake tunnel under Lake Michigan, a distance of 6 000 ft., for the City of Milwaukee, numerous blow-outs caused its temporary abandonment. The work was finally completed successfully under Mr. Benzenberg's direct charge.

After the intake water tunnel, crib, and shaft for the water-works of the City of Cleveland had been lost, he took the work under his immediate direction, and recovered them.

His plan for keeping the waters of the Milwaukee and Kinnickinnic Rivers free of sewage pollution by means of fresh lake water pumped into them near the city limits, was a bold undertaking and was condemned by many prominent engineers as impracticable. Mr. Benzenberg built the works and they have been eminently successful. To no other man is the City of Milwaukee so much indebted for its excellent water, sewer, and street improvements, all of

* Memoir prepared by the following Committee: John W. Alvord, M. Am. Soc. C. E., *Chairman*, C. F. Loweth, Past-President, Am. Soc. C. E., and T. Chalkley Hatton, M. Am. Soc. C. E.

which he originally planned, no material changes in these plans having since been found necessary.

In 1911, the University of Wisconsin awarded him the degree of Doctor of Science. In 1893 and 1894, he was President of the American Water Works Association, and, in 1895, became the first President of the American Society of Municipal Improvements.

In addition to his very active professional work, Mr. Benzenberg was quite as active in Masonic circles in which he was widely known throughout his own State and the country. He was a member of all the Masonic organizations, in many of which he had served in official capacities. He received his Thirty-third Degree in 1886 and, at the time of his death, was one of the three active members of the Supreme Council from Wisconsin.

On December 16, 1879, Mr. Benzenberg was married to Allie Wolfrum who survives him; his daughter, Mrs. May Benzenberg Mayer, of New York, N. Y., also survives him.

Throughout his life, Mr. Benzenberg was one of those engineers who, having broadness of vision, bold initiative, and high ideals, commanded the respect and confidence of the community in the improvement of which he took such an active part, and of those with whom he came in contact.

Mr. Benzenberg was elected a Member of the American Society of Civil Engineers on May 2, 1883. He served as Director of the Society from 1895 to 1897; as Vice-President in 1901 and 1902; and as President in 1907.

CHARLES HINCKLEY BAKER, M. Am. Soc. C. E.*

DIED APRIL 18, 1924.

Charles Hinckley Baker, the eldest of the five children of William Taylor and Eliza (Dunster) Baker, was born on November 30, 1864, at Chicago, Ill. His parents were natives of the States of New York and New Hampshire, respectively, and were both of English descent. Henry Dunster, the founder of the Dunster family in America, was the first President of Harvard University.

Charles Hinckley Baker was graduated with honors from Cornell University in 1886 with the degree of Bachelor in Civil Engineering. After one year of work on railroad construction in Dakota for the Chicago Northwestern Railway Company, he accepted a position in Seattle, Wash., in the same kind of work, with the Seattle, Lake Shore, and Eastern Railroad Company.

After three years Mr. Baker resigned from this position and opened a private engineering office in Seattle, and soon developed a contracting business for the construction of railroads and water-works. In 1891, he built, by contract, the Third Street and Suburban Electric Railway, in Seattle, and an electric light and power plant in connection therewith. Later, he was appointed Receiver of the Merchants National Bank of Seattle.

In 1898, he planned and constructed at Snoqualmie Falls the first hydro-electric power plant in the Northwest and organized the Snoqualamie Falls Power Company (later, the Seattle Tacoma Power Company), of which he was President, General Manager, and Chief Engineer until the end of 1904. This plant was, and still is, extremely novel for the reason that the power-house consists of a cavern hewn in the rock about 300 ft. below the bed of the river. The Niagara Plant had just been constructed with the turbines placed in the shaft underground, and Mr. Baker carried his work even further than that at Niagara by placing the whole plant underground.

In 1904, he moved to New York, N. Y., and was one of the organizers of the Muscle Shoals Hydro-Electric Power Company and the Alabama Interstate Power Company and, as Engineer, made the first surveys and plans for the Muscle Shoals Project. He also organized and served as Vice-President of the American Cyanamid Company. He was largely interested in reclaiming about 100 000 acres of land in the Everglades District in Florida, on which tract the City of Moorehaven is now located.

Mr. Baker's principal professional interest was in the development of new, original, and unusual projects. The fact that these enterprises were carried through to a successful conclusion is due in large measure to his clearness of vision and careful organization.

Four children survive him, namely, William T. Baker, Leslie D. F. Baker, Mrs. Dorothy Baker Calhoun, and Ruth Baker.

Mr. Baker was elected a Member of the American Society of Civil Engineers on June 4, 1902.

* Memoir compiled by C. B. Wing, M. Am. Soc. C. E., with the assistance of H. L. Gray, M. Am. Soc. C. E., William W. Best, Esq., of Seattle, Wash., and H. R. Hoffeld, Esq., of Buffalo, N. Y.

WILLIAM JAMES BALDWIN, M. Am. Soc. C. E.*

DIED MAY 9, 1924.

William James Baldwin, the son of Captain John and Giovanna Caterina (San Giovanni) Baldwin, was born on June 14, 1844, on shipboard, and his birth was recorded at Waterford, Ireland. He was educated in the schools of Boston, Mass., and of Charlottetown, Prince Edward Island.

In 1862, he entered the office of his father who was a Naval Architect and Surveyor for Lloyds (British), to study naval architecture, including navigation, drawing, engineering, and physics.

In order to acquire a practical knowledge of the details of ship construction, Mr. Baldwin, in 1864, accepted a position in the iron shipyard of Donald McKay, at East Boston, Mass., where he was engaged for two years on the construction of three monitors and the conversion of several blockade runners into cruisers. In 1866, he was made Assistant under Stephen Gates, Civil Engineer, in the construction and repair of iron ships at East Boston, but, in 1868, on account of the decline in iron shipbuilding, he turned his attention to general machinery. From 1870 to 1874, he served as Manager and General Superintendent of the Detroit Novelty Works, in charge of about 100 men, in the foundry, forge, and finishing shops.

From 1874 to his death, Mr. Baldwin engaged in professional practice as a Consulting and Designing Engineer, principally in connection with the construction and design of engineering plants for large public buildings, which included boilers, engines, elevators, pumps, small water-works, and ventilating and heating systems. Many of these buildings are of prominence in New York, N. Y., where he had established his headquarters, for example, the College of Physicians and Surgeons, the Vanderbilt Clinic, Sloane Maternity Hospital, and the old Tribune Building. He also installed engineering plants in many apartment houses; a department store; the Government buildings on David's Island in New York Harbor; the New York State Reformatory, Elmira, N. Y.; the Insane Asylum, Wilmington, Del.; Clayton Block, Denver, Colo.; and the elementary schools at Honesdale, Pa. In the State of Michigan, he installed engineering plants in the State Prison at Jackson, the prison at Ionia, the State School at Coldwater, and the State Insane Asylum at Kalamazoo. Altogether, the number of boilers designed and set under his supervision exceeded five hundred.

He was also Consulting Engineer for the Department of Health, City of New York; Consulting Engineer and Designer for the United States War College at Washington, D. C.; for the United States Immigrant Station in New York Harbor; and for the United States Soldiers' Home in Tennessee; as well as the New York Telephone Company and the Empire City Subway Company.

From 1880 to 1889, Mr. Baldwin was Associate Editor of *Engineering Record* in its Department of Sanitary Engineering. He also served as Lecturer

* Memoir compiled from information on file at the Headquarters of the Society.

and Professor of Thermal Engineering at the Brooklyn Polytechnic Institute, Brooklyn, N. Y.

He was an Honorary Member of the American Society of Heating and Ventilating Engineers; a Member of the American Society of Mechanical Engineers and the Telephone Pioneers of America; and a Life Member of the Brooklyn Chapter of the American Institute of Architects. Mr. Baldwin was also a member of the Committee of the American Society of Mechanical Engineers that formulated standard pipe threads (known as the "Briggs formula"), for the United States and Canada in 1886, and of the Special Committee for Electric Screw-Thread Standards, as well as of the International Committee for the Formulation of an International Standard for Pipes and Fittings.

He was the author of a number of books, of which the following have been published: "Steam Heating for Buildings" (1881); "Hot Water Heating and Fitting" (1887); "Baldwin on Heating" (1890); "Data for Heating and Ventilation" (1897); "An Outline of Ventilation and Warming" (1899); and "The Ventilation of the School Room" (1901). He was also a contributor to the Dictionary of Architecture and Building.

Mr. Baldwin was elected a Member of the American Society of Civil Engineers on September 5, 1888.

FRANK VALENTINE ERHARD BARDOL, M. Am. Soc. C. E.*

DIED APRIL 9, 1925.

Frank Valentine Erhard Bardol was born on June 12, 1869, at Erie, Pa. He received his early education in the public schools of Buffalo, N. Y., where his parents moved when he was a child. At the age of 16 he entered Cornell University as a student in Civil Engineering and was graduated with the degree of Civil Engineer in 1889.

In July, 1889, Mr. Bardol began his engineering career as Draftsman and Principal Assistant Engineer with the Lehigh Valley Railroad Company, at Buffalo, N. Y., under Division Engineer, S. T. M. B. Kielland, M. Am. Soc. C. E., where he was engaged in terminal improvements, including the construction of docks, warehouses, and yards at Buffalo, as well as at Duluth, Minn., and Chicago, Ill. He remained in the service of the Lehigh Valley Railroad Company until the end of 1891.

In January, 1892, he was appointed to and accepted the position of Assistant Engineer in the Bureau of Engineering, Department of Public Works at Buffalo, in direct charge of the construction and maintenance of bridges, street pavements, and also of general surveys and other miscellaneous work in the Bureau. He held this position until the end of 1897.

In January, 1898, at the age of 29, Mr. Bardol was appointed City Engineer of the City of Buffalo, and in this capacity was Chief Engineer of the Bureau of Engineering, Department of Public Works, in charge of the supervision of all construction and maintenance of public work in the city, except water. During his term of office as City Engineer, he was confronted with a number of the city's municipal problems, among which were the increase of the navigability of the Buffalo River; the solution of the railroad terminal problem for the city's various incoming lines; the Hamburg Canal nuisance; and various harbor improvements, which he was instrumental in obtaining through his efforts before the River and Harbors Committee at Washington, D. C.

On the expiration of his term of office as City Engineer of Buffalo at the end of 1901, Mr. Bardol retired from public service and immediately opened an office in Buffalo as Consulting Engineer and afterward as Construction Engineer. The years that followed in his comparatively short life were eventful in the extreme; his natural gifts along many different lines were outstanding not only in the formation of successful construction companies, but also as a keen investor which enabled him in a few years to become a multimillionaire.

In 1902, Mr. Bardol formed the Eastern Construction Company, and as its President built a number of asphalt streets in Buffalo and adjacent towns. This Company was dissolved at the end of two years. In 1904, he organized the Eastern Concrete Steel Company, and as President built many structures, among which were a number of bridges, viaducts, and subways along the line of the New York Central and Hudson River Railroad; also similar structures

* Memoir prepared by F. J. Tresise, M. Am. Soc. C. E.

for the Lehigh Valley and Delaware, Lackawanna and Western Railroads, the State Normal School in Buffalo; the Buffalo City Hospital; and many other large buildings for Cornell University; Syracuse University; the Eastman Kodak Company, of Rochester, N. Y.; and the Larkin Company, of Buffalo.

In 1911, he became associated with the Rock Asphalt and Cement Corporation of Buffalo, and as Vice-President and General Manager was in charge of the construction of many millions of dollars of public improvements throughout the city. He was engaged in this capacity until his death.

In 1916, Mr. Bardol added to his many other interests the Erie Beach Amusement Park. In connection with the ownership of the Park, he purchased the Niagara Ferry and Transportation Company, operating between Buffalo and Fort Erie, Ont., Canada, across the Niagara River; the Buffalo and Fort Erie Railway Company, operating between Fort Erie and the Park; and a steamship line operating summer excursion boats from the foot of Main Street, Buffalo, to the Lake Dock at Erie Beach Park. This large enterprise, which for years had continued as a failure was, through his executive ability and business acumen, put on a highly paying basis within the nine years of his ownership and personal management, and enlarged and improved to such an extent that at the time of his death it ranked second to none in the United States.

In describing the career of Mr. Bardol the editor of a local paper stated:

"Frank V. E. Bardol was a most successful, if not conspicuous figure in the life of Buffalo. As Vice-President and General Manager of one of the two large local Paving Companies, he had the spending of millions of the taxpayers' money in the course of 25 years, but the politics inevitably connected with such a business he apparently left to his associates.

"Mr. Bardol was a thoroughly practical engineer with a keen business mind, and was one of those men who could make money out of his hobbies for he made a hobby of operating the Erie Beach, which he bought out of the Bankruptcy Court."

He was married to Katharine M. Bott, of Buffalo, on May 11, 1910, who, with three children, Franklin V. E., Robert George, and Edgar John, survives him.

Mr. Bardol was of a kindly disposition, and was eager to help those who were worthy of assistance. This was fully shown in his $250 000 endowment for scholarships in the University of Buffalo, and many other smaller gifts.

He was a member of the Buffalo Club; Buffalo Athletic Club; the Rotary Club; Buffalo Orpheus; Washington Lodge, F. and A. M.; the Buffalo Consistory; Ismailia Temple; and the Buffalo Fraternal Order of Eagles.

Mr. Bardol was elected a Member of the American Society of Civil Engineers on May 2, 1900.

GUION VICTOR BARRIL, M. Am. Soc. C. E.*

DIED APRIL 17, 1924.

Guion Victor Barril was born in New York, N. Y., on December 25, 1879. His paternal grandfather, John Joseph de Barril, came from Madrid when a young man, as Spanish Consul to Baltimore, Md., and remained in the United States, where he was well known in diplomatic and business circles. His maternal grandfather, William H. Guion, of the old "Guion Steamship Line," was one of the pioneers of the fast-going ocean steamers.

Mr. Barril was graduated from St. Francis Xavier College in New York, in 1898, and began his professional career as Rodman, with the City Surveyor of Baltimore, Md., remaining in this position approximately one year. From 1899 to 1901, he served as Rodman, Transitman, and Resident Engineer on preliminary and location surveys and on grade reduction for the Baltimore and Ohio Railroad Company.

In 1901, he held the position of Transitman and Chief of Party on grade reduction surveys for the Southern Railroad Company, and, in 1902, acted as Chief of Party on location surveys for the Baltimore and Ohio Railroad Company. From 1902 to 1904, he was Assistant to the Chief Engineer on general construction work for the Northampton Portland Cement Company, and for a few months served as Topographical Draftsman with the Erie Railroad Company.

On July 19, 1904, Mr. Barril went to the Isthmus of Panama as a Civil Engineer with the Isthmian Canal Commission. He was first engaged on general engineering work and, later, was made Assistant in Charge of Street Improvement and other municipal work in the City of Panama.

Leaving the Canal Zone in 1909, Mr. Barril spent one year in private practice in New York and vicinity, and from 1910 to 1911, served as Assistant Superintendent on double-track construction for the New York, Ontario, and Western Railroad, for the MacDonald Construction Company of New York. He devoted the next three years to railway construction work in Brazil. Returning to the Panama Canal in 1914, he was associated with the Engineering Staff of the Fortification Division, and was in direct charge of the construction of some important works.

In November, 1921, Mr. Barril left the service of the Panama Canal to take charge of the Panama Construction Company, in the Republic of Panama. He served as Chief Engineer of that Company which was engaged on the construction of about 50 km. of highway for the Republic. While on this work Mr. Barril showed extreme tact and resourcefulness in dealing with a class of labor unfamiliar with the various phases involved in the construction of a first-class highway.

On the completion of the highway contract, Mr. Barril became associated with the United States Army on the Canal Zone, where he was engaged on miscellaneous construction projects until his death on April 17, 1924.

* Memoir prepared by E. S. Randolph, Assoc. M. Am. Soc. C. E.

He was one of the pioneer builders of the Panama Canal, having arrived when the sanitary conditions were appalling and before modern science had dealt with the fevers and other diseases which are so prevalent in tropical countries. The improvements which have been wrought during the last twenty years, and have made the Canal Zone a winter resort visited by many tourists, are an enduring credit to, and an undeniable proof of, the efficacy of science, and much credit is due Mr. Barril for lending his assistance at a time when it was so greatly needed.

In August, 1900, Mr. Barril was married to Virginia Alexander, of Winchester, Va. He is survived by his widow, a son, John J. Barril, and his mother, Mrs. Benton Gauldre-Boilleau, of Washington, D. C.

He was an able constructor, a diligent student, and very thorough in planning and keeping records of his construction work. He was prominent in his community, a member of the Panama Rotary Club, of the Engineers Club of the Canal Zone, of the Knights of Columbus, and of the Society of the Incas, the latter being composed only of pioneers in the construction of the Panama Canal.

Mr. Barril was elected a Member of the American Society of Civil Engineers on July 11, 1921.

EDWARD ADAM BECK, M. Am. Soc. C. E.*

DIED OCTOBER 18, 1925.

When Edward A. Beck, City Manager of Lynchburg, Va., died in that city on October 18, 1925, the profession to which he had dedicated his life lost one of its ablest and most successful members. Although he was only forty years of age at the time of his death, his accomplishments were already conspicuous in the field of municipal administration, and he had reached the position where he was regarded as a leader in this important branch of modern city government.

Edward Adam Beck was born on January 19, 1885, in Peru, Ind., the son of John Henry and Catherine Magdelan (Silberman) Beck.

After attending the local elementary schools, he entered Purdue University, from which he was graduated in 1913 with the degree of Bachelor of Science. In 1917, he received the degree of Civil Engineer.

From 1906 to 1914, prior to and during his college course, Mr. Beck was engaged on engineering work, ranging from preliminary surveys for local electric lines to the construction of important railroad terminals. During five years of this period he served as Division Engineer of the Wabash System and had supervision of the location and construction of the important terminal at Detroit, Mich., including the negotiation for municipal franchises, a valuable training for the profession which he afterward adopted, and in which he was a real pioneer.

In 1914, he was called to the Borough of Edgeworth, Pa., as Borough Manager. He was the first to hold such a position in Pennsylvania and was able to do so only under a very liberal construction of the then existing laws. The experiment was, however, so successful, that general laws were promptly enacted by the State, permitting the adoption by municipalities of the manager form of government. During 1917, he was also Manager of Sewickley, a borough contiguous to Edgeworth. It is doubtful whether this unique distinction of serving two cities in such capacity at the same time has been enjoyed by any other City Manager.

When Mr. Beck resigned these positions, he was elected, from among a large number of aspirants, to the position of City Manager of Goldsboro, N. C., being the first official in this position for that city. During the period of approximately two years that he served in this capacity, he displayed marked ability as an administrative officer, and gained much valuable experience in the conduct of municipal affairs.

In 1918, he resigned his civil position to enter the Army during the World War. Commissioned first as Captain in the Sanitary Corps, he was later promoted to the rank of Major in the Engineer Reserves of the United States Army and was placed in charge of water supply, sewage disposal, and mosquito control, at Camp Shelby, Miss. He was under orders for overseas service when the Armistice was signed.

* Memoir prepared by R. W. B. Hart, M. Am. Soc. C. E.

After his discharge from military service Mr. Beck again entered the field of municipal management and was elected as City Manager of Auburn, Me., where he also held the position of Chief Engineer of the Water and Sewer District. So marked was his success in this position that the municipal authorities of Lynchburg, Va., after a careful survey of the entire field, of available candidates, elected him as its first City Manager. He entered upon the duties of his office in Lynchburg on September 1, 1920, and continued in this service until his death, which was caused by pneumonia, following an operation.

Mr. Beck brought to the administration of the affairs of Lynchburg an experience that peculiarly fitted him for the tasks there presented, having displayed such unusual ability, untiring energy, and genius for organization and management that he won the entire community to the ardent support of the new plan of government, although a number of its citizens had at first bitterly opposed the change. His record of accomplishment in Lynchburg not only gained recognition from the citizens of that community, but attracted the attention of those interested in city government in all parts of the United States, and he became definitely established as a distinguished leader in this new profession.

At the time of his death Mr. Beck had just completed the preliminary work necessary for the immediate rebuilding of Rivermont Bridge in Lynchburg. The work involved the change from an iron structure to one of reinforced concrete. The idea was his own and resulted in the saving of many thousands of dollars to the city. Because of the unusual height and length of the viaduct it is, perhaps, the most monumental work of this character ever undertaken and has attracted the interest of engineers in all sections of the country. In grateful recognition of the valued services of its first City Manager, and as a tribute that will most appropriately perpetuate his memory, the City Council has formally designated this structure, "Beck Bridge," and on its completion a suitably inscribed tablet will be unveiled with appropriate public ceremonies.

Mr. Beck was noted for the thoroughness of his knowledge of all the details incident to the administration of municipal affairs. He was, therefore, a man of prompt decision and of unusually sound judgment. The City Council soon learned that it could with confidence follow his recommendations in all matters with the assurance that they had been most carefully and intelligently considered by him.

He was a man of unfailing courtesy and consideration, and of absolute fairness. The humblest citizen could always gain access to him and receive an attentive hearing. The most influential person could not secure a privilege or a concession that he did not deem fair and just to all.

Possessing vigorous mentality and dauntless moral courage, he could maintain his conclusions with forceful ability in any forum, for he was a strong debater. He was often called on to address audiences interested in civic affairs and his utterances were controlled by a lofty conception of the true ideals of citizenship and public service.

Mr. Beck found a most congenial atmosphere in Lynchburg and frequently stated that he hoped to spend his life among its people. Such a pur-

pose was itself proof of the sincere affection in which he was held by them, and it was further attested when the news of his untimely and unexpected death shocked the city, for the whole community was grief-stricken and expressions of personal sorrow were heard from those in every walk of life.

Business was suspended throughout the city during the funeral ceremonies, and when his body was borne to its last resting place in Spring Hill Cemetery, it was laid in a grave that was banked with beautiful floral designs, tributes from nearly every organization in the city and from hundreds of individuals, while throngs of people assembled to share in this expression of a common sorrow for a great loss.

Although young in years, his life had been rich in service and filled with the joy of worth while achievement. On the people of every community in which he had lived, he left the impress of a clean and exemplary life devoted to the highest ideals. The influence of such an example never dies; no man can leave a richer legacy to posterity.

On June 26, 1907, he was married to Hazel Maude Locke, of Peru, Ind., and is survived by her and one son, Edward Adam Beck, Jr., as well as by a sister Mrs. Catherine Dilly, of Cleveland, Ohio, and a brother, Albert J. Beck, of Indianapolis, Ind.

Mr. Beck was a man of rare personal charm and one who delighted in the intimate associations of his fellows. He was a member of the Masonic Fraternity, having been a Thirty-second Degree Mason, a Knight Templar, and a Shriner. He was also an Elk and a Rotarian and was very active in the latter organization.

Mr. Beck was elected an Associate Member of the American Society of Civil Engineers on May 6, 1914, and a Member on November 26, 1918.

EDMUND HAMILTON BOWSER, M. Am. Soc. C. E.*

DIED MAY 8, 1925.

Edmund Hamilton Bowser was born at Louisville, Ky., on January 13, 1856. His father was John W. Bowser, formerly of Baltimore, Md., and his mother before her marriage was Anna Culver, of New Orleans, La. She was gifted as a poetess and wrote a prize-winning poem for the Nashville Centennial.

Mr. Bowser received his engineering education at Vanderbilt University, Nashville, Tenn., which he attended from 1878 to 1882. At the time of his graduation he was awarded a scholarship for the highest standing in the engineering class. For a year after graduation he remained at the University as Physical Instructor.

In 1883 he entered the service of the Cincinnati Southern Railway Company as Assistant Engineer, and continued in this position for two years. For three years thereafter he was in Florida, engaged in general engineering work. On April 15, 1888, he entered the service of the Newport News and Mississippi Valley Railroad Company as Assistant Engineer. This railroad later became the Chesapeake, Ohio, and Southwestern, and is now a part of the Illinois Central System. While with the Chesapeake, Ohio, and Southwestern Railroad Company he was stationed at Louisville, Ky., with the title of First Assistant Engineer, and was in charge of the work in the Bridge, Building, and Track Departments.

In 1898, a year after the Chesapeake, Ohio, and Southwestern Railroad was incorporated with the Illinois Central System, Mr. Bowser was made Roadmaster, with headquarters at Memphis, Tenn. He resigned this position in July, 1899, to become Assistant Engineer with the Louisville and Nashville Railroad Company, with headquarters at Pensacola, Fla. While in this position he had charge of several large dock projects. In October, 1903, he left the employ of the Louisville and Nashville Railroad Company to become General Manager of the Southern Creosoting Company, at Slidell, La.

On August 1, 1907, Mr. Bowser re-entered the employ of the Illinois Central Railroad Company as Chief Timber Inspector and on January 1, 1912, was promoted to the position of Superintendent of the Timber Department. On January 1, 1922, he became Superintendent of Ties and Treatment, which position he held until his death.

He was an active member of the American Railway Engineering Association and the American Wood Preservers' Association. He had served as a member of the Committee on Wood Preservation of the American Railway Engineering Association for about fifteen years, having been Vice-Chairman in 1912, 1913, and 1914. In the last report of this Committee,† Mr. Bowser contributed a paper entitled "The Effect of Preservatives on the Inflammability of Wood." He was a recognized authority on wood preservation and his papers on the subject attracted wide attention. For more than twenty years

* Memoir prepared by A. L. Dabney, M. Am. Soc. C. E.

† *Bulletin 270*, Am. Ry. Eng. Assoc., October, 1924.

he made a close study of wood preservation, and it is largely due to his efficient and conscientious work that the present high standard of wood treatment has been attained.

Mr. Bowser was a Charter Member of the Engineers Club of Memphis, a member of its first Board of Directors, and during 1922 served as President. Under his guidance the Club was inspired with a new enthusiasm and grew in membership and influence. At the close of his administration he delivered an address which is notable in the Club's history. It was immediately ordered published for distribution, and has the distinction of being the only paper that the Club has put into print. The following quotation from this address has, since its utterance, appeared on every *Bulletin* of the Club:

"The true Engineer is one who sees visions, and following his visions beyond the beaten path of common practice, does something that has not been done before, or does it in a better way."

From his boyhood Mr. Bowser had been an active member of the Unitarian Church. For five years he was President of the Board of Trustees of the of the Memphis Church, and was Chairman of the Building Committee for the new church building erected in 1923.

He was married to Alice Fairfax Smith, daughter of Judge James Power Smith and Julia Robert Smith, of Chattanooga, Tenn., on June 20, 1889. He is survived by his widow, three nephews, Allen F. Owen, of Chicago, Ill.; and Wilfred and David Bowser, of Anchorage, Ky., and two nieces, Miss Ethel Owen, of Washington, D. C., and Mrs. Helen Owen Lee, of Pasadena, Calif.

Mr. Bowser was drowned with the sinking of the Steamer *Norman* in the Mississippi River, near Memphis, Tenn., on May 8, 1925. He had with him a great-nephew, a boy ten years old. A small power boat came to the aid of the *Norman's* passengers, and Mr. Bowser swam to the side of this boat and placed the boy in it, but refused to get in himself until the women and weaker swimmers were saved. He swam away to assist others and was not again seen alive.

His sincerity, optimism, enthusiasm, and geniality won and held his fellow engineers who had for him a strong esteem and genuine affection.

Mr. Bowser was elected a Member of the American Society of Civil Engineers on June 1, 1904.

THOMAS PETTUS BRANCH, M. Am. Soc. C. E.*

DIED MAY 28, 1923.

Thomas Pettus Branch, the second son of Franklin Addison and Margaret Antonia Branch, was born at Tallahassee, Fla., on May 20, 1864. His father and grandfather were both Methodist ministers.

After completing his public school education he entered Vanderbilt University in September, 1881, and was graduated from that institution as a Civil Engineer in June, 1885.

In September, 1885, Mr. Branch assumed the duties of Professor of Mathematics and Engineering in the State Agricultural College at Corvallis, Ore., to which position he had been elected on the recommendation of Chancellor Garland of Vanderbilt University. At the end of his first year the ownership of the institution changed and the entire faculty resigned. Professor Branch served the Northern Pacific Railroad for the next three years, surveying and locating line and superintending the building of track through the mountains.

In 1889, he returned to Georgia and taught school for a few months in Meriwether County. Then, with a college friend, he surveyed and platted several towns in Tennessee, including Harriman. Between 1890 and 1894, he was in the service of the Savannah, Americus and Montgomery Railroad (now the Seaboard Air Line Railroad) and, as Field Engineer, built twenty miles of the line between Americus and Savannah, Ga. On the completion of this work, he entered the service of the Louisville and Nashville Railroad and was Resident Engineer at Guntersville, Ala., but his health having failed, he had to give up active field work.

In September, 1894, Professor Branch took charge of the Shellman, Ga., Public School and served in that position for one year.

In September, 1895, he became associated with the Georgia School of Technology as Adjunct Professor of Mathematics, and until his death was in charge of the Civil Engineering Department. For many years during this period he served as Secretary of the Faculty and Registrar. In 1920, the University of Georgia conferred on him the degree of Doctor of Science.

Dr. Branch had high ideals and lived a clean Christian life. He was an active member and for many years an officer in the Methodist Episcopal Church. Devotion to duty and loyalty to principle were the outstanding traits of his character. His long years of service in educational institutions developed in him a clear and balanced judgment of human nature, which, combined with a kindly and friendly personality, compelled the confidence and admiration of his associates and students.

At the Georgia School of Technology he shares the credit of having inaugurated the Co-Operative Educational Courses, and no member of its Faculty

* Memoir prepared by the late Paul H. Norcross, M. Am. Soc. C. E.

deserves more praise for its growth and influence. Dr. Branch's life was one of intensive and unselfish service. His influence on the undergraduates and their subsequent careers during the twenty-eight years that he was connected with the Georgia School of Technology is reflected in the admiration with which his name is held by those who knew him. He lived and died beloved by all.

In 1895, he was married to Mary Susie Pharr who died in 1907. In 1909, he married Mina Pharr Thweatt who, with a son, Thomas Pharr Branch, and a daughter, survives him.

Dr. Branch was elected an Associate Member of the American Society of Civil Engineers on February 5, 1902, and a Member on May 5, 1915.

FRED AZDELL BRYAN, M. Am. Soc. C. E.*

DIED JULY 23, 1924.

Fred Azdell Bryan, a prominent citizen and one of the leading business men of South Bend, Ind., died suddenly, of septic poisoning, on July 23, 1924. Although he was but little past middle age, he had achieved a remarkable record and left a considerable sphere in the world better for his having lived in it.

Fred Azdell Bryan was born in Mechanicstown, Carroll County, Ohio, on December 21, 1867, the son of John and Anna Azdell Bryan, the latter of Columbiana County, Ohio. He attended the common schools and High School in Wellesville, Ohio. In 1886, he entered the Pennsylvania State College, from which he was graduated in 1890, as a Civil Engineer. As a student he was diligent, bright, and prompt, and his sunny disposition and staunchness of character endeared him to a circle much larger than his own class. He was never ostentatious, nor would he seek to gain favor or preferment through rivalry with his peers and associates. Friendship was to him above all considerations of personal advantage. Unselfishness was an outstanding trait of his beautiful character; and, for such a personality, a useful and successful career was to be foretold during his student days. His whole life was singularly free from those antagonisms and personal animosities which men of affairs so commonly encounter in conducting large business enterprises.

Soon after leaving college, in 1890, Mr. Bryan was engaged as an Assistant in an engineering party with the Pennsylvania Railroad Company. During the next two years he was employed as a Draftsman and Transitman with the Denver and Rio Grande Railroad Company. From 1893 to 1896 he was with the Edison General Electric Company, as Engineer and Superintendent of Construction in the building of approximately 75 miles of city and interurban electric railroads. During the next five years, 1896-1901, he served as Assistant Engineer with the Michigan Central Railroad Company and was stationed at Niles, Mich. In this connection his work had to do with extensive improvements and relocation of tracks for reduction of grades and curvature, bridge renewals, etc., carried out by the late Augustus Torrey, then Chief Engineer of the Michigan Central Railroad.

About this time (1901) Mr. Charles A. Chapin, owner of the South Bend, Ind., Electric Company, undertook the promotion of an extensive scheme of hydro-electric power development on the St. Joseph River, to supply energy for electric lighting, electric railway operation, and general power purposes throughout the river valley. After searching for an engineer qualified by training and experience to take charge of this work, Mr. Chapin selected Mr. Bryan, and made him General Manager of the South Bend Electric Company, with headquarters at South Bend.

Mr. Bryan at once took charge of the design and construction of two dams and hydro-electric plants on the St. Joseph River, costing approxi-

* Memoir prepared by the late Walter Mason Camp, M. Am. Soc. C. E.

mately $1 500 000, followed by the construction of a steam plant, in South Bend, costing $250 000, to take the peak of the load. In addition, he had charge of the operation of two hydraulic plants and another steam plant, of similar capacity, already constructed and supplying power to South Bend.

These extensive power developments for the operation of varied public utilities comprised hydro-electric plants at Twin Branch, Elkhart, Ind., Buchanan and Berrien Springs, Mich., with a power distribution extending from Michigan City, Ind., and St. Joseph and Benton Harbor, Mich., on the west, to Ligonier, Ind., on the east—a densely populated and manufacturing district approximating 2 000 sq. miles in area—and constituting one of the largest electrical utilities of the Middle West.

So faithfully and successfully did Mr. Bryan prosecute the purposes of this public utility enterprise that he remained with it as engineering and executive head for twenty-two years. In 1908 the South Bend Electric Company became the Indiana and Michigan Electric Company, with Mr. Bryan as its President. It was through his vision, initiative, and executive ability that the original plant of the South Bend Electric Company was expanded to cover the additions previously mentioned.

In 1923 the controlling interest in the Indiana and Michigan Electric Company was sold to the American Gas and Electric Company. The new organization desired Mr. Bryan to retain the management of the property, but, largely owing to the condition of his health, he declined. Close application to the exacting requirements of multitudinous affairs had worn upon him, and he decided to seek relief in travel, with a view of engaging later in a business that would permit more time for recreation than he had previously been able to enjoy.

Early in 1923, shortly after severing his connection with the Indiana and Michigan Electric Company, Mr. and Mrs. Bryan started on an extended tour of the Orient. They visited Hawaii and Japan and traveled far into the interior of China. While in China, Mr. Bryan received a cablegram offering him an important position as a Consulting Engineer with the Studebaker interests, to serve in an advisory capacity with the Harris Trust and Savings Bank, of Chicago, Ill., which was financing the Studebaker enterprises, consisting of traction, gas, and water utilities in Illinois and Missouri. He accepted this offer, and on his return to the United States in the fall of 1923, became actively engaged in his new occupation, with an office in South Bend, spending a part of his time, more or less regularly, in Chicago. He was thus engaged at the time of his death.

In his civic activities Mr. Bryan was hardly less distinguished than in his engineering and executive achievements. His admirable qualities of character, taken with his trained intellect and genius for organization, made him a splendid type of public-spirited citizen. Besides being a Director in the Citizens National Bank, of South Bend, he was a member of the Chamber of Commerce, the South Bend Country Club, the Knife and Fork Club, the Indiana and University Clubs, the Phi Gamma Delta Fraternity, and of the Union League and the Indiana Society in Chicago. He also served as President of the South Bend Boy Scouts' Council, to which he gave much of

his time and energy, and as a Director of the City Reserve Mission, and was a member of the First Presbyterian Church.

During the World War Mr. Bryan was placed in charge of the Third Liberty Loan for Northern Indiana, and the alacrity with which his district subscribed its full quota was credited to the executive ability and good will which he brought to bear upon this very pressing and indispensable financial project of the nation.

It is exceptional that the life of one engaged with as many business affairs as Mr. Bryan could be occupied so largely with the public enterprises of the community in which he lived. His masterful organization and management of the Boy Scout movement won for him not only the love and veneration of the boys but the admiration of the general public. In South Bend, with a population of about 80 000, the proper rearing of children has been studied as one of the most serious social problems of the community, and the zeal which Mr. Bryan put into this work was greatly appreciated and commended. His efforts in this service, in which he became so eminent, may have been prompted by recollections of the early years of his own life, or by the fact that he himself was childless. He had, by nature, a warm spot in his heart for boys. It is said, further, that he was constantly on the lookout for worthy boys ambitious for an education but lacking opportunities to acquire it; and that, in many instances, he found ways and means to place such young men in a position to realize their objective without letting them know their benefactor. It is deserving of all commendation that in his will he provided a substantial trust fund the income from which was designed to meet the cost of constantly maintaining four young men in college, the favored ones to be selected from among worthy boys of South Bend identified with the Boy Scout organization.

It would have been incompatible with Mr. Bryan's success as a public utility executive and leader in social service had he not been a man of independent opinions on public questions and policies. Although he was habitually frank and outspoken in the expression of his views, yet his manner of address was always so considerate and kindly that he seldom, if ever, offended. This can be said notwithstanding that there often fell to him the difficult task of conducting negotiations with State commissions, municipal councils, and other civic authorities regarding matters of franchise rights or rate adjustments. He had a way of presenting his case that seldom failed to carry conviction. An open manner and conservative habit of statement, a conspicuous sense of fair dealing, a direct method of getting to the point of a question were characteristic of Mr. Bryan and accomplished for the interests that he represented what most men of affairs would not attempt to carry through without the employment of qualified legal talent. His willingness to consider broadly the rights of others and a dignified manner of dealing were the criteria of his business methods.

In his social life, as in his business career, Mr. Bryan always held to high ideals and his tastes were cultural. He was married on June 9, 1896, to Estelle M. McVicker, of Lisbon, Ohio, who survives him.

Mr. Bryan was elected a Member of the American Society of Civil Engineers on December 1, 1908.

ENRIQUE BUDGE, M. Am. Soc. C. E.*

DIED JUNE 2, 1924.

Enrique Budge was born in Valparaiso, Chile, on November 15, 1844, the son of Robert Forbes Budge, a native of Scotland, and Mercedes Prats y Urizar. Robert Forbes Budge was an *ex*-officer of the British Navy, who after the surrender of Napoleon Bonaparte retired from the service. He made two voyages to Chile, the first in 1818 and the second in 1820. He then remained in that country and, after a few years, married Mercedes Prats, the daughter of the Postmaster-General of Chile. Enrique Budge was the seventh son of a family of ten. He began his education at the English school of Messrs. Bloom and Goldfinch in Valparaiso, specializing in mathematics.

Mr. Budge began his technical career in 1863 at the Victoria Foundry and Iron Works of Messrs. Henderson, Flynn and Company, of Valparaiso, where he acquired a practical knowledge of structural engineering that was of great assistance to him in his subsequent work. In 1864, he joined the forces of the Valparaiso and Quillota Railway, then under construction by the renowned English engineer, George Lloyd. In 1865, he was appointed by the Chilean Government as Assistant Engineer on the San Fernando and Curico Railroad and, in 1867, undertook the work of turning the course of the Rio Teno near Curico. This river, due to extraordinary freshets, had inundated the neighboring farms, and Mr. Budge opened a new course for its flow by cutting the railway embankment, thereby completely isolating the railway bridge. This work was most successful.

From 1867 to 1869, he was Resident Engineer of the work at Paso Hondo, on the Valparaiso and Santiago Railroad. This work included difficult alignment of the track, which crossed the Paso Hondo River five times in a distance of 800 m. It also included the construction of heavy embankments, cuts, and tunnels to obtain the most economical layout which called for a single bridge and a tunnel to cross the stream, at a consequently low maintenance cost, which was the end desired. As the old bridges had been built ten years previously of timber with arched spans and were of different sizes, a serious difficulty was encountered because of the enormous expense entailed, including the delay of traffic. The new location, therefore, was accepted as a solution inasmuch as it did not interfere with the traffic. Having completed this work, he was appointed in 1869 as Resident Engineer of the Valparaiso and Santiago Railroad.

From 1870 to 1872, as Resident Engineer, Mr. Budge surveyed and constructed the Los Vegas and San Felipe Railroad, a branch of the main line from Valparaiso to Santiago, Chile, and from 1872 to 1874, he extended that branch line 45 km. to Los Andes, Chile, the starting point of the Chilian Transandine Railway to Argentina through the Cordillera de los Andes, *via* Mendoza. This railroad completed, he was again in charge, as Resident Engineer, of the Valparaiso and Santiago Railroad, including the Los Andes Branch.

* Memoir prepared by Edward B. Budge, M. Am. Soc. C. E.

In 1875, the Chilian Government awarded by contract the construction of the Curico and Angol Railroad and the Chillan, Concepcion and Talcahuano Railroad, a total length of 425 km. Technical and financial difficulties began with the contractors and the bankers who were to provide the funds for the work, and, consequently, with the Government itself. Mr. Budge, whose sound judgment and knowledge of railway construction had already greatly helped the Government on similar occasions, was proposed as Chief Engineer and Controller of Finances of the enterprise, and was immediately accepted in this capacity by mutual agreement of the three parties concerned. Work on this part of the Chilian Railway System was most difficult and expensive, as the railway extends to the southern part of the Republic and crosses all the rivers of the country, which flow from east to west and are subject to heavy freshets in winter. Moreover, as the large bridges included in the contract were to be temporary on account of the cost involved and the time required for permanent structures, and yet had to be of sufficient strength to withstand the traffic during the winter, the contractors were at a loss as to how to comply with the Government requirements; but, fortunately, Mr. Budge's perseverance and sound advice overcame all difficulties.

In 1878 and 1879, he was commissioned by the Chilian Government to study the construction and working of railroads in Europe and the United States, but on account of the war with Peru and Bolivia, he was obliged to return to Chile before finishing his commission, having visited only European countries. He was again appointed Resident Engineer of the Valparaiso and Santiago Railroad in 1880, and in 1881 he was appointed Chief Engineer and Director of the new Government mole at Valparaiso, a very important and complicated work which required a foundation of fifty-three caissons, from 20 to 100 ft. long, and 11½ ft. in diameter, sunk by the pneumatic process and filled with mortar. On this foundation iron bridge girders were anchored, and a system of smaller girders supported a concrete pavement laid on steel buckle-plates. In this pavement were embedded the tracks for the heavy hydraulic cranes used for unloading material directly from the ships, and also tracks for hauling loaded and empty cars. A 60-ton hydraulic crane was erected for handling extra heavy loads. This mole has stood as a monument to Mr. Budge's enterprise, for neither storms nor the great earthquake of 1906 did the least damage to the solid foundations of this unique structure. During these years, he also studied and carried out plans for the water-works systems of the Towns of Curico, San Felipe, and Los Andes.

In 1884, he was called upon by the Chilian Government to take charge as Engineer in Chief of the whole plan for Government railways from Valparaiso to Talcahuano and Angol. He, therefore, undertook the work of reconstructing all the important temporary bridges, which, having been built under economical conditions, were actually a great hindrance to railway traffic, particularly as in winter heavy freshets were sure to destroy some of the bridges and embankments. Among many others, he reconstructed the Nuble Bridge, the Laja Bridge, and the Bio-Bio Bridge, each 500 m. long, in 50-m. spans. The abutments and piers were built by the pneumatic process because these rivers

carry such a volume of water that no other form of foundation was considered satisfactory.

Mr. Budge continued his connection with the Chilian State Railways until 1894, and, during that time, he served as Director-General of the State Railways in 1885, 1891-92, and 1894. His work on these railways was of the utmost importance, and involved, in addition to the construction of bridges as previously mentioned, new lines, stations, etc., making improvements in accordance with the latest engineering practice.

He had been a member of the Sociedad de Fomento Fabril (an organization of merchants and manufacturers formed to promote Chilean business interests), since 1883, and served as Counselor in 1889; Vice-President in 1898; and President from 1902 to 1908. Due to his important and long service he was nominated in 1909 as Honorary President, which office he held until his death. During his term of office, he organized three technical schools in Santiago, and one in each of the following towns: Talca, Chillan, Concepcion, Valdina, Serena, and Iquique.

In 1901, Mr. Budge undertook the work of preparing an exhibit to be entered by the Chilian Government at the Pan-American Exposition, at Buffalo, N. Y., and was given charge of this task as General Commissioner for Chile. It was due to his unfailing energy and propaganda in favor of this plan that his country was able to hold a most merited and flattering position in that World's Fair.

In 1904, he was again appointed by the Government to take charge of the Chilean National Exposition, which was carried out principally by the Sociedad de Fomento Fabril and was a great triumph for Chilean industries. The thanks of these industries were tendered Mr. Budge who had devoted all his experience to the success of the undertaking.

He acted as Consulting Engineer to Messrs. Clarke Brothers who had concessions from the Chilean and Argentina Governments for building the Transandine Railway to Argentina. When the proposed plans were submitted to him, he at once offered a complete modification of them. In order to use the common steam locomotive to reduce the general gradient of 8%, the proposed plans had included a series of costly loops and tunnels. Mr. Budge at once recommended the higher grade and the use of the Abt rack system. The contractors immediately grasped the idea and accordingly new plans were drawn using a grade of 8%, with rack line, which resulted in great economy in cost and time of construction. This line crosses the Andes at a height of 10 500 ft., through a tunnel 3 144 m. long.

Mr. Budge acted on behalf of the Government as Consulting Engineer to report on the proposal for the construction of the graving dock at Talcahuano; he also reported on the construction of wharves and quay walls for the Port of Iquique and a mole for Pisagua.

The terrible earthquake at Valparaiso in August, 1906, which destroyed this and neighboring cities and resulted in enormous losses through death and fires caused by falling houses, introduced a most serious problem as regards the reconstruction of the city. Mr. Budge was called on by the Government to undertake this work which required an experienced engineer.

New plans had to be laid out, new grading from hills to the sea-front, new outlets for the streams, new streets—in short, a new city had to be designed with modern causeways and footpaths, with an expenditure of £3 000 000 gold.

Having completed this work of reconstruction, Mr. Budge was nominated in 1910 as Inspector General of Materials to be furnished to the Chilean Government by European and North American firms. He devised a sound method of financial accounting for materials provided to the railways and Government contractors, including those used in the construction of quay walls and breakwaters at Valparaiso, by the firm of S. Pearson and Son, Limited. He held that office until 1920, during which time he visited the principal manufacturing establishments of Europe and the United States.

On his return to Chile, Mr. Budge was appointed Director of various industrial corporations, which positions he held until his accidental death. In his endeavor to pass through a crowd in one of the banks in Santiago, he fell into a deep trench recently opened for the layout of electric cables, striking his chin. This blow caused concussion of the brain which gradually developed into paralysis and subsequent death.

Mr. Budge was very active, never tiring of any work that he undertook. He was the first President of the Portland Cement Factory, "El Melon", the only factory of its kind in Chile, and placed it on a sound financial basis. It can be truly said of him, that up to 1910 no engineering plan was carried out by the Government of Chile or by Chilean contractors without his endorsement, so that his name is connected with almost all the public works constructed in Chile during the last fifty years.

Mr. Budge is survived by his widow, Mrs. Carolina Zañartu de Budge, and three children, Olga, wife of Augustin Edwards, Chilean Minister to Great Britain, Carlos, and Enrique.

He was elected a Member of the Institution of Civil Engineers of Great Britain and also a Member of the Institution of Mechanical Engineers in 1879. He was a faithful member of the Roman Catholic Church.

Mr. Budge was elected a Member of the American Society of Civil Engineers on February 1, 1882.

WALTER MASON CAMP, M. Am. Soc. C. E.*

DIED AUGUST 3, 1925

Walter Mason Camp, the son of Treat Bosworth and Hannah A. (Brown) Camp, was born at Camptown, Pa., on April 21, 1867. He came of Colonial ancestry; his earliest paternal American ancestor was Nicholas Camp, who came from Essex County, England, in 1631, with the Rev. John Elliott, and settled in Massachusetts. Both his paternal and maternal ancestors served in the Revolutionary War. His father who was an insurance surveyor and the author of insurance literature, served as Captain of Company F, 52d Pennsylvania Infantry, during the Civil War and was once confined in Libby Prison.

Mr. Camp received his preliminary education by winter attendance at the public schools in Wyalusing, Pa. At the age of nine he was employed as a fireman in a planing mill at Wyalusing and, later, for four years, he worked on farms and at lumber camps. From 1882 to 1887, he was with the Lehigh Valley Railroad Company as Trackman, Chainman, and Rodman on preliminary and location surveys, and double-track construction, and, later, on extensive river surveys for bridge location. During this time he acquired also a working knowledge of telegraphy.

In 1887, Mr. Camp entered Pennsylvania State College, from which he was graduated in 1891 as Civil Engineer. For a few months following his graduation he served as a Surveyor with the San Joaquin Valley Railway Company, at Fresno, Calif., and as Draftsman in the office of the Chief Engineer of the Southern Pacific Company, at San Francisco, Calif. From 1892 to 1894, he was Engineer and Superintendent of the Rainier Avenue Electric Railway, Seattle, Wash., which extended eight miles into the suburbs; he had full charge also of the location and construction of an uncompleted part of the road. He designed and built rolling stock for heavy freight traffic, repair shops, station buildings, and wharves for the steamers operated in connection with the railway and also designed and rebuilt a counterweight system for operating electric cars on a 17% grade.

Mr. Camp was with the Seattle, Lake Shore, and Eastern Railway Company as Engineer in charge of building spur-tracks into lumber camps in 1894 and 1895. His work here consisted in relocating and rebuilding parts of the main line, widening cuts and embankments, and reballasting parts of the roadbed. He served also as Work Train Foreman, Surveyor, and Section Foreman. In 1895, in addition to other work, Mr. Camp studied as a post-graduate student in electrical and steam engineering at the University of Wis-

* Memoir compiled from information on file at the Headquarters of the Society.

consin, and, in 1896, taught for a short time in the National School of Electricity in Chicago, Ill. He then accepted the position of Receiver and Inspector of Materials for construction, and served as Superintendent of Construction, for the Englewood and Chicago Electric Railway Company (a storage battery road), in Chicago. He also had charge of building the counterweight system for this Company in Morgan Park, Ill.

In the spring of 1897, Mr. Camp became Engineering Editor and, later, Managing Editor of the *Railway and Engineering Review* (now the *Railway Review*), in Chicago. He served also as Consulting Engineer to the Englewood and Chicago Electric Railway Company, with reference to the operation of the counterweight system and in repairing it.

He acted as Consulting Engineer to capitalists investigating the cost of construction, operation, and traffic possibilities of an electric railway for Towanda, Pa., in 1898, and, in 1899, to the Automatic Rail Joint Spring Company of Chicago in reference to track fastenings and the manufacture of concrete ties. In 1900, he was consulted by the Kobu Railway Company of Japan in the matter of the selection of track materials.

Mr. Camp was admired as a writer by members of the railroad fraternity because of the clear and forceful presentation of his opinions. He was particularly well versed in matters relating to track maintenance and automatic train control. On learning of his death, one of his associates remarked: "He was the best-informed man I ever knew". His published works, excluding his editorials, comprise "Notes on Track," which is used as a textbook in colleges having Railroad Departments; "Life of Samuel F. Patterson"; and "Railroad Transportation at the Universal Exposition, St. Louis 1904". He was also the author of numerous papers published by engineering and historical associations. No one was ever in doubt as to the exact meaning of his editorials, which were based on thorough investigation and sound knowledge. He understood railroad conditions thoroughly.

Although devoted to the work of his profession, Mr. Camp was a man of varied interests. He was fond of outdoor life and owned and operated a farm at Lake Village, Ind., where he became a successful farmer. He also had a quiet retreat in the Michigan woods. He was a student of Indian life, customs, and history. He spoke a number of Indian languages and dialects and had made a study of their wars with the white man from the standpoint of the Indian. His vacations were often spent in research work on their reservations in the West where he was a welcome visitor, and it was not unusual to see a group of Indians in his office seeking advice. Mr. Camp was also a student of Astronomy and was interested in the question of whether or not the planets are inhabited. In his will he provided a fund which is to be used by the Northwestern University to carry on research work to investigate this problem.

He was greatly concerned in the welfare of young men, many of whom profited by his kindly counsel. His cabin in Michigan and his farm were always at their disposal. He was married, at Blue Island, Ill. on May 2, 1898, to Emeline L. F. Sayles, who survives him.

He was a member of the American Railway Engineering Association; American Railway Bridge and Building Association; Roadmasters' and Maintenance-of-Way Association; Society for the Promotion of Engineering Education; Permanent Way Institution (Great Britain); Railway Signal Association (now Signal Section of the American Railway Association); Mississippi Valley Historical Association; Western Society of Engineers; Chicago Engineers Club, and others.

Mr. Camp was elected a Member of the American Society of Civil Engineers May 1, 1901.

CHARLES EDWARD LAW BALDWIN DAVIS, M. Am. Soc. C. E.*

DIED JUNE 4, 1925.

Charles Edward Law Baldwin Davis, the son of Charles S. A. and Mary Jeanette (Downs) Davis, was born at New Haven, Conn., on February 16, 1844. After spending one year in the Academic Department of Yale College in 1861-62, he was appointed a Cadet at the United States Military Academy, at West Point, N. Y., on July 1, 1862. He was graduated from West Point on June 18, 1866, and commissioned a Second Lieutenant of the Corps of Engineers, United States Army. He was promoted to the rank of First Lieutenant, Corps of Engineers, on March 7, 1867; Captain on September 12, 1877; Major on April 7, 1888; Lieutenant-Colonel on May 3, 1901; Colonel on October 10, 1905; and Brigadier-General U. S. Army, on January 29, 1908.

Lieutenant Davis served on various military duties at Willets' Point and West Point, N. Y., and as Assistant Engineer in the construction of fortifications at Boston, Mass., Harbor, from 1866 to 1872. He was Assistant Engineer on the improvement of the mouth of the Mississippi River, from December 9, 1872, to May 1, 1876, and of Galveston Harbor, Texas, from 1876 to 1881. Captain Davis was then assigned to duty with the Light House Establishment, as Engineer of the Tenth Lighthouse District (1882), and afterward of the Eleventh District (1882 to 1886), and was also in charge of the improvements of the rivers and harbors of the Milwaukee, Wis., Engineer District, from 1882 to 1892.

From 1892 to 1896, Major Davis was on duty at Washington, D. C., in charge of the defensive works of the Potomac River, of certain river and harbor improvements in Maryland and Virginia, of the construction of a sewerage system at Fort Monroe, of the reconstruction of the Washington end of Long Bridge, of repairs to the Aqueduct Bridge across the Potomac, and of a survey and plan of a bridge across the Eastern Branch of the Potomac at the foot of South Capitol Street, Washington. He was also, during this period, a member of the Board of Officers to report on the various types of disappearing carriages for seacoast fortifications, and, from 1895 to 1896, was in charge of increasing the water supply of Washington, D. C.

From 1896 to 1902, Colonel Davis was stationed at San Francisco, Calif., in charge of the fortifications in that locality and in the Hawaiian Islands, as Engineer of the Twelth Lighthouse District. He also served as member of the California Débris Commission to regulate hydraulic mining. During part of this time he had charge of the improvements of the harbors of Wilmington, San Diego, and San Luis Obispo, Calif., as well as of the construction of the breakwater at San Pedro, Calif.

From 1902 to 1904 he was Division Engineer of the Philippine Division, and was in charge of the designing of fortifications and of military map-making in the Philippine Islands. From 1904, until his retirement from active service, on February 16, 1908, General Davis was in charge of the improvement of the channels connecting the waters of the Great Lakes, between Chicago,

* Memoir prepared by C. McD. Townsend, Col., U. S. A. (*Retired*), M. Am. Soc. C. E.

Ill., Duluth, Minn., and Buffalo, N. Y., his work including the design and commencement of the construction of the third lock in the canal at Sault Ste. Marie, Mich., the enlargement of the channel of St. Marys River, and the construction of the Livingston Channel.

In 1907 and 1908, he was also Division Engineer of the Lakes Division, Detroit River. After his retirement from the military service, General Davis was employed in 1910 and 1911, as United States Agent in charge of fortifications and river and harbor work in the San Francisco District.

General Davis had a most pleasing personality, and, although he never posed as a specialist in any particular branch of engineering, he had executive ability which enabled him to accomplish successfully the numerous and varied duties to which he was assigned during his professional career.

His fame as an engineer will be perpetuated by the inauguration of the lock which bears his name at Sault Ste. Marie. The Davis Lock is a concrete structure, 1 350 ft. long, between miter sills 80 ft. wide, and has 24½ ft. of water on its miter sills at low water, with a lift varying from 17 to 21 ft., dependent on the varying stages. With its companion, the Sabine Lock, which has recently been constructed, it is one of the largest lift locks in the world, being exceeded only by those of the Panama Canal.

General Davis was elected a Member of the American Society of Civil Engineers on July 12, 1877. He was also a member of the Army and Navy Club of Washington, D. C.

GEORGE DEVIN, M. Am. Soc. C. E.*

DIED MAY 28, 1925.

George Devin, the son of M. L. and Amanda J. Devin, was born in Macon County, Illinois, on February 27, 1847. The family afterward removed to a farm near Des Moines, Iowa, where he attended the public schools.

In June, 1864, Mr. Devin enlisted in Company B, 48th Regiment, Iowa Infantry, and served until he was mustered out in the following October. In 1871, he entered Cornell University, from which he was graduated in 1873, with the degree of Bachelor in Civil Engineering.

Following his graduation, Mr. Devin was employed on construction work by the firm of T. B. White and Sons, of New Brighton, Pa. (afterward the Penn Bridge Company, of Beaver Falls, Pa.), and served as Superintendent of Shops for that firm.

He was afterward a member of the firm, and Manager, of the Pittsburgh Bridge Company, of Pittsburgh, Pa. Among his associates in the organization of this Company was Mr. John A. Nichols, formerly Master Mechanic of the Keystone Bridge Company. One of the notable works of that firm was the reconstruction of the Railway Suspension Bridge at Niagara Falls. In 1881, he engaged in independent contracting for bridges in connection with the Delaware Bridge Company, of Athens, Pa., of which Charles Macdonald, Past-President, Am. Soc. C. E., was then President.

During practically all his subsequent work, Mr. Devin was engaged in the design and construction of bridges, in contracting in connection therewith, or in a consulting capacity. He was identified at different times with the Wrought Iron Bridge Company, of Canton, Ohio, as Assistant Superintendent; the King Bridge Company, of Cleveland, Ohio, as Engineer; the Atlanta Bridge Company, of Atlanta, Ga., as Superintendent of Erection; with the late Theodore Cooper, M. Am. Soc. C. E., of New York, N. Y.; the Pottsville Bridge Company, of Pottsville, Pa., as Chief Engineer; the Penn Bridge Company, of Beaver Falls, Pa.; the American Bridge Company; the Kansas City Terminal Company, Kansas City, Mo., etc. He also was engaged in private practice as a Consulting Engineer in Chicago, Ill., New York, N. Y., and St. Louis, Mo.

On July 25, 1876, Mr. Devin was married to Emma R. Lowry, of New Brighton, Pa., who died in St. Louis, Mo., in 1922.

Shortly after his wife's death, Mr. Devin retired from active work and, later, became an inmate of the National Home for Disabled Volunteer Soldiers at Sawtelle, Calif., where he remained until his death. He was buried in the Home Cemetery with full military honors on June 1, 1925.

* Memoir prepared by T. Sidney White, M. Am. Soc. C. E.

He was the eldest of six brothers, only one of whom, James Devin, of Des Moines, Iowa, and a sister, Mrs. Sarah D. Clarke, of Redwood City, Calif., survive him.

Mr. Devin had a great liking for mechanical problems, and took much pleasure in mathematical calculations, investigations, and analyses, and, also, in studies of stresses in iron and steel structures.

He was a man of absolute integrity, considerate of others, and possessed of a strong sense of loyalty to and affection for his friends.

Mr. Devin was elected a Member of the American Society of Civil Engineers on September 7, 1887.

JOHN HOFFMAN DUNLAP, M. Am. Soc. C. E.*

DIED JULY 29, 1924.

John Hoffman Dunlap, son of George Harlan and Mary Catherine (Folger) Dunlap, was born in Harrisville, N. H., on September 9, 1882. He was educated in the public schools of Concord, N. H., and in Dartmouth College, from which he received the degree of Bachelor of Arts in 1905, and the degree of Civil Engineer from the Thayer School of Civil Engineering in 1908. Between 1905 and 1908, besides doing the work for his engineering degree, he served with the United States Reclamation Service, in Nevada, and for a time with the Pennsylvania Lines West, in Columbus, Ohio.

During the summer of 1908, Mr. Dunlap was Field Instructor in the Thayer School, and in the fall of that year he was engaged as Instructor of Descriptive Geometry and Drawing by the College of Applied Science of the State University of Iowa, with which institution he remained until his appointment as Secretary of the American Society of Civil Engineers, in 1922. He was made Instructor in Civil Engineering in 1909, Assistant Professor of Hydraulics and Sanitary Engineering in the Department of Civil Engineering in 1911, Associate Professor in 1917, and Professor in 1920.

During his stay in Iowa Mr. Dunlap was engaged in engineering work, as time permitted, was for seven years Secretary-Treasurer of the Iowa Engineering Society, and for one year its President, and he wrote and published more than twenty-five papers on various engineering matters, principally in the field of sanitary engineering practice.

He was also active in the formation of the Iowa Section of the American Society of Civil Engineers and was its President for one year.

Mr. Dunlap was a member of the Society for the Promotion of Engineering Education, the New England Water Works Association, the American Water Works Association, the American Public Health Association, the Proportional Representation League, Gamma Alpha, Phi Beta Kappa, Sigma Xi, and of the Sigma Nu Fraternity. During the later years of his residence in Iowa City, he was President of the Social Service League of Iowa City, and a member of the Executive Committee of the Boy Scout Council.

On June 29, 1910, in Franklin, Vt., he was married to Fanny Maria Gates. She and three sons, George Spaulding, Richard Folger, and Clark Gates, are living.

Mr. Dunlap's death resulted from injuries sustained in a railroad wreck near Chicago, Ill., on June 30, 1924, as he was returning from the Annual Convention of the Society that had been held in Pasadena, Calif. He was buried in Franklin, Vt., on July 31, 1924.

Mr. Dunlap was elected an Associate Member of the American Society of Civil Engineers on April 17, 1917, and a Member on June 6, 1921. He was elected Secretary of the Society on June 19, 1922.

* Memoir prepared by Anson Marston, R. W. Crum, and the late William G. Raymond, Members, Am. Soc. C. E.

As almost all Mr. Dunlap's professional life was lived in the State of Iowa, it seems fitting to record first something of the position he won for himself there.

A session of the Iowa Section of the American Society of Civil Engineers was held in Iowa City on October 7, 1924, to honor his memory. At this session addresses were made by C. S. Nichols, M. Am. Soc. C. E., Professor at Iowa State College, and Past-President of the Iowa Section, Lloyd A. Canfield, Assoc. M. Am. Soc. C. E., a former student under Mr. Dunlap and Secretary of the Iowa Engineering Society, Anson Marston, M. Am. Soc. C. E., Dean of Engineering, Iowa State College, then Vice-President of the American Society of Civil Engineers, and the late William G. Raymond, M. Am. Soc. C. E., Dean of the College of Applied Science of the State University of Iowa. Excerpts from the addresses of Professor Nichols and Dean Raymond will indicate Mr. Dunlap's position in the State as an engineer, as a teacher, and as a citizen.

Professor Nichols spoke, in part, as follows:

"We are gathered here to-night, a company of loyal friends, with hearts and minds in perfect harmony, to pay honor to a departed comrade. How empty must sound any words we shall utter in our feeble attempts to express our admiration for and appreciation of John Dunlap. Much more appropriate would be a session of silent meditation together, mentally building him into our midst once more, and reviewing in retrospection the pleasant and profitable association we have had with him.

"John Dunlap was first of all a man, a Christian man whose every activity was in keeping with his noble principles and high ideals. He was an engineer, true, but only because through this profession he found the best opportunity to use his talents for service. To this end he became the best engineer possible. This was to him but natural, because it was fundamentally essential to insure a full measure of usefulness.

"He looked well to his tools, both as to manufacture and condition, and he became proficient in their uses. In his tool chest were some most valuable pieces of equipment. If we were to pick them out, one by one, we would probably choose first of all his Refinement of Thought. Who could ever forget his masterful address at Sioux City, in 1922, as President of the Iowa Engineering Society, on 'The Engineer's World', and this earnest appeal to his fellow engineers: 'Shall we not each of us dedicate ourselves, our lives, our training, our property, all that we have and are, in order that we may serve as good citizens of a noble country, and of a world growing steadily better, because it is an engineer's world in which each engineer is playing his part, with a firm and steadfast faith that the soul of civilization is not meat and raiment, but life and spirit.' Then and there he probably as nearly as ever bared his soul to his fellow men. This took courage, indomitable courage, because it was a radical departure from the ordinary, and it was only because he was sound personally that he could say those things.

"And right there he showed the handiwork of another important tool, Definiteness of Purpose. Never, under any circumstances, could John Dunlap be swerved from the line of duty which he had set up for his life. Even as he pioneered, he kept the line straight, his compass in his hand. He had the courage to speak his convictions even though he stood alone.

"Did any of you ever fail to understand him when he spoke to you either in public address or in private conversation? I cannot conceive of it. He was master of those exceptional tools, Clearness of Thought and Clearness of Presentation. He had a wonderful ability to integrate ideas and impressions

and to rationalize them into concrete expression, whether in speech, in illustration, or in construction. Through these characteristics he gained the finest confidence of those with whom he dealt.

"Diligence also was his constant aid, and explains in large part his wonderful capacity for accomplishment. Most things did not come easy for him; he worked as probably few of us ever will know how.

"Of him it can be said truthfully that he accepted Personal Responsibility for his every action. There was no passing of responsibility by John Dunlap. If he made mistakes, he acknowledged them, and made amends so far as it was physically possible. He was thoughtful and deliberate in his preparations, but when the time came to act there was brought into play an irresistible force for accomplishment. But his zeal for accomplishment and his earnestness in putting things through were ever tempered by an abundance of Consideration of Others. Even his opponents in controversy considered him a clean and honorable fighter for the principles he championed.

"Above all we like to recall his Cheerful Disposition and his saving Sense of Humor. Of his perplexities he kept us in complete ignorance, except as they were common to a considerable group and called for group discussion. Always bright and happy, his presence added joy to any gathering, large or small.

"These are the things which made him a man among men, and, coupled with native ability, made him a good engineer. We are but incidentally interested in the details of his professional work, as they contributed to his success only in the experience they afforded, and the opportunity they offered for contact with men and for the rendering of personal or community service. It is of interest, however, to follow through the high points of his work, and catch the point of view throughout.

"It was while acting as Field Instructor in the summer of 1908 at his Alma Mater that he saw the large opportunity for service in the intimate contacts which are possible between instructor and student, and decided to devote himself to educational work. He joined the Engineering Faculty of the State University of Iowa, where he rose in rank and influence until his call to the executive leadership of our National Society.

"During the summer of 1915, while engaged in a water-works survey for the Extension Division of the State University, he caught his first real perspective of conditions surrounding the administration and operation of municipal utilities, and saw the opportunity of and great need for those improvements which only the experienced engineer can direct properly. It was then that his personal opportunity and duty became plain. During that summer he visited about forty of our best Iowa cities, and while employed only to gather data for a report, he was diligent in his efforts to assist the local officials in remedying their difficulties. The full value of this voluntary advice and personal service will never be known, but there is much evidence that through his report and recommendations there resulted not only local improvements in many of the towns visited, but State legislation as well, which through the years to come will safeguard the interests of both municipal and private water-works companies.

"During the summer of 1916, he was Engineer in charge of the construction of the Sewage Treatment Plant for the State Hospital and Colony for Epileptics at Woodward, and there became impressed that an engineer's work was not complete until he had provided adequately for the future operation of the plant by the layman operator. He made a remarkably clear perspective sketch of this plant, showing every operating detail, accompanied by detailed instructions for its operation, and had them framed and hung in the plant. He also spent hours of his own time in providing for satisfactory supervision and maintenance.

"The summers of 1917 and 1918 were spent under the auspices of the State Board of Health inspecting sewage treatment plants in all parts of the State. In this capacity, he rendered the same valuable service to the municipalities that had characterized his work two years before in connection with the water-works plants. Ever on the outlook for methods of making improvements, he formulated excellent recommendations for more complete supervision and control by the State Board of Health of the design, construction, and operation of municipal sanitary works.

"He was one of the most ardent supporters of the annual conferences of sewage treatment plant operators and contributed much to their success.

"In his consulting practice he invariably rendered his clients a service of high character. The esteem of a man by his own townspeople usually tells the true story. One instance in the case of John Dunlap will suffice. In 1915, a controversy arose between the Iowa City Water Company and the City as to water rates. The City retained John Dunlap as its engineer, and after investigation and consultation a satisfactory rate was agreed upon. For various reasons this rate continued throughout the World War period, although during this time the company is reported to have sustained a considerable loss. With continued high prices, therefore, some adjustments in the rate became necessary, and the Company chose John Dunlap to make a complete valuation of its properties and determine a proper rate for water service. As a result of his study and recommendation a rate was established satisfactory to both parties. Here was the case of a man who could serve either party to a controversy and satisfy both. Why was it? Because he was fair and square, and both sides knew that his decision would be rendered purely on the basis of what was right. And, of course, there was no question as to his technical ability.

"Several years ago, while in a large hospital, he watched a patient in the use of a drinking fountain. What he saw caused him to watch others, there and elsewhere. As he turned over in his mind these facts of observation he became convinced that elements of danger existed in the usual type of vertical jet drinking fountain because of lip contact with the jet spindle and the possibility of pathogenic bacteria remaining in the water column, much as a ball can be balanced on a vertical column of air. Examination proved his fears to be well founded, and inquiry through State boards of health revealed the fact that suspicion was arising elsewhere. He gathered all the information he could and presented a paper before the Iowa Section of the American Water Works Association, recommending the appointment of a committee to thoroughly investigate this problem of public health. The committee was appointed, Mr. Dunlap was made Chairman, served for three years, made a thorough investigation, enlisted the support of various agencies, gathered all available information and substantiating evidence on the subject, and submitted constructive recommendations for the adoption of a specific principle in drinking fountain design which would eliminate all possibility of disease transmission in the manner mentioned. These recommendations were published in the *Proceedings* of the American Water Works Association, have been commented on very favorably by sanitary authorities, and will result in much improvement in this important matter.

"How many of us watching the apparently little things and after turning them over in our minds assign to them a proper valuation? That was what John Dunlap was doing all the time. His mind carried over from a demonstration or experiment in physics to the drinking fountain jet of water, and he saw there an element of danger.

"Many of the substantial things which have been accomplished in this State could be traced back to their initiation in the mind of John Dunlap. He was an inspiring and a directing genius, but in his characteristic quiet way. In many instances others received the credit that should have gone to

him. We have been wont to claim credit for the passage of the Engineers' Registration Law, but we should not forget that John Dunlap was the quiet director of our efforts and to him should go a very large measure of credit for the creation of a Board of Engineering Examiners which will justify itself in the years to come after the trying early years have passed.

"John Dunlap was an example of the best type of engineer. He was active and able in the profession; he gave to his fellow engineers the benefit of his experience and observations through papers and otherwise; he entered heart and soul into the activities of those technical societies with which logically he should affiliate; he was a worker in public undertakings for the benefit of society, whether local, State, or National; he took active part in all the worth-while local affairs of his community; he was a pillar in his church; and, above all, he was loyal and true to his home and his family.

"What more can we say of John Dunlap than that he was a real man, a real engineer, and a real friend? He left us while yet a young man and his work but nicely started, yet the place he filled will long be vacant. The lesson he left us is this: Engineering provides an excellent opportunity for service of the highest type, if we but have the point of view. May we not all make honest endeavor to get more nearly the point of view which John Dunlap had, to the end that we may make our profession nobler and worthier of universal respect."

The late Dean Raymond spoke of Mr. Dunlap's work as a teacher and a citizen of Iowa City, saying:

"Mr. Dunlap was one of those teachers who are keenly alive to the desirability of improving curricula and teaching methods, particularly to the desirability of so improving them as better to develop the man in the student, by giving him breadth of vision and wholesome example. Clean in his living before these students, conscientious in his preparation for his appearances before them in the classroom, sympathetic, generous, and sound with his counsel as they came to him with their personal problems, or their problems of class, or general student activities, interested in their sports as well as in their work, Mr. Dunlap was an invaluable influence for good in the college.

"In the University organization, too, he was recognized to be one of the most useful of men, holding positions on the more important committees. Uniformly courteous even when differing radically with his colleagues, always finding good in the weakest of human beings, the right arm of his pastor in the work of his church, leader in the social service work of the community, John Dunlap unconsciously exerted an influence in University and community life that rarely is contributed by any one man. Not soon will he or his work in Iowa City and the University be forgotten."

In the brief period that Mr. Dunlap was permitted to serve the American Society of Civil Engineers as its Secretary, he made a most remarkably favorable impression on the Board of Direction to which he was immediately responsible, and gave evidence of a real vision of future Society usefulness, to the realization of which he was giving of the best that he had.

One who was familiar with his work as Secretary has this to say of him:

"Fresh from the activities of university life, enthusiastic for his new work, Mr. Dunlap threw himself into the service of the Society. There he found an outlet for his utmost efforts. For two short years he gave all his energies to the many-sided tasks that confront the Secretary of a National engineering organization. Wherever it extended, the Society work felt the impetus of his dynamic energy. This resulted in improvements in the internal machinery of the Soceity, in the quickening of the Society in its relations

with its members, and in the furthering of its external co-operation with other activities.

"It was at Mr. Dunlap's suggestion that comprehensive sub-committees of the Society's Committee on Technical Activities and Publications were established. He felt that thereby this important technical work would gain the interest, the counsel, and the assistance of a widespread membership. On his recommendation the Board of Direction adopted changes in the method of handling membership applications. These improvements included the establishment of local membership committees to secure more effective assistance in determining the qualifications of applicants, the revision of the application form, and the classification of applicants for action by the Board, whereby the unquestioned cases were approved more expeditiously and the intermediate to doubtful cases could thus receive a more careful attention.

"Following Mr. Dunlap's suggestion, a questionnaire was sent to members of the Society to learn their special engineering interests. When these forms were catalogued they served to indicate the direction and extent of the major activities of the membership and to provide useful lists of individuals to serve the Society in their particular fields.

"Perhaps his external relations—with Local Sections, Student Chapters, and individual members—were even more successful in their results. For example, he visited every Local Section; in some cases he was enthusiastically urged to return for the second and even the third conference. Similarly, he talked to the Student Chapters, with which his long teaching experience stood him in good stead. Mr. Dunlap's theory was that the administrative office of the Society, its individual members, and the Local Sections, could best co-operate as they knew each other better and appreciated the other's problems; that he could take the Society to the Local Sections and, in return, bring back timely suggestions for its improvement, thus cementing the two parts of the work through mutual understanding and interchange of ideas. These visits had the further effect of strengthening the Society before the public through the favorable presentation of its ideals which he made as an effective speaker.

"Finally, Mr. Dunlap's efforts were marked by a cordiality in the Society's relations with other organizations. Early in his official work he evinced a warm interest in the efforts of the Engineering Societies' Employment Service. He regarded this as a most potent force for assisting engineers and for improving the standards of engineering employment. At a critical period in the existence of this activity, he won over for it the Society's continued support. Even before his death the recovery of the Employment Service had vindicated his judgment.

"Another co-operative effort from which Mr. Dunlap expected great things and to which he gave much study was the Joint Conference Committee (comprising the Presidents and Secretaries of the Founder Societies) formed to consider important matters of common interest to the four National Societies. During its first year, Mr. Dunlap, as Secretary of this Committee, was active in formulating policies under which it has since continued to prosper.

"In still another direction—engineering education—he played an important part. From its inception he was closely connected with the investigation of engineering colleges and curricula carried on by the Society for the Promotion of Engineering Education: First, as a member of the Development Committee of that Society, which formulated the project; then, as a member of its Board of Investigation and Co-ordination, which superseded in the actual conduct of the work; and, finally, as an *ex officio* representative of the American Society of Civil Engineers on the Board of Educational Councillors, a committee of the National Engineering Societies co-operating in the investigation. Thus, in his dual capacity, he was of invaluable help to the work, forming a

strong point of official contact between engineering teaching and engineering practice.

"In all Mr. Dunlap's official relations—with his confrères at Headquarters, with other officers, with members or students, with outside organizations—his motto was 'Service'. Whether it was a fundamental policy of the Society or a courtesy to a single distant member, the matter received his closest attention. He gave his all, without stint; this service was not only the last, but the greatest of his life—great in its conceptions and great in its accomplishments."

On August 1, 1924, the Executive Committee of the American Society of Civil Engineers adopted resolutions, as follows:

"Whereas, it has pleased Almighty God to take from us our beloved Secretary,

JOHN HOFFMAN DUNLAP

"Be It Resolved: That the President and the Executive Committee, in behalf of the Board of Direction and the members of the American Society of Civil Engineers, express their sense of the great loss which the Society and the Engineering Profession of the United States has suffered by the death of our Secretary, who by his labors in our behalf; by his faithfulness, efficiency and courtesy; by his advocacy of the highest ideals for the advancement of the Profession; by his unselfish efforts to serve every member of the Society; and by the example he set us as a Christian citizen, has made a lasting impression in our hearts.

"Be It Further Resolved: That our heartfelt sympathy be extended to his family, that his family be furnished with a copy of this resolution, and that it be spread upon our records, printed in the *Proceedings* and *Transactions,* and that copies be furnished to the technical press and to other Engineering Societies."

It seems to be fitting to close this memoir with a repetition of the inspiring words of Mr. Dunlap spoken as he retired from the Presidency of the Iowa Engineering Society and quoted by Professor Nichols in his address before the Memorial Session of the Iowa Section of the American Society of Civil Engineers:

"'Shall we not each of us dedicate ourselves, our lives, our training, our property, all that we have and are, in order that we may serve as good citizens of a noble country, and of a world growing steadily better, because it is an engineer's world in which each engineer is playing his part, with a firm and steadfast faith that the soul of civilization is not meat and raiment, but life and spirit.'"

CHARLES WELLS EDDY, M. Am. Soc. C. E.*

DIED JUNE 18, 1925.

Charles Wells Eddy was born on November 8, 1875, at Brunswick, N. Y. He was of Pilgrim stock, having been a descendant of Samuel Eddy, who came to America in 1630, and Kenelm Winslow, who came in 1629, both joining the Pilgrim Company at Plymouth, Mass. He was also a descendant of Robert Collins, of Sandown, N. H., who served as Captain of the New Hampshire Volunteers in the Revolutionary War.

In 1884, Mr. Eddy moved with his parents to Simsbury, Conn., where he received his early education at the public schools and McLean Seminary. He then entered the Connecticut Agricultural College and was graduated in the Class of 1893.

From 1893 to 1900, he was engaged in land surveying and in surveying for and inspection of State roads for various towns of Connecticut. From November 14, 1900, until his death, on June 18, 1925, with the exception of about a year, Mr. Eddy was employed by the City of Waterbury, Conn., in the Bureau of Engineering. During 1900 and 1902, he was engaged as Surveyor on the completion of the Wigwam Reservoir of the Waterbury Water Supply. In 1906 and 1907 he served as Resident Engineer in charge of the construction of the dam of the Coe Brass Company, at Torrington, Conn.

From 1908 to 1916, Mr. Eddy was engaged in preliminary surveys for, and was Resident Engineer in charge of, the construction of a dam for the Morris Reservoir which forms a 2 000 000 000-gal. storage reservoir for the City of Waterbury. Following this engagement he was employed in other water supply work and in making preliminary surveys for the 7-mile tunnel which the City of Waterbury is now constructing to enlarge its present water system, and also in designs for future controlling works for this system. At the time of his death, he was Resident Engineer at Thomaston, Conn.

Mr. Eddy was always loyal and keenly interested in his work, never sparing himself when it required unusual attention.

That he was willing to devote some of his time for the benefit of others is shown by the fact that besides being a member of the Society, he had also been a member of the Connecticut Society of Civil Engineers since January, 1900, having been its President at the time of his death. He was also a member of the New England Water Works Association, and of the American Society for Municipal Improvements; Director of the Thomaston Water Company; President of the Thomaston Sewer Company, the Thomaston Business Men's Association, and of the Masonic Temple Corporation of Thomaston.

Mr. Eddy was a member of the Congregational Church of Thomaston, and a Thirty-second Degree Mason.

He was married at Simsbury, Conn., on July 31, 1901, to Gertrude Holcombe, who survives him.

Mr. Eddy was elected an Associate Member of the American Society of Civil Engineers on March 2, 1909, and a Member on June 24, 1914.

* Memoir prepared by R. A. Cairns, M. Am. Soc. C. E.

MARTIN RYERSON EVERETT, M. Am. Soc. C. E.*

DIED FEBRUARY 4, 1926.

Martin Ryerson Everett was born at Frankford Township, near Branchville, N. J., on August 21, 1867. He was the son of Martin Ryerson and Julia (Roe) Everett, both of whom died before he attained the age of ten. He attended the public schools of Branchville and, after graduating, entered the employ of one of the merchants of that town.

In 1890, becoming dissatisfied with the limited opportunities of a rural community, Mr. Everett went to New York, N. Y., and accepted a clerical position with the Jackson Architectural Iron Works. While thus employed he met many civil and mechanical engineers and after learning of their achievements became so interested that he decided to become an engineer. He immediately entered the evening sessions at Cooper Union and received the degree of Bachelor of Science in 1898. At this time he was in charge of the Structural Shop of the Jackson Architectural Iron Works.

In 1900, Mr. Everett went to Newark, N. J., to be Shop Manager and Designer of Steel Structures for Cooper and Wigand, Contractors and Engineers. After receiving the degree of Civil Engineer from Cooper Union on May 28, 1904, he became connected with the Cooper-Wigand-Cooke Company.

From 1906 to 1908, Mr. Everett was General Manager of the Cooper Iron Works, of Newark. Resigning from this position he engaged in private practice as a Contractor and Engineer, continuing as such until 1910, when he became Vice-President of the Hedden Iron Construction Company, in charge of operation. After serving as Vice-President with this Company for two years he resigned and again engaged in private practice.

In 1914, he incorporated the Company known as the Martin R. Everett, Incorporated, Engineers and Contractors, with headquarters in Newark, and served as President until his death from a lingering illness on February 4, 1926.

Mr. Everett had a wonderful and unique personality, made many friends, and was a favorite with those whose good fortune it was to know him well. Always interested in sports and athletics as a young man, he was an enthusiastic supporter of them throughout his life and was never happier than in witnessing some big game.

Mr. Everett had resided in East Orange, N. J., for twenty-five years. He was an active member of the Arlington Avenue Presbyterian Church of that city, and was also a member of many fraternal and community organizations.

He is survived by his widow, Lillian Robertson Everett, his son, Martin Ryerson, Jr., and his sister, Miss Jennie H. Everett, of Bayonne, N. J.

Mr. Everett was elected a Member of the American Society of Civil Engineers on April 14, 1919.

* Memoir prepared by Victor Graff, Pres., Martin R. Everett, Inc., Newark, N. J.

GEORGE THOMAS FORSYTH, M. Am. Soc. C. E.*

DIED AUGUST 31, 1925.

George Thomas Forsyth, the son of Elizabeth and Jonathan Forsyth, was born in Providence, R. I., on January 19, 1876. In his early childhood his parents moved to Salinas, Calif., where his boyhood was spent and where he received his public school education.

In 1895 he entered Leland Stanford, Jr., University. There he pursued a special engineering course until the spring of 1899, when he accepted a position with the Union Iron Works at San Francisco, Calif., where he was engaged until December, 1900, in ship-fitting and the erection of marine engines.

In January, 1901, Mr. Forsyth entered the service of the Southern Pacific Company at San Francisco, as Draftsman in the office of the Engineer of Bridges. This was on the eve of the great physical development which was to take place in that railroad and in other lines then being associated with it by the late E. H. Harriman. In 1903, Mr. Forsyth was promoted to the position of Chief Draftsman, in which capacity he was in direct charge of bridge and miscellaneous structural and mechanical design. He remained in this position until 1905, during which period he took an important part in the preparation of the common standard plans of the Harriman System. These plans touched almost every physical detail of railroad construction and maintenance, and their value is evidenced by the fact that most of them are still in use on the Union Pacific and Southern Pacific Systems. Perhaps his most effective efforts were directed to the standard bridge designs, in which he showed ingenuity and clearness of thought that seemed sometimes to approach genius.

In 1905, Mr. Forsyth was appointed Bridge Engineer of the Oregon Railroad and Navigation Company and of the Oregon Lines of the Southern Pacific System. In the great program of extensions and improvements which had just begun, and during the next eight years, he was continuously occupied with the design and construction of many bridges, but he also found time to devote himself to power-plant and wood preservation problems. His most important work in this period was in connection with a large double-deck lift-bridge over the Willamette River, at Portland, Ore., which was constructed under his supervision, and to the design of which he contributed largely.

In 1913, Mr. Forsyth decided that better opportunities existed outside of railroad service, and with the regret and good wishes of his associates he joined the Union Bridge Company of Kansas City, Mo., and later the contracting firm of Diecks and Diecks. In these engagements he successfully carried to completion a number pneumatic caisson and subaqueous tunnel projects.

* Memoir prepared by S. Murray, M. Am. Soc. C. E.

In 1920, Mr. Forsyth returned to Portland, Ore., where he opened an office and engaged in independent practice. Early in 1924, he was called to take charge of a tunnel beneath the Duwamish River at Seattle, Wash., in the construction of which difficulties had developed that appeared to be insurmountable. Under his direction, this project, which had seemed palpably a failure, was carried to a swift and successful completion. This was his last important work.

Mr. Forsyth was a brilliant mathematician and a leader in bridge design and construction. Loyal and steadfast, he was a man of independent thought and of original and whimsical humor. His passing leaves a gap which his friends and associates can never fill.

He was married on June 16, 1902, to Florence Ligon, who, with one son, George Thomas, Jr., survives him.

Mr. Forsyth was elected a Member of the American Society of Civil Engineers on October 1, 1912.

HENRY FOX, M. Am. Soc. C. E.*

DIED JUNE 8, 1925.

Henry Fox was born at Troy, N. Y., on September 12, 1864, and was educated at the Baraboo, Wis., High School.

He entered the service of the Chicago and Northwestern Railway Company in the spring of 1885, as Rodman, and remained in that position during the construction of the Northern Illinois Railway. In 1886, he was employed as Levelman, on the survey and construction of the Maple River Railway, at Onawa, Iowa. During 1887, he was in camp in the woods of Northern Michigan, while in charge of a division of the Iron River Railway then being built between Iron River and Watersmeet.

In 1889, Mr. Fox entered the University of Wisconsin as a Special Student, changing in the Sophomore year to the Mechanical Engineering Department, and was graduated in 1892 with the degree of Bachelor in Mechanical Engineering. During his summer vacations he was employed by the Chicago and Northwestern Railway Company, on general maintenance-of-way work. As soon as he left the University, he was engaged as Inspector by the Chicago and Northwestern Railway Company, during the erection of the Hall System of automatic block signals on the Galena and Wisconsin Divisions. In 1893, he became Mechanical Draftsman at St. Joseph and Hermann, Mo., with the Missouri River Commission.

In 1895, Mr. Fox was employed on the Illinois and Mississippi Canal (Hennepin Canal) and the Chicago and Northwestern Railway, and in 1896, served as Resident Engineer on railway construction for that Company at Gillett, Wis. From 1896 to 1897, he served as Assistant Engineer for the Railway Company at Manitowoc, Wis., in charge of the construction of tracks, buildings, docks, and car-ferry slips. From 1898 to 1900, Mr. Fox was employed as U. S. Inspector for the Illinois and Mississippi Canal, in charge of the construction work of the Second Division, Eastern Section.

From 1900 to 1902, he held the position of Locating Engineer on the Choctaw, Oklahoma and Gulf Railroad, at Hartshorne, Okla.; and from 1902 to 1905 that of Junior Engineer with the U. S. War Department, on field work, in charge of secondary triangulation on seven deep waterway surveys on the Illinois and Des Plaines Rivers from Lockport to Grafton, Ill., a distance approximating 300 miles. On the completion of this work, he was transferred from Chicago, Ill., to Baltimore, Md.

From 1905 to 1909, Mr. Fox served as Assistant Engineer in the U. S. Engineer's Office at Baltimore in local charge of river and harbor improvement and fortifications, as well as ship channels in the Chesapeake and Patapsco Rivers, leading to Baltimore. From 1909 to 1915, he was connected with the Maryland Dredging and Contracting Company, (now the Arundel Corporation of Baltimore), as Superintendent and Engineer of general river and harbor improvement work along the Atlantic seaboard, and was engaged

* Memoir prepared by the Misses Pearl C. and Esther Fox, Baltimore, Md.

in construction work at Buzzards Bay and on the Cape Cod Canal. He also supervised the contract work on the New York State Barge Canal, and on bridges, locks, and dams on the Mohawk River.

From 1916 to 1917, he held the position of U. S. Assistant Engineer at Little Rock, Ark., in charge of field parties for the survey of the Arkansas River, from Little Rock to the mouth of the river, a distance of approximately 150 miles. In 1917, he was relieved from duty and ordered into active military service as Captain of Engineers and served during the World War until 1919.

During the latter part of 1919 and part of 1920, Captain Fox was employed with the Baltimore and Ohio Railroad as Assistant Engineer to the District Engineer of Construction, on new tracks, bridges, and engine terminal facilities, such as roundhouses and machine shops, at Grafton, Fairmont, and Clarksburg, W. Va.

In 1920, he returned to the U. S. Engineer Department as Assistant Engineer on construction, Division No. 1, Wilson Dam, Muscle Shoals Project, on the Tennessee River at Florence, Ala. When this work was discontinued on May 1, 1921, he entered the State Highway Department of Florida at Tallahassee, as Location and Construction Engineer, and built concrete bridges and 20 miles of State Highway No. 8.

During 1922, Captain Fox was employed as Engineer with Trammell and Holway, Engineers, on the Spavinaw Water Project for the City of Tulsa, Okla., and located 55 miles of 54 and 60-in. conduit lines at Spavinaw Dam and Tiawah Tunnel. In 1923, he accepted a position with the Walbridge-Aldinger Company, Contractors for the Spavinaw Water Project, at Tulsa, for which Company he constructed 55 miles of standard-gauge railway.

During 1924, he entered the State Highway Department of Oklahoma as a Federal Engineer in charge of field parties in the northern part of the State, which position he held at the time of his sudden illness and death on June 8, 1925.

On October 18, 1893, he was married to Pearl Irwin Clark, of St. Louis, Mo. He is survived by a son, Henry Clark Fox, a student in Civil Engineering at the University of Maryland, and two daughters, Pearl C. and Esther Fox, of Baltimore.

Captain Fox had been a member of the Western Society of Engineers since 1900, and of the American Association of Engineers (Washington, D. C., Chapter), since 1921. He was also a member of Lodge 34 A. F. and A. M.; of Lodge 49, of Baraboo, Wis.; a Thirty-second Degree Mason; and a Member of Zamora Temple and Alabama Consistory, of Birmingham, Ala.

Captain Fox was elected a Member of the American Society of Civil Engineers on July 1, 1909.

STEPHENSON WATERS FOX, M. Am. Soc. C. E.*

DIED MAY 8, 1925.

The unfortunate sinking of the Steamer *Norman* in the Mississippi River below Memphis, Tenn., on May 8, 1925, took a large toll from the membership of the Society. Of this number was Stephenson Waters Fox, of Paducah, Ky.

"S. Waters" Fox, as he preferred to be known, was born in Natchez, Miss., on August 3, 1856. He was graduated from the Rensselaer Polytechnic Institute, Troy, N. Y., in 1876 as a Civil Engineer. Previous to and following his graduation, Mr. Fox, like many young engineers of that day, obtained his early experience on railroad surveys, these engagements being on several railroads in Southern Indiana, Kentucky, and Tennessee.

In 1876, he was for a short period with the Snead and Company Iron Works, of Louisville, Ky., as Computer. In 1878, he became an Assistant to the late Brig.-Gen. Charles R. Suter, U. S. A. (*Retired*), M. Am. Soc. C. E., then Major, Corps of Engineers, U. S. A., and was engaged on improvement work on the Mississippi River. He remained as an Assistant Civil Engineer with the U. S. Corps of Engineers, until 1903. During this period he had a large degree of responsibility for the design and construction of many important river improvements, principally of revetments for the protection of caving banks and levees, and the control of the Mississippi and Missouri Rivers and their tributaries, including works at Choctaw Bend, Mississippi River, and at Glasgow, Cedar City, St. Charles, St. Joseph, Kansas City, Mo., and other points on the Missouri River.

In 1903, Mr. Fox resigned from the Government service and opened an office at Kansas City, Mo., as a Consulting Engineer, specializing in river improvements, the construction of levees, and land reclamation. In this capacity he served, with much credit, several railways, municipalities, and other corporations, on difficult projects of considerable magnitude and also designed several large suction dredges. During this period, he was for several years Chief Engineer for the Union Terminal Railway Company, of St. Joseph, Mo. Later, he moved to St. Louis, Mo., where his engineering activities were of similar character.

In 1917, when the United States entered the World War, Mr. Fox was commissioned a Major in the Engineer Reserve Corps, and served with credit in the Division of Inland Waterways of the United States Railroad Administration as Engineer of Contracts and Construction. After the close of the war Major Fox became associated with Cox and Stevens, Marine Engineers, of New York, N. Y., and while with this firm designed a new type of boat for use on the Erie Canal.

In 1922, he became Consulting Engineer for the Joyce-Watkins Company, large producers of railroad ties, telegraph poles, etc. This engagement necessitated his removal to Paducah, Ky. In addition to the questions of timber

* Memoir prepared by Charles F. Loweth, Past-President, Am. Soc. C. E.

treating and of transporting ties and poles by river, he had presented to him many new and novel problems, such as the design of special machines and floating equipment for handling a special product. To these he brought to bear those qualities of patient, thorough, and open-minded investigation which were characteristic of him and which had contributed largely to his success and had won for him the hearty commendation of many employers. His zeal and interest in new and novel, yet always practical, problems kept him much younger than his years.

Major Fox was considered an authority on levees, bank protection, and other kindred problems relating to the control of the Missouri River, to which he had given much careful and enthusiastic study. In 1905, he presented to the Society a noteworthy paper entitled "Technical Methods of River Improvement as Developed on the Lower Missouri River, by the General Government, from 1876 to 1903."*

Outside his professional work, Major Fox always found time and interest for many worth-while activities and lent his assistance to many enterprises that promised good to the community. Although a resident of Paducah for only three years, he was, at the time of his death, President of the local Lions Club; he had been for some time a member of the City Planning Commission and had been appointed its Chairman within a month of his unfortunate death. For many years he was an active member of the Presbyterian Church.

Major Fox was a man of culture and appreciation, fair-minded, broad, and kindly. He had a genial and happy disposition and made and retained many friends. If one may be judged by his aspirations and ideals, as well as by his accomplishments, the sentiments in the following verses, a typewritten copy of which was found on his remains, reflects in some degree the measure of the man:

"Carve your name high o'er the shifting sands,
Where the steadfast rocks defy decay.
All you can hold in your cold, dead hand
Is what you have given away.

"Count your wide conquest o'er sea and land,
Heap up your gold, hoard as you may.
All you can hold in your cold, dead hand
Is what you have given away.

"Build your pyramids, skyward, let them rise,
Stand gazed at by millions, cultured they say.
All you can hold in your cold, dead hand,
Is what you have given away.

"Silver and gold and jewels, all so grand,
King of the solon, a mart, a day.
Yet all you can hold in your cold, dead hand
Is what you have given away."

The great river on whose banks Major Fox was born and spent his boyhood days, and near which most of his life was spent, finally claimed him

* *Transactions*, Am. Soc. C. E., Vol. LIV (1905), p. 280.

as its own; but the works in the design and construction of which he had so large a share, built to control the river, are yet unconquered by it.

He is survived by his widow, a son, Burton C. Fox, of Portland, Ore., and three daughters, Mrs. Jett Blackburn, of Portland, Ore., Mrs. Cleveland X. Hening, of San Francisco, Calif., and Mrs. Harvey J. Schmelzer, of Kansas City, Mo.

Major Fox was elected a Junior of the American Society of Civil Engineers on July 7, 1880, and a Member on October 6, 1886.

MILTON HARVEY FREEMAN, M. Am. Soc. C. E.*

DIED MARCH 24, 1925.

Milton Harvey Freeman was born at Crary Mills, St. Lawrence County, N. Y., on October 12, 1871. He was the son of the late Noel O. Freeman whose family came from Vermont, having settled there in Colonial days. His mother was Mary E. Harvey, whose family on the maternal side settled at Gill, near Northfield, Mass.

Mr. Freeman was graduated from the Potsdam Normal School, at Potsdam, N. Y., in June, 1895, and for the next two years, taught in district schools in New York State. In September, 1897, he became principal of the Heuvelton Union Free School at Heuvelton, N. Y., where he remained until June, 1899. In the fall of that year, he entered the University of Michigan, with the idea of specializing in mathematics and physics for the purpose of teaching these subjects. While at college, his interests were directed toward engineering as a profession, and he was graduated from the University in June, 1903, with the degree of Bachelor of Science in Civil Engineering.

Immediately after his graduation, Mr. Freeman was employed by the Michigan Central Railroad Company as Assistant Engineer in charge of party, locating branches for logging, double-tracking the main line, laying out freight yards, and maintenance-of-way work. He continued in this position until September, 1905, when he returned to the East and was employed by the Pennsylvania Tunnel and Terminal Railroad Company on the East River Section of the Pennsylvania Tunnel, where he remained as Inspector of shield tunnels until 1907. From 1907 until May, 1909, he served as Assistant Engineer in charge of a shift on shield-driving, building iron lining, and placing the concrete lining in four tubes westward from the Long Island City Shaft.

In May, 1909, Mr. Freeman accepted a position with the Board of Water Supply of New York as Inspector in rock tunnel and in charge of a shaft field party at the Rondout Siphon, where he continued until December of that year, when he was made Assistant Engineer in the Designing Department on studies of reinforced concrete. In March, 1910, he became Assistant Engineer-Chief Inspector on placing concrete tunnel lining. He was engaged in this work until March, 1911, when he became Section Engineer at the Rondout Siphon in charge of concreting and grouting the tunnel lining, which included tests and experimental work in grouting porous rock. He continued at this work until April, 1914, when he was made Section Engineer on the Narrows Siphon in charge of experimental work on the flexible water-tight joint for the 36-in. cast-iron pipe.

In September, 1914, Mr. Freeman entered the employ of the New York State Public Service Commission for the First District as Resident Engineer on the Brooklyn side of the Old Slip-Clark Street Tunnel, from mid-river

* Memoir prepared by James F. Sanborn and Ole Singstad, Members, Am. Soc. C. E.

to the Brooklyn Approach. These were shield-driven tunnels under the East River and the streets of Brooklyn and were to be included in the Rapid Transit Railroad System of New York. In December, 1916, he became Resident Engineer in charge of the entire River Section and Manhattan Approach of the 60th Street, New York-North Jane Street, Long Island City Tunnels, where he remained until the completion of excavation in October, 1918. He was then made Resident Engineer on the Manhattan shore of the 14th Street Eastern-North 7th Street Brooklyn Tunnel, and was employed on this work until June 30, 1919. During the latter period, he was also busy devising grouting methods and was in charge of the field work of stopping leakage at the Canal Street Station of the Brooklyn Rapid Transit Company in New York.

On July 1, 1919, he was appointed Resident Engineer to the New York State Bridge and Tunnel Commission and the New Jersey Interstate Bridge and Tunnel Commission under the late Clifford M. Holland, M. Am. Soc. C. E., Chief Engineer. He served with Mr. Holland and his associates in the work of investigation and preliminary design, and on January 1, 1921, became Division Engineer. On July 1, 1921, he was promoted to the position of Engineer of Construction, and remained in active charge of the construction work until the death of Mr. Holland, whom he succeeded as Chief Engineer on December 1, 1924.

The construction of The Holland Tunnel, formerly known as the Hudson River Vehicular Tunnel, is a monument to the high talent and painstaking ability of Mr. Freeman as a subaqueous tunnel engineer. Under Mr. Holland, Mr. Freeman had personal charge of construction on this pioneer project which far exceeds in proportions any tunnel built under similar conditions. The successful driving of five shields of a diameter of 30 ft. 4 in. and one of 31 ft. in itself suffices to mark him as a great engineer. The difficulties encountered in shield-driving, together with the problems of sinking five large shafts, one—the New York River Ventilation Shaft—surpassing in size any similar construction; the water-tightness of these enormous tubes; and the intricate concrete work on the inner lining, were problems that called for his best efforts. The success attained in these various branches of the work stand out as distinct accomplishments of high engineering ability.

Weakened by close application for many years to the numberless details of the work under his charge, his career was prematurely ended. He died at his home in Valhalla, N. Y., on March 24, 1925, of pneumonia after an illness of eight days.

Mr. Freeman was noted for a thoroughness in his knowledge of every detail and for his integrity of purpose and judgment in the solution of the intricate problems which were his responsibility. His devotion to his professional activities was intense and no labor was too great for him to give to the correct solution of whatever problem might be presented to him. He had a fine sense of justice and was noted for his consideration in never doing anything to hurt the feelings of his associates or subordinates. He had high ideals and was fearless in carrying through his purpose with standards of action that marked him among men. His influence was keenly felt by

those who came in contact with him, and his death leaves with them a sense of irreparable loss. With his ability was an unusual modesty which always marked his relations with others. One of the outstanding accomplishments of his professional career was the success that attended his work in stopping the leakage at the Canal Street Station of New York's Rapid Transit System. He prepared a paper on this subject, which he read before the Brooklyn Engineers' Club on April 15, 1920, and for which he was awarded the Alfred T. White Prize by the Club.

In 1917, Mr. Freeman was married to Gertrude Vandermark, of Stone Ridge, N. Y., who survives him.

The New York State Bridge and Tunnel Commission and the New Jersey Interstate Bridge and Tunnel Commission, of which Commissions Mr. Freeman was the Chief Engineer, adopted unanimously by a rising vote the following resolution as a mark of respect to the memory of their late Chief Engineer:

"Again the New York and the New Jersey Bridge and Tunnel Commissions meet in special session with a vacancy in the office of Chief Engineer. Milton H. Freeman, Chief Engineer to the Commissions, answered the final summons on Tuesday, March twenty-fourth, nineteen hundred and twenty-five, midway in the fifty-fifth year of an active, useful life. Appointed Chief Engineer December 1, 1924, to succeed the late Clifford M. Holland, he brought to the office eminent qualifications of education, trained judgment and wide practical experience. It may be said that he gave himself too unsparingly to the building of the Holland Tunnel. His service was intensive and untiring, his ways so quiet and unobtrusive, his words so considerate and his manner so courteous as to inspire respect and affection, making every member of our Commissions sensible of a personal loss and of the loss to the great undertaking in which we are engaged."

He was a member of the Brooklyn Engineers' Club and the University of Michigan Club of New York.

Mr. Freeman was elected an Associate Member of the American Society of Civil Engineers on July 1, 1909, and a Member on September 9, 1919. He served as a representative of the Society on the Joint Committee on Specifications for Concrete and Reinforced Concrete.

FRANKLIN IDE FULLER, M. Am. Soc. C. E.*

DIED DECEMBER 16, 1925.

Franklin Ide Fuller, the son of Leonard F. and Mary I. Fuller, was born on May 29, 1858, in Providence, R. I., of an old New England family. He attended the public schools, afterward becoming a student of civil engineering while working in the offices of the City Engineer of Providence. He spent four years in this office, winning several promotions, and mastering much of the technique of his chosen profession. At the end of his term, he entered the railroad service and was engaged in location and construction work on the New York, West Shore and Buffalo Railroad.

In 1883, Mr. Fuller went to Oregon and there was connected with railway work as representative of the Northern Pacific Terminal Company. The failure of the Northern Pacific Railway Company and the cessation of railroad construction in the Northwest at that time forced him to enter the contracting business in which he continued for four years, devoted largely to railway and heavy timber work. The next three years were spent with the Oregon Iron and Steel Company at Oswego, Ore., first, as Assistant to the Manager and, later, as Manager of the Company. Following this engagement he devoted a year to the real estate business.

In 1892, he became connected with the Portland Cable Railway Company, and remained with this Company and its successors, through several consolidations and mergers, until his death. He was active in the transportation development of the city, having done more perhaps to increase street-car lines in Portland than any other man. When the Cable Company became the Portland Traction Company, Mr. Fuller was made Manager and continued in this office until the Company was merged with the Portland Railway Company, when he was made General Manager.

By his forethought and hard work he saved his Company from failure during the panic of 1893. Banks, stores, hotels, and all lines of business were being closed. It was the blackest hour of Portland's history. Mr. Fuller at once put the line on as low an operating basis as possible. In order to get sufficient cash to keep it going, he sold books of street-car tickets among the business houses, even taking them into the residence districts and selling them from house to house. In 1904, came the amalgamation of the Portland Railway Company and the City and Suburban Railway Company, forming the Portland Consolidated Railway Company. Mr. Fuller was General Manager for a year, then the properties were purchased by the Clark and Seligman interests of Philadelphia, Pa., and New York, N. Y., and the Portland Railway Company was re-organized with Mr. Fuller as President.

The incorporation of the Portland Railway, Light, and Power Company came in 1906, with the addition of the lines and other property of the Oregon Water, Power, and Railway Company. Mr. Fuller became Vice-President in charge of the railways of the Corporation.

* Memoir prepared by J. P. Newell and D. C. Henny, Members, Am. Soc. C. E.

One of the monuments to his engineering ability and progressiveness is the Portland Heights Street Car Line. At the time it was projected, it was said to be impossible, and impracticable, but his professional eye saw the way clear to the fulfillment of his plan, and for years this was the most scenic city street-car line in America.

At the time of his death, Mr. Fuller was the oldest official in electric transportation work on the Pacific Coast, both in length of service and in age. He was active in the affairs of the Chamber of Commerce, at times serving as Director. He was also a member of the Woodmen of the World, Modern Woodmen of America, Royal Arcanum, Arlington Club, and Multnomah Club. He was a Director of the Lumbermen's Trust Company and of the Oregon Portland Cement Company.

On April 14, 1886, he was married to Anna Jessie Parrish, of Portland, who survives him, with one son, Leonard F. Fuller, Chief Engineer of the Thompson Radio Corporation, of Ridgewood, N. J.

The outstanding feature of Mr. Fuller's professional career was his work as an executive, with the task of administering the intricate affairs of a great public utility. In this position the many-sidedness of the man was apparent. In the settlement of the innumerable controversies which arise between city officials and a street railway company, Mr. Fuller's word was always conclusive. Whatever he promised he performed.

The esteem in which he was held by the City Council of Portland was expressed in the following resolution:

"On account of his long service in this community and his intimate knowledge of local transportation and traffic problems, the counsel of Mr. F. I. Fuller was always valuable in assisting us to arrive at a fair and reasonable solution. He was a man of exceptional ability, sound judgment, and high moral character, always patient and considerate of the rights of others. Even in the face of strenuous objections he was always calm, deliberate, and courteous in presenting his views, or in pressing the claims and wishes of the large utility he represented. We voice our deep regret at his passing, because we realize that Portland has lost a valuable citizen, and the profession of engineering an honorable and worthy member."

Modern conditions of employment offer many opportunities for friction with subordinates, but the character of Mr Fuller's relations with his men is best evidenced by their spontaneous tribute in a memorial addressed to Mrs. Fuller, in which they stated, in part:

"In the passing of Franklin Ide Fuller, we, his co-workers in the Portland Electric Power Company, feel keenly the loss of a faithful, sympathetic friend who was ever ready to advise and help us.

"His high sense of honor and loyalty to us and to the Company he so generously served have been an inspiration to all with whom he came in contact. We know no act of his life that does not express kindness, tenderness, and generosity.

"Any one who could occupy, with such gentle humility, the position he has so long and honorably filled is an example of the highest type of Christian character and is indeed a great man."

The esteem in which Mr. Fuller was held by his associates was voiced by the President of his Company as follows:

"He was well known and admired for his staunch qualities and rugged character by many thousands of citizens in all walks of life. He was quick to realize the needs of Portland and he always endeavored to serve his city and his fellow-men to the best of his ability.

"During the many years in which I have been associated with Mr. Fuller, his calm serenity, sure poise, and good judgment in the face of perplexing problems have always been a source of deep inspiration to his associates in our company. I have never known a more faithful friend, a more lovable companion or a more conscientious, loyal business associate."

The Directors of the Lumbermen's Trust Company expressed their sorrow in a memorial which is, in part, as follows:

"In the passing of Mr. Franklin Ide Fuller, the Lumbermen's Trust Company and its affiliated corporations have suffered the loss of a valued adviser, who, by reason of his broad acquaintanceship, his acute analysis of problems, his matured judgment and his unwavering integrity, has contributed largely to the material progress of our institutions. Beyond and above this his unselfishness, cheerfulness, modesty of character and boundless faith in his fellow men have endeared him to each one of us and have left to us a loving memory which we will always retain."

Mr. Fuller's whole life was permeated by principles to which his pastor and intimate friend, Dr. H. L. Bowman, bears testimony:

"Mr. Franklin I. Fuller during his residence in Portland was associated with the Church and with Christian work. In 1909 his membership was placed in the First Presbyterian Church with which he was actively affiliated. For the last thirteen years of his life he was a member of the Board of Trustees of the Church and for the last two years its President. He gave generously and loyally of his time and interest to the work of the Church.

"But Mr. Fuller's life was not one in which Christian character simply showed in Church activity. His life was an outstanding instance of the principles of the Master interpreted in terms of practical business life. In the beautiful and considerate fellowship of the home, in devoted friendship, in the sterling integrity of business conduct, in unselfish consideration of coworkers, in the activities of citizenship Mr. Fuller revealed the rich fruitage of Christian character. His was a life in which religion was woven so thoroughly as to become instinctive; with no impulse for display his life was yet a vocal testimony of the value of Christian ideals in building character and enriching personality.

"Such a life so mastered by the spirit of Jesus Christ becomes in memory as it was in fellowship a summons to the noblest life."

To the members of his profession Mr. Fuller's conspicuous success as an executive was a source of pride, his cordiality and kindness a constant encouragement, his fine public spirit an example, and his upright life an inspiration.

Mr. Fuller was elected a Member of the American Society of Civil Engineers on January 6, 1886. He was one of the first Presidents of the Portland Section of the Society.

JOB ROCKFIELD FURMAN, M. Am. Soc. C. E.*

DIED SEPTEMBER 7, 1925

Job Rockfield Furman, the son of John M. and Virginia Holmes Furman, was born in Westchester, N. Y., on June 21, 1865. His early schooling was received at the Friends School in New York, N. Y. He then entered the Stevens Institute of Technology at Hoboken, N. J., from which he was graduated in 1885 with the degree of Mechanical Engineer.

After a brief experience in the rebuilding of marine engines, in 1887, he entered the employ of Otis Brothers and Company, manufacturers of elevators and hoisting machinery. Working first as a Draftsman, he early acquired a considerable knowledge of elevator apparatus and was generally assigned to take charge of elevators of an unusual character.

In 1888, he went to Paris, France, as Resident Engineer in charge of all work in foreign countries. There he was in charge of the erection of the two elevators in the Eiffel Tower. This undertaking, at a time when prevailing low, brick-walled buildings did not call for any long lifts, presented a unique problem, since the Eiffel Tower elevators ascend 420 ft., with a load equal to 50 persons.

Mr. Furman remained in Paris during the time of the Exposition in 1889 and then returned to the United States where he took an important part in the development of high-pressure hydraulic apparatus, which was in vogue at that time. As Assistant Engineer, he co-operated in the design of three 25 000-lb. passenger elevators, as well as towers and viaducts, for the Weehawken Viaduct at Weehawken, N. J., and superintended its erection. About this time he was also engaged on the Catskill Mountain Incline. In 1891, he assumed full charge of the Engineering Department of Otis Brothers and Company with headquarters in New York.

In 1894, Mr. Furman again crossed the ocean, this time to Glasgow, Scotland, where, as Resident Engineer, he superintended the installation of twelve high-pressure 15 000-lb., vehicle elevators for the Glasgow Harbor Tunnel, the most powerful lifts attempted to that date.

In 1896 he severed his connection with Otis Brothers and Company to become Secretary and Treasurer of the C. F. Parker and Company until 1898. During this time, in addition to other works, he was in charge of the deepening of 6½ miles of the Erie Canal. He then went to London, England, where he engaged in professional engineering work as a Consulting and Contracting Engineer. As Consulting Engineer to the Sprague Electric Elevator Company, he designed forty-eight 17 000-lb. electric passenger elevators for the $15 000 000 London Central Railway and supervised their installation.

In 1904, Mr. Furman again became associated with the Otis Brothers Elevator Company as Assistant Chief Engineer and was placed in charge of the installation of the 170 electric lifts in the new London Underground Railways. He laid out the twelve large elevators installed in the Elbe Tunnel

* Memoir prepared by Theodore V. Purcell, Vice-Pres., The Peoples Gas Light & Coke Co., Chicago, Ill.

at Hamburg, Germany, and shortly afterward left the Otis Company to join the well-known architects, D. H. Burnham and Company.

With this firm, which later assumed its present name of Graham, Anderson, Probst, and White, Mr. Furman remained until his death. As Chief Mechanical Engineer he assisted in the design of many large buildings for which the firm acted as Architect, among which were the Peoples Gas Building, Wrigley Building, Continental and Commercial National Bank Building, Straus Building, the Federal Reserve Bank Buildings, the Field Museum of Natural History, and the Chicago Union Station, in Chicago, Ill., which was the last large work with which he was associated, and the Equitable Building, in New York, N. Y.

Mr. Furman died at the Presbyterian Hospital in Chicago on September 7, 1925, following a brief illness. His wife, Florence O'Mullen Furman, whom he married in London, had died in 1908, leaving no children. He is survived by three sisters and two brothers.

Although Mr. Furman's professional life was one of distinction and outstanding success, his private life was perhaps even a greater achievement. He was a man of strict integrity and of a notably affable and kindly nature. His constant cheerfulness and good humor, and this despite a chronic illness in his later years, won for him a host of intimate and devoted friends.

He was a member of the University Club of Chicago and the Engineers Club of New York.

Mr. Furman was elected a Junior of the American Society of Civil Engineers on July 2, 1890, and a Member on December 7, 1904.

JOACHIM GOTSCHE GIAVER, M. Am. Soc. C. E.*

DIED MAY 29, 1925.

Joachim Gotsche Giaver, the son of Jens Holmboe Giaver and Hanna Birgit Holmboe, was born on August 15, 1856, in Gjovik, a small town owned by his family, near Tromso, Norway. His father, of a prominent Norwegian family, was a large landholder and one of the leading figures in the fishing industry in Northern Norway. Tutored at home, Mr. Giaver entered the Trondhjems Technical College, at Trondhjem, Norway, from which he was graduated in 1881 with the degree of Civil Engineer.

As the engineering field in Norway was limited at that time he came to America in 1882, and found employment as Draftsman for the Northern Pacific Railroad Company at St. Paul, Minn. At the close of 1883, Mr. Giaver left for Pittsburgh, Pa., to become a Draftsman for the Shiffler Bridge Company and from 1885 to 1890 served as Chief Engineer of that Company. During that period he designed several of the bridges over the Allegheny and Monongahela Rivers at Pittsburgh, and by odd coincidence one of his first engagements was the design of the structural framework for the Statue of Liberty in New York Harbor.

Mr. Giaver forged rapidly to the front as one of the leading structural designers of the United States and, in 1891, he went to Chicago, Ill., to become Assistant Chief Engineer of the famous World's Columbian Exposition. Having been one of the first to recognize the importance of wind bracing in the structural framework of buildings, he was in charge of the perfecting of this feature in the structural design of the buildings for the Exposition. He designed the three-hinge arch in the dome of the Liberal Arts Building which at that time was the largest truss of its kind in the world, having a span of 368 ft. 0 in.

Following the Exposition, Mr. Giaver was engaged in the general contracting business from 1893 to 1896 and served as Bridge Designer for the Sanitary District of Chicago from 1896 to 1898. In this latter position he designed the various bridges over the Chicago Drainage Canal.

During the World's Columbian Exposition, Mr. Giaver had become acquainted with and worked with Chicago's famous architect, the late Daniel H. Burnham, and, in 1898, he became Chief Engineer for the firm of D. H. Burnham and Company, which position he held for eighteen years, until 1915. During this time, Mr. Giaver's engineering work was in the practical development of the modern skyscraper from the old spread-footing foundation, cast-iron column, wrought-iron structural framework to the modern caisson foundation and structural steel framework.

As Chief Engineer for the Burnham Company, he had charge of more than four hundred of the largest buildings in the United States, among which were the Flat Iron, Gimbel, Maiden Lane, and Equitable Buildings in New

* Memoir prepared by Amasa C. Bull, Esq., Chicago, Ill.

York, N. Y.; the Field Museum, Continental and Commercial National Bank, Railway Exchange, Peoples Gas, Conway, Field, and other buildings in Chicago, Ill.; the Union Station and Post Office in Washington, D. C.; the Frick, Oliver, Smithfield, and First National Bank in Pittsburgh, Pa.; the May Company in Cleveland, Ohio; and the Wanamaker and Land Title Buildings in Philadelphia, Pa. One of his most interesting designs was that for the dome of the Mount Wilson Observatory in California.

In 1915, Mr. Giaver interested himself in the question of licensing structural engineers in the State of Illinois. He was the leader in securing the passage in 1915 of a bill to this effect by the Illinois Legislature, as the result of which structural engineers are allowed to practice their profession on equal terms with the architects in that State. Prior to this it had been impossible for an engineer to practice in building operations, except as an employee of an architect, as under the former law, building plans could not be lawfully approved unless bearing the signature of a licensed architect.

In 1920, Mr. Giaver was decorated with the order of Knight of St. Olaf, 1st Class, by King Haakon of Norway, in recognition of his prominence as an engineer and his activities on behalf of Norwegians in the United States. This is the highest honor accorded by the Norwegian Government to a non-citizen of that country, Mr. Giaver having received his final papers as a citizen of the United States in 1896.

After the death of Mr. Burnham in 1915, Mr. Giaver established offices in Chicago as a Consulting Engineer, and, in 1916, organized the architectural and engineering firm of Giaver and Dinkelberg. His last work as a member of this firm, finished only a few months before his death, was the engineering design of the 42-story Jewelers Building being erected at Wabash Avenue and Wacker Drive, Chicago, which will be the tallest office building west of New York.

Mr. Giaver was married in New York on September 3, 1883, to Louise C. Schmedling, of Trondhjem, Norway. He is survived by his widow, two daughters, Astrid (Mrs. Ralph Holmboe), Birgit (Mrs. A. C. Bull), and three sons, Erling, Finn J., and Einar W. Giaver.

In private life, Mr. Giaver was an enthusiastic yachtsman. It was his principal hobby and he won many prizes racing his boats on Lake Michigan. He was a Trustee of the Norwegian American Hospital in Chicago; President of the Structural Engineers Association; former Director of the Western Society of Engineers; President of the Norwegian Engineers Association; Past-President of the Chicago Norske Klub; former Vice-Commodore of the Columbia Yacht Club; and a member of the Engineers Club, Svenska Klubben, Chicago Athletic Club, and Chicago Yacht Club.

Mr. Giaver was elected a Member of the American Society of Civil Engineers on September 12, 1916.

WILBUR FRANCIS GOODRICH, M. Am. Soc. C. E.*

DIED FEBRUARY 26, 1925.

Wilbur Francis Goodrich, the son of Charles A. and Eleanor M. Goodrich, was born on August 18, 1853, at Lunenburg, Mass. He was educated at the Cambridge, Mass., Grammar and High Schools and at Comers' Commercial College at Boston, Mass. In 1873, he entered the Massachusetts Agricultural College, but remained only one year, having been compelled to leave because of illness.

On April 27, 1874, he began his engineering studies in the office of Shedd and Sawyer, Civil Engineers of Boston, where he remained for three years. His first work was that of surveying house lots at Chestnut Hill Reservoir, Newton, Mass. In 1877, he was employed on the Newton, Mass., Water-Works.

In 1880, Mr. Goodrich was engaged from June to September by the Nashua and Lowell Railroad Company as Assistant to the late Frederick Brooks, M. Am. Soc. C. E.; in September and October, he was with the late James B. Francis, Past-President, Am. Soc. C. E., on work on the Locks and Canals at Lowell, Mass., and in November and December, he was in the service of the Massachusetts Central Railroad Company as Assistant, in charge of a party, laying out work for construction.

From December, 1880, to March, 1881, Mr. Goodrich was employed by the City of Lawrence, Mass., as Transitman and Levelman on a survey of a section of the Valley of the Spicket River. From April, 1881, to January, 1883, he served the Fitchburg Railroad Company as Assistant in charge of a party, under the late Edmund K. Turner, M. Am. Soc. C. E., Chief Engineer, his principal work being the laying out of work for construction.

From January, 1883, to April, 1884, he was employed by the New York and New England Railroad Company as Assistant Engineer in charge of construction. This was contract work for which Mr. Goodrich prepared all the plans and made all the estimates. From April to December, 1884, he was with Whitman and Breck, Civil Engineers, of Boston.

From April to August, 1885, he served with the late William S. Barbour, M. Am. Soc. C. E., Chief Engineer of the City of Cambridge, Mass., as Inspector on the laying of 3 miles of 30-in., iron, water pipe and also in the removal and rebuilding of a drawbridge. From October to December, 1885, he was with the Massachusetts State Drainage Commission and for the six months following with Mr. E. Bowditch, of Boston, on topographical surveys at Ipswich, Mass.

In 1886 Mr. Goodrich was appointed Resident Engineer on the Toledo, St. Louis, and Kansas City Railroad, in charge of changing 63 miles of road from narrow to standard gauge and the building of a stone bridge foundation, a pier, and two abutments over the Big Wildcat River, two miles west of Kokomo, Ind. Following this engagement he was employed as Resident Engineer on the Mackinaw, Toledo, and Saginaw Railroad, and, subsequently, he was engaged

* Memoir compiled from information on file at the Headquarters of the Society.

for six months as Consulting Engineer on water-works in Vermont and on plans for electric street railways at Steubenville, Ohio, and Newport, R. I., as well as a reservoir for the United States Government on Coasters Harbor Island, Rhode Island.

As a Contractor with Mr. J. F. Barry, of New York, N. Y., Mr. Goodrich constructed three miles of electric railroad at Steubenville, one mile of street-railroad on Southern Boulevard, New York, and two miles at Richmond, Va. He also installed a 34-in. pipe line through ledge rock in Rockville, Conn. He then accepted the position of Resident Engineer with the Kansas City and Toledo Railroad Company, his work consisting of change of line covering 76 miles, from Ramsey, Ill., to St. Louis, Mo., including the construction of bridges, etc. Subsequently, he became Assistant Engineer with the Old Colony Railroad Company at Boston, and, later, was engaged in private engineering work, including the construction of the sewer system for the Metropolitan Park Commission of Massachusetts and change of line of the Revere Beach Railroad at Crescent Beach, Revere, Mass. His work also included change of grade at street crossings at Lowell, Mass., for the Nashua and Lowell Railroad Company.

From 1891 to 1894, Mr. Goodrich served as Assistant Engineer of the Metropolitan Sewerage Commission of Massachusetts. During this period he also constructed a wharf at Deer Island, in Boston Harbor, and for ten months following this engagement he served as Construction Engineer of a reservoir for an additional water supply at Yarmouth, Mass. He also held the position of Consulting Engineer for the Portland Stoneware Company's Plant, at Portland, Me., and was engaged for six months on inspection work at Payson Park Reservoir, at Cambridge, Mass. He also acted as Consulting Engineer for the construction of a water-works system at Edwardsville, Ill., and for the Coffeen Coal and Copper Company, on the construction of its plant at Coffeen, Ill.

In July, 1896, Mr. Goodrich became Assistant Engineer of the Boston Terminal Company in charge of laying out foundations, tracks, etc., for the New South Station, Boston. In 1900, he served for six months with the New York Central and Hudson River Railroad Company at Buffalo, N. Y., and also as Consulting Engineer of coal and iron property at Bedford, Pa.

In 1901, he had charge of construction at the Stanley Electric Plant, at Pittsfield, Mass., and, at the same time, acted as Consulting Engineer for the Austin Street Railroad, at Austin, Tex. He later served as Consulting Engineer for the Westfield Water-Works, Westfield, N. J., and the Piqua, Ohio, Electric Street Railroad Company, and made surveys and estimates for an electric railway from Washington, D. C., to Great Falls and Fairfax Court House, Va.

In 1902, Mr. Goodrich acted as Consulting Engineer for mill works at Adams, Mass., and examined coal property at Riddlesburg, Pa., and Middlesboro, Ky. He was also engaged as Consulting Engineer for mills at Saugus and Lawrence, Mass.; for the electric street railway at North Andover, Mass.; the Revere Boulevard, Revere, Mass.; and made plans and estimates for the Montreal Terminal Water Power Plant at Kinderhook Creek, N. Y.

In the same year he was placed in full charge of the work at the plant of the Gale Creek Coal Company, at Wilkeson, Wash., which comprised opening the mine, installing new machinery, and constructing new buildings. He also made examinations and prepared reports on the Denver, Lakewood, and Golden Railroad. In the State of Washington, he reported on the water power of the Nesqually River, the coal property of the Burnett Coal Company at Carbonado, and coal mines at Franklin, Fairhaven, and Whatcom, as well as coal property at Fairfax and Mulmont. He also made an examination and reported on the Tacoma and Eastern Railroad at Tacoma.

In 1903 and 1904, Mr. Goodrich served as Engineer in charge of pumping out and installing machinery in a coal mine in Gayton, Va., and as Consulting Engineer on work in other cities in New England and the South. He also served as Consulting Engineer and reported on the construction of a plant for the Amalgamate Graphite Company at Johnsburg, N. Y., and, in 1905, made examinations and reports on the Connecticut River below Springfield, Mass., as well as for the Newfoundland, Labrador, and Sandwich Bay Company.

From July, 1905, to July, 1906, he was engaged as Consulting Engineer for the Nevada Northern Railroad Company and from 1906 to 1908 in private work in Boston. In 1909, he was employed as Consulting Engineer to the Boston Finance Commission and, in 1910, was Engineer in charge of pumps and engines in mills at Adams, Mass. He also held the position of Engineer of the Municipal Filtration Company at Boston.

In 1911, he served as President of the T. M. Filter Company of Boston and, until 1915, carried on a private consulting practice in several cities in Massachusetts. In 1916, he was again Consulting Engineer for the Boston Finance Commission; was engaged in machine-shop designing at Fitchburg, Mass.; and served as Engineer of the Shirley Peat Works at Shirley, Mass.

In 1917, and 1918, Mr. Goodrich held the positions of Engineer for the Elm Street Bridge at Manchester, N. H.; Resident Engineer on the Fort Smith and Western Railroad; and Consulting Engineer for the Boston and Maine Railroad Company. During this period he was also Inspector of the building of a wharf, pier, and pier shed at the Army Base at South Boston, Mass., and, in 1919, he became Inspector at the Army Base and also held the position of Construction Engineer for the Eastern Massachusetts Street Railway Company, with work at Chelsea, Randolph, Salem, Peabody, Revere, Weymouth, Lynn, and Lowell, Mass.

In 1921, he was appointed Inspector of a bridge over the Connecticut River at Springfield, Mass., and from 1922 to 1923 was engaged in private work for the Boston Finance Commission and was also Engineer in charge of the relocation of railroad yards and tracks at Springfield, Mass.

From 1924 until his death on February 26, 1925, Mr. Goodrich was engaged in private practice which included the building of a coal trestle, water tanks, and Colonial filling stations for the J. F. Kennedy Company, of Boston.

He is survived by his widow, Mary L. Goodrich, two sons, and two daughters.

Mr. Goodrich was a member of, and Treasurer of, the Railway Appliance Association of Providence, R. I., and an Associate Member of the Alumni of the Massachusetts Agricultural College. He was a member of the First Methodist Episcopal Church and also of Soley Lodge, F. and A. M., of Somerville, Mass.

He was essentially a home man, taking more pleasure with his family than with social clubs and organizations. Having a kindly disposition and a ready smile he made friends easily and retained such friendships throughout his life.

Mr. Goodrich was elected a Member of the American Society of Civil Engineers on May 4, 1887. He was also a member of the Northeastern Section of the Society.

JOHN LYELL HARPER, M. Am. Soc. C. E.*

DIED NOVEMBER 28, 1924.

The importance of hydro-electric power development was just beginning to be realized when Mr. Harper came to Niagara Falls, N. Y., in 1902. With his broad vision and ability and situated as he was at this mightiest source of power, he was destined to take the lead in an industrial achievement which remains unsurpassed anywhere in the world. At his death he was recognized as one of the greatest engineering minds of the age and the famous development in the Gorge below the Falls of Niagara will always stand as a monument to his genius.

John Lyell Harper, the younger of the two sons of Joseph F. and Quintilla K. Harper, was born on a farm in a hilly section of New York State called Harpersfield, on September 21, 1873. Like most farmers' sons, he had to work long, hard hours in the fields as well as climb the mountain every day to attend the country school in the next valley; but the rigors of country life served only to develop his keen mind and foster in him a passionate love of Nature which he retained throughout his life.

He loved books and applied himself diligently to the pursuit of knowledge, very early showing more than ordinary interest in experimenting with running water. There were plenty of mountain streams around his home and it was his special study, with a degree of success beyond his years, to attempt to control the powers of Nature, damming up one stream and diverting the course of another to turn mills and water-wheels of his own invention. Thus, his boyhood was spent and at the age of twenty he had finished the course of study at Stamford Seminary; whence he followed his own particular star to Cornell University, from which he was graduated with the degree of Mechanical Engineer.

From Cornell, Mr. Harper went to Seattle, Wash., as a Draftsman with the Oregon Improvement Company, making designs for the repair and construction of railway equipment. After four months, however, he resigned this position to enter the employ of the Union Electric Company where the field of practical experience included the repair and testing of motors, transformers, and general electrical apparatus.

In June, 1898, he secured the position of Operating and Construction Engineer of the Twin City Rapid Transit Company, of Minneapolis, Minn., and remained with that Company a year. In this position his training was broader and more along the lines in which he was most interested. Among other things he had a great deal of experience in designing and constructing substations, installing underground cables, testing water-wheels, and similar work in connection with the operation of the 10 000-h.p. water-power plant of the Company.

* Memoir prepared by N. R. Gibson, and O. D. Dales, Members, Am. Soc. C. E., and G. R. Shepard, Esq.

At the end of the year Mr. Harper transferred his interests to the firm of I. W. Gray and Company and was responsible for the design of an electric railway and steam power plant for Mankato, Minn.

Steadily, he was increasing his experience and his value, and, in 1899, he took his first important position, as Field Representative for Messrs. Floy and Carpenter, Consulting Engineers, of New York, N. Y., on their project to build a hydro-electric power plant for the St. Croix Power Company of Wisconsin at Appleriver, Wis. He was in entire charge of the construction of this development which was completed in three years.

It was the turning point in his career when in 1901 he came East, to Buffalo, N. Y., to represent certain manufacturing and engineering interests at the Pan-American Exposition, for there he got into touch with the Niagara Falls Hydraulic Power and Manufacturing Company and in 1902 accepted a position with that Company as Assistant to the Engineer, the late Wallace C. Johnson, M. Am. Soc. C. E. It was there that his principal work began, and from that time until his death in 1924, the story of his life is intimately associated with the history of the development of power from the waters of the Niagara River.

This Company in 1881 had begun the construction of a power-house called Station No. 1 for supplying electric power for commercial purposes. In February, 1902, when Mr. Harper came to Niagara Falls, this plant had a capacity of 2 000 h.p., and a new power-house called Station No. 2 had been partly constructed, having at that time a capacity of about 20 000 h.p. In two years he became Chief Engineer and prepared plans for the completion of Station No. 2 and, in 1904, when this was accomplished, it had an installed capacity of 33 000 h.p. Station No. 1 was then abandoned and the construction of Station No. 3 begun, which was ultimately to have a capacity of 130 000 h.p. From that time rapid improvement was made and further plans for the future were under way when, in 1909, the Governments of Canada and the United States set a limit to the quantity of water that could legally be diverted for power purposes. Plans already made were carried out, but incentive to greater achievement was thus temporarily removed.

During the World War emergency, the Niagara Falls Power Company, Hydraulic Power Company, and Cliff Electrical Distributing Company, were merged under the name of The Niagara Falls Power Company, owning all developments on the American side and controlling the Canadian Niagara Power Company on the Canadian side. The combined plants had a total installed capacity of 350 000 h.p., and it was a fitting tribute to Mr. Harper's vision and ability that the developments of the consolidated companies were placed in his hands as Chief Engineer, and that, in 1919, the responsibility of Vice-President was added.

The shortage of power was serious in this period and when the War Department requested plans for the most economical development for utilizing the remaining water available for diversion, it was the Harper plan that was accepted. This provided not only for immediately placing all developed power

at the disposal of the Government agents for distribution to specified customers who were making the materials vital for war work, but also for the rush development of another 100 000 h.p. without sacrifice of permanency of work or efficiency of conversion.

These construction plans were being rushed to completion when the signing of the Armistice made speed less important; however, the whole 100 000 h.p. was put in operation within twenty-two months from the time it was authorized. Shortly afterward, the Government of the United States established the Federal Power Commission and its approval was given to the engineering plans of The Niagara Falls Power Company. A license for fifty years was issued for all the water permitted to be diverted from the American side of the Falls in accordance with the terms of the existing treaty, with the exception of 500 cu. ft. per sec., which was reserved for diversion through the New York State Barge Canal.

As no further water was available, the only source of additional power was the use of some of the water under increased head. Plans accordingly went forward for the construction of Station No. 3-C which was begun in April, 1921, and completed in two years. The water for this Station was brought from the upper river under the city in a tunnel 32 ft. in diameter. Each of the three units in this plant has a capacity of 70 000 h.p.—at present, the largest installation in the world.

In connection with the Niagara development, Mr. Harper was especially interested in the question of the preservation of the scenic beauty of the Cataract in its relation to diversion of water for industrial purposes. For the scientific engineering study of this problem he designed and had built a model of the Niagara Falls and Rapids on a scale large enough to determine with a reasonable degree of accuracy the effect of diversion on the Falls and Rapids and to actually demonstrate the feasibility of the various plans for remedial works which are necessary to prevent the present rapid and irregular recession of the Horseshoe Falls. He also wrote several papers on this subject and thus brought his opinions and the results of his research to the attention of the public.

Mr. Harper found time to interest himself in chemical and metallurgical research, particularly along the lines of the application in these industries of electric service. He was successful in inventing and patenting the Harper electric furnace, for the commercialization of which he organized, shortly before his death, the Harper Electric Furnace Corporation.

He also initiated, in 1924, the organization of the firm of Harper and Taylor, Incorporated, to carry on a consulting engineering practice in conjunction with his Engineering Staff at the Power Company.

In 1898, he was married to Linda E. Wheeler, of Ithaca, N. Y., who, with two sons and three daughters, survives him.

Mr. Harper was a man of high ideals and absolute integrity, a loyal friend, and devoted to his family. A lover of Nature and of the out-of-doors, his chief hobby was the cultivation of flowers and fruits and his gardens were always a delight.

Cut off in the prime of life, in the midst of plans and visions for the future, it was some consolation to those who knew him that he lived to see the completion of his two latest projects. Throughout his work he maintained the highest engineering and moral standards, and his premature death was a loss to the profession.

Mr. Harper was elected a Member of the American Society of Civil Engineers on June 5, 1907.

JOHN FILLMORE HAYFORD, M. Am. Soc. C. E.*

DIED MARCH 10, 1925.

John Fillmore Hayford, the son of Hiram and Mildred Hayford, was born at Rouses Point, N. Y., on May 19, 1868. His early education was secured, while a farm boy, at the small stone schoolhouse not far from the farm.

Mr. Hayford was graduated from Cornell University in 1889, with the degree of Civil Engineer, and immediately entered the service of the United States Coast and Geodetic Survey at Washington, D. C., as a Computer. In 1892, he was made Assistant Astronomer in charge of field parties on the International Boundary Commission, laying out the boundary between the United States and Mexico, on which work he was engaged during 1892 and 1893.

On the completion of the Boundary Survey, he returned to the Coast and Geodetic Survey as Aid, and, later, as Assistant in 1894 and 1895. He then resigned to become Instructor in Civil Engineering at Cornell University, which position he held until 1898, when he again entered the Coast and Geodetic Survey as Expert Computer. In 1900, he was made Chief of the Computing Division, succeeding Mr. Charles A. Schott, and, in addition, was appointed Inspector of Geodetic Work. In this dual capacity he served the Nation until 1909, when he resigned to become Director of the College of Engineering at Northwestern University at Evanston, Ill., a position which he held until his death. He died of a complication principally dropsical in character, after an illness of three months.

While carrying on the regular work of the two departments of the Survey over which he had supervision, Mr. Hayford found time to conduct many original investigations. His theory of isostasy brought him international reputation, for which he was awarded, in 1924, the Victoria Medal of the Royal Geographic Society of Great Britain, an honor accorded to only two other Americans. His work with the transit micrometer and precision level revolutionized longitude determinations and precise level work, and he was a great factor in the adoption of the U. S. Standard Geodetic Datum, which has now become the North American Datum.

During the time that he was developing his theory of isostasy, Mr. Hayford made computations as to the figure of the earth, and his values of the lengths of the earth's semi-major and semi-minor axes, together with the ratio of polar flattening, were adopted in October, 1924, by the International Geodetic and Geophysical Union, as being the values which best fitted the various continents of the earth. He was twice selected to represent the United States as Geodetist at International Geodetic Conferences, once at Budapest, Hungary, and again at London, England.

* Memoir prepared by William H. Burger, Prof. of Civ. Eng., Northwestern Univ., Evanston, Ill.

As an active member of the Philosophical Society and of the Engineers' Club of Washington, he presented many papers. He was a member of the Cosmos Club of that city and took great delight in there meeting friends and visitors to the Capitol City.

At Northwestern University, as in Washington, Professor Hayford found time to do a prodigious amount of work other than that connected with his regular duties as Director of the College of Engineering. Especially notable was his work as Research Associate of the Carnegie Institution, studying the laws of evaporation, etc., underlying changes in lake levels on the Great Lakes, and in investigating factors affecting stream flow.

In 1912, Professor Hayford was selected by the Arbitrator, the late Chief Justice White, to head the Commission of Engineers to survey the disputed boundary between Panama and Costa Rica, and, later, was a member of the Commission of Engineers and Geologists sent to study the slides at the Panama Canal. In 1915, he was appointed by the President as a member of the National Advisory Committee for Aeronautics, and during the World War he was engaged with a group of scientists at the U. S. Bureau of Standards in devising instruments for recording airplane performance.

Professor Hayford was elected a member of the National Academy of Sciences, one of the highest honors granted to scientists in this country. He was also a member of the American Astronomical Society, American Association for the Advancement of Science, National Research Council of the American Geophysical Union, Western Society of Engineers, and the Society for the Promotion of Engineering Education. In 1918, George Washington University conferred on him the degree of Doctor of Science.

Although he was a scientist of the highest attainment, there was probably no field in which he took keener delight than in considering the problems of engineering education. He was honored by being elected President of the Society for the Promotion of Engineering Education for 1918-19.

As a citizen of the community in which he passed the last fifteen years of his life, Professor Hayford was most highly esteemed, and took an active interest in civic problems and affairs. He was a member of the Rotary Club, University Club, Northwestern Chapter of Sigma Xi, and the Unitarian Church of Evanston, and of the Chaos Club of Chicago, Ill.

Professor Hayford was the author of "Geodetic Astronomy", a joint author of "The Adjustment of Observations by the Method of Least Squares," and author of numerous special reports, monograms, and articles dealing with geodetic, geophysical, geological, and educational subjects.

More than a quarter of a century of very intimate association with Professor Hayford, first as a member of the U. S. Coast and Geodetic Survey, and, later, as a member of the Faculty at Northwestern University, revealed the following characteristics to be uppermost in his life: Intense energy of body and mind, strict honesty of thought and purpose, and a hatred of all sham and pretense. He had the great faculty of making and holding friends; he seldom entered into anything without giving to it his fullest energies; and he obtained the keenest enjoyment out of life.

He was married on October 11, 1894, to Lucy Stone, of Charlotte, N. Y., who survives him, with four children, Walter S., Maxwell F., John B., and Phyllis Hayford. The latter is a Junior in the College of Engineering at Northwestern University, and the three sons have engineering degrees from the same institution.

Professor Hayford was elected an Associate Member of the American Society of Civil Engineers on May 6, 1896, and a Member on April 2, 1907.

CURTIS HILL, M. Am. Soc. C. E.*

DIED OCTOBER 18, 1925.

Curtis Hill, the son of William Moberly and Ann (Gossett) Hill, was born near Independence, Mo., on September 4, 1870, on a farm acquired by his grandfather, Adam Hill, who came from Kentucky to Missouri on horseback in 1830. The Hill family is of sturdy pioneer stock and is one of the oldest in the United States, the first of that name having emigrated from England to America in early Colonial times.

Curtis Hill's early education was obtained in the public schools of Jackson County, Missouri, and at Woodland College, at Independence. He was graduated from the University of Missouri in 1896 with the degree of Bachelor of Science in Civil Engineering, and a year later received the degree of Civil Engineer from Cornell University. While at the University of Missouri he was prominent in athletics and played three years on the football team, serving one year as Captain.

Mr. Hill's professional work was connected chiefly with railroads, waterways, highway bridges, highways, and municipal construction. He was engaged as Instrumentman on railroad construction in Mexico in 1892, prior to his graduation from the University of Missouri, and after leaving Cornell University, in 1897, he was employed by the Illinois Central Railroad Company for a few months on track construction. He again engaged in railroad work in 1899, this time with the Chicago and Northwestern Railway Company.

From September, 1897, to August, 1899, Mr. Hill was with the United States Deep Waterways Commission, engaged in making hydrographic and topographic surveys and estimates for a proposed canal outlet for the Great Lakes. During the next two years he was employed by the Central Bridge Company, of St. Louis, Mo., as Designer, Contracting Agent, and Erection Superintendent.

Beginning in October, 1901, Mr. Hill spent almost six years with the Sewer Department of St. Louis, eighteen months as Assistant Engineer and the remainder of the time as Engineer of Sewers. In the latter position he had charge of all field surveys and construction work pertaining to sewers, the cost of the work done under his direction amounting to about $5 000 000.

In July, 1907, Mr. Hill was appointed State Highway Engineer of Missouri by Governor Folk, being the first to hold that office. Two years later, on account of the highly satisfactory character of his work, he was re-appointed by Governor Hadley and remained in office until April, 1913. During his encumbency he did considerable pioneer work in connection with highway construction, laying the foundation for the extensive development later author-

* Memoir prepared by Victor H. Cochrane, M. Am. Soc. C. E.

ized by vote of the people. His valuable services in this office were commemorated by Mr. J. M. Lowe in the dedication of his book, "National Old Trails Road", as follows:

"To Curtis Hill, the first State Highway Engineer of Missouri, whose spirit rose to a full comprehension of the duties and obligations of his high office, and whose intelligence and lofty vision enabled him to grasp with full appreciation the mighty pioneer problems which he alone had to solve, this book is respectfully and affectionately dedicated."

In April, 1913, Mr. Hill became City Engineer of Kansas City, Mo., and during the next five years he had charge of a great variety of municipal construction aggregating about $10 000 000 in cost, and also made plans for proposed sewers, flood protection works, trafficways, and other improvements. For one year during this period he acted as Advisory Engineer to the Board of Park Commissioners of Kansas City.

On June 7, 1918, occurred the tragic event that caused Mr. Hill to be incapacitated for the remainder of his life. He was shot in the head by a disgruntled contractor, a man of violent temper and unsound mentality. Some time previously, this contractor had been awarded the contract for the construction of a small bridge, which he had failed to complete satisfactorily. Later, he submitted a bid for a much larger bridge, and because of the previous trouble and the fact that the bid seemed to be much too low, Mr. Hill recommended that the contract be not awarded to him; however, the contractor persuaded the Board of Public Works that he could build the bridge properly and expeditiously, and he was given the work. He was wholly without experience in bridge construction and had no adequate equipment. After he had worked several months without making any progress, the Board annulled his contract and gave the work to a responsible and experienced firm. The contractor was present at the meeting of the Board when this action was taken, and he immediately shot Mr. Hill.

At first it was thought that Mr. Hill's injury would prove fatal, but after undergoing an operation for the removal of a piece of the bullet from the brain, he seemed considerably improved; however, the wound had caused a partial amnesia, from which he never fully recovered. He was retained by the City as Assistant City Engineer until April, 1925, and during 1923 and 1924 he was able to work occasionally, although for only a few days at a time. In April, 1925, he was pensioned by the City. At this time his health was in such a critical state that death seemed imminent, but he lived until October 18, 1925.

During the entire period of his illness Mr. Hill was uniformly cheerful, and accepted his fate without bitterness and with a degree of fortitude that was the marvel of his friends.

On account of his pleasing personality, his kindliness, his helpfulness, and his fair-mindedness, Mr. Hill was well liked by his associates and respected by every one. He was a man of good intellectual attainments and excellent judgment, and possessed the ability to analyze a given situation or problem and to determine the best course of action under the circumstances. His honesty and integrity were recognized by all who came in contact with him, and his ability and trustworthiness as an engineer and public servant were amply

demonstrated by the fact that although he was a lifelong Democrat, he was retained in office by Republican officials as readily as if he had belonged to that party.

In 1899, Mr. Hill was married to Flora Lewis, of Missoula, Mont., who was the daughter of a pioneer missionary to the Northwest Territories. He is survived by his widow and two children, William Moberly Hills, an engineering student in the University of Missouri, and Elizabeth Hill Gordon.

He was a member of the Presbyterian Church, the Beta Theta Pi Fraternity, the Kansas City Engineers' Club, and was a Thirty-second Degree Scottish Rite Mason.

Mr. Hill was elected an Associate Member of the American Society of Civil Engineers on October 2, 1901, and a Member on January 3, 1907.

HENRY CLARENCE HODGKINS, M. Am. Soc. C. E.*

DIED NOVEMBER 8, 1925.

Henry Clarence Hodgkins was born at Carthage, N. Y., on December 12, 1854, the son of Ezra and Amelia (Fuller) Hodgkins. His mother's ancestry has been traced to Edward Fuller, who came to America in the *Mayflower*. The Hodgkins and Fuller families belong to a group of miners and manufacturers who were attracted to Northern New York by the mineral wealth and splendid water-power facilities of the Black, Grass, and Oswegatchie Rivers. The Towns of Fullerville and Rossie were the sites of foundries and furnaces operated by Fuller Brothers, one of whom, Ashbel, was the maternal grandfather of Henry Clarence Hodgkins.

Mr. Hodgkins was graduated from Union College with the degree of Civil Engineer in 1875. He then began a long and useful career as Engineer in the survey and improvement of the islands of the St. Lawrence River, including the following: Thousand Island Park, Westminster, Round Island, Hillcrest, Grenell Island, and Central Parks. He also did considerable landscape work in parks and cemeteries at Watertown, N. Y., and extensive surveys in a long litigated dispute between the Rossie and Karney Ore Mines.

In 1880, he became Assistant Engineer on the Water Power Pumping Station at Watertown, and in 1881 held the position of Topographer on the New York and Boston Inland Railroad surveys. In 1882, he designed and supervised the construction of three masonry arch bridges across the Grass River in St. Lawrence County, New York, with an aggregate length of 850 ft., and also served as Transitman on the location of the Carthage and Adirondack Railroad.

From 1883 to 1885, Mr. Hodgkins served as City Engineer of Watertown, and in this capacity designed and supervised the construction of the Court Street Bridge and approaches with a clear span of 170 ft. This bridge in 1921 was replaced by a double-deck concrete structure.

In 1884, he entered into partnership with Hinds, Moffett, and Company, which a short time afterward became the firm of Moffett, Hodgkins, and Clarke. As the engineer member of this firm, he had charge of the plans and construction of seventy-five water-works including those at Duluth, Minn., Butte, Mont., and Peoria and Galesburg, Ill. In connection with the firm, he had supervision of the surveys of the Norwood and Massena Railroad, the Kinderhook and Hudson Railroad, and took personal charge of the location and construction of the Gouverneur and Oswegatchie Railroad. The firm also acted as Engineers of the Lake Street Elevated Railroad in Chicago, Ill., installed and operated three gas plants and ten central station electric lighting plants, and installed one of the first central alternating-current plants at Rochester, Minn., in 1887.

* Memoir prepared by William M. Booth, Esq., Syracuse, N. Y.

Mr. Hodgkins served as President of the Montreal Water and Power Company from 1890 to 1893 and until 1900 was President of the Electric Engineering and Supply Company, manufacturers on a large scale of electric lighting supplies. At this time he was also interested in the appraisal, arbitration, negotiation, and sale of the properties of many water companies to towns.

In 1902, he served as Engineer for the Town of Waterloo, Ont., Canada, in the construction of its water-works, and, also, as Engineer for the Village of Carthage, N. Y., in obtaining its water supply by gravity from the Adirondacks, fifteen miles away.

From 1904 to 1907 Mr. Hodgkins was employed in connection with the transmission lines of the Northern Ontario Light and Power Company, Limited, and as Consulting Engineer for the City of Berlin, Ont., Canada, in the installation of electric pumping, filtration, and a concrete water tower. He also served as Consulting Engineer for the City of Brantford, Ont., on the revision of its distribution system and increased water supply, including reservoir and electric pumping; as well as Engineer for the Galion Water Company, for an increased water supply and a pumping plant. From 1907 to 1914, Mr. Hodgkins was engaged in general consulting work.

From 1923 until his death he held the position of Engineer for the City of Cortland, N. Y., studying the remodeling and enlarging of the water distribution system, the new water supply, the question of pumping by electric *versus* steam power, and the designing and constructing of a complete new pumping station. In 1914, he served as Engineer for the proposed suburban water supply at Toronto, Ont., Canada, and as Consulting Engineer for the City of Kitchener, Ont., for its additional water supply, pumping mains, and reservoir. Mr. Hodgkins also served as Engineer for the Village of Manlius, N. Y., from 1918 to 1921, on its additional water supply works, electric pumping station, and conduit.

On June 11, 1889, Mr. Hodgkins was married to Mary Ida Follett, of Newark, Ohio, and in the autumn of that year they removed to Syracuse, N. Y., where they have since resided. Two of their three children are living, Henry Follett Hodgkins and Marian Hodgkins Baxter.

In June, 1925, Mr. Hodgkins joined the surviving members of his Class at Union College in the celebration of its Fiftieth Anniversary. This was a very happy occasion. He was a member of the Technology Club of Syracuse and was elected a member of the American Water Works Association on April 18, 1888.

His contributions to the literature of his profession were made through papers presented at the annual meetings of the Societies of which he was a member. He was an authority on franchises, and this subject was the title of a much discussed paper before the American Water Works Association, in Cincinnati, Ohio, in 1915. The Syracuse Technology Club received his hearty support and co-operation. He attended the meetings with great regularity, contributed papers, and served on its committees. His knowledge of the topography of Central and Northern New York always surprised his associates.

For nearly a quarter of a century Mr. Hodgkins was a member and officer of the First Presbyterian Church of Syracuse, and to these duties he brought the dignity, integrity, and accuracy of the engineer. As a companion he was genial, fond of anecdote, and ready to draw from a broad personal experience and careful reading. His interest in the comprehensive physical development of the City of Syracuse was unusual. His civic pride was of the highest order. In his death, the Engineering Profession has lost a useful member. He will be missed by many friends.

Mr. Hodgkins was elected a Member of the American Society of Civil Engineers on June 6, 1911.

OLAF HOFF, M. Am. Soc. C. E.*

DIED DECEMBER 24, 1924.

Olaf Hoff, the son of Martin C. and Gunhild Hoff, was born in Smaalenene, Norway, on April 2, 1859. He received his elementary education in the Real-gymnasium at Ringerike, Norway, and entered the Polytechnic Institute at Christiania (now Oslo), Norway, from which he was graduated in 1879 with the degree of Civil Engineer and with the highest honors ever granted by the Institute.

In 1879, Mr. Hoff came to the United States, and in the spring of 1880 entered the service of the Keystone Bridge Company as Assistant Foreman in one of the shops of the Company at Pittsburgh, Pa. In a few months he was transferred to the Drafting-Room, where he was gradually advanced, eventually becoming Assistant Chief Engineer.

From 1881 to 1883, he was in the employ of the Mexican Central Railroad, first as Bridge Engineer and, later, as Locating Engineer on the Tampico Division. From 1883 to 1885, Mr. Hoff served as Chief Engineer of the Shiffler Bridge Company, of Pittsburgh.

From 1885 to 1901, he was engaged in consulting and contracting practice in Minneapolis, Minn., acting, at the same time, as Western Representative of the Shiffler Bridge Company. In 1891, he built a steel railroad bridge across the Mississippi River at Minneapolis for the Minneapolis and Western Railway Company, and devised the method of erecting this bridge without falsework. He designed a highway bridge across the Mississippi River at Muscatine, Iowa, 2 000 ft. long, with a 400-ft. cantilever span. He also acted as Consulting Engineer on several bridges crossing the Mississippi River at St. Paul, Minn., and made competitive designs for others, such as the Washington Bridge across the Harlem River, New York, N. Y., and two arch bridges across the Mississippi River, one at Nicolet Avenue and another at Lake Street, Minneapolis.

From 1901 to 1905, Mr. Hoff served as Engineer of Structures for the New York Central and Hudson River Railroad Company, in charge of bridges and buildings, and was identified with the renewal and reconstruction of more than four hundred bridges.

From 1905 to 1908, he was a member of the contracting firm of Butler Brothers-Hoff Company. This Company, as contractors, built the Michigan Central Railway Company's tunnel under the Detroit River between Detroit, Mich., and Windsor, Ont., Canada. The Railroad Company's plans called for a new type of subaqueous tunnel construction, consisting of twin steel tubes to be placed in a dredged trench and encased in concrete under water. Mr. Hoff gave all his exceptional inventive and sound engineering talent to the solution of a suitable design for the steel tubes and practical methods for floating and lowering them in place to correct alignment and

* Memoir prepared by Sverre Dahm, M. Am. Soc. C. E., and J. L. Holst, Chf. Engr., Todd, Robertson, Todd Eng. Corporation, New York, N. Y.

grade, and for an adequate and economical encasement of these tubes with concrete placed in deep, swiftly flowing water.

From 1908, Mr. Hoff was engaged in consulting practice in New York, N. Y. The success of his past engineering performances had brought him an enviable reputation and he was therefore consulted by individuals and corporations regarding large problems of building construction, including subways, tunnels, harbors, and bridges.

The type of subaqueous tunnel used under the Detroit River and the methods employed in its construction had proved to be so satisfactory in every respect that Mr. Hoff secured, in competition with other tunnel types, the contract for the construction of the four-track tunnel carrying the Lexington Avenue Subway under the Harlem River in New York. This contract, awarded the McMullen-Hoff Company of which he was a member, was undertaken in 1912 and satisfactorily completed in 1915.

He acted as Chief Consultant to the Cunard Steamship Company, Limited, in connection with its proposed steamship terminal in the Port of New York and as Consulting Engineer to the New York Central Railroad Company on the design and construction of the A. H. Smith Memorial Bridge across the Hudson River at Castleton, N. Y., and of the Michigan Central Railroad Arch Bridge across the Niagara River, at Niagara Falls, N. Y.

Mr. Hoff's inventive mind concerned itself with many subjects. He invented methods for submarine pile-driving, reinforced concrete piles, grain-bin construction of reinforced concrete, fireproof floor construction, etc. His studious nature kept him fully abreast with the progress of his profession, and he possessed the inventiveness combined with a clear, analytical mind and the sense of fairness required for a successful engineer. These qualities made for him a great many friends and admirers in his profession, to whom he added many more among the laymen through his winning personality, his kind helpfulness, and his attention to other people's needs.

In 1885, Mr. Hoff was married to Josie Johnson Vea, of Stoughton, Wis. He is survived by Mrs. Hoff and five children, two daughters, Mrs. Olga Hoff Fernald and Mrs. Borghild Hoff Lyman, and three sons, Matthew, Olaf, and Trygve Hoff.

He was a member of the American Society for Testing Materials, American Association for the Advancement of Science, Engineers' Club of New York, National Geographic Society, Colony Club of New York, and Commonwealth Club of Montclair, N. J.

Mr. Hoff was elected a Member of the American Society of Civil Engineers on May 6, 1885.

CLIFFORD MILBURN HOLLAND, M. Am. Soc. C. E.*

DIED OCTOBER 27, 1924.

Clifford Milburn Holland was born at Somerset, Mass., on March 13, 1883. He was the only child of Edward John Holland and Lydia Frances Hood. His father's ancestors lived in Maine, where they had moved from Massachusetts soon after the Revolution. On his father's side he was descended from Dr. Francis Le Baron, the "nameless nobleman" of early Plymouth. His mother's family came from about Providence, R. I., and Fall River, Mass., and was descended from Roger Williams and others of the earliest settlers.

He attended public schools in Somerset until 1892, then six years in St. Joseph, Mich., and then one year at the High School in Fall River, after which his family moved to Cambridge, Mass. He was graduated from the Cambridge Latin School in 1902 and entered Harvard University, receiving the degree of Bachelor of Arts in 1905 and the degree of Bachelor of Science in Civil Engineering in 1906. It is noteworthy that he completed the work for both degrees in four years.

Before leaving Harvard, he had successfully passed civil service examinations and had been appointed Assistant Engineer in the Engineering Forces of the Rapid Transit Railroad Commission of New York, N. Y. In June, 1906, he began his engineering career with an assignment to the construction of the Battery-Joralemon Street Tunnel, in which position he continued until its completion in 1908. The work was of an especially difficult and trying character, but he met every task with enthusiasm and liking and performed it very quickly and completely. He had decided to specialize in compressed-air tunneling, but no one could foresee that six years of political delay would elapse before the start of new river tunnels. He was engaged at first in completing the construction records in connection with extra claims, then, for a time, in field charge of a portion of the Fourth Avenue Subway in Brooklyn, N. Y., and, later, the new tunnels coming into prospect, he was assigned to a study of the sections to be recommended. For that use he gathered exhaustive data as to the dimensions, depths, and construction conditions of all similar tunnels that had been built in America and Europe, made a comparative analysis of their stresses and parts, and compiled the results. The work was admirably done and proved most useful, besides giving Mr. Holland a breadth of recorded information on the subject few could equal.

When, in 1914, contracts were let for the Old Slip-Clark Street and Whitehall Street-Montague Street Tunnels, Assistant Engineer Holland was entrusted with direct charge. His age was then thirty-one, but there was no question as to his fitness. His responsibilities were afterward increased when contracts were let for the Willoughby Street, Fourteenth Street, and Sixtieth Street Tunnels, making the contract value of work under his charge about $26 000 000. With five double-tube tunnels, of which four were subaqueous, in progress at once, involving a vast amount of engineering and official detail;

* Memoir prepared by the following Committee: Robert Ridgway, Past-President, Am. Soc. C. E., *Chairman*, and Charles Gilman and Frederick C. Noble, Members, Am. Soc. C. E.

with frequent emergencies requiring him to visit widely separated points and render quick, sure decisions at any time of the day or night, the situation would have dismayed any but a man of great courage, ambition, and exceptional ability. In 1916 he was accorded the long desired title and emoluments of Division Engineer. The entire work was carried along according to the best engineering precedents and without serious mishap. At the end of this connection, in June, 1919, the amount and range of Mr. Holland's subaqueous tunneling experience and his close relation to it, as well as his marked success in such work, placed him uniquely in his chosen specialty.

The New York State and New Jersey Interstate Bridge and Tunnel Commissions having been empowered by their respective State Legislatures to cooperate in the construction of a vehicular tunnel under the Hudson River, and realizing, as the Commissioners express it in their Annual Report, "that it was of vital importance to the project to secure the highest degree of technical guidance to develop and carry out this great engineering project", appointed Mr. Holland as their Chief Engineer, which office he assumed July 1, 1919. The report further says:

"After a most careful deliberation the Commission was unanimous in inviting Mr. Clifford M. Holland of Brooklyn, New York, to accept its appointment as chief engineer. The New Jersey Interstate Bridge and Tunnel Commission participated in this action.

"In securing Mr. Holland's acceptance of this position, the Commission felt that it had been singularly fortunate. At the time of his appointment he was tunnel engineer of the Public Service Commission, First District, State of New York, and was in immediate charge of the construction of all the subway tunnels under the East River. Mr. Holland has probably had a greater and more successful experience in the work of subaqueous tunnel construction than any other member of his profession."

The problem Mr. Holland was given to solve involved a number of features without precise precedent and the expenditure of about $42 000 000. An effective engineering organization had to be built up of men selected for their technical and personal qualifications rather than for political reasons, and it is greatly to the credit of the Commissioners that this right was accorded. Many plans for construction and traffic handling had been proposed and these were carefully studied and investigated. The plan finally recommended by Mr. Holland will provide a pair of cast-iron tubes, shield-driven, each measuring 29 ft. 6 in. in outside diameter and accommodating two lines of traffic in the same direction in each tube. The adoption of this plan, which has been vindicated, cost the Chief Engineer a severe struggle in which he was obliged to defend his recommendation, with all the grit and resources at his command, against the criticisms and rival plans brought forward by other engineers.

To the question of ventilation of the tunnel after completion Mr. Holland gave a great deal of personal attention, carrying out experiments in conjunction with the University of Illinois, Yale University, and the United States Bureau of Mines, and making frequent trips for that purpose, as well as an examination of ventilation conditions in European tunnels for vehicular traffic. These investigations and others relating to design, power requirements, prob-

able traffic and operating income, and other features, are fully described in his Annual Reports, published by the Commissions, and form an unusually fine record.

While Mr. Holland appeared to be generally in vigorous health, it is probable that he suffered acutely at intervals during several years. His sudden, unexpected death occurred on October 27, 1924, at Battle Creek, Mich., where he had gone for rest and treatment.

Mr. Holland was married on November 5, 1908, to Anna Coolidge Davenport, of Watertown, Mass., who survives him, as do four children, Anne, Clarissa, Benita, and Lydia.

At the time of his death Mr. Holland was a member of the Board of Direction, which adopted the following resolution:

"On October twenty-seventh, nineteen hundred and twenty-four, Clifford Milburn Holland, obedient to the call of the Divine Providence, entered the great Beyond. His friends and associates cherish his memory. He was strong, active, and fearless in the right, kind and considerate of all, a man of attainment, of initiative, and of native ability. Young in years, but old in accomplishment, he lived intensively, yet the joy of living was always his.

"Few, indeed, are endowed with the qualities of leadership, with the ability which inspires confidence and with the inborn knowledge of men and things as was he. His future lay great before him. His past was one of inspiration. His work stands as an enduring monument.

"We, his fellow members on the Board of Direction of the American Society of Civil Engineers assembled in meeting at Pittsburgh, Pennsylvania, on this first day of December, nineteen hundred and twenty-four, desire to record our sorrow which is beyond the power of words to express. The Society and the profession have sustained an incalculable and inexpressible loss.

"In mute and reverent silence we stand in the presence of an inscrutable Providence. Yet in a measure we find our consolation in being the better and the richer because in the fullness of his life we knew him, and knowing, did love and honor him.

"Our sympathy goes out to his family and to all he held dear and it is the desire of this Board of Direction, speaking for all the membership of the Society, that this expression be entered upon its minutes, be printed in its *Proceedings* and be, in suitable form, sent to the family of Clifford Milburn Holland."

He was active in the affairs of the Harvard Club of New York and the Harvard Engineering Society. Following is the minute passed by the Board of Managers of the Harvard Club:

"In the death of Clifford Milburn Holland, the Harvard Club of New York has lost a loyal and earnest member. He took a deep interest in the work of the Club, serving faithfully as a member of the Committee on Admissions and as a member of the Board of Managers. He was prominent in the affairs of the Harvard Engineering Society during the past twelve years, in which time he held all of its offices. He was equally interested in the new Harvard Engineering School, a member of its Visiting Committee, and an inspiration to the undergraduate studying engineering.

"He fought squarely and with all the energy at his command for whatever he deemed to be right. A loyal and unselfish friend he was always ready to help those who needed his assistance. Forceful and diplomatic, the benefit of his counsel was sought by all. Acknowledged to be an authority in that

branch of engineering to which he had devoted his working life, his career was cut short at the moment of bringing to fruition the monumental work that bears his name.

"In grateful recognition of his services to Harvard and to his fellow men, we—the Board of Managers of the Harvard Club—spread upon our records this minute to his memory."

At a meeting of the Harvard Engineering Society, held for memorial exercises, action was taken to rename the engineering scholarship stipend established at Harvard by that Society, to be known thereafter as the Clifford M. Holland Memorial Aid in Engineering. A resolution was adopted, from which is excerpted the following:

"In the death of Clifford Milburn Holland of the Class of 1906, The Harvard Engineering Society has lost one of its most active and useful members, loved and respected by all. He served the Harvard Engineering Society of New York as Treasurer in 1914-15, Secretary in 1915-19, and Vice-President in 1919-20, and The Harvard Engineering Society as Second Vice-President, 1920-21, and as President, 1922-23, in all of which positions he left a distinguished record. He gave unstintingly of his time and strength in the discharge of his duties and his support and counsel were ever available in the conduct of the affairs of the Society. He took a very active part in the consolidation of the Association of Harvard Engineers, the Lawrence Scientific School Association, and the Harvard Engineering Society of New York, which resulted in the present strong National Society. He took a great interest in the new Harvard Engineering School and, as Chairman of the Committee on University Relations of the Society, directed the influence of the alumni body in its co-operation with and assistance to the School."

Mr. Holland was also a member of the American Association of Engineers and of the Engineers' Club of New York.

As a tribute to the Chief Engineer who gave so much to make the tunnel a success, and as a lasting monument to his memory, the New York State and the New Jersey Interstate Bridge and Tunnel Commissions, within a short time after his death, officially named the tunnel "The Holland Tunnel". This fitting and magnanimous act is deeply appreciated by Mr. Holland's associates and doubtless will be by engineers generally.

Engineering work with which he was entrusted was always managed and executed according to the best standards, that result being secured in a manner that won him the regard, as well as the respect, of the contractors who executed the work, as witnessed by the following tribute:

"The Executive Council of the General Contractors' Association desires to place on record, as a minute of this meeting, its expression of sorrow over the death of Clifford M. Holland, Chief Engineer of the Hudson Vehicular Tunnel.

"Mr. Holland possessed with his extraordinary ability as an engineer those qualities of tact, courtesy and fairness that won the respect and affection of all those with whom he came in contact, socially or professionally.

"In his death the community has lost a public servant of inestimable value; the engineering profession, a brilliant leader; his intimates and associates, a beloved friend.

"As contractors, many of whom were privileged to come in contact with him, we honor ourselves in paying this tribute to Clifford M. Holland, who combined unswerving loyalty to his professional duty, with that spirit of honesty that conceded even-handed justice in all his dealings."

Resolutions of sorrow and appreciation were adopted by the New York State and New Jersey Interstate Bridge and Tunnel Commissions, and by the Board of Transportation of New York City which had succeeded to the control of the rapid transit work with which Mr. Holland was formerly connected.

It was characteristic of him that, even when his professional burdens were the heaviest, he found time to serve on committees of the Society and to share generously in its counsels. His earnest sympathies and activities were always to be enlisted in any movement he felt to be for the real benefit of engineers and the profession.

The loss of Clifford Holland was felt inconsolably by his many sincere friends, and to a peculiarly notable degree even by those whose acquaintanceship was slight. His rare personal qualities, so manifestly genuine, attracted friendship at once. In any gathering, his presence would be a buoyant, lively influence, or his absence would be noted and regretted. In every way he was human, sociable, likable. His place in many hearts will hardly be filled.

The following verse is a tribute *in memoriam* by Mrs. Marguerite Janvrin Adams, the wife of one of Mr. Holland's classmates:

"He is not dead who leaves on earth enshrined
His image in a work of faith and skill;
He has not died, when we can listen still
To words of praise that everywhere we find
On comrades' lips. Each day will subtly bind
Us closer to him, whose untiring will
Struggled thus valiantly and nobly 'til
The task completed lay,—to aid mankind.
He who has made on earth one work complete
To live beyond this little span of days
That we call life—at parting of the ways
Has left a monument before our feet
At which we bow in reverence. Thus we stand
So near in homage we can clasp his hand."

Mr. Holland was elected an Associate Member of the American Society of Civil Engineers on October 29, 1912, a Member on April 18, 1917, and served as a Director from 1922 until his death.

MINARD LAFEVER HOLMAN, M. Am. Soc. C. E.*

DIED JANUARY 4, 1925.

Minard Lafever Holman, the son of John Henry and Mary Ann (Richards) Holman, was born in Mexico, Oxford County, Me., on June 15, 1852.

He attended Washington University in St. Louis, Mo., and was graduated from that institution in 1874 with the degree of Bachelor of Arts. The honorary degree of Master of Arts was conferred on him by his Alma Mater in 1905.

Mr. Holman served as Assistant Engineer in the Office of the Superintendent of Architecture of the United States Treasury Department for two years after his graduation; and then one year with the firm of Flad and Pfeifer, Engineers, at St. Louis. In 1877, he was appointed Assistant Engineer to the late Thomas J. Whitman, M. Am. Soc. C. E., Water Commissioner at St. Louis. He retained this position for ten years and was then appointed Water Commissioner by Mayor D. R. Francis, serving in this capacity for three terms of four years each. At that time, the Water Commissioner not only had full charge of the design, construction, and operation of the water-works but was also a member of the Board of Public Improvements, which was charged with the design and construction of all public works for the City of St. Louis.

As Water Commissioner, Mr. Holman displayed rare ability in designing and constructing the extension of the water-works at the Chain of Rocks and the extension of the distribution system, involving an expenditure of about $10 000 000. This extension included a new intake located in the channel of the Mississippi River at the Chain of Rocks, about seven miles above the former intake at Bissell's Point; a tunnel under the river from the intake to the pumping station; a new low-service pumping plant having a capacity of about 100 000 000 gal. per day; six settling basins with a combined capacity of 180 000 000 gal.; and a brick and concrete conduit from the Chain of Rocks to Bissell's Point.

Mr. Holman's untiring efforts to reach the correct solution of the various problems encountered in this work, his honesty of purpose, his ready recognition of the meritorious work of his assistants, and his freedom from political influence in the organization of his Department, inspired those who served under him. As a member of the Board of Public Improvements, he upheld the traditions which enabled this Board to transact the public business with honesty and fairness. His broad knowledge of engineering helped to guide the Board in the proper solution of each new engineering problem. As Chairman of the Committee on City Lighting, he had charge of the conduit construction for the electric wires, and the preparation of the specifications and letting of contracts for the lighting of streets, alleys, and public buildings.

Mr. Holman kept well informed regarding engineering developments, especially those tending to improve the water-works. He installed high-duty

* Memoir prepared by Edward Flad, Baxter L. Brown and James C. Travilla, Members, Am. Soc. C. E.

triple-expansion pumping engines and down-draft boilers in the pumping plants, thereby materially reducing the cost of operation. Under his direction, electric cranes were installed in the engine houses and electric drives in the machine shop.

After the expiration of his third term as Water Commissioner, Mr. Holman accepted the position of Superintendent of the Missouri Edison Electric Company, which he held for five years. In 1904, he organized the firm of Holman and Laird, Consulting Hydraulic and Mechanical Engineers, and continued in active practice until ill health forced his retirement. He died at his home in St. Louis, on January 4, 1925, after a prolonged illness.

Mr. Holman was recognized as an authority in the field of water-works engineering, and was frequently consulted by various cities in connection with water-works problems. He served on the Board of Appraisers of Denver, Colo., which determined the value of the water system of that city and fixed the schedule of water rates.

He was actively interested in the American Society of Mechanical Engineers, having served as Vice-President in 1894-96 and in 1903-05, and as President in 1908-09. He was also an Honorary Member of the American Water Works Association and of the Engineers' Club of St. Louis, having served as President of the latter in 1888 and 1910.

On September 7, 1879, Mr. Holman was married to Margaret H. Holland, of St. Louis. His widow and three sons, Charles H., Minard H., and George R., survive him, one daughter, Mary Holman, having died in 1918.

The City of St. Louis is indebted to Mr. Holman for twenty-two years of faithful and unselfish service. He lived a noble and useful life; a seeker after truth, patient and untiring in his investigations, with a keen analytical mind, combined with unusual engineering ability, his acts inspired public confidence and general recognition of his honesty, ability, and fidelity.

Mr. Holman was elected a Member of the American Society of Civil Engineers on April 2, 1884, and served as Vice-President of the Society in 1905 and 1906.

CLARENCE LORING HUSSEY, M. Am. Soc. C. E.*

DIED DECEMBER 5, 1925.

Clarence Loring Hussey was born in Milford, Mass., on November 16, 1884. He was the son of the late Albertus and Effie A. (Holmes) Hussey, both of whom were born in Milford, where Mrs. Hussey lived until a few years ago.

Mr. Hussey was graduated from the Massachusetts Institute of Technology in June, 1908. He immediately entered the employ of the Providence Engineering Works as Foreman in charge of automobile experiments and testing, and remained with that firm until March, 1909, when he became a member of the Engineering Staff of F. P. Sheldon and Sons, of Providence, R. I., Mill Construction Engineers. From April, 1910, to July, 1912, Mr. Hussey was employed by the Rhode Island Electric Car Company on designs and experimental work relative to street railway power plants. While with this company he designed and supervised the construction of extensive car barn facilities and several bridges.

From July, 1912, until his death Mr. Hussey served as Bridge Engineer for the Rhode Island State Board of Public Roads and as such supervised the design and the construction of all bridges on the State Highway System. In addition, he conducted the investigation for and designed the proposed $3 000 000 Washington Bridge over the Seekonk River, which is to connect the City of Providence with the Town of East Providence. Previous to his death he was directing the examination and preliminary design for a proposed toll bridge across Mount Hope Bay to connect the Island of Rhode Island, on which is located the City of Newport, with the mainland at Bristol. The construction of this bridge is being promoted in order to provide a direct route between the Cities of Providence and Newport.

Among the many original ideas which Mr. Hussey introduced in the construction of concrete arch bridges in Rhode Island was the design which allowed approximately a 50% reduction in the volume of abutments and arch rings for a given width of road. This design called for the carrying of a sidewalk and railing on brackets anchored to the arch ring. On the back of these brackets lay inclined spandrel walls which materially reduced the dead load due to the spandrel fill. This form of construction made necessary light sidewalk railings in the design of which Mr. Hussey again exhibited striking originality.

He was particular in the selection of mineral aggregates to insure high strength concrete. His special mixtures for contraction joints permitted a larger range of use of these troublesome features of construction, and whenever it became necessary to combine concrete with other materials his designs were unique and effective.

By nature Mr. Hussey was of a quiet, studious disposition. This manifested itself in his investigations of concrete mixtures as applied to the State

* Memoir prepared by G. H. Henderson, M. Am. Soc. C. E.

bridge work and to the construction of his own home which was of concrete. He originated most of the standards now in vogue in his Department; these included an elaborate file of records, photographs, and cost data. In fact, the whole conduct of the State bridge work became highly systematized under his direction and this in large measure accounted for its success.

He was also a keen student of structural design. His ideas, although marked by striking originality, had the saving virtue of reasonableness. In numerous instances he put them into practice to the great economy of the State bridge work. In his conduct of his Department, he was noted for his rigid, almost inflexible, principles applied alike to his colleagues, to contractors, and to himself.

In spite of a reserved outward manner, within he was most friendly and companionable. This became very evident to those who knew him in social or professional organization work. For several years he was Secretary of the Technology Club of Rhode Island (graduates of the Massachusetts Institute of Technology). He was also active in the Providence Engineering Society. His attitude in these services was always one of helpfulness and his efforts were characterized by an innate thoroughness and conscientiousness. Few demands were made on his help without a willing response and no duties did he undertake without their full accomplishment.

As a mark of respect to the memory of its late Bridge Engineer, the Rhode Island State Board of Public Roads, at its meeting on December 9, 1925, adopted unanimously the following resolution:

"*Whereas,* it hath pleased Almighty God in his divine wisdom to call from his earthly labors Clarence L. Hussey, our friend and co-worker; therefore be it

"*Resolved* that the State Board of Public Roads record its sense of severe loss in the decease of a highly efficient and conscientious member of its staff. Throughout his thirteen years of association with the State Board of Public Roads he performed faithfully and well every task to him assigned. He rendered a public service which may justly be pointed to with pride and held up as an inspiration to others called into public service; and be it further

"*Resolved* that this Board does hereby express to the family of the late Clarence L. Hussey its most sincere sympathy in their bereavement, which loss is shared by the members of this Board in a very personal sense; and be it further

"*Resolved* that this resolution be spread upon the minutes of this meeting and that the Clerk be and he hereby is directed to transmit a copy hereof to the family of our late respected associate and Chief Bridge Engineer."

For several years previous to his death Mr. Hussey, who was unmarried, resided with his parents in Providence. Besides his mother he is survived by a sister, Mrs. John R. Frazier, of Providence.

He was a member of the Masonic fraternity, in addition to local technical societies, and served on various bridge structure committees with the American Association of State Highway Officials and the National Research Council.

Mr. Hussey was elected a Member of the American Society of Civil Engineers on April 7, 1924.

THOMAS NOTTINGHAM JACOB, M. Am. Soc. C. E.*

DIED MAY 25, 1925.

Thomas Nottingham Jacob, the son of W. W. and Helen Williams Jacob, was born at Washington, D. C., on February 6, 1866. His maternal grandfather, Col. John Stutt Williams, of Virginia, served with distinction in the War of 1812.

Mr. Jacob's early engineering experience was gained by service in minor capacities in the Engineering Departments of the Chicago, Milwaukee and St. Paul Railway Company, the Illinois Central Railway Company, and the Union Pacific Railroad Company, from 1886 to 1891. He was then appointed Assistant Engineer on the Kanawha and Michigan Railroad and was engaged on location and construction work for four years, principally in West Virginia.

In April, 1895, he became Assistant Engineer with the United States Engineer Corps, and was assigned to the Rock Island (Illinois) District, where he remained until November, 1897. His work while with the Government was in connection with surveys and plans for the flood control of the Mississippi River.

Mr. Jacobs left the Government service to return to railroad work with the Wabash Railway Company and, later, with the Illinois Central Railway Company, the Chicago, Milwaukee and St. Paul Railway Company, and the St. Louis Valley Railway Company (now the Illinois Division of the Missouri Pacific Railroad).

In 1903, he formed a partnership with Frank Hutchinson, M. Am. Soc. C. E., for the general practice of Civil Engineering, with offices at East St. Louis, Ill., and, in 1904, he conducted the field investigations relative to flood protection and control at Kansas City, Mo. This partnership continued until the early part of 1909, when Mr. Jacob was appointed Chief Engineer of the East Side Levee and Sanitary District, which was organized for the purpose of protecting about 150 sq. miles of industrial territory in St. Clair and Madison Counties, Illinois. As Chief Engineer of this District, he worked out plans of flood prevention and drainage, in conjunction with the late John A. Ockerson, Past-President Am. Soc. C. E., as Consulting Engineer, involving an expenditure of about $7 000 000. Mr. Jacob served in this capacity until January, 1914, when he again entered general engineering practice, first, in East St. Louis, and, later, in St. Louis, Mo., making a specialty of drainage and levee work, in which he was quite successful. His practice covered important work in Indiana, Illinois, and Missouri, and, at the time of his death, he was Chief Engineer of the Peru, Ind., Flood Control District.

* Memoir prepared by Baxter L. Brown, M. Am. Soc. C. E.

Mr. Jacob was a man of high moral character and commanded the respect of all with whom he came in contact. On November 29, 1893, he was married to Margaret T. Leonard, of Hugheston, W. Va., who survives him.

He was a member of the Engineers' Club of St. Louis, the St. Louis Railway Club, and the St. Louis Institute of Consulting Engineers.

Mr. Jacob was elected a Member of the American Society of Civil Engineers on May 4, 1909.

WILLIAM SIMPSON KELLER, M. Am. Soc. C. E.*

DIED SEPTEMBER 9, 1925.

William Simpson Keller was born, on February 20, 1874, at Tuscumbia, Ala., and was a son of Arthur Henley and Sally (Simpson) Keller. His father who was a native of Tuscumbia, was Captain and Paymaster in Roddy's Command, Confederate Army, was promoted to the rank of Major just as the war closed, and was a prominent lawyer and editor. He was a grandson of David and Mary (Moore) Keller, of Tuscumbia, the latter a direct descendant of Governor Spottswood, first Colonial Governor of Virginia, and a second cousin to Gen. Robert E. Lee.

Mr. Keller received his early schooling at Tuscumbia, attended the State Normal School at Florence, Ala., and was graduated with the degree of Bachelor in Civil Engineering from the University of Alabama in 1893.

He was County Surveyor of Colbert County, Alabama, during 1897 and 1898; Assistant Engineer of Shiloh National Park from 1900 to 1905; Chief Engineer of the Good Roads Commission, Madison County, Tennessee, from 1905 to 1910; with the United States Office of Public Roads from 1908 to 1909; Engineer in charge of road construction in Dallas County, Alabama, in 1910 and 1911. In 1911, Mr. Keller was appointed State Highway Engineer of Alabama, which important post he held until his death.

The love, respect, and esteem in which Mr. Keller was held could not be better stated than in the following editorial that appeared in the *Montgomery Advertiser,* Montgomery, Ala., on September 10, 1925:

"SIMPSON KELLER

"The death of William Simpson Keller, in the prime of his life, is a distinct loss to the people of Alabama, and to the State government which is engaged in administering the benefits to the people which they have claimed. Both the people and the State government are poorer by the death of this high-minded and efficient public servant who had demonstrated his capacity for a highly specialized public service.

"His friends, those who knew him well and intimately, find inexplicable the decree of Providence which removed from his place in the activities of mankind this efficient and faithful public servant who through many years of apprenticeship and experience had demonstrated that he possessed in a rare degree the ability and energy which his high professional position required. He was essentially a product of Alabama. He was born among the people of Alabama, lived among them all his life, was educated among them and among them did his work of life. The death of this man of ability and character recalls the life of his father, who was so well known and admired by the members of a past generation. Captain A. H. Keller was a member of the Civil War generation and he faithfully served the Confederacy as a soldier till the collapse of the Confederacy, which involved the people of the South in financial and material ruin. For many years afterwards Captain Keller was the brilliant editor of a paper in Tuscumbia, which exerted no little influence in the affairs of the State. There in clean-minded, high-thinking, his children grew up about him. It is a tribute to a fine example of an older generation merely

* Memoir prepared by R. P. Boyd, M. Am. Soc. C. E.

to say that he was the father of that famous genius, Helen Keller, who courageously overcame the handicaps of Nature, and of Simpson Keller, whose expert professional skill inaugurated the era of good roads in Alabama.

"The younger Keller prepared for his professional career at the University of Alabama and in him was mingled the love and pride in his chosen profession and a love for and an appreciation of his native State. Alabama was preparing to enter upon an era of good road building, but the sentiment had not found adequate expression. The first stage of the campaign to give Alabama good roads was the bonding of several counties to finance the great undertaking in those counties. It was natural that Dallas, a leader of this movement of progress, should call in for direction a native of Alabama who had previously demonstrated his energy and ability, and so Simpson Keller was called in as County Engineer to undertake this great work. Much depended upon the integrity and professional skill of these early county engineers. The taxpayers had taxed themselves to pay for a great public work the nature of which they had not known, as no work of this kind had been done by previous generations, and they scrutinized closely this heavy expense and the results that it brought. Dallas County entered the work in a whole-hearted way, but somewhat timidly and nervously. But under the direction of Mr. Keller, this work of road building was an unqualified success and when the taxpayers of Dallas viewed and used the good roads they were eminently satisfied that they had authorized the bond issue. In time the creation of good roads became an activity of the State and the able Engineer who had been so successful in Dallas County, was called to superintend the greater work of building good roads for the State. He made his job one of the most important in the State, and although the law relating to the State Highway Commission was inadequate, this law was never altered without some of the men of influence considering its possible effect on Mr. Keller's office.

"As to how a road should be built and where it should be located, there are many differences of opinion among men. Through the many years he was in office, Mr. Keller was successful in composing these difficulties, not merely because he was diplomatic and polite, but because he was just and impartial and guided by the highest principles of integrity. Many of our States have had highway scandals over the misuse of the large sums of money involved in their construction, but there has been none in Alabama, where the funds were administered by an honest, high-minded gentleman, who strove always to administer the trust that was imposed in him for the good of the people. The work for good roads in Alabama is far-flung and comprehensive. Much of it has already been accomplished and in these splendid highways will be found the monument of Simpson Keller, who was the right man in the right place. It is the beginning of an era and a man who was conspicuous in ushering in the era will not be forgotten by a grateful people when they travel the smooth and beautiful highways which they owe in a large degree to his skill.

"Mr. Keller, in his dependability, his faithfulness to his duty, and his grasp of the problems of his office was an inspiration to his associates, both his superiors and his subordinates.

"He was a man of striking personality, warm, genial and cordial, and considerate of the rights of all. He leaves a most pleasant memory to all who knew him and tender and pleasant memories to those who were admitted to his friendship. A faithful public servant, a patriotic citizen, a loyal friend, and a helpful neighbor who loved his fellow men passed when he died. He went in the prime of life and when he had every reason to expect many years of useful activity. He died literally 'with his harness upon his back.'"

Mr. Keller was an Elder in the Presbyterian Church, a Rotarian, a Fellowcraft Mason, but was taken ill before receiving his master's degree. He served as Director and Past-President of the American Association of State Highway Officials, and was a Director of the American Road Builders Association.

In 1902, he was married to Aline Moore, of Columbia, Tenn., who died in February, 1911. In 1913, he was married to Annie Searcy, of Tuscaloosa, Ala., who survives him. Other relatives are two sisters, Mrs. L. Warren Tyson, of Montgomery, Ala., and Miss Helen Keller, of New York, N. Y.; one brother, Phillips B. Keller, of Mineral Wells, Tex., and an aunt, Mrs. L. K. Lasseter, of Selma, Ala.

Mr. Keller was elected a Member of the American Society of Civil Engineers on July 2, 1913.

WALTER GILL KIRKPATRICK, M. Am. Soc. C. E.*

DIED MAY 8, 1925.

Walter Gill Kirkpatrick was born in Canton, Miss., on September 6, 1859. His early education was acquired in the public schools of his native town. He was also a student of the University of Mississippi for one session, 1877-78.

After filling a subordinate position in 1881 with a surveying party of the Denver, South Park, and Pacific Railroad Company, Mr. Kirkpatrick entered Vanderbilt University where he received four degrees, Bachelor of Engineering, in 1886, Civil Engineer, in 1887, Bachelor of Science and Master of Science, in 1889, and was for two years Fellow and Assistant in Engineering.

He spent several years in municipal, bridge, and railroad engineering in Tennessee, partly as Assistant Engineer and partly in responsible charge of work, and was then, successively, United States Assistant Engineer in charge of a party surveying the Cumberland River for a system of locks and dams, locating Lock A; Secretary of the Engineering Association of the South; Acting Professor of Civil Engineering, for a short time, in Union College, Schenectady, N. Y.; and Engineer in charge of the Tennessee Centennial Exposition, for a period of six months.

For several years, Mr. Kirkpatrick's work was largely in Mississippi, where he was engaged as a Designing and Constructing Engineer for electric light plants, water-works, and sewerage systems in Canton, Holly Springs, West Point, Crystal Springs, and other of the more important towns of the State. In 1898, he was made the first Municipal Engineer of the City of Jackson, Miss.

In 1899, Mr. Kirkpatrick constructed a Melan arch bridge in Jackson, the first of its kind to be built south of the Mason and Dixon Line and the sixth of its kind in the world. It was only 6 in. thick at the crown, and was built of reinforced concrete. The old surface wore away and a new one has replaced it, yet the body of the structure, enwrapped in the strength given it by its designer, stands to-day and is giving continuous service.

A few years later, he constructed the first concrete sidewalks to be laid in Mississippi. With opposition on all sides, he built these walks confidently and unhesitatingly. He also constructed the first radial brick chimney in Mississippi. It was built in ten days by four men, and is still giving continuous service. He also drew up the first plumbing sanitary ordinances in Mississippi —ordinances which have been adopted by nearly every town of any size in the State and by many towns in other Southern States.

The facts thus briefly enumerated, and others which might be mentioned, abundantly justify the claim that Mr. Kirkpatrick did genuinely pioneer work in his profession and that it was work of the highest grade.

From Jackson, he went to Birmingham, Ala., as Municipal Engineer of that city, after which he was with The Alabama Power Company and then

* Memoir prepared by Alfred Hume, Chancellor, Univ. of Mississippi, University, Miss.

became City Engineer of Monroe, La. Later, still, he was engaged in private practice as a Consulting Engineer in Atlanta, Ga. He served as Designing and Consulting Engineer for more than a score of towns and cities in nearly all the Southern States, building water-works, drainage systems, electric light plants, etc. He also served as Consulting Engineer for a number of the largest Southern cities. For two years prior to his death, he had been Professor of Municipal Engineering at the University of Mississippi.

The following is quoted from resolutions adopted after Professor Kirkpatrick's death by the Faculty of the University of Mississippi:

"As a student at Vanderbilt University, he was conspicuous for his fine scholarship and fine manhood. A student of deep and earnest purpose, he was an inspiration to the younger men. He made it a principle to master each subject from day to day; he hesitated at no task; he found no problem too difficult. His splendid success in no way spoiled the lovableness of his young manhood. The faculty early showed its high esteem by appointing him to a responsible teaching fellowship and by the bestowal of various other honors.

"His later life was of a piece with his college years. Always and everywhere he was a Christian gentleman.

"As a practicing engineer he had the highest regard for professional thoroughness and professional ethics. He worked with a patience and accuracy worthy of all emulation. In Nashville, Tenn.; in Jackson, Miss.; in Monroe, La.; in Birmingham, Ala.; and in many other Southern cities, he held positions of the highest responsibility and held them with honor to his profession. He loved his work.

"As a University professor he was a deep student, an enthusiastic teacher, and a warm friend. Even as a practical engineer he was a valued teacher, for he was always an inspiration to the young engineers under him.

"We mourn his loss,
A beautiful life has ceased to be,
A gentle star has been blotted from the galaxy."

The writer was intimately acquainted with Professor Kirkpatrick from the time of their college days at Vanderbilt University. In a recent report to the Board of Trustees of the University of Mississippi, referring to the Faculty, he said:

"The University has suffered a heavy loss in the death of Professor Walter Gill Kirkpatrick of the Chair of Municipal Engineering. He died on the 8th of May, when the Government Steamboat, *Norman,* sank in the Mississippi River, about sixteen miles south of Memphis. The Engineers of the Mid-South had been holding a two days' session in the City of Memphis, and had been down the river on the fatal day, inspecting Government work. Members of the American Society of Civil Engineers were in conference on the boat at the time of the disaster.

"Although Professor Kirkpatrick had been connected with the University only two years, he had made a profound impression on his students. While his loss seems simply irreparable, yet the moral and spiritual effect of his tragic death is one of the finest things I have ever witnessed. The affection and esteem in which he was held has created a spirit and atmosphere of

incalculable value. I have never known a more admirable man, one whose life was whiter or more straight. He was the very soul of honor. Not a single blemish marked his record. I think that no mere man ever had a more nearly flawless character or led a more blameless life."

The same disaster that caused the death of Professor Kirkpatrick resulted in the drowning of his wife, *née* Willie Jones, of Canton, Miss. They had no children.

Professor Kirkpatrick was elected an Associate Member of the American Society of Civil Engineers on April 6, 1892, and a Member on October 5, 1898.

CHARLES WILLIAM KNIGHT, M. Am. Soc. C. E.*

DIED MAY 20, 1923

Charles William Knight, the son of Joseph H. and Julia E. (Butts) Knight, was born at Stanwix, near Rome, N. Y., on October 26, 1847. He was graduated from the Rome Academy, having had among his teachers the Hon. Elihu Root and Professor Oren Root. He also attended the Eastman Business College.

Mr. Knight's father was a land surveyor and carpenter and under his instruction the son learned the elements of surveying and gained a thorough knowledge of the carpentry trade, which stood him in good stead in his later engineering work. He familiarized himself with the cabinet maker's trade also, in which he was able to do highly creditable work. His next earliest experience in civil engineering was under the late George W. Chase, City Engineer and Surveyor of Rome, N. Y. While with Mr. Chase, he determined to make civil engineering his life work and applied himself to such studies as would fit him for that profession. He was associated with the late Peter Hogan, a Civil Engineer of Albany, N. Y., on several projects, among which were the water-works systems of Grand Rapids and Muskegon, Mich., and the original water-works system of Gloversville, N. Y.

After the completion of these works, Mr. Knight, as Contractor, built considerable parts of the sewerage system of Gloversville, and, later, became associated with the late Adam Miller, of Saratoga Springs, N. Y., and built under contract many systems of water-works. In 1881-82 he served as Engineer for R. D. Wood and Company, of Philadelphia, Pa., Contractors, on 6 miles of 48-in. pipe line from the Kensico Dam of the Bronx River supply. The partnership with Mr. Miller continued until about 1885, after which Mr. Knight discontinued contracting to devote his time to the practice of his chosen profession, making a specialty of Hydraulic and Sanitary Engineering.

A partnership was formed with Charles C. Hopkins, M. Am. Soc. C. E., in December, 1887, which continued until January 1, 1890, when J. W. Ledoux and J. W. Kittrell, Members, Am. Soc. C. E., were added to the partnership, forming The Stanwix Engineering Company, of Rome, N. Y. Mr. Ledoux retired in the fall of 1890, and Mr. Kittrell in 1896, the firm then becoming Knight and Hopkins. This latter association continued until 1910. In 1912, with his son, Arthur P. Knight, Mr. Knight organized the engineering firm of C. W. Knight and Son, which continued until the death of the elder Mr. Knight.

During his long career, Mr. Knight served as Chief Engineer of many complete systems of water-works, among which were those of Sayre and Athens, Scottdale, Uniontown, Muncy, Marietta, Tunkhannock, Connellsville, Greenville, and Uniontown, in Pennsylvania; Cambridge, Cazenovia, Frankfort, Ilion, Jordan, Ossining, Skaneateles, Wellsville, and Cleveland, in New York; and Newcomerstown, in Ohio. He was also Chief Engineer of the storage and distribution reservoirs of the Altoona, Pa., Water-Works; of the Westmoreland

* Memoir prepared by Charles C. Hopkins, M. Am. Soc. C. E.

Water Company's System supplying Greensburg and other Pennsylvania boroughs; of the Bear Rock Run and the Bell's Gap Run Reservoirs for the Pennsylvania Railroad Company; of the new water supply for DuBois, Pa.; and of improvements to the supplies of many other municipalities. In his work he was called upon to appraise the value of many existing water-works systems and to act as expert in many legal cases involving the diversion of water.

Those who were acquainted with Mr. Knight knew him to be a man of innate honesty and fairness and of a most genial nature, modest and unassuming. He never allowed his temper to get the better of him, nor did he show umbrage at provocation. His early experience as a contractor taught him fair dealing in his later work, and in all his engineering engagements his well formulated opinions carried great weight. Temperamentally, he was an optimist, always seeing the good and the bright side of life and its affairs, and a lover of the beautiful in human character and in Nature. He took a great interest in all matters pertaining to his native town and city.

Mr. Knight was a devout and consistent member of the Presbyterian Church, and was also a member of the Rome (N. Y.) Club.

He was twice married, his first wife having been Altay Elizabeth Potter, of Cleveland, N. Y., whom he married on October 18, 1875. Of the four children born of that marriage, one son, Arthur P. Knight, survives him. Mrs. Knight died on December 18, 1884. His second wife was Charlotte L. Lyndon, of Eaton, N. Y., whom he married on October 16, 1907, and who, with two children, Dorothy L. and Edgar Franklin, survives him.

Mr. Knight was elected a Member of the American Society of Civil Engineers on July 9, 1906.

CHARLES WALTER LAWRENCE, M. Am. Soc. C. E.*

DIED APRIL 1, 1925.

Charles Walter Lawrence, the son of C. F. and Mary A. Lawrence, of Claremont, Calif., was born at Plain Grove, Lawrence County, Pa., on June 24, 1872.

He received his engineering education at Pennsylvania State College, from which he was graduated in 1897, with the degree of Bachelor of Science in Civil Engineering. He returned to his Alma Mater in 1904 to receive the degree of Civil Engineer.

In September, 1897, Mr. Lawrence was appointed Instructor in Civil Engineering at Pennsylvania State College, where he remained until June, 1899, when he accepted a position as Draftsman with the Pennsylvania Steel Company, at Steelton, Pa.

In October, 1901, he entered the employ of the Brown-Ketchum Iron Works, at Indianapolis, Ind., as Structural Engineer on the design of steel building and bridge work. In September, 1904, he returned to Pennsylvania State College as Assistant Professor of Structural Engineering, which position he retained until September, 1906, when he became Professor of Civil Engineering at the James Milliken University, Decatur, Ill., which Department he established and directed until June, 1909. His summer vacations from 1906 to 1909 were spent in practical work as Engineer with the Brown-Ketchum Iron Works, on steel design, as Designing Engineer with the Westlake Construction Company, St. Louis, Mo., and as Engineer with the American Bridge Company, Lassig Plant, on design and checking of details.

For fourteen years prior to his death, Professor Lawrence served as Professor of Civil Engineering and as Dean of that Department at the University of Southern California, Los Angeles, Calif. It was largely through his efforts that the Department was developed into one of the leading engineering colleges in the United States.

In Professor Lawrence's early demise, the Engineering Profession loses an able and clear-headed thinker, and the community and the educational institution which he served, an honest, public-spirited man, and an able teacher.

To those of his many friends who came in contact with him, the outstanding features of his character were his whole-souled and unselfish devotion to his profession, his loyalty and friendship for those with whom he was associated, simplicity of character, and forgetfulness of self.

Professor Lawrence was a Charter Member and Vice-President of the Los Angeles Chapter of the American Association of Engineers, later becoming a

* Memoir prepared by Henry Z. Osborne, M. Am. Soc. C. E.

member of its Board of Directors; a member of the Society for the Promotion of Engineering Education; and a member of Chi Epsilon, the National Honorary Civil Engineering Fraternity.

He is survived by his widow and two sons, Harold Glover and Charles Oliver Lawrence.

Professor Lawrence was elected an Associate Member of the American Society of Civil Engineers on January 8, 1908, and a Member on January 20, 1922.

JAMES ALBERT McDONOUGH, M. Am. Soc. C. E.*

DIED AUGUST 8, 1925.

James Albert McDonough was born at Knoxville, Tenn., on December 17, 1869, the son of Dr. James S. and Mary Lenorr McDonough.

He received his education in the schools of his native city, completing the course in Civil Engineering at the University of Tennessee and taking post-graduate work there in 1894 and 1895.

In June, 1895, Mr. McDonough entered the service of the Ohio River, Knoxville, and Tidewater Railway Company and between this date and 1897 was employed by that company and others on topographic surveying and drafting work. From 1897 to 1901, he was engaged as Inspector and Draftsman at Port Royal, S. C., on the design and construction of buildings, dry docks, and other naval yard works, and from 1901 to 1904, was employed at the Navy Yard at Boston, Mass., and the United States Bureau of Yards and Docks, Washington, D. C., as a Designer on similar work.

In November, 1904, he entered the United States Engineer Department at Wheeling, W. Va., as a Junior Engineer engaged on the design of locks, dams, etc., for the canalization of the Ohio River, and from this time was employed on this work in various capacities. He was promoted to be Assistant Engineer in 1914, and in 1915 was transferred from Wheeling to the office of the Division Engineer at Cincinnati, Ohio, where, at the time of his death, he was acting as Head of the Designing Department and Assistant to the Division Engineer.

Mr. McDonough was married in 1916 to Martha Longley, of St. Clairsville, Ohio, who with his mother and three sisters survives him.

He was deeply interested in his profession and devoted much of his time to its study and to participation in its activities. While well versed along broad engineering and scientific lines, Mr. McDonough possessed special knowledge of the inland waterways of America and Europe and was regarded as an authority on that subject. Always a firm believer in the ultimate success of the Ohio River improvement, on which he spent so many years of his life's work, his inability to see its approaching completion is particularly regrettable.

Mr. McDonough was elected an Associate Member of the American Society of Civil Engineers on April 3, 1907, and a Member on May 3, 1910. He had also served as President of the Cincinnati Section of the Society and was an active member of the Engineers Club of Cincinnati.

* Memoir prepared by C. W. Kutz, M. Am. Soc. C. E.

JOHN HENRY MADDEN, M. Am. Soc. C. E.*

DIED JANUARY 22, 1925.

Within twenty-seven years after he had received his degree as Civil Engineer at Columbia University, New York, N. Y., John Henry Madden became Chief Engineer of the Transit Commission, after an active career in the supervision of construction of the New York underground railroads which made him an expert and well qualified him for such a responsible position. Few engineers can equal such a record of achievement in a similar period of time. His untimely death cut short a career which promised a brilliant future.

Born in New York, N. Y., on October 18, 1875, John Henry Madden received his preliminary education in the public schools and then entered Columbia University, from which he was graduated in 1897 with the degree of Civil Engineer. His first work after his graduation was with William Barclay Parsons, Hon. M. Am. Soc. C. E., who designed and directed the construction of New York's first subway. Young Madden worked as a Surveyor and Draftsman for the new underground railroad and became so interested in the work that he made it virtually his life's pursuit.

For only a few years did he interrupt his work on underground railroad construction. He devoted two years to service with the United States Geological Survey as Field Assistant on topography and another year to engineering design and construction under Charles W. Leavitt, M. Am. Soc. C. E., to whom he was Assistant Engineer. In 1901, he returned to his first love and became Assistant Engineer on the construction of a section of the first subway in Broadway between 60th Street and 82d Street, under the Board of Rapid Transit Railroad Commissioners of New York. When he had been on this work for six years, a change in the law replaced the Board of Rapid Transit Railroad Commissioners with the Public Service Commission for the First District. In 1907 he was transferred to the staff of the new Commission with the title of Assistant Engineer and remained in its employ and on the same kind of work for eleven years.

In those eleven years Mr. Madden showed the stuff of which he was made. He handled the most difficult and dangerous kind of subway construction, always with credit to himself and to the great satisfaction of his superior officers, among whom were Alfred Craven and Daniel L. Turner, Members, Am. Soc. C. E., and Robert Ridgway, Past-President, Am. Soc. C. E., each in turn the Chief Engineer of the Commissions conducting subway construction. Included in this period of Mr. Madden's activity were supervision of extensions of stations in the first subway between 157th Street and Dyckman Street, Manhattan—a delicate piece of work because it had to be done during the daily operation of the subway trains; the construction of four sections of the new subways—Dual System—in Lexington Avenue between 53d and 104th Streets, Manhattan; the construction of two more sections in Park Place, Beekman and William Streets, Manhattan; and four more sections in Flatbush Avenue, Brooklyn, between Atlantic Avenue and Malbone Street. All

* Memoir prepared by Le Roy T. Harkness, Transit Commr., New York, N. Y., and Frederick L. Cranford, M. Am. Soc. C. E.

this was difficult underground construction which taxed the skill of the most experienced engineers. His work on the William Street Subway is historic in its success in underpinning lofty buildings and avoiding damage to abutting property.

In 1917, as a reward for his faithful and efficient services, Mr. Madden was appointed by the Public Service Commission to be Engineer of Rapid Transit Operations, in charge of expenditures and distribution of revenues under the operating contracts of the subways of the Dual System. The World War brought the United States into armed conflict in 1917, and the Government called many engineers into its service. Mr. Madden was one of them, receiving and accepting in the summer of 1918 an appointment from the War Department as Assistant Director of Construction of Nitrate Plants. The Director was Frederick L. Cranford, M. Am. Soc. C. E., and in association with him, Mr. Madden was in charge of the construction of the three Government nitrate plants at Muscle Shoals, Ala., and Toledo and Cincinnati, Ohio.

At the conclusion of the war Mr. Madden became a member of Mr. Cranford's Company—Frederick L. Cranford, Inc.—where he remained until 1921. In that year the Transit Commission was created by law, and empowered to make a valuation of all the street railroad properties in New York City. The Commission chose Mr. Madden for this work and appointed him its Valuation Engineer. He organized the work and made a complete valuation of the subways, elevated roads, and street surface lines, submitting to the Commission a monumental report, which is one of the greatest of its kind and is recognized as a model throughout the United States. In 1924, the Transit Commission appointed him its Chief Engineer, a position which he held at the time of his death.

Mr. Madden was highly talented and in addition was gifted with a personality which attracted warm friendships. Great ability in an engineer combines many attributes—technical knowledge, practical experience, common sense, courage, vision, fairness, scientific honesty, executive ability—all of which John Madden had, together with a positive and forceful directness which, although outspoken, never offended. Where he was known he was always respected and loved, and the sense of loss when his death occurred was marked by unusual and spontaneous expressions of regret containing warm tributes to his character and ability.

Mr. Madden was elected a Junior of the American Society of Civil Engineers on October 7, 1902, an Associate Member on May 4, 1904, and a Member on June 16, 1919.

VIRGIL GEORGE MARANI, M. Am. Soc. C. E.*

DIED NOVEMBER 2, 1925.

Virgil George Marani was born on July 4, 1868, at Regio, in the Province of Emilia, Italy. He was left an orphan when very young and, with his two brothers, was removed to Edinburgh, Scotland, where he received a common school education and was taught to forget his native tongue. At the age of eleven, young Marani ran away from school, but was brought back and by way of discipline and punishment was sent to a naval schoolship known as H.M.S. *Conway,* located at Liverpool, England. He served on this ship for two years and then was given an honorable discharge because he was considered to be of insufficient physical strength to complete the naval course. Determined to build up his physical strength and to see the world, the thirteen-year old boy shipped on a full rigged sailing ship of the British Merchant Marine which, at that time, was one of the largest in the world, and began a career as a sailor which lasted until he was twenty-one years of age. In that time, he had served on sailing vessels and steamships, had circumnavigated the globe twice, and had sailed many times around Cape Horn and the Cape of Good Hope.

On coming of age, in 1889, Mr. Marani inherited his share of his mother's estate and had the good judgment and the ambition to use this money for his education through high school and university. Within four years he had completed both high school and university work, having been graduated from the University of Toronto in 1893, with the diploma of Civil Engineer. In June, 1924, the degree of Civil Engineer was conferred on him by his Alma Mater.

During his vacations, from 1889 to 1893, Mr. Marani held the position of Chief Inspector of Sewerage Works at Brantford, Ont., Canada, for Willis Chipman, M. Am. Soc. C. E., Sanitary Engineer of Toronto, Ont., Canada, and immediately following his graduation he was engaged as Transitman on survey work for Mr. W. J. White, at Buffalo, N. Y., and, also, as Chief of Field Party and General Inspector for Mr. Edward F. Ball.

Within a year after his graduation from the University of Toronto, however, Mr. Marani went to Cleveland, Ohio, where he was employed in the City Engineer's Office, successively, as Assistant City Draftsman, Chief Draftsman, and Inspector and Engineer of Sanitation. In 1895, he became Assistant Engineer with the Cleveland Gas Light and Coke Company, and in 1900 was promoted to be Engineer of Construction, holding this position for six years. In this capacity he detailed and planned all buildings, machinery, foundations, etc., for a plant covering more than eleven acres.

From 1907 to 1909, he was Engineer Superintendent of the Cuyahoga County Court House, at Cleveland, Ohio, a monumental structure costing $5 000 000. Following the completion of this building in 1910, he was appointed Building Commissioner of the City of Cleveland, which office he

* Memoir compiled from information furnished by H. H. Macdonald, Secretary, The Gypsum Industries, Chicago, Ill.

held for one year. During this time he revised the City Building Code, established a fee system, etc.

In 1912, Mr. Marani engaged in private practice and designed and superintended the construction of buildings of all types. He also served as Consulting Engineer for the National Fireproofing Company, of Pittsburgh, Pa., and until 1918 as Consulting Engineer for the United States Gypsum Company, leaving this Company to serve during the World War as Representative of the War Service Committee on Gypsum. From 1918 until his sudden death at his home in Evanston, Ill., Mr. Marani was engaged as Chief Engineer of The Gypsum Industries.

Mr. Marani was among the foremost authorities on gypsum in the United States. He was a prolific writer on this subject as well as on construction materials, building codes, fire-protective requirements, etc. Endowed with a genial and friendly personality, he was regarded with affection and admiration by all with whom he came in contact.

Mr. Marani was a member of the American Society for Testing Materials, the American Association of Engineers, Western Society of Engineers, Cleveland Engineering Society, and the National Fire Protection Association. He was also a member of the Masonic Fraternity, and of the Lake Shore Athletic Club and the Old Colony Club.

Mr. Marani was elected a Member of the American Society of Civil Engineers on February 4, 1914.

CHARLES STEWART MAURICE, M. Am. Soc. C. E.*

DIED FEBRUARY 20, 1924.

Charles Stewart Maurice was born at Perth Amboy, N. J., on June 29, 1840, the son of Charles Frazier and Cornelia Joline Maurice. On his father's side, the family was of Welsh and Scotch-Irish descent, and on his mother's, a combination of Huguenot French and Holland Dutch, the latter having been among the earliest settlers of New Amsterdam.

In 1842, Charles Frazier Maurice, at the earnest solicitation of a number of friends, opened a small private school at Napanock, N. Y. Several years later, the growing school needing enlargement, was moved to Sing Sing (now Ossining), N. Y., and was merged with the old Mount Pleasant Military Academy, which became one of the best known private schools of its time. It was here that the son received his early education. In 1858, he entered Williams College at Williamstown, Mass., as a Sophomore, and was graduated in 1861, the Salutatorian of his Class, and a member of the honorary society, Phi Beta Kappa.

In order to serve his country in the Civil War which by this time had come to be considered as more than a "90-day affair," Mr. Maurice decided to enter the Navy. In order to render effective service, however, he desired to prepare himself for a Commission on the Engineer Staff, and began the study of Marine Engineering, entering Rensselaer Polytechnic Institute, Troy, N. Y., during the fall of 1861, where he covered the subjects allotted to the Sophomore and Junior years in one year. On November 17, 1862, he was commissioned Third Assistant Engineer (Midshipman), U. S. Navy, and reported for duty at the Washington Yard, where he was assigned to the *Ossipee,* which later joined the North Atlantic Squadron, doing duty on the blockade of the Southern States on the Atlantic seaboard.

Never having had robust health, the restricted ration available in those days brought on severe and prolonged attacks of indigestion, and in the summer of 1863, Mr. Maurice was ordered home on sick leave. He never completely recovered from the effects of this illness, and was subject to indigestion, more or less severe, for the remainder of his life. On rejoining the Navy in the autumn of that year, he was ordered to the Portsmouth Navy Yard, and assigned to the fitting out of the *Agawam,* one of the "90-day" gunboats, built "double ended" for service on the narrow, crooked rivers of the South. Before her entire completion the *Agawam* was ordered to sea in chase of a Confederate privateer known to be off the New England coast, and was very nearly lost in a gale off Cape Sable, Nova Scotia. After her refitting, she joined the North Atlantic Squadron in the spring of 1864, Mr. Maurice going with her as Second Assistant Engineer, having been promoted to that rank on March 23, 1864.

The spring and early summer of 1864 was spent by Mr. Maurice on the James River, where a small fleet of ironclads and gunboats was keeping open

* Memoir prepared by George H. Maurice, M. Am. Soc. C. E.

a line of communication with General Grant, and was under frequent fire from enemy batteries and sharpshooters along the banks, notably at Dutch Gap and Malvern Hill. He was then transferred to the steam frigate, *Colorado*, of the West Gulf Squadron, doing blockade duty while Admiral Farragut was fighting in Mobile Bay, the *Colorado* not taking part in the fight, as she drew too much water to cross the bar. It is interesting to note that the late Admiral George Dewey was a Lieutenant on the *Colorado* at this time. Mr. Maurice was later transferred to the flagship, *Malvern*, of the North Atlantic Squadron, but had returned to the *Colorado* when she took part in the second attack on, and the capture of, Fort Fisher, on January 15, 1865. He remained in the Navy until December 21, 1865, on which date he resigned and was honorably discharged after declining the appointment as Assistant Professor of Mathematics at the United States Naval Academy at Annapolis, Md.

The problems of rapid transit in New York, N. Y., were even then demanding solution; the railway termini—both the Hudson River Railroad on West 30th Street, and the Harlem and the New Haven Railroads on Park Avenue, between 26th and 27th Streets, were far removed from the business center on Lower Broadway. Several lines of river steamers designed to run a "commuter's service" were planned to operate on both the East River and the Hudson River, and Mr. Maurice was employed as Engineer by the Lower Hudson Steamboat Company. He designed the engines for the two boats of that Company, the *Sleepy Hollow* and the *Sunnyside*, built during the spring of 1866. He then trained the engine-room crews of these boats, which proved to be the fastest boats on the river with the exception of the famous *Mary Powell*, a much larger boat. They ran from the Battery, New York, to Sing Sing (now Ossining) and Peekskill, N. Y., but needless to say rapid transit around New York by water was not a success, and the Company went into bankruptcy. It was probably during the latter part of the winter of 1865-66 that Mr. Maurice found time to work for about two months in a foundry and machine shop at Peekskill, thus acquiring shop experience.

During the fall of 1866, he became interested, with his boyhood friend, Eugene Underhill, in the tanning business, and they spent the winter of 1866-67 at Liberty, N. Y., and also at Great Bend, Pa., investigating and observing the operations of the large tanneries in those sections. In the spring of 1867, they built their tannery from the plans of, and under the supervision of, Mr. Maurice, at Athens, Pa., this section of Bradford County then being in the center of large forests of original growth of hemlock, the bark of which was necessary for their operations. In the autumn of 1869, he disposed of his interest to his partner, and returned to the family home at Briarcliff, N. Y.

During the next few years the little village of Athens, situated on the banks of the North Branch of the Susquehanna and the Chemung Rivers, instead of being at the junction of the Pennsylvania and New York State systems of canals (now abandoned in that section), was in the close vicinity of the junction of four railroads. This rapid transition from water to rail transportation had been brought about by the work of the late Col. C. F. Welles,

a man of vision, sprung from the best pioneer stock in the valley of the North Branch. This change appealed so strongly to the business sagacity of the late Mr. N. C. Harris, a leading merchant and banker of this section, that he introduced Mr. Maurice to the late Charles Kellogg, M. Am. Soc. C. E., who had a patent on combination wooden and iron bridges and having already started works at Athens for their fabrication, was desirous of extending them. Mr. Maurice secured a contract from the old Oswego Midland Railroad covering timber bridges in which more than 1 000 000 ft. of timber was used, and afterward, in 1871, the partnership of Kellogg and Maurice was formed. An office was maintained in New York, of which Mr. Maurice was in charge until 1874, when he moved to Athens with his family, and there he continued to reside until his death.

The firm of Kellogg and Maurice was one of the pioneers in the building of iron bridges, and the second to build those of steel. Its first bridge of any note was one over the Tombigbee River in Southern Alabama, with a draw-span which was erected without falseworks, an unprecedented feat at that time, and one which was the subject of a paper* presented to the Society by Mr. Maurice on December 17, 1873. Among other early works of this firm may be mentioned a section of the Third Avenue Elevated Railroad in New York, noteworthy for the methods of erection, bridges in Nova Scotia and Brazil, one over the Snake River, in Idaho, and one over the Platte River at Plattsmouth, Nebr. Later, came the Thames River Bridge, at New London, Conn., with the longest swing-span of its day, 500 ft., double-tracked, but a pigmy beside present-day structures.

The partnership continued until 1884, when a merger was formed with other firms in the same business, and the Union Bridge Company was organized. The firm was composed of Charles Macdonald, Past-President, Am. Soc. C. E., formerly of the Delaware Bridge Company; the late Thomas C. Clark, Past-President, Am. Soc. C. E., formerly of Clark, Reeves and Company; the late Edmund Hayes, M. Am. Soc. C. E., and George S. Field, M. Am. Soc. C. E., of the Central Bridge Company; the late Charles Kellogg, M. Am. Soc. C. E., and Mr. Maurice. Shops were maintained at Buffalo, N. Y., as well as at Athens, Pa., and the Company became internationally famous for the number and importance of the bridges it built in all parts of the world, and for the excellence and integrity of its work.

Pin-connected bridges were the standard in those days. With the introduction of steel, special methods and plant became necessary and all these requirements were met by the partners with either original designs or improvements on tools already in use. As an instance, for the famous Hawkesbury River Bridge, in New South Wales, the Bessemer steel for the eye-bars came from England and was manufactured into bars at Athens; before they were accepted, however, these bars had to be annealed in special furnaces built for the purpose; and full-sized tests were made in the 600-ton hydraulic testing machine also built at Athens for this purpose. The sinking of the

* "An Account of the Erection of a Draw-Bridge without False Works", *Transactions*, Am. Soc. C. E., Vol. XI (1873), p. 330.

caissons for the piers of this bridge in the rapid tidal flow of the river was probably the most hazardous construction work undertaken by the Company; the spans were floated into position on pontoons.

Other noteworthy bridges built by the Company were the Poughkeepsie Bridge over the Hudson River, the Cantilever Bridge over the Niagara River, the Cairo Bridge over the Ohio River, the Memphis Bridge over the Mississippi River, and many other bridges over the Mississippi, Ohio, and Missouri Rivers.

In 1895, Mr. Maurice was obliged to retire from business on account of ill-health, and spent the summer of 1896 in Europe. He continued to reside in Athens, taking daily drives over the hills of which he was so fond, and spending the winters at Jekyl Island, Georgia.

On April 28, 1869, at St. George's Church, New York, he was united in marriage to Charlotte Marshall, daughter of John G., and Marian Marshall Holbrooke, both of whom were descended from the oldest Pilgrim and Puritan stock of New England. Of this union nine children were born: Archibald S., Marian B., and Margaret S., residing at Athens, Pa.; George H., of Eagle Springs, N. C.; Charles F., of Glen Ridge, N. J.; Cornelia (Mrs. Robert Wilkinson), of Poughkeepsie, N. Y.; Albert T., of Rye, N. Y.; Emily M. (Mrs. C. W. Dall), of Cedarhurst, N. Y.; and Charlotte, who died in infancy. Mrs. Maurice, who had endeared herself to the community by her many kind deeds and acts of charity, and was beloved by all who knew her, died in 1909. Besides the sons and daughters, there survives a brother, Benjamin Maurice, of Mamaroneck, N. Y., and eleven grandchildren.

Of a retiring and quiet disposition, never voluntarily speaking of himself, Mr. Maurice's outstanding characteristics were absolute accuracy of mind and justice to all by thought and deed. He was a lover of Nature, and sought his pleasure and recreations in the woods and fields, as botanist, trout fisherman, wing shot, or as rider or driver, as the seasons and chance offered. During his early manhood, the flights of wild ducks and geese were especially alluring, and many a shooting trip was planned in connection with the building of bridges over Western rivers. In middle life, fly-fishing claimed a larger share in his recreation, and his usual summer vacations were spent somewhere in the wilds of Canada, accompanied by one or two of his sons, where his delight and pride was to land a 4-lb. trout with a 4-oz. rod from swift water. He rode horseback until he was over seventy years of age. To the time of his final illness he kept up his daily drives with fine horses, driven by himself until failing strength compelled him to relinquish the reins, and afterward by automobile.

At the time of his death, he was a member of the Loyal Legion, the Phi Beta Kappa Society, President of the Spaulding Memorial Library, of Athens, member of the American Forestry Association, the Jekyl Island Club, and the University and Union Clubs of New York City.

Mr. Maurice was elected a Member of the American Society of Civil Engineers on May 15, 1872.

JOSEPH MAYER, M. Am. Soc. C. E.*

DIED DECEMBER 13, 1924.

Joseph Mayer was born at Schriens, Vorarlberg, Austria, on February 11, 1855. His father, Anton Mayer, a native of Bludeur, Austria, was the owner of a flour mill and bakery and also of a hotel. His mother was Marie Bickel, also of Bludeur. Joseph Mayer was graduated from the Polytechnicum of Vienna, Austria, in 1880, with the degree of Professor of Mathematics and Descriptive Geometry for Realschulen.

After coming to the United States, Mr. Mayer was employed in the Chicago Office of the Delaware Bridge Company, serving for two years as Draftsman and for two years as an Assistant Engineer. Following a trip to Europe he was with the Union Bridge Company from September, 1885, to September, 1896, as Assistant Engineer in the New York Office from 1885 to 1890; in the Buffalo Office from 1890 to 1895; and as Chief Engineer in the New York Office in 1895 and 1896.

During this time he made designs, calculations, and estimates for innumerable bids, mainly for superstructures, and superintended detail plans for successful bids, among which were those for the following four large cantilever bridges: The Kentucky and Indiana Bridge across the Ohio River at New Albany, Ind. (Delaware Bridge Company); the Poughkeepsie Bridge across the Hudson River; the Kanawha River Bridge in West Virginia; and the Kentucky River Bridge.

From 1896 to 1908, Mr. Mayer practiced as a Consulting Engineer in New York, N. Y. Until 1901, he worked on a proposed railway suspension bridge across the Hudson River at 59th Street, making designs, plans, and estimates therefor for the Union Bridge Company. The late Alfred Noble, Past-President, Am. Soc. C. E., had agreed to be its Chief Engineer if the design met with his approval and if the project were successfully financed, but being a member of the Isthmian Canal Commission, he was unable then to give the study of the design his personal attention. Subsequently, he spent approximately two weeks in going over this work, personally re-checking all Mr. Mayer's assumptions and formulas and many of the stresses. He approved entirely of Mr. Mayer's design and plans, and so reported to the proposed financiers. The project failed to materialize, however, not because of difficulty in financing, but because of inability to secure the necessary legislation. The bill passed the State Legislature, but was vetoed by Governor Odell for the stated reason that it was a Special Bill disguised in the form of a General Bill.

After the failure of the Hudson River Bridge project, in May, 1901, Mr. Mayer was engaged as Consulting Engineer for the Union Bridge Company, making plans and estimates for a proposed suspension bridge for Sydney Harbor, Sydney, Australia, both as to substructure and superstructure. This design won a prize. Later, under a second competition, Mr. Mayer (as Consulting Engineer for the Pennsylvania Steel Company) made another design

* Memoir prepared by Edwin Duryea, M. Am. Soc. C. E.

and estimate of cost for a suspension bridge for Sydney Harbor, but neither of these bridges has been built.

Shortly thereafter (beginning in August, 1901), Mr. Mayer made a considerable study of the Brooklyn Bridge, New York, jointly with Edwin Duryea, M. Am. Soc. C. E. Following the alarming discovery by a man traveling across the bridge that seven short suspenders near the middle of the north main cable had failed, the District Attorney for New York County, the late Eugene Philbin, appointed Mr. Duryea to make an examination of the bridge and report on its safety and on the responsibility for its neglect. All traffic was stopped for a day or two while temporary repairs were being made. After a hasty inspection of the structure had been made, the District Attorney consented to the appointment of Mr. Mayer as an Associate Consulting Engineer. In the ensuing detailed and protracted study of all parts of the bridge, Mr. Duryea made the field examination and Mr. Mayer the stress analyses. The joint report was placed by the District Attorney before the Grand Jury, which ordered more careful and more frequent inspections of all parts of the bridge. This report was censured severely by the Bridge Department of New York City and its friends, and the Consulting Engineers spent much time for several months in refuting the attacks in various technical periodicals, many of which had published the report in full.

Subsequently, Mr. Mayer was Consulting Engineer on the construction of the power plant of the Ontario Power Company at Niagara Falls. He designed all the bridges and viaducts for the Buffalo and Susquehanna Railway Company and was engaged in other miscellaneous engineering work.

Leaving his consulting work in 1908, Mr. Mayer was Principal Assistant Engineer of the Quebec Bridge until its completion in November, 1917. He then retired from active practice and moved to Pasadena, Calif., where he made his home, occupying his time in the study of economics and social science in which he was much interested, until his sudden death.

Mr. Mayer was married in October, 1900, to Catherine Proescher, of New York, who, with their son and only child, Joseph Edward Mayer, a Teaching Fellow in Chemistry in the University of California, Berkeley, Calif., and a brother in Austria, survives him.

Charles Macdonald, Past-President, Am. Soc. C. E., and for many years President of the Union Bridge Company, writes regarding Mr. Mayer as follows:

"I can say of his [Mr. Mayer's] standing as a designer of bridges that it was of the highest order of merit. I cannot call to mind the many plans he worked out for us, but I remember one particular design for which he deserves all the credit, namely, a suspension bridge to cross the Hudson River at 59th Street, New York. It was accepted by the New York and New Jersey Commissioners, but the project failed to secure the necessary capital.

"The liquidation of the Union Bridge Company in its later years made it necessary to dispense with Mr. Mayer's services, but I shall always remember with pride and gratitude his never failing attention to duty while I had the pleasure of being associated with him."

Ralph Modjeski, M. Am. Soc. C. E., one of the Engineers and Commissioners who had charge of the reconstruction of the Quebec Bridge, writes about Mr. Mayer's work there, as follows:

"He was employed as Principal Assistant Engineer in the Montreal office of the Board of Engineers and was particularly in charge of all calculations and designs. He was connected with that work from October, 1908, until its completion. His great usefulness came from the fact that he was a most unusual mathematician as well as an excellent designer."

Mr. Mayer was a frequent contributor to the *Transactions* of the Society, having been the author of six papers and having contributed many discussions which dealt not only with the subject of bridges, but also with other subjects, such as "Canals Between the Lakes and New York", "The Just Value of Monopolies and the Regulation of the Prices of Their Products", catenary trolley construction, centrifugal pumps and fans, principles of valuation, etc. He was nothing less than a genius in the practical application of higher mathematics to bridge design, particularly to the design of suspension bridges.

Mr. Mayer was elected a Member of the American Society of Civil Engineers on October 3, 1894.

MANSFIELD MERRIMAN, M. Am. Soc. C. E.*

DIED JUNE 7, 1925.

Mansfield Merriman, the son of Mansfield and Lucy Merriman, was born at Southington, Conn., on March 27, 1848. His father was a descendant of Captain Nathaniel Merriman, who came from England to Boston, Mass., in 1632, settled in New Haven, Conn., about 1640 and was, in 1670, one of the founders of Wallingford, Conn. His mother was a descendant of John Hall, who came from England to Boston in 1633, settled in New Haven in 1639, and who also was one of the founders of Wallingford.

Mansfield Merriman received his early education at the South Center District School and Lewis Academy, both situated at Southington. In 1866, he left school and went to work on his father's farm. He was employed later in a factory and printing office, taught a country school, studied geometry, surveying, and mathematics in his spare time, and on April 5, 1867, was appointed a County Surveyor. Playfair's Euclid was his first technical book, but he early formed the habit of regularly buying books. During 1866 on a total income of $172 he acquired thirty books at a cost of $27.50 These books included such titles as "Elements of Philosophy," "Letters on Astronomy," "First Lessons in Greek," "Latin Prose Composition," "Lady of the Lake," Blair's "Rhetoric," and Flint's "Surveying." While engineering and mathematics claimed most of his effort, yet his leisure time was devoted in a surprising degree to literature, to history, and to poetry. Along these lines as well as in botany, geology, physics, chemistry, and genealogical research, he found pleasure and recreation.

In spite of his father's stern opposition, but with the sympathetic aid of his mother, he went to New Haven and passed the entrance examinations at Sheffield Scientific School in 1868. He received second prize for excellence in studies during his Freshman year and a prize for excellence in botany in his Senior year, was President of his Class in his Junior and Senior years, and was elected a member of Book and Snake.

After graduating with the degree of Bachelor of Philosophy, in 1871, Mr. Merriman studied as a Graduate Student and received a second degree, Civil Engineer, in 1872. He became an Assistant in the United States Engineer Corps, in which position he remained for a year. Later, he spent six months studying in Germany, at Hanover, Dresden, and Berlin. Returning to New Haven, he was engaged on bridge construction for several months and became Assistant in Civil Engineering at Yale University in 1874, which position he held until 1877. During these years he resumed his studies at the Yale Graduate School, receiving the degree of Doctor of Philosophy, in 1876. In 1877 and 1878 he was Instructor in Civil Engineering and Astronomy at Yale.

In the fall of 1878, Mansfield Merriman went to Lehigh University, Bethlehem, Pa., as Professor of Civil Engineering. Engineering education was

* Memoir prepared by the following Committee: Horace Andrews, *Chairman*, Walter R. Okeson and A. B. Hill, Members, Am. Soc. C. E.

then in its infancy, with few textbooks and with little known as to the proper technique of teaching. With his advent at Lehigh began Professor Merriman's great contribution to engineering education, first, in the form of teaching methods that rendered a complicated subject so simple that boys in his lecture-room were astonished to find his conclusions already formulated in their minds before he stated them; and, second, by the creation of a series of textbooks quite unequalled at the time of their publication, and many of which, after thirty or forty years, are still in daily use in colleges. This latter fact was made possible by the careful and constant revision given to these textbooks, many of which ran through numerous editions.

Perhaps no other man contributed so much to the success of Lehigh University as an engineering school, or had so great an influence in the establishment of her ideals and practices. His own rapidly advancing reputation as a teacher and engineer did much to attract favorable attention to this young school, and the high standards set in his Department induced similar standards in all the other Engineering Departments.

At the Annual Convention of the Society in the City of Mexico, in 1907, Professor Merriman was present, and his reception must have proved very gratifying to him, as it was most enthusiastic, especially at the assemblages in the Mineria, where many of his former pupils had again the privilege of meeting him. Many of the engineering works shown to the members present at that Convention were carried out by the Spanish speaking students who had formerly been under Professor Merriman at Lehigh University.

Lorenzo Perez Castro, M. Am. Soc. C. E., in *The Revista Mexicana de Ingenieria y Arquitectura* for August 15, 1925, wrote:

"During half a century, Merriman, as a great educational force, unceasingly expounded the principles of directness and simplicity in teaching and instruction. * * * Through natural inclination he loved study from the years of his youth. * * * By his own efforts he became an engineer, a mathematician and a man of science. * * * He had recourse to all the fountains within his reach in the effort to satisfy the divine desire. As steps in the ladder he acquired Latin and Greek as well as French and German, all of which he could read fluently. * * * The remembrance of Merriman and his pure constructive work, so beneficial to his fellow men, will be permanent and lasting. It will live in his pupils and students as in all those who, through generations to come, are to profit by his instruction and writings; and when, in the course of time, his books of applied studies become antiquated through the natural evolution of human knowledge, the master will remain embodied in his subjective life, or life of remembrance, with that worthy legion of men who live for humanity."

What Mansfield Merriman did for Lehigh University in a large way, he did for many other engineering schools indirectly through his textbooks, and the whole Engineering Profession owes a great debt to him.

In 1903, while still at Lehigh, he was a member of a Special Commission appointed by Mayor Low, of New York, N. Y., to report on plans for the Manhattan Bridge across the East River. Professor Merriman was also for four years, 1881-85, Acting Assistant in the United States Coast and Geodetic Survey, in charge of summer triangulation work in Pennsylvania.

In 1906, the University of Pennsylvania conferred on Professor Merriman the degree of Doctor of Science, and, in 1913, the degree of Doctor of Laws was conferred on him by Lehigh University.

In 1907, he resigned his position as head of the Civil Engineering Department at Lehigh University and engaged in consulting practice in New York, specializing in hydraulic problems and bridge design and construction.

Professor Merriman's work as an author and editor began in 1876, his thesis for the degree of Ph.D. at Yale, "Elements of Least Squares", having been published in 1877. He served as Editor-in-Chief of the "American Civil Engineers' Pocket Book," beginning in 1908 (first issued in 1911; Fourth Edition, 1920); Associate Editor of Johnson's (now Appleton's) Universal Encyclopedia, 1892-98, having written and edited the articles on Civil Engineering; Joint Editor with Mr. R. S. Woodward of "Mathematical Monographs" (twenty-one volumes), published from 1896 to 1921; author of "The Theory and Calculation of Continuous Bridges" (1876); "Elements of the Methods of Least Squares" (London, 1877); "The Figure of the Earth" (1881); "Method of Least Squares" (1884; Eighth Edition, 1911), translated into Spanish; "Mechanics of Materials" (1885; Eleventh Edition, 1914); "Treatise on Hydraulics" (1889; Tenth Edition, 1916); "Strength of Materials" (1897; Sixth Edition, 1912); "Elements of Sanitary Engineering" (1898; Fourth Edition, 1918), translated into Japanese; "Precise Surveying and Geodesy" (1899; Second Edition, 1912); "Elements of Mechanics" (1905); "Elements of Hydraulics" (1912); "Mathematical Tables" (1915); "Recreations in Mathematics" (1917), published under the pseudonym of "H. E. Licks"; joint author with John P. Brooks, Assoc. M. Am. Soc. C. E., of "Handbook for Surveyors" (1895; Fourth Edition, 1912); and with Henry S. Jacoby, M. Am. Soc. C. E., of "A Textbook on Roofs and Bridges", in four volumes (1888-98; Fourth Edition, 1917). He also contributed on many occasions to the papers and discussions of the Society and to various technical magazines. *Engineering News-Record,* in its number for April 17, 1924, celebrating the completion of fifty years in its engineering work, contained (page 649), a specially prepared paper from Professor Merriman. The total number of his books printed to 1925 was 340 000.

As a member of various scientific and mathematical organizations, Professor Merriman contributed to their proceedings and also prepared numerous articles for the engineering and technical press. He was one of the founders of the Society for the Promotion of Engineering Education and its President in 1896; one of the original organizers, in 1898, of the American Section of the International Association for Testing Materials, holding the office of Chairman of the organization until 1900; he was, therefore, one of the charter members of the American Society for Testing Materials at the time of its organization in 1902, and was elected President in 1915. He was a Fellow of the American Association for the Advancement of Science and Vice-President of the Engineering Section in 1894; member of the American Philosophical Society; American Institute of Mining and Metallurgical Engineers; American Mathematical Society; New York Academy of Sciences; Mathematical Asso-

ciation of America; Yale Engineering Association; Wallingford Historical Society; Engineers' Club, New York, N. Y.; re-elected President of the Class of 1871, Sheffield Scientific School, in 1921; member of H. Stanley Goodwin Lodge, F. and A. M., Bethlehem, Pa., and former District Deputy Grand Master.

Professor Merriman's own evaluation of the science of teaching and of his philosophy of life was expressed in an address delivered in 1911 at Southington, Conn., before the Alumni Association of Lewis Academy, when he said:

"Benjamin Franklin published about 1780 an essay on the 'Usefulness of the Mathematics'. This is one of those simple and wonderful articles which that great man prepared for the express purpose of influencing the young men of America, and it explains how mathematics lies at the foundation of surveying, engineering, and astronomy. In it occurs a reference to the 117th proposition of the 13th book of Euclid's Geometry, and Plato is quoted as saying that a man who could not understand this proposition ought not to be ranked among rational creatures. This essay first came into my hands in 1864, and its perusal greatly impressed me, especially the reference to Euclid, and accordingly I sent to New Haven by my aunt, who went there twice a year to buy supplies for her millinery trade, to purchase a copy of Playfair's Euclid, which was then used as a textbook at Yale College.*

"During an experience of more than thirty years, I have had good opportunities for estimating the influence which teachers actually exert. Time only allows me here to say that such influence seems to be slight outside of the class exercises. Teachers who go out upon the bleachers and swing their hats usually lose the respect of students in the class room. Teachers who select students as favorites under the idea of influencing them, put themselves in a most awkward position, which injures all parties concerned. The influence of teachers upon pupils seems to lie almost entirely in training them to proper habits of order and obedience, and in securing from them satisfactory attention to the lessons assigned. Special interest in particular lines of study, high ambitions, and resolves toward forming better habits of life appear to come to a boy not from teachers but from parents, friends, fellow students, and books. On the whole then, it seems to me wise for a teacher to confine himself zealously to the work of instruction, being always strictly impartial in attempting to influence his pupils, if he desires to attain the highest degree of success.

"A good definition of success lies in its contrast with another word. Proceed means to go ahead, but succeed means to go ahead and to get there. With all young men and young women who have formed a purpose, there exists an ambition to succeed. To this end hard work is necessary, and also the results of this work must be a little better than those of the average men and women who are engaged in the same kind of labor. But even these cannot bring success unless the worker rejoices in his work and sees that it brings each year better and better results. True success is not the attainment of money or fame, but rather the joy and satisfaction of the work itself and the consciousness that its results are regarded as valuable by those competent to judge.

"And the message that I would fain try to bring is this: Have joy in your labor, and then your work shall be well done; perhaps also some of the results of that work shall exert inspiring influences in directions of which you yourself do not dream."

* This incident played a direct part in formulating Professor Merriman's determination to go out into that great world which lay beyond the horizon of his ancestral home.

Professor Merriman was married on June 5, 1875, to Wanda Kubale, of Posen, Germany. He is survived by five children, Thaddeus, Lucile, Alice Pauline, Richard Mansfield, and Norman Nathaniel, issue of this marriage; two of the sons, Thaddeus and Richard Mansfield, are members of the Society. Mrs. Merriman died on March 10, 1889, and Professor Merriman was married on May 24, 1891, to Anna Rosina Godshalk, of Bethlehem, Pa., who died on January 8, 1907. He was married for the third time on June 6, 1910, in New York City, to Bazena Treat Downes. There were no children by the second or third marriage.

Professor Merriman was elected a Junior of the American Society of Civil Engineers on May 12, 1875, and a Member on September 3, 1884.

CHARLES HENRY MILLER, M. Am. Soc. C. E.*

DIED MAY 8, 1925.

Charles Henry Miller was born at Strasburg, Lancaster County, Pa., on November 30, 1866, the son of Henry B. and Elizabeth (Bartholomew) Miller, who were also natives of Lancaster County. He received his education in the public schools of Strasburg, and was graduated from the High School in 1884 and from Lehigh University, with the degree of Civil Engineer, in 1888. Immediately following his graduation, Mr. Miller became identified with the improvement of the Mississippi River under the direction of the United States Corps of Engineers. While in this employ he conducted surveys, dredge work, bank revetment, etc., and held the titles of Instrumentman, Draftsman, Assistant Engineer, Chief of Survey Party, and, finally, Superintendent of Construction, his employment having extended over a period of thirteen years.

In 1901, Mr. Miller was made Superintendent of Construction of the McClintic-Marshall Construction Company at Pittsburgh, Pa., and had charge of the building of the Pittsburgh plant of that Company. He maintained this connection for four years. In 1905 he was made Engineer of River Protection for the Missouri, Pacific, and Iron Mountain Railway System, but resigned this position in 1911, to organize and become President of the Miller Engineering Company, Little Rock, Ark. This Company was engaged in both engineering and construction work, and Mr. Miller had personal charge of the Engineering Branch. During the six-year period ending in 1917, he served as Chief Engineer of the Cairo Levee District, the St. Johns Levee and Drainage District, and the Indian Bayou Drainage Districts Nos. 1 and 2; he was a member of the Board of Consulting Engineers for several railroad companies on river bank protection and drainage work; and Consulting Engineer for the United States Government on the "sunk land" cases in Eastern Arkansas. His Company acted as Contractors for municipal improvements in Batesville and Little Rock, Ark.; built five miles of river-bank protection work for the Caddo Levee Board on Red River, Louisiana; Dam No. 2 in the Ouachita River, Louisiana; and extensive dike and bank protection work on the Mississippi River, the last two having been constructed for the U. S. Corps of Engineers.

In the midst of these activities as an Engineer and Contractor, Mr. Miller put aside all thought of personal interest on the entrance of the United States into the World War, and enlisted in June, 1917, receiving a commission as Major of Engineers. He acted as Constructing Quartermaster in charge of the building of Camp Cody at Deming, N. Mex. In December, 1917, he was appointed Commanding Officer of the 2d Battalion of the 23d Engineers (Highway Regiment). Major Miller arrived in France in April, 1918, and was placed in charge of railway, highway, camp, motor-park, and hospital construction, dealing with as many as 15 000 troops, including service battalions, prisoners, and Chinese laborers.

* Memoir prepared by Alfred M. Lund, M. Am. Soc. C. E.

In February and March, 1919, he commanded the 2d Battalion of the 307th Engineers of the 82d Division. In April, 1919, he returned to the United States, and received an honorable discharge on April 5, 1919.

On his return to civil life, Major Miller re-organized the Miller Engineering Company as the Miller-Butterworth Company, Engineers and Contractors. This firm immediately launched into the active road-building program of Arkansas and built thirty-three miles of "Warrenite" pavements in Arkansas County. Major Miller still continued to devote his attention to the engineering part of the Company's work. He was appointed by the United States Supreme Court as a Commissioner to decide on twelve miles of the boundary between Arkansas and Mississippi, and he also served on a commission to establish a portion of the boundary between Oklahoma and Texas. He acted as Consultant or as Arbiter in many suits on highway work about which he was recognized as an authority. At the time of his death he was serving as representative of the State of Arkansas on a Board appointed to equalize the taxes of Conway County.

Besides being President of the Miller-Butterworth Company, Major Miller was Vice-President of the Southern Sand Company, President and General Manager of the Allen Gravel Company, and Vice-President of the Southern Material and Construction Company.

Major Miller was also active in scientific and civic affairs. He was a member of the American Association of Engineers, the American Railway Engineering Association, and the Little Rock Engineers' Club, and a Director of the Federated Engineering Council, and the Little Rock Chamber of Commerce.

In 1900, he was married to Edna Ward, of Luna Landing, Ark., by whom he is survived. He is also survived by two daughters, Mrs. Edgar A. O'Hair wife of Capt. E. A. O'Hair, F. A., U. S. Army, Fort Sill, Okla., and Mrs Stacy C. Howell, wife of Dr. S. C. Howell, Little Rock, Ark.

Major Miller was very popular with all his associates and acquaintances and highly respected by his fellow engineers. He was generous with his means and his time and always active in any undertaking that concerned the welfare of individuals or of the community in which he lived.

He lost his life on May 8, 1925, when the U. S. Steamer *Norman* sank in the Mississippi River, carrying to their death twenty-three of the passengers and crew. The passengers were engineers and their families in attendance at the Mid-South Convention of Engineers, at that time meeting at Memphis, Tenn.

Major Miller was elected a Member of the American Society of Civil Engineers on May 2, 1899.

FRANK MILLER, M. Am. Soc. C. E.*

DIED MAY 29, 1923.

Frank Miller the fourth son of Matthew and Sara Anne (Thompson) Miller, of Manchester and Sale, Cheshire, England, was born in Manchester, England, on June 23, 1862. After a preparatory education in private schools in Sale and Southport, he went to Germany and continued his education, attending schools in Lahnestein, Düsseldorf, and Dresden, where he was a student at the Polytechnicum. During the summer vacations he traveled with a private tutor and studied art in many cities of Europe.

On his return to England, his father wanted him to enter the British Army, or to join him in his business of cotton manufacturing, as he had large interests in Lancashire, but the boy preferred to follow his own inclination in the choice of a profession and, finally, decided to enter Owen's College, Victoria University, England, to continue his study of engineering. He was for some time a lecturer at Owen's College.

For one year Mr. Miller served as Machinist and Draftsman at Appleby Brothers' Bridge and Crane Works, near London. From 1883 to 1885, he was Assistant in the office of Sir John Fowler and the late Sir Benjamin Baker, Hon. M. Am. Soc. C. E., Consulting Engineers, London, making plans for bridges, masonry construction, railroads, and details for the Forth Bridge in Scotland. From 1885 to 1888, he held the position of Assistant Engineer of the Lancashire and Yorkshire Railway Company, in charge of design and construction for four-tracking 10 miles of a difficult section of the road, including bridges, station buildings, retaining walls, etc. He also made marine surveys for deepening and improving the channel and harbor at Fleetwood, England.

In 1888, Mr. Miller went to Montreal, Que., Canada, to gain wider experience and to find a larger field in engineering work, and until 1889, he was connected with the Grand Trunk Railway Company of Canada, making surveys for railroad extensions. From 1889 to 1891, he served as Draftsman, Designer, and Estimator for Riter and Conley Steel Works, Pittsburgh, Pa., and from 1891 to 1892, he was engaged in designing new structures for the Brooklyn Elevated Railroad under the late O. F. Nichols, M. Am. Soc. C. E., Chief Engineer. From 1892 to 1895, he was with Cooper, Hewitt and Company, at Trenton, N. J., at the plant of the New Jersey Steel and Iron Company, in charge of draftsmen, designing bridges, buildings, and structural steel work of all kinds, and making estimates.

From 1895 to 1903, Mr. Miller was in private practice as Consulting and Constructing Engineer and as a member of the Structural Engineering Com-

* Memoir prepared by John B. Humphreys, Esq., Paterson, N. J.

pany, of New York, N. Y. During this time a large number of difficult engineering works were designed and constructed successfully, including the plant of the Central Lard Company at Jersey City, N. J., a large building filled with heavy oil tanks on every floor. This building, the cost of which was about $250 000, was constructed on a swamp. The Passaic Iron Foundry was also built under the supervision of this Company, as well as the steel work of numerous office, loft, and mill buildings and bridges, including their foundations. The Long Beach, N. Y., Draw-Bridge and various pile protection works were also designed and their construction superintended by the Company. In 1902, Mr. Miller designed, estimated, and superintended the construction of the floating derrick of 120 tons capacity for the United States Navy Yard at Portsmouth, N. H.

From 1903 to June, 1909, he was Chief Engineer of the Snare and Triest Company, Contracting Engineers, during which period he designed and superintended the construction of the coffer-dams for the building of the foundations for, and also superintended the erection of the steel work of, the highway draw-span of the Flushing Bridge, Flushing, N. Y., built at a cost approximating $280 000. Mr. Miller designed and estimated the cost of large fireproof warehouses, and railroad locomotive and repair shops, built for the United Railways, at Havana, Cuba, at a cost of $500 000. He also designed and estimated the cost of three concrete and steel ocean piers, built for the United Railways of Havana and the Cuban Central Railway, at Havana and Cienfuegos, Cuba, as well as numerous bridges (for railroads), and coal-handling plants. He planned the method of building the Manhattan Subway Terminal of the Williamsburgh Bridge, New York, N. Y., the contract price of which was about $1 100 000; he also designed and superintended the shoring and underpinning of the adjoining buildings, as well as the shoring of the street railway tracks and temporary roadways.

Mr. Miller designed the plant and method of erecting the steel work of the approaches of the Queensborough Bridge at 59th Street, New York, and Long Island City, N. Y., over the East River. He designed the coffer-dams, pile and concrete construction of the tunnel connecting the Delaware, Lackawanna and Western Railway Station at Hoboken, N. J., with the Hudson and Manhattan Tunnel, and arranged the method of construction. He also designed the coffer-dams, foundations, and steel work for the Perth Amboy Bridge across Raritan Bay and superintended the construction. He designed, estimated and superintended the construction of sixteen large floating derricks for The Merritt and Chapman Derrick and Wrecking Company and The M. P. Smith and Sons Company, both of New York. These derricks had a lifting capacity of from 25 to 150 tons, and were built at a total cost of about $500 000. This work, executed for The Snare and Triest Company, represents some of the large contracts, but, in addition, Mr. Miller had charge of all engineering matters affecting designs and erection, and of a large part of the estimates.

He subsequently entered into partnership under the firm name of Long and Miller, Consulting and Contracting Engineers, New York. While a member of that firm, he was interested in the development of Larchmont Gardens, reconstruction work at Fort Slocum, and the widening of a part of Fifth Avenue, New York, and was also engaged on many other pieces of engineering and construction work in and around that city. The firm of Long and Miller dissolved partnership in 1915, and Mr. Miller continued the practice of his profession alone, as a Consulting Engineer with an office in New York.

At the outbreak of the World War, he offered his services to both the United States and Great Britain, but was not accepted on account of his health. He had hoped that his experiences as an engineer would be of service to the Allied Forces.

For many years Mr. Miller served as Consulting Engineer for Messrs. Merritt and Chapman, and was associated with that Company on several of its large contracts, notably the laying of the Narrows Siphon. It was Mr. Miller who raised the American liner *St. Paul* after she had turned over at her pier at the foot of West Twenty-second Street, New York, and had lain fast in the mud for some time.

In April, 1920, he visited the West Indies, in order to make plans and specifications for harbor works at East Harbour in the Turks and Caicos Islands group.

In June, 1920, he became seriously ill, but, after being in a critical condition, he finally recovered sufficiently to resume his business. From time to time, he suffered greatly, but never allowed his health to deter him, to curb his ambition, or to handicap him in the execution of any piece of work he had in hand.

At different times, Mr. Miller did work for and in connection with the Keystone Cement and Fireproofing Company. He was also interested in the development of Belwood Park, a part of the Hewitt Estate, near Newark, N. J.

The last work that Mr. Miller undertook and executed was the design for the great dome on the new Roman Catholic College, Marywood, at Scranton, Pa., which is considered a very beautiful piece of work, the second or third largest dome in the United States, and, as one of his business associates said, a "lasting tribute to his wonderful talent for design, and to his memory". He had the gratification, in the last few weeks of his life, of knowing that it had been successfully erected.

His profession was to him an art, and his genius comprised a diversified talent for design, but he also inherited from his Scotch ancestry his strongly practical characteristics.

In the autumn of 1922, Mr. Miller suffered a severe recurrence of his previous illness, and notwithstanding a very brave effort on his part to overcome his malady, he never regained his normal strength, although striving to the last to continue active work. He died on May 29, 1923, after two weeks of acute illness.

In 1889, Mr. Miller was married to Ada Elizabeth Sawyer, the only daughter of Thomas Spencer Sawyer, of Deeplish, Rochdale, England, who, with one daughter, Dorothy Ethel, survives him.

In social life Mr. Miller was popular and beloved by many. He had a high sense of humor, not boisterously exhibited, but when greatly amused, a sort of concealed laughter permeated his entire body. His life was a kindly and a useful one.

He was a member of the Canadian Club, and the Engineers Club of New York. In religion, he was a member of the Protestant Episcopal Church, and usually attended a daily service at one of the city churches. He was an ardent lover of the best in music, this taste having been fostered and cultivated during his long residence in Germany.

Mr. Miller was elected a Member of the American Society of Civil Engineers on October 5, 1904.

CHARLES MILLER MORSE, M. Am. Soc. C. E.*

DIED JUNE 18, 1921.

Charles Miller Morse was born in Buffalo, N. Y., on January 11, 1854. He was the son of David R. Morse, a financier and merchant in that city, who for several years was President of the Erie County Savings Bank.

In 1870, Mr. Morse entered Yale University, intending to take the Classical Course, with a view to a literary career, but he developed a taste for applied science and studied Mechanical Engineering for two years. He supplemented his study at Sheffield Scientific School by practical shop training in the puddling mill of the Union Iron Works at Buffalo.

After leaving Yale, he was employed, in 1873, on a survey for the Buffalo and Jamestown Railroad, and from 1873 to 1874, was Transitman with the U. S. Engineer Corps on work along Lakes Erie and Ontario. In 1876, he went to Dunkirk, N. Y., where he learned the trade of machinist at the Brooks Locomotive Works, and in 1877 and 1878, he served in that capacity in the shops of the New York, Lake Erie, and Western Railroad Company at Buffalo, and also in the shops of the Lake Shore and Michigan Southern Railway Company at Cleveland, Ohio.

Mr. Morse was afterward engaged as Draftsman and Assistant to the Superintendent of Motive Power with the Erie Railroad Company. In the latter position he had charge of the reconstruction of the locomotives of the Company from broad gauge to standard gauge, the work having been done at the Locomotive Shops of The Dickson Manufacturing Company at Scranton, Pa. From 1878 to 1881, he designed and superintended the construction of railroad shop equipments for the New York, Lake Erie, and Western Railroad Company at Buffalo and at Hornellsville, N. Y. From 1881 to 1882, he was Superintendent of the Crown Point Iron Company, at Crown Point, N. Y., and, in 1882, entered private practice in New York, N. Y. From 1883 to 1888, he also acted as New York representative for several manufacturers of machinery.

In 1888, he returned to Buffalo where he was engaged in private practice as a Mechanical Engineer and became head of the Buffalo Engineering Company. From 1890 to 1894, he acted as representative of Otis Brothers and Company, of New York, and, subsequently, as representative of the Babcock and Wilcox Boiler Company.

Mr. Morse was City Engineer of Buffalo for seven years. On December 26, 1901, he was appointed Deputy Engineer Commissioner in charge of the Bureau of Engineering of the Department of Public Works, serving until December 31, 1908, and receiving much commendation for his honest and efficient work. During his administration, the overflowed lands in South Buffalo were reclaimed, and an extensive report on the subject written by him in 1903, was published† by the Buffalo Society of Natural Science in 1908.

* Memoir prepared by Charles Warren Hunt, Secretary Emeritus, Am. Soc. C. E.

† *Bulletin*, Buffalo Soc. of Natural Science, Vol. VIII (1908), No. 2.

On August 6, 1907, the United States Government issued to him a license as Chief Engineer, and he was attached to the Naval Militia Steamer *Hawk* in the District of Buffalo.

In December, 1890, Mr. Morse was married to Kathleen Edgar, of Easton, Pa. Owing to the ill health of his wife, he bought a small fruit farm in Mount Dora, Fla., where he spent several winters, meanwhile retaining his business connections in Buffalo and New York. Mrs. Morse died on May 11, 1916. In 1917, he disposed of his home in Buffalo and moved to Florida permanently. At this time he gave up engineering practice and engaged in farming.

On April 3, 1918, Mr. Morse was married to Mary A. Donohue, of Boston, Mass., who survives him.

He died at Mount Dora on June 18, 1921, and was buried at Forest Lawn Cemetery at Buffalo.

He was a member of the American Society of Mechanical Engineers, of the Sons of the American Revolution, the Army and Navy Club of America, Cosmos Club of Washington, D. C., Engineers Club of New York, Buffalo Club, and Columbia Yacht Club, and his genial personality and social characteristics endeared him to all with whom he came in contact.

Mr. Morse was elected a Member of the American Society of Civil Engineers on January 2, 1895.

PAUL HOWES NORCROSS, M. Am. Soc. C. E.*

DIED MAY 8, 1925.

Paul Howes Norcross was born in Atlanta, Ga., on January 19, 1880, the eldest son of Dr. Virgil C. and Fanny Howes Norcross, and a grandson of Jonathan Norcross, the first Mayor of Atlanta and one of its pioneer citizens.

After his preparatory education in the schools of Atlanta, Mr. Norcross entered the Georgia School of Technology and was graduated, in 1902, with the degree of Textile Engineer. Immediately afterward, he spent about six months in one of the large Southern cotton mills, but did not find this branch of engineering especially attractive to him. Accordingly, he obtained a position with the Carolina Bridge Company, of Burlington, N. C., first in the Drafting and Designing Department, but because of his sales ability, which was even then apparent, he was soon transferred to the Contract Department and, later, made Special Sales Representative in charge of territory covering eleven of the Southern States.

The Carolina Bridge Company was later merged into the Virginia Bridge and Iron Company and Mr. Norcross continued with this organization, having been most successful in the promotion and sale of a large number of highway and railway bridges throughout the South. In 1905, he accepted a position with the Westinghouse, Church, Kerr and Company, in connection with the structural design of the Pennsylvania Station in New York, N. Y. After the completion of this work, he became a member of the Staff of Herbert C. Keith, M. Am. Soc. C. E., Consulting Structural Engineer, on special bridge design for the New York Central Railroad, in the electrified zone between Harmon, N. Y., and the Grand Central Station, New York.

In 1906, Mr. Norcross formed a partnership with Gabriel R. Solomon, M. Am. Soc. C. E., under the firm name of Solomon-Norcross Company, with principal offices in Atlanta. The firm engaged in general consulting practice throughout the South, covering all classes of municipal and hydraulic work. In 1910, a branch office was opened in Watervliet, N. Y., and Francis J. Keis, M. Am. Soc. C. E., was taken into the partnership, under the firm name of Solomon, Norcross and Keis, on Northern work. Mr. Solomon retired from the organization in 1918, on being commissioned an officer in the Army, and the firm operated under the name of Norcross and Keis until January 1, 1923, when Mr. Keis retired. Mr. Norcross continued the Atlanta Office, as a Consulting Engineer, until the time of his death.

Mr. Norcross' engineering activities, during the last few years of his life, were largely concentrated on water supply and filtration projects, his firm having served more than fifty municipalities on this particular type of work. At the time of his death, he was Consulting Engineer to the City of Atlanta, where the $3 500 000 pumping and filtration plant was nearing completion, and was in charge of the design and supervision of the new $1 300 000 water supply for Spartanburg, S. C.

* Memoir prepared by George W. Fuller and Gabriel R. Solomon, Members, Am. Soc. C. E., and Frederick H. McDonald, Assoc. M. Am. Soc. C. E.

Among the many projects designed and supervised by Mr. Norcross and his partners might be mentioned: Water supplies at Prattville and Talladega, Ala.; DeFuniak Springs, Milton, Haines City, and Sebring, Fla.; Cartersville, Rome, Jefferson, and Madison, Ga.; Amory, Miss.; Watervliet, N. Y.; Aiken, S. C.; and Cleveland and Morristown, Tenn.; also, special work for the Island of Bermuda; sewage disposal plants at Lexington, Ky., and a number of other cities; water power reports for the Broad River Power Company, Alabama Electric Company, and Selma Electric Company; and flood-control works at Watervliet, N. Y.

At the outbreak of the World War, Mr. Norcross was appointed as Supervising Engineer at Camp Jackson, Columbia, S. C., and, on the completion of this work, served in a similar capacity at Camp Humphreys, Accotink, Va. He was later made Acquisition Agent for Camp Benning, Georgia, and purchased approximately 30 000 acres of land for the Government. He was then called to Washington, D. C., and became a member of the Civilian Staff of the Water Supply Branch of the Construction Division of the Army, in charge of Dabney H. Maury, M. Am. Soc. C. E., then Lieutenant-Colonel, and while on this work, was given many assignments in connection with the water supplies of the camps, cantonments, and Regular Army posts. After the Armistice, Mr. Norcross returned to Atlanta and, with Mr. Keis, continued his consulting practice.

Among his last professional engagements was that as a member of a group of twenty-eight engineers constituting a Board of Review to pass on numerous aspects of the problems of the Sanitary District of Chicago, Ill. He served as Secretary of the Board and as a member of its Committees on Water Supply and on Water Power. The unanimity and promptness with which the Board reported its findings and conclusions on this many-sided proposition of unusual local, interstate, National, and international significance was due in no small measure to the organizing ability, tactfulness, and energy of Mr. Norcross.

Mr. Norcross' activities outside his engineering office were numerous and varied. He had unusual social talents, was thoroughly at home in any company, and was an exceptionally interesting and ready speaker. His keen wit caused him to be sought on all occasions as a presiding officer, and his appropriate remarks greatly enlivened every meeting at which he was present. No one could place a body of men more quickly at their ease, nor overcome a formal reserve more easily, than he.

For many years he was active in the Atlanta Rotary Club and served the Club as President. In 1924, he had the distinction, unique for an engineer, of being made President of the Atlanta Chamber of Commerce, having served that organization as Director, and as Chairman of a number of important committees. He was also President of the Alumni Association of the Georgia School of Technology.

Mr. Norcross lost his life, with twenty-two others, when the steamer *H. E. Norman*, carrying delegates of the First Annual Convention of the Association of Engineers of the Mid-South, sank in the Mississippi River south of Memphis, Tenn., near Kohoma Landing. His attendance at the Convention was in performance of his duty as a Director of the Society in aiding the con-

sideration of the formation of a Local Section. His last speech was on the ill-fated *Norman,* in the course of a river inspection trip forming part of the Convention program, during which he urged that the Engineering Profession should organize so as to be of as much aid as practicable to young engineers.

He was a Scottish Rite Mason, a Shriner, and an Elder in the First Presbyterian Church of Atlanta.

Mr. Norcross was married, in November, 1907, to Mattie L. Holt, of Graham, N. C., who died in October, 1920, leaving a daughter, Catherine Holt Norcross, who survives her parents.

His many friends among the engineers of the United States were particularly gratified at the appreciation of his worth as a citizen shown at Atlanta on May 31, 1925, by a Community Memorial Service, when leading public officials, representatives of important associations, and of his church, characterized him as:

"A Dutiful Son—A Devoted Husband
A Loving Father—A Loyal Friend
A Public-Spirited Citizen
A True Christian".

Mr. Norcross was elected an Associate Member of the American Society of Civil Engineers on July 9, 1906 and a Member on May 7, 1913. At the time of his death he was serving as a Director of the Society to which office he had been elected on January 21, 1925.

CHARLES EZRA PERKINS, M. Am. Soc. C. E.*

DIED JULY 17, 1925.

Charles Ezra Perkins, the son of Simon and Grace Ingersoll (Tod) Perkins, was born at Akron, Ohio, on May 7, 1850. His grandfather, Simon Perkins, went to Ohio from Connecticut in 1800 and settled at Warren, Trumbull County, as the Representative of the Western Reserve Land Company. His maternal grandfather was Judge Tod, father of Governor David Tod, a well-known name among Ohio jurists fifty years ago.

Mr. Perkins was educated at the public schools of Akron and at the Western Reserve Academy, at Hudson, Ohio. He also attended Rensselaer Polytechnic Institute, at Troy, N. Y., from 1868 to 1870, where he took a special course in Civil and Mining Engineering. In 1871, he studied at the School of Mines, Columbia University, New York, N. Y.

On leaving the School of Mines, Mr. Perkins was appointed Assistant City Engineer of Akron and served in that capacity until he was appointed City Engineer in the summer of 1871, to which position he was afterward elected, serving as such until 1877. His work in this connection comprised all kinds of municipal engineering, particularly grading and paving. He was also engaged in the settlement of street improvement claims against the city in process of adjudication. During his term of office preliminary surveys were made for the establishment of sewerage and water systems for the city.

From 1878 to 1883, he was engaged in private practice, including ditch work, bridges, and road improvement, but the greater part of this time was spent in the agricultural implement business.

In addition to his private practice Mr. Perkins served as Surveyor and Engineer of Summit County, Ohio, from 1883 to 1892, in charge of all County engineering work, highway bridges, ditches, highway improvements, and coal mine surveys.

In 1892, he was appointed to the official position of Chief Engineer of Public Works of Ohio, an office created by statute in 1878. As Executive Officer of this great system of waterways, consisting of about 600 miles of canals, he served the State faithfully for sixteen years, during which time the canal properties under his control proved to be of inestimable value to the people, and a distinct and important asset to the State.

After his retirement from his official position in 1908, Mr. Perkins engaged in private practice as a Civil, Hydraulic and Mining Engineer, with an office in Akron. He was a Republican in politics and a party leader in Ohio.

He was married, in 1880, to May Adams, daughter of Mr. Frank Adams, a prominent manufacturer of Akron, who survives him.

Mr. Perkins was elected a Member of the American Society of Civil Engineers on June 6, 1906. He was also a member of the International Association of Navigation.

*Memoir prepared from information on file at the Headquarters of the Society.

CHARLES OSCAR POOLE, M. Am. Soc. C. E.*

DIED APRIL 2, 1925.

Charles Oscar Poole, the son of Reuben and Mary Agnes (Gorace) Poole, was born at Saulsbury, Mass., on June 17, 1859. His father was a Mechanical Engineer and came to the United States from Yorkshire, England.

When Mr. Poole was ten years of age, the family moved to San Francisco, Calif. He attended the public schools there and later took special studies in Mechanical Engineering, which profession he intended to follow.

From 1880 to 1884, he served as Machinist and Blacksmith for the Oregon Improvement Company in the State of Washington, leaving to work for four years with the Newcastle Coal Mine near Seattle, as Steam Engineer. In 1888, he returned to the Oregon Improvement Company as Master Mechanic of the Seattle Division, which position he filled for three years.

In 1891, Mr. Poole returned to San Francisco as First Operator and, later, as Foreman of the Dynamo Room of the California Electric Light Company of San Francisco. In 1894, he was made General Foreman of Shop and Maintenance for the Company, which position he held until, after a consolidation of interests in 1897, he was made General Superintendent of the Electrical Department of what was then known as The Edison Light and Power Company of San Francisco. During the next three years, he was in charge of the management of the entire electric light and power business of San Francisco, except the street railways.

In 1900, he resigned from this position to become General Superintendent and Assistant Manager of the Standard Electric Company of California. In this capacity, Mr. Poole had entire charge of the Construction and Operating Departments and under his direction the early hydro-electric development at Electra was undertaken and the first high-tension transmission line was constructed into San Francisco. During this time, Mr. Poole was interested, also, in the United Gas and Electrical Company which held the electric and gas supply franchise in San José, Calif., and much of the business of this Company was under his supervision as Manager and Supervising Engineer.

In 1902, he accepted a position as Sales Engineer for the Stanley Electric Company of Pittsfield, Mass., and had his headquarters with the Hendrie and Bolthoff Manufacturing and Supply Company, at Denver, Colo. Through this connection, he became acquainted with a Denver group of capitalists who, in 1904, began to construct a hydro-electric power development on Bishop Creek, California, with a power transmission line to Tonopah, Nev. From 1904 to 1905, Mr. Poole was Designing Engineer, with offices in Denver, for the Company, then known as the Nevada Power Mining and Milling Company, and, from 1905 to 1907, he served as Assistant General Manager with offices in Goldfield, Nev.

* Memoir prepared by E. J. Waugh and F. O. Dolson, Members, Am. Soc. C. E.

In 1907, The Nevada California Power Company was organized and from August, 1907, until October, 1923, Mr. Poole was its Chief Engineer. Several allied companies were afterward formed, of which he was also Chief Engineer, notably The Southern Sierras Power Company, The Sierras Construction Company, the Hydro-Electric Company, the Pacific Power Company, and many others.

In 1910, Mr. Poole formed a partnership with Mr. R. G. Manifold, the firm being known as Manifold and Poole, Consulting Engineers. They were the joint authors of "Straight Line Engineering Diagrams". The firm designed the steel tower transmission line of The Southern Sierras Power Company, a pioneer in length and voltage. It is 237 miles long and operates at 87 000 volts from Bishop Creek, in Inyo County, to San Bernardino, Calif.

As Chief Engineer in charge of design, construction, and, for many years, of operation, Mr. Poole was, in a great measure, responsible for the growth and development of the systems of The Nevada California Power Company, The Southern Sierras Power Company and Affiliated Companies, and for the development of the territory which these Companies serve.

In October, 1923, he resigned as Chief Engineer, moved to Los Angeles, Calif., and resumed general practice as Consulting Engineer with Mr. Manifold, as Manifold and Poole. He continued as Consulting Engineer for The Nevada California Power Company group until his death.

Mr. Poole was married in 1913 to Mrs. Cora Weston, of Los Angeles, who survives him. A son, Walter, in the United States Marine Corps, at Mare Island, Calif., and a daughter, Mrs. Erma Lorenson, of Oakland, Calif., children by a former marriage, are also living.

His work covered a period when great strides were made in the electrical industry, and he did as much as any one man in California to extend and simplify the electric and especially the hydro-electric industry of the State. He was foremost in the development of outdoor sub-stations, and long, high-voltage transmission.

Mr. Poole had a fine mind, a faculty of keen observation, and an exceptional memory. He was a clear and forceful conversationalist and speaker, with a ready fund of anecdote and quick wit. He was a truly strong, constructive personality. There were two outstanding characteristics that set him apart from ordinary men. One was a driving energy of mind and body that would not be satisfied with anything partly known or partly done. He typified the dynamos with which he worked and understood. The other characteristic was loyalty and steadfastness. Once having started on a course, he did not waver and he was ever a loyal friend. He will be greatly missed in the Engineering Profession both for his work and for his good fellowship and social qualities.

At its meeting of August 12, 1925, the following resolution was adopted by the Los Angeles Section of the Society of which Mr. Poole was a member:

"Charles Oscar Poole, M. Am. Soc. C. E., a man from our midst, died on April 2, 1925. We, his friends, can but offer to his memory the highest possible tribute for a life, highly successful as the world terms success, but more,

invaluable for its contribution to the advancement of his chosen profession, and for its personal inspiration, which we who knew him well so greatly appreciate and will so greatly miss.

"Since a young man Charles Oscar Poole was closely identified with electrical and hydro-electrical development. His work covered a period when great strides were made in this field, and he did as much as any one man in California to extend and simplify the electrical, especially the hydro-electrical, industry of the State. He was a leader in the development of outdoor sub-stations, and of transmission over long distances. To his efforts can be credited the first high-tension transmission line into the City of San Francisco, where he had for some years control of the entire electric light and power business of that City, excepting only the street railways. The steel tower transmission line of the Southern Sierras Power Company, a pioneer of length and voltage, is a monument to his efforts.

"Courage, untiring energy, vision and love of pioneering work, and loyalty to high ideals were his, and it is these traits of character, as well as his accomplishments, which lead us, his fellows in endeavor, to tender our respect to a man whose memory can only lead to greater progress, such progress as was so strongly the motive of his life."

Mr. Poole was a Fellow of the American Institute of Electrical Engineers and was elected a Member of the American Society of Civil Engineers on October 21, 1924.

AMBROSE VINCENT POWELL, M. Am. Soc. C. E.*

DIED DECEMBER 5, 1924.

Ambrose Vincent Powell, the only child of Col. George W. and Mary (Dunbar) Powell, was born at Laurens, N. Y., on May 4, 1847. He attended the Cooperstown, N. Y., Seminary and, later, studied at Rensselaer Polytechnic Institute at Troy, N. Y., from which he was graduated in 1868 with the degree of Civil Engineer.

In 1869 and 1870, Mr. Powell was engaged in general engineering work, and, in 1871, became Assistant Engineer on the Oil Creek and Alleghany River Railroad. From 1872 to 1875, he held the position of Engineer of the Michigan and Ohio Railroad and in 1876 and 1877, was engaged in general engineering practice at Battle Creek, Mich.

In 1878, he became Engineer for the Receiver of the Chicago and Northwestern Grand Trunk Railroad Company and, in 1879, was Principal Assistant Engineer of the Chicago and Grand Trunk Railroad. In 1880 and 1881, he was Principal Assistant Engineer of the Chicago and Western Indiana Railroad and in 1882 and 1883, Engineer of the South Chicago Dock Company.

In 1884 and 1885, he was Chief Engineer and Superintendent of the Calumet and Chicago Canal and Dock Company, and from 1886 to 1888, he served as Engineer and Commissioner of Public Works of Hyde Park, then a suburb of Chicago, Ill.

From 1889 until his death, Mr. Powell was engaged in private practice as a Consulting Civil Engineer in Chicago, making a specialty of dock and harbor work. He built the lake front harbor improvements at Ashtabula, Ohio, as well as harbor improvements on the Calumet River for the South Chicago Dock Company and for the Calumet and Chicago Canal and Dock Company. He constructed a yard and dry dock for the Chicago Shipbuilding Company, a dry dock for the Superior Shipbuilding Company, and coal-handling plants for E. L. Hedstrom and Company, George Lill Coal Company, O. S. Richardson and Company, and others, at Chicago and elsewhere. He designed and constructed dry docks for several large shipbuilding companies on the Great Lakes, at Buffalo, N. Y., Toledo, Ohio, and at other lake cities. He was the first to use a concrete portal for a dry dock.

Mr. Powell was a successful engineer. His keen observation and logical thinking resulted in sound engineering judgment. His citizenship was always thoughtful and constructive; for many years he was an active member and officer of the Hyde Park Improvement Association, a voluntary organization which was concerned with matters relating to the welfare of that section of Chicago.

* Memoir compiled from information on file at the Headquarters of the Society.

He was a member of the Western Society of Engineers for forty-three years having served as Treasurer in 1887. In 1898, Mr. Powell became Vice-President, and, in 1900, was elected President. He was a member of the Calumet Country Club, the Chicago Engineers Club, and the Union League Club of Chicago. He is survived by his widow, Mrs. Hannah Belle Clark Powell.

Mr. Powell was elected a Member of the American Society of Civil Engineers on February 6, 1901.

WILLIAM LeROY REYNOLDS, M. Am. Soc. C. E.*

DIED APRIL 16, 1925.

William LeRoy Reynolds, the son of Charles N. and Minnie C. Reynolds, was born at Dundas, Ohio, on April 26, 1886. He began his elementary schooling in McArthur, Ohio. In 1895, his parents moved West, settling in Colorado, first in La Junta, then in Monument, and, finally, in 1900, locating permanently in Denver, in which city Mr. Reynolds finished his public school education preparatory to entering college. He was graduated from the University of Colorado, at Boulder, in 1909, with the degree of Bachelor of Science in Civil Engineering.

Soon after his graduation he spent several months at Palisades, Colo., where he was engaged in both office and field work for the Orchard Construction Company. He next accepted the position of Office Engineer for the Antlers Orchard Development Company, at Silt, Colo., an irrigation system for 4 000 acres. From March until December, 1910, he served as Construction Engineer for the Glenwood Light and Power Company, and was in charge of the construction of nine miles of steel pole transmission lines, substations, power-house, and water-works improvements at and near Glenwood Springs, Colo. This work cost $40 000. From December until June, 1911, he made various estimates, surveys, and designs for the Antlers Orchard Development Company, the Glenwood Light and Power Company, the Colorado Orchard Development Company, and the United States Forest Service.

From June, 1911, to July, 1912, Mr. Reynolds was Division Engineer on the construction of Willcox Canal System covering 10 000 acres near Rifle, Colo. From August, 1912, to June, 1913, he was employed by the Board of Public Works, at Denver, as Assistant Engineer, in charge of the Cherry Creek reconstruction work, bridges, walls, sewers, etc., the aggregate cost of which was about $300 000.

In July, 1913, he was appointed Construction Engineer on the replacement of outlet works, at Big Horn Reservoir, near Sheridan, Wyo., the cost of which was estimated at $40 000. After its completion, in December, 1913, he was engaged in private work in Glenwood Springs and Denver, until April, 1914, when he became Assistant Engineer for the Public Utilities Commission of the City and County of Denver, on construction and water-rate cases. He completed this work in August, 1915, and immediately afterward accepted the position of Assistant Engineer for the Van Sant-Houghton Company, and was engaged in estimating the cost of a new water plant for the City of Denver.

In July, 1916, and for the next two years, Mr. Reynolds served in the capacity of Field Engineer for the American Smelting and Refining Company, at Leadville, Denver, Pueblo, and Durango, Colo. During this time he was in charge of the design and erection of smelter structures worth about $350 000.

In March, 1918, he had the honor of being appointed by the Government to a position of great responsibility for the period of the World War. He was

* Memoir prepared by M. C. Hinderlider, M. Am. Soc. C. E.

sent to Nitro, W. Va., where he was made Supervising Engineer, in the Office of the Director of the U. S. Explosives Plant C. Mr. Reynolds was in direct charge of most of the construction and, later, was given the management of the operation of the water system (capacity 60 000 000 gal. per day, the cost of which was about $2 000 000) and also of the sewer and gas systems.

His work at Nitro was valued very highly by the Government which showed its appreciation by appointing him Chairman of the Plant Salvage Board, at the signing of the Armistice. The confining work, long hours, and stern responsibilities during the war, however, had left their imprint upon him. His health began to fail and before the Salvage Board had finished its task of disposing of the plant, Mr. Reynolds was obliged to resign and return to Colorado. He established himself on a small fruit farm near Silt, in 1920, and for the greater part of the next four years devoted most of his energies toward regaining his health. During this period he could engage only in advisory work, and although he was elected President of the Farmers Irrigation Company, his activities were limited.

In April, 1924, he felt that his health had been sufficiently restored to enable him to accept an appointment from the State of Colorado, that of Hydraulic and Railway Engineer, with headquarters in Denver. He held this position for almost a year, when he suffered a physical relapse and, after a short illness, died at his home in Denver on April 16, 1925.

Mr. Reynolds was a member of the Capitol Heights Presbyterian Church, of Denver, and of the Glenwood Springs Lodge No. 65, Royal Arch Masons No. 22.

He was married to Luellen Comer, of Glenwood Springs, Colo., on May 5, 1912. Besides his widow and a daughter, his parents and a sister survive him.

Mr. Reynolds was elected an Associate Member of the American Society of Civil Engineers on February 4, 1914, and a Member on April 14, 1919.

LINDLEY MARSHALL RICE, M. Am. Soc. C. E.*

DIED AUGUST 21, 1925.

Lindley Marshall Rice, the son of James Walker and Bell (Wilson) Rice, was born at Lawrenceburg, Ky., on September 1, 1868. His forebears were of hardy pioneering Scotch-Irish stock and were among the first to cross the Appalachian Mountains to settle in Kentucky. His parents were slave holders at the time of the Civil War.

Little is known of Mr. Rice's boyhood, but his technical education was obtained at the Georgetown College, Georgetown, Ky., where he took a scientific course.

On leaving college in 1888, he went to the Northwest and began the practice of engineering in the State of Washington. His first work consisted of assisting in the construction of a slaughter-house for the firm of Rice and Gardner. When this work was completed, Mr. Rice found employment in the office of Mr. A. Gardner, then City Engineer of Seattle, Wash., as Chainman and Rodman, and he was also engaged in general municipal engineering work. In the spring of 1889, he served with the Fairhaven and Southern Railway Company of Washington as Rodman and was afterward promoted to the positions of Leveler and Transitman. He remained with this Company until the fall of 1890.

At this time the Fairhaven and Southern Railway Company was merged with the Great Northern Railway Company, and Mr. Rice was engaged as Assistant Engineer in charge of construction on the Seattle-Montana Branch, under the late William Parsons Watson, M. Am. Soc. C. E. Although he was only 22 years of age, he carried out the work efficiently.

On the completion of this project, Mr. Rice was transferred to the main line of the railroad, in the Cascade Mountains, where he served as Assistant Engineer, reporting to John F. Stevens, Hon. M. Am. Soc. C. E. This work was finished in the early part of 1893, and he then engaged in private practice for two years.

In 1895, Mr. Rice was employed on the Kalso and Slocan Railroad in British Columbia as Assistant Engineer, but early in 1898 entered the service of the Canadian Pacific Railway Company as Locating Engineer and Division Engineer on Construction. He remained on this work until June, 1900, when he resigned to enter private practice in Seattle. He devoted the following year to the location and construction of logging railroads and subsequently, until 1907, built nearly all the railroads of this type that were constructed in the State of Washington.

In 1905, he organized the Klickitat Irrigation and Power Company and served the Company at various times as Chief Engineer, General Manager, and Director. In 1907, he organized the L. M. Rice and Company, an engineering and contracting corporation, and was at various times its President, Chief Engineer, and General Manager. This Company carried out various

* Memoir prepared from information furnished by L. M. Rice, Jr., and from data on file at the Headquarters of the Society.

contracts, including the completion of about 100 miles of the Kettle Valley Railroad between Midway and Penticton, British Columbia. It had as its principal object the irrigation of arid lands, and as Mr. Rice had studied this subject thoroughly he was recognized as an authority in matters relating to irrigation.

He was connected with many of the large engineering projects of the West and was the first to conceive the idea of irrigating the Horse Heaven Plateau, a vast stretch of level, fertile land located in Southeastern Washington which on account of scarcity of water in summer had not been settled. The Horse Heaven Irrigation District was organized to take over the water rights on the Klickitat and Big Muddy Rivers for use in irrigating the Plateau and Mr. Rice became Chief Engineer of the District.

From 1915 to 1918, his great interest in reclamation work led to his appointment as Chairman of the Executive Committee of the Western States Reclamation Conference, and he was also active in the work of the Washington Irrigation Institute.

During his tenure of office with the Western States Reclamation Conference, Mr. Rice was in constant contact with many of the large Western projects. At this time he also took an active interest in the Wenatchee Heights Irrigation District and the Yelm Irrigation District, both of which were in the State of Washington. In 1918, he served as Chief Engineer of the Quincy Irrigation District.

In recent years, he had spent a great part of the winter months in Eastern cities in an untiring effort to secure financial support for the Horse Heaven Irrigation Project. This work weakened his vitality, and it was shortly after his return to Seattle from one of these trips that his sudden death from apoplexy occurred.

Mr. Rice was married at Edmonds, Wash., on September 9, 1891, to Adelaide E. Leonhardt who, with two daughters, Ophelia and Miriam E., and three sons, Lindley M., Jr., Carleton L., and James W., survives him.

Mr. Rice was elected a Member of the American Society of Civil Engineers on May 31, 1916. He was also a member of the Rainier Club of Seattle and the American Geographical Society.

GEORGE THOMAS ROBERTS, M. Am. Soc. C. E.*

Died September 2, 1924.

George Thomas Roberts, the son of George and Jennie (Whiteman) Roberts, was born in Buffalo, N. Y., on September 23, 1871. He attended the public schools and the Central High School of that city until 1891.

At the age of 20 years, Mr. Roberts began his engineering work as a Draftsman with the Lehigh Valley Railroad Company at Buffalo. In 1894, he became a Draftsman in the office of the City Engineer, and, later, was appointed Assistant City Engineer.

In 1901, he formed a partnership with F. V. E. Bardol, M. Am. Soc. C. E., who was at that time City Engineer of Buffalo. The organization was called the Eastern Concrete Steel Company, Building and Engineering Contractors. Mr. Roberts continued in that business until his death.

Among the buildings erected by this Company are the New York State Normal School, City Hospital Buildings, Majestic Theatre, Elmwood Theatre, St. Margaret's Church, Canisius College, all at Buffalo; also, several of the Eastman Kodak Company Buildings, at Rochester, N. Y., the Drill Hall at Cornell University, Ithaca, N. Y., and the Public Library at Westfield, N. Y.

He was a member of the Buffalo Consistory, Ismalia Temple, of Buffalo; Washington Lodge No. 240, F. and A. M.; the Buffalo Club, the Athletic Club, the Park Club, and the Automobile Club; and, also, of the Buffalo Builders' Exchange.

Mr. Roberts was elected an Associate Member of the American Society of Civil Engineers on September 3, 1902, and a Member on March 3, 1908.

* Memoir prepared by Emile Low, M. Am. Soc. C. E.

WILLIAM ROTCH, M. Am. Soc. C. E.*

DIED AUGUST 14, 1925.

William Rotch was born at New Bedford, Mass., on July 22, 1844, the eldest son of William J. and Emily (Morgan) Rotch. On both sides of his family, he came from distinguished ancestry. His great-great-great-grandfather, Joseph Rotch, already a successful shipowner, came to New Bedford from Nantucket, Mass., in 1765. Joseph Rotch and his sons, William and Francis, found New Bedford a farming town doing a little whaling off the coast, and by their energy they established the industry that made the town the great whaling center of the world. Francis Rotch built and owned the *Dartmouth*, the best known of the three ships of the Boston "Tea Party". William Rotch (1738 to 1828) owned the ship, *Bedford*, which first displayed the American flag in foreign waters in 1783. After the Revolution, he established the whale fishery in France, but returned to the United States following the French Revolution. With the decline of the whaling industry, the Rotchs turned their energies to cotton manufacturing. At various times Mr. Rotch's father, William J. Rotch, was President and Director of most of the older New Bedford cotton mills. On his mother's side, Mr. Rotch was a grandson of Charles W. Morgan, one of the great whaling merchants of New Bedford, whose whaling vessel, the *Charles W. Morgan*, has been preserved as a memorial to the whaling industry.

William Rotch was educated at the Friends Academy of New Bedford, and was graduated from Harvard University in 1865, with the degree of Bachelor of Arts. He then entered the Ecole Centrale des Arts et Manufactures, in Paris, France, from which he was graduated in 1869, ninth in a class of 225, with the degree of "Ingenieur Civil".

Mr. Rotch began his professional career in 1871 as Assistant Engineer of the Fall River, Mass., Water-Works, then in course of construction, and served as Chief Engineer from 1874 to 1880, when the works were completed. From 1880 to 1882, he was Consulting Engineer and Purchasing Agent of the Mexican Central Railway Company, the Sonora Railway Company, the Atlantic and Pacific Railroad Company and the California Southern Railroad Company, the lines of the last two now forming the western portion of the main line of the Santa Fé System. From 1884 to 1891, he was Consulting Engineer of the Cleveland, Canton, and Southern Railroad Company, the Lakeside and Marblehead Railroad Company, the New Bedford and Fall River Railroad Company, and other railroad companies in Ohio and Massachusetts.

In 1881, Governor Long of Massachusetts appointed Mr. Rotch as Engineer and Member of the Commission which finally established the boundary line between Massachusetts and Rhode Island that had been in dispute for more than 250 years. He served on this Commission until 1883.

In 1894 and 1895 he was a member of the Re-Organization Committee of the Atchison, Topeka, and Santa Fé Railway, and from 1895 to 1900, he served as

* Memoir prepared by Albert B. Drake, M. Am. Soc. C. E.

a Director of that road. He was also a Director of the Mexican Central Railway Company from 1880 to 1900, and had charge of the first train which ran from Boston to the City of Mexico, in April, 1884. From 1886 to 1894, Mr. Rotch was President of the Tremont Electric Lighting Company and the Consolidated Electric Manufacturing Company, and a Director of its successor, the Walker Company, which Company installed the complete electric plant in the Waldorf-Astoria Hotel in New York, N. Y., then the finest electric lighting plant in America.

Mr. Rotch was an official in many Massachusetts corporations. From 1913 to 1922, he was President of the Federal Wharf and Storage Company; from 1918 to 1922, Vice-President of the State Wharf and Storage Company; and from 1918 to 1922, President of the Fibre Manufacturing Company. He was also President of the Railroad Wharf and Storage Company, the Denbigh Mining Corporation, the Terrible Dunderberg Mining and Powder Company; Vice-President of the Bonanza Development Company; Treasurer of the Broadway Storage Company; Trustee and Treasurer of the Nonquitt Real Estate Trust; Managing Director of the Rotch Travelling Scholarship for Architects; and a Director of the Infants Hospital and the Adams Nervine Hospital. In 1870, he was elected the first President of the Union for Good Works in New Bedford.

Mr. Rotch was President of the Alliance Francaise of Boston and Cambridge, Mass.; Governor of the Massachusetts Society of Colonial Wars; an Honorary Member of the Boston Society of Architects; a member of the Association des Anciens Eleves de l'Ecole Centrale of Paris, the Bostonian Society, the Somerset and Harvard Clubs of Boston, and the Wamsutta Club of New Bedford. He was also a member of the Arlington Street (Unitarian) Church, of Boston.

He was married in 1873 to Mary Rotch Eliot, who survives him, together with three children, Edith Eliot, Charles Morgan, and Clara, the wife of Dr. Channing Frothingham.

Mr. Rotch had both his office and residence in Boston for many years. In spite of his eighty-one years, he had been in excellent health and had attended the Harvard-Yale Baseball Game in June, 1925, at which he was the oldest graduate present. Taken ill at his summer home at Nonquitt, Mass., he was taken to the hospital at New Bedford on June 25, where following an operation his condition was thought to be improving until he suffered a heart attack which caused his death. Possessed of a wonderfully genial personality, Mr. Rotch leaves a host of friends.

Mr. Rotch was connected with the American Society of Civil Engineers for fifty-two years, having been elected a Member on March 5, 1873.

HENRY NORLANDE RUTTAN, M. Am. Soc. C. E.*

DIED OCTOBER 31, 1925.

Henry Norlande Ruttan was born on May 21, 1848, at Cobourg, Ont., Canada, where he received his early education. His parents were Henry Jones and Margaret (Pringle) Ruttan. His grandfather was the Hon. Henry Ruttan, formerly Speaker of the Legislative Assembly of Ontario.

Without the advantages of a college education or a technical training, Mr. Ruttan started while quite young in what became his life work and profession. Beginning in 1867 on the Engineers' Staff of the Grand Trunk Railway Company, he had a varied experience for sixteen years in railway work, as follows: 1869, on Engineers' Staff, Intercolonial Railway Company; from 1870 to 1874, on construction, Intercolonial Railway, and, later, in charge of a Division on Baie de Chaleur, including much heavy bridge work; in 1874, on preliminary surveys, Canadian Pacific Railway, north of Lake Superior; from 1875 to 1877, in charge of a party locating the line of the Canadian Pacific Railway from the summit of the Rocky Mountains, eastward; from 1877 to 1880, Engineer for the Contractor on the construction of the Canadian Pacific Railway between Lake of the Woods, Ontario, and Winnipeg, Man.; and from 1880 to 1885, designed and built bridges over the Red River at Emerson, Man., and over the Assiniboine at Winnipeg. As Engineer and Contractor Mr. Ruttan built the first forty miles of the Manitoba North Western Railway, and as Contractor, the first fifty miles of the Manitoba South Western Railway.

On June 29, 1885, Mr. Ruttan was appointed City Engineer of Winnipeg, and held this position for twenty-nine years, resigning in May, 1914. Among the outstanding features of his work as City Engineer was the design and construction of the Artesian Well System in 1899, when the City purchased the plant of the Winnipeg Water-Works Company which obtained its supply from the Assiniboine River. In 1898, he remodeled the asphalt plant which the City had purchased from contractors, which plant has supplied all the requirements of the City since that time for paving purposes.

In 1905, Mr. Ruttan was instrumental in the installation of a Fire Service Water-Works System with a separate distribution system. This plant is capable of pumping 18 000 000 gal. per 24 hours at 300-lb. pressure. He also designed and constructed the sewerage system of Winnipeg, a difficult task when the flatness of the territory is considered. In 1906, he made the original survey for the hydro-electric system of the City of Winnipeg and, later, acted as Consulting Engineer.

His success as an Engineer is a good example of what can be accomplished by hard work combined with a good mind. Practically all his spare time was given to reading and self-education.

In addition to his engineering achievements, he had an enviable military record, as follows: He enlisted in the Cobourg Rifle Company of the 40th Regiment, Canadian Militia, in 1863, and served with that unit for seven

* Memoir prepared by J. G. Sullivan, M. Am. Soc. C. E., and William P. Brereton, City Engr., Winnipeg, Man., Canada.

years. On November 9, 1883, he was appointed Captain in the 90th Regiment (Winnipeg Rifles), and was promoted to the rank of Major on February 14, 1887. Subsequently, on December 25, 1895, he was appointed Lieutenant-Colonel in command of the unit, which command he held until December 27, 1900.

From 1900 to 1910, Colonel Ruttan was a Reserve Officer, but in April, 1910, he was appointed Lieutenant-Colonel to command the 100th Regiment (Winnipeg Grenadiers), in which appointment he remained until October, 1912, when he was selected to command the 20th Infantry Brigade, with which he served until September 5, 1918. He was appointed Temporary Brigadier General on July 8, 1916, and assumed command of Military District No. 10 (in addition to his duties as Commander of the 20th Infantry Brigade), during the World War period. General Ruttan was one of the first officers in Military District No. 10 to pass the Militia Staff Course.

His war service record is, as follows: Fenian Raid, 1866, for which he received the General Service Medal with one clasp (90th Winnipeg Battalion); North West Rebellion, 1885, actions at Fish Creek (April 24), and Batoche (May 9 to 12), and operations against Chief Big Bear's Band (June-July), for which he received a medal with clasp. He was made a Companion of the Order of St. Michael and St. George for services in connection with the World War (1914-18).

General Ruttan's military training developed a manner that was somewhat awe-inspiring to the young clerk, but his sense of justice and kindness of heart was such that he was admired and respected by all who knew him. He never attempted to shift any responsibility; on the contrary, he often assumed full personal obligation for the mistakes of his subordinates.

As is the case with all public servants he had adversaries and many men were elected to the City Council at Winnipeg and even as Mayor as an opponent to General Ruttan only to become his supporter in a short time.

In many cases the good judgment he displayed in disputed questions became apparent only after a long period of time, such as his decision to use municipal forces only in the construction of improvements during the boom days of 1905 to 1913, which saved the City of Winnipeg from a condition of over-development reached by many other towns in the Prairie Provinces at that time. If 50% of the public officials had the same character and determination to do what was right as General Ruttan, the public operation of utilities, especially those with a low operating ratio, would not be such a bugbear to the ordinary taxpayer.

General Ruttan served as President of the Winnipeg Branch of the Navy League for fifteen years and was granted the Special Service Decoration. He was a Past-President of the Canadian Society of Civil Engineers (now the Engineering Institute of Canada) and a member of the Royal Society.

General Ruttan is survived by his widow, Mrs. Andrian Ruttan, *nee* Barberie, and the following children: Maj. H. A. Ruttan, of Port Arthur, Ont.; Lieut.-Col. C. M. Ruttan, Capt. F. N. Ruttan, and Capt. J. D. Ruttan, all of Winnipeg; and Mrs. H. L. Wethey, of Neepawa, Man., Canada.

General Ruttan was elected a Member of the American Society of Civil Engineers on January 4, 1893.

AMOS SCHAEFFER, M. Am. Soc. C. E.*

DIED OCTOBER 3, 1924.

Amos Schaeffer, the son of Augustus and Amanda (Leibensberger) Schaeffer, was born on a farm at Fleetwood, Berks County, Pa., on February 17, 1867. He was of German ancestry, and of the people who populated a large part of the State of Pennsylvania and made it the flourishing and prosperous section that it is to-day.

His early education was obtained in the public schools, after which he was graduated from the Kutztown, Pa., Normal School. Following a few years of teaching, he entered Franklin and Marshall College at Lancaster, Pa., from which he was graduated in 1889 with the degree of Bachelor of Arts and finished with a year at Sheffield Scientific School, Yale University, from which he obtained the degree of Bachelor of Philosophy in 1890.

Following his graduation, Mr. Schaeffer was employed by the Lehigh Valley Railroad Company on surveys and field work. In April, 1892, he entered the service of Rodgers and Clement, Contractors for the Niagara Falls Power Tunnel, in charge of portal masonry and stone-cutting for the tunnel, which is about 1⅓ miles long, and is the tail-race for the water that enters the turbines after dropping perpendicularly about 150 ft. at the forebay at Niagara River above the Falls. The tunnel was built under the City of Niagara Falls for the first utilization of the power of the Falls by electricity (1895), and is still in use.

During parts of 1893 and 1894, Mr. Schaeffer was engaged with the United States War Department at Jonesport, Me., in charge of the removal of ledge rock under water in the ship channel and for a short time with the Lehigh Valley Railroad Company on a contour survey for railroad location in Maryland.

In May, 1894, he returned to Niagara Falls, entering the City Engineer's Office as Assistant Engineer, in charge of the design and construction of sewers. He completed the construction of a tunnel trunk sewer in rock; made plans for and superintended the construction of brick and asphalt pavements; designed and constructed the skewbacks and abutments for a 150-ft. span skew steel arch bridge; and located and established street lines and grades. The City of Niagara Falls had grown rapidly in size and population after the introduction of cheap electric power and many new industries had settled there, necessitating additional residential facilities, and during the latter part of his stay, Mr. Schaeffer engaged in private practice on property sub-division and general engineering in connection with the paper and pulp mills which had located there.

In November, 1897, he entered the employ of the City of New York, which, with one short exception, was destined to be his last change. His first work was in the Topographical Bureau, Department of Sewers, Borough of the Bronx. In 1900, on the inception of the work of the Rapid Transit Railroad,

* Memoir prepared by William J. Boucher, M. Am. Soc. C. E.

he transferred to the Rapid Transit Commission where he made a specialty of the sewer problems. The construction of this railroad, built as it is, almost entirely below the streets of the City, caused a most elaborate re-design of the sewer system, necessitating many re-arrangements, new construction, and reconstruction of the trunk sewers and lateral systems. On the resignation of Mr. Calvin W. Hendrick in 1905, Mr. Schaeffer was appointed Engineer of Sewers. The careful investigation, study, design, and construction done under his supervision is amply attested by the results after practically twenty-five years' growth of the great city at a rate never anticipated.

In 1909, Mr. Schaeffer resigned from the Rapid Transit Commission and Sewer Division to become Chief Engineer of Gore-Meenan Company, Sub-Contractors for a section of the Catskill Aqueduct on the east side of the Hudson River, near Peekskill, N. Y. Although he worked hard and gave his best thought and efforts to the success of the work, financial difficulties caused a suspension. This, however, was through no fault of Mr. Schaeffer.

In 1910, Mr. Schaeffer was appointed Consulting Engineer, Borough of the Bronx, New York, a new position, reporting to the President of the Borough. Some years later, he was appointed to a similar position—Consulting Engineer, Borough of Manhattan—which he held until his death, serving in the two Boroughs under five different Presidents and three Mayors. While in this position, Mr. Schaeffer's work was of a most varied character and comprised largely, improvements planned for the future betterment of the fast growing Boroughs to accommodate the increasing traffic and population. Vision, forethought, and sound planning were essential, and with his experience and training, he was well equipped to cope with the many problems which were constantly arising. His intimate knowledge of the laws and regulations of municipalities enabled him to frame his recommendations so that they secured speedy adoption. One of the Borough Presidents has expressed the opinion that he was one of the ablest engineers in the City of New York.

Mr. Schaeffer was of a most pleasing and cordial personality; once he had been gained as a friend, he remained such. His contacts were with all classes, and his tact and diplomacy as well as his thorough knowledge of the facts regarding his work and projects enabled him to present the matter in hand in a pleasing and convincing manner.

He was an ardent church worker, having been affiliated with the Fordham Manor Reformed Church, Bronx, New York, of which he was a member and Elder, and a Member of the Classis. In 1924, he was elected one of three Trustees of the Board of Direction of the Reformed Church of America. Shortly after his death, the Consistory of that Church adopted a resolution, which was spread on its minute book and published in the "Monthly Leaflet", part of which reads, as follows:

"Mr. Schaeffer's character was above reproach. His virtues were the virtues of the stoical philosophy as expounded by Gilbert Murray. 'God, the eternal dramatist', says Professor Murray, 'has cast you for some part in his drama, and hands you the rôle. It may turn out that you are cast for a triumphant king; it may be for a slave who dies of torture. What does that matter to the good actor? He can play either part; his only business is to accept the rôle

given him and perform it well. * * * Do you ever find that history praises a man because he was healthy, or long-lived, or because he enjoyed himself a great deal? History never thinks of such things. The thing that lives is a man's goodness, his great deeds, his virtues, or his heroism. * * * The only good for man is to be good. And when Zeno says "good," he means good in the ultimate Day-of-Judgment sense.'

"Mr. Schaeffer accepted his rôle in life in this spirit. He was not concerned about his ease or his pleasure—only his duty. That is why he held his position, unprotected by Civil Service, through all the mutations of political strife. In the City Government he was a Gibraltar that could not be moved by corruption, flattery, or threat. To him the only question concerning any measure was, 'Is it right, and in the public interest?' In the Consistory, these same traits were always in evidence. His goodness was not expressed by words, but by deeds. He had vision and courage, and boldly faced the future even when the outlook was darkest; much of the progress of the church in recent years we owe to him. He served on many important committees, and always with fidelity and distinction. Everything that came under his scrutiny received thorough and conscientious consideration.

"As a friend and neighbor Mr. Schaeffer was kind and true, ready to help in time of need. He was a charming host and had—what is rare with men as serious as he was—a delightful sense of humor. He was a real man, without sham of any kind. 'The reality of things,' says L. P. Jacks, 'is inversely proportional to the noisiness of the self-announcement.' Mr. Schaeffer was as modest as he was efficient. He never sought the spotlight, but was content to play his rôle even when others took the applause.

"The Consistory as a body hereby expresses its regret at the loss of one of its most useful members. As individuals, the members have lost a genuine friend and lovable companion. They offer their profound sympathy to the widow and orphans left by the deceased, and incorporate this expression of their respect and loss into the official record of the church."

Mr. Schaeffer's health began to fail during the summer of 1924; a long vacation seemed to restore him somewhat, but he died of typhoid fever on October 3, 1924. He was married to Florence Harriot Messmore who, with a son and a daughter, survives him.

Mr. Schaeffer was elected an Associate Member of the American Society of Civil Engineers on February 3, 1904, and a Member on February 6, 1906. He was also a Past-President of the Municipal Engineers of the City of New York.

GRANVILLE WHEATON SHAW, M. Am. Soc. C. E.*

DIED JANUARY 6, 1925.

Granville Wheaton Shaw, the son of Theodore Granville and Matilda (Ward) Shaw, was born in Chillicothe, Ohio, on April 27, 1849. He moved to Louisville, Ky., with his parents in 1851, where he resided for the remainder of his life. His education was obtained in the public schools of Louisville with the exception of five years spent in private schools before he reached his fifteenth year.

In January, 1868, Mr. Shaw entered the service of the Louisville, Cincinnati and Lexington Railroad Company, then building a line between La Grange, Ky., and Cincinnati, Ohio, under the direction of the Chief Engineer, the late I. M. St. John, M. Am. Soc. C. E., and was engaged as Rodman on location surveys and as Assistant Engineer on the construction of shops, roundhouses, etc.

In May, 1869, he went with General St. John to the Louisville, New Albany and St. Louis Railroad (now a part of the Southern Railway System) as Leveler on preliminary surveys of the line from New Albany, Ind., to Mt. Carmel, Ill. In 1870, he was employed as Assistant Engineer on the construction of a bridge across Licking River, at South Covington, Ky., and in building a railroad from that point through Newport, Ky., to connect with the Ohio River Bridge. He was also employed as Draftsman on a Government survey of the Upper Cumberland River under the late S. T. Abert, M. Am. Soc. C. E., U. S. Engineer. In 1872, he was appointed Assistant Engineer on the Louisville, Cincinnati and Lexington Railroad under General St. John and was engaged in the reconstruction of bridges and surveys for railroad extensions.

From 1873 to 1875, Mr. Shaw served as Assistant City Engineer of Louisville, under the late Thomas P. Shanks, M. Am. Soc. C. E., in charge of sewer construction and the extension of a short-line railroad along the river front to East Louisville, and had supervision of all city bridges. He subsequently went to Vincennes, Ind., in the employ of Cunningham and Mettesser, manufacturers of glucose, and for one season commanded the steamer *Vigo*, a Wabash River packet running in the interests of the firm. He also designed a drainage system for the property of the Company and rebuilt its plant.

In 1878, Mr. Shaw returned to Louisville and was employed as Assistant Engineer on the preliminary survey of the Falls of Ohio River with Mr Thomas H. McMichael and, in 1879, was in charge of a survey and cross-sections of the Falls of the Ohio River under Gen. G. Weitzel, Corps of Engineers, U. S. A., completing the field work by the end of the year.

Early in 1880, he accepted the position of correspondent for the *Louisville Commercial* at Frankfort, Ky., during the session of the Kentucky Legislature.

* Memoir compiled from information on file at the Headquarters of the Society.

When the latter adjourned, he was appointed City Editor of the *Commercial* and remained with that newspaper until the fall of 1882, when he established a Bureau for the Testing of Hydraulic Cement at Louisville. From 1883 to 1885, he was engaged in various surveys and works in connection with the Louisville and Portland Canal and the Falls of the Ohio River under the late Col. William E. Merrill, Corps of Engineers, U. S. A., M. Am. Soc. C. E.

In March, 1886, Mr. Shaw accepted the position of U. S. Assistant Engineer in charge of improving the Falls of the Ohio River, at Louisville, and the enlargement of the Louisville and Portland Canal, under Maj. Amos Stickney, Corps of Engineers, U. S. A. He also constructed the new river wall, began the work of enlarging the harbor in front of the city, and was engaged in excavating a channel through rock. He remained in this position until December 31, 1894.

In December, 1895, he accepted the position of Chief Clerk in the U. S. Engineer's Office at Columbus, Ohio, under the late Brig.-Gen. H. M. Chittenden, U. S. A., M. Am. Soc. C. E. (at that time, Lieutenant, Corps of Engineers, U. S. A.), and was engaged in survey work for a canal from Lake Erie to the Ohio River. This work was finished in February, 1896.

Mr. Shaw then went to Milwaukee, Wis., to test cement for the Government and, later, to Missouri to inspect timber. In January, 1897, he was re-appointed by Capt. J. G. Warren, Corps of Engineers, U. S. A. (now Colonel, U. S. A. (*Retired*)), M. Am. Soc. C. E., to a position in the U. S. Engineer Office at Louisville, and again placed in charge of the improvements on the Ohio River. He remained on this work until his retirement in 1912, on account of failing health.

On January 14, 1873, Mr. Shaw was married to Adeline Orpha Wolford, the daughter of Henry Wolford, for many years City Treasurer of Louisville. Mrs. Shaw died on January 1, 1926. A son, George Wolford Shaw, survives him.

Mr. Shaw's standards of conduct were high, especially when pecuniary transactions were involved. To whatever work he was assigned he gave close attention, and did his duty to the satisfaction of those above him. He was a member for several years of the Engineers and Architects Club at Louisville, helping it financially and taking part occasionally in the discussions of technical papers.

After entering the service of the U. S. Engineer Department, Mr. Shaw became interested in the local politics of Louisville, for which his previous employment under the City Engineer and as a reporter for a newspaper had prepared him. This intimacy, however, was not carried so far as to be detrimental to his work under the Engineer Department.

His social life found an outlet in his fondness for vocal music, especially when rendered by male voices. These musical activities were begun as a young man in church choirs, and were continued in wider circles until he was well beyond fifty years of age. Beginning with a quartet of singers and with a fifth man as leader and pianist, he watched a small organization grow to

become a noted institution in the musical life of Louisville. This assemblage of young men had various names—the Arion Club, the Apollo Club, the Louisville Male Chorus—and of all of them he was President for a time. He took charge of arrangements for public performances and many other matters not connected with actual training for concerts, serenades, etc.

Throughout his life, Mr. Shaw was a devoted member of the Protestant Episcopal Church.

Mr. Shaw was elected a Member of the American Society of Civil Engineers on October 5, 1887.

JOSEPH THOMPSON STUART, M. Am. Soc. C. E.*

DIED FEBRUARY 13, 1925.

Joseph Thompson Stuart was born at Port Royal, Pa., on May 28, 1858, and received his early education at the public schools in Allegheny, Pa.

On August 4, 1881, he entered the employ of the Pennsylvania Railroad Company (Pittsburgh Division), as Chainman, but, in October, was transferred to the Construction Department.

In 1886, Mr. Stuart was promoted to be Assistant Engineer of Construction, in charge of change of line and the construction of a four-track arch bridge at Brinton, Pa., on the Pittsburgh Division. In 1887, the construction of the 33d Street Bridge and the extension of the train-shed at Pittsburgh, Pa., was effected under his direction.

From 1888 to 1890, Mr. Stuart was engaged in reconstruction work, for the Pennsylvania Railroad Company, building shops and yards and rebuilding line and the South Fork and Viaduct Bridges after the Johnstown flood. He also rebuilt the Portage Tunnel at Gallitzin, Pa.

In 1891, he had charge of the change of line at Principio, Md., and also at Washington, D. C., including the elimination of the grade crossing for the Union Station. During this time he also rebuilt the Ebensburg Branch of the Pennsylvania Railroad from Cresson to Kaylor, Pa.

From 1892 to 1895, Mr. Stuart was in charge of the construction of the Broad Street Station at Philadelphia, Pa., and, in 1895 and 1896, of the Delaware Bridge for the Pennsylvania Railroad at Philadelphia, which included the caisson work and superstructure. During 1897 and 1898, the change of line at Bacon Hill and Northeast, Md., of the Philadelphia, Baltimore, and Washington Railroad, was under his direction. In 1899 and 1900, he built the Bond Street Terminal Warehouse and Pier at Baltimore, Md., and in 1900 and 1901, he constructed the Camden Terminal at Camden, N. J.

Mr. Stuart was in charge of the elevation of the railroad through Chester, Pa., from 1901 until 1903. On August 1, 1903, he was appointed Engineer of Construction in charge of all construction work of the Pennsylvania Railroad Company between Philadelphia and Washington, D. C., under the supervision of the Chief Engineer. This included the building of the Wilmington and Washington Elevated Railroads.

On January 31, 1904, he resigned his position with the Pennsylvania Railroad Company and entered the contracting business with Mr. Andy Brann. This partnership afterward became Brann and Stuart Company, Incorporated, of which Company Mr. Stuart was President at the time of his death which took place at Dunedin, Fla., on February 13, 1925.

He was a member of the Engineers Club and the Union League of Philadelphia, the Scotch-Irish Society of Pennsylvania, the Philadelphia Yacht Club, and an Honorary Member of the Philadelphia Lehigh Club.

Mr. Stuart was elected a Member of the American Society of Civil Engineers on May 4, 1904.

* Memoir compiled from information furnished by Joseph J. Vogdes, M. Am. Soc. C. E.

JUDSON GILMAN TABLER, M. Am. Soc. C. E.*

DIED DECEMBER 28, 1924

Judson Gilman Tabler, the son of William D. and Mary (Reader) Tabler, was born on August 4, 1877, in Washington, D. C., where he received his education and grew to manhood.

Mr. Tabler was a Draftsman in the United States Patent Office at Washington when war was declared against Spain in 1898 and he promptly resigned his position to enlist. He served during that war with Company F of the 3d Volunteer Engineers and was in Cuba when mustered out of service. He remained on the Island until July, 1902, being engaged during this time as a Instrumentman on highway and railway construction.

In the summer of 1902, he returned to the United States and for the remainder of that year was located in Jennings, La., on irrigation work, going from there to Mexico in December to accept a position with the National Railways of Mexico. He remained in Mexico for more than eleven years, or until 1914, when the revolutionary state of the country had paralyzed all construction work. During this time Mr. Tabler rose from Instrumentman to the position of Chief Construction Engineer for the Company, having held many responsible and important positions in the interim. When forced out by the Revolution in 1914, he was engaged in building a railroad from Tampico to Vera Cruz, involving very difficult construction problems.

After his return to the United States, Mr. Tabler was employed for a time as Assistant Resident Engineer on maintenance of way by the International and Great Northern Railroad Company. In January, 1916, he became Principal Assistant Chief Engineer of the Berthe Engineering Company of Charleston, Mo., Consulting Engineers for various drainage and flood protection projects and other work. Except for a few months with the United States Engineers in Texas, in 1916, and a year spent in laying out and supervising the construction of a plant at Penniman, Va., including buildings, sewers, railroads, and yards, for the Dupont Powder Company, in 1917 and 1918, during the World War, he remained with the Berthe Engineering Company until June, 1921, when he resigned to accept a position with the Virginia State Highway Department. While with the Berthe Engineering Company, he had charge of construction work valued at more than $3 000 000, including ditches, levees, pavements, flood-gates, sewers, and reinforced concrete and sub-foundation work.

Mr. Tabler remained with the Virginia State Highway Department until January, 1924, when he resigned to accept the position of County Engineer of Bedford County, Virginia, with headquarters at Bedford, where he had made many friends.

In the comparatively short span of life alloted to him Mr. Tabler had a wide and varied experience. The writer knew him intimately for more than twenty-seven years, having first met him in the Army in 1898, and after-

* Memoir prepared by L. T. Berthe, M. Am. Soc. C. E.

ward worked with him for years in Cuba, Mexico, and the United States. He was a man of real ability, at his best when conditions were most difficult, never losing his temper or his judgment in times of emergency and always ready to expend his last ounce of energy to "put the job through". Men who worked with him invariably not only admired and respected him as an Engineer, but valued him as a friend. He inspired true and lasting friendships among his fellow workers. Mr. Tabler was in every way the personification of an engineer and a gentleman and his memory will live long in the hearts of all who knew him. He was married in El Paso, Tex., to Dora M. Wrieth, who survives him.

He was a member of the American Association of Engineers, of the Masonic Fraternity, and of the Christian Church.

Mr. Tabler was elected a Member of the American Society of Civil Engineers on December 6, 1920.

JAMES TOWNSEND TAYLOR, M. Am. Soc. C. E.*

Died March 17, 1924.

James Townsend Taylor was born in Waiaha, Kona, Hawaii, on March 19, 1859. He was the son of the Rev. Townsend Elijah and Persis Goodale (Thurston) Taylor.

California having become the home of the family, he attended Golden Gate Academy, Washington College, and the University of California.

In 1877, Mr. Taylor began his engineering work with the South Pacific Coast Railroad Company and remained in its service until 1879. He was with the California State Engineering Department in 1880 and 1881; with the Pomona Land and Water Company in 1883; and served as City Engineer of Pomona, Calif., in 1887 and 1888. Mr. Taylor was engaged in public and private practice in Southern California from 1889 to 1896, and was connected with work on an irrigation project in New Mexico from 1896 to 1898. From 1899 until his death he was engaged in public works and carried on a private practice in Hawaii.

He was a member of the Honolulu Chamber of Commerce, the Honolulu Commercial Club, the Independent Order of Odd Fellows at Pomona, Calif., and the American Association of Engineers.

He was married on October 31, 1894, to Ethel May Webster, who, with a son, Thurston W. Taylor, survives him.

Mr. Taylor was elected a Member of the American Society of Civil Engineers on January 3, 1894.

* Memoir prepared by Charles H. Kluegel, M. Am. Soc. C. E.

TOBIAS TONNESEN, M. Am. Soc. C. E.*

DIED FEBRUARY 22, 1925

Tobias Tönnesen, the son of the Captain of the Harbor of Mandal, Norway, was born in Mandal on January 10, 1867.

After finishing school at home, he went at the age of 18 to Gothenburg, Sweden, to study engineering at the Chalmerska Institut of Technology. In his veins, however, ran the blood of the old Vikings and, in 1887, he left the Institute to embark for the United States and eventually landed at New York, N. Y.

At first he found it difficult to secure a position to his liking, but he finally succeeded in obtaining employment in a bridge shop, where he painted bridge girders. Not long afterward Mr. Tönnesen was promoted to a position in the Designing Department as a Draftsman.

In 1889 he went West, where he held several positions, first in the City Engineer's Office in Seattle, Wash., and, later, as a Topographer and Instrumentman on the Great Northern System. Subsequently, he became Assistant Engineer on the Everett and Monte Christo Railway; the California Midland Railroad, at San Francisco, Calif., and the San Francisco and Great Salt Lake Railroad. Mr. Tönnesen was also employed on irrigation and municipal work in the State of Washington; with the Nelson and Fort Sheppard Railroad, at Nelson, B. C., Canada; with the Great Northern Railroad; and also on the Portland, Ore., Water-Works.

Leaving the Northwest, Mr. Tönnesen went, in 1895, to Guatemala where he was engaged as an Assistant Engineer on the Ferro Carril Verapaz, Ferro Carril del Norte, and Ferro Carril Central. While thus employed he decided that it would be to his advantage to complete his theoretical knowledge of engineering and, therefore, he returned to Gothenburg in 1897 to enter again the Chalmerska Institut of Technology, from which he was graduated in 1899.

During his stay in Guatemala, Mr. Tönnesen had made the acquaintance of the owner of large coffee plantations, a Norwegian, Carlo Z. Thomsen, a resident of Hamburg, Germany, who introduced him to members of the large German Company, Otavi Minen und Eisenbahngesellschaft, of Berlin. This firm engaged him as a Consulting Engineer during the construction of its 350-mile narrow-gauge railway (0.60 m.) called the Otavi Railway, in South-West Africa, at the time the longest narrow-gauge railway in the world. The actual work was started in 1903, and was completed as far as the Tsumeb Copper Mine in 1906. During this period the Germans were at war with the Hereros.

On this railway Mr. Tönnesen had an opportunity to show his great ability and experience in railway construction. It was not only the longest, but also the most carefully constructed narrow-gauge railway in the world and

* Memoir prepared by H. S. Hanssen, Esq., Marseilles, France.

gained wide renown. The contractors were Arthur Koppel and Company, of Berlin, which firm was also in charge of the location of the railway.

After the completion of the Otavi Railway, Mr. Tönnesen was engaged in 1907 as General Manager to the South-West Africa Company, Limited, of London, England, for its large estates and mining properties at Grootfontein in the northern part of South-West Africa. With the exception of a few years spent in Norway during the World War, when he was interested in mining in Sweden, he remained in this position until his death.

While in South-West Africa Mr. Tönnesen made numerous expeditions to its northern part to investigate the country, which was little known to white men, and he lectured on these expeditions and on the country before the Royal Geographical Society, London, to a large audience. On one of his trips he penetrated into Portuguese Angola, where he contracted sleeping sickness which ultimately caused his death. He died on shipboard on his way to his home in Oslo, Norway, on February 22, 1925, and was buried at sea. He is survived by his wife and two children, who live in Oslo.

As a memorial to him the South-West Africa Company, Limited, has erected at Grootfontein, a bronze bust, which stands on a 6-ft. base of Norwegian Laborador syenite. The bust was made by a young Norwegian artist.

Mr. Tönnesen was a real man, of strong opinions and principles, who did not hesitate to fight for what he thought to be right. He was unusually energetic and industrious, never sparing himself. He was gifted with common sense and had a wide, practical knowledge for which he was probably indebted to his experience in the United States, a country which he admired and often mentioned.

In addition to being the recipient of the Royal Norwegian Order of St. Olav, Mr. Tönnesen was also decorated with the Prussian Order of the Red Eagle for his services rendered in the construction of the Otavi Railway in South-West Africa.

Mr. Tönnesen was elected a Member of the American Society of Civil Engineers on May 1, 1901.

SAMUEL WHINERY, M. Am. Soc. C. E.*

DIED JANUARY 14, 1925.

By the death of Samuel Whinery, at East Orange, N. J., on January 14, 1925, the Society lost one of its oldest and most distinguished members, and the profession lost one of its notable and patriotic figures, one whose devotion to its highest ideals will ever stand as a shining example for its younger members.

Samuel Whinery, the son of Robert W. and Susan (Enlow) Whinery, was born near Salem, Ohio, on November 20, 1845. In 1853, the family moved to a farm in Jennings County, Indiana, where young Samuel attended the public schools. Later, he attended the Friends School near Salem, Ind., and continued his education by two terms at Indiana University.

Between the ages of eighteen and twenty-one, he taught in the county schools. During this period he devoted all his leisure time to systematic study, especially of Civil Engineering. It was in this wise that young Whinery obtained that firm and solid foundation upon which his great scientific and engineering career was built. One must pause to compare the very limited opportunity then afforded for professional education in the Middle West with its present-day colleges and universities equipped with large, highly trained and experienced staffs. Can one gainsay that young Whinery, with only a few textbooks and no teachers, but with a studious inclination, a character formed upon the simple and strong faith of the Friends, the mental solitude of a sparsely settled country, reached a professional goal that is seldom equalled by those fortunate enough to have the advantages of a modern technical education?

About 1868, the Indianapolis and Vincennes Railroad was being promoted to develop the part of Indiana in which Mr. Whinery lived. When the locating party took the field, he joined it, and thus gained his first experience, interest, and apprenticeship in railroad engineering. Within a year (1868), he became Resident Engineer in charge of construction on a 13-mile section of this road.

In 1869, he was appointed Resident Engineer of Construction on the Indianapolis and St. Louis Railroad at Terre Haute, Ind. Following the completion of his residency, he built the railroad shops at Mattoon, Ill., and was then appointed First Assistant Engineer on the location and construction of the Cincinnati, Rockport and Southwestern Railroad in Southern Indiana, on which he was engaged for two years.

In 1873, he accepted a position with the Cincinnati Southern Railroad, notable then as now as being an enterprise built and owned by the City of Cincinnati, Ohio. The survey of this railroad was a distinct advance in the location of railroads. The theory of the economics of railway location was used throughout in the far-sighted policy of its Engineering Staff of which Mr. Whinery was an influential member, the Chief Engineer having been the late L. G. F. Bouscaren, M. Am. Soc. C. E. After the completion of the loca-

* Memoir prepared by Frederic Molitor, M. Am. Soc. C. E.

tion, Mr. Whinery had charge of a residency and finally was made Division Engineer in charge of the Chattanooga Division. Thus, at thirty years of age, Samuel Whinery had local charge and the responsibility of the heavy construction of this Division.

Upon the completion of the Cincinnati Southern Railroad, in 1878, Mr. Whinery was induced to accept a position as Civilian Assistant Engineer in charge of the Muscle Shoals Improvement of the Tennessee River. In this position, his experience in charge of railroad construction, his executive ability, and his diplomatic talents were of inestimable value to the officers of the Corps of Engineers, U. S. Army.

In 1881, his services were again sought by railroad companies. He was offered and accepted the charge of the location and construction of the Northern Division of the New Orleans and Northeastern Railroad. During the first year of its operation he acted as Superintendent, and it was at this time that the telephone was first used in train dispatching. Mr. Whinery early had seen the many advantages of this new invention, and, consequently, made use of it during the construction stages of the work. When train service was inaugurated, its use was continued as the means of communication between dispatchers and train crews, and he, therefore, was the first to use the telephone for this purpose.

In 1884, the first period of Mr. Whinery's career ended. Upon the location and construction of railroads he had served his apprenticeship in practical engineering and had risen to responsible charge of work. His services and fidelity had been of the greatest aid to his chiefs, his talents had impressed the profession, and his administrative ability had been established. In short, he had acquired that firm basis which qualified him to "go on his own" as a consultant in private practice. For the following three years his work was in the South and he executed many important commissions of varied character, which called for a broad knowledge of engineering. His most important achievement during this period was the design and construction of the Incline Railroad up Lookout Mountain, at Chattanooga, Tenn., rising 1 200 ft. in less than 1 mile.

In 1887, he received a flattering offer from the Warren-Scharff Asphalt Company to direct its installations. For one year, he served as Assistant Manager in charge of its road pavements and engineering work. Here his professional talents found a large field of usefulness in the growing public demands for better streets, and highways. At the end of a year he was appointed Vice-President and General Manager of the Company, which position he held for twelve years (serving as President for the last six months), or until the Company was merged with the "Asphalt Trust".

Notwithstanding the responsibilities of his office with this Company, Mr. Whinery managed to be of service in other fields of engineering. Among other activities, he served as a member of the Engineering Commission to investigate a new water supply for the City of Cincinnati (1896-97), and as President of the Water Commission of Wyoming, a suburb of Cincinnati. In these undertakings his researches and studies again showed his great power of analysis,

and his sometimes new and always advanced professional ideas. His conclusions have long since been proved correct.

In March, 1901, Mr. Whinery opened an office in New York, N. Y. He had not become known to the general public, but his professional standing among his associates had placed him among the leaders of engineering thought and accomplishment. He practiced in New York, with marked success, for twenty-two years, retiring in 1923, with fifty-five years of active professional work to his credit and to the good of his clients and the public.

Among the many important works with which he was identified or directed during this last epoch of his career were the following: Reports (1903) on the transportation problems of New York, and on the extension of the pierhead lines in the North River; Consulting Engineer to the Finance Commission of Boston, Mass.; Consulting Engineer (1907-08) to the Commission on City Expenditures of Chicago, Ill.; Consulting Engineer (1905) to the President of the Borough of Manhattan, New York; Chairman of the New Jersey Commission to re-appraise railroads and canals; President of the Water Commission of East Orange, N. J.; report on the elimination of railroad grade crossings, at East Orange. He also assisted Gustav Lindenthal, M. Am. Soc. C. E., in research work in connection with some of the details of the famous New York Connecting Railroad Bridge over Hell Gate.

Although beyond the age of military service at the time of the World War, Mr. Whinery offered his services, and was placed by the U. S. Navy Department on the Reserve Officers' Examining Board of the Bureau of Yards and Docks. He spent an arduous term in the civil side of prosecuting the war. His ready and patriotic encouragement was extended to those emergency Engineer Officers who were in active service. The writer cherishes the simple and beautiful words he had from Mr. Whinery, both on leaving for active service in France and on his return. In a word, Mr. Whinery was a patriot and a militant Quaker during the country's experience in the World War.

He was always an advocate of the Engineer taking his part in public affairs, and lending his experience and advice to the proper solution of public questions, where engineering and economics were factors. His was not a passive part; on the contrary he gave liberally of his time and vast experience as evidenced by the fact that for many years he was Chairman of the Committee of Engineering and Sanitation of the Merchants' Association of New York. He also served for several years as President of the Board of Education of East Orange.

His active interest in engineering education in this country, particularly for a broader and better curriculum, led the Universities of Cincinnati, Illinois, and Cornell to invite him to lecture before their engineering students.

Many of the older members of the Society feel that Mr. Whinery's best example to the younger members, and one of his leading contributions to the profession, was expressed in his determined and devoted stand for an established Code of Ethics. The writer believes that it is largely, if not wholly, due to Mr. Whinery that professional conscience was awakened on this sub-

ject. At least, it is believed that he was the first to bring the subject before the profession in an explanatory and constructive way.

On his retirement from the Presidency of the Cincinnati Engineers' Club at the Annual Meeting on December 15, 1892, the subject of his address was "Ethics of Civil Engineers", and this exhaustive and constructive paper has been preserved to the profession.* It is worthy to be read by every engineer, because even now, after all the engineering societies have finally adopted Codes, it will be found that few of them have neglected at least some of his early suggestions. In this able address, Mr. Whinery refers to ethics as "being the science of right conduct". Can it be better expressed? Then, again, he said: "Such a code must be built up along the lines of the system of morals, which is universally recognized as the outgrowth and sequence of the Ten Commandments". Could the construction of a code of ethics be builded on a firmer or safer foundation? And, further, he said: "No class of men has the moral right to set up itself a standard of conduct, which, however, advantageous it may be for their selfish ends, shall disregard the rights and courtesies due to those who are not members of their particular calling". Could there be a better interpretation? He said, also: "It is feared that civil engineers are sometimes remiss in that scrupulous regard for the rights and reputations of their professional brethren, which is so carefully observed in other professions". Did he then speak wrongly? Is this statement not as true to-day as it was in 1893?

The duty of the engineer to the public as then expressed by Mr. Whinery is as follows:

"The engineer owes to the public his earnest services as a good citizen in the highest sense that the word implies. He has no right to neglect his share of those political and social duties which fall to the lot of every patriotic citizen. It is a fact frequently remarked that the civil engineer of the better class is not often found in the ranks of the leaders of civic and social life".

This seems unhappily as true to-day as when Mr. Whinery made the statement thirty-two years ago. It was these sentiments that Mr. Whinery spoke among his associates and professional friends. They bore fruit when in June, 1911, the American Institute of Consulting Engineers adopted a Code of Ethics (the first engineering code in the United States, it is believed), prepared by a committee of which the late John F. Wallace, Past-President, Am. Soc. C. E., was Chairman. As a member of this Committee the writer can now testify that Mr. Whinery's address of December, 1892, was one of the guiding factors in its preparation. His influence was doubtless again felt when, on September 2, 1914, the Society adopted a Code of Ethics.

From 1915 to 1922, Mr. Whinery was Chairman of the Committee on Professional Practice and Ethics of the American Institute of Consulting Engineers. During this time the Committee passed upon many cases of the Institute's Code and upon interpretations of it. His work was accomplished so splendidly and carefully that the Institute in 1923, upon his retirement from this important office, named him Chairman Emeritus. He continued to give

* *Engineering News*, January 26, 1893.

valuable advice to the Committee, notwithstanding his increasing years. Only a short time before his death he stated that he felt his life work would be incomplete until the Engineering Profession had adopted a Code of Ethics, and it was his hope that engineers would abide by it as the medical and legal professions have done.

Mr. Whinery was the author of "Municipal Public Works" (1903), and of "Specifications for Pavements and Roads". He also contributed largely to the publications of various professional societies and to technical magazines.

He was a Charter Member of the American Institute of Consulting Engineers (1910) and served on its Council in 1916, and from 1919 to 1921, and was also Vice-President for two years, 1920 and 1921. He was also a member of the American Society of Mechanical Engineers, American Institute of Mining and Metallurgical Engineers, American Public Health Association, and the American Academy of Political and Social Science.

Mr. Whinery was married on November 10, 1875, to Elizabeth A. Crawford, daughter of John Crawford, of Somerset, Ky. Besides his widow, he is survived by the following children: Samuel Brent Whinery, an engineer, Charles C. Whinery, Maurice R. Whinery, Andrew J. Whinery, a lawyer, John E. Whinery, and a daughter, Elizabeth Whinery. For the last twenty years of his life, he resided in East Orange, N. J.

It is by the wide variety of his professional work, the detailed research and analyses of his conclusions, and the thoughtful expression of his reports that his professional career is best measured, marking him as outstanding on the long roll of membership in the Society.

This memoir may best be concluded by a statement of Mr. Whinery's worth as a man and as an engineer, contributed by one of the leaders of the profession and one whose gift of expression is so well known and appreciated, William J. Wilgus, M. Am. Soc. C. E.:

"I had for him a rare combination of affection for his lovable qualities and respect for his professional attainments and judicial character. In fact, his sweetness of disposition, ability as an engineer, and devotion to the highest ideals of our calling, marked him as a man out of the ordinary, whose passing from our midst leaves a niche that will be most difficult, if not impossible, to fill. At least, he is enshrined in our memory as one whose example we should all aspire to follow".

Mr. Whinery was elected a Junior of the American Society of Civil Engineers on April 1, 1874, and a Member on May 4, 1881. He also served on the Board of Direction in 1891 and from 1899 to 1901, and as Vice-President in 1892 and 1893.

CHARLES SUMNER WILLIAMSON, M. Am. Soc. C. E.*

DIED MARCH 31, 1923.

Charles Sumner Williamson was born at Cleveland, Ohio, on April 12, 1874, the son of Samuel Williamson and Stella E. (Sumner) Williamson. His mother was a direct descendant of Increase Sumner, one of the early Governors of the State of Massachusetts. His father also came from a long line of Colonial families in the United States.

Following his preparatory education in the public schools of Cleveland, Mr. Williamson received his technical and engineering training in the Case School of Applied Science, from which he received the degree of Bachelor of Science in 1895 and that of Civil Engineer in 1898.

From 1894 to 1895, he worked in the office of a local Civil Engineer. From 1896 to 1898, he was in the employ of the King Bridge Company and the Variety Iron Works of Cleveland. Except for this minor work, Mr. Williamson was employed in the hoisting and conveying machinery business throughout all his comparatively brief career. One of his earlier engagements was with the Brown Hoisting Machinery Company of Cleveland, with which he served, successively, as Draftsman, Squad Foreman, and Designing and Erecting Engineer, his work having to do with the design and installation of car dumpers for transferring coal from railroad cars to Lake vessels, ore and coal-handling bridges, and other special types of material-handling equipment.

Following this period with the Brown Hoisting Machinery Company, covering the principal part of the time from 1895 to 1904, he was made Contracting Sales Engineer of Heyl and Patterson, Incorporated, of Pittsburgh, Pa., which position he held for seven years. In this capacity Mr. Williamson played an important part in establishing and building up the ore and coal-handling machinery department, which specialized in the development, production, and installation, at the Lake coal and ore docks and at blast furnaces, of ore and coal-handling equipment of the bridge and hoisting tower type.

In 1911, he became associated with the Mead-Morrison Manufacturing Company, Constructing Engineers, of Boston, Mass., as Western Manager, with headquarters at Chicago, Ill., his duties being those of Managing and Contracting Engineer for iron ore and coal-handling machinery. He was later promoted to be Vice-President, still retaining charge of the Western District, and remained with the company in this capacity until he was compelled to retire from active business on account of ill-health. While with the Mead-Morrison Company, Mr. Williamson's activities were not limited to America, but extended to Europe and Australia.

Mr. Williamson was fortunate during his early practical experience, while with the Brown Hoisting Machinery Company, in being brought in contact

* Memoir prepared by a Committee consisting of G. H. Hutchinson, *Chairman*, W. H. Hoyt, J. N. Hatch, and William S. Monroe, Members Am. Soc. C. E., and Messrs. A. M. Merriweather and C. C. Brooks.

with Mr. Alex E. Brown, the pioneer in the development and introduction of the mechanical handling of materials, a man of remarkable personality, whose clear vision, inventive genius, indomitable will, and persistent effort had much to do in making possible the world supremacy of the Great Lakes, in the handling and transportation of bulk cargo material.

Although the pioneering work had been done, and much accomplished in the line of the mechanical handling of bulk materials before Mr. Williamson entered the field, he had been intimately associated with the progress made in the previous twenty-five years. His activities covered the entire period of development of the more modern type of material-handling equipment. Various improvements in mechanical detail and in methods of material handling stand to his credit, some of which he protected by letters patent.

Mr. Williamson was a great lover of fine horses and possessed several prize winners.

On August 14, 1914, he was married to Grace Elizabeth Meigs, of Madison, Wis., who, with one child, survives him.

Mr. Williamson was elected an Associate Member of the American Society of Civil Engineers on February 7, 1906, and a Member on October 31, 1911.

FREDERIC IRVING WINSLOW, M. Am. Soc. C. E.*

DIED FEBRUARY 21, 1924.

Frederic Irving Winslow was born at New Bedford, Mass., on January 30, 1863, the son of George William and Jane Lucretia (Southwick) Winslow.

He received his early education in private and public schools with one year in High School. In 1877, he removed to Boston and attended the English High School from which he was graduated in 1881, having received the Lawrence Prize and the Franklin Medal. Subsequently, he was graduated from the four-year course in Mechanical Design in the South Boston School of Art.

Mr. Winslow's first position was in the City Surveyor's office at Boston, in 1881, and he continued in the employ of that city for about thirty-eight years. At first, he was engaged in surveys and, later, entered the City Engineer's office as Transitman in the Water Department and from 1885 to 1911 was Senior Assistant Engineer of the Boston Water-Works. From 1911 to 1916, he served as Engineer in charge of all the extension work, and then became Assistant Engineer on street construction for the City. From October, 1917, to August, 1918, he held the position of Supervising Engineer in charge of the office and the construction at the Squantum Destroyer Plant for the United States Navy Department.

From April, 1919, until his death, Mr. Winslow was Division Engineer of the Sudbury Department of the Metropolitan Water-Works, stationed at Framingham, Mass., and also carried on a consulting practice.

His specialties were hydraulics and water-works. Among his other accomplishments are the extension of water service to Long Island, Boston Harbor; the design and construction of Deer Island Reservoir; and the design and construction of two tunnels in compressed air.

Mr. Winslow was the author of numerous papers on engineering topics, which appeared chiefly in *Engineering News,* for which he was for a short time Boston correspondent, in the *Journal* of the New England Water Works Association, and in the *Harvard Engineering Journal;* and of a chronology of the Boston Water-Works (1912).

He was a member of the Boston Society of Civil Engineers from 1885, having served as Librarian from 1907 to 1913, of the New England Water Works Association from 1892 (he had been Treasurer from 1922 until his death), of the American Water Works Association, of the Boston Congregational Club, and of the Royal Arcanum. He was a Republican in his political affiliations.

Mr. Winslow was married on June 15, 1892, to Myrtle S. Smith, of South Newbury, Ohio, who survives him, with their son, Irving H. Winslow.

* Memoir prepared by John C. Chase, M. Am. Soc. C. E., and Charles W. Banks, Assoc. M. Am. Soc. C. E.

His death occurred at his home in Framingham on February 21, 1924, after an illness of only a few days.

He was a great student, making a practice to read about two hundred volumes annually for many years. He was a frequent attendant at engineering meetings in Boston and had a wide acquaintance. He was a genial, companionable man, highly esteemed by all who knew him—one whose passing has left a noticeably vacant place, and whose memory will long be cherished by his many friends.

Mr. Winslow was elected a Member of the American Society of Civil Engineers on May 6, 1914, and was a Charter Member of the Northeastern Section of the Society. During the last year of his life he had rendered the Society valuable service in the preparation of memoirs of a number of New England members for whom none had ever been written.

JOHN ROSS CHAMBERLIN, Assoc. M. Am. Soc. C. E.*

DIED DECEMBER 15, 1925

John Ross Chamberlin was born in Rochester, Ohio, on March 15, 1876. He was the son of Dr. Charles S. Chamberlin, a practicing physician, who died when John was five years of age. His mother, Mrs. Tillie (Kisinger) Chamberlin, is still living.

Mr. Chamberlin received his early education in district schools and worked on the farm during vacations. When he was seventeen years of age the family moved to Tiffin, Ohio, and he entered the Academy of Heidelberg University in the fall of 1894. During the summer months he worked for the County Engineer, Mr. Charles J. Peters, and it was here that he acquired his first knowledge and experience as a civil engineer. In the fall of 1898, he entered the Ohio State University where he remained until April, 1902. He then withdrew from the University and went to Peru where he spent a year with the Engineering Corps in charge of the construction of the Cerro de Pasco Railway.

On his return to the United States in 1903, Mr. Chamberlin accepted a position with the Baltimore and Ohio Railroad, with headquarters at Baltimore, Md. In the fall of 1905, he returned to the Ohio State University as Instructor in Civil Engineering, and was, successively, Instructor and Assistant Professor of Civil Engineering until 1912. He was graduated from the Ohio State University with the degree of Civil Engineer as of the Class of 1902.

In 1912, he was appointed Division Engineer in the Bureau of Bridges of the Ohio State Highway Department, and, in 1913, Chief Engineer of Bridges, in which capacity he served until 1919.

During this time Mr. Chamberlin was quite active in the study of the economics of highway bridge construction, and several *Bulletins* pertaining to bridge and pavement design were published by the Highway Department under his direction. In 1915 to 1918 he made a study of highway bridge specifications and, in 1918, the specification for highway structures of the Ohio State Highway Department was published. Many new and important provisions in this specification, especially with reference to reinforced concrete floors, and the determination of the required waterway for bridges, were the work of Mr. Chamberlin.

In October, 1919, he received an appointment as Senior Highway Bridge Engineer in the United States Bureau of Public Roads and spent a year in Washington, D. C. He was then transferred to District No. 5 at Omaha, Nebr., where he served until his death.

He was married in 1907 to Ada Maurer, of Chillicothe, Ohio, who, with five children, two sons and three daughters, survives him.

* Memoir prepared by Clyde T. Morris, M. Am. Soc. C. E.

Mr. Chamberlin was appointed a member of the Special Committee on Concrete and Reinforced Concrete Arches of the Society in May, 1923. On learning of his death the following minute was signed by all the members of the Committee and sent to the Board of Direction:

"It was with deep regret that we, the members of the Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers, learned of the death of John R. Chamberlin.

"Mr. Chamberlin was appointed on the Committee in May, 1923, and was a faithful and interested participant in its work until his death. He brought a wealth of good judgment and wise counsel to our work and his quiet courteous manner and lovable disposition endeared him to all.

"We feel keenly the loss of his good counsel in our proceedings and of his genial presence, and wish to express our sorrow at his loss, and our sympathy to his family which has lost a kind and loving father.

"(Signed) W. M. Wilson,
Geo. E. Beggs,
E. H. Harder,
A. C. Janni,
Clyde T. Morris."

Mr. Chamberlin was elected an Associate Member of the American Society of Civil Engineers on May 6, 1914.

JOHN STERLING DEANS, Assoc. M. Am. Soc. C. E.*

DIED AUGUST 18, 1924.

John Sterling Deans, the son of the late John Sterling Deans, M. Am. Soc. C. E., and Clara Barr Deans, was born in Phœnixville, Pa., on July 5, 1891. His father was for many years prominently connected with the Phoenix Bridge Company of Phœnixville, as Vice-President and Consulting Engineer.

Mr. Deans received his degree of Bachelor of Philosophy from the Sheffield Scientific School in 1912, and was in the employ of the Pennsylvania Railroad Company from 1912 to 1916, first, in the Construction Department and, later, in the Maintenance-of-Way Department.

From 1916 to 1917, he was connected with the National Aniline and Chemical Company as Superintendent of Engineering Construction of the Abbott Road Plant at Buffalo, N. Y.

Mr. Deans enlisted in the Ordnance Corps of the United States Army, and was commissioned a First Lieutenant on October 5, 1917, and assigned to the Construction Section of the Supply Division at Washington, D. C., where he was in charge of the design of railroad and civil engineering work. He was promoted to the rank of Captain in June, 1918, and in the following September was sent to France. There he was assigned to the Construction and Maintenance Division of the Ordnance Department.

Mr. Deans was honorably discharged from the Army in January, 1919, and then entered the construction business as General Superintendent for the J. N. Byers and Son, Incorporated, of Buffalo. At the time of his death, he was Secretary of the firm.

He was married in 1916 to Harriet Gertrude Byers, a sister of Mr. J. Newton Byers. Mrs. Deans survives him, with a son and two daughters; he also leaves his mother, brother, and two sisters.

Mr. Deans was elected an Associate Member of the American Society of Civil Engineers on September 9, 1919.

* Memoir prepared by Emile Low, M. Am. Soc. C. E.

RAYMON EDGAR FULCHER, Assoc. M. Am. Soc. C. E.*

DIED JULY 14, 1925.

Raymon Edgar Fulcher was born at Jackson, Calif., on August 4, 1879. His early education was received in the public schools and High School of Oakland, Calif. During vacations and at other times until he was nineteen years of age, he was engaged in and about the gold mines and ore-dressing plants of the Mother Lode District of California, with his uncle, Wilton E. Darrow, and it was after this experience that his leaning toward an engineering career developed and became the most prominent factor in his life's work. Mr. Fulcher was largely self-educated in engineering, having been a keen observer and a persistent reader and student of technical civil engineering literature.

In 1904, he passed an examination at Sacramento, Calif., and was appointed Deputy County Surveyor of Tuolumne County, California. He also served as Deputy United States Mineral Surveyor and was engaged in underground mine surveying on the Mother Lode Mines in the vicinity of Sonora, Calif.

Mr. Fulcher's work as an engineer was varied in character, embracing mining, railroad engineering, tunnel construction, road work, earth-fill dams, rock-fill dams with concrete face, high-tension transmission-line construction, pumping plants, electric car lines, and oil pipe-line construction. As a result of his experience in mining and kindred activities he examined and reported on many mining properties and oil prospects in California, and, later, in Arizona.

In 1905, Mr. Fulcher was engaged by Sanderson and Porter as Field and Construction Engineer on work for the Stanislaus Electric Power Company, in California, which included the location of a construction railroad, flume and dams, and new road work and transmission lines. In 1911, he built an electric car line between the towns of Raymond and Southbend, Wash., on Willapa Harbor. He continued with Sanderson and Porter on hydro-electric development construction in British Columbia and Vancouver and on a pumping plant at Burbank, Wash., until 1914, when he was appointed Field Engineer in charge of the construction of the Valley Oil Pipe Line, extending 170 miles from Coalinga to Martinez, Calif., on the Bay of San Francisco.

From 1916, until the summer of 1918, Mr. Fulcher was engaged in a similar capacity on the construction of an oil pipe line across the States of Oklahoma and Missouri, terminating on the east bank of the Mississippi River in Illinois, a few miles above St. Louis, Mo. The crowning achievement of his career as an Engineer was probably the laying of this pipe line across the Mississippi, which was an important link in supplying fuel oil to the Allied fleets during the latter period of the World War.

During this work, which was continued without interruption through the excessively cold winter of 1917-18, Mr. Fulcher contracted pneumonia, which made it necessary for him later to live in Tucson, Ariz. While there, he acted as Engineer in charge of the installation of a 1 000-h.p. Diesel engine electric

* Memoir prepared by Wynne Meredith, M. Am. Soc. C. E.

unit for the Tucson Gas, Electric Light, and Power Company. He then engaged in building construction and general engineering practice, and, later, specialized in roofing and roof covering, developing a successful business of which he was in charge at the time of his death. He died on July 14, 1925, at Phœnix, Ariz., of bronchial pneumonia, caused by the extreme heat.

Mr. Fulcher was married on March 8, 1901, to Blanche E. Divoll, of Sonora, Calif. His widow and three daughters survive him.

As an engineer of ability, Mr. Fulcher combined with his technical attainments, an open mind, executive qualities, and a winning personality. To his more intimate friends he will be remembered for his modest ways, his fine sense of humor, and those rare qualities of mind and heart that so endeared him to all with whom he came in contact.

Mr. Fulcher was elected an Associate Member of the American Society of Civil Engineers on March 4, 1913.

JOHN STANTON GOODELL, Assoc. M. Am. Soc. C. E.*

DIED NOVEMBER 13, 1925

John Stanton Goodell was born at Amherst, Mass., on January 20, 1875. He was the elder son of the late Henry Hill Goodell, formerly President of the Massachusetts Agricultural College, and Helen Stanton Goodell. He was descended from Puritan ancestors, the first of the family to come to America having been Robert Goodale who settled in Salem, Mass., in 1634. Eighty or more members of the family spelling the name either "Goodale" or "Goodell" served in the Revolutionary War.

Mr. Goodell received his education at the Massachusetts Agricultural College, at Amherst, and, later, at the Rensselaer Polytechnic Institute, Troy, N. Y., where he completed his course in Civil Engineering.

From March 1, 1899, until February, 1908, he was connected with the Gulf, Colorado and Santa Fé Railway Company, in Texas, as Chainman, Rodman, Instrumentman, Levelman, and Transitman on locations at different times. He also served as Assistant Engineer on the Northern Division of the railway, under F. Merritt, Division Engineer, at Clebourne, Tex., and as Office Engineer under C. F. W. Felt, M. Am. Soc. C. E., Chief Engineer, at Galveston, Tex.

In February, 1908, Mr. Goodell resigned, to accept a position with The Kwong Tung Yueh-Han Railway Company, Limited, at Canton, China, serving as Principal Assistant Engineer and, later, until August, 1910, as Engineer of Maintenance of Way, under the late Sir Chentung Liang Cheng, former Ambassador to the United States, who at that time was President of the Company.

From August, 1910, to October, 1911, he was engaged in private practice at Amherst, Mass., principally on land surveys, work on sewer lines, replacing lost highway monuments, etc. From October, 1911, to July, 1916, Mr. Goodell served as Manager of a rubber plantation in Nahiku, Island of Maui, Hawaii, and from July, 1916, to September, 1921, he was employed as Assistant Engineer in the Construction Department of the Atchison, Topeka and Santa Fé Railway Company, under C. F. W. Felt, Chief Engineer, at Chicago, Ill.

While in this position Mr. Goodell was engaged in checking estimates for proposed construction work in Missouri and later was in charge of a field office at Brunswick, Mo., where estimates, location and right-of-way maps, profiles, plans for the elimination of highway grade crossings, etc., were made for a proposed line from Standish to Moberly, Mo.

From July 21, 1917, to June 1, 1921, he was in charge of the construction of the Barton County and Santa Fé Railway from Holyrood to Galatia, Kans. On the completion of this work he returned to Amherst, Mass., where he engaged in private practice, continuing in this work until 1924 when his health became impaired.

* Memoir compiled from information on file at the Headquarters of the Society.

Mr. Goodell was married on May 29, 1906, to Edith Friese, at Galveston, Tex., who survives him. He is also survived by his mother, Mrs. Helen S. Goodell, of Amherst, Mass., and a brother, Dr. William Goodell, of Springfield, Mass.

Mr. Goodell was elected an Associate Member of the American Society of Civil Engineers on March 7, 1906. He was also a member of the American Railway Engineering Association.

ROBERT HUGH McNEILLY, Assoc. M. Am. Soc. C. E.*

DIED MAY 8, 1925.

Robert Hugh McNeilly was born on September 30, 1879, near Hustonville, Lincoln County, Ky. His father was Dr. James H. McNeilly, a Presbyterian minister and a former Captain and Chaplain in the Confederate Army. His mother was Mary Weatherford McNeilly.

Mr. McNeilly received his engineering education at Vanderbilt University, Nashville, Tenn., from which he was graduated with the degree of Bachelor in Engineering, in 1902 (conferred in 1908). He received the degree of Civil Engineer from the University in 1915.

He was employed as a Draftsman in the Ordnance Department, United States Army, until 1904. From 1904 to 1905 he was Constructing Engineer for several water supply and hydro-electric plants in Tennessee and Mississippi, and from 1905 to 1906, he was Assistant Professor of Civil Engineering at the University of Mississippi, after which he was again employed by the U. S. Army in the capacity of Engineer and Superintendent of Construction on the Columbus, Ohio, Barracks from 1906 to 1907. During 1907, he held the position of Assistant Engineer for the Muscle Shoals Hydro-Electric Power Company.

From 1907 until his death, Professor McNeilly was a member of the Engineering Faculty at Vanderbilt University and during that time he also conducted a private engineering practice.

It was in his capacity as a teacher that Professor McNeilly accomplished most for his students and friends. He had a singularly happy and cheerful disposition, was efficient, earnest, and just, and always ready to help in any task or problem where he might be needed. He was the moving spirit in the organization of the Student Chapter of the Society at Vanderbilt University, which has been named the Robert H. McNeilly Chapter in honor of him.

Professor McNeilly was drowned in the *Norman* tragedy in the Mississippi River, near Memphis, Tenn., on May 8, 1925. He was married to Marian Brown Stockell on June 8, 1904, and is survived by Mrs. McNeilly and a son, Stockell McNeilly.

In his address at the memorial services to the Vanderbilt University Alumni who perished in this disaster, Chancellor Kirkland said with respect to Professor McNeilly: "He helped lift the burdens of others. He did not put his burdens on you."

Professor McNeilly was elected an Associate Member of the American Society of Civil Engineers on November 26, 1923. He was also a member of the Engineering Association of Nashville.

* Memoir prepared by Hunter McDonald, Past-President, Am. Soc. C. E., and L. R. Currey, Jr., Assoc. M. Am. Soc. C. E.

RICHARD EDWARD PHILLIPS, Assoc. M. Am. Soc. C. E.*

DIED AUGUST 8, 1925.

Richard Edward Phillips was born in Lockport, N. Y., on November 23, 1880, and while still in his early youth moved to Buffalo, N. Y. At the age of seventeen he terminated his attendance at High School and started his engineering career in the employ of Mr. Henry L. Lyon, a civil engineer, engaged in lot surveying.

In 1899, Mr. Phillips entered the employ of the New York Central Railroad Company at Depew, N. Y. It was in the shops of this Company that he acquired a knowledge of practical mechanics.

In 1902, he accepted an appointment as Chainman in the Engineering Department of the State of New York. At that time the State was extensively engaged in work on the Barge Canal and highways preliminary to the large scale construction which later was undertaken. Mr. Phillips was engaged in highway work for about a year when he was transferred to work on the Barge Canal, with which he was identified for the remainder of his life.

He rose in the Department of the State Engineer and Surveyor from Chainman to Supervising Engineer, and for some years prior to his death held the title of Engineer of Claims. Until the Barge Canal was completed, he was actively engaged in the design and supervision of important features of that project. Later, many large damage claims resulting from extra compensation demanded by contractors and by private property and water-power owners were filed against the State. Mr. Phillips' duties consisted of the investigation and technical defense of these claims.

It was in the latter connection that he displayed his unusual ability. His exceptional knowledge of the relation of law to engineering and his information relative to the cost of construction and water power values made his services highly valuable to both the Engineering and Legal Departments of the State of New York.

Mr. Phillips' health failed about a year prior to his death and he was granted a leave of absence from his duties in the hope of effecting his recovery. All efforts toward this end were unavailing, and he died at Saranac Lake, New York, on August 8, 1925.

He was married in 1904 to Emma Christine Reed, of Albany, N. Y., by whom he is survived, with two daughters, Mrs. Louise Carey and Miss Marion Phillips.

Mr. Phillips was elected an Associate Member of the American Society of Civil Engineers on May 7, 1913.

* Memoir prepared by F. P. Williams, M. Am. Soc. C. E.

FRANK LAWRENCE SHELDON, Assoc. M. Am. Soc. C. E.*

DIED JUNE 22, 1925

Frank Lawrence Sheldon, the son of Frank Perry and Nellie (Noyes) Sheldon, was born in Providence, R. I., on August 1, 1883. He was a direct descendant of Samuel Winsor, who married Mercy Williams, a sister of Roger Williams. His father, the founder of the firm of F. P. Sheldon and Son, was a noted mill engineer, having designed and re-organized many of the most prominent textile establishments in the United States.

Mr. Sheldon's earlier education was received in the public schools of Providence, and at Riverview Military Academy, Poughkeepsie, N. Y. In 1903, he entered Rensselaer Polytechnic Institute, Troy, N. Y., as a student in Civil Engineering, but left that institution in 1905, to enter his father's employ as Resident Engineer on mill construction, his first work having been on the buildings for the Lorraine Manufacturing Company at Pawtucket, R. I.

After serving for several years in this position, he entered the office of the Company and assisted in the design of various industrial plants. In 1912, he went to Knoxville, Tenn., to represent the firm of F. P. Sheldon and Son and to supervise the building of a large mill and weave shed for the Brookside Mills, under its plans. On the completion of this construction, he again took up office work, and, at the time of his death, was second in charge of the office, planning the work for the drafting force, checking plans and computations, and assisting in textile research.

Mr. Sheldon was a member of the Rensselaer Society of Engineers and of the American Society for Testing Materials, having served on its Committee on Textile Materials, as he was familiar with textile manufacturing and had made considerable research in this art.

Having been a proficient athlete in his school days—he had been a member of the football and hockey teams at Riverview Military Academy and at Rensselaer Polytechnic Institute—he always maintained a great interest in sports and was a keen student of modern football tactics. He was also fond of boating, hunting, and fishing, being at the time of his death a member of the Rhode Island Fish and Game Protective Association. Among the fine arts, music was his greatest delight, and while at Riverview Military Academy he was a member of the Glee Club.

Mr. Sheldon died, after a brief illness, at his home in Providence, on June 22, 1925, and is survived by his widow, Mrs. Jetty L. (Wilson) Sheldon, to whom he was married on March 12, 1907, a son, Frank L., Jr., a sister, Mrs. James B. Barrett, and a brother, Arthur N. Sheldon, of the firm of E. P. Sheldon and Son.

Mr. Sheldon was elected an Associate Member of the American Society of Civil Engineers on June 6, 1921.

* Memoir prepared by Frederick H. Paulson, Jun. Am. Soc. C. E.

ALBERT FRANKLIN SICKMAN, Assoc. M. Am. Soc. C. E.*

DIED OCTOBER 17, 1925.

Albert Franklin Sickman, son of John and Martha Pomerene Sickman, was born at Snowden, Pa., on December 10, 1856. He received his technical education at the University of Pittsburgh, Pittsburgh, Pa., from which he was graduated in 1878.

In July, 1878, Mr. Sickman went to Holyoke, Mass., and entered the service of the Holyoke Water Power Company, remaining in the continuous employ of that Company until his death—a period of forty-seven years.

He went to Holyoke at the invitation of his brother, Mr. James M. Sickman, then the chief engineering member of the office force of the Holyoke Water Power Company. This organization, however, at that time, did little engineering work, being mainly engaged in giving lines and grades for new streets and in general surveying. Soon afterward, in 1879, the Company undertook the organization of a systematic control and lawful allotment to each mill owner of the quantity of water he should draw from the Company's canals—the common receptacle from which they all drew the water that was then converted into power by each mill owner.

For lack of such control and allotment, water was being largely overdrawn, wasted, and was even at work in a manner to cause the destruction of some of the canals. This called for the services of an hydraulic engineer, and, as a pupil of the late James B. Francis, Past-President, Am. Soc. C. E., of Lowell, Mass., Clemens Herschel, Past-President, Am. Soc. C. E., was chosen for this work.

Mr. Herschel went to Holyoke in 1879, and "Frank" Sickman, as he was generally called, was designated to be Mr. Herschel's first Assistant. At that time, at twenty-three years of age, Mr. Sickman was the same quiet, conscientious worker that he remained to the day of his death. He had been firmly grounded in the elements of his profession, by faithful study at the University of Pittsburgh, and Mr. Herschel takes pleasure in recalling his ready response, as a most apt pupil, to every demand made on him.

When Mr. Herschel resigned his position as Hydraulic Engineer of the Holyoke Water Power Company, in 1889, Mr. Sickman became his successor. At that time the system of charging for excess drafts of water by the mill owners, and thus keeping control of the draft by each one of them, had become firmly established, but naturally required expert continuance of the work. The far-famed Holyoke Testing Flume had been established in 1881, and called for skilled work in meeting the needs of American wheel builders. No one thing has contributed so much, probably, to the present eminent and useful state of the art of hydraulic turbine building in the United States, as the work done in this flume since its foundation, and, to a large extent, by Mr. Sickman after 1889.

* Memoir prepared by Clemens Herschel, Past-President, Am. Soc. C. E., and Robert E. Barrett, M. Am. Soc. C. E.

In October, 1913, he was elected Assistant Treasurer of the Company and, in November, 1919, was also made Clerk, both of which positions he held at the time of his death.

He was a master of hydraulic problems and familiar with every detail of the history and developments of the work at Holyoke, and in his death, the Company that he had served so long and faithfully suffered an irreparable loss.

Mr. Sickman had varied interests outside his profession. He was a keen student of Nature, an authority on the New England trees, and knew every wild flower of the section, and where and when to find it.

He was always courteous and affable and his genial disposition endeared him to his associates; he was ever fair and considerate in his relations with those under him.

Mr. Sickman was twice married. His first wife died many years ago, leaving four children, all of whom survive him. In 1909, he married Margaret Griffin, of Holyoke, who also survives him.

He was a member of the New England Water Works Association and of the Engineering Society of Western Massachusetts.

Mr. Sickman was elected an Associate Member of the American Society of Civil Engineers on March 4, 1903.

NATHANIEL AUGUSTINE THAYER, Assoc. M. Am. Soc. C. E.*

DIED SEPTEMBER 10, 1925.

Nathaniel Augustine Thayer was born in South Boston, Mass., on November 4, 1880. He was the son of the late Edmond G. Thayer, a Civil Engineer and Architect of Boston, and Florence Hamilton Thayer.

Mr. Thayer was graduated from the Quincy, Mass., High School, and from the Lawrence Scientific School of Harvard University, in 1902, with the degree of Bachelor of Science. For several years after his graduation from college he was a Rodman and, later, an Instrumentman with the Cleveland, Chicago, Cincinnati, and St. Louis Railroad Company, after which, he spent a year as Structural Draftsman for J. R. Worcester and Company, Consulting Engineers, Boston, engaged in detailing and designing steel and reinforced concrete structures.

From 1906 to 1909, he served in the Bridge Engineering Department of the New York Central and Hudson River Railroad Company; as Structural Draftsman for Vielé, Blackwell, and Buck, New York, N. Y., on the design of a wooden flume and steel and wooden head-gate for water-power development at Cobalt, Ont., Canada; and as Structural Detailer on buildings for Post and McCord, and, later, for Gunvald Aus, M. Am. Soc. C. E. In 1909 and 1910, he was employed as a Structural Steel Draftsman with the New York Public Service Commission, First District, on designs for subways and elevated railways for New York City.

During the latter part of 1910 and throughout most of 1911, Mr. Thayer was engaged by the Erie Railroad Company to work on plans for the wooden dock and for a grade-crossing elimination at Youngstown, Ohio, following which, he served as Structural Steel Designer for the Board of Education, School Building Department, New York City, and also held the position of Assistant Engineer with the Board of Transportation of the City of New York, Robert Ridgway, Past-President, Am. Soc. C. E., Chief Engineer.

Through intensive study Mr. Thayer became a Certified Public Accountant in the State of New York. He undertook this work in order to be thoroughly grounded for an independent study which he was making of industrial corporation finance.

He died of pneumonia on September 10, 1925, at St. Luke's Hospital, New York City, after an illness of only four days, and is survived by his mother, Mrs. Edmond G. Thayer, of Wellesley Hills, Mass., a sister, Mrs. Raymond E. Huntington, and two brothers, Frederick G. Thayer, of Wellesley Hills, and Lucien H. Thayer, of Dorchester, Mass.

He was a member of The Harvard Engineering Society, and the Unitarian Church.

Mr. Thayer was elected a Junior of the American Society of Civil Engineers on April 5, 1910, and an Associate Member on April 2, 1912.

* Memoir prepared by Theodore Reed Kendall, Assoc. M. Am. Soc. C. E.

EDMUND BOYD ULRICH, Assoc. M. Am. Soc. C. E.*

DIED NOVEMBER 20, 1925.

Edmund Boyd Ulrich was born at Reading, Pa., on September 25, 1871 the son of Dr. Daniel and Mary (Boyd) Ulrich. He received his education in the public schools of his native city, and was graduated from High School in 1885.

Immediately after his graduation, Mr. Ulrich entered the service of William H. Dechant, Civil Engineer, and after serving successively as Rodman and Transitman for several years, accepted a position as Transitman on a railroad construction operation in North Carolina. He then returned to Reading and served as Instrumentman on the construction of the Neversink Mountain Railroad, the Mohnton and Adamstown Electric Railroad, and other local construction operations.

In 1894, Mr. Ulrich entered the employ of the City of Reading, as Transitman, attached to the City Engineer's Staff, and served in this capacity during the construction of the trunk line intercepting storm sewer and further work for the initial district of municipal sanitary sewers, including a pumping station.

In 1903, he established an engineering office, and engaged in surveying and general engineering practice until April 12, 1909, the date of his appointment to the office of City Engineer of Reading, for a term of three years. He was re-appointed to this office, successively in 1912, 1916, and 1920, serving continuously for fifteen years until May 7, 1924. Mr. Ulrich then re-established a surveying and engineering office, and in conjunction therewith served as City Engineer for Mahanoy City, Pa.

He was actively affiliated with and served as Secretary of the American Turbit (pigeon) Club. He owned select pigeons and enjoyed a National reputation as a pigeon fancier and competent judge, thus becoming a factor in promoting pigeon and poultry exhibits.

Mr. Ulrich was in ill-health for seven weeks before his death on November 20, 1925. He is survived by his widow who was Bessie Chamberlain, of Winston-Salem, N. C.

Mr. Ulrich was elected an Associate Member of the American Society of Civil Engineers on April 6, 1909.

* Memoir prepared by E. Clinton Weber, Esq., Reading, Pa.

FRANKLIN JAMES VAN HOOK, Assoc. M. Am. Soc. C. E.*

DIED JULY 26, 1925.

Franklin James Van Hook, the son of Frank and Matilda (Wallace) Van Hook, was born at Lockport, N. Y., on July 31, 1881. Prepared for college in local schools, he entered the Massachusetts Institute of Technology, and was graduated in 1906 with the degree of Bachelor of Science (in Civil Engineering).

Employment by the "Big Four" Railroad Company, at Wabash, Ind., the Charles River Basin Commission, the New York State Water Supply Commission on the Racquette River, and the Commissioners of Sewerage at Louisville, Ky., occupied the first five years of Mr. Van Hook's technical career. Later, he was engaged on grade-crossing elimination at Nashville, Tenn., for the Louisville and Nashville Railroad Company.

In 1912, Mr. Van Hook entered the service of the City of Cincinnati, Ohio, in connection with the preparation of a plan of comprehensive sewerage. In this work he made a particular study of rainfall and run-off, and outlined requirements for relief drainage. Following this engagement, he served as Resident Engineer for the Louisville and Nashville Railroad Company on the construction of the First Avenue Viaduct at Birmingham, Ala. During 1915, 1916, and 1917, he was Superintendent of Construction for the Jefferson Construction Company of New Orleans, La. In this capacity, and in addition to other work, he supervised the installation of the Warrior River steam power plant of the Alabama Power Company.

The call of the World War found Mr. Van Hook debarred from active service. His desire to help took him into cantonment construction at Camp Meigs and elsewhere for the Construction Division of the United States War Department. In 1918 and 1919 he was Field Engineer for Westinghouse, Church, Kerr and Company on the construction of the United States Government Nitrate Plant at Muscle Shoals, Ala.

He spent the period from 1919 to 1924 in Cleveland, Ohio, where he served chiefly as Designer and Estimator of industrial buildings for the Boldt Construction Company and the Sam W. Emerson Company.

In 1924, Mr. Van Hook was made General Manager of the Oil City Wood Working Manufacturing Company, Oil City, Pa., and, later, became a Director of the Company. At the time of his death, he was engaged on plans for enlarging the activities of that firm.

Mr. Van Hook was a member of the Baptist Church. He was a Mason, a Rotarian, a Director of the Western Pennsylvania Builders' Supply Association, and held membership in other organizations, both civic and social.

* Memoir prepared by Frank C. Tolles, M. Am. Soc. C. E.

He was married in 1916 to Madge Nave, of Louisville, Ky. Mrs. Van Hook and two children, Robert Wallace and Annie Laurie, survive him.

His career is typical of many who make up the profession and help to formulate and sustain its best traditions. It is a career which subordinates self in loyal devotion to the interests of employer or client. Mr. Van Hook was such an engineer. His abilities were not inconsiderable. His life was useful.

Mr. Van Hook was elected an Associate Member of the American Society of Civil Engineers on May 7, 1913.

WILLIAM OLIN WALKER, Assoc. M. Am. Soc. C. E.*

DIED MAY 8, 1925.

William Olin Walker was born at Chuckey City, Tenn., on July 18, 1890. His father, Thomas S. Walker, and his mother, Mary (Dooley) Walker, were descendants of the sturdy early settlers of Tennessee and Virginia.

Mr. Walker was educated at the Morristown, Tenn., High School, De Pauw University, Greencastle, Ind., where he took a Liberal Arts Course, and in the Extension Division of the University of Wisconsin. He was not graduated from De Pauw University, although for several years he alternated periods of college work there with periods of engineering employment for which he early showed an inclination.

During the summer of 1907, he was Rodman on the Cincinnati, New Orleans and Texas Pacific Railway Company (Southern Railway System), at Oakdale, Tenn., and during the summer of 1908, was employed in a similar capacity with the County Highway Engineer at Greenville, Tenn. From January to September, 1909, he served as Rodman with the Southern Railway System, and as Rodman and Instrumentman with the County Highway Engineer at Morristown, Tenn. During the summer of 1911, and part of 1912, he was Masonry Inspector with the Louisville and Nashville Railroad Company, resigning to accept a position as Building Inspector with the Illinois Central Railroad Company on construction at Paducah, Ky., and other locations.

After another year at De Pauw University (1913-14), Mr. Walker re-entered the service of the Illinois Central Railroad Company, and attained a varied and broad experience in railroad engineering—maintenance of way work, location, construction, and office practice—as follows: From October, 1915, to February, 1918, he was Instrumentman on maintenance of way, and had charge of laying out work on the construction of a new yard at Baton Rouge, La., and terminal facilities at Mattoon, Ill. From February to August, 1918, he had charge of a field party on a 100-mile preliminary survey for a new trunk line. In August, 1918, he became Acting Resident Engineer at Champaign, Ill., in charge of the construction of a classification yard. From January, 1919, to January, 1920, he was employed as Engineer Accountant at various points on the Illinois Central System, reclassifying accounts to prorate the cost of construction and improvements between the Railroad Administration and the Railroad Corporation.

In January, 1920, Mr. Walker entered the employ of the Sinclair Refining Company, at Coffeyville, Kans., as Cost Engineer. His work consisted of keeping detailed costs on the construction of a $5 000 000 refinery, also of making appraisal of an old refinery plant. His ability and thorough, painstaking work won recognition and he was transferred to the general office of that Company at Chicago, Ill., in April, 1921. In this position he checked cost reports from all the Company's refineries, and also assisted in reclassifying accounts for insurance. In December, 1921, he was given charge of the cost work in

* Memoir prepared from information on file at the Headquarters of the Society.

connection with the construction of new refineries at Coffeyville, Kansas City, Mo., Houston, Tex., and Marcus Hook, Pa., with headquarters at Coffeyville. During October, 1923, he was transferred to the refinery at East Chicago, Ind., as Assistant Engineer of Construction, in addition to his duties of Cost Engineer at the East Chicago Plant. The completion of the construction of the new refinery at East Chicago, in November, 1924, marked the approximate completion of the $40 000 000 construction program of the Sinclair Refining Company, in which Mr. Walker took an active part from its beginning.

On the completion of this work he accepted a position as Engineer for the H. W. Nelson Construction Company of Kentucky, with headquarters at Fulton, Ky., where he had charge of the construction of approximately thirty miles of the new line of railroad being built by the Illinois Central Railroad Company, from Edgewood, Ill., to Fulton.

A Convention of the Association of Engineers of the Mid-South was held at Memphis, Tenn., on May 8, 1925. Many prominent engineers were brought together at this meeting, especially men of the profession from the Central Southern States, the majority of whom were members of the Society. The United States Government Engineers extended the Association an invitation to visit the Government revetment plant about twelve miles south of Memphis, near Coahoma Bend. Soon after leaving the revetment plant on their return to the city, one of the boats, the United States Steamboat *Norman,* sank in the Mississippi River. Mr. Walker and twenty-two others lost their lives in this disaster.

Perhaps two of Mr. Walker's most striking characteristics were his enthusiasm for his work and his keen interest in civic affairs. He was highly esteemed by his employers, respected and admired by his associates and subordinates, and known by his friends to have had fine ideals and deep convictions. He was a man of strong character, who never flinched from his duty, and yet was possessed of a tender devotion to his family and an innate courtesy and kindliness toward all.

Early in life he became a member of the Methodist Episcopal Church, of which he remained a faithful and consistent adherent. As a Knight Templar, Mr. Walker was active in Masonic circles.

In 1922, he was married to Mattie Sue Berry, of Barnesville, Ga., who, with two children, Meriam and Edith, survives him. He is also survived by four sisters, Mrs. E. T. Hall, of Brookfield, Mo., Mrs. R. B. Walker, of Walla Walla, Wash., and the Misses Clara and Eunice Walker, of Morristown, Tenn., and one brother, F. C. Walker, of Chicago, Ill.

Mr. Walker was elected a Junior of the American Society of Civil Engineers on March 7, 1921, and an Associate Member on May 28, 1923.

GEORGE BURNHAM, JR., Affiliate, Am. Soc. C. E.

DIED NOVEMBER 22, 1924.

George Burnham, Jr., the son of George and Anna (Hemple) Burnham, was born in Philadelphia, Pa., on November 30, 1849. His father was associated with Matthias Baldwin in the early days of locomotive building and, later, was Senior Member of the firm of Burnham, Parry, and Williams, predecessor of the Baldwin Locomotive Works.

George Burnham, Jr., was educated in the public and private schools of Philadelphia, preparatory to entering the Rensselaer Polytechnic Institute, at Troy, N. Y., from which he was graduated in 1872 with the degree of Civil Engineer.

During the summer vacation of 1871, he was employed as Rodman with a field party on railroad work for the Bennett's Branch Extension, connecting the Allegheny Valley Railroad with the Philadelphia and Erie Railroad.

After his graduation in 1872, Mr. Burnham was engaged on surveys in Fairmount Park, Philadelphia, and as Leveler on a railroad survey from Paoli to Chester, Pa., for the Pennsylvania Railroad Company, under the late James McCrea, M. Am. Soc. C. E., who afterward became President of the Company. Samuel Rea, Hon. M. Am. Soc. C. E., was also a member of this party.

In 1873 and 1874, Mr. Burnham was Rodman on the location and construction of a branch of the Pennsylvania Railroad from Williamsburg, Pa., to the Springfield, Pa., iron ore mines, and in 1874 and 1875 served as Leveler on the Bridge Division of the Bound Brook Railroad, Philadelphia to New York, N. Y. (now the Reading Central).

With the late D. McN. Stauffer, M. Am. Soc. C. E., he had entire charge of the construction of the bridge over the Delaware River which, with trestle approaches and bridges over canals on the Pennsylvania and New Jersey sides, was more than a mile in length. During this work he was stationed at Yardleyville, Pa.

On the completion of the bridge, Mr. Burnham returned to Philadelphia and, after 1875, when he entered into business with Mr. Nelson Stowe in the manufacture of the Stowe flexible shaft, never again engaged in the practice of his profession. From 1875 to 1880, he was Treasurer of the Barr Pumping Engine Company and Managing Director of the Spiral Weld Tube Company.

On September 1, 1880, he became associated with the firm of Burnham, Parry and Williams and served in the Treasurer's Department until January 1, 1896, when he became a member of the firm. On December 31, 1906, he retired from active business. His other business connections during these periods of active work and after retirement were, as follows: Director of the Central National Bank; Keystone Telephone Company; Keystone Coal and Iron Company; C. S. Wheeler Manufacturing Company; United Utilities and Service Company; Mineral Development Company; and the Integrity Trust Company.

* Memoir prepared by Edward B. Guthrie, M. Am. Soc. C. E.

Mr. Burnham was deeply interested in civic affairs, both municipal and Federal, and for many years was a leader in movements for the betterment of Philadelphia, in the words of one of his associates, "not asking for office, emolument, not even fame, only to be allowed to serve the city he loved." It was in this work and other civic matters that Mr. Burnham was truly notable, rather than in his career as an engineer which covered a period of only four years.

He was connected with various civic organizations, as follows: In 1880, he became a member of the Citizens Municipal Association and was also a Charter Member of the Philadelphia Civil Service Reform Association of which he was President in 1904. In 1890, he became a Charter Member and for ten years was President of the Philadelphia Municipal League. In 1894, this organization with the City Club of New York issued a call for the foundation of the National Municipal League of which Mr. Burnham was Treasurer for twenty-five years. In 1892, he was appointed a member of the Committee of 100 in Philadelphia and took a prominent part in its activities. In 1904, he became a member of the Committee of 70 and was engaged in its work at the time of his death. In 1905, he became a Charter Member of the City Club of Philadelphia and served as its President for ten years. In 1913, he served as a member of the Committee of 100, the purpose of which was to forward the election of Mayor Blankenburg. In 1921, he was appointed by the City Club of Philadelphia as its representative on the Board of the Thomas Harrison Municipal Trust. He was made Chairman of the Board and held this office until his death.

On January 15, 1924, a Memorial Meeting was held at the Social Service Building in Philadelphia, at which addresses were delivered on the character and activities of Mr. Burnham by the following co-workers in reform matters: Messrs. Thomas Raeburn White, whose subject was, "Services in Politics"; Samuel S. Fels, who spoke on "Municipal Research"; W. W. Montgomery, on "Civil Service Reform"; Sherman C. Kingsley, on "Philanthropy"; J. Henry Scattergood, on "The Man"; Richard S. Childs, on "Municipal Reform"; and James G. McDonald, on "The City in Foreign Affairs".

To depict the fine character of Mr. Burnham the following editorial is quoted from the *Philadelphia Public Ledger:*

"George Burnham

"By the death of George Burnham, the city has lost one of its best friends. He stood confidently for everything that was honorable and upright and law-abiding in municipal government. He devoted time and strength and liberally gave of his own means in the furtherance of causes in which he believed, not asking if they were popular or likely to be victorious. There was in the man a fine spirit of chivalry and loyalty that endeared him to all who strove beside him for better things. Modest to the point of diffidence, he never put himself forward, he never asked the credit for anything he did. He disliked to make speeches or even to preside at meetings, but he never shirked his duty in movements for the general welfare. In fact, there was no undertaking for the good of the city with which his name and his effectual effort were not identified.

In these he was the friendly, kindly, devoted counselor and partner. He was an example of the political crusader who is not a fanatic: he was that rare phenomenon, a reformer with a sense of humor and a dispassionate willingness to hear the other side. Important factor that he was in business life, it is as civic patriot that Philadelphia will gratefully remember him; and his city will commend him to posterity as a shining pattern of high-minded citizenship."

To these tributes, the writer wishes to add his own, speaking of Mr. Burnham as a friend and acquaintance of many years, dating from 1871. His gift of friendship equalled, or even surpassed, all others, and those whose privilege it was to know him feel that the world is made poorer by his death.

Mr. Burnham was married to Anna B. Lewis, on April 14, 1881, who, with two sons, Enoch Lewis Burnham and George Burnham 3d, and two daughters, Mrs. Arthur Peck, and Mrs. Albert DeSilver, survives him. He is also survived by two sisters, Miss Mary A. Burnham, and Mrs. Theodore J. Lewis.

Mr. Burnham was elected a Junior of the American Society of Civil Engineers on January 6, 1875, and an Affiliate on July 2, 1890.

COLEMAN MERIWETHER, Affiliate, Am. Soc. C. E.*

DIED OCTOBER 8, 1924.

Coleman Meriwether, the son of Charles and Patti (Barbour) Meriwether, was born in Louisville, Ky., on July 14, 1875. He received his early education in private schools of that city and in the Lawrenceville Preparatory School, at Lawrenceville, N. J.

In September, 1894, Mr. Meriwether entered the Engineering Department of Snead and Company Iron Works, at Louisville, as Draftsman. In March, 1896, he resigned this position to become Manager of the Louisville Branch of the Pope Manufacturing Company of Hartford, Conn. In April, 1899, he entered the office of the Louisville and Nashville Railroad Company, at Louisville, as Assistant Engineer, where he remained until, in June, 1900, he became Draftsman in the office of the Chief Engineer of the Baltimore and Ohio Railroad, at Baltimore, Md.

In April, 1901, he was appointed Chief Draftsman in the office of the Chief Engineer of the Southern Railway Company (St. Louis-Louisville Lines), at Louisville, but in January, 1904, he was made Land and Industrial Agent and transferred to Birmingham, Ala. From January to September, 1904, he served as Chief Draftsman in the Chief Engineer's office of the St. Louis-San Francisco Railway Company, at Cape Girardeau, Mo., resigning to accept a similar position with the Seaboard Air Line Railway Company at Portsmouth, Va.

About this time, Mr. Meriwether became interested in protecting timber piles by using lock-joint pipe, and developed the sewer pipe joint known as the Meriwether system of lock-joint pipe. From April, 1905, until August, 1917, he served as President and Chief Engineer of the Lock Joint Pipe Company of New York, N. Y., and Ampere, N. J., and from August, 1917, to January, 1919, as Manager of the Cement Products Bureau of the Portland Cement Association, of Chicago, Ill. From January, 1919, to April, 1920, he was associated with Mr. H. R. Duckwall, of Indianapolis, Ind., in the development of high-pressure reinforced concrete water pipe to a satisfactory conclusion, and for almost a year thereafter he was interested in the promotion of this type of pipe at Louisville. In January, 1921, he organized the Meriwether Pressure Pipe Company, of which he was Vice-President and Engineer until his death.

In his efforts to popularize reinforced concrete pipe, Mr. Meriwether traveled throughout much of the civilized world and made many friends wherever he went. He was widely known as an inventor of concrete pipe joints, having obtained seven patents on reinforced concrete specialties, and as a manufacturer of reinforced concrete pipe.

On June 21, 1919, he was married to Jessie Burks Scott, of Lynchburg, Va., who survives him.

* Memoir prepared from information on file at the Headquarters of the Society.

He was a member of the American Society for Testing Materials, American Concrete Institute, Concrete Products Association, and the Engineering Institute of Canada. He was also a member of the Engineers Club of New York and of the Country Club at Louisville, Ky.

In an obituary notice published elsewhere,* it was said that:

"A fitting epitaph for Coleman Meriwether may well include these thoughts: 'He achieved success in that he lived well, laughed often, was loved by many. He left the world better in that his work for civilization endures and has been an inspiration to others. His memory is revered by many friends'."

Mr. Meriwether was elected an Affiliate of the American Society of Civil Engineers on February 7, 1906.

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* *Concrete Products*, November, 1924, p. 72.

WILLARD ADELBERT SMITH, Affiliate, Am. Soc. C. E.*

DIED NOVEMBER 29, 1923.

In the death of Willard Adelbert Smith, the nation has lost one of its best-known authorities on the history and status of railroad transportation in the United States. Although he had passed the traditional age of three score years and ten he had in no sense lost interest in the pressing questions of the day, his activity in portraying the truth of railroad conditions having been maintained to the time of his sudden failure in health, about a year before his death.

His acquaintance and influence in the railroad industry were far-reaching. He had met railway men in all grades of service and authority, from division operating officials to the highest executives, and was constantly being called into their councils. There were but few men, outside of those actually employed in railway service, to whom was given the opportunity to know as much of the railway industry from the inside as was the case with Mr. Smith throughout his long career.

Willard Adelbert Smith was born on a farm, near Kenosha, Wis., on September 20, 1849. He was the son of William Harrison and Mehitabel (Allen) Smith, who emigrated to Wisconsin from New Hampshire in 1841. His paternal ancestors came from England in 1640, and his mother's family came also from England at an early day and settled in the State of New York. His father was an Abolitionist whose home was a station on the "Underground Railway", serving as a hiding place for fugitive slaves.

Mr. Smith was educated in the common schools of Kenosha, the High School of Rockford, Ill., and Shurtleff College, at Upper Alton, Ill., from which he was graduated, in a literary course, in 1869. He next entered the Law Department of Washington University, at St. Louis, Mo., from which he was graduated in 1872. He was admitted to the Bar, in the State of Missouri, in 1870, and to practice in the United States Courts, in 1871. In later years, the degrees of Master of Arts and Doctor of Laws were conferred on him by Shurtleff College.

From his early choice of occupation it was the destiny of Willard Smith to become a publisher; and his ability to master the varied scope of his activities, both private and public, throughout his life, developed from his experience with the publishing business as the nucleus. He first entered publication work in 1871, as Editor and Publisher of the *St. Louis Railway Register.* Three years later he acquired the ownership of the *Railway Review,* of Chicago, Ill., which he retained until he died. Thus, at the time of his death, his ownership and management of the *Railway Review* lacked but a few months of a full half century. In 1897 the name of the publication was changed to the *Railway and Engineering Review,* and in 1914 it was changed back again to the *Railway Review.*

In his charge and direction of the publication of this weekly railway newspaper and magazine Mr. Smith was, except during a few short periods, edito-

* Memoir prepared by the late W. M. Camp, M. Am. Soc. C. E.

rial writer as well as the head of the business management. Liberally educated and versatile, he early acquired a broad knowledge of the railroad transportation development of the country, in the technique of construction, equipment and operation, as well as of the financial and executive management of the properties. In such an experience he saw the railroad mileage of the nation expand from about 55 000 to 260 000; and the progress of Government regulation of the railroads he had followed, studied, and written of, critically and ably, from its birth through the subsequent involved and shifty situation.

Under the guidance of Willard Smith the attitude of the *Railway Review* on railroad questions was independent, espousing the cause of the railroads against the many unjust assaults of legislative authority, in the past, and freely criticizing railroad officials for errors of management or policy whenever that view seemed sufficiently well supported by the facts. In his editorial ideas and expressions the man was entirely frank and fearless. The motto of his editorial attitude on all questions was "to think clearly, to speak plainly, and to say the thing that ought to be said." His command of English was excellent, he had the habit of speaking plainly, and when his thought was written down it was so clearly and so courageously expressed that there was no need for "interpretation." Being an accomplished public speaker his voice, as well as his pen, was often called into service.

It was these abilities and qualifications that brought about Mr. Smith's appointment on July 27, 1891, as Chief of the Department of Transportation at the World's Columbian Exposition, held in Chicago in 1893. A broad and careful canvass of the views of railroad executives, by the Exposition authorities, as to the logical man for this work decided the matter for Willard A. Smith, and he undertook the work enthusiastically. For a whole year he traveled through Europe making arrangements for the foreign exhibits. How well he succeeded has become a matter of history. While his editorial ability had gained for him a national reputation, the organization which he developed to display the material progress of railroads throughout the world made him a man of international renown. Among the outstanding features of the result were the historical exhibit of equipment of the Baltimore and Ohio Railroad; the extensive display of track and equipment by the Pennsylvania Railroad; and the great track exhibit of Mr. A. Haarmann, of Germany.

After the Chicago World's Fair, Mr. Smith temporarily laid aside active management of the *Railway Review* to engage in manufacturing. In 1894, he became President of the Cloud Steel Truck Company of Chicago, and, later, Vice-President of the National Malleable Castings Company.

In 1899, he went to France as Chief of the Department of Transportation and Engineering for the American Commission to the Paris Exposition of 1900. His previous travels in Europe on exposition business, the acquaintanceships formed, and his eminent success as head of a department at the Chicago Exposition seven years earlier, were experiences that served him to much advantage in his connection with this foreign Exposition. An innovation of the American exhibit that had not been anticipated by the French,

but which was one of the purposes of Mr. Smith, was the display of American-built steel cars, placed on the grounds with some difficulty by means of barges on the River Seine, of hoisting machinery, and of special tracks. It has been said that this exhibit led to the introduction of steel cars on European railways. On his return from Paris that year he again became actively engaged with the *Railway Review,* continuing as President, which office he had never relinquished while engaged with outside enterprises; and it should not be overlooked that for many years he was also the publisher of the *Railway Master Mechanic* and of the "Railway Official List."

In the organization for the Louisiana Purchase Exposition, in St. Louis, Mo., in 1904, again he was sought as Chief of the Department of Transportation, and again he accepted the office. The exhibit was housed in a single building covering sixteen acres. It included, as in 1893, the historical railway exhibit of the Baltimore and Ohio Railroad, and examples of the heaviest railway rolling stock of the day, locomotives and cars; but perhaps the most worthy effort and result of the exhibition of rolling equipment was the display of the latest refinements of design and service. Notable exhibits of this kind were the numerous designs of modern draft gear; the practical operation and tests of air-brake equipment for an 80-car freight train, by the Westinghouse Air Brake Company; and the locomotive testing plant of the Pennsylvania Railroad, maintained in continuous operation during several weeks. The thoroughness with which this plant was designed and installed within the short time available, and its very satisfactory operation as a testing device, won great praise for the motive power officials of the railroad and for the staff of experts who conducted the tests.

It was through the suggestion and strong appeal of Mr. Smith that the management of the Pennsylvania Railroad had been led to decide on a locomotive testing plant as the distinguishing feature of its exhibit, and he was made a member of the Advisory Board which co-operated with the Railroad Company in conducting the important series of tests that were there carried out. The other members of this Board were the late Theodore N. Ely, M. Am. Soc. C. E., and J. J. Turner. The data and results of these tests were published in a separate volume by the Railroad Company. The plant was subsequently removed to the Pennsylvania Shops at Altoona, Pa.

Perhaps the most spectacular feature of the Transportation exhibit—at any rate so regarded from the popular standpoint—was a 120-ton, Atlantic type locomotive and tender of the "Big Four" Railway, mounted on a 70-ft. turn-table displayed by the Chicago Bridge and Iron Works. The turn-table was carried on a "center" supported on a concrete pier, in the ordinary way, with the lower flanges of the girders 4 ft. clear of the floor. The locomotive was blocked clear of the turn-table rails, so that its wheels could revolve; and by means of an electric drive both the turn-table and the locomotive drivers were kept continually in motion, the table swinging on its center.

A remarkable feature of the Transportation exhibit, not so well appreciated at that time but now of historic interest, was a Department of Aeronautics, conducted under the immediate direction of the late Octave Chanute, Past-

President, Am. Soc. C. E. For this purpose a field of ten acres was fenced off as part of the Exposition Grounds, and gliding experiments were conducted at frequent intervals. The motive power was a stationary street-car motor geared to a drum carrying 1 000 ft. of light steel cable. In these experiments the biplanes would often rise to a height of 60 ft. above ground, when the cable would be detached and the pilot would steer the plane around the grounds and make a landing. The attendance of those interested in aviation was large, including some of the foremost inventors of France, Italy, Brazil, and other parts of the world, as well as those of the United States, the latter including notably the Wright brothers, who not long after this exhibition achieved epoch-making progress.

At the conclusion of the Exposition, the *Railway Review,* under the direction of Mr. Smith, published a volume covering the railroad exhibits of the Exposition, in much detail. The book contains more than 600 illustrations.

In recognition of Mr. Smith's technical and executive abilities as an organizer of railroad exhibitions, he became the recipient of many honors and decorations. He was appointed and served as a Delegate of the State Department of the United States Government to the International Railway Congress which met in Paris, in 1900, and in Washington, in 1905. He was honored with medals by the Governments of France, Germany, and Italy, for services in connection with the World's Columbian Exposition; and was presented with the famous Tiffany "Transportation Vase" by the American exhibitors in the Department of Transportation, World's Columbian Exposition, as a testimonial of his eminent services as Chief of that Department. He was decorated Chevalier of the Legion of Honor, by France, and, in 1907, promoted to Officer of the Legion of Honor. He was decorated with the Royal Order of the Crown by Germany; and with the Imperial Order of the Rising Sun by Japan. Mr. Smith was the last surviving Chief of Department of the Chicago World's Fair.

He had long been an active member of several railroad and engineering associations, including the Master Car Builders' Association; the American Railway Master Mechanics Association; the Western Society of Engineers; the Western Railway Club; and the American Society of Railroad Superintendents. Locally, he was a member of the Union League Club, the Chicago Engineers' Club, and had been a Trustee of the University of Chicago from 1894 until he felt obliged, by failing health, to tender his resignation, a few months before his death. For a long time he served as the Chairman of the Press and Extension Committee of that institution. He was a Patron and Honorary Curator of the Field Museum of Natural History and, for many years, a Trustee of the Memorial Baptist Church. He also served as President of the Chicago Baptist Social Union for two years.

Mr. Smith was married on May 1, 1872, to Maria C. Dickinson, of St. Louis, Mo., who passed away suddenly about five years before his death. He is survived by three children: Mrs. B. V. Crandall, of Kenilworth, Ill.; and Miss Edith and Harold A. Smith, of Highland Park, Ill., the last named becoming President of the *Railway Review* on the death of his father.

In his private and social life, as well as in his business career, Mr. Smith was widely recognized as a fine type of gentleman. He was a man of extraordinary intellectual force and culture, with broad and charitable views; and to know him was to admire him. His impulses were humanitarian, and the serious bent of his nature was educational. He had been a student all his life, and a constant reader of the best literary talent of his time.

Schooled by contact with some of the best engineering minds and methods of the country, and having intimate knowledge of the progress of the design of railway rolling stock, shop, and other equipment, he was not infrequently called into consultation for opinions or advice on engineering enterprises.

Mr. Smith was elected an Affiliate of the American Society of Civil Engineers on September 2, 1914.

GEORGE LEE ANDERSON, Jun. Am. Soc. C. E.*

DIED MAY 8, 1925.

George Lee Anderson was born in Poland, Ind., on October 7, 1894. As his mother died when he was about two years of age, he was sent to live with his grandparents near Sunman, Ind.

He was graduated from the district school near Sunman in 1908 and entered the High School at Greensburg, Ind., from which he was graduated in 1913. In September, 1913, he entered Purdue University, West Lafayette, Ind., taking the course in Civil Engineering. After two years at Purdue, he entered the employ of the Surveyor of Benton County, Indiana, as Draftsman and Computer.

In September, 1916, Mr. Anderson entered the United States Government service as a Surveyman, attached to the U. S. Engineer's Office at Memphis, Tenn. He remained in this position until 1918, when he entered the military service as a Private in the 472d Engineers, serving in the capacity of Levelman and Plane-Table Operator until his discharge in February, 1919.

In March, 1919, he returned to the U. S. Engineer's Office at Memphis as a Surveyman, remaining there until September of that year, when he returned to Purdue University to continue his course in engineering. During the summer of 1920, Mr. Anderson was employed as a Draftsman and Computer by Ayres and Miller, Engineers, of Memphis. He returned to Purdue University in September, 1920, where he completed his course and received the degree of Bachelor of Science in Civil Engineering, in June, 1921.

From July, 1921, until August, 1922, Mr. Anderson was employed in the U. S. Engineer's Office, at Memphis, as a Structural and Mechanical Draftsman. He then entered the employ of the firm of Ayres and Miller, as Office Engineer, retaining that position until his death.

Mr. Anderson was of a modest and retiring disposition and spent his spare time in reading both engineering and classical literature, possessing a remarkable memory of both. He was thorough and painstaking in every detail of his work, and, although only a young man, was an engineer of ability and possessed a high standard of integrity and professional conduct.

He was a member of the Engineers Club of Memphis, and was among those who lost their lives when the Steamer *Norman* sank in the Mississippi River on May 8, 1925. On board this steamer was a party of engineers who had been on an inspection trip down the river as part of the program of the Convention of the Association of Engineers of the Mid-South held in Memphis on May 7 and 8, 1925. He is survived by his father, Mr. Oliver C. Anderson, of Sunman, Ind.

Mr. Anderson was elected a Junior of the American Society of Civil Engineers on June 19, 1922.

* Memoir prepared by Garner W. Miller, Assoc. M. Am. Soc. C. E.

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OF THE

American Society of Civil Engineers

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